Advances in Resummed Calculations

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In processes involving disparate scales $Q \gg Q_0$, higher-order corrections are enhanced by large logarithms

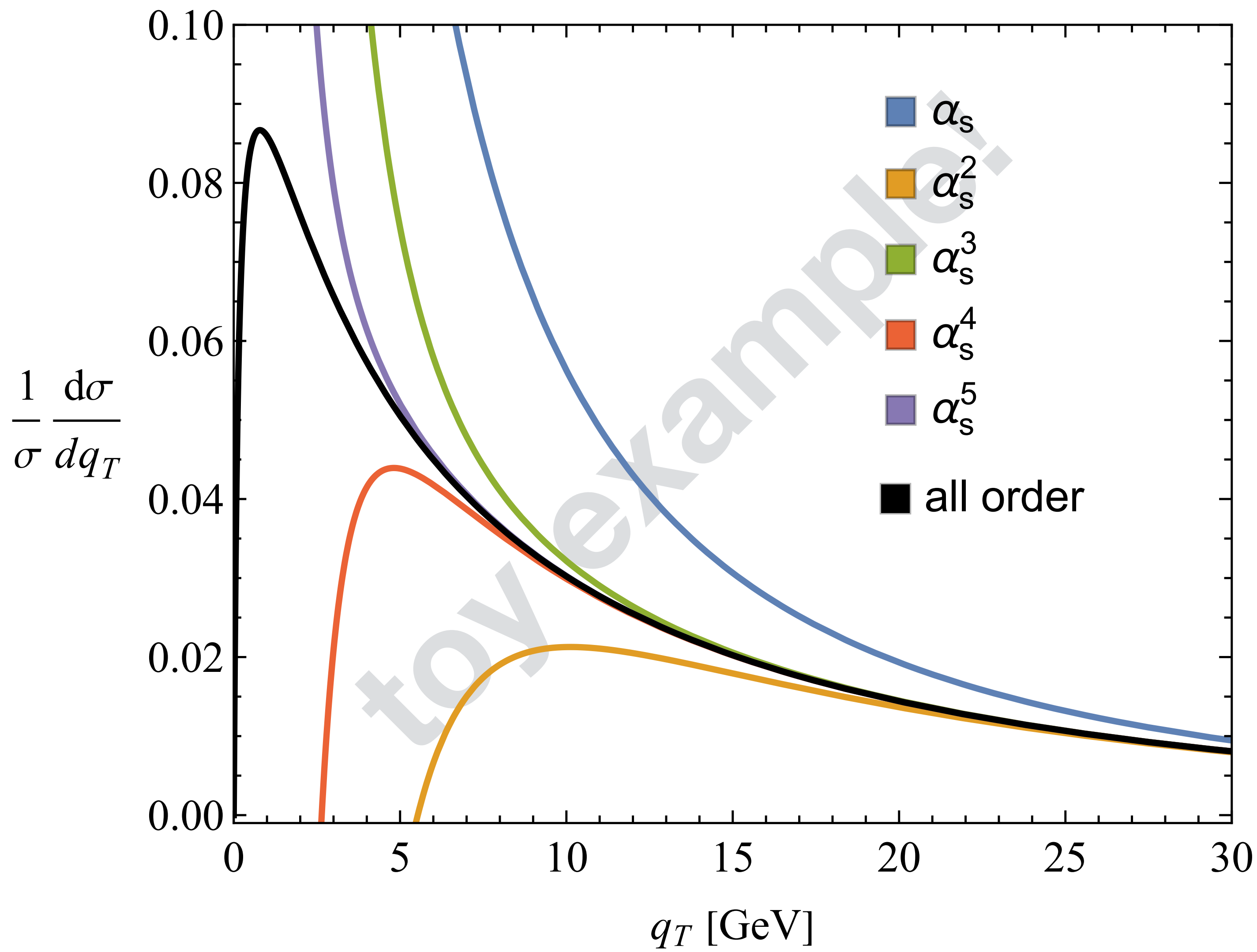
$$\alpha_s^n \ln^m Q/Q_0$$

which can spoil perturbative expansion. Maximum power of logarithms depends on problem

- Single logarithmic: $m \leqq n$
- Sudakov (soft + collinear): $m \leqq 2n$

Resum enhanced contributions to all orders.

- Count $\ln(Q/Q_0) \sim 1/\alpha_s$
- Systematic expansion: LL, NLL, NNLL, …



Classic example is Z- or W-production at low transverse momentum $q_T \ll M_Z : L = \ln(M_Z/q_T)$

Need all-order resummations for reliable extractions of α_s and M_W from the spectrum.

→ talks by Florencia Castillo, Valentina Guglielmi, Oleg Kuprash, Giulia Marinelli

Exponentiation

One can show that cross section has the form

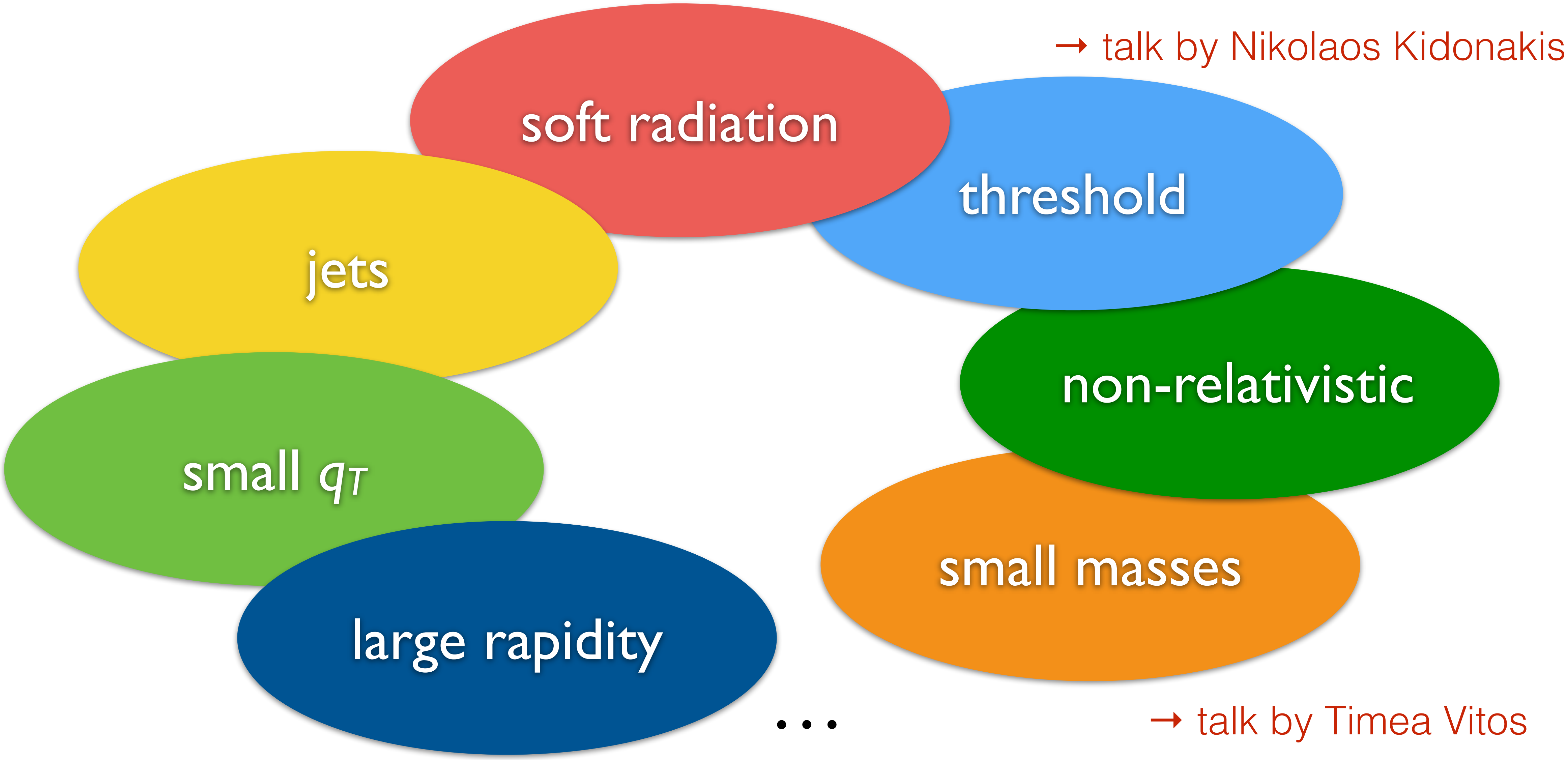
$$\Sigma(p_T) = \exp\left(L\, g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \alpha_s^2 g_4(\alpha_s L) + \dots\right)$$

$\sim 1/\alpha_s$ ~ 1 $\sim \alpha_s$ $\sim \alpha_s^2$

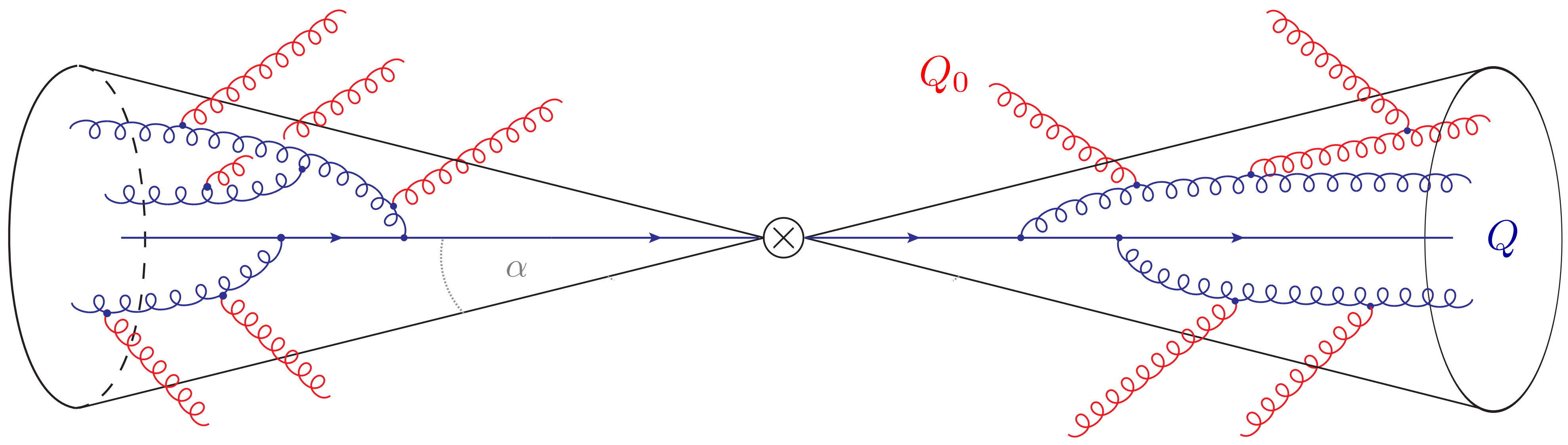
Accuracy:

- LL: g_1; NLL: g_1, g_2; NNLL: g_1, g_2, g_3

Expand in α_s but count $\alpha_s L$ as $O(1)$



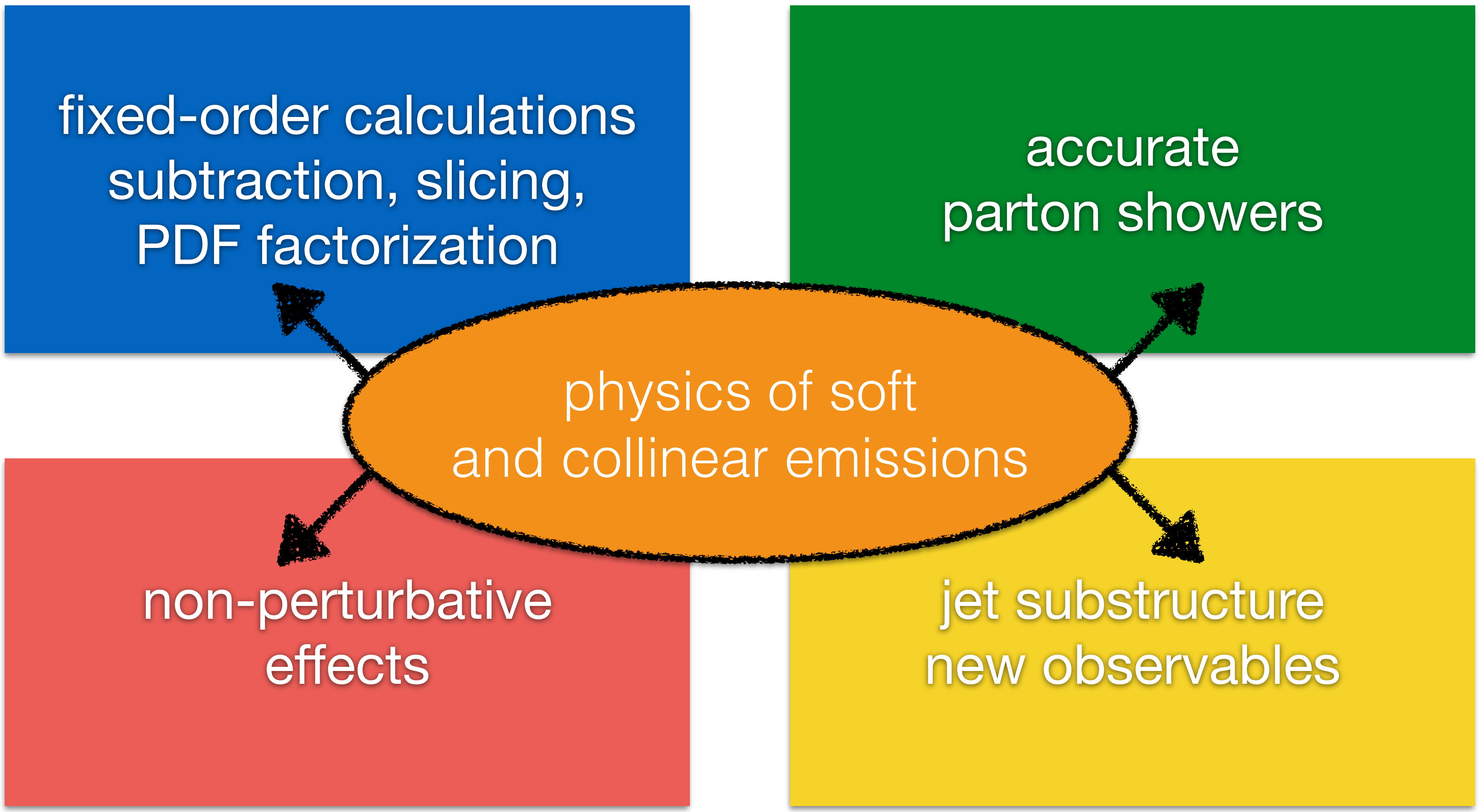
Many types of scale hierarchies, many different types of resummations … and by now many different EFTs



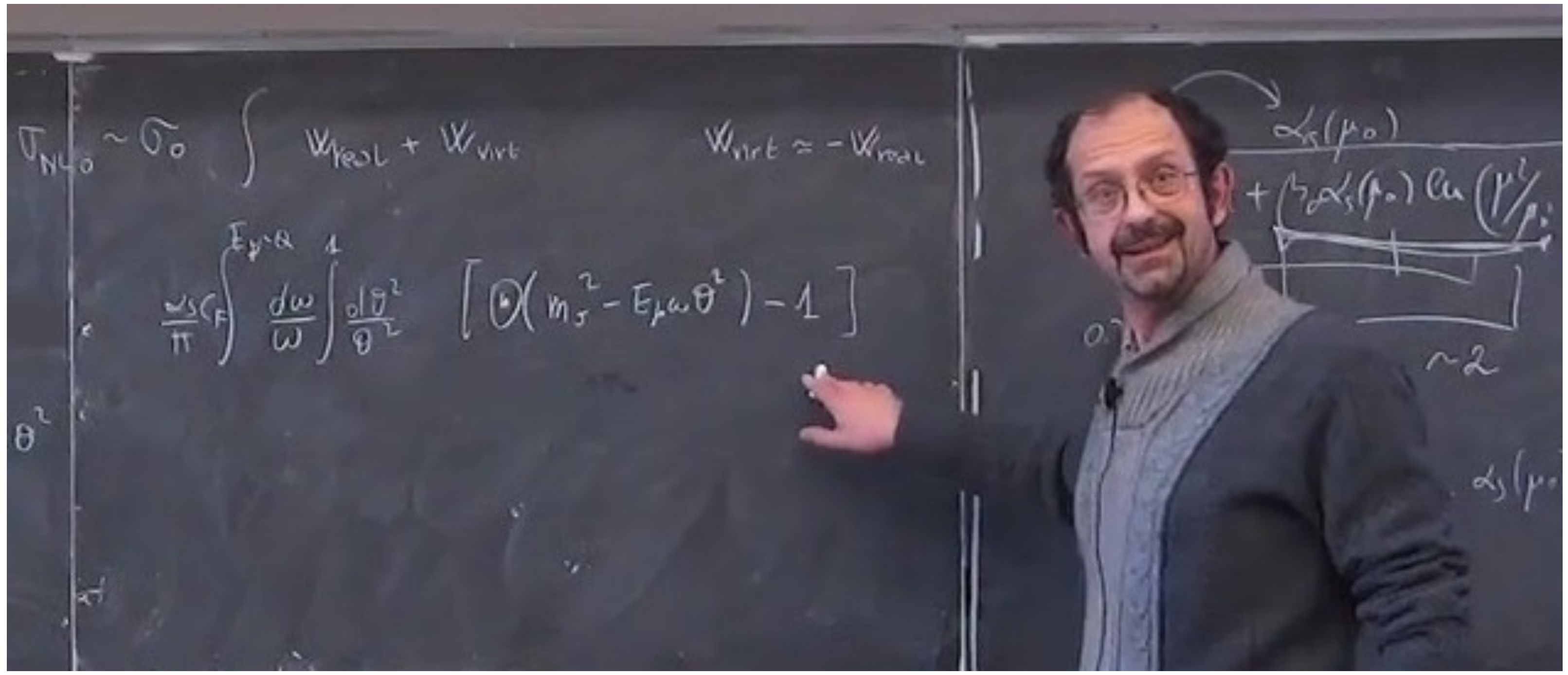
Large logarithms arise due to soft and collinear emissions.

