Muon g-2 Theory

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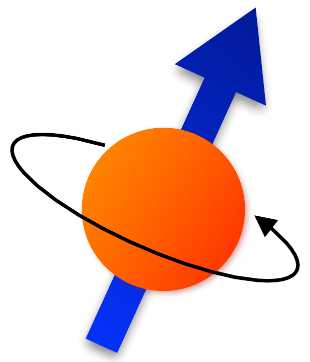
Outline

- Introduction
- Muon g-2 Theory Initiative
- data-driven HVP
- lattice HVP
- HLbL
- Summary and Outlook
- Appendix



- ``The anomalous magnetic moment of the muon in the SM":
  1st White Paper published in 2020 (132 authors, 82 institutions)
  [T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166.]
- ``Prospects for precise predictions of a_μ in the SM":
  2022 Snowmass Summer Study, arXiv:2203.15810
- Summary statement on the status of Muon g-2 Theory in the SM: https://muon-gm2-theory.illinois.edu

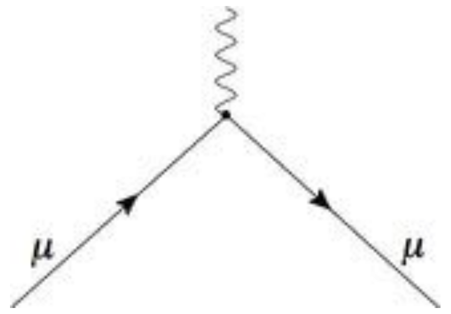
➠ Lattice 2024 conference

➠ Muon g-2 TI workshop @ KEK (9-13 Sep 2024)

Anomalous magnetic moment

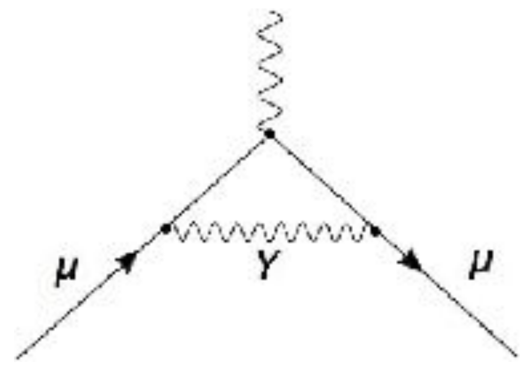
The magnetic moment of charged leptons (e, μ, τ): $\vec{\mu} = g\frac{e}{2m}\vec{S}$

Dirac (leading order): $g = 2$



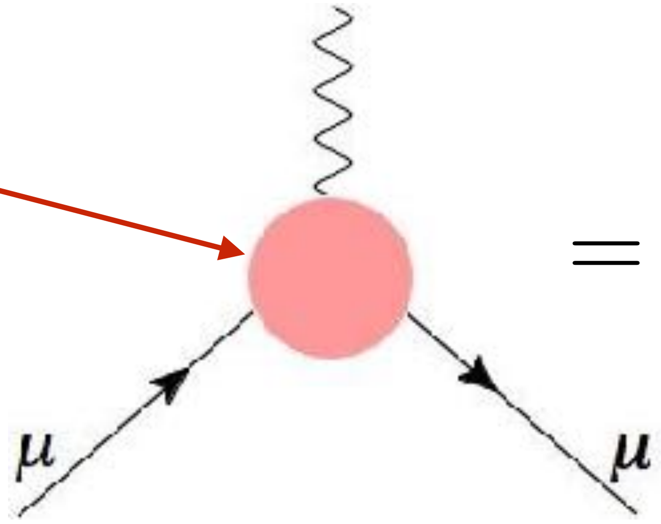
$$= (-ie)\,\bar{u}(p')\gamma^{\mu}u(p)$$

Quantum effects (loops):



$$\Rightarrow \quad g = 2\left(1 + \frac{\alpha}{2\pi}\right)$$

All SM particles contribute



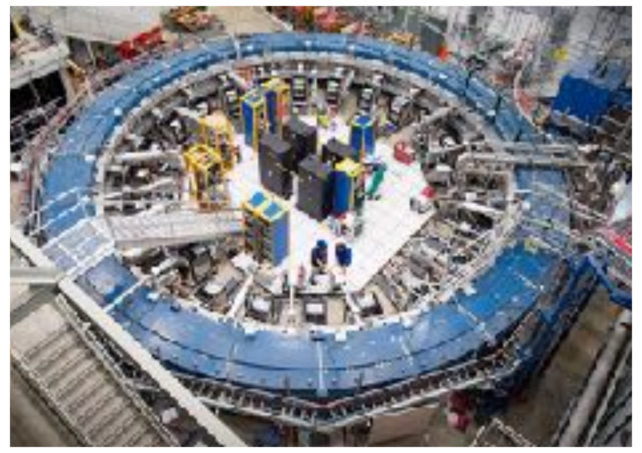
$$= (-i\,e)\,\bar{u}(p')\left[\gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2(q^2)\right]u(p)$$

