Selected

Recent Advances in PDFs From Lattice QCD

Martha Constantinou

Temple University

Disclaimer

Mistakes attributed to Wine Tasting Excursion in Breisach

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Before…

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Before…

After…

OUTLINE

A. Lattice QCD @ LHC

B. Methods to access PDFs (+ GPDs, TMDs) from lattice QCD

C. Selected recent results on:
- Quark PDFs
- Gluon PDFs
- Novel Developments

D. Synergy with phenomenology

E. Concluding remarks

Lattice QCD @ LHC

★ Inputs for interpreting high-energy collisions at LHC for SM and Beyond.

M. Seidel, Thu 9:00 am | M. Muskinja, Thu 9:30 am | J. M. Cruz Martinez, Thu 10:00 am

Predictions of the Higgs boson production cross section in pp collisions

G. Heinrich, Tue 9:30 am

★ Lattice QCD-derived PDFs complement phenomenological fits to experimental data sets. Better constraints on the isovector proton PDF for high-x region at ~10% level will impact predictions for new-physics searches at ATLAS and CMS [2017 PDFLattice Report, Prog. Part. Nucl. Phys. 100, 107 (2018)]

★ Lattice can complement experimental data in kinematic regions where data are sparse or unavailable. Essential for heavy ion collisions, small-x physics, and precision Higgs and top-quark physics

★ Lattice results on spin-dependent PDFs interest for experiments at the LHC and other spin-polarized collision experiments.

★ Strange PDFs have important role in precision EW physics can be coupled with W + c data from LHC (unpolarized PDFs).

T. Hobbs, Tue 2 pm

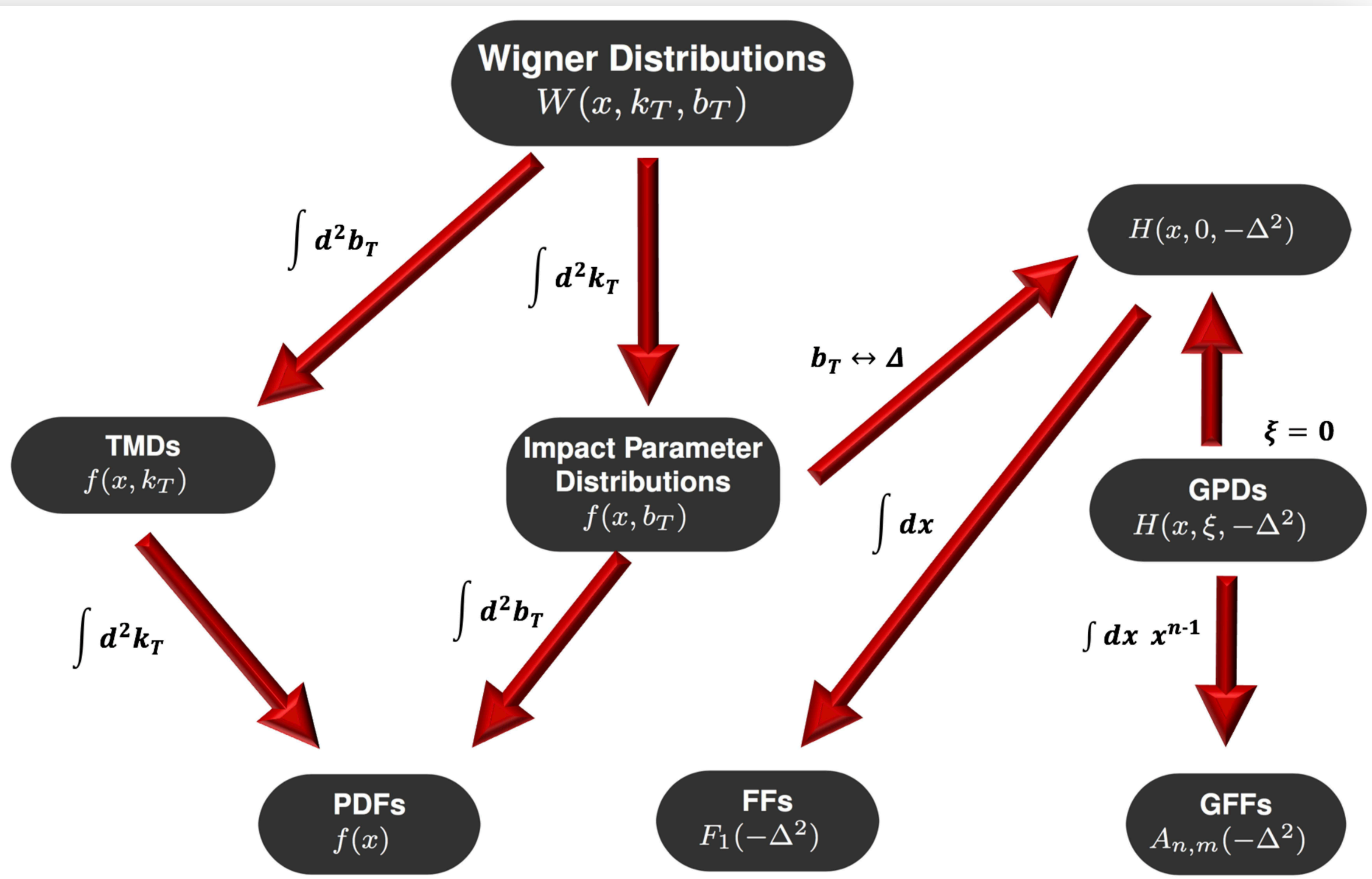
Accessing PDFs/GPDs from lattice QCD

Nucleon Characterization

Wigner distributions

★ Fully characterize partonic structure of hadrons

★ Provide multi-dim images of the parton distributions in phase space



★ Correlations between momenta, positions, spins

★ Information on the hadron's mechanical properties (OAM, pressure, etc.)

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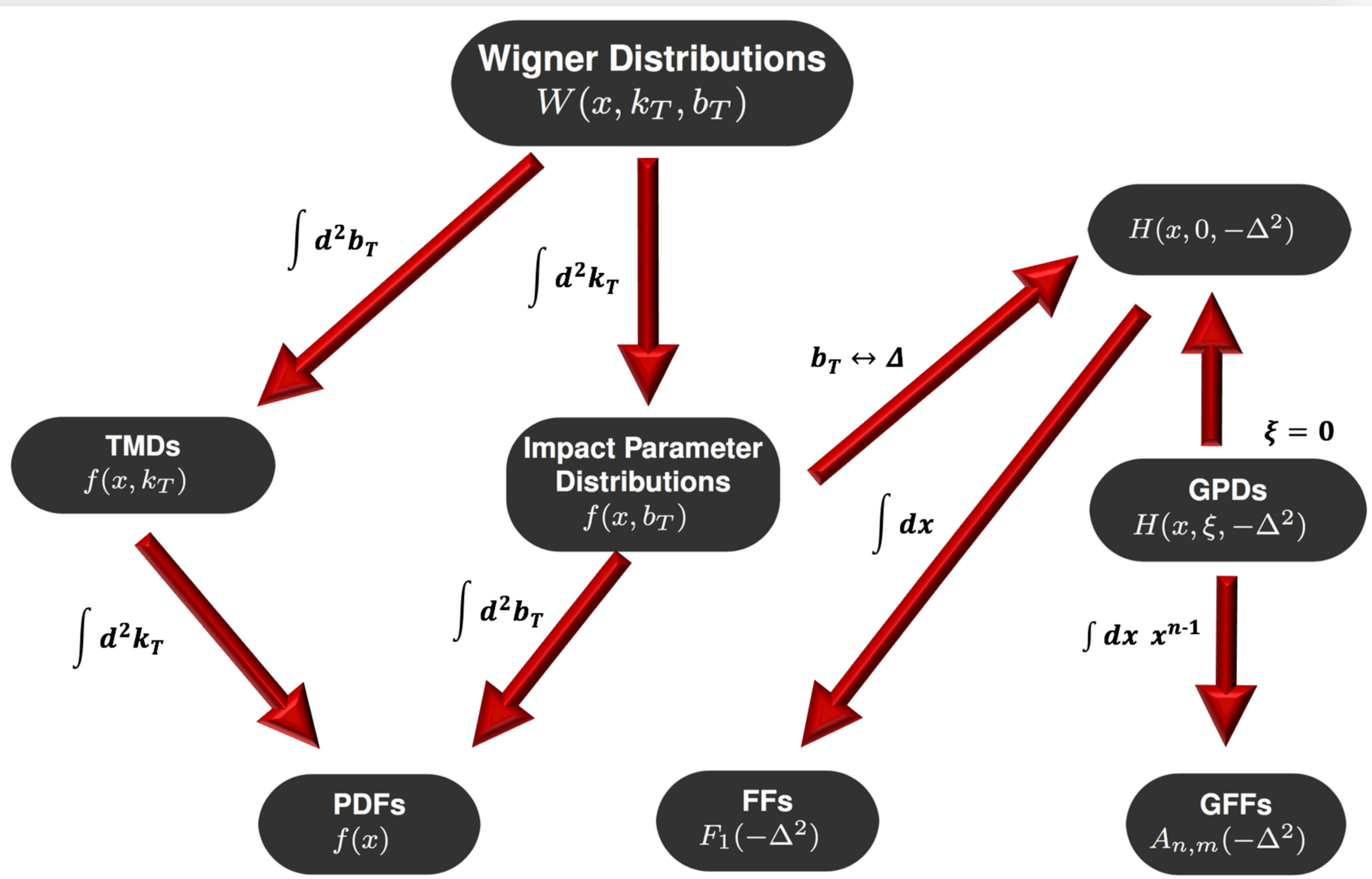
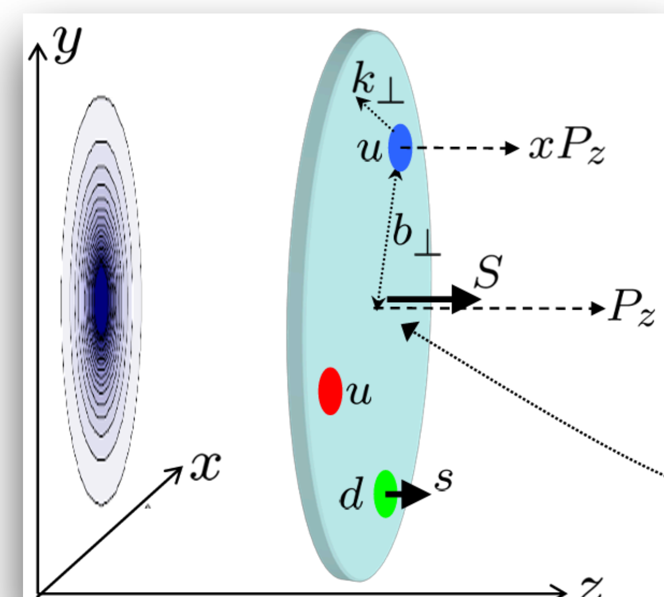


★ Partons contain information on
x: longitudinal momentum fraction
k_T: transverse momentum
$b_\perp$: impact parameter



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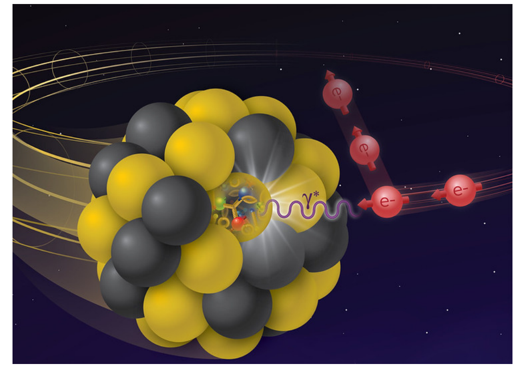
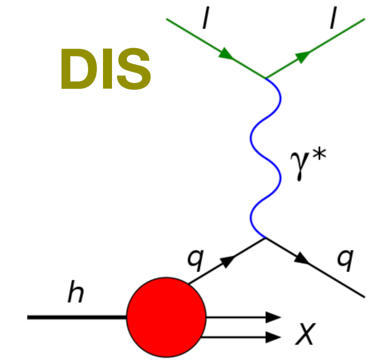
Hadron Structure

★ Structure of hadrons explored in high-energy scattering processes



Collisions @ EIC

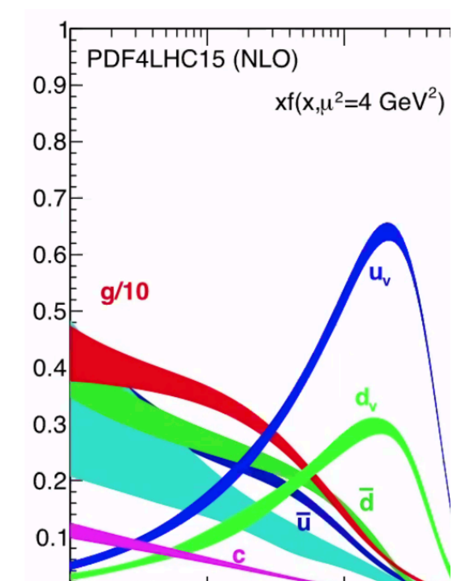


★ Processes cross-section contains information on hadron

$$\sigma_{\rm DIS}(x, Q^2) = \sum_i \left[H^i_{\rm DIS} \otimes f_i\right](x, Q^2)$$

$$[a \otimes b](x) \equiv \int_x^1 \frac{d\xi}{\xi}\, a\left(\frac{x}{\xi}\right)\, b(\xi)$$

★ Hadron structure expressed in terms of distribution functions of partonic constituents (PDFs, GPDs, TMDs)



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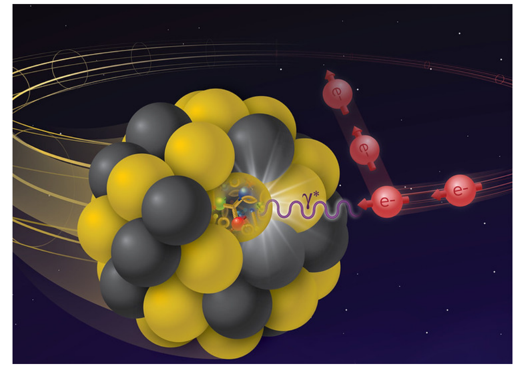

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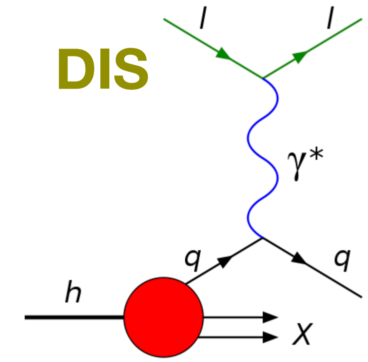

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Perturb. part
(process dependent)

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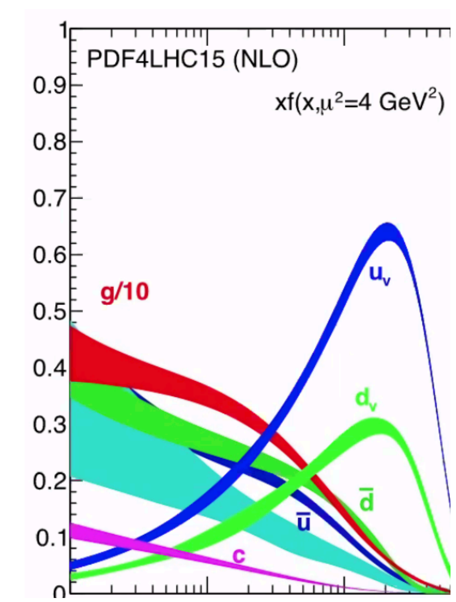

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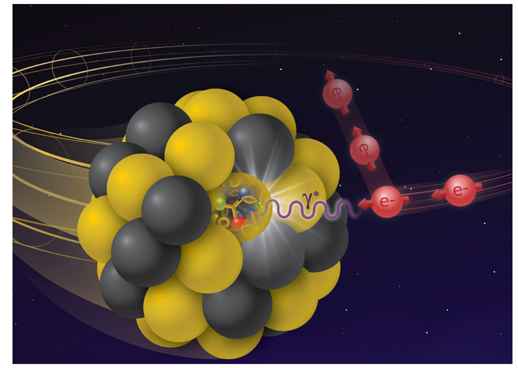
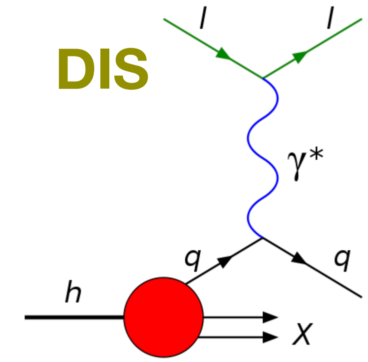


Collisions @ EIC

DIS



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Perturb. part
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Non-Perturb. part
(process “independent”)

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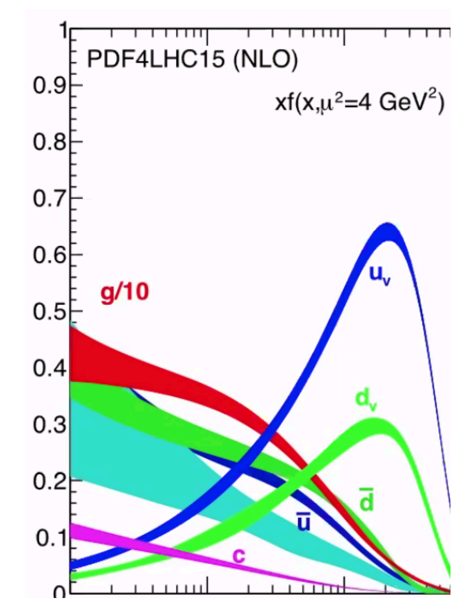

Accessing information on PDFs/GPDs

★ In parton model, physical picture valid for infinite momentum frame

[R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969)]

★ PDFs parameterized via matrix elements of nonlocal light-cone operators

$$f(x) = \frac{1}{4\pi} \int dy^- e^{-ixP^+ y^-} \langle P, S | \bar{\psi}_f \gamma^+ \mathcal{W} \psi_f | P, S \rangle$$

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★ **Mellin moments**
(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\, \gamma^\sigma W[-\tfrac{1}{2}z, \tfrac{1}{2}z]\, q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty} \frac{1}{n!}\, z_{\alpha_1} \ldots z_{\alpha_n} \underbrace{\left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1} \ldots \overleftrightarrow{D}^{\alpha_n} q\right]}_{\text{local operators}}$$

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1} \left\{ \gamma^{\{\mu}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}} A_{n,i}(t) - i\frac{\Delta_\alpha \sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}} B_{n,i}(t) \right\} + \frac{\Delta^\mu\Delta^{\mu_1}\ldots\Delta^{\mu_{n-1}}}{m_N} C_{n,0}(\Delta^2)\Big|_{n\ \text{even}} \Big) \Big\} + \frac{\Delta^\mu\Delta^{\mu_1}\ldots\Delta^{\mu_{n-1}}}{m_N} C_{n,0}(\Delta^2)\Big|_{n\ \text{even}} \Big] U(P)$$

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Reconstruction of PDFs/GPDs very challenging

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(quasi-GPDs, pseudo-GPDs, …)

$$\langle N(P_f) \,|\, \underbrace{\bar{\Psi}(z)\, \Gamma\, \mathcal{W}(z,0) \Psi(0)}_{\text{Nonlocal operator with Wilson line}} \,|\, N(P_i) \rangle_\mu$$

$$\langle N(P') | O_V^\mu(x) | N(P) \rangle = \overline{U}(P') \left\{ \gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N} E(x,\xi,t) \right\} U(P) + \text{ht}\,,$$

$$\langle N(P') | O_A^\mu(x) | N(P) \rangle = \overline{U}(P') \left\{ \gamma^\mu \gamma_5 \widetilde{H}(x,\xi,t) + \frac{\gamma_5 \Delta^\mu}{2m_N} \widetilde{E}(x,\xi,t) \right\} U(P) + \text{ht}\,,$$

$$\langle N(P') | O_T^{\mu\nu}(x) | N(P) \rangle = \overline{U}(P') \left\{ i\sigma^{\mu\nu} H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N} E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2} \widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N} \widetilde{E}_T(x,\xi,t) \right\} U(P) + \text{ht}$$