Resummations are based on the factorization of cross sections in soft and collinear limits.

This factorization is at the heart of collider physics and has implications in all its areas.



Insights into all-order structure of cross sections have lead to progress in all of the above areas.

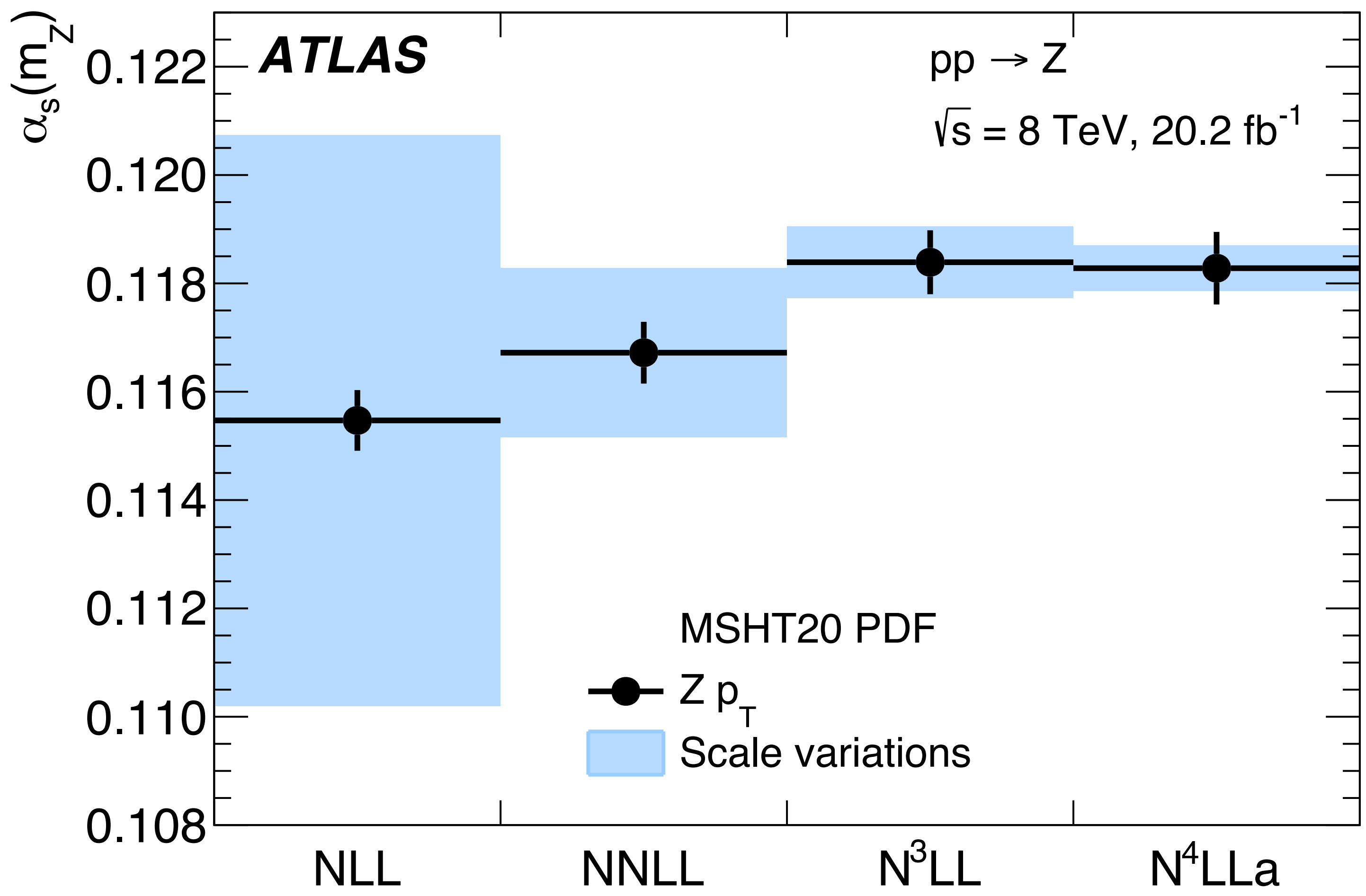


These connections are exemplified by the many scientific achievements of Stefano Catani (’58-’24).

A pioneer of resummation, but his deep insights into soft and collinear dynamics led him to contribute to all areas of collider QCD.

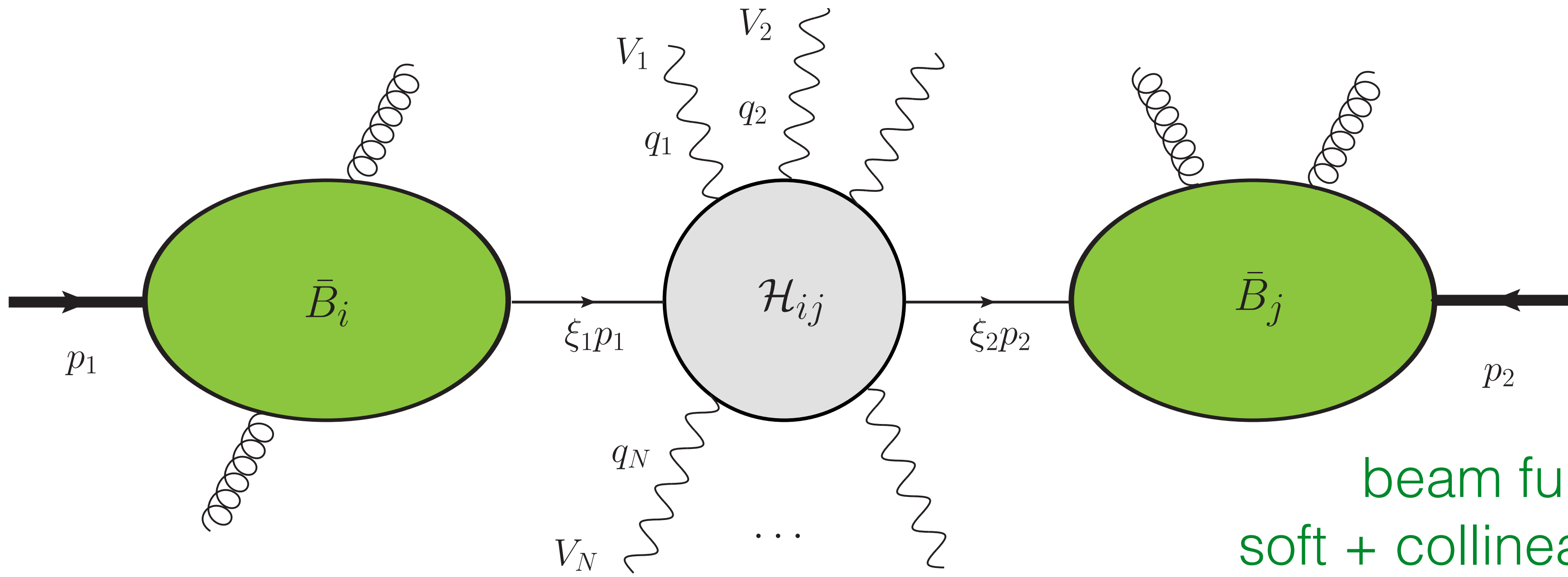
Outline

- High-precision resummations
  - q_T spectrum in Drell-Yan production; α_s and M_W determinations from ATLAS and CMS
- New observables for jet substructure
  - energy-energy correlators and α_s and m_t determination
- Resummation of jet observables and parton showers
  - non-global logs, clustering logs, super-leading logs
- Back to the basics
  - all-order structure of wide-angle scattering
  - collinear factorization violation vs. PDF factorization



α_s and m_t from precision resummations of transverse momentum spectra

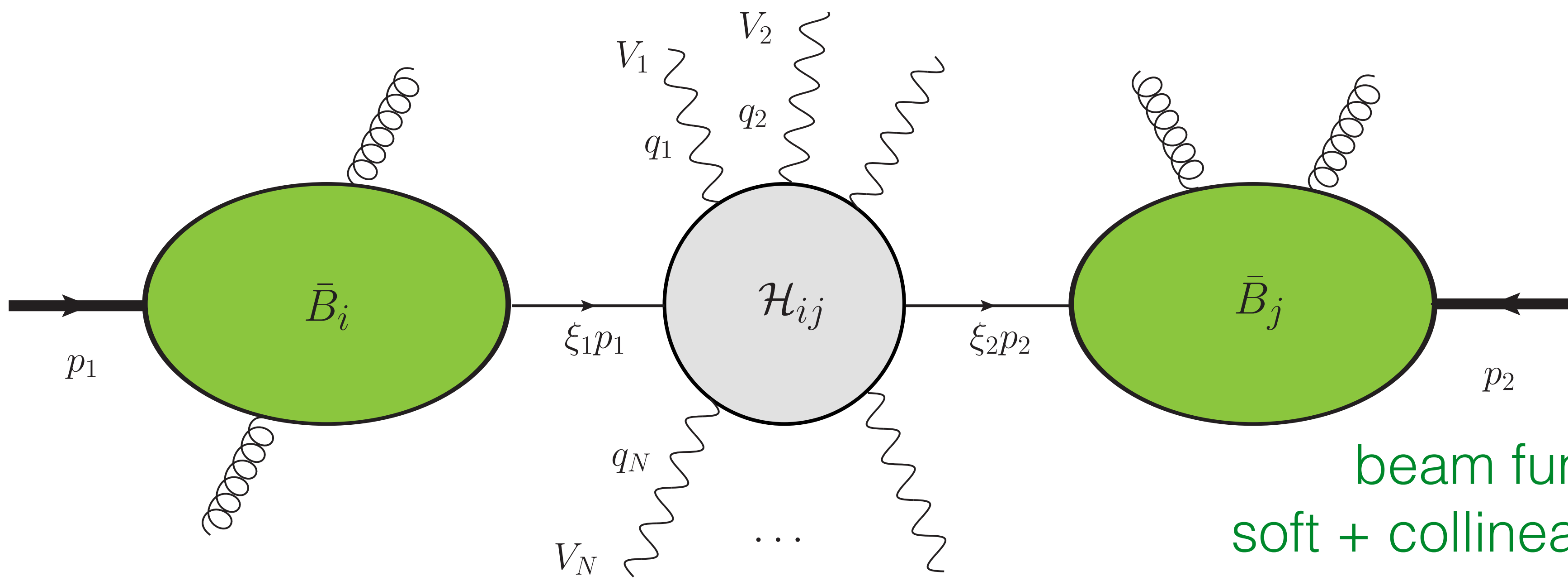
$pp \to$ "EW bosons" $+ X$ at low q_T



- Ingredients known to very high accuracy
  - three-loop beam functions Ebert, Mistlberger, Vita '20
  - three-loop hard functions for Z/W/γ (with singlet contributions Gehrmann, Primo '21 with top mass Chen, Czakon, Niggetiedt '21), two-loop for diboson processes
  - four-loop hard anomalous dimensions Manteuffel, Panzer, and Schabinger '20 and anomaly exponent Duhr, Mistlberger, Vita '22; Moult, Zhu, Zhu '22
  - fixed-order matching to α_s^3 from MCFM and NNLOJet

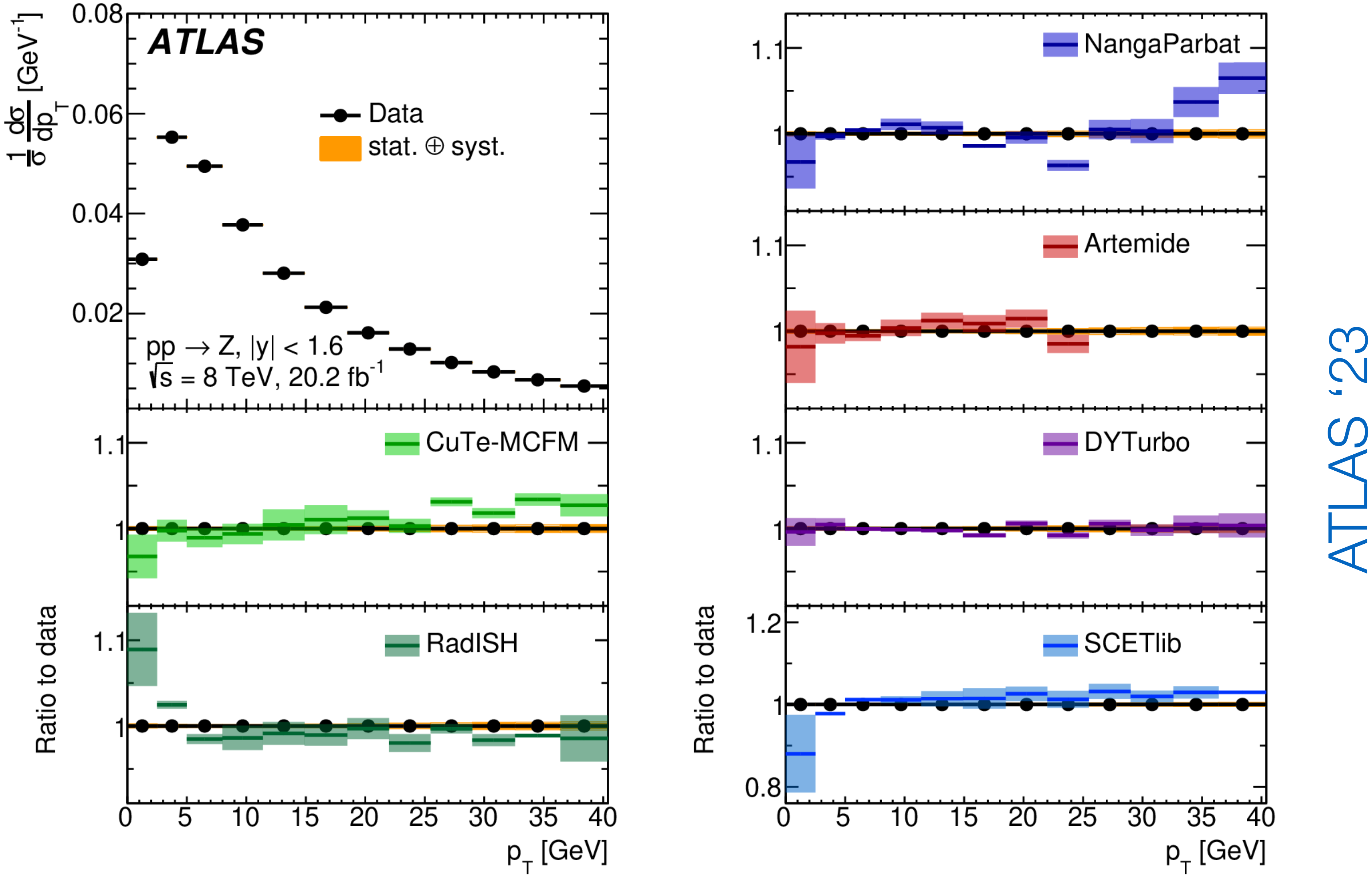
Almost N⁴LL accuracy

hard function: Born + virtual corrections



beam functions
soft + collinear emissions

- Missing for full N⁴LL + accuracy
  - four-loop PDF evolution, i.e. N³LO PDFs. Note: approximate N³LO PDF exist → talks by Sven Moch and Tongzhi Yang
  - five-loop cusp anomalous dimension (likely numerically irrelevant)