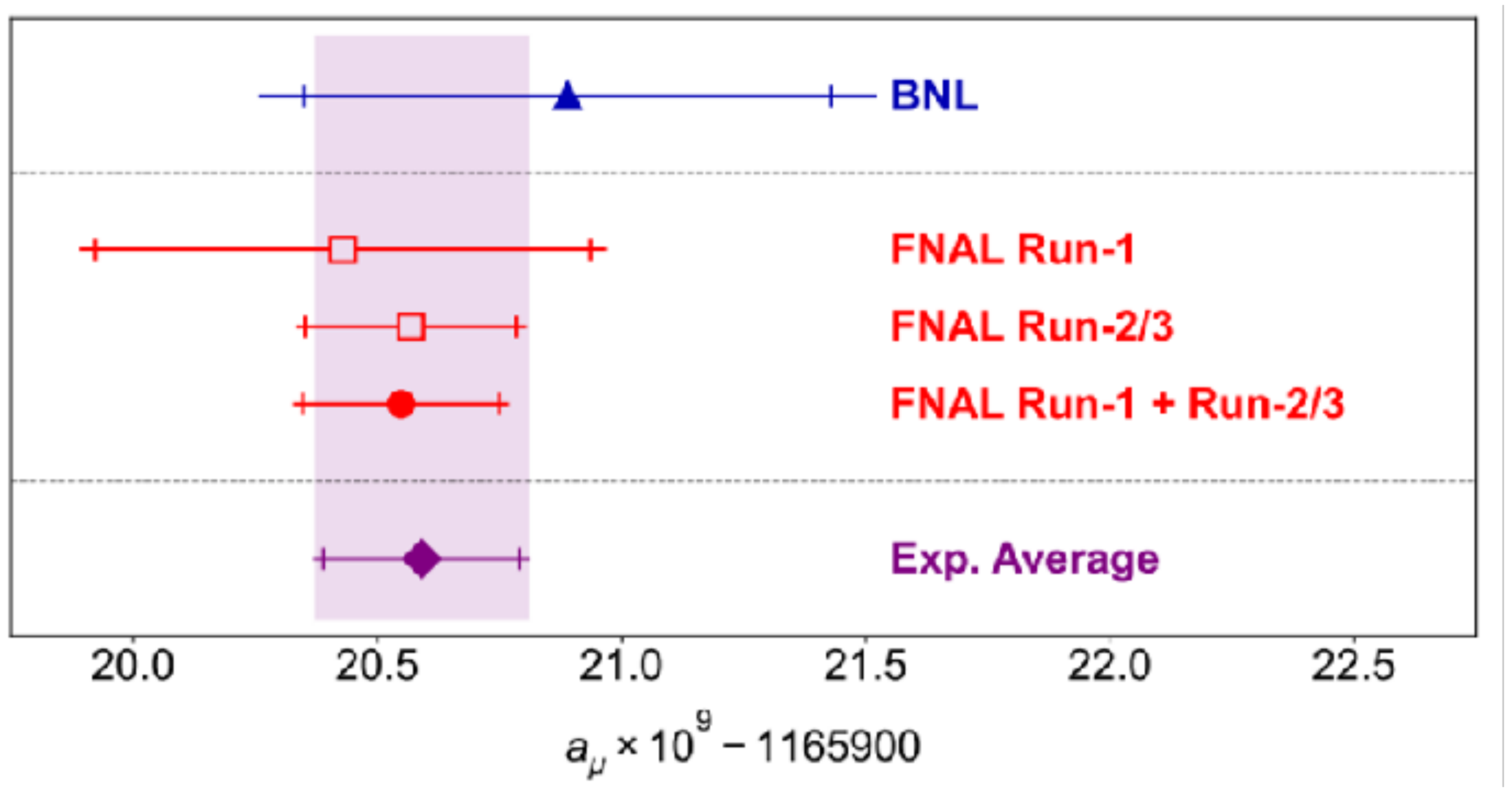
Note: $F_1(0) = 1$ and $g = 2 + 2\,F_2(0)$

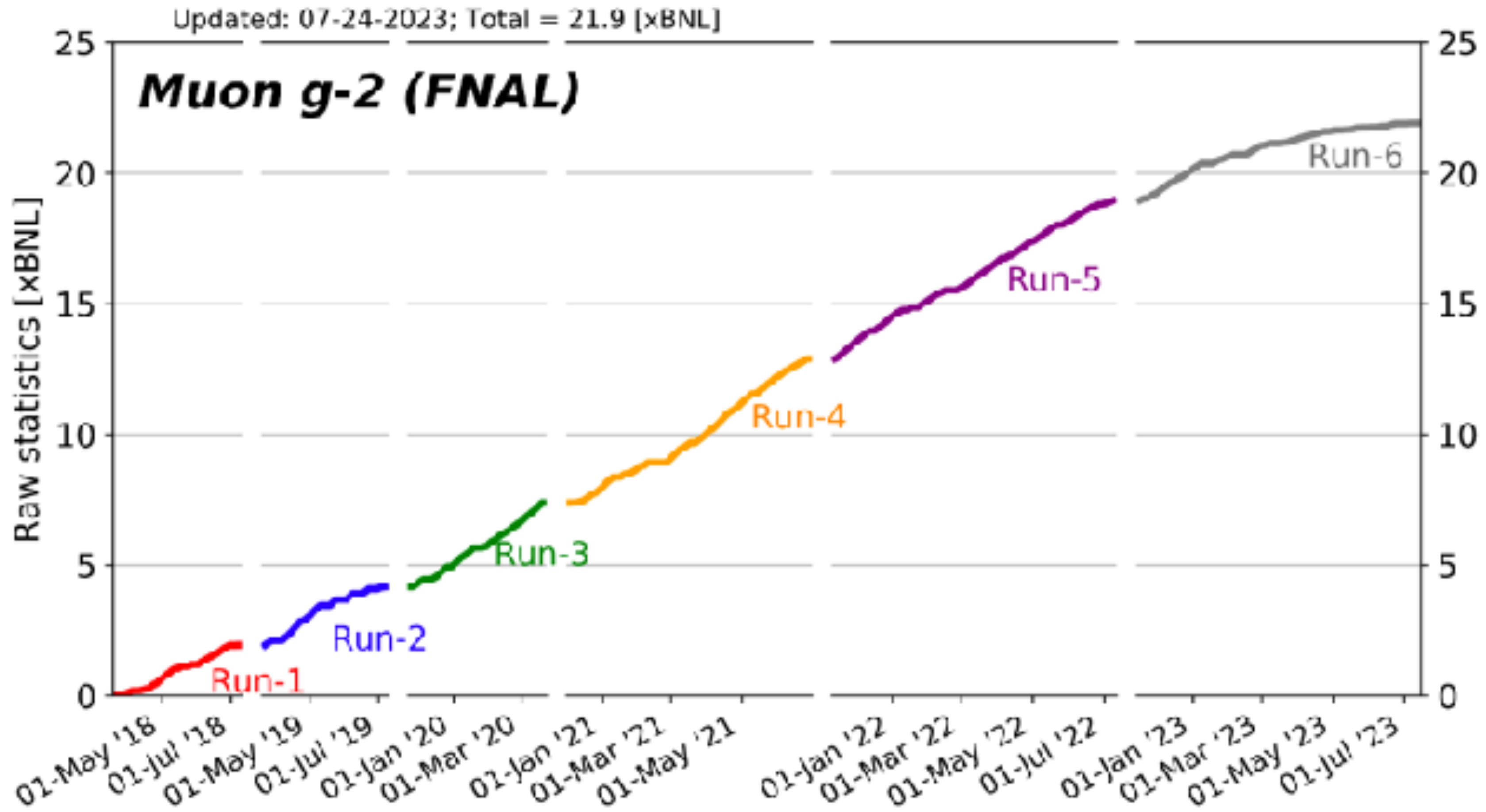
Anomalous magnetic moment: $a \equiv \frac{g-2}{2} = F_2(0)$ $= \frac{\alpha}{2\pi} + O(\alpha^2) + \ldots = 0.00116\ldots$