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This talk

Novel Approaches

★ Hadronic tensor	[K.F. Liu, S.J. Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]
Auxiliary scalar quark	[U. Aglietti et al., Phys. Lett. B441, 371 (1998), arXiv:hep-ph/9806277]
Fictitious heavy quark	[W. Detmold, C. J. D, Lin, Phys. Rev. D73, 014501 (2006)]
Auxiliary scalar quark	[V. Braun & D. Mueller, Eur. Phys. J. C55, 349 (2008), arXiv:0709.1348]
Higher moments	[Z. Davoudi, M. Savage, Phys. Rev. D86, 054505 (2012)]
Quasi-distributions (LaMET)	[X. Ji, PRL 110 (2013) 262002, arXiv:1305.1539; Sci. China PPMA. 57, 1407 (2014)]
Compton amplitude and OPE	[A. Chambers et al. (QCDSF), PRL 118, 242001 (2017), arXiv:1703.01153]
Pseudo-distributions	[A. Radyushkin, Phys. Rev. D 96, 034025 (2017), arXiv:1705.01488]
Good lattice cross sections	[Y-Q Ma & J. Qiu, Phys. Rev. Lett. 120, 022003 (2018), arXiv:1709.03018]
PDFs without Wilson line	[Y. Zhao Phys.Rev.D 109 (2024) 9, 094506, arXiv:2306.14960]
Moments of PDFs of any order	[A. Shindler, Phys.Rev.D 110 (2024) 5, L051503, arXiv:2311.18704]

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★ Reviews of methods and applications

- ***A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results***
  K. Cichy & M. Constantinou (invited review) Advances in HEP 2019, 3036904, arXiv:1811.07248

- ***Large Momentum Effective Theory***
  X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang, and Y. Zhao (2020), 2004.03543

- ***The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD***
  M. Constantinou (invited review) Eur. Phys. J. A 57 (2021) 2, 77, arXiv:2010.02445

Novel Approaches

quasi-PDFs

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pseudo-PDFs

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x-dependent distribution functions

current-current correlators

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Other methods

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Novel Approaches

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Same raw lattice data

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Well-studied "novel" methods for PDFs/GPDs in LQCD

Matrix elements of non-local operators (space-like separated fields) with boosted hadrons

$$\mathscr{M}(P_f, P_i, z) = \langle N(P_f) \,|\, \bar{\Psi}(z)\, \Gamma\, \mathscr{W}(z,0) \Psi(0) \,|\, N(P_i) \rangle_\mu$$

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quasi-PDFs

pseudo-ITD

[A. Radyushkin, PRD 96, 034025 (2017)]

$$\tilde{q}_\Gamma^{\mathrm{GPD}}(x, t, \xi, P_3, \mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z}\,\mathcal{M}(P_f, P_i, z)$$

$$\mathfrak{M}(\nu, \xi, t; z_3^2) \equiv \frac{\mathcal{M}(\nu, \xi, t; z_3^2)}{\mathcal{M}(0,0,0; z^2)} \qquad (\nu = z \cdot p)$$

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Matching in momentum space
(Large Momentum
Effective Theory)

pseudo-ITD [A. Radyushkin, PRD 96, 034025 (2017)]

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Matching in ν space

$$Q(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{i\nu x} q(x, \mu^2)$$

Light-cone PDFs & GPDs

Well-studied "novel" methods for PDFs/GPDs in LQCD

Matrix elements of non-local operators (space-like separated fields) with boosted hadrons

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Effective Theory)

Matching in ν space

$$Q(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{i\nu x} q(x, \mu^2)$$

Light-cone PDFs & GPDs

Calculation very taxing!
- length of the Wilson line (z)
- nucleon momentum boost (P_3)
- momentum transfer (t)
- skewness (ξ)

Well-studied "novel" methods for PDFs/GPDs in LQCD

Matrix elements of non-local operators (space-like separated fields) with boosted hadrons

$$\mathcal{M}(P_f, P_i, z) = \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathcal{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]
[X. Ji, Sci. China Phys. M.A. 57 (2014) 1407]

quasi-PDFs

$$\tilde{q}^{\rm GPD}_\Gamma(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z}\,\mathcal{M}(P_f,P_i,z)$$

Matching in momentum space
(Large Momentum
Effective Theory)

pseudo-ITD [A. Radyushkin, PRD 96, 034025 (2017)]

$$\mathfrak{M}(\nu,\xi,t;z_3^2) \equiv \frac{\mathcal{M}(\nu,\xi,t;z_3^2)}{\mathcal{M}(0,0,0;z^2)} \qquad (\nu = z\cdot p)$$

Matching in ν space

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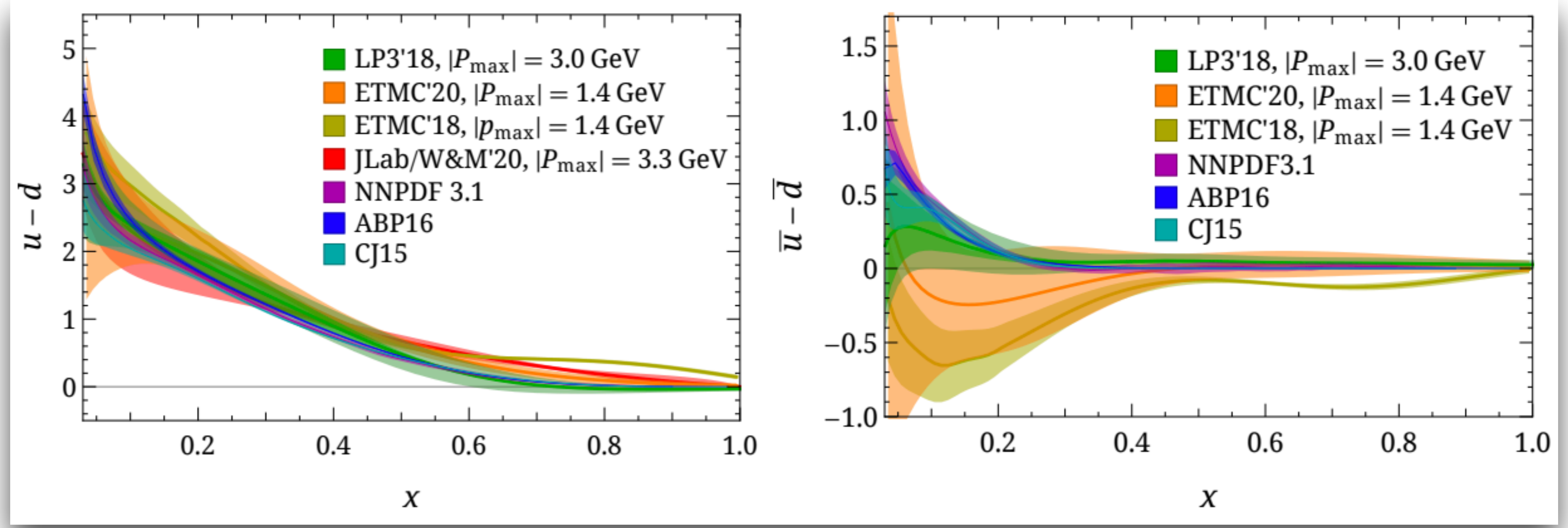
$\left.\begin{array}{l}\text{- length of the Wilson line (} z \text{)}\\ \text{- nucleon momentum boost (} P_3 \text{)}\end{array}\right\}$ PDFs, GPDs

$\left.\begin{array}{l}\text{- momentum transfer (} t \text{)}\\ \text{- skewness (} \xi \text{)}\end{array}\right\}$ GPDs

Quark PDFs:

The unpolarized case

Collection of results



[M. Constantinou et al. (2020 PDFLattice Report), Prog.Part.Nucl.Phys. 121 (2021) 103908]

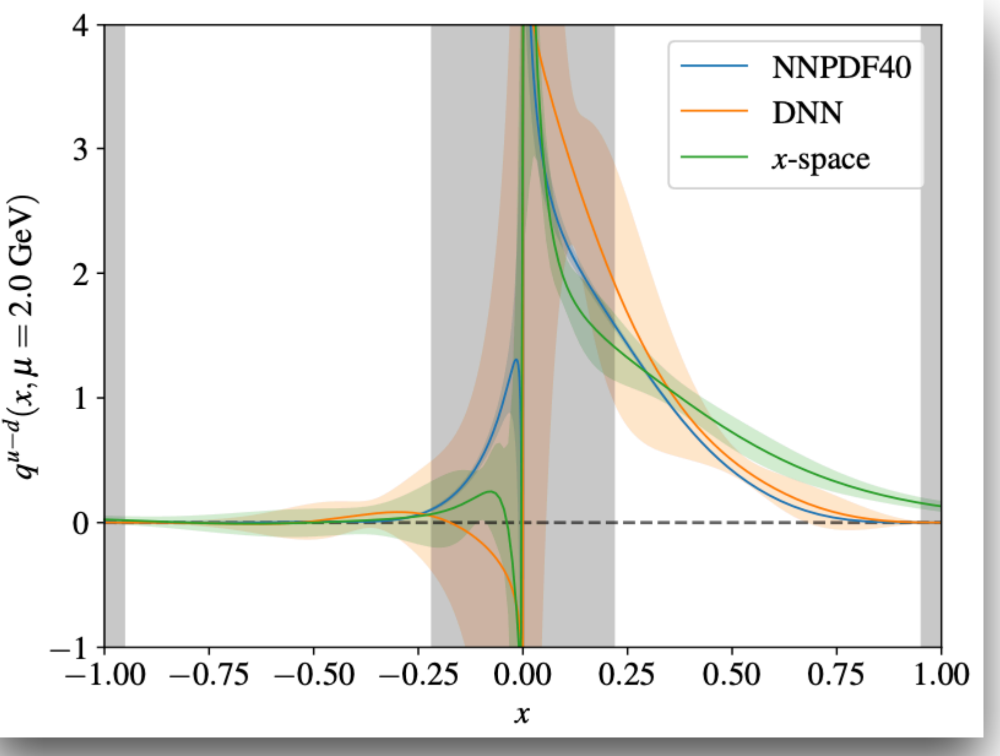
★ Several improvements:

- More calculations at physical quark masses
- Ensembles at various lattice spacings
- Addressing systematic uncertainties due to methodologies

Refining the unpolarized proton PDF (u-d)

★ Physical quark masses

- HISQ, a=0.076 fm, P~1.5 GeV
- Deep Neural Network for inverse problem
- NNLO for matching



[X. Gao et al., PRD 107 (2023) 7, 074509]

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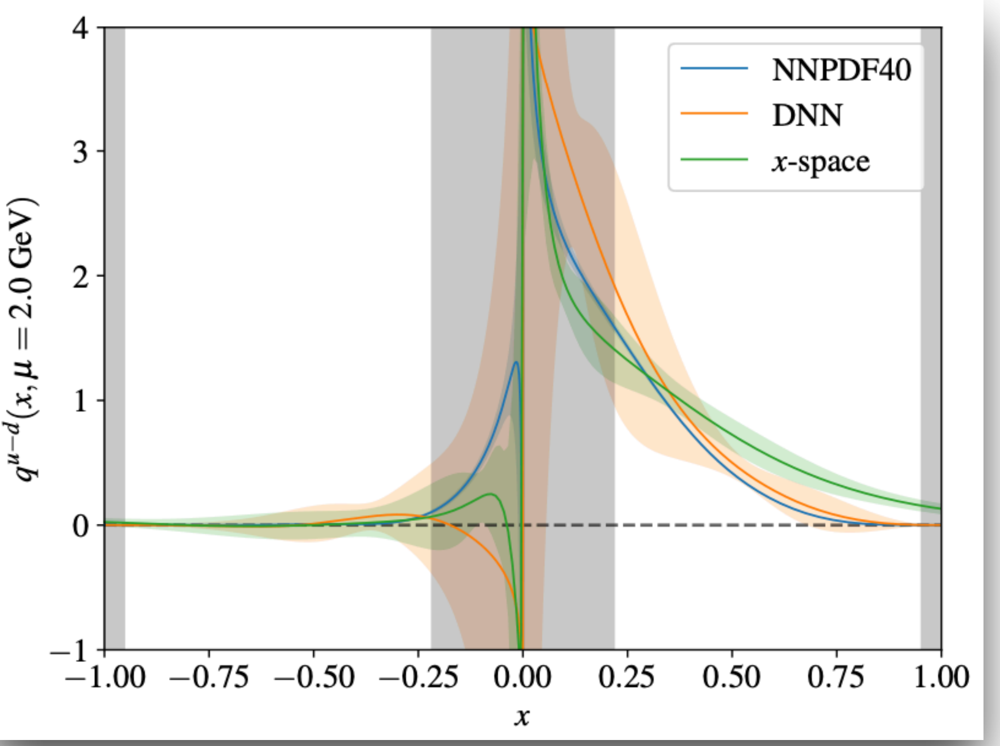
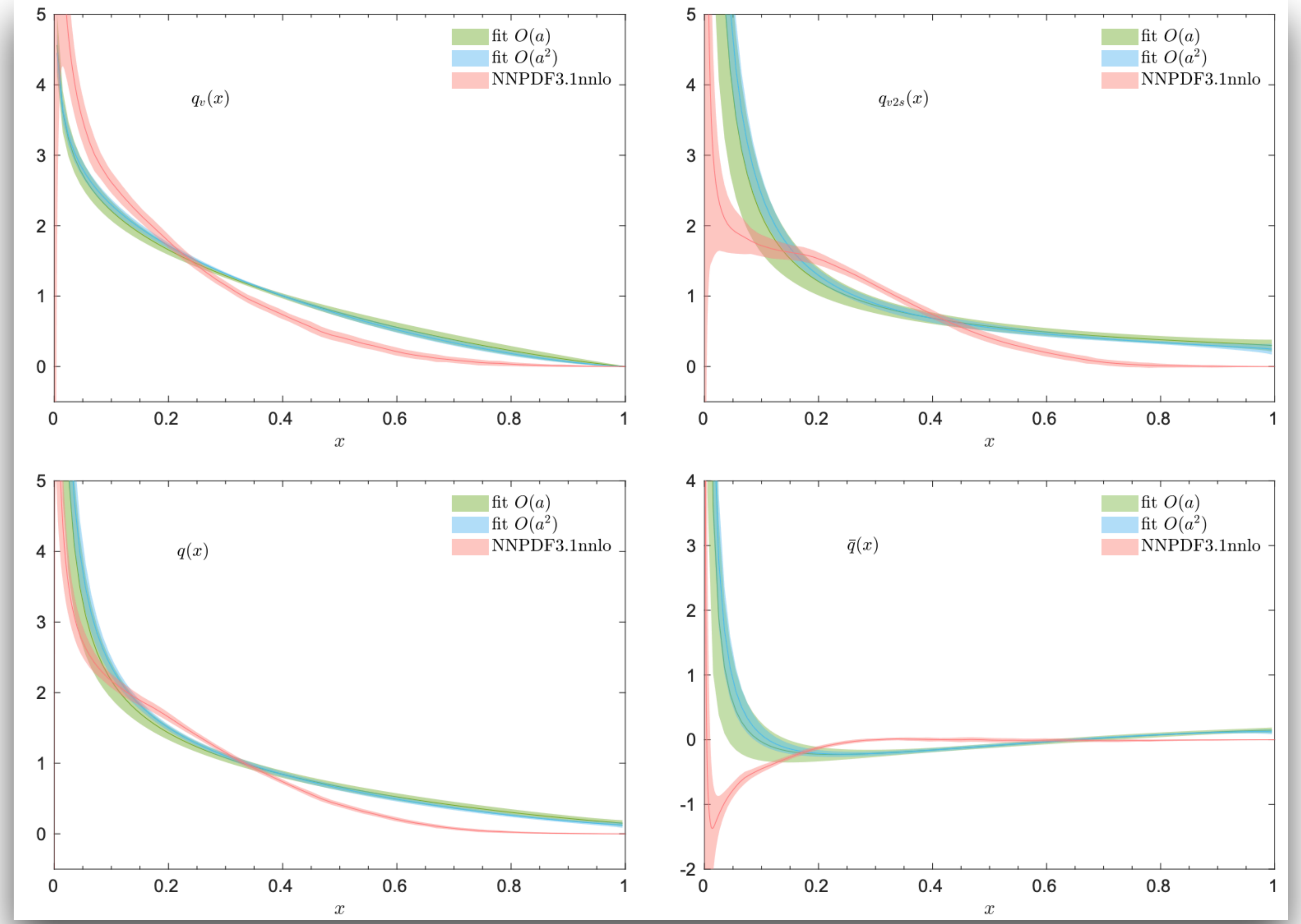


[X. Gao et al., PRD 107 (2023) 7, 074509]

★ **Continuum limit**

- TM&clover, a=0.09 fm, m_π=350 MeV
- P~1.8 GeV
- NNLO for matching

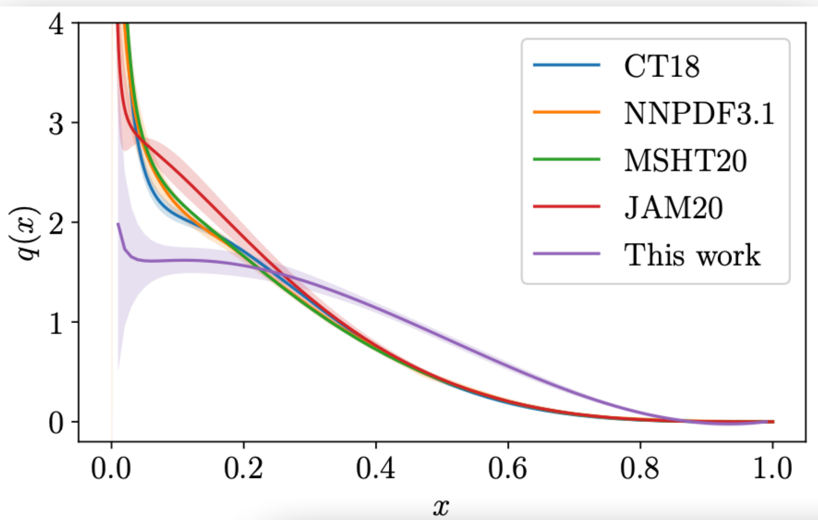


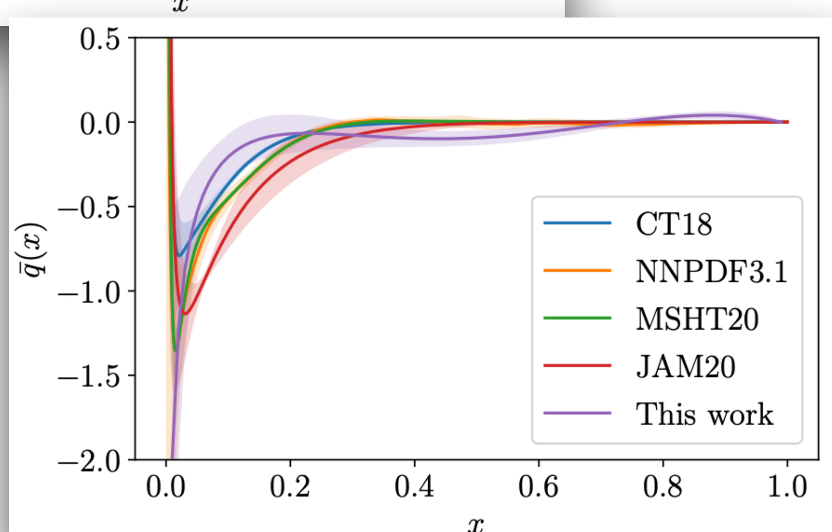
[Bhat et al., PRD 106 (2022) 5, 054504]