- aN^{4}LL resummations from several groups with different formalisms
- Results include $\alpha_s{}^3$ fixed order from MCFM

Comparison and uncertainties

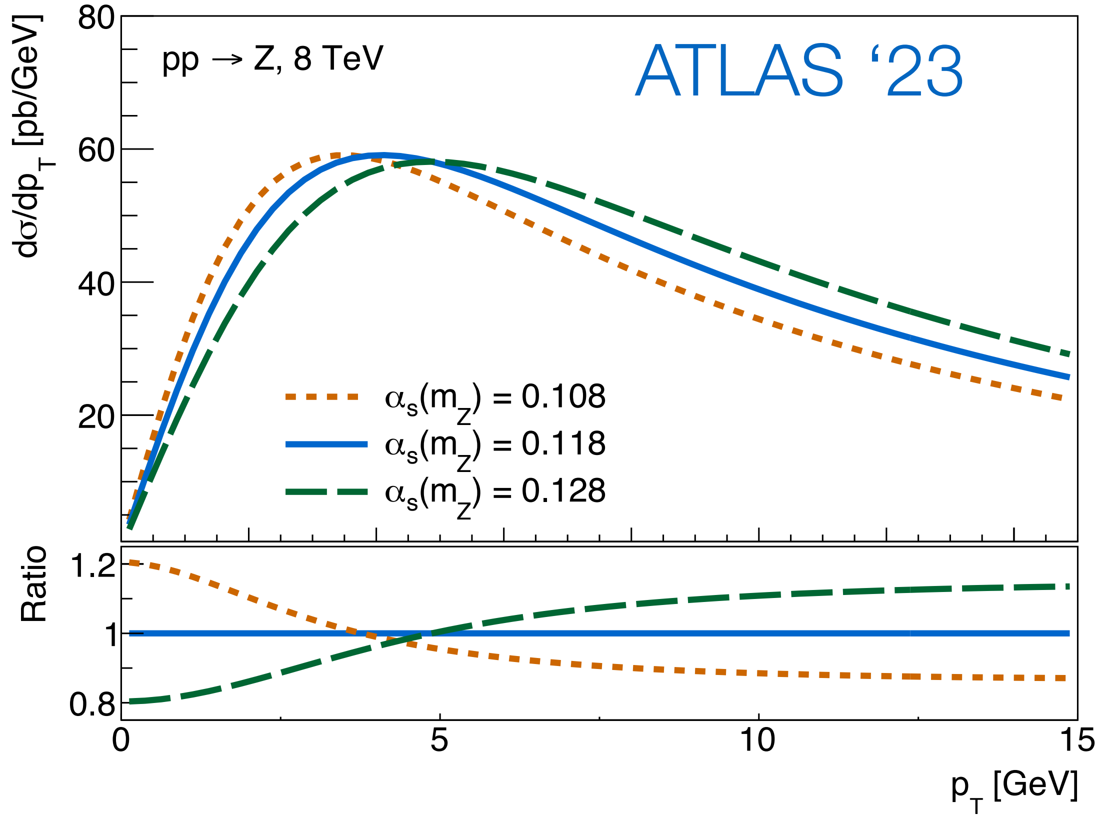
Resummed computations are performed in a variety of (equivalent) formalisms and with different of scheme choices

- Scale setting in momentum space (CuTe, Radish) versus impact parameter space (everyone else)
- Different formalisms for rapidity logs (CSS, collinear anomaly, RRG) and associated uncertainty
- Different matching schemes / transition to fixed order

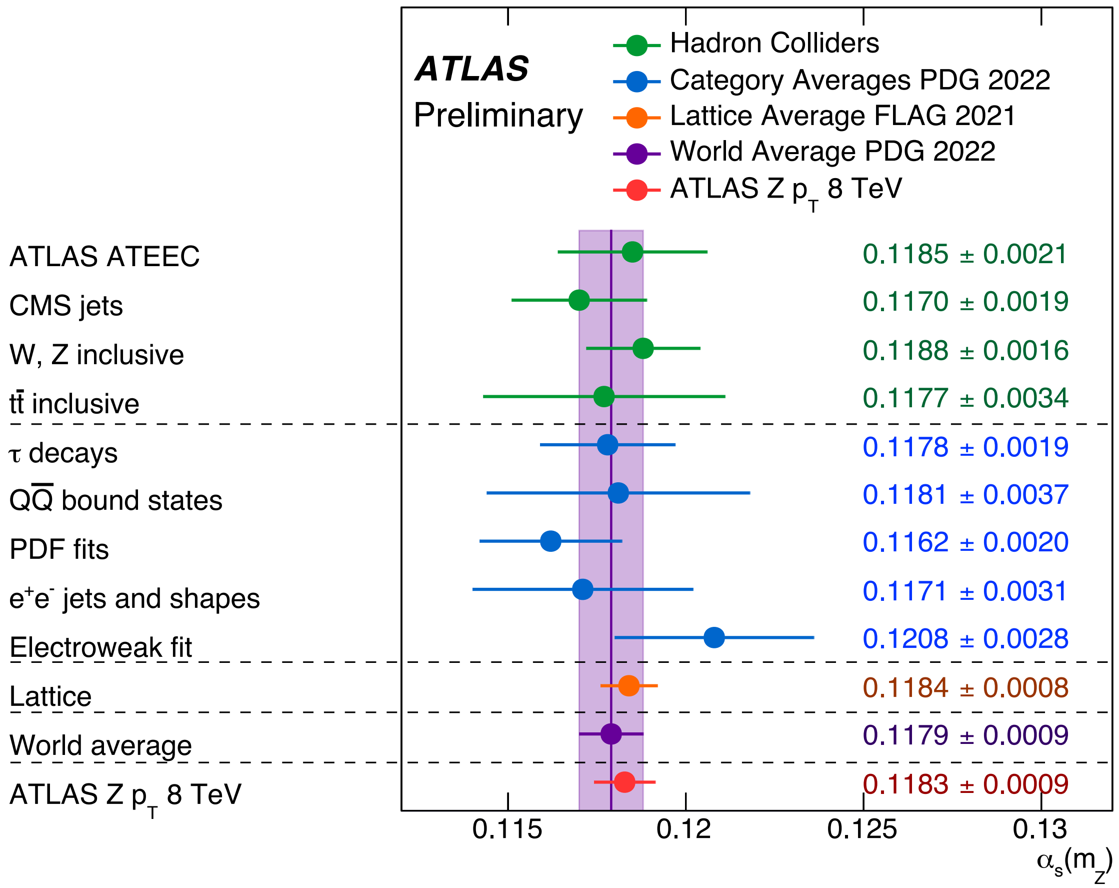
Uncertainty estimates are much less standardized than for fixed-order computations!

- Ongoing comparison/benchmark efforts by LHC EW sub-group (since 2018, to be completed this year!)

ATLAS α_s extraction



- Reconstruct inclusive spectrum rate from angular coefficients
- α_s from fit to DYTurbo
- N3LO fixed order MCFM and NNLOJet
- MSHT20 approximate N3LO PDFs
  - cross checks with NNLO sets
- Non-perturbative effects based on two-parameter ansatz by Collins Rogers '14



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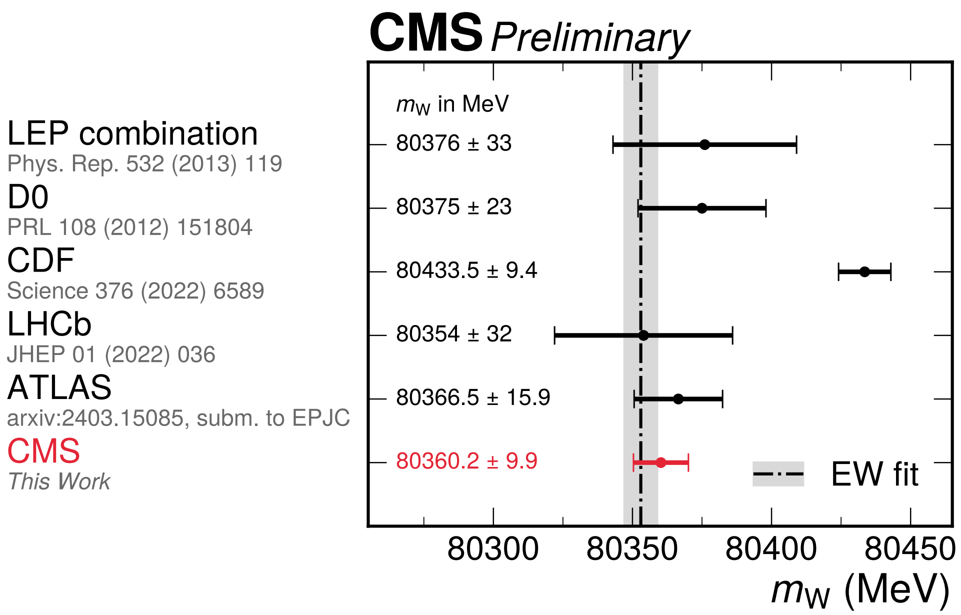
Experimental uncertainty	+0.00044	-0.00044
PDF uncertainty	+0.00051	-0.00051
Scale variations uncertainties	+0.00042	-0.00042
Matching to fixed order	0	-0.00008
Non-perturbative model	+0.00012	-0.00020
Flavour model	+0.00021	-0.00029
QED ISR	+0.00014	-0.00014
N4LL approximation	+0.00004	-0.00004
Total	+0.00084	-0.00088

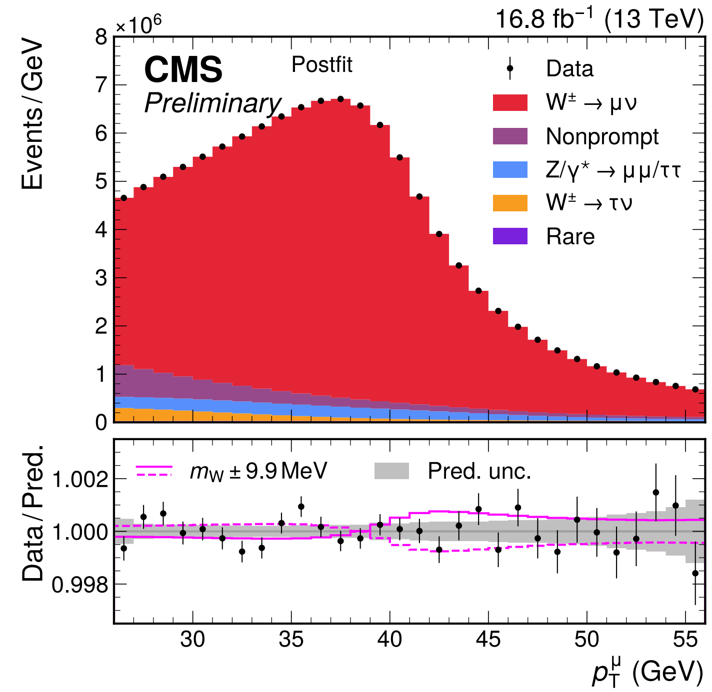
With these high-order resummed and matched computations, we have entered a new regime of precision collider calculations.

Unprecedented precision, but also difficult to be sure the uncertainties are reliably estimated … no previous experience with this level precision at hadron colliders!

→ talk by Giulia Marinelli

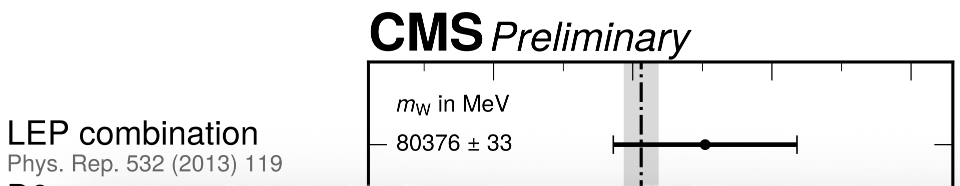
CMS M_W extraction

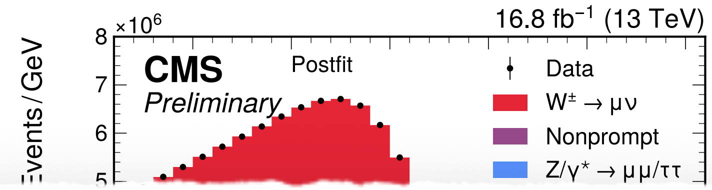


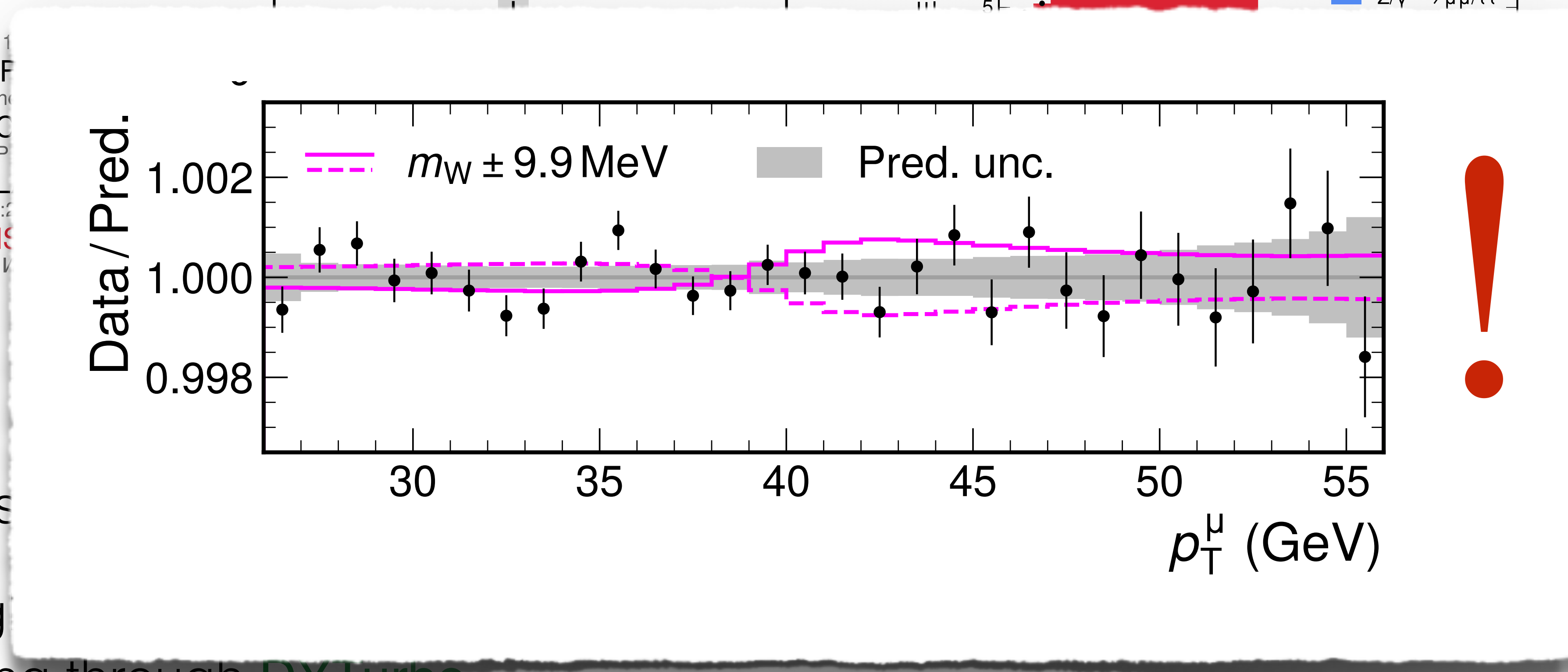


- W-mass from Jacobian peak in charged-lepton p_T spectrum and rapidity distribution.
- Reweigh MiNNLO$_{PS}$ with resummation from SCETlib at N^{3}LL, $\alpha_s{}^2$ fixed-order matching through DYTurbo
- Higher-order coefficients in resummation as theory nuisance parameters Tackmann, unpublished → talk by Giulia Marinelli
  - use statistical model for their distribution; can capture correlations since same coefficient enters different observables
  - values extracted from fit to data