Muon g-2 experiment

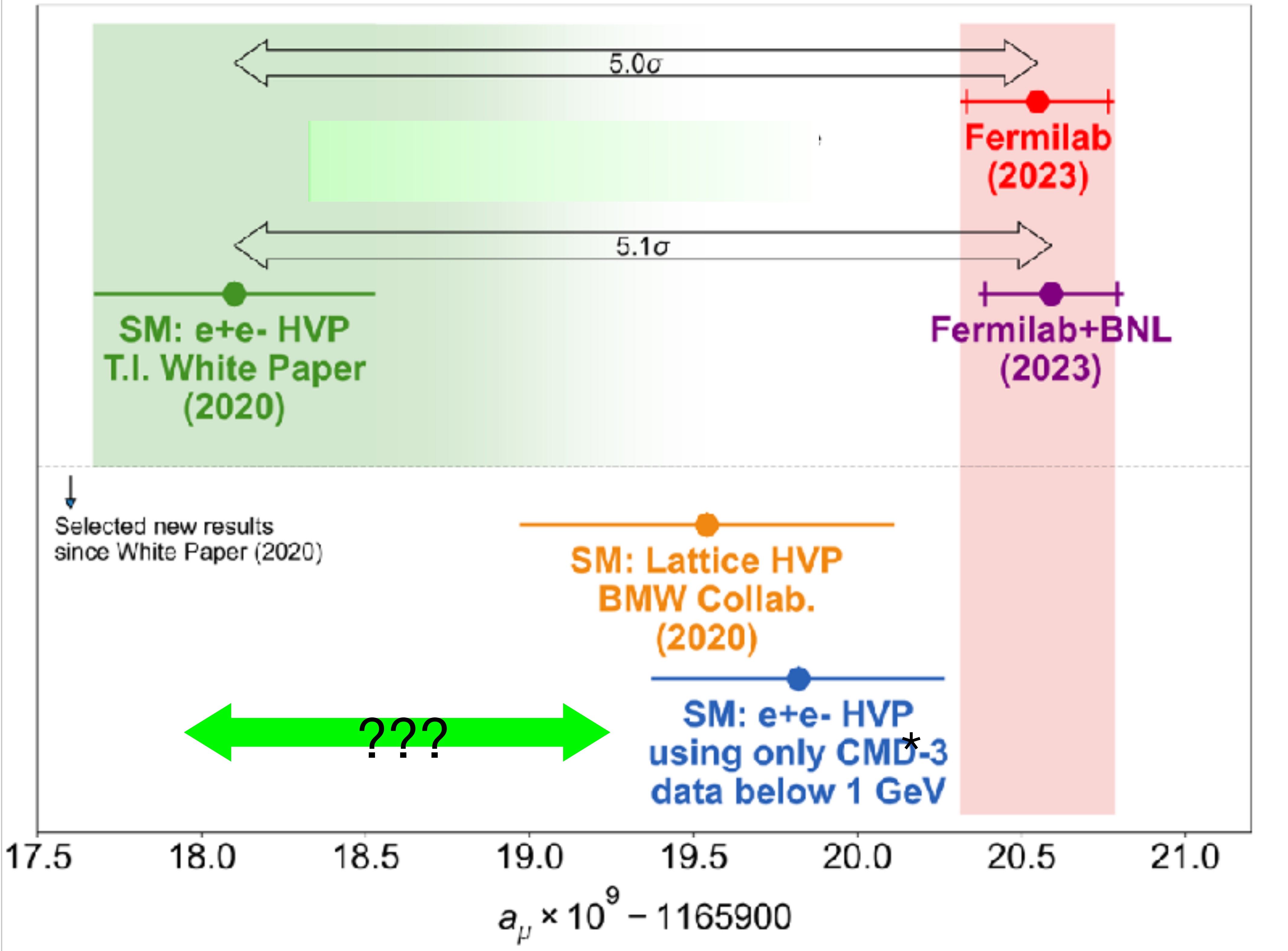
- The Fermilab experiment released the measurement result from their run 2&3 data on 10 Aug 2023. [D. Aguillard et al, 2308.06230]
- Run 6 completed summer 2023.
- Release of final measurement result expected in 2025







adapted from J. Mott @ Scientific Seminar, 10



Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

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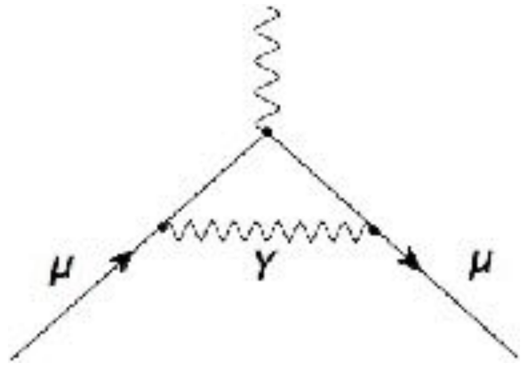
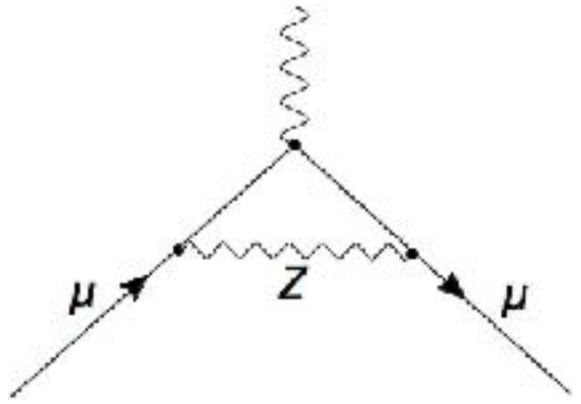
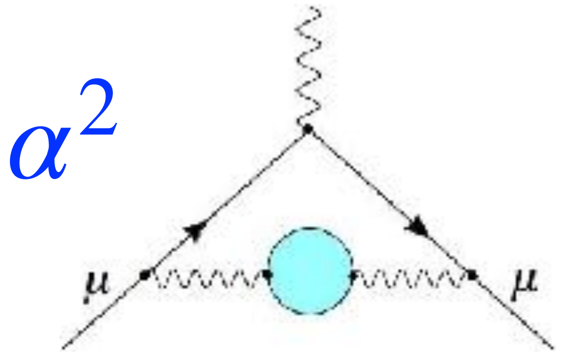
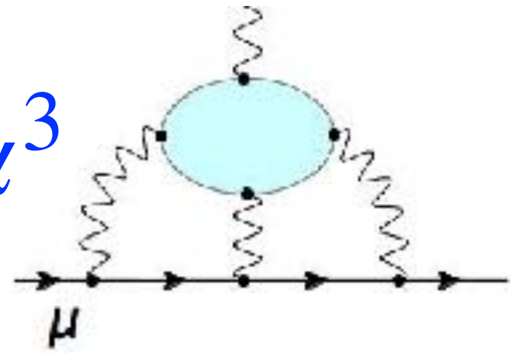
QED		+… (5 loops)	$116\,584\,718.9\,(1) \times 10^{-11}$	0.001 ppm	
EW		+… (2 loops)	$153.6\,(1.0) \times 10^{-11}$	0.01 ppm	
HVP	α^2 	+… (NNLO)	$6845\,(40) \times 10^{-11}$ [0.6%]	0.34 ppm	Hadronic corrections
HLbL	α^3 	+… (NLO)	$92\,(18) \times 10^{-11}$ [20%]	0.15 ppm	