Improving evolution of PDFs

★ Continuum limit - higher twist effects

- Clover, a=0.075, 0.065, 0.048 fm
- m_π=440 MeV
- Jacobi polynomials for controlling finite-a & higher twist





[Karpie et al., JHEP 11 (2021) 024]

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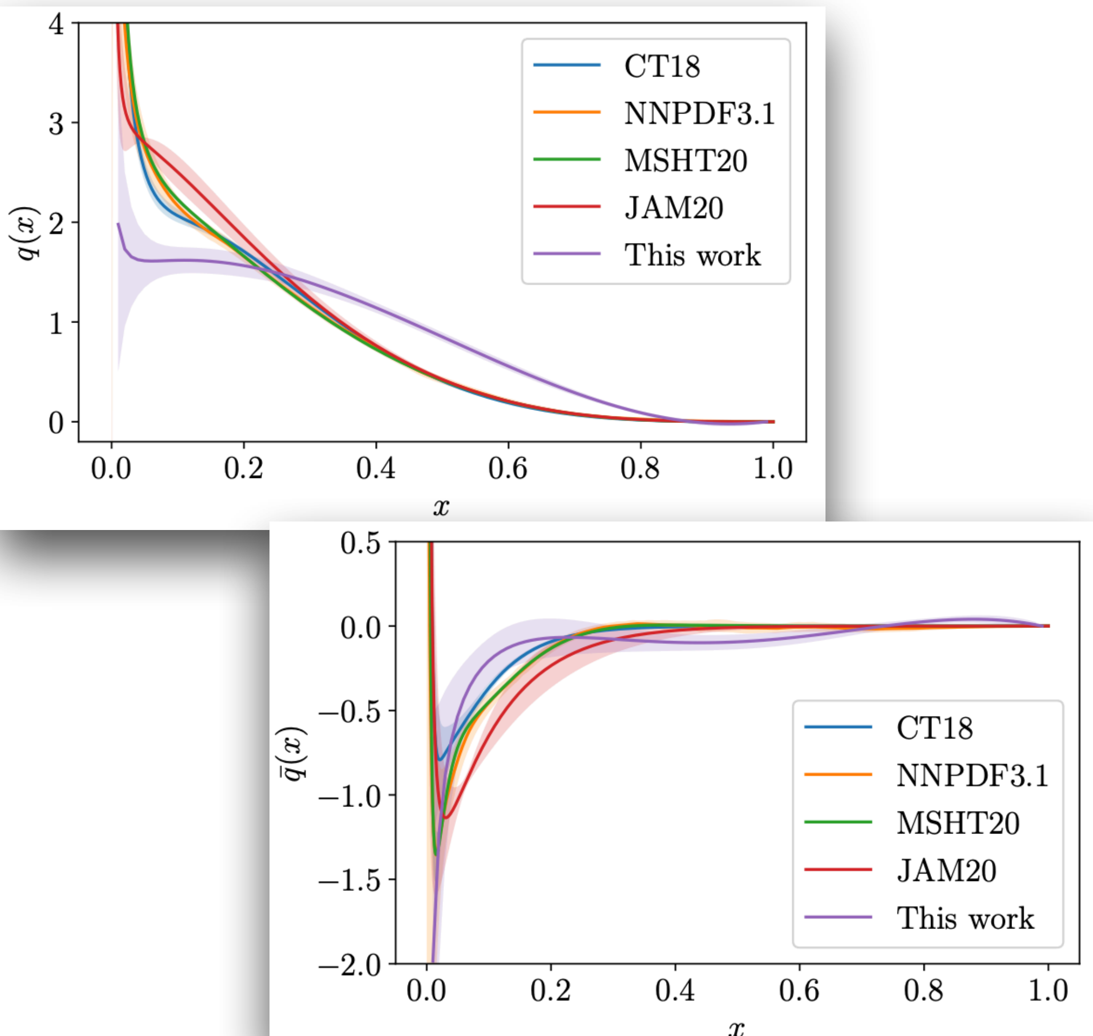
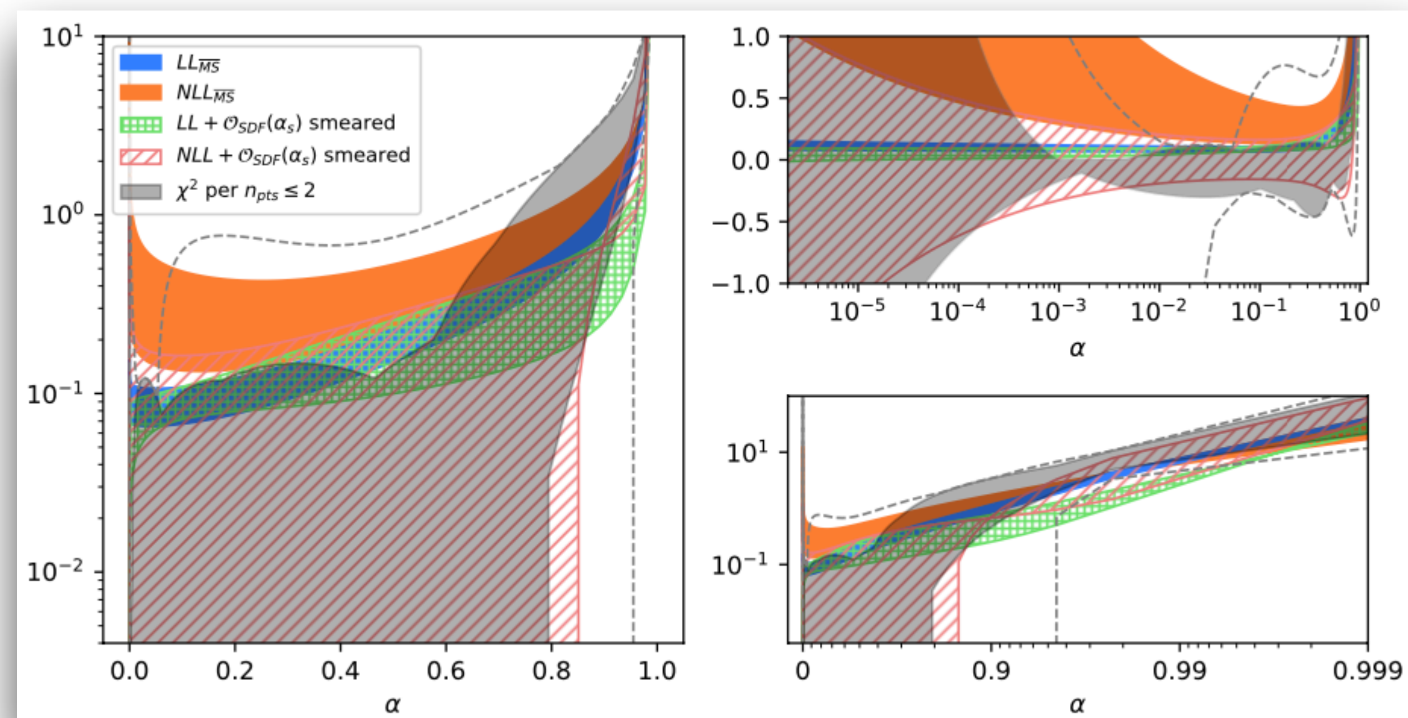


[Karpie et al., JHEP 11 (2021) 024]

★ Non-perturbative scale evolution of pseudo distributions:

- lattice scale much different than scale for light-cone PDFs
- addresses the subtle z^2 behavior of matrix elements
- reduces fluctuation of lattice data



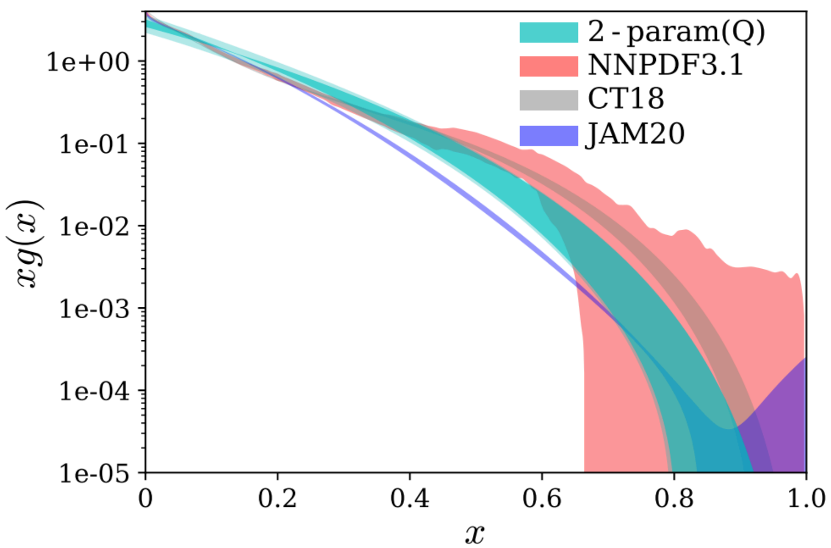
[H. Dutrieux et al. (HadStruc), JHEP 04 (2024) 061]

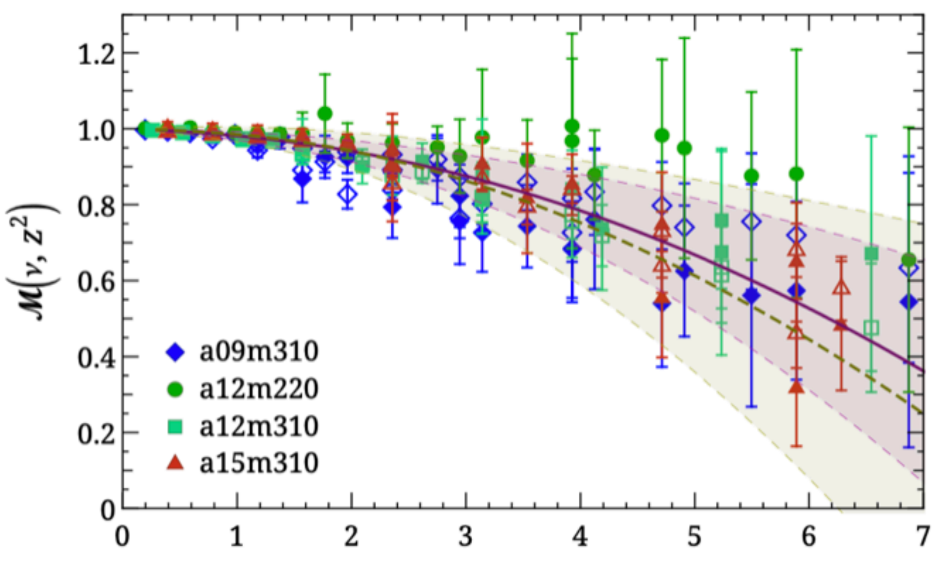
Evolution of vector operator much larger than anticipated

Gluon PDFs for the proton

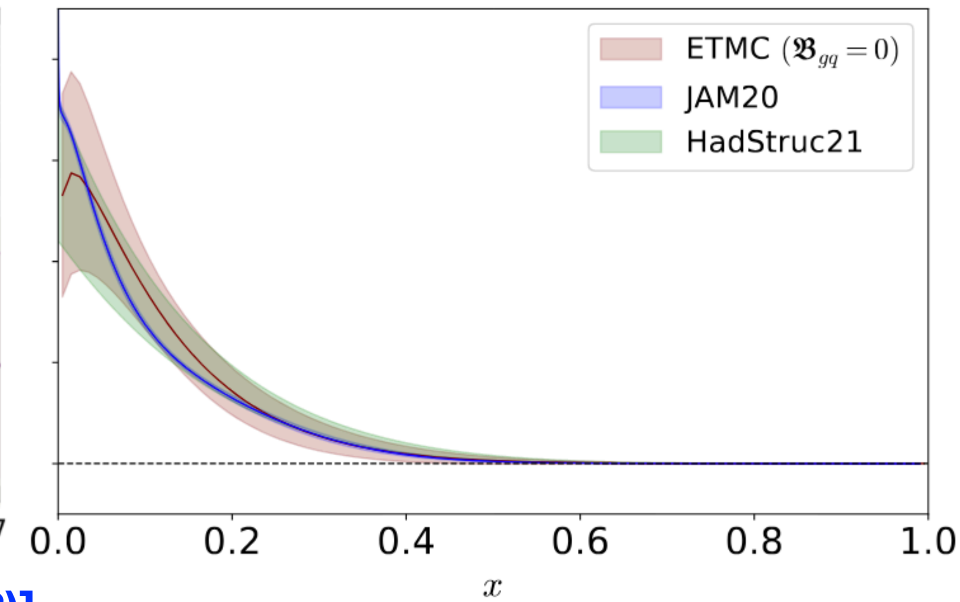
Gluon PDF

[Khan et al., JHEP 11, 148 (2021)]





[Delmar et al., PRD 108 (2023) 9, 094515]



[Fan et al., Phys. Rev. D 108, 014508 (2023)]

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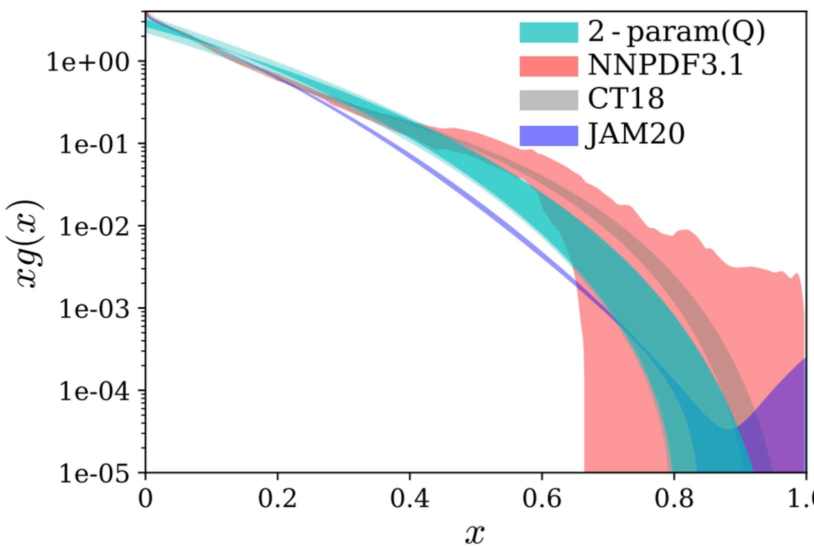