CMS M_W extraction







- W-mass … stribution.
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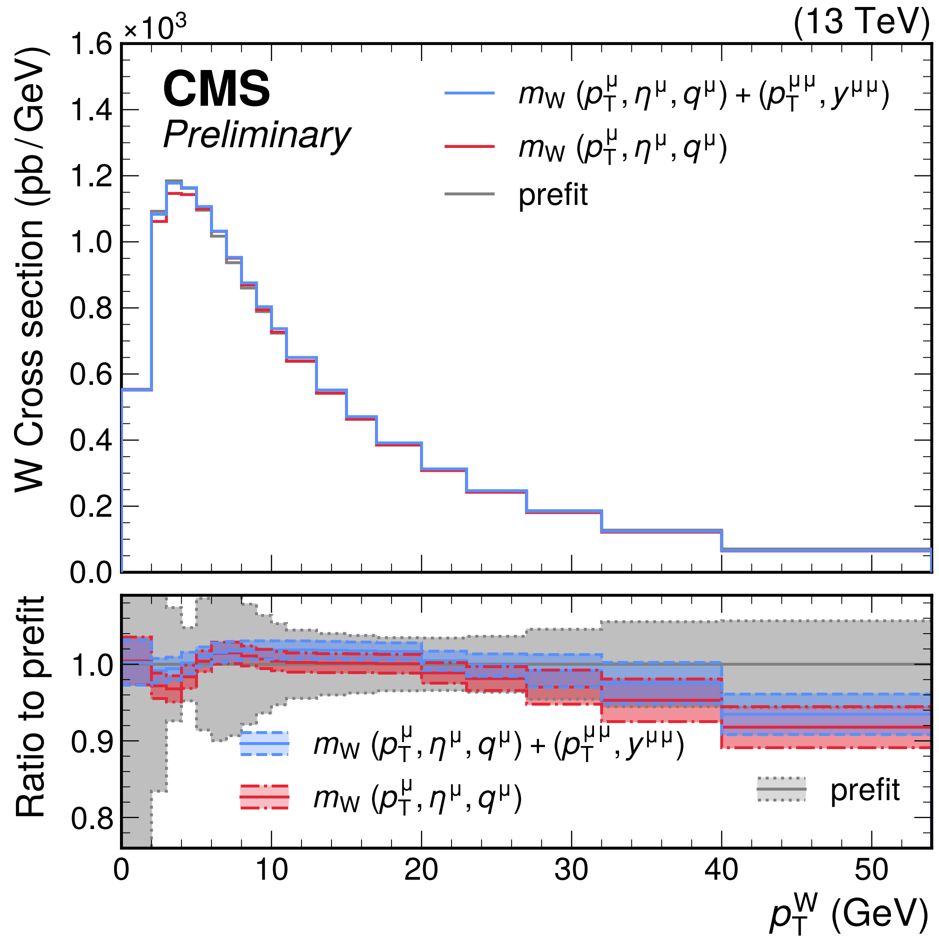


Theoretical uncertainties before and after fit of nuisance parameters to data using W measurements or W and Z measurements.

- Crucial new element of analysis is that **theoretical predictions are fit to data to significantly reduce their uncertainties**.
  - CMS validates this procedure on Z-production, but does not use Z-data for W-mass extraction
  - Effects not accounted for in the original theory prediction (higher-order electroweak corrections, quark masses,…)?

N-jettiness event shape

- 3-loop soft function for 0-jettiness Baranowski, Delto, Melnikov, Pikelner, Wang '24
    - Also 3-loop beam functions are known Ebert, Mistlberger, Vita '20, Baranowski, Behring, Melnikov, Tancredi, Wever '22 → all ingredients for 3-loop jettiness slicing available
- New precise representations of the two-loop N-jettiness soft function Bell, Dehnadi, Mohrmann, Rahn'23; Agarwal, Melnikov, Pedron '24
- N^3LL resummation of 1-jettiness for Z-boson plus jet in Geneva Alioli, Bell, Billis, Broggio, Dehnadi, Lim, Marinelli, Nagar, Napoletano, Rahn '23

Goal of is not comparison with measurements, but use for fixed-order computations and simulations of jet processes (slicing,…)

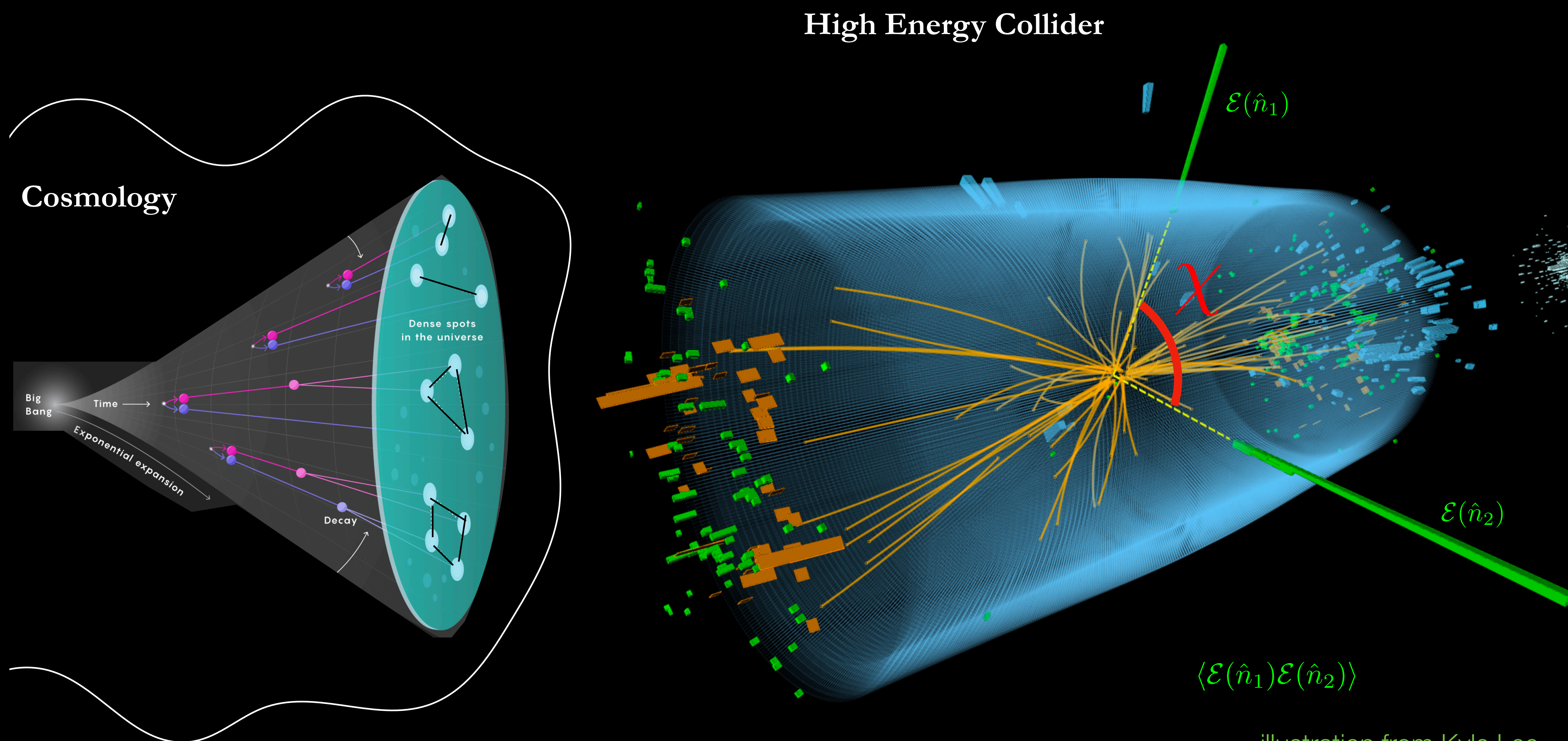
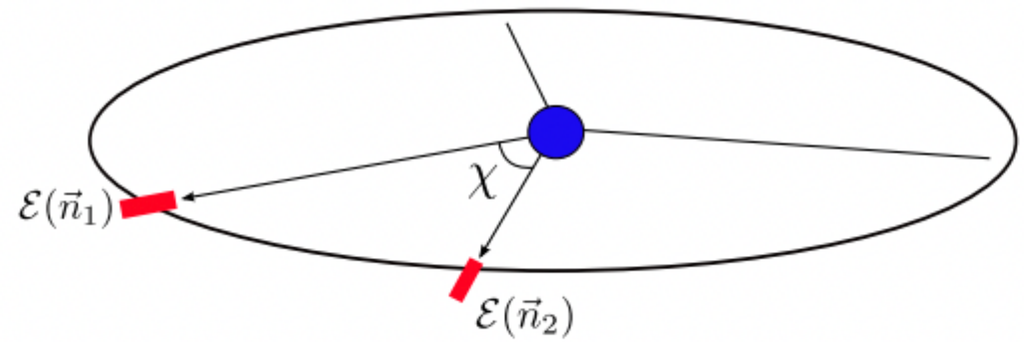


illustration from Kyle Lee

Energy-energy correlators and jet substructure



Matrix elements

Energy-flow operator

$$\langle\Psi|\mathcal{E}(\hat{n}_1)\cdots\mathcal{E}(\hat{n}_k)|\Psi\rangle \quad \text{with} \quad \mathcal{E}(\hat{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\hat{n})$$

Sveshnikov, Tkachov '95

characterize energy flow into the detector

A lot of new interesting developments in using these energy-energy correlators to study jet subtructure, determine α_s and m_t, …

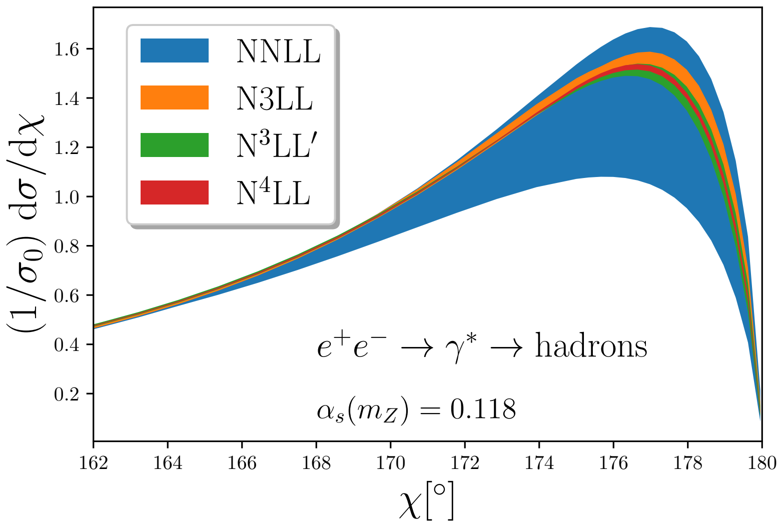
Correlators have many good properties

- weighted by energy: insensitive to soft radiation:
- factorization, light-ray OPE, CFT techniques Hofman, Maldacena '08

Energy-Energy Correlator

Simplest example is the two-point function

$$\mathrm{EEC}(\chi) = \sum_{a,b} \int \mathrm{d}\sigma_{e^+e^- \to a+b+X} \frac{E_a E_b}{Q^2} \delta(\cos\chi_{ab} - \cos\chi)$$



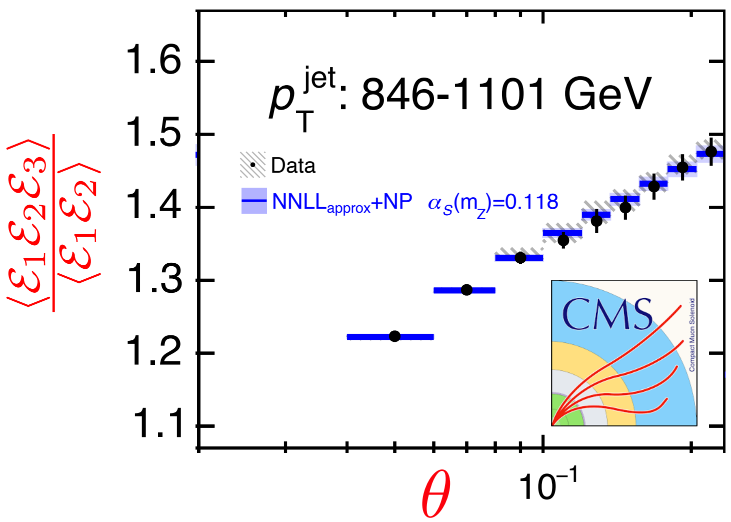
Duhr, Mistlberger, Vita '22

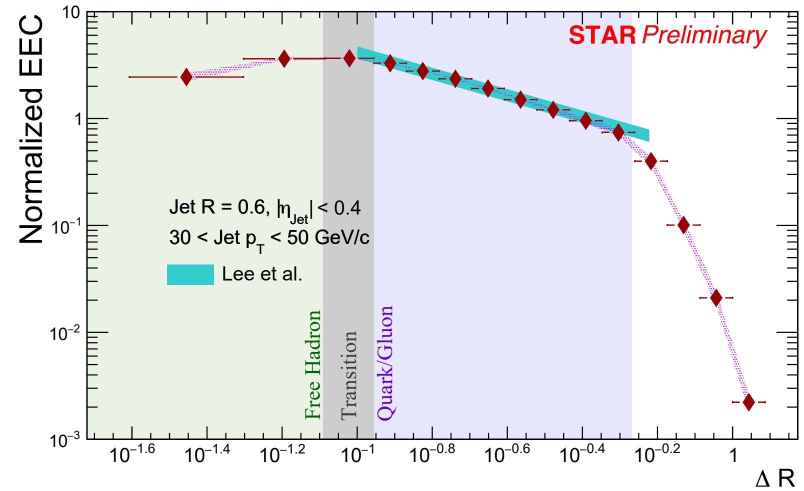
large logarithms both for small and large angles. For large angle N^{4}LL is known!