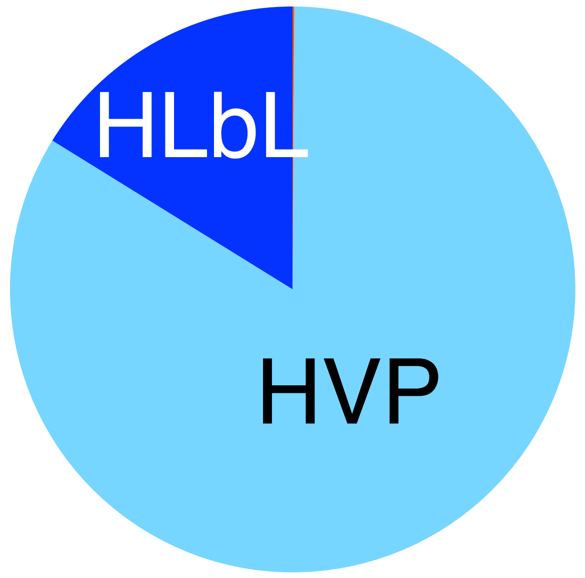
Muon g-2: SM contributions

$$a_\mu = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{EW}) + a_\mu(\mathrm{hadronic})$$

contribution

- QED
- EW
- HVP
- HLbL

error^2

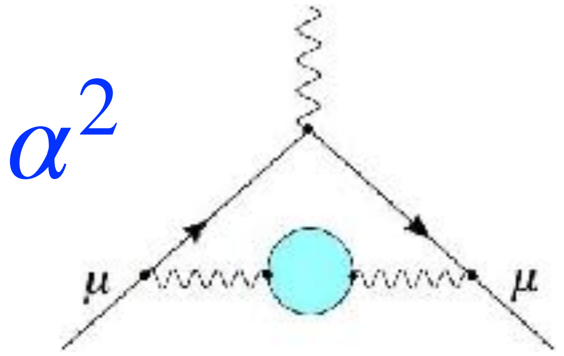


$116\,584\,718.9\,(1) \times 10^{-11}$ — 0.001 ppm

$153.6\,(1.0) \times 10^{-11}$ — 0.01 ppm

Hadronic corrections

HVP α^2



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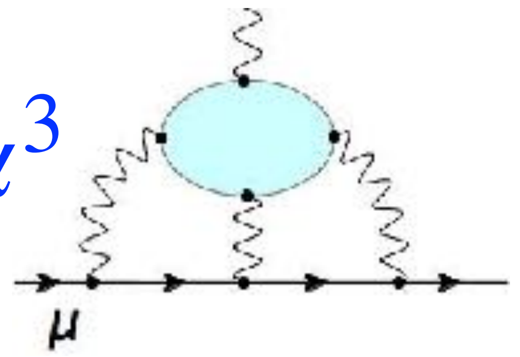
HLbL α^3



+… (NLO)

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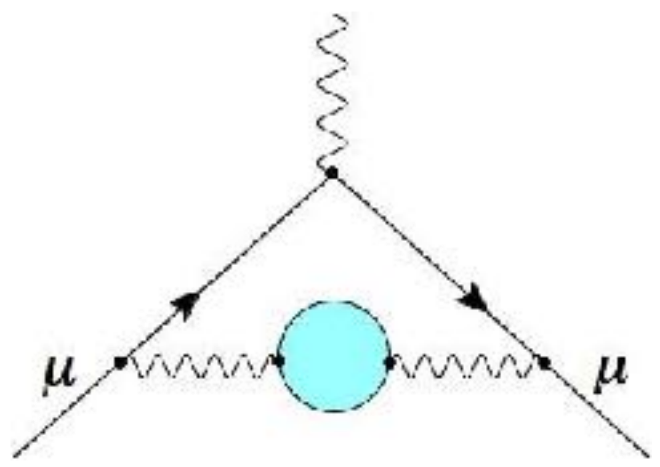
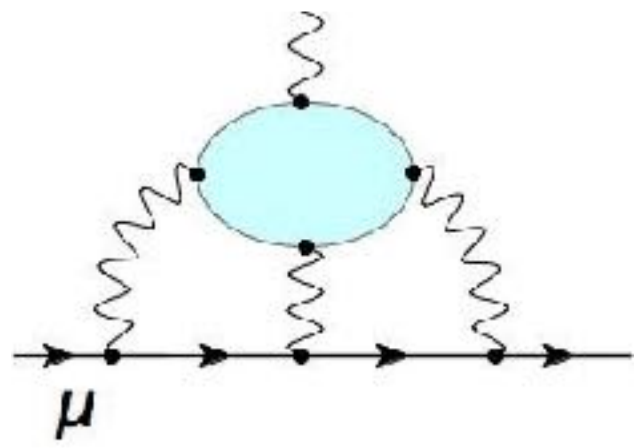
Muon g-2: hadronic corrections