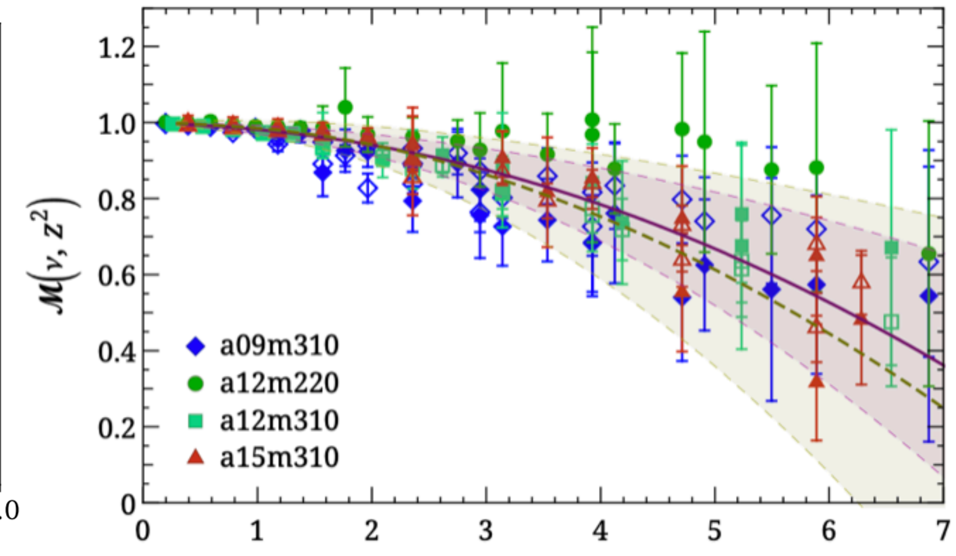

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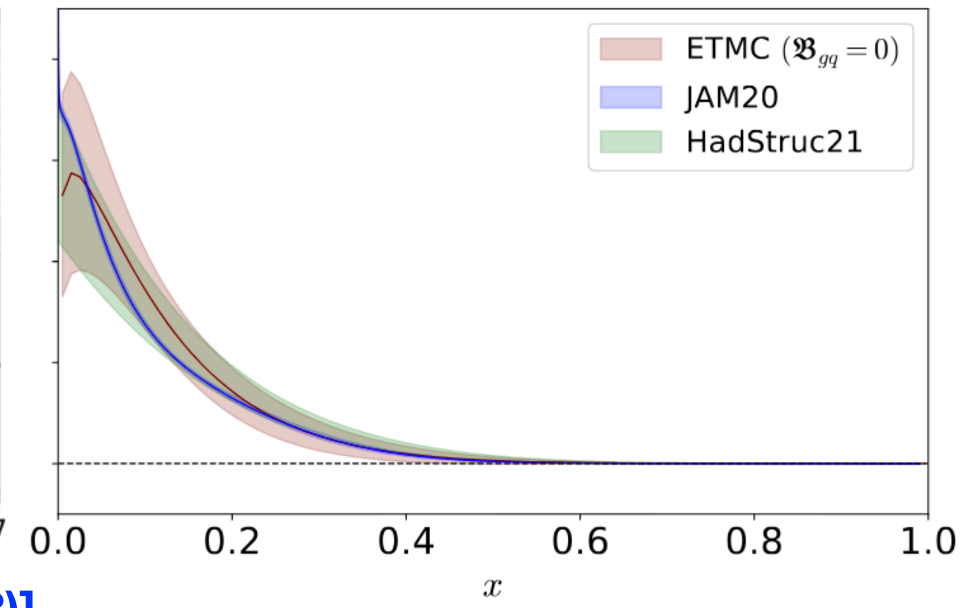
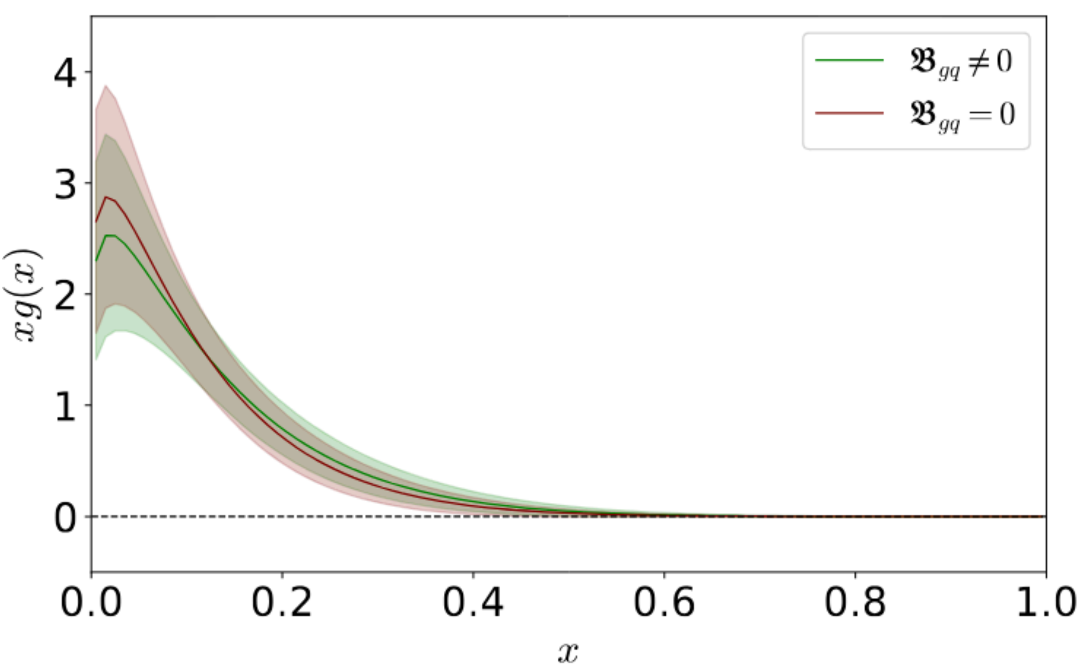
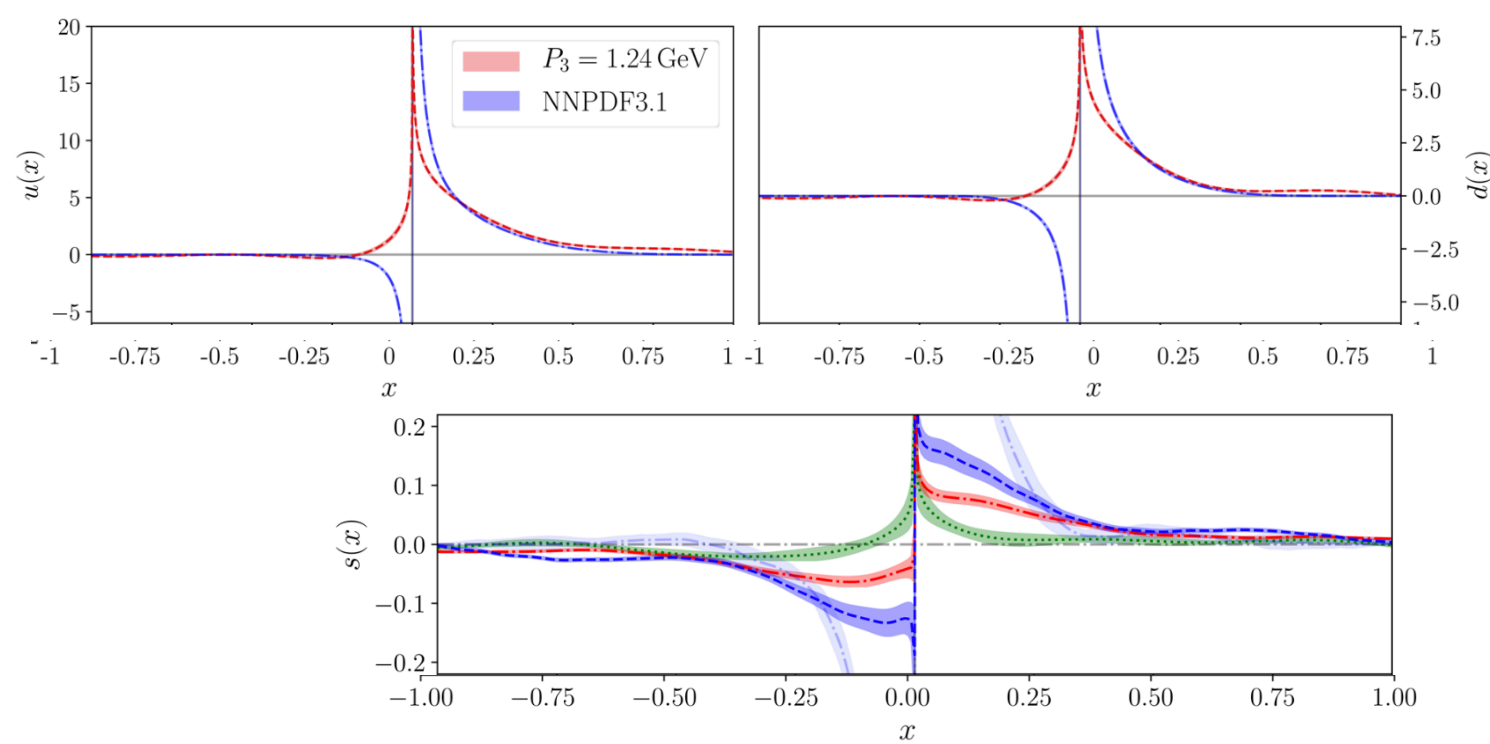


★ Elimination of Mixing with Quark Singlet PDFs

[J. Delmar et al., PRD 108 (2023) 9, 094515]



Dedicated calculation of quark-single PDFs

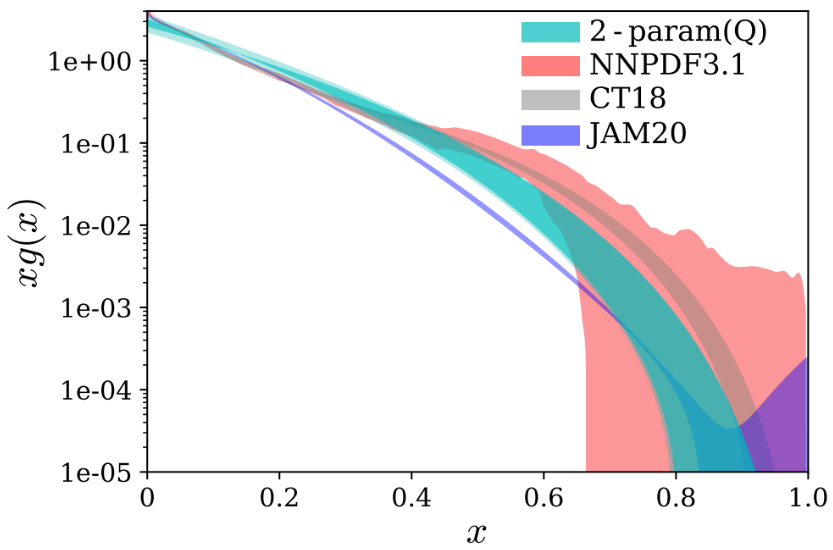


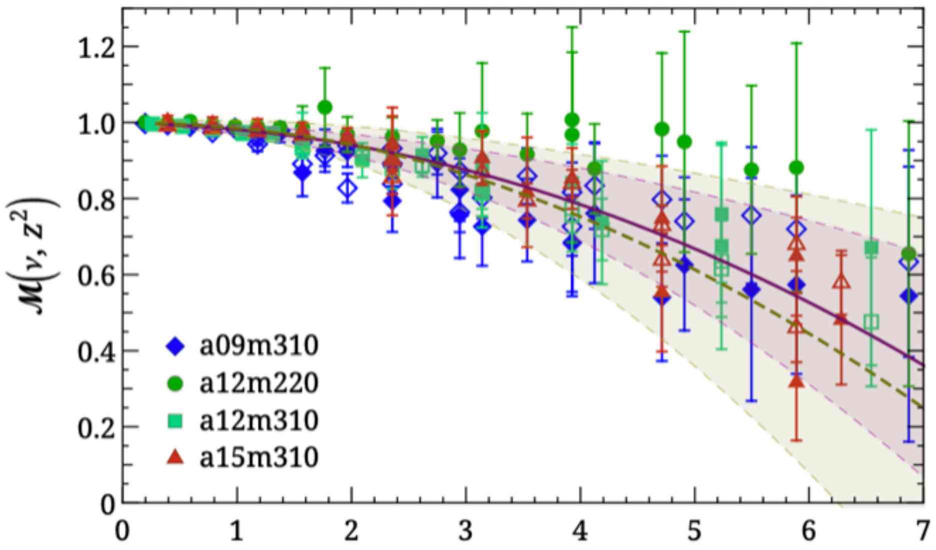
[C. Alexandrou, PRD 104 (2021) 5, 054503]

Lattice QCD can provide key information

Gluon PDF

[Khan et al., JHEP 11, 148 (2021)]





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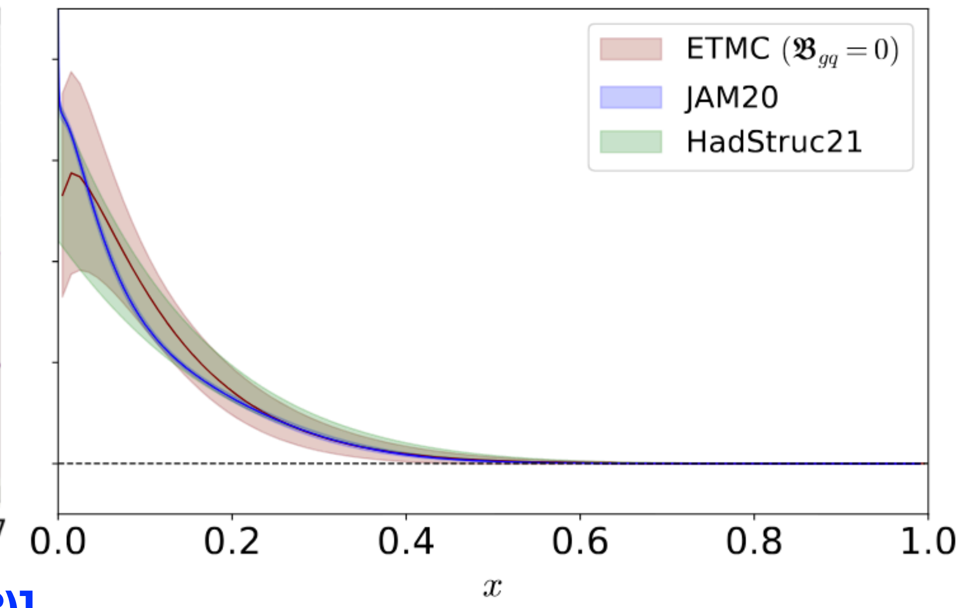

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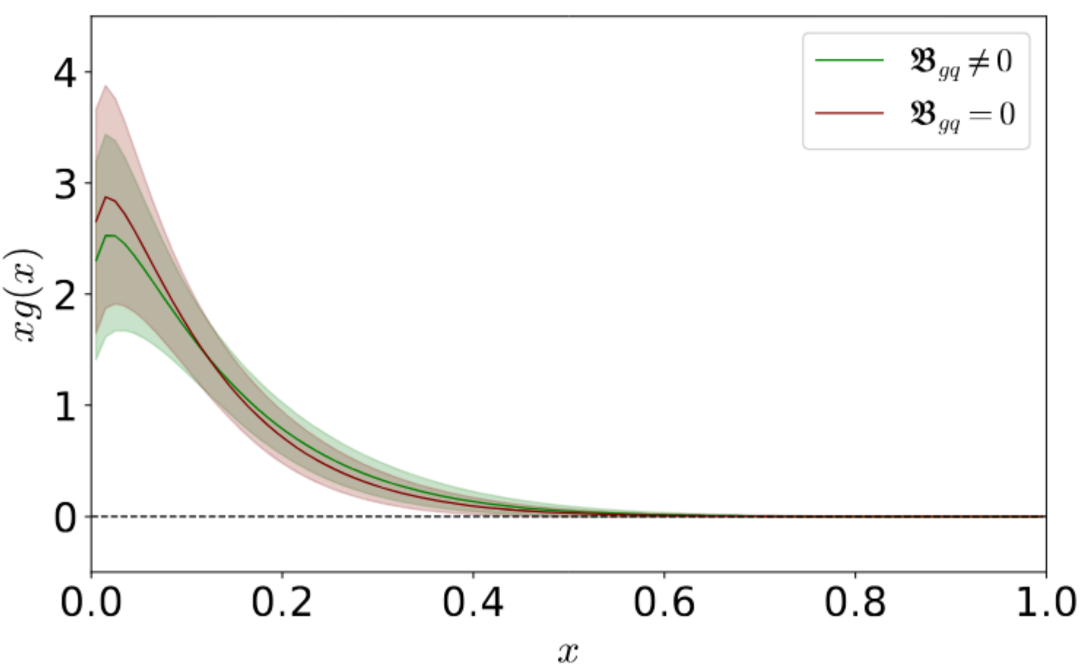

Mixing

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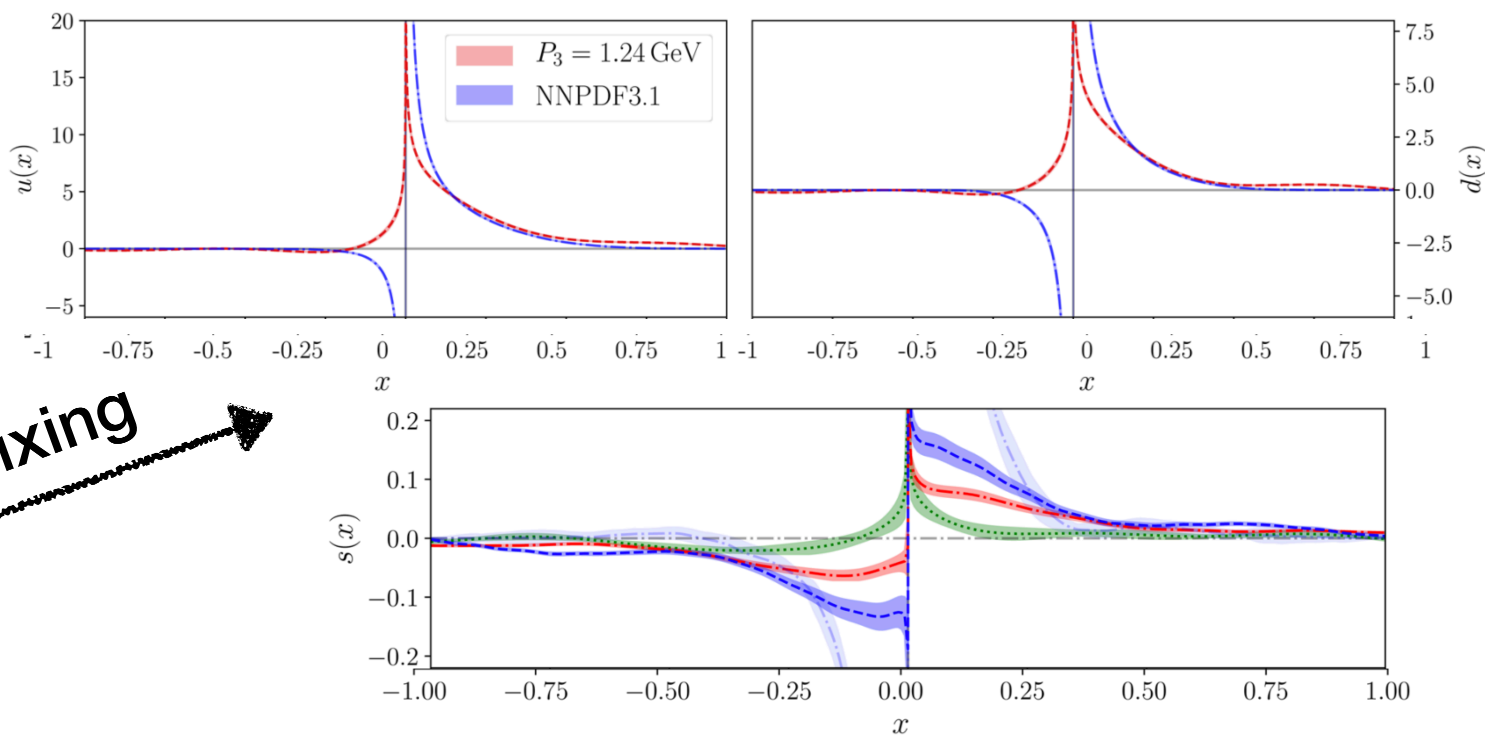

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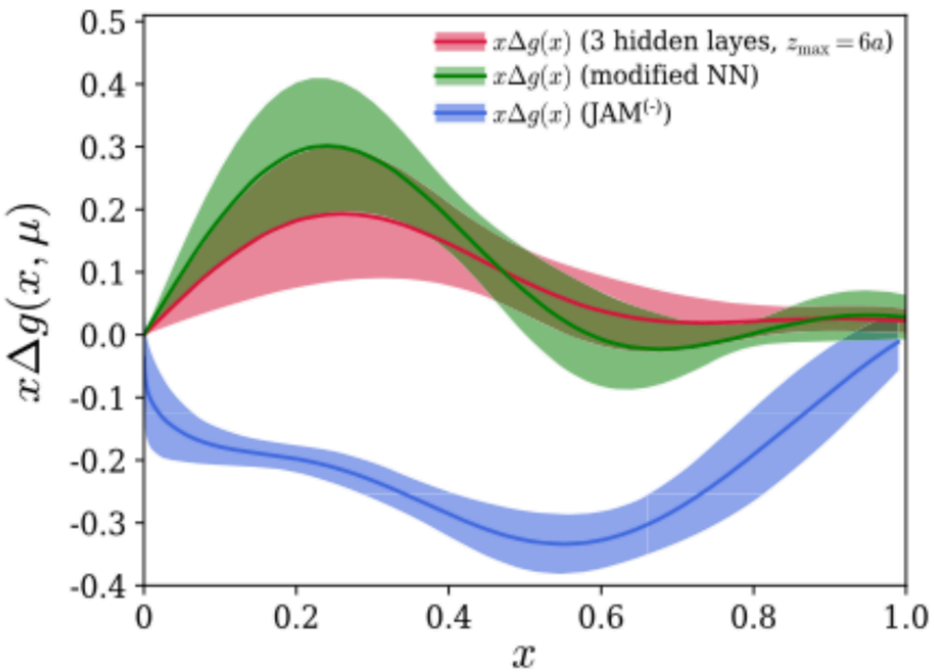
Lattice QCD can provide key information

Gluon Helicity PDF

- Neural network analysis of lattice calculation disfavors negative gluon polarizability



[T. Khan et al., PRD 108, 074502]