Ingredients:

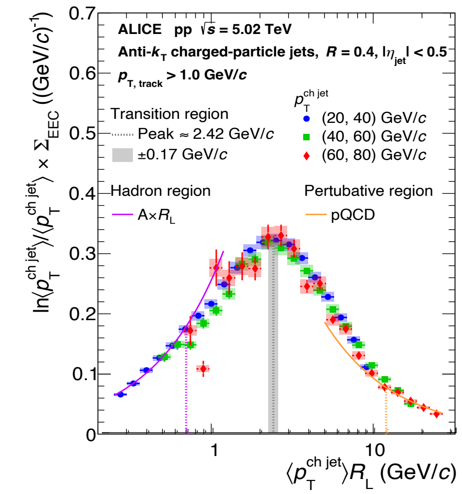
- 3-loop jet functions Ebert, Mistlberger, Vita '20
- 4-loop rapidity anomalous dimension Duhr, Mistlberger, Vita '22, Moult, Zhu, Zhu '22.
- four-loop hard anomalous dimensions Manteuffel, Panzer, and Schabinger '20; Lee, Manteuffel, Schabinger, Smirnov, Smirnov, and M. Steinhauser '22.
- four-loop cusp Henn, Korchemsky, Mistlberger '19; Manteuffel, Panzer, and Schabinger '20 + … 5-loop cusp is missing, estimated to have very small effect.

“Conformal Colliders Meet the LHC”





NLL: Lee, Mecaj, Moult ‘22

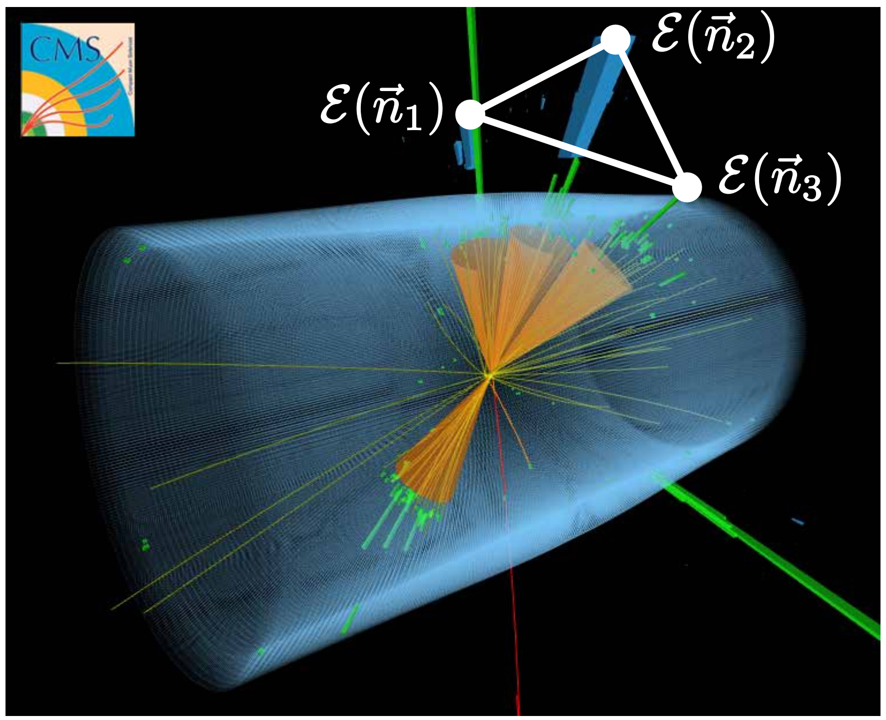


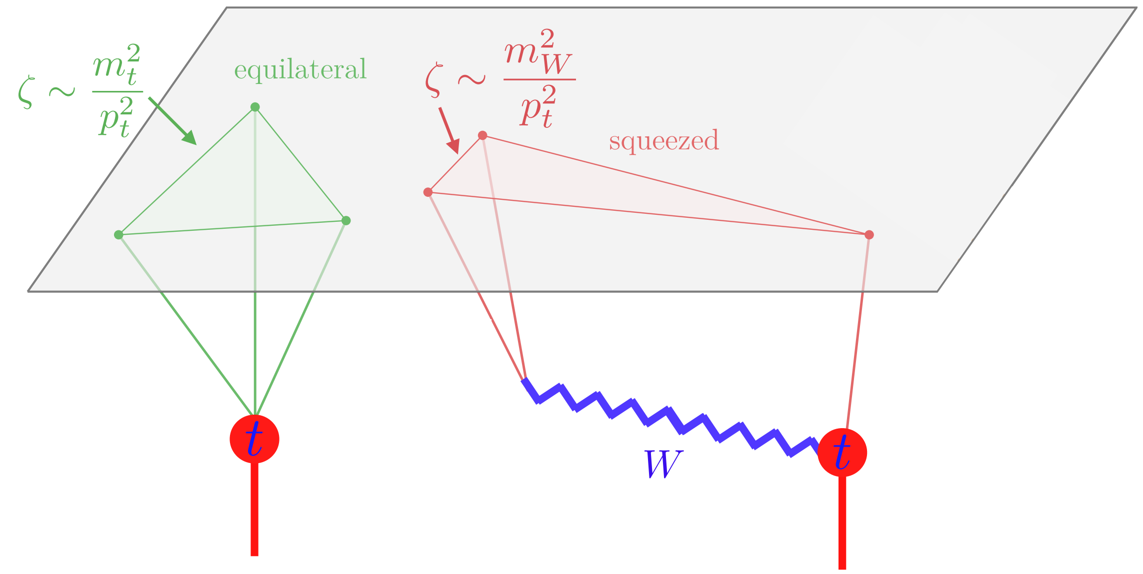
- By now, several measurements of transverse EECs within jets at hadron colliders (ALICE, ATLAS, CMS, STAR).

- CMS α_s determination based on aNNLL resummation Chen, Gao, Li, Xu, Zhang, Zhu ‘23

- $\alpha_s(M_Z) = 0.1229^{+0.0014}_{-0.0012}\,(\text{stat})^{+0.0030}_{-0.0033}\,(\text{theo})^{+0.0023}_{-0.0036}(\text{exp})$

Top quark mass from EECs



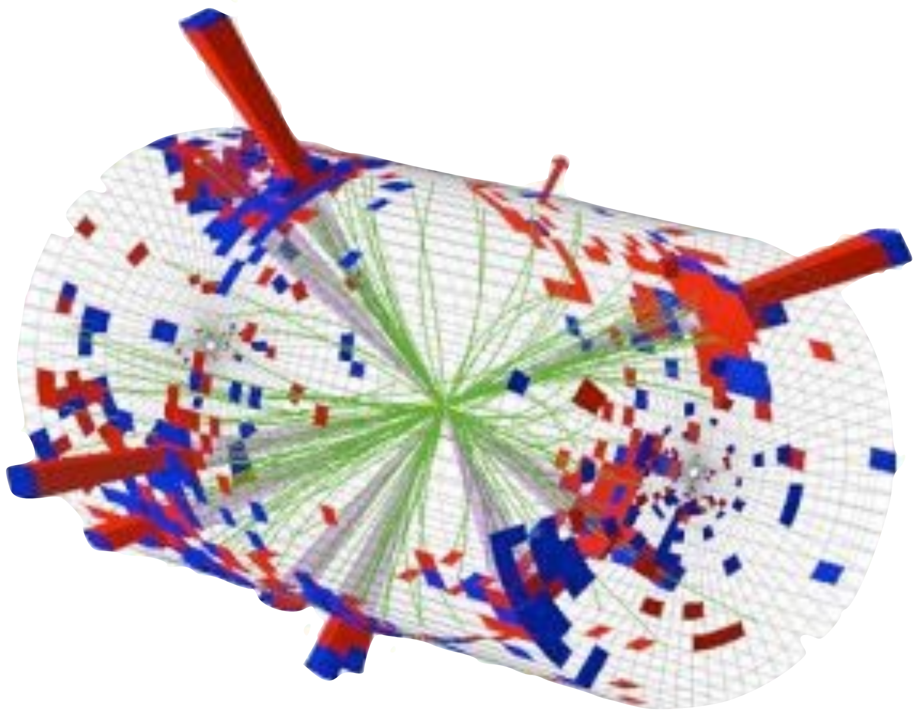


Proposals to extract ratio m_t/m_W from 3-point correlator in top decays Holguin, Moult, Pathak, Procura, Schöfbeck, Schwarz '23, '24; Xiao, Ye, Zhu '24

→ talk by Adi Pathak

Many new ideas and results

- EECs for *b*- and *c*-quarks Lee, Mecaj Moult ’22
- Non-Gaussianities in collider energy flux Chen, Moult, Thaler, Zhu ’22
- Nucleon energy correlators Liu, Zhu ’22, Cao, Liu, Zhu ‘23
- TMDs from Semi-inclusive Energy Correlators Liu, Xhu ‘24
- EECs for nuclear matter at the electron-ion collider (EIC) Devereaux, Fan, Ke, Lee, Moult ’23
- EECs for studying the quark-gluon plasma Andres, Dominguez, Holguin, Marquet, Moult ’23, ‘24; Liu, Liu, Pan, Yuan and Zhu ’23
- Non-perturbative effects in EECs Schindler, Stewart, Sun ’23; Lee, Pathak, Stewart, Sun ’24; Chen, Liu, Ma ’24, Chen, Monni, Xu, Zhu ’24;
- v-point energy correlators Budhraja, Chen, Waalewijn ’24
- Higgs decay EECs Yang, Zhang ’24
- EECs on tracks Lee, Moult ’23; Jaarsma, Moult, Waalewijn, Zhu ’23
- N^3LL for transverse EEC in back-to-back limit Gao, Li, Moult, Zhu ‘23
- …



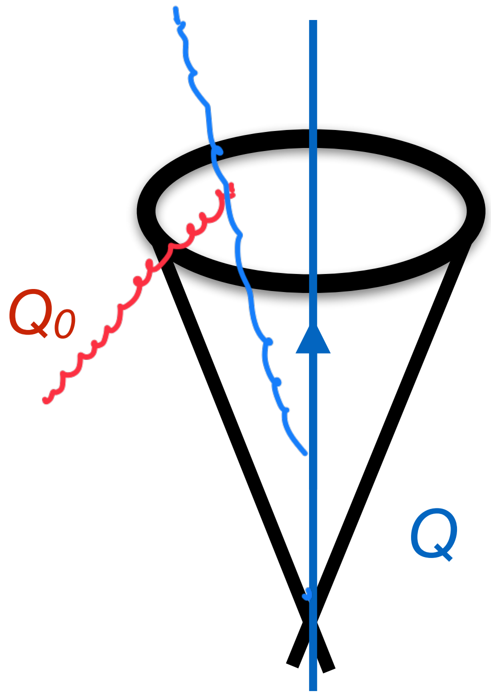
Resummation for jet processes: non-global, clustering and super-leading logarithms

Traditional resummation methods limited to a small set of simple, inclusive (``global'') observables.

Any observable with angular cuts is non-global:

- isolation cones (e.g. in photon production)
- exclusive jet cross sections
- gaps beween jets, veto regions

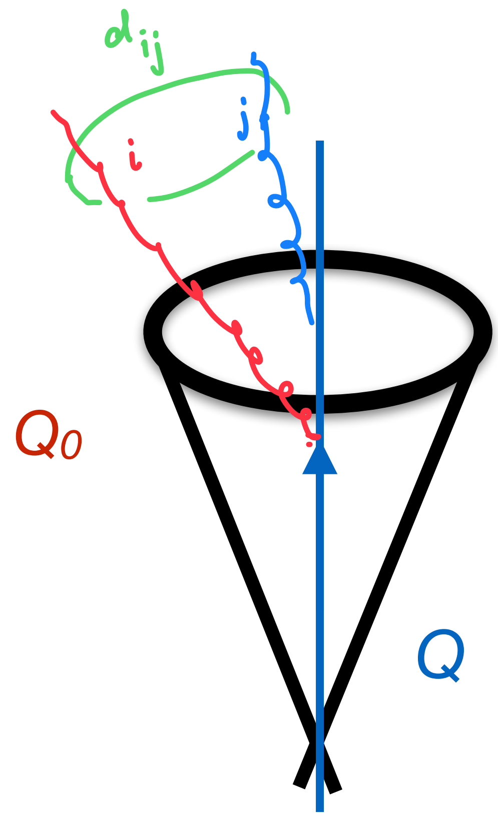
A lot of progress in resumming logarithms in more complicated observables.



Non-global logs (NGLs)

- soft gluons from secondary emissions
- QCD only

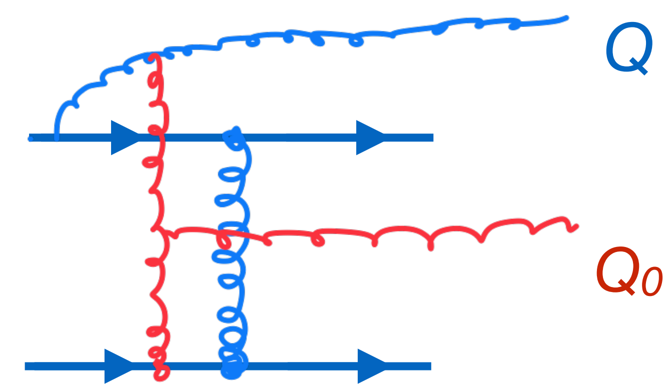
Dasgupta, Salam '00



Clustering logs (CLs)

- phase-space constraints of new emissions depends on all existing partons
- even in QED

Appleby, Seymour '02



Super-leading logs (SLLs)

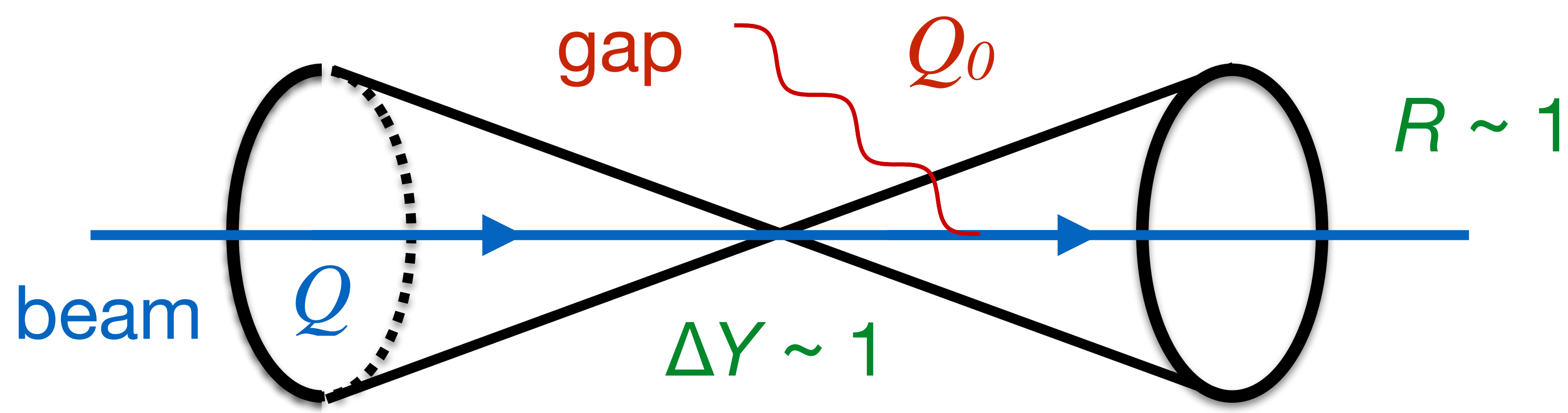
- Glauber phases spoil collinear cancellations
- QCD only
- hadron-hadron colliders only

Forshaw, Seymour '06

Super-Leading Logs (SLLs)

Forshaw, Kyrieleis, Seymour ’06 ’08

Consider gap between jets at hadron collider, cone around beam direction



Large logarithms $\alpha_s^n L^m$ with $L = \ln(Q/Q_0)$

- e^+e^- : $m \leq n$, leading logs $m = n$
- $pp:\ \alpha_s L,\ \alpha_s^2 L^2,\ \alpha_s^3 L^3\ ,\alpha_s^4 L^5 \ldots\ ,\alpha_s^{3+n} L^{3+2n}$

LL effect, but vanishes in large-N_c limit!