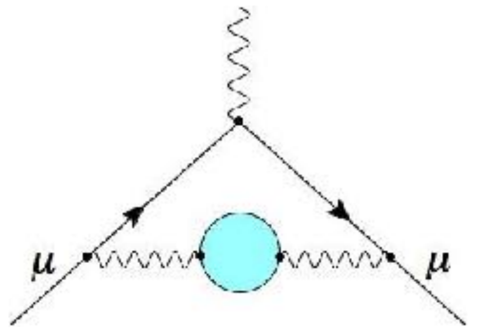


- Hadronic contributions are obtained by integrating over all possible virtual photon momenta, integral is weighted towards low q^2.
- Cannot use perturbation theory to reliably compute the hadronic bubbles
- Two-point & four-point functions:

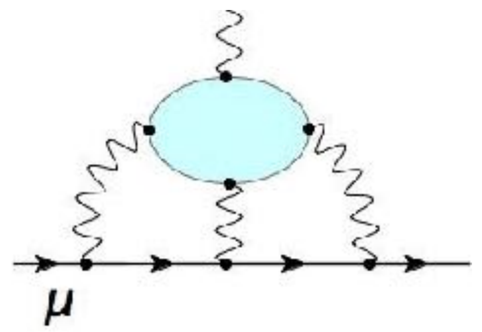
HVP: $\langle 0 | T\{j_\mu j_\nu\} | 0 \rangle$ HLbL: $\langle 0 | T\{j_\mu j_\nu j_\rho j_\sigma\} | 0 \rangle$

Two independent approaches

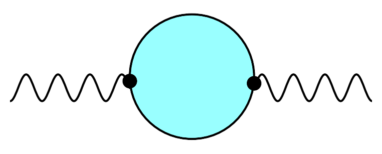
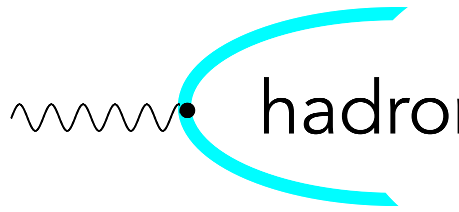
1. Dispersive, data-driven
2. Lattice QCD



Hadronic Corrections

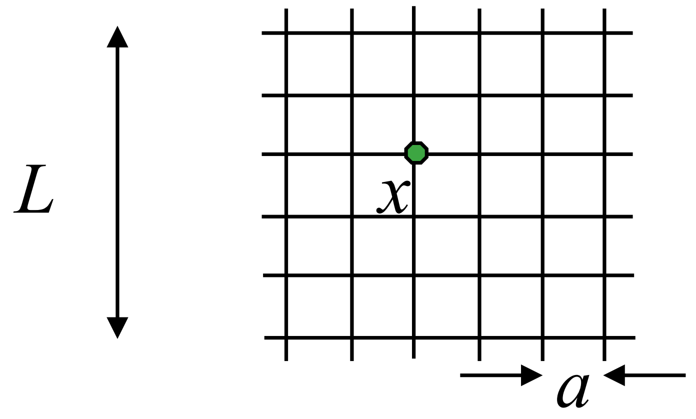


- Dispersive, data-driven:
  HVP: integrate hadronic cross section over CM energy:

Im[] ~ |  |^2 ➠ $a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2) = \frac{m_\mu^2}{12\pi^3} \int ds \frac{\hat{K}(s)}{s} \sigma_{\exp}(s)$

Many experiments (over 20+ years) have measured the e^+e^-cross sections for (almost) all channels over the needed energy range with increasing precision.
For HLbL: new dispersive approach

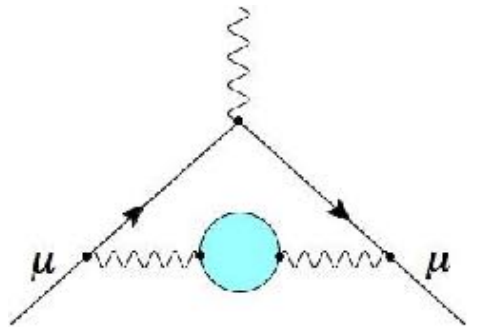
- Direct calculation using Euclidean Lattice QCD ➠ $a_\mu^{\mathrm{HVP,LO}} = 4\,\alpha^2 \int_0^\infty dt\, C(t)\, \tilde{w}(t)$



Approximations:
  discrete space-time (spacing a)
  finite spatial volume (L), and time extent (T)
  …

  - *ab-initio* method to quantify QCD effects
  - already used for simple hadronic quantities with high precision
  - requires large-scale computational resources
  - allows for entirely SM theory based evaluations

Integrals are evaluated numerically using Monte Carlo methods.



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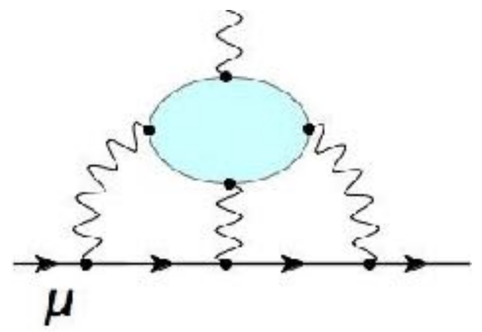

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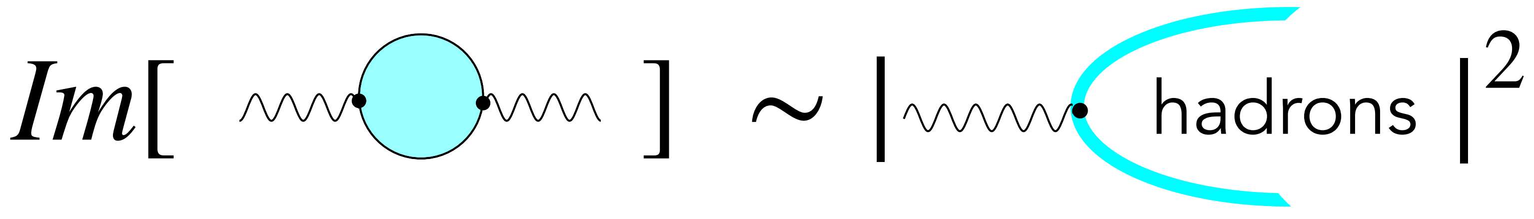