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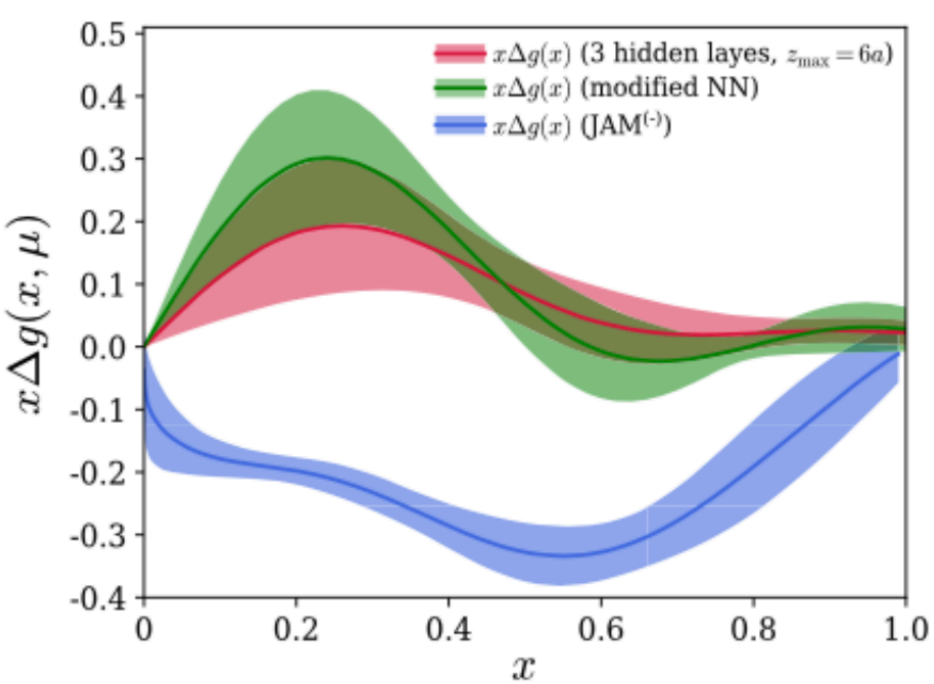
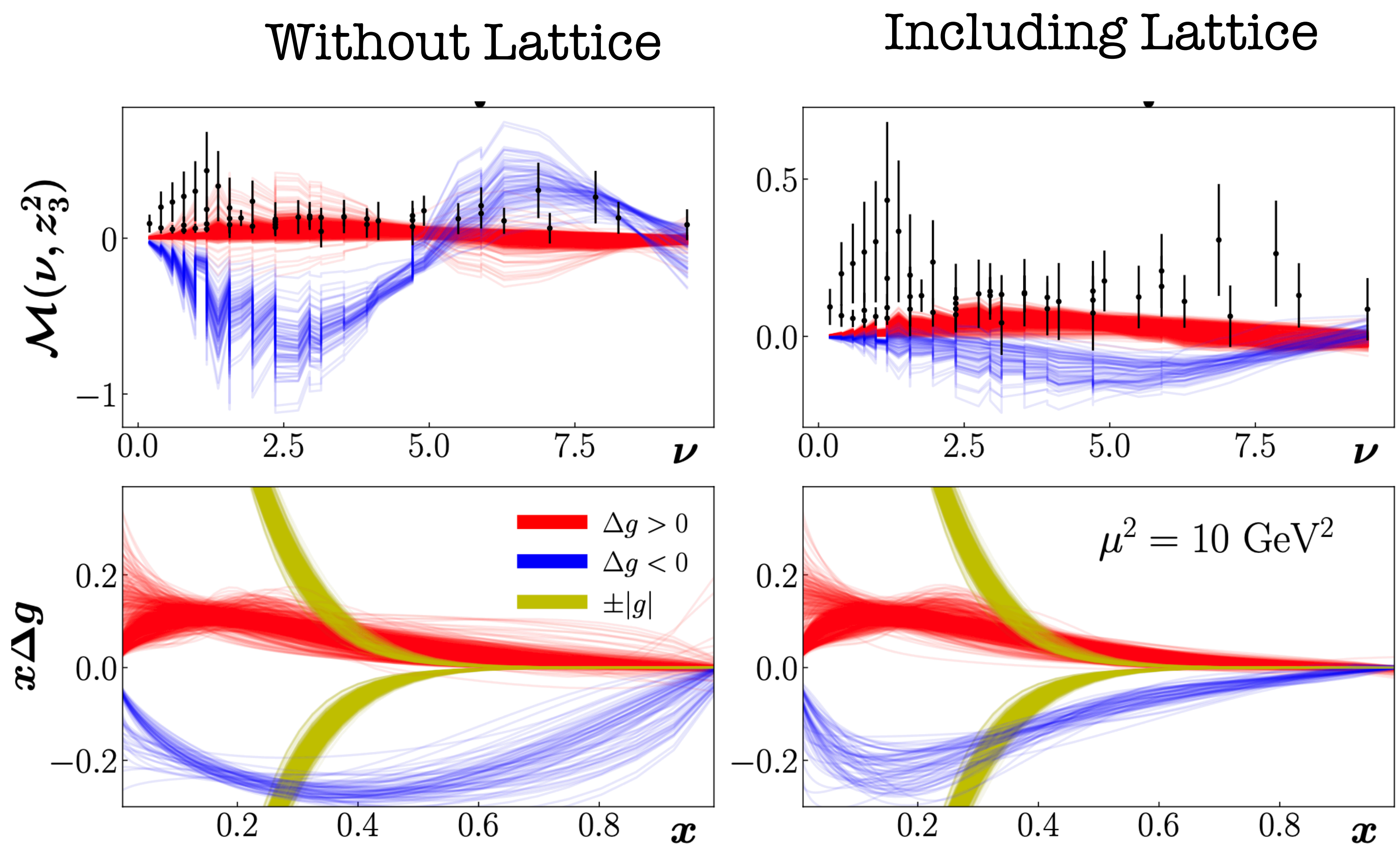


[T. Khan et al., PRD 108, 074502]

[J. Karpie et al., PRD 109 (2024) 3, 036031]



LQCD: Hint for a nonzero gluon spin (proton)
JAM analysis: No positivity constraint
(Δg > |g| for some regions of x)

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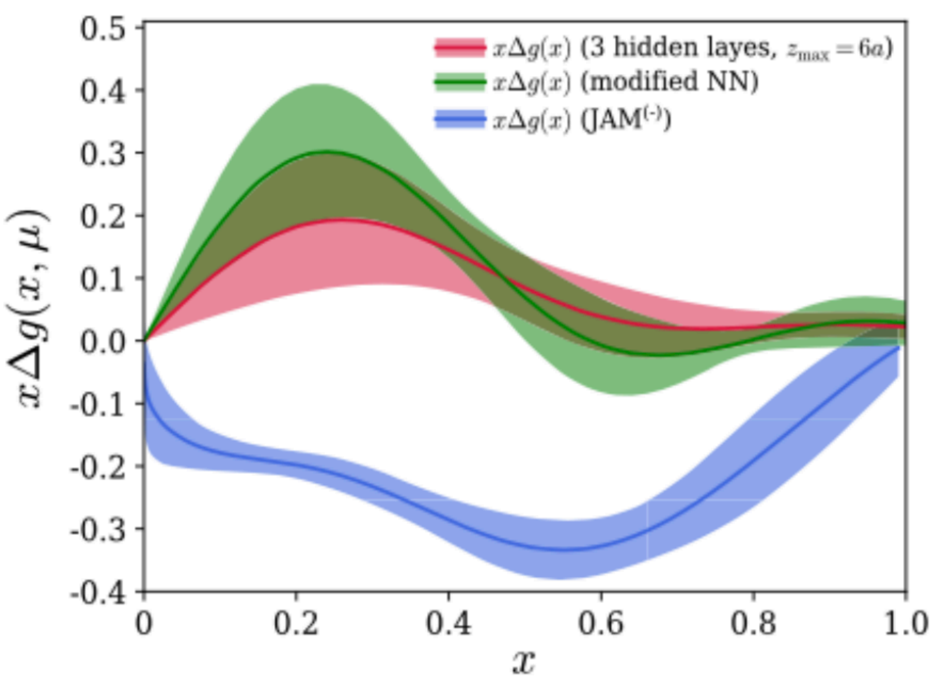

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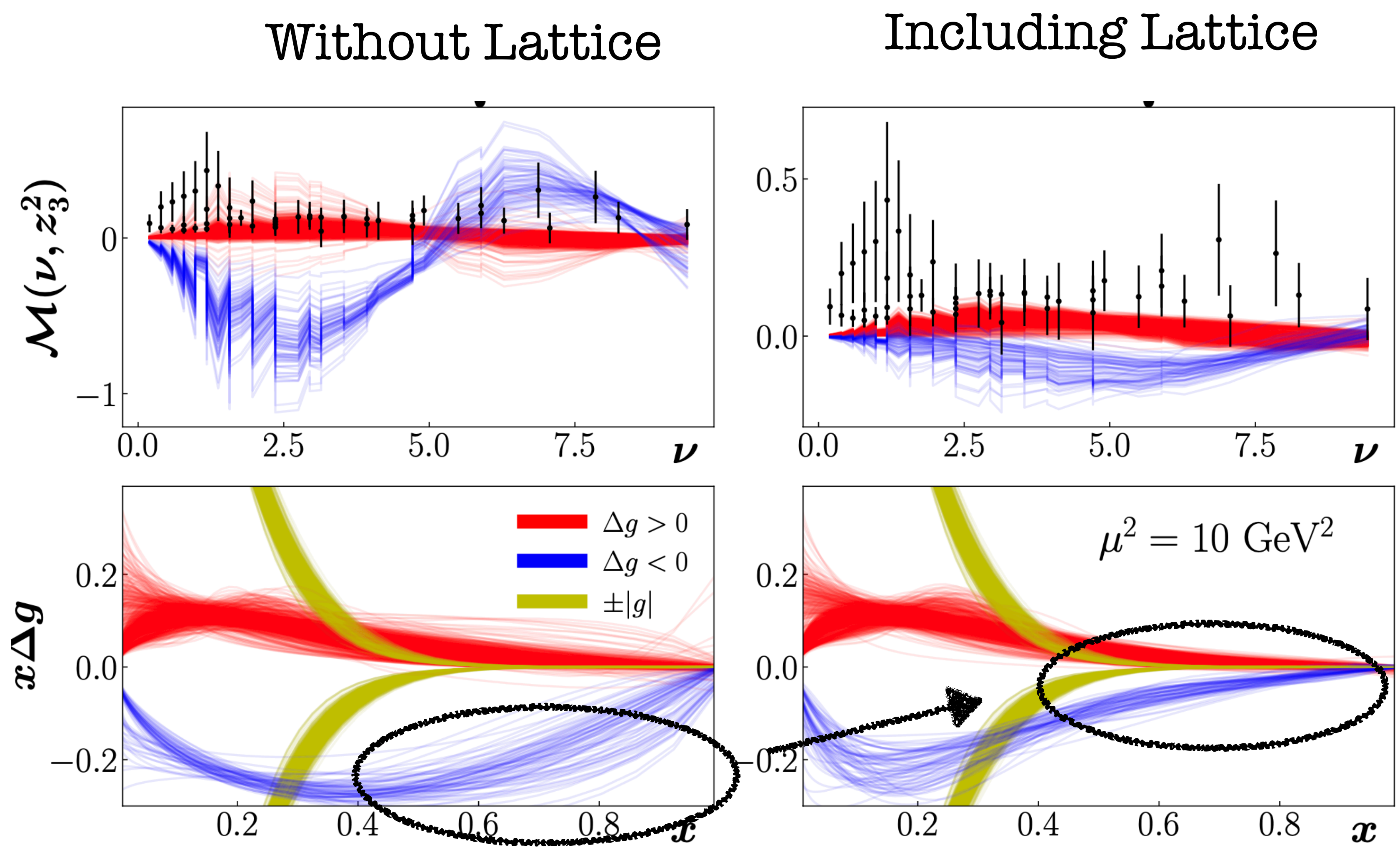

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New Developments

★ Twist-3 PDFs

★ GPDs

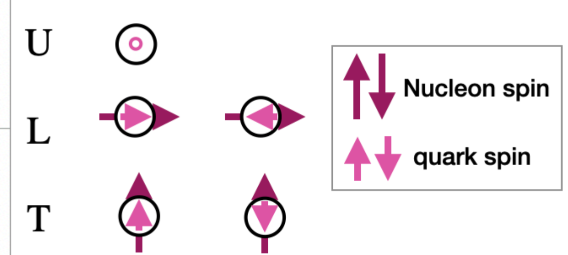
Twist-classification of PDFs, GPDs, TMDs

★ Twist: The order in Q^{-1} entering factorization

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L ($\gamma^+\gamma^5$)	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity



(Selected) Twist-3 $(f_i^{(1)})$

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U	G_1, G_2 G_3, G_4		
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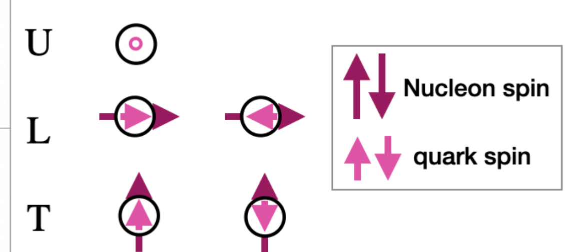

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★ Twist-2: probabilistic densities - a wealth of information exists (mostly on PDFs)

★ Twist-3: poorly known, but very important and have physical interpretation:
- as sizable as twist-2
- contain information about quark-gluon correlations inside hadrons
- appear in QCD factorization theorems for various observables (e.g. g_2)

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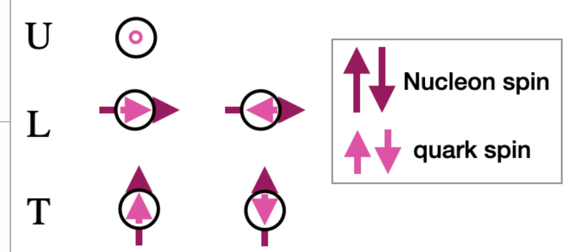

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The extraction of twist-3 is very challenges both experimentally and theoretically

Theoretical setup

★ Correlation functions in coordinate space

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization of coordinate-space correlation functions

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$\begin{aligned}F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[&P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)\\ &+\Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)\\ &+\Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)\end{aligned}$$

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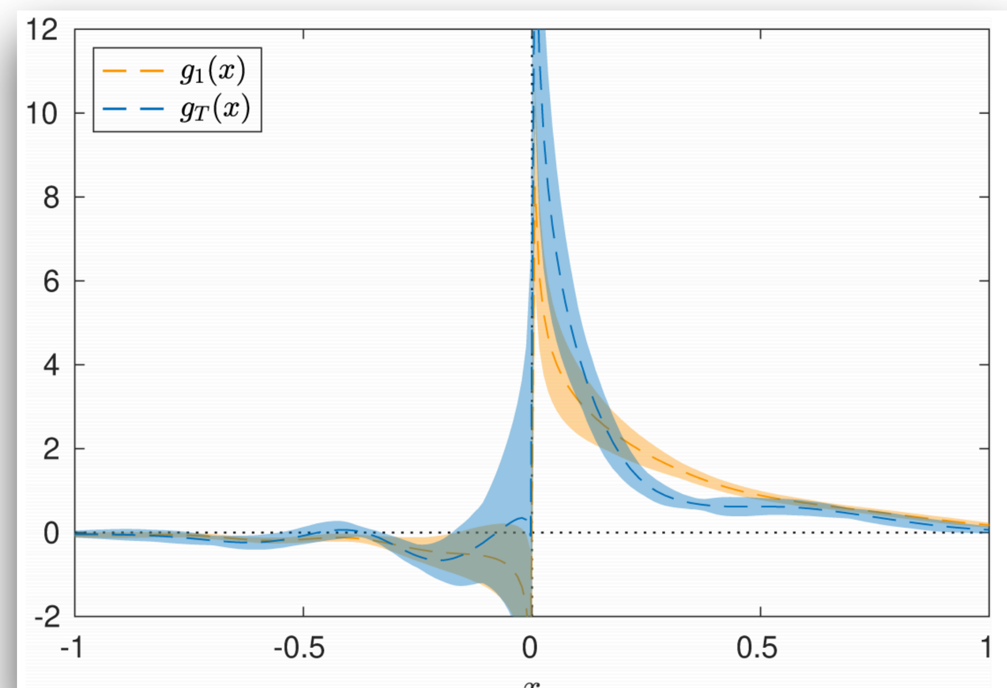
[S. Bhattacharya et al., PRD 102 (2020) 11 (Editors Selection)]



Pion mass:	260 MeV
Lattice spacing:	0.093 fm
Volume:	32^3 x 64

Twist-3 counterpart as sizable as twist-2

Burkhardt-Cottingham sum rule important check

$$\int_{-1}^{1} dx\, g_1(x) - \int_{-1}^{1} dx\, g_T(x) = 0$$

Theoretical setup

★ Correlation functions in coordinate space

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar\psi(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec z_\perp=\vec 0_\perp}$$

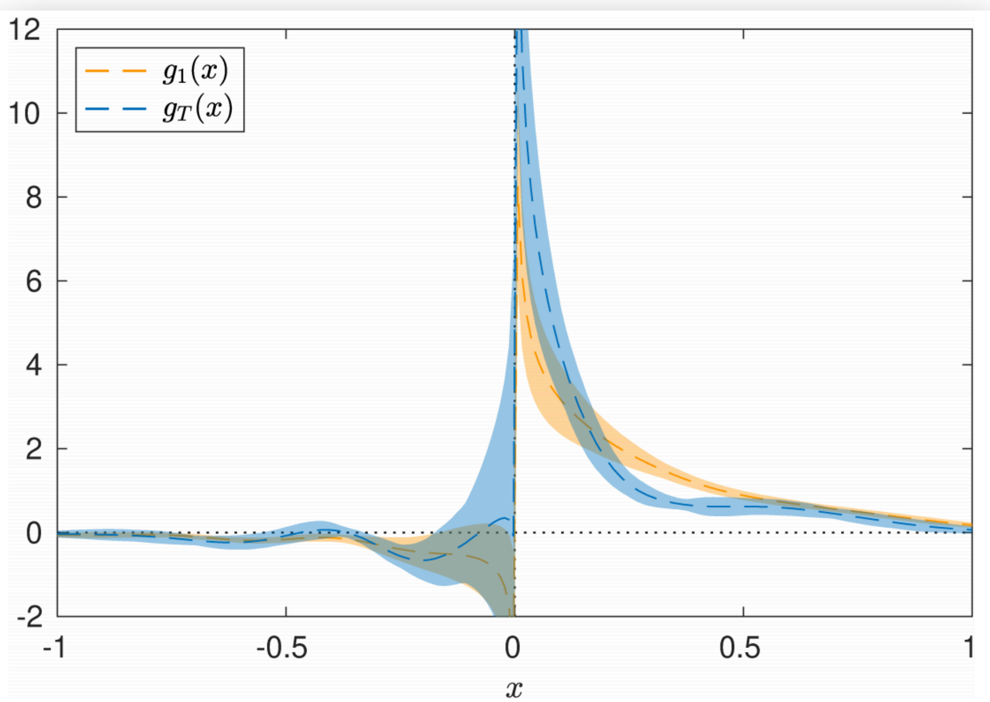
★ Parametrization of coordinate-space correlation functions

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$\begin{aligned}F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar u(p_f,\lambda')\Big[&P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde H}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde E}(x,\xi,t;P^3)\\ &+\Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde E+\widetilde G_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde H+\widetilde G_2}(x,\xi,t;P^3)\\ &+\Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde G_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde G_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)\end{aligned}$$

★ Forward limit for twist-3: only $\widetilde{H} + \widetilde{G}_2 \equiv g_T$ survives

[S. Bhattacharya et al., PRD 102 (2020) 11 (Editors Selection)]