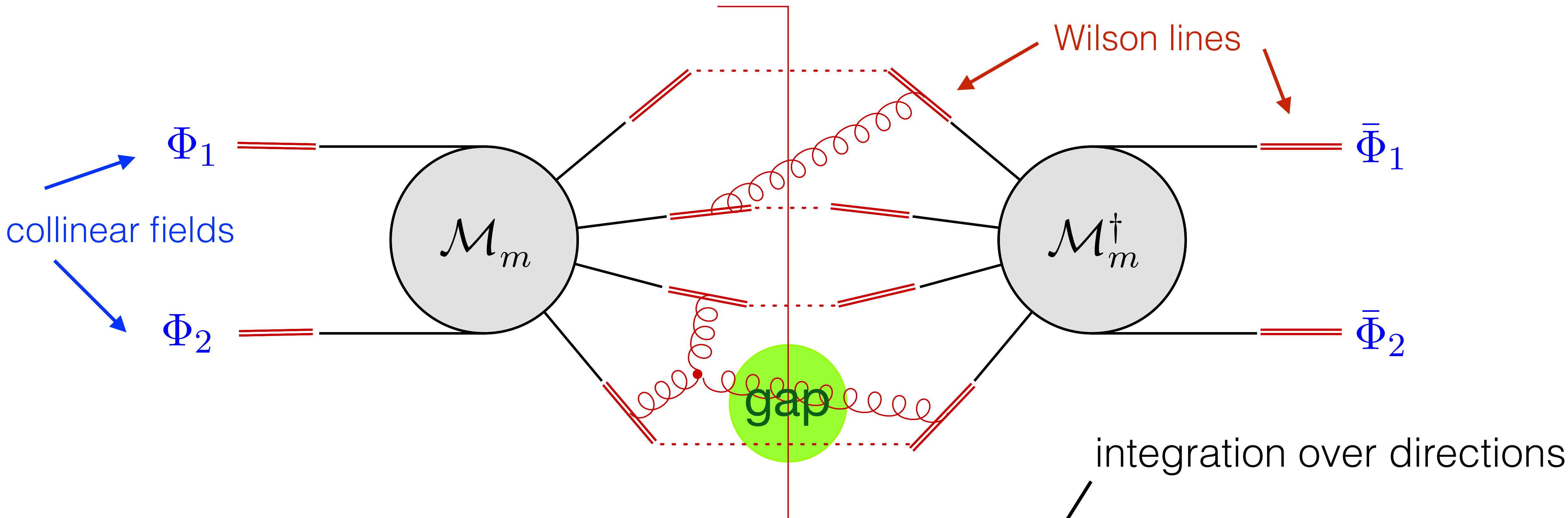
By now, we have all-order factorization theorems, which enable resummation for

- **NGLs** in e^+e^- TB, Neubert, Rothen, Shao '16
- **SLLs** TB, Neubert, Shao '21 + Stillger '23
- **CLs** TB, Haag '23

New results

- First resummations of **SLLs** TB, Neubert, Shao '21 + Stillger '23 and **Glauber phases** Böer, Neubert, Stillger '23 + Hager, Xu '23, '24
- Resummation of **subleading NGLs** TB, Schalch, Xu '23

Factorization for gaps between jets



$$\sigma(Q_0) = \sum_{m=m_0}^{\infty} \int d\xi_1 d\xi_2 \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \xi_1, \xi_2, \mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, \xi_1, \xi_2, \mu) \rangle$$

Hard functions
m hard partons along
fixed directions $\{n_1, \ldots, n_m\}$
$\boldsymbol{\mathcal{H}}_m \propto |\mathcal{M}_m\rangle\langle\mathcal{M}_m|$

Soft + collinear function
squared amplitude
for *m* Wilson lines
+collinear fields

RG evolution

Renormalized hard functions fulfill RG equation

$$\frac{d}{d\ln\mu}\boldsymbol{\mathcal{H}}_m = -\sum_{l=m_0}^{m} \boldsymbol{\mathcal{H}}_l \, \boldsymbol{\Gamma}_{lm}^H$$

matrix in multiplicity and color space

One-loop hard anomalous dimension:

cusp-piece soft+collinear

purely soft

$$\boldsymbol{\Gamma}^H = \gamma_{\rm cusp}(\alpha_s)\left(\boldsymbol{\Gamma}^c \ln\frac{\mu^2}{Q^2} + \boldsymbol{V}^G\right) + \frac{\alpha_s}{4\pi}\,\overline{\boldsymbol{\Gamma}} + \boldsymbol{\Gamma}^C$$

generates SLLs

$\propto i\pi$ Glauber

purely collinear

generates NGLs and CLs

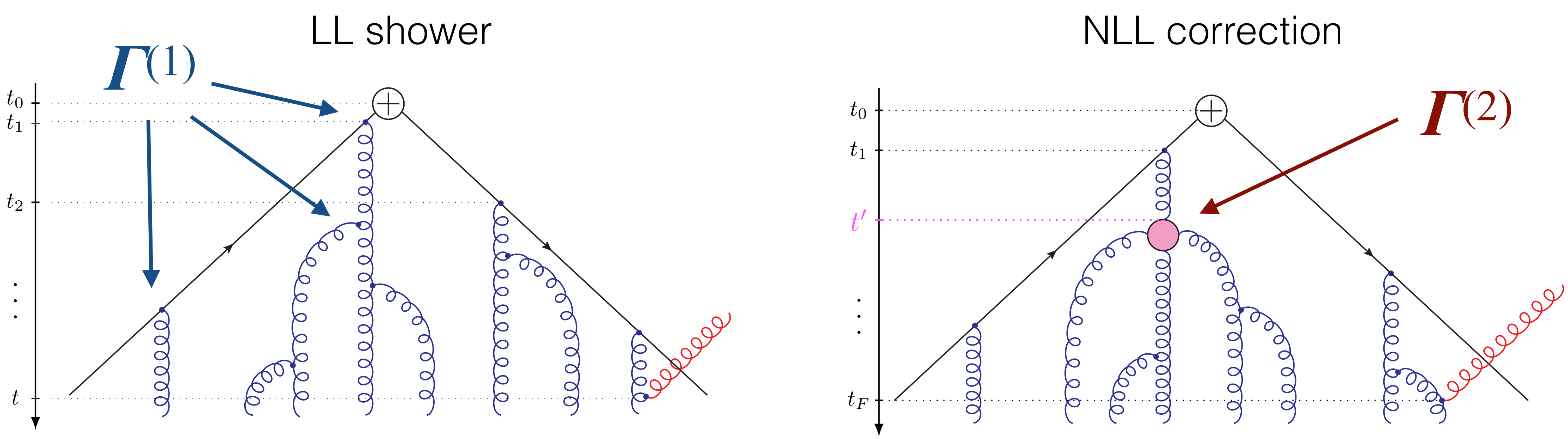
In this framework resummation is obtained by solving the associated RG equations.

Challenge: $\boldsymbol{\Gamma}^{(1)}$ is infinite matrix in the space of particle multiplicities and colors!

- NGLs: implemented $\boldsymbol{\Gamma}^{(1)}$ and $\boldsymbol{\Gamma}^{(2)}$ in the large N_C limit into MC framework $\mathtt{MARZILI}$ to solve RG equation numerically
- SLLs: need full color to see effect. Compute leading SLLs order by order, sum up series → talk by Philipp Böer

MARZILI

TB, Schalch, Xu ‘23



Monte Carlo code to solve the RG equation in the large N_c limit

- LL equivalent to Dasgupta-Salam shower. Full-color LL is available Hatta, Ueda ’13; De Angelis, Forshaw and Plätzer ’20
- NLL has one insertion of $\Gamma^{(2)}$, which includes double-real, real-virtual, and purely virtual terms

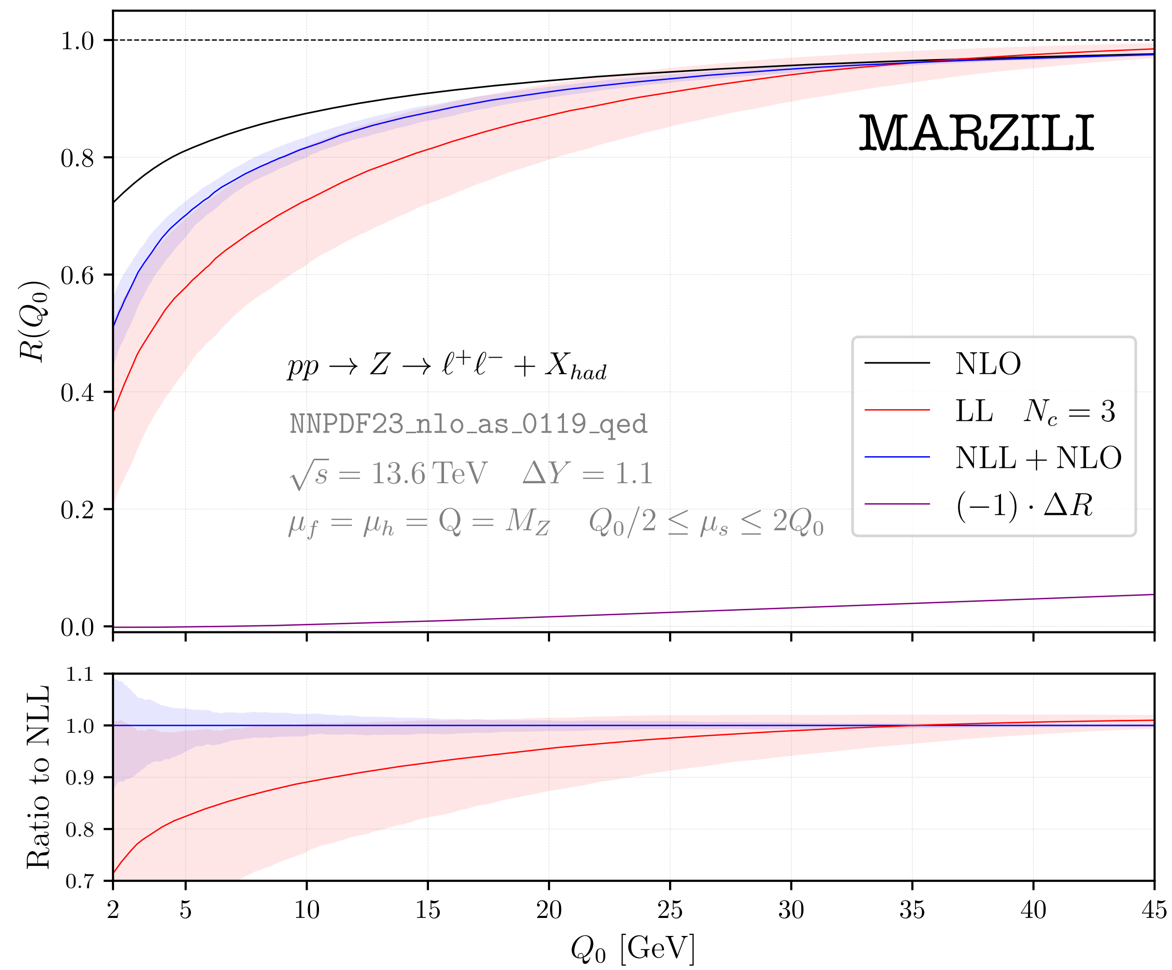
resummation with a parton shower

Subleading NGLs at the LHC

TB, Schalch, Xu ‘23

Z-production with veto $p_T < Q_0$ on radiation in rapidity slice around Z.

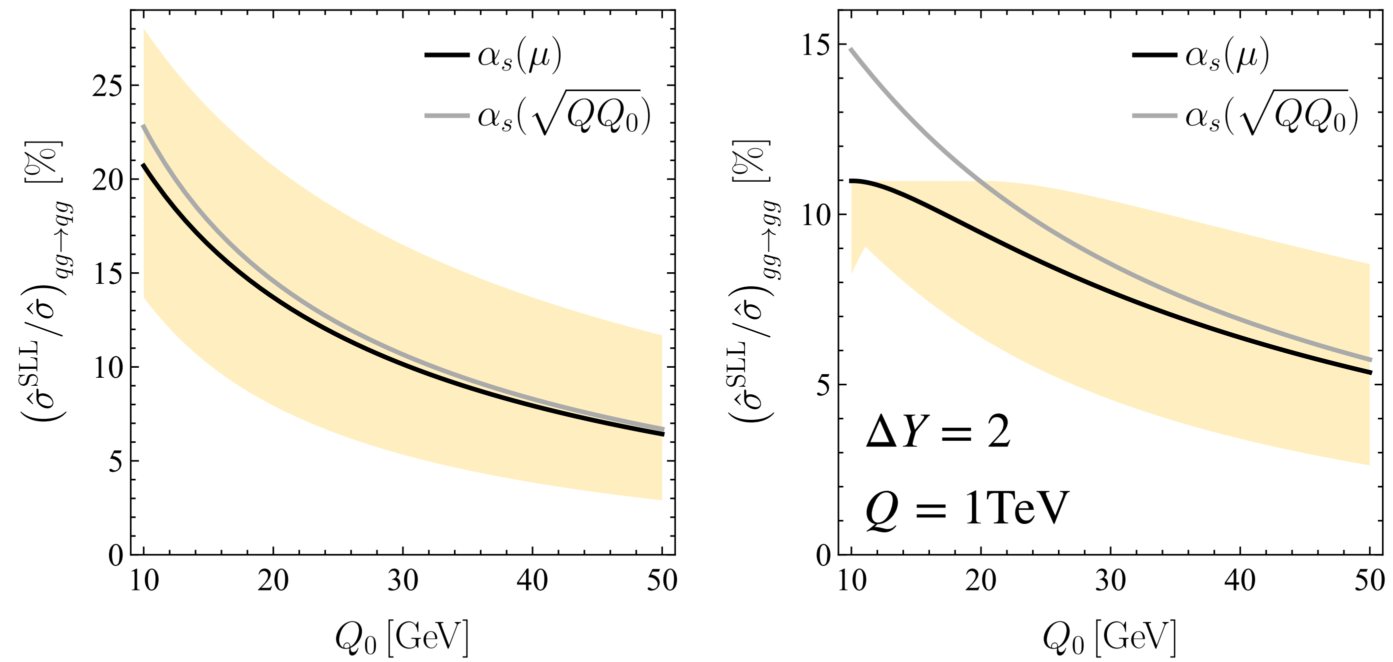
Compute fraction of $R(Q_0)$ of events which pass the veto.



Corrections scale as $\mathcal{O}(\alpha_s^2)$ or $\mathcal{O}(\alpha_s/N_c^2)$

First NGL resummation at this accuracy level!

First resummations for SLLs



- Small effects for $pp \to Z/H$, $pp \to Z/H + j$, but sizable for dijet production TB, Neubert, Shao '21 + Stillger '23
  - Similar-size effects in hadronic cross sections, but will need to combine SLL+NGLs to assess phenomenolgical significance
- By now also resummation including higher Glauber terms Böer, Neubert, Stillger '23 + Hager, Xu '24 and running coupling Böer, Hager, Neubert, Stillger, Xu '24 is available.