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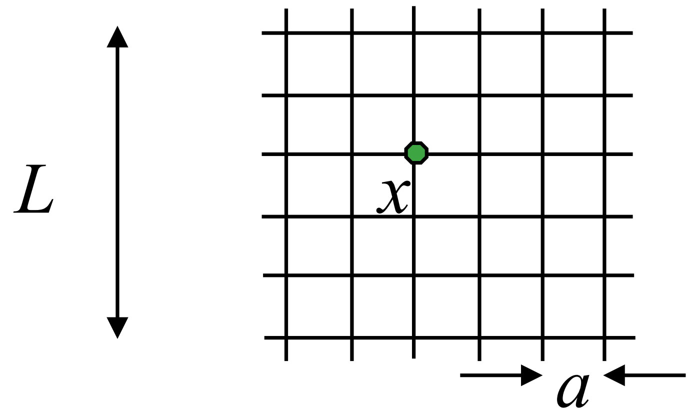

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Muon g-2 Theory Initiative

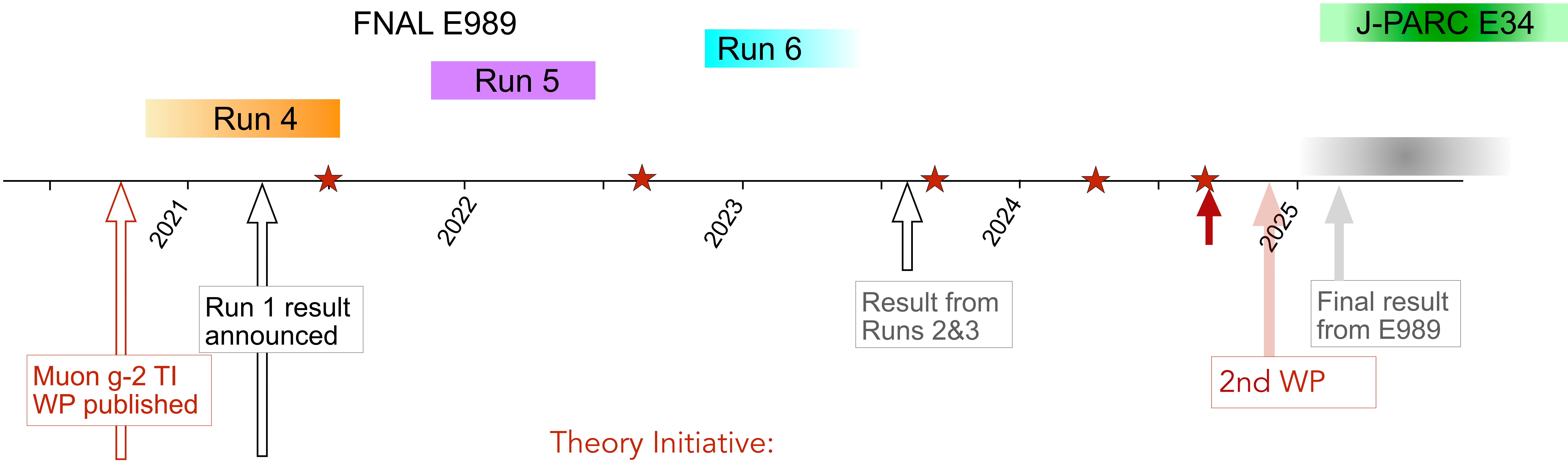
Steering Committee

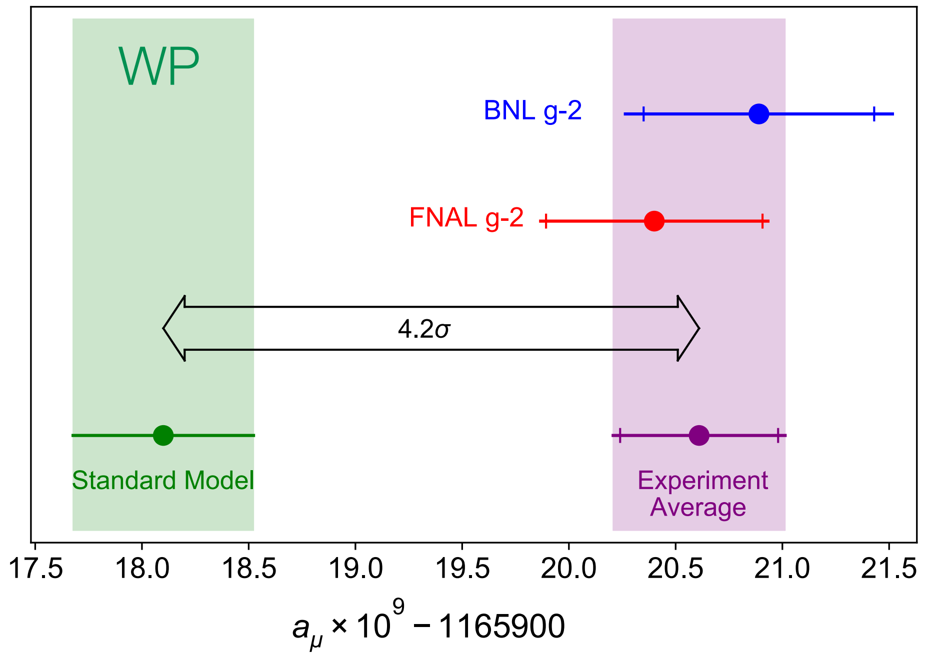
- Gilberto Colangelo (Bern)
- Michel Davier (Orsay) co-chair
- Aida El-Khadra (UIUC & Fermilab) chair
- Martin Hoferichter (Bern)
- Christoph Lehner (Regensburg University) co-chair
- Laurent Lellouch (Marseille)
- Tsutomu Mibe (KEK)
  J-PARC Muon g-2/EDM experiment
- Lee Roberts (Boston)
  Fermilab Muon g-2 experiment
- Thomas Teubner (Liverpool)
- Hartmut Wittig (Mainz)

https://muon-gm2-theory.illinois.edu

- Maximize the impact of the Fermilab and J-PARC experiments
  ➠ quantify and reduce the theoretical uncertainties on the SM prediction
- assess reliability of uncertainty estimates
- summarize the theory status: White Papers
- organize workshops to bring the different communities together:
  - First plenary workshop near Fermilab: 3-6 June 2017
  - …
  - Virtual Spring 2024 TI workshop hosted by UIUC: 15-17, 23-24 Apr 2024
  - Seventh plenary workshop hosted by KEK/KMI (Japan): 9-13 Sep 2024
  - Eight plenary workshop: Orsay (France), 8-12 Sep 2025
  - Ninth and tenth plenary workshops: US, UK