Pion mass:	260 MeV
Lattice spacing:	0.093 fm
Volume:	32^3 x 64

Twist-3 counterpart as sizable as twist-2

Burkhardt-Cottingham sum rule important check

$$\int_{-1}^{1} dx\, g_1(x) - \int_{-1}^{1} dx\, g_T(x) = 0.01(20)$$

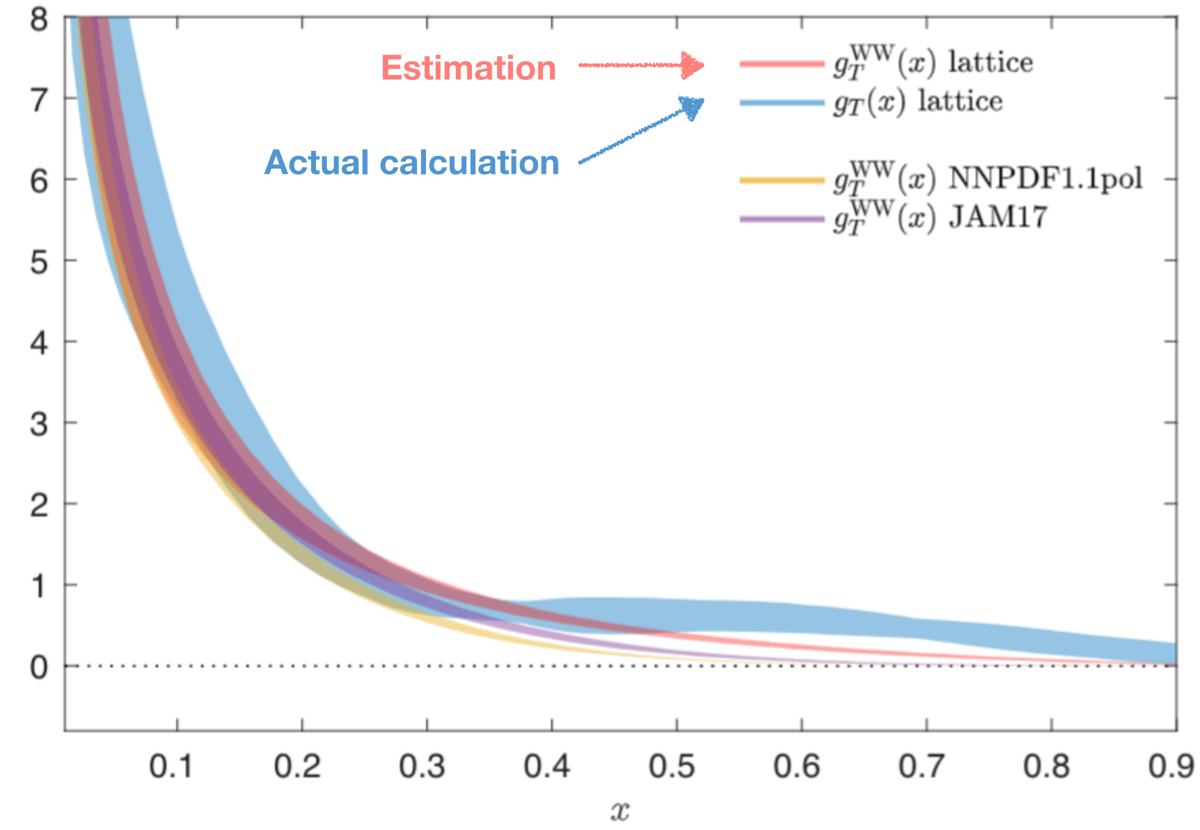
WW approximation

[S. Bhattacharya et al., PRD 102 (2020) 11 (Editors Selection)]

WW approximation: $g_T^{\text{WW}}(x) = \int_x^1 \frac{dy}{y} g_1(y)$

twist-3 $g_T(x)$ determined by the twist-2 $g_1(x)$

- $g_T(x)$ agrees with $g_T^{\text{WW}}(x)$ for $x < 0.5$ (violations up to 30-40% possible)
- Violations of 15-40% expected from experimental data
[A. Accardi et al., JHEP 11 (2009) 093]



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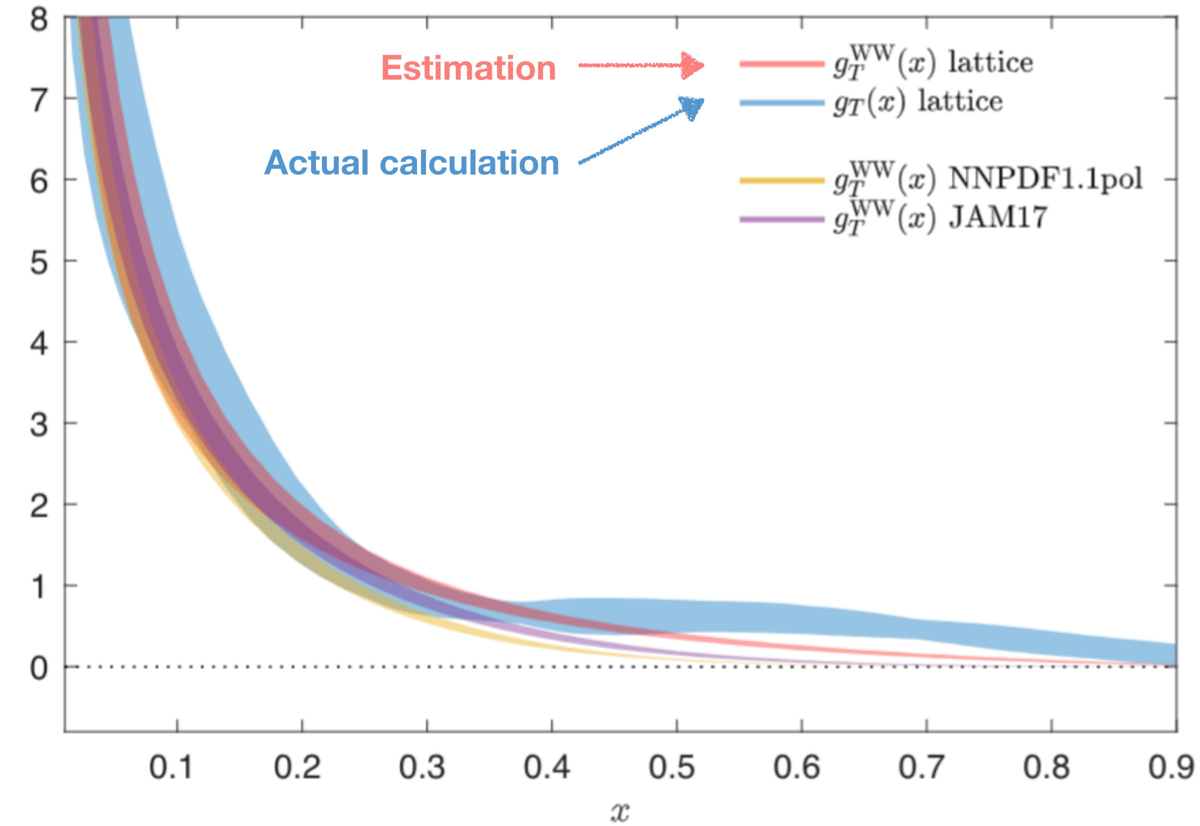
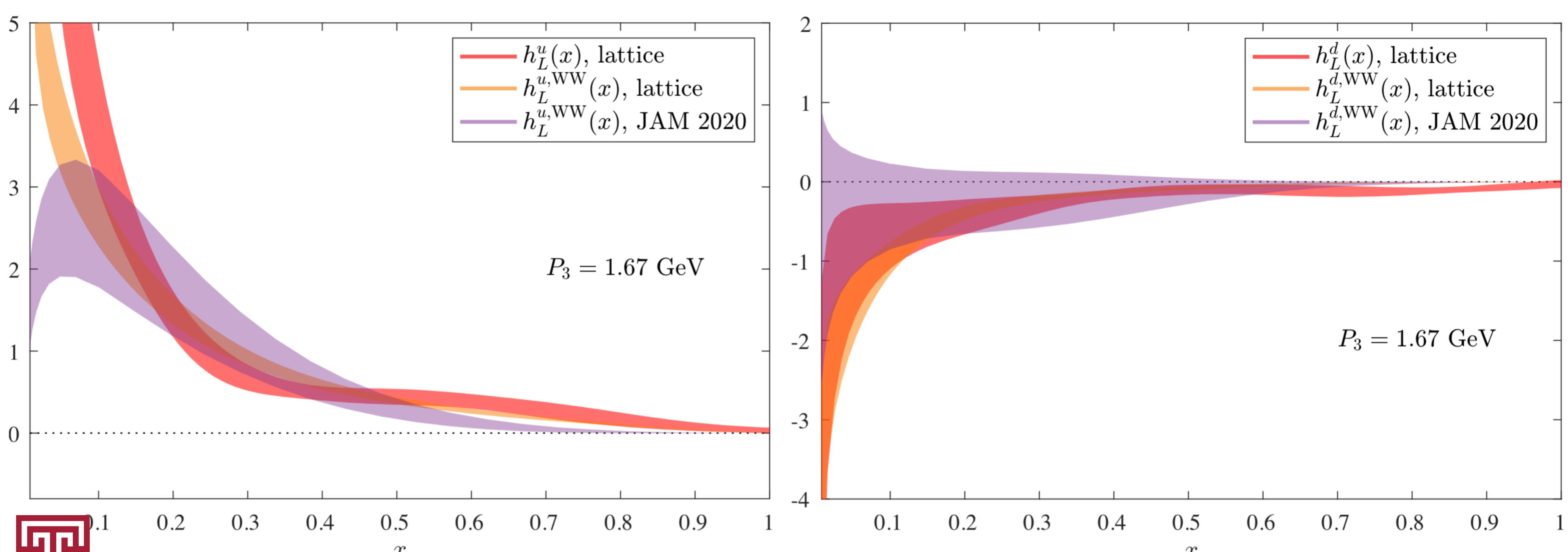


Twist-3 $h_L(x)$ PDF

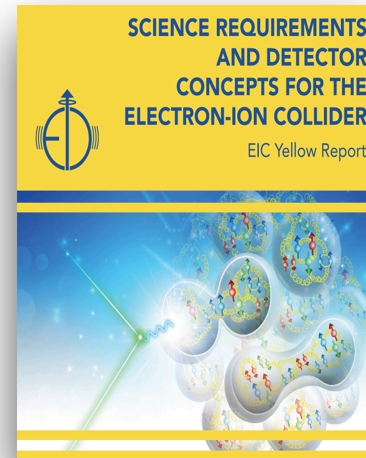
[S. Bhattacharya et al., PRD 104 (2021) 11, 114510]

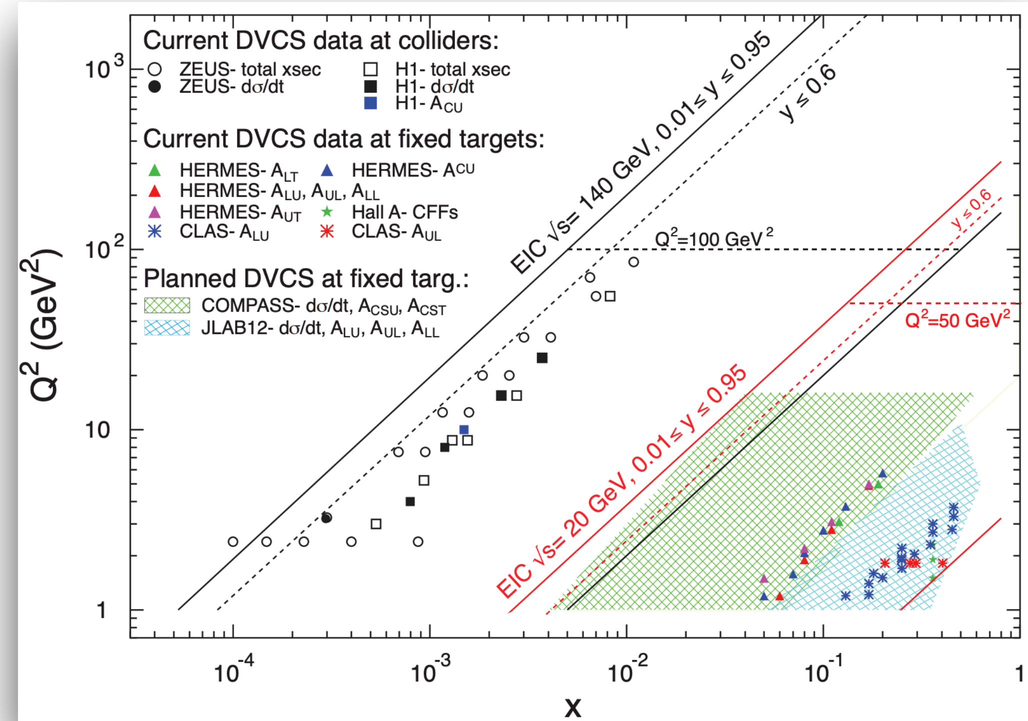


- h_L^u dominant - tension between h_L & $h_L^{\rm WW}$
- h_L^d <0 and decays faster then h_L^u

Proton GPDs

★ Tomographic imaging of proton has central role in the science program of EIC
GPDs, FFs, GFFs, TMDs, …
[R. Abdul Khalek et al.,
EIC Yellow Report 2021, arXiv:2103.05419]





★ GPDs are not well-constrained experimentally

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}$$

$$\langle N(P')|O_A^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5 \widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \text{ht}$$