→ talk by Philipp Böer

resummation with parton showers with

Connection to parton showers

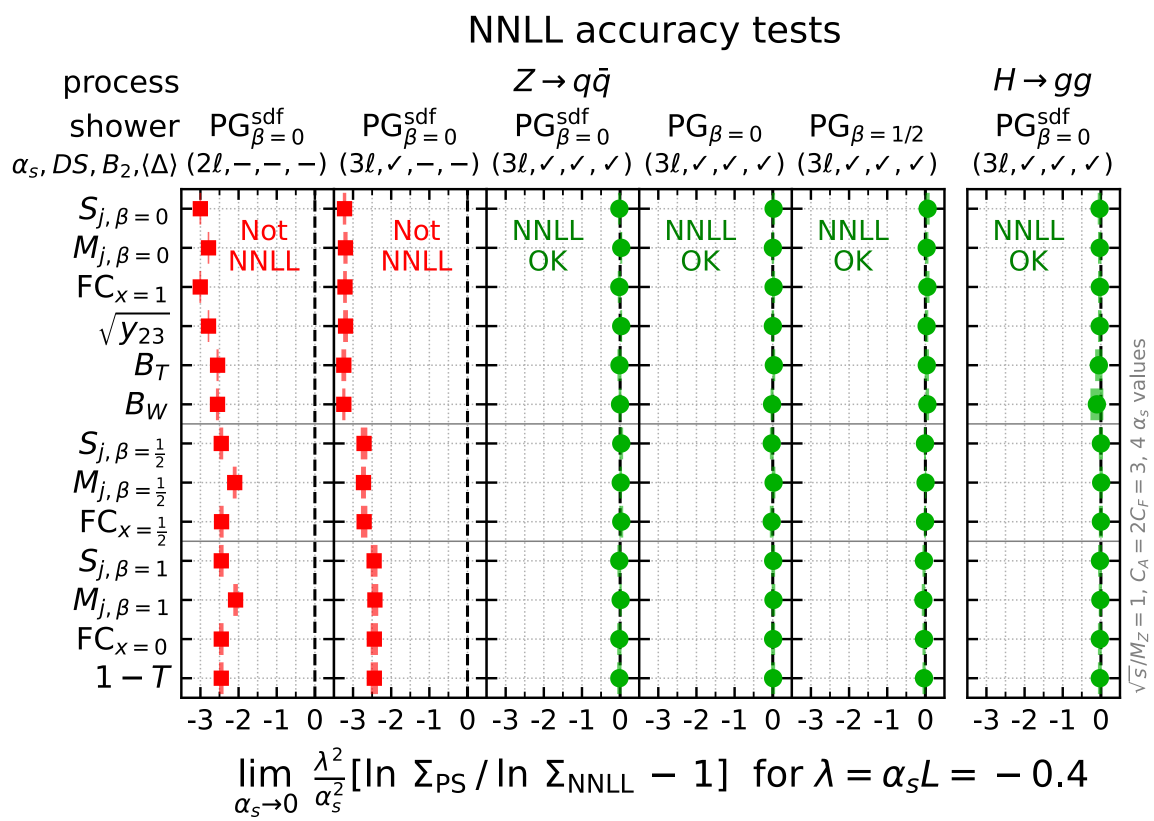
In the past, not much cross talk between parton shower MCs and resummation

- analytical
- very simple observables
- $\mathbf{N}^n\mathbf{LL}$ accuracy (by now up to n=4!)
- exact color
- non-perturbative matrix elements, fits

- numerical
- fully general
- $\mathbf{LL}$ + many subleading effects + tuning
- large-N_c limit + some
- hadronization models

As discussed, resummation is being extended to more complicated observables and is using MC methods…

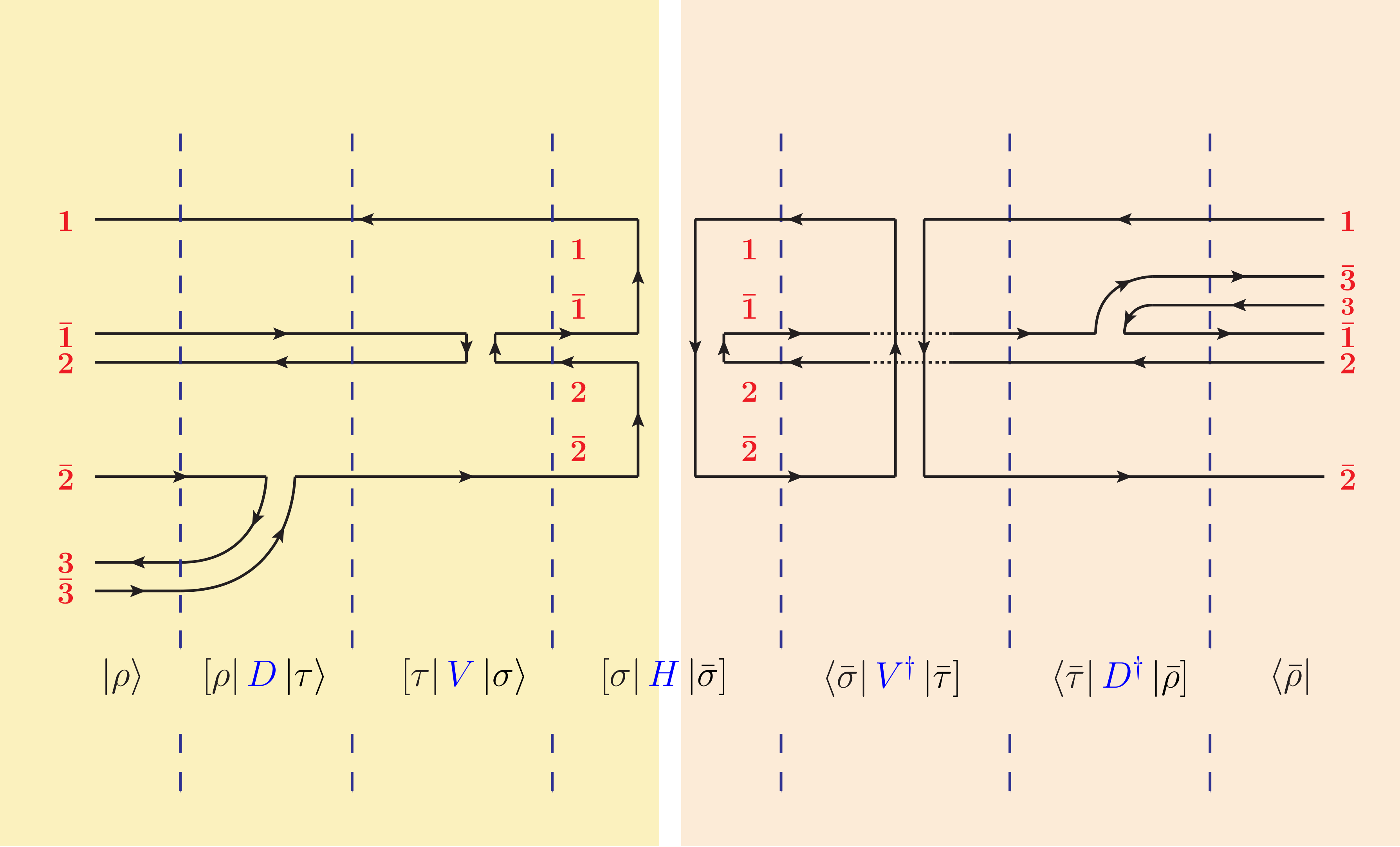
… and parton shower are moving to higher accuracy!



PanScales shower: van Beekveld, Dasgupta, El-Menoufi, Ferrario Ravasio, Hamilton, Helliwell, Karlberg, Monni, Salam, Scyboz, Soto-Ontoso, Soyez '24

Development of parton showers which systematically include higher-log effects PanScales, Alaric, … → talk by Alexander Karlberg

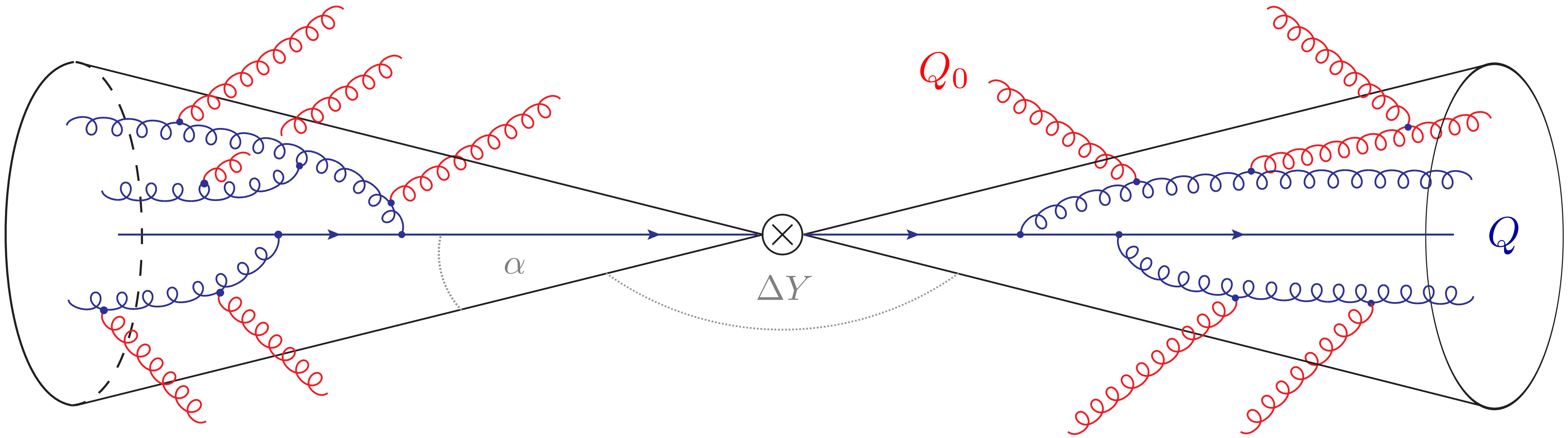
parton showers with resummation



De Angelis, Forshaw, Plätzer '21

Amplitude evolution, development of full color showers.
Deductor Nagy, Soper, CVolver Plätzer, Sjodahl, De Angelis, Forshaw, Holguin, …

→ talks by Simon Plätzer and Fernando Torre González



Resummation of subleading soft logarithms in jet processes using MC method: **Gnole** Banfi, Dreyer, Monni '21 **Marzili** TB, Schalch, Xu, '23 …

… and same result from **PanScales** MC Ferrario Ravasio, Hamilton, Karlberg, Salam, Scyboz, Soyez '23.

Numerical agreement to better than 1% among the three approaches.

1. Momentum regions in wide-angle scattering
2. Collinear factorization violation and PDF factorization?

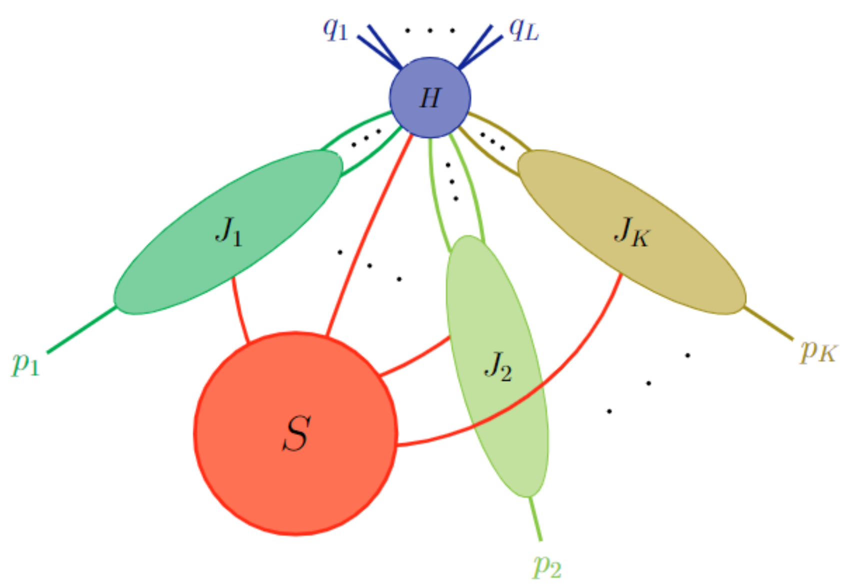
1.) New insights into the MoR

Soft-Collinear Effective theory is based on method of regions (MoR) expansion of loop (and phase-space) integrals.

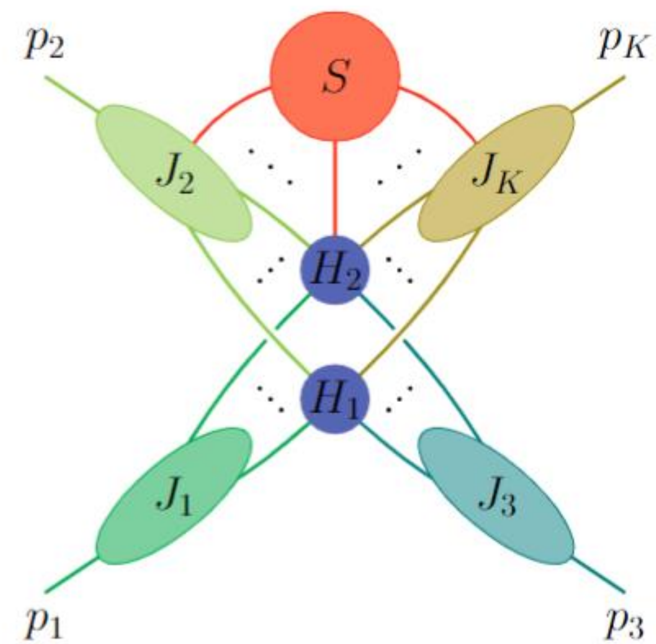
Long-standing open question whether usual soft and collinear regions are sufficient to all orders?

- Yes, for massless wide-angle scattering! Gardi, Herzog, Jones, Ma, Schlenk, ’22; proof: Ma ’23
- but proof only applies to “facet regions” and Gardi, Herzog, Jones, Mao ‘24 have identified a set of “inside regions” for special topologies.

Gardi, Herzog, Jones, Mao ‘24 conjecture



facet

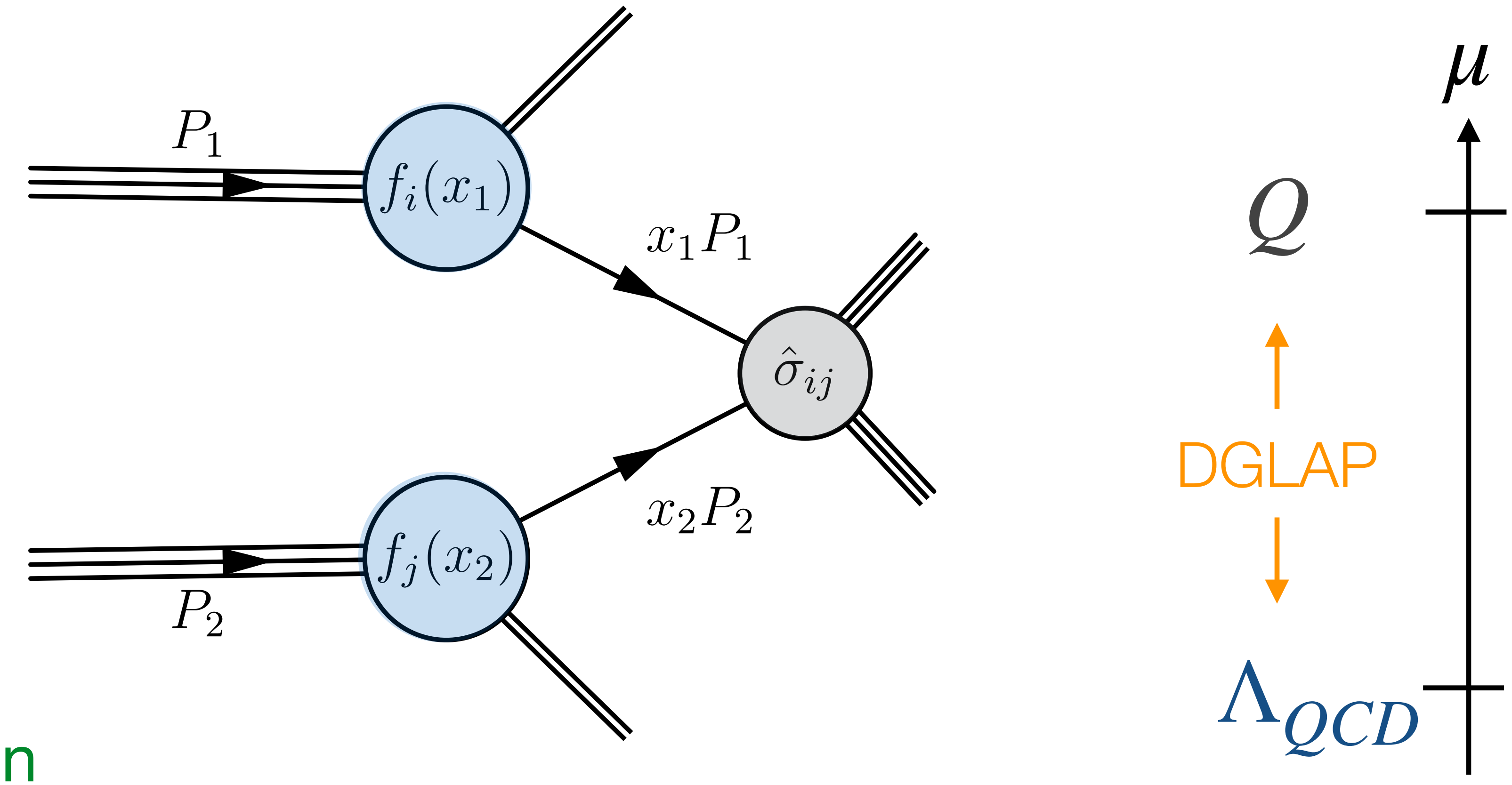


inside

Hidden inside region correspond to multiple hard scattering. Compatible with SCET and power suppressed in QCD.