★ Can be accessed also at the twist-3 level

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
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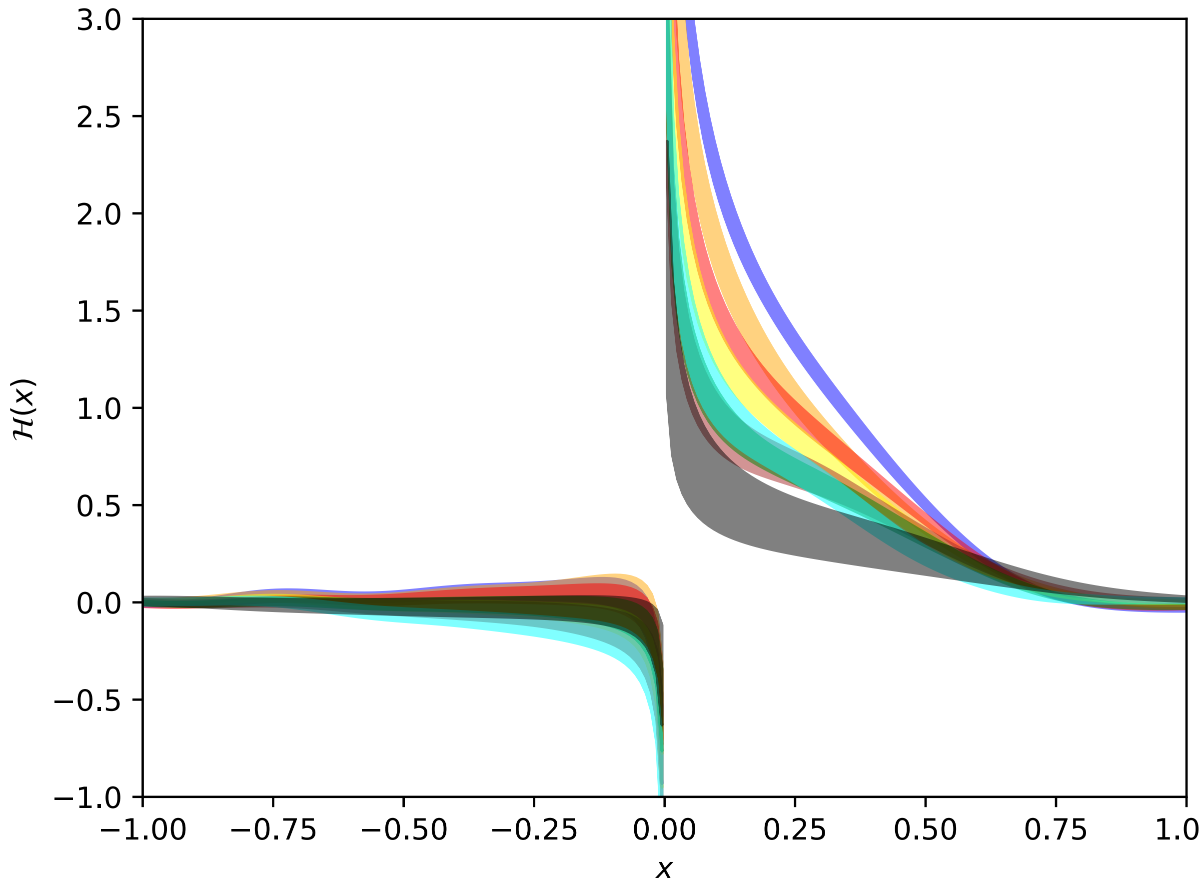
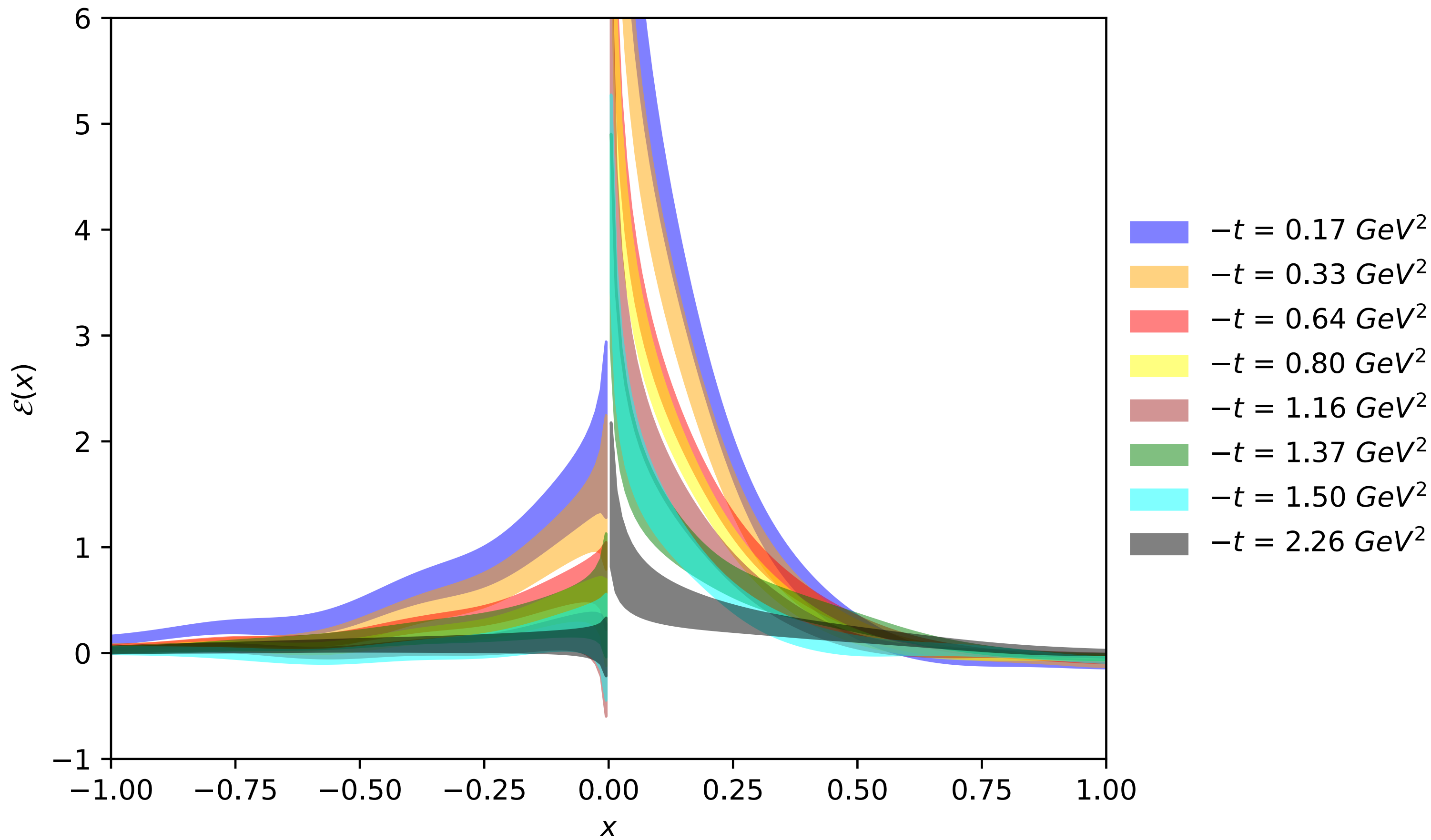
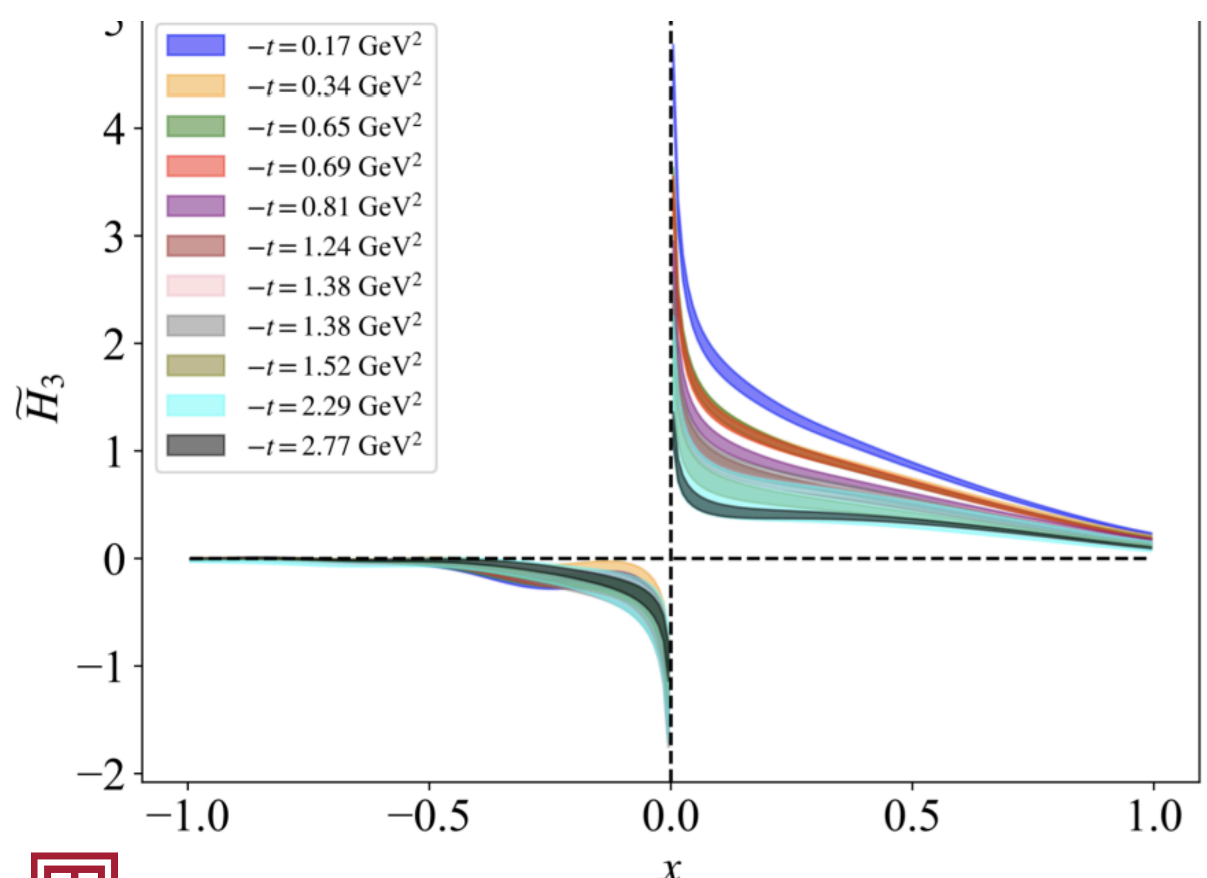
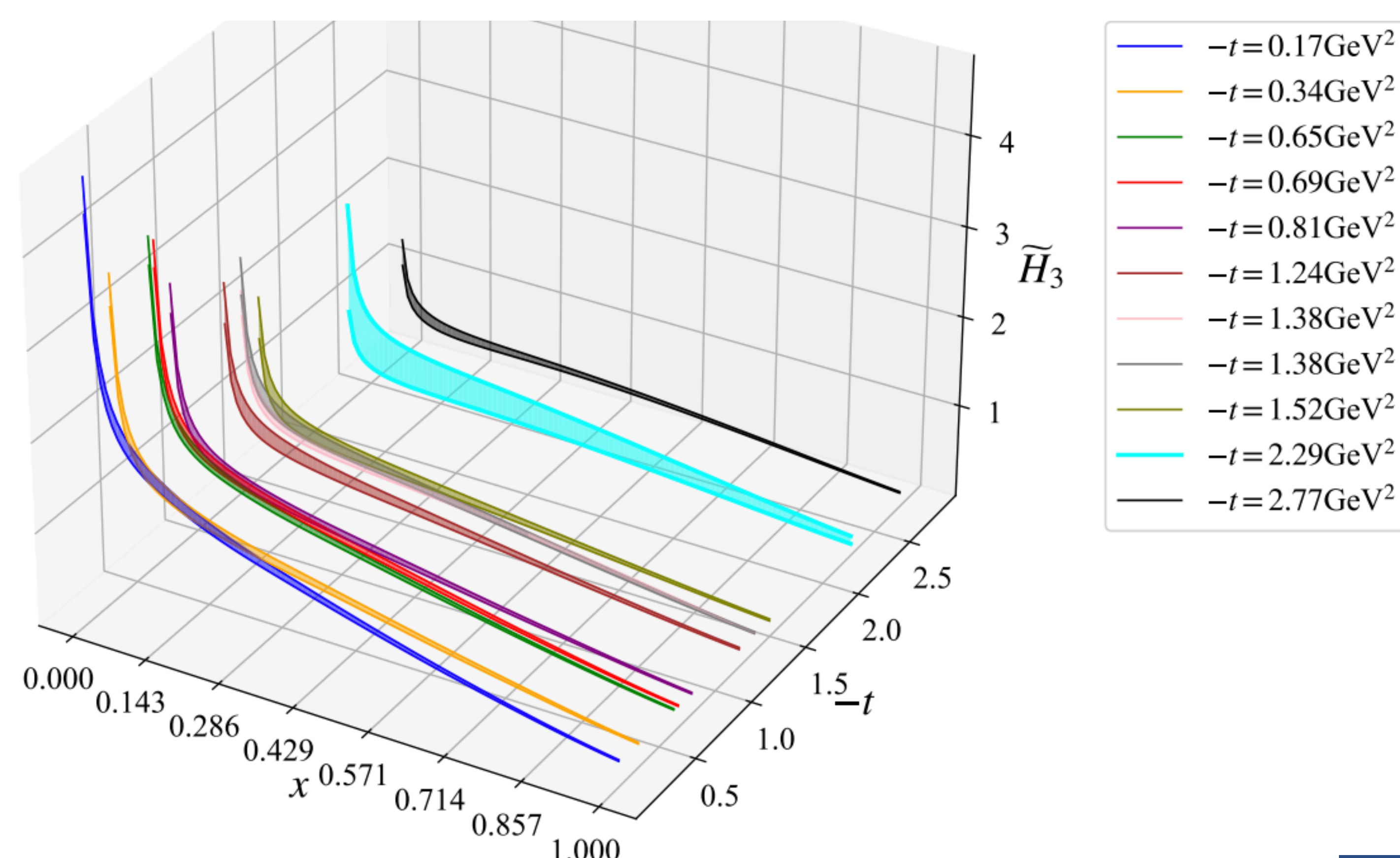
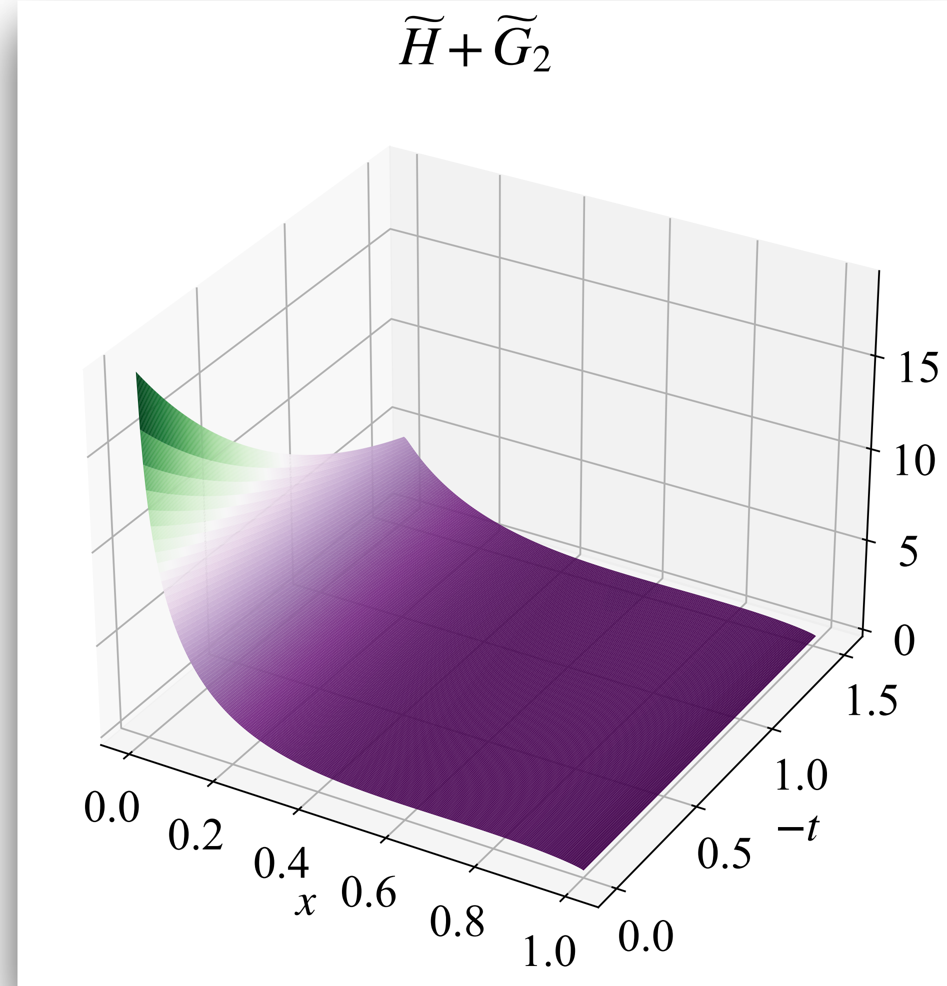
Light-cone GPDs

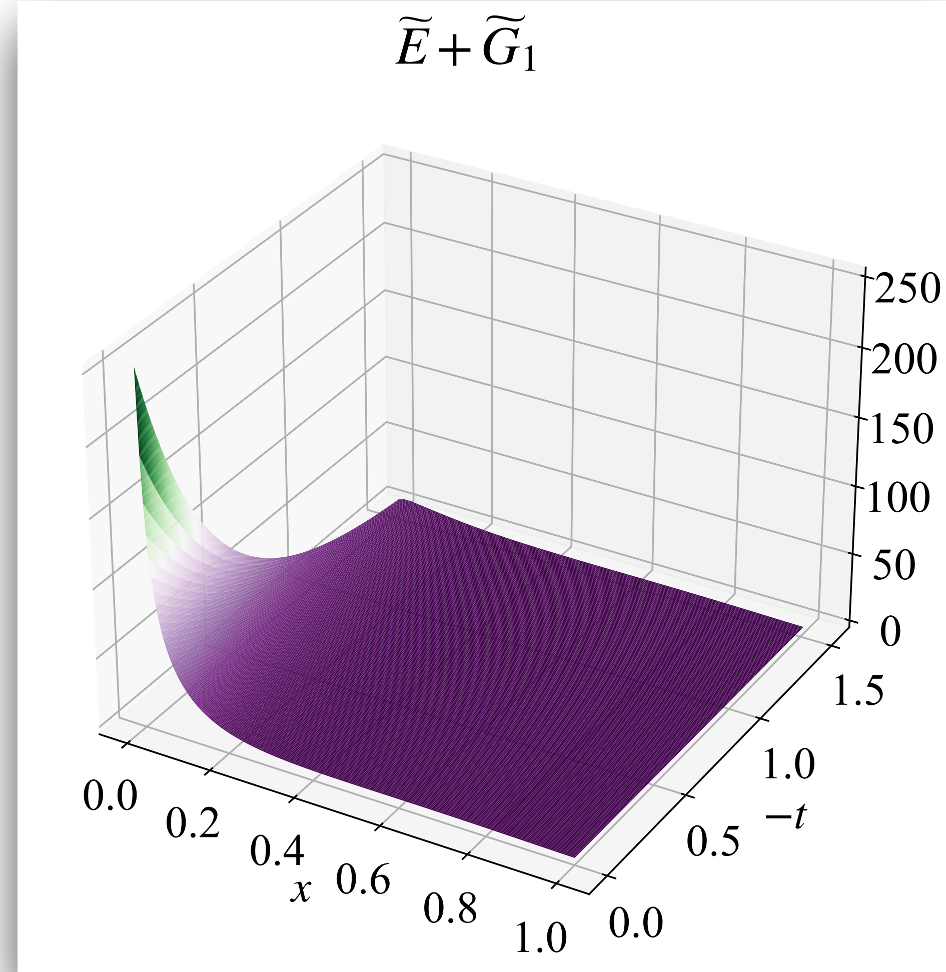












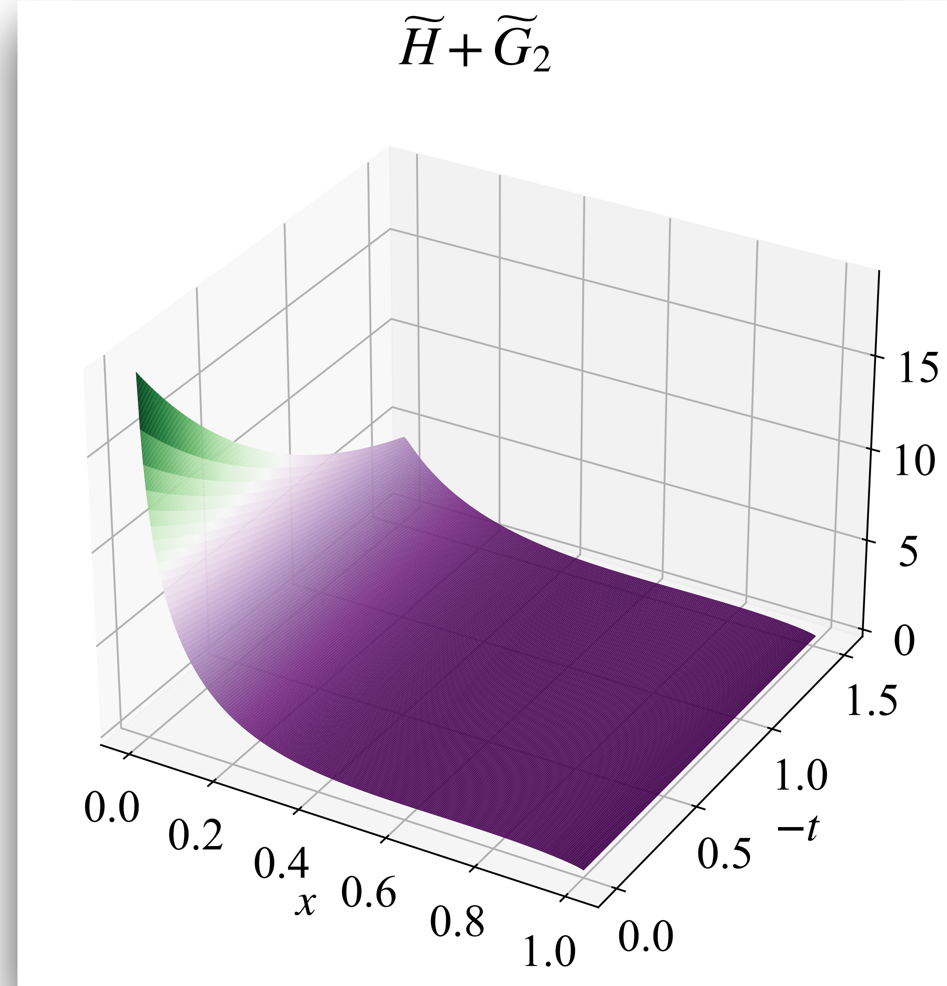
- ★ Direct access to $\widetilde{E}$-GPD not possible for zero skewness

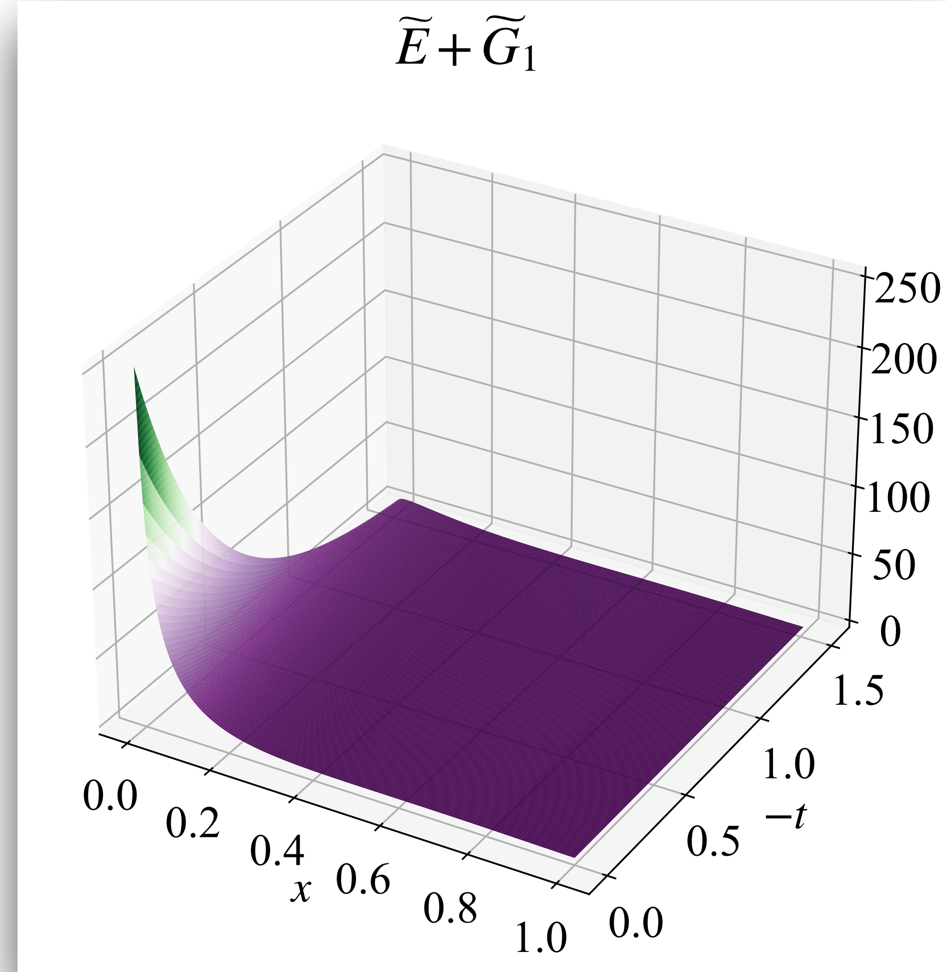
$$P^\mu \frac{\Delta^3 \gamma_5}{2mP^0} F_{\widetilde{E}}(x,\xi,t;P^3)$$

- ★ Glimpse into $\widetilde{E}$-GPD through twist-3 :