Hidden regions in forward scattering correspond to Glauber modes.

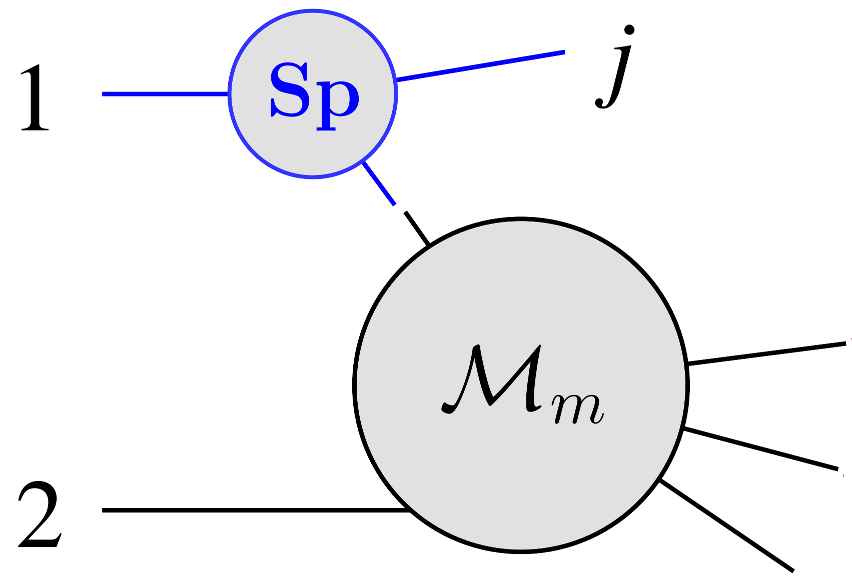
2.) PDF Factorization vs SLLs



- Scale separation
  - perturbative hard-scattering $\hat{\sigma}_{ij}$ at scale Q
  - non-perturbative PDFs $f_i(x)$ at scale Λ_{QCD}
- No low-energy interactions between incoming hadrons
  - cancellation of soft and Glauber physics CSS '85 (for DY)
- Purely collinear, single logarithmic DGLAP evolution

Collinear Factorization Violation

Catani, de Florian, Rodrigo '11; Forshaw, Seymour, Siodmok '12;



New results for **Sp**
Henn, Ma,Xu, Yan, Zhang, Zhu '24
Guan, Herzog, Ma, Mistlberger, Suresh '24

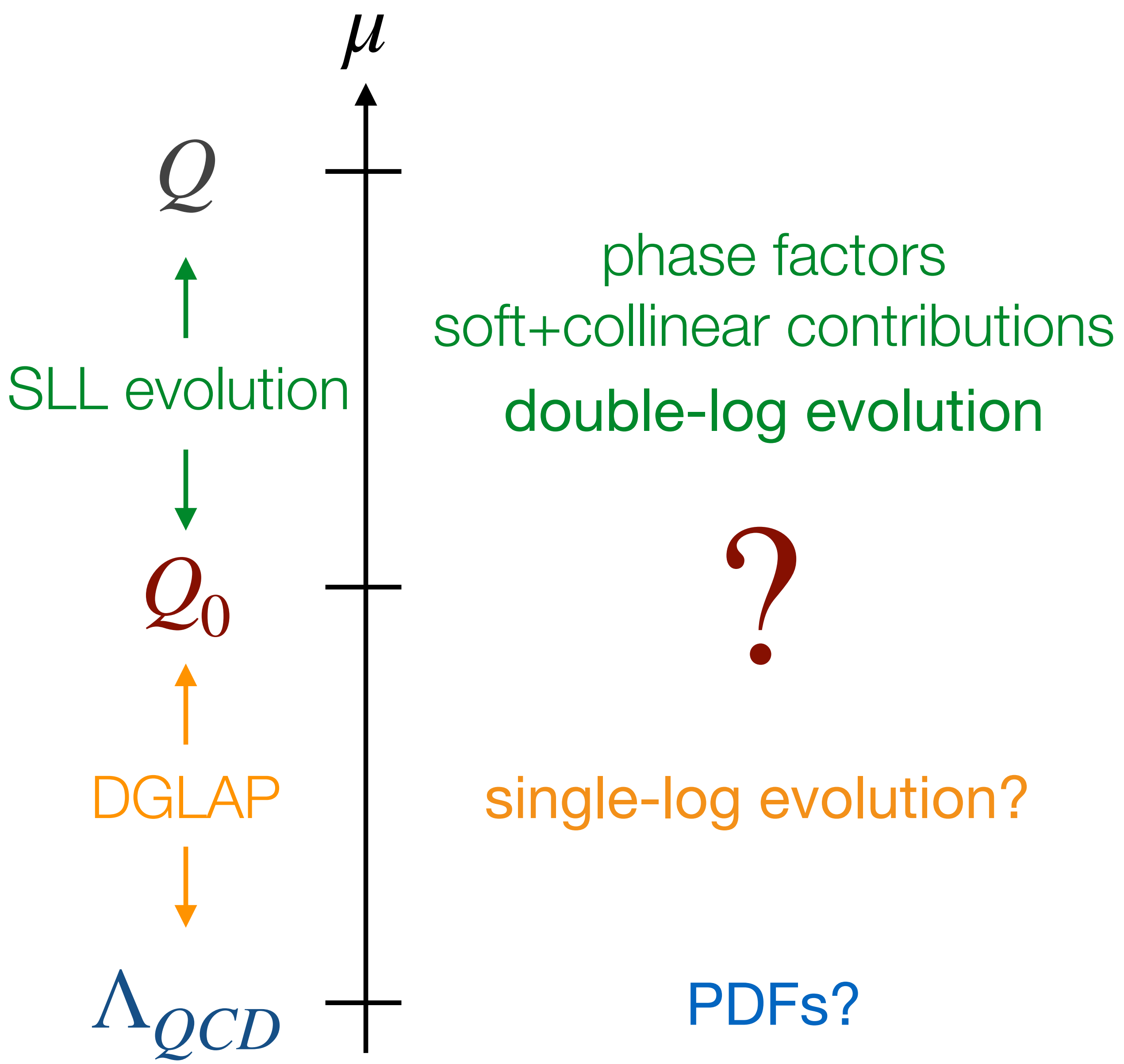
For space-like collinear limit $1 \parallel j$ the splitting amplitude **Sp** depends on the colors and directions of the partons not involved in the splitting!

- Related to non-cancellation due to soft phases

Implications for PDF factorization?

SLLs vs DGLAP

- Double-logarithmic SLLs directly related to collinear factorization breaking
- SLLs generated from double logarithmic running
- If PDF factorization holds, something interesting must happen at scale Q_0 which converts between the two evolutions



EFT analysis

TB, Hager, Jaskiewicz, Neubert, Schwienbacher '24

In our effective theory framework questions about PDF factorization on previous slides can be formulated concisely and answered

- Use RG to predict the form of the low-energy matrix elements $\mathscr{W}_m$ required for consistency with PDF factorization
  - Show that form of $\mathscr{W}_m$ requires soft-collinear interactions at three-loop order
- Perform a systematic method of region analysis of the relevant diagrams contributing to $\mathscr{W}_m$
  - Identify a hidden active-active Glauber region, well defined in dimensional regularization
  - Computing this Glauber contribution, we find that it indeed has the required form to convert the double logarithmic SLL to single-log running

Collinear factorization breaking at $\mu = Q$

x

soft-collinear factorization breaking by Glauber modes at $\mu = Q_0$

=

PDF factorization for $\mu < Q_0$

“factorization restoration”

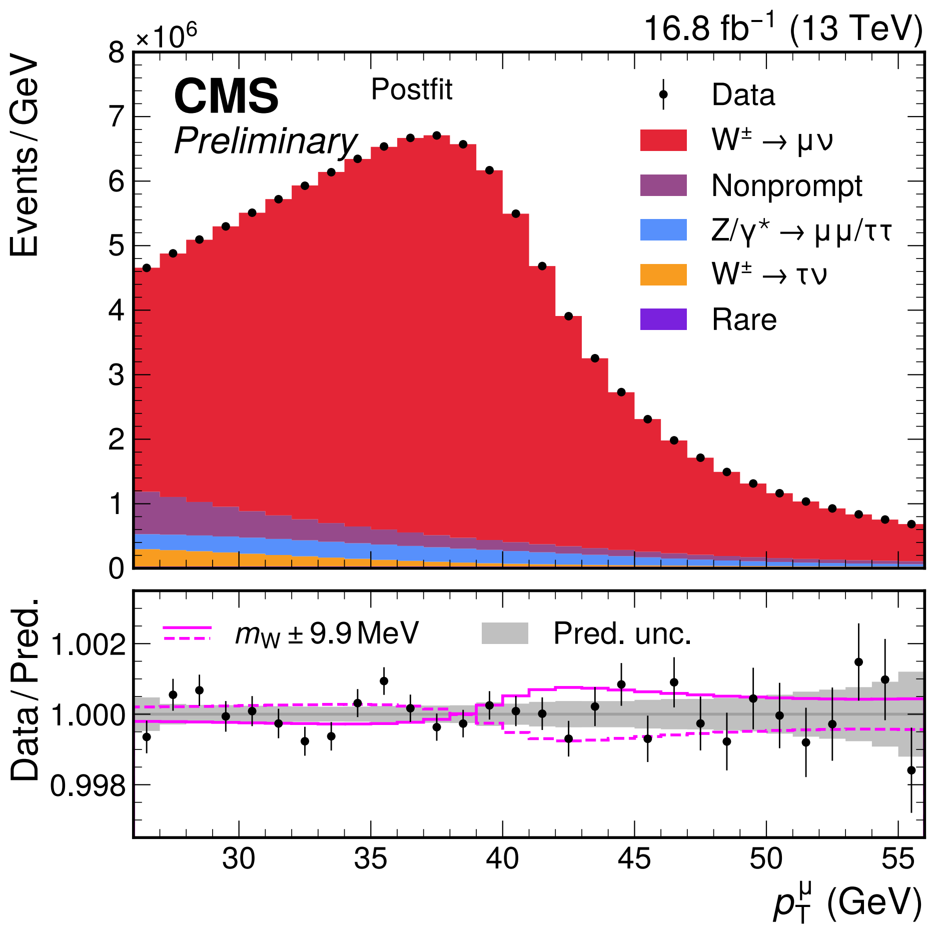
TB, Hager, Jaskiewicz, Neubert, Schwienbacher ‘24

Note: Analysis is a consistency check at 4-loop order, not a factorization proof. Nevertheless remarkable that factorization survives; all elements for the breaking are present at this order.

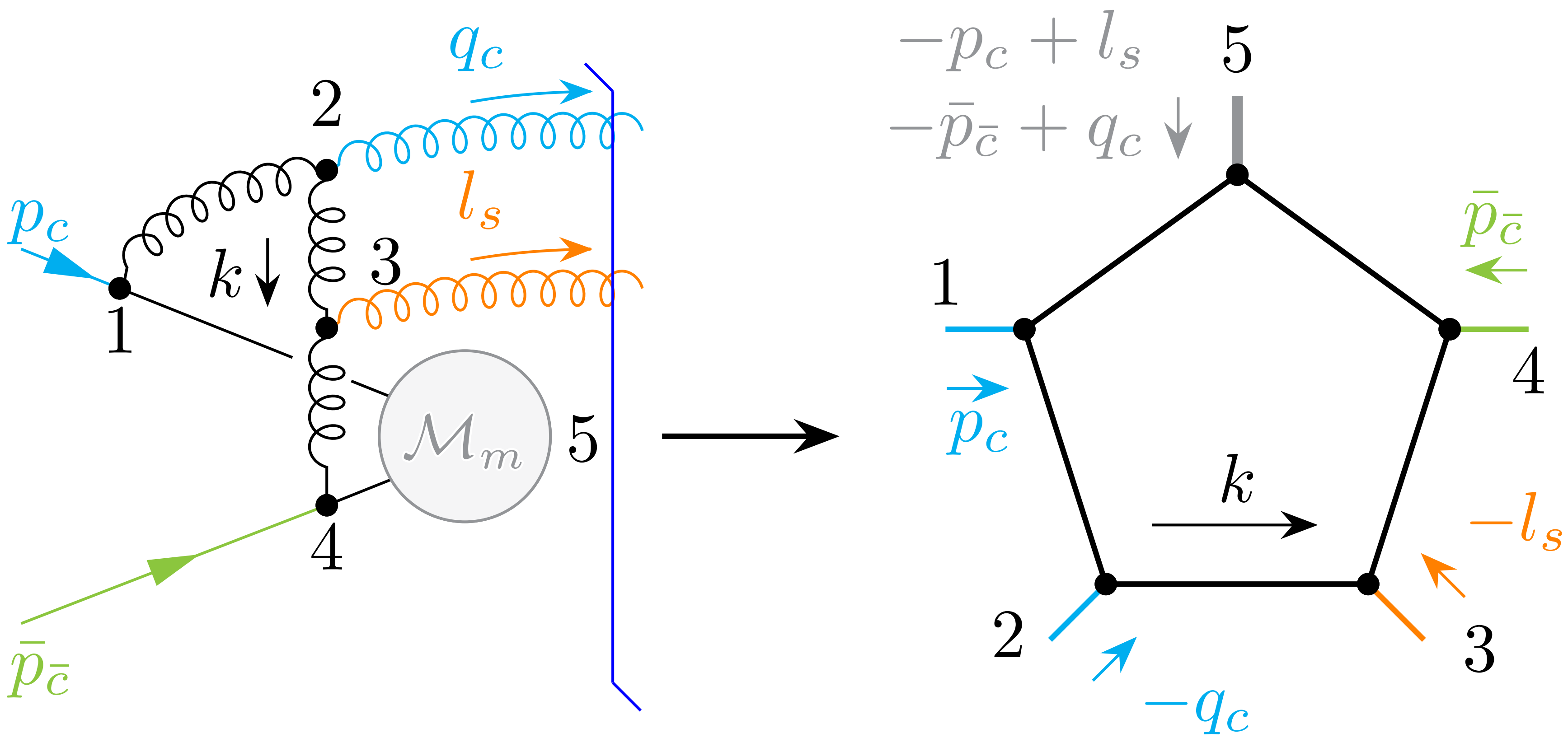
Summary

- High-precision resummations up to N^4LL for q_T spectra, but experiment is far ahead of theory!
- Energy-energy correlators are a promising new class of LHC observables
  - many new ideas and new measurements
- Resummations for logarithms in jet observables are becoming available: CLs, NGLs, SLLs
  - new insights into soft-collinear interplay and factorization
- New partons showers with higher-logarithmic resummations

Extra Slides



Source of uncertainty	Impact (MeV) Nominal	Impact (MeV) Global
Muon momentum scale	4.8	4.4
Muon reco. efficiency	3.0	2.3
W and Z angular coeffs.	3.3	3.0
Higher-order EW	2.0	1.9
p_T^V modeling	2.0	0.8
PDF	4.4	2.8
Nonprompt background	3.2	1.7
Integrated luminosity	0.1	0.1
MC sample size	1.5	3.8
Data sample size	2.4	6.0
Total uncertainty	9.9	9.9



q_c
2
l_s
p_c
k
3
1
$\mathcal{M}_m$
5
4
$\bar{p}_{\bar{c}}$
$-p_c + l_s$
$-\bar{p}_{\bar{c}} + q_c$
5
$\bar{p}_{\bar{c}}$
1
4
p_c
k
$-l_s$
2
3
$-q_c$