$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$





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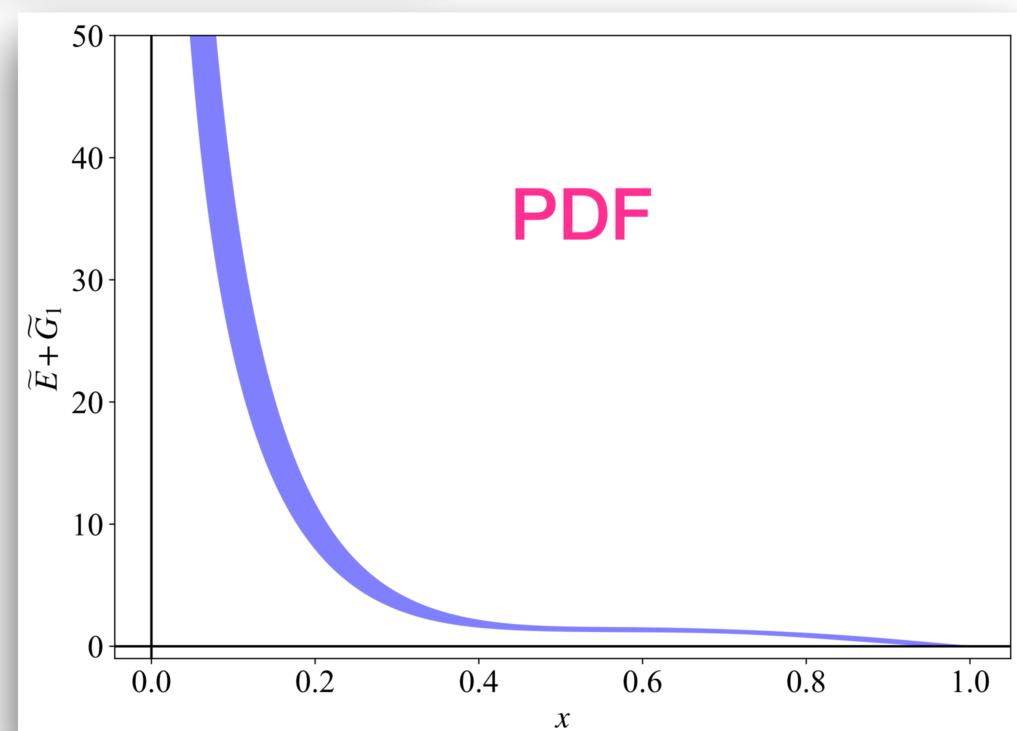


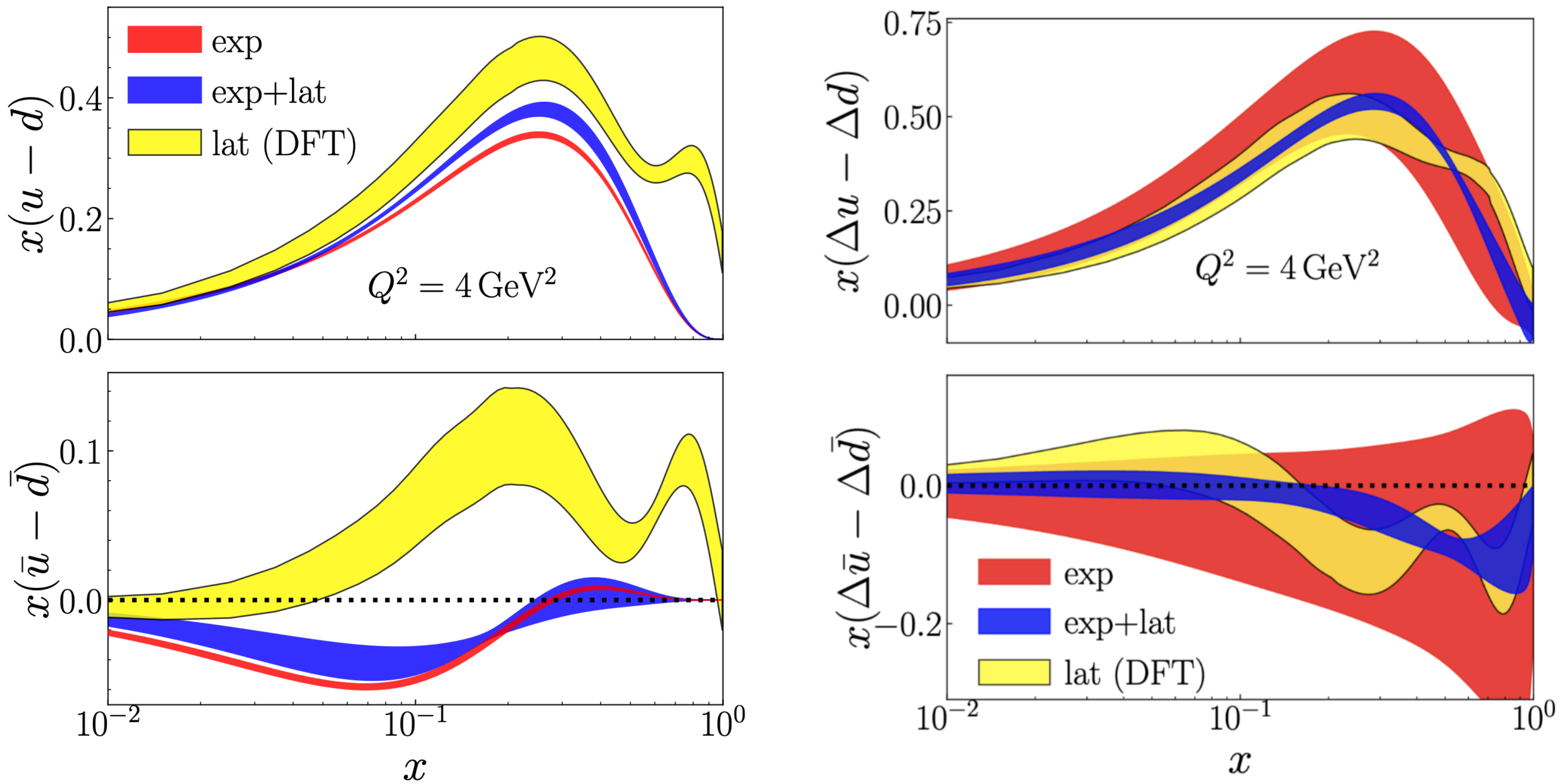
Synergy/Complementarity of lattice and phenomenology

Incorporating lattice PDFs in global analyses

Synergy between lattice and phenomenology

★ Lattice and experimental data sets data within the same global analysis (JAM framework)

[J. Bringewatt et al., PRD 103 (2021) 016003, arXiv:2010.00548]



- Consistent picture with JAM for unpolarized PDF

- Significant impact for helicity PDF

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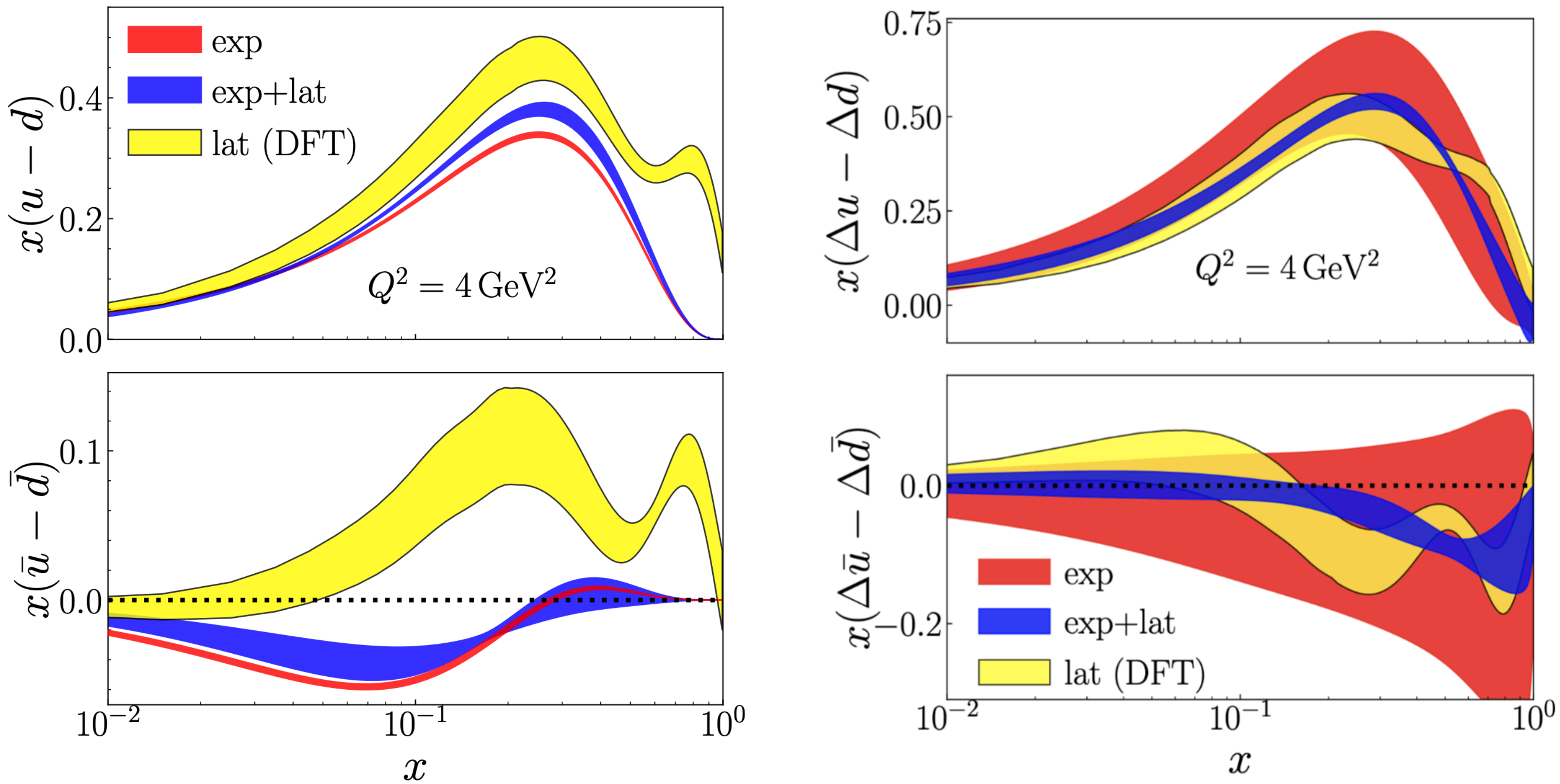

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★ Other efforts within NNPDF framework

[K. Cichy et al., JHEP 10 (2019) 137, arXiv:1907.06037]
[L. Del Debbio et al., JHEP 02 (2021) 138, 2010.03996]

★ Interest in applying similar approach to quantities that are more challenging to extract experimentally (GPDs, twist-3 distributions, …)

How to lattice QCD data fit into the overall effort for hadron tomography

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QUARK-GLUON TOMOGRAPHY COLLABORATION

U.S. DEPARTMENT OF ENERGY | Office of Science

Award Number: DE-SC0023646

1. Theoretical studies of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties
2. Lattice QCD calculations of GPDs and related structures
3. Global analysis of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

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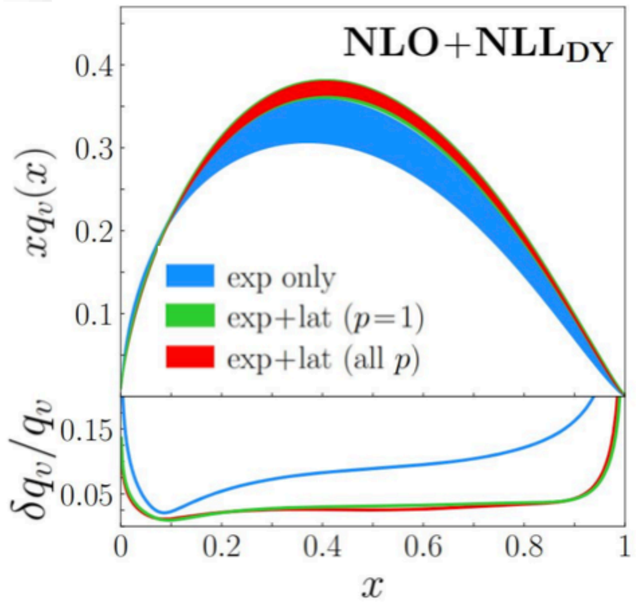
Other GPD global analysis efforts:

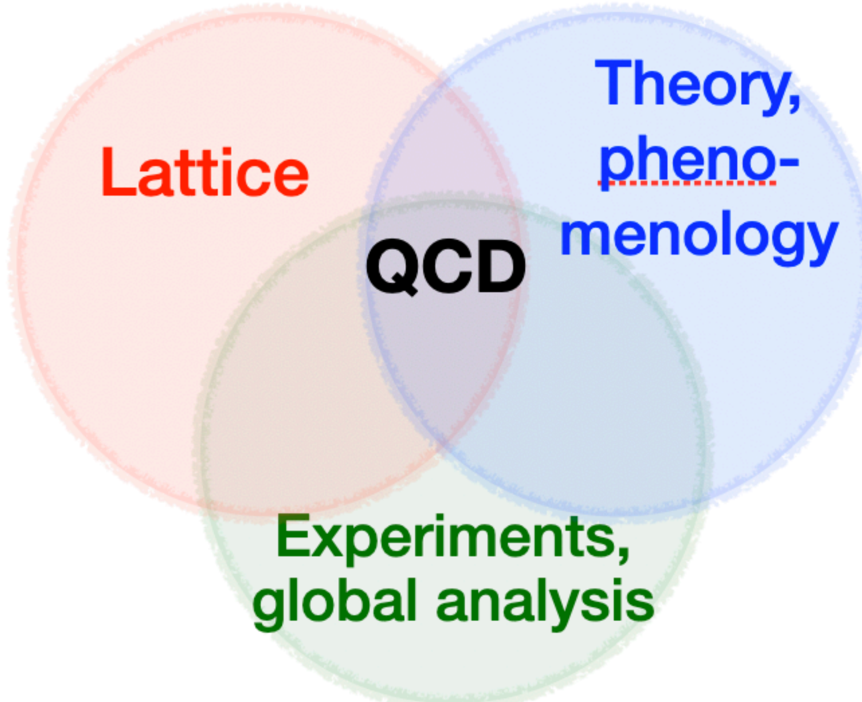
- Gepard [https://gepard.phy.hr/]
- PARTONS [https://partons.cea.fr]
- EXCLAIM [https://exclaimcollab.github.io/web.github.io/#/]

Synergies: constraints & predictive power of lattice QCD

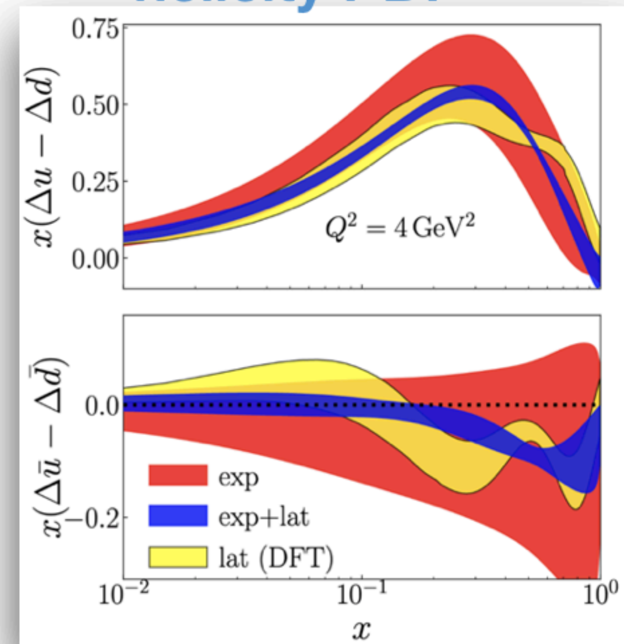
pion PDF



[JAM/HadStruc, PRD105 (2022) 114051]

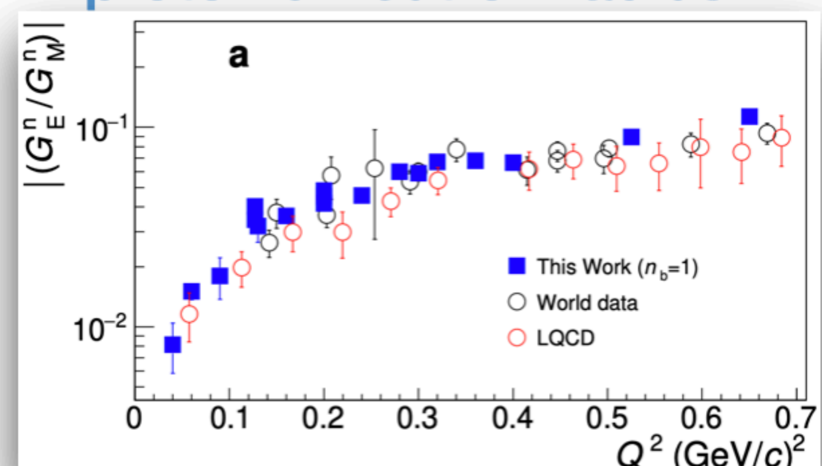


helicity PDF



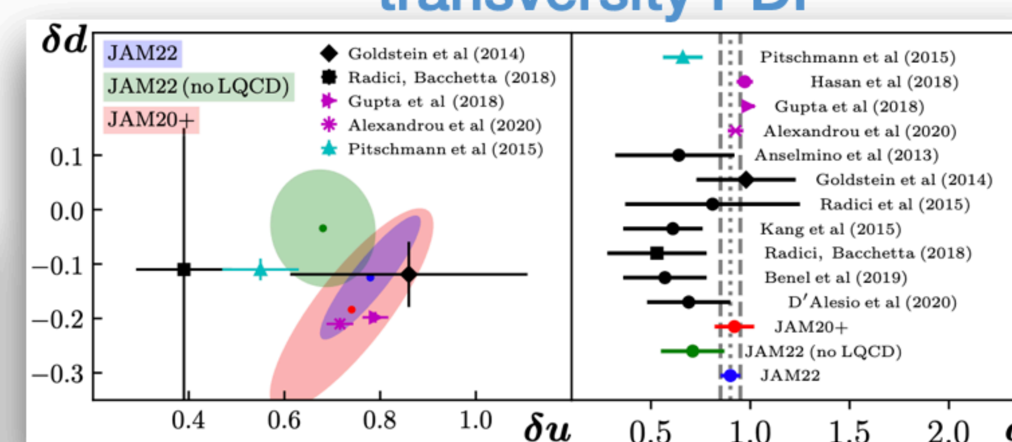
[JAM & ETMC, PRD 103 (2021) 016003]

proton & neutron radius



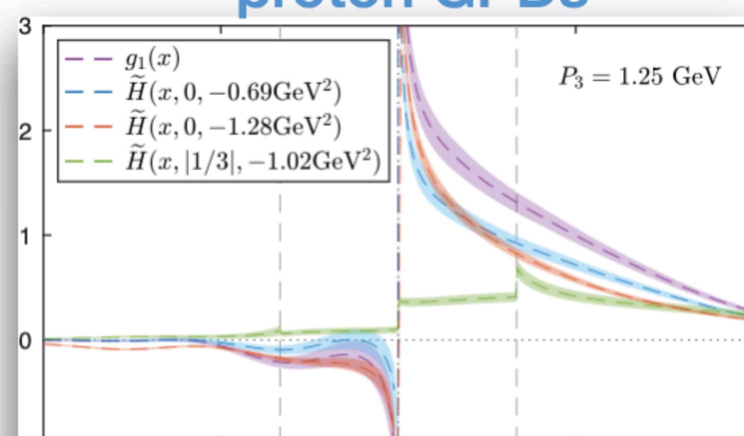
[Atac et al., Nature Comm. 12, 1759 (2021)]

transversity PDF



[JAM, PRD 106 (2022) 3, 034014]

proton GPDs



[ETMC, PRL 125 (2020) 262001]

And many more!

Concluding remarks

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- ★ New Developments in several promising directions:

DA	GPDs	TMDs
M-H Chu, Thu 3:30 pm	H. Dutrieux, Thu 4:30pm	M. Wagman, Thu 5 pm

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- ★ Synergy with phenomenology has the potential to enhance the impact of lattice QCD data and complement data sets

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Thank you

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DOE Early Career Award (NP)
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NERSC

ASCR Leadership Computing Challenge ALCC