Near-term timeline





Theory Initiative:

- ☆ CMD-3 seminar (virtual): 27 March 2023 at 8:00am US CDT
- ☆ 2nd CMD-3 discussion meeting
- ☆ 8/9/2023: Status of Muon g-2 Theory in SM

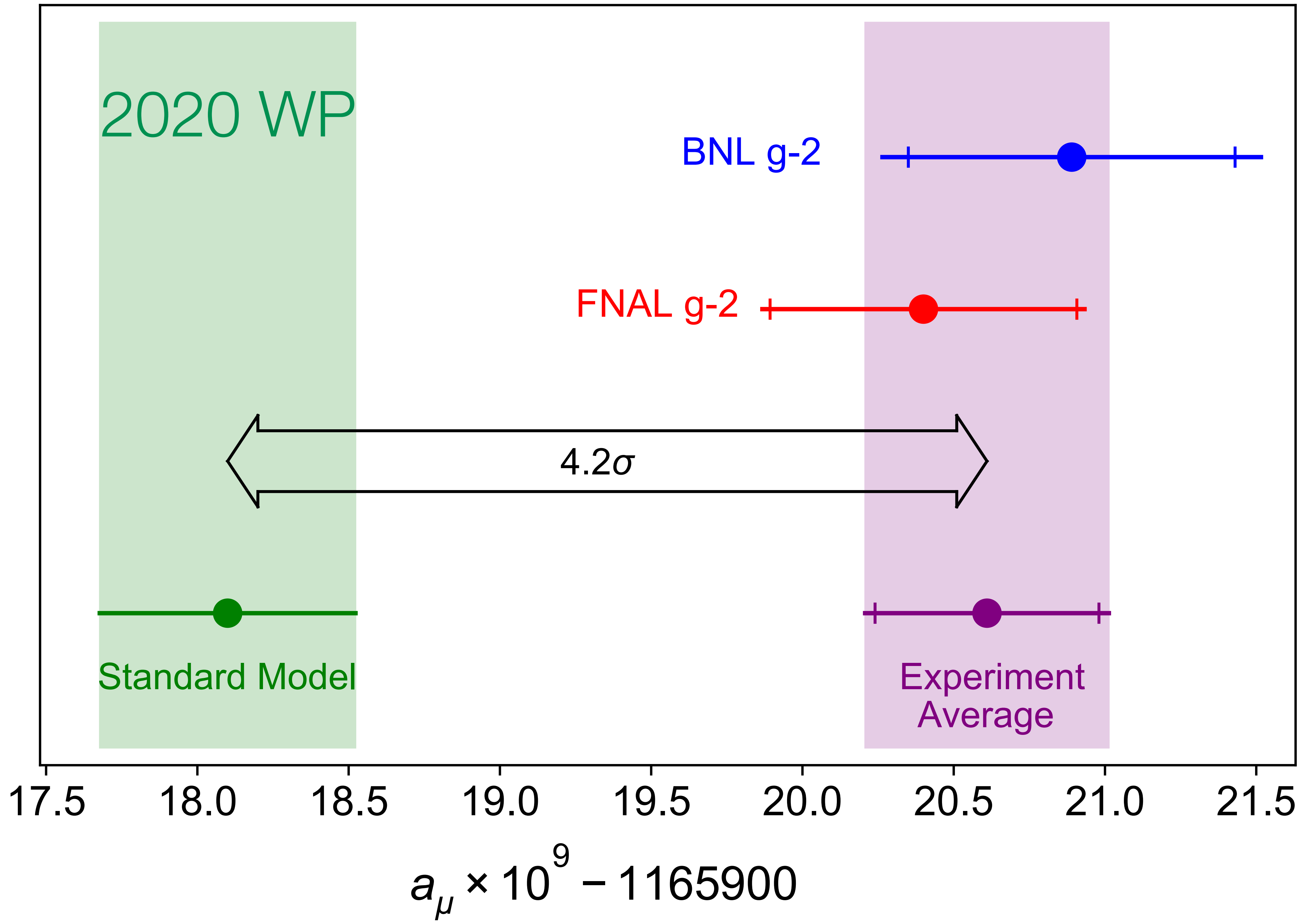
★ TI workshops:

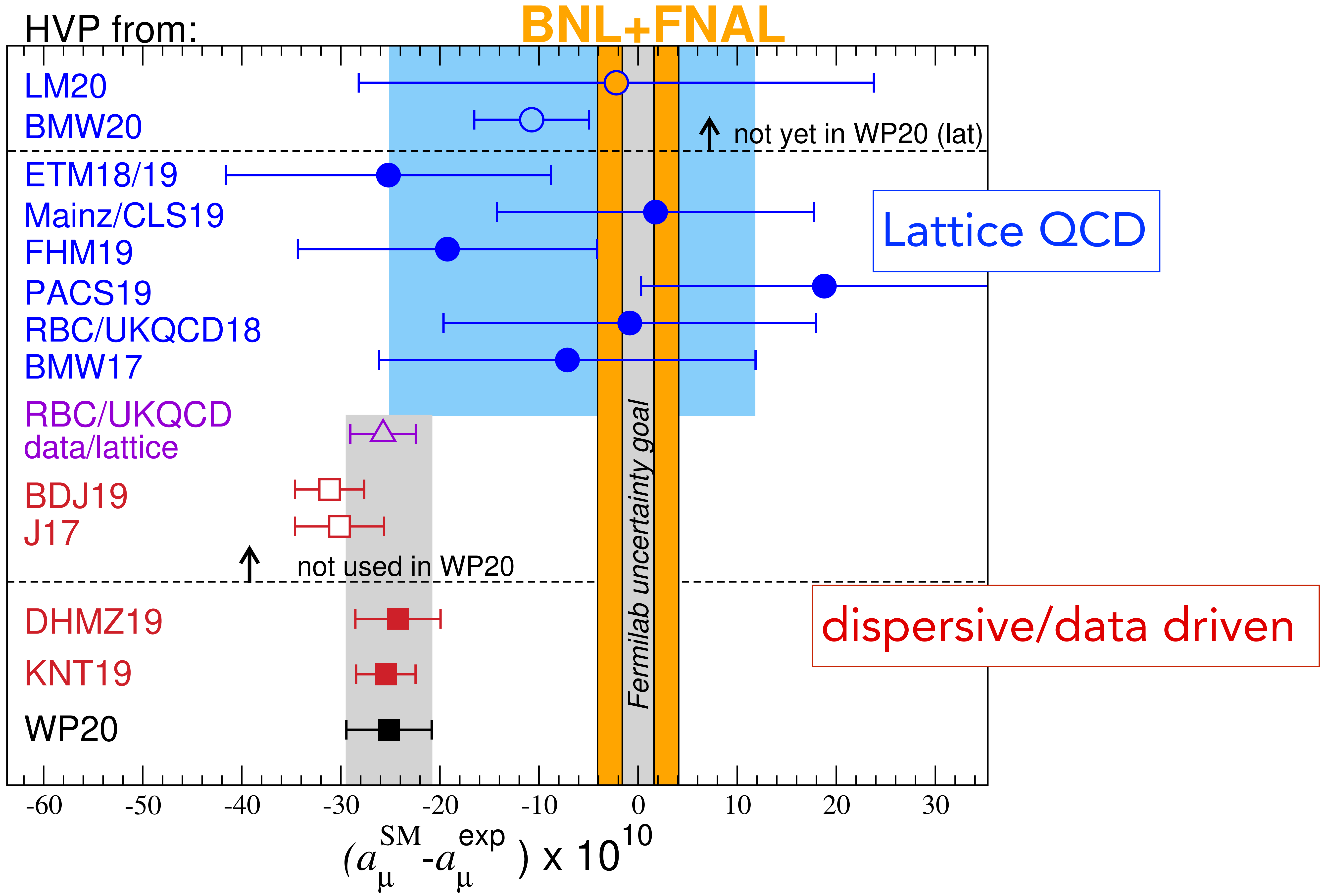
Jun 2021 @ KEK (virtual) | Sep 2023 @ Bern | Sep 2024 @ KEK & KMI

Sep 2022 @ Higgscentre | Apr 2024 (virtual)

Experiment vs SM theory

- 2021

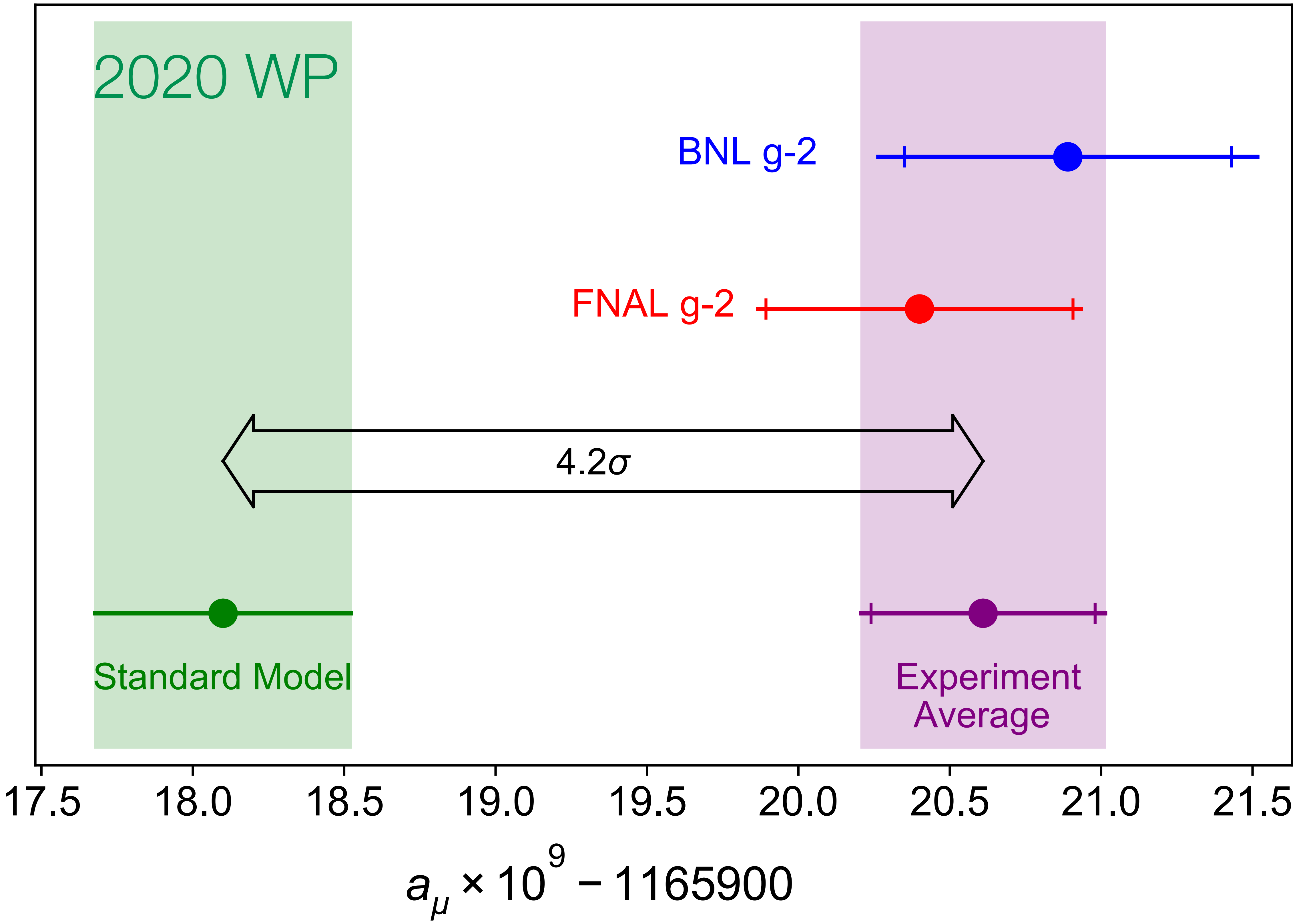




$$a_\mu^{\rm SM} = a_\mu^{\rm HVP} + [a_\mu^{\rm QED} + a_\mu^{\rm Weak} + a_\mu^{\rm HLbL}]$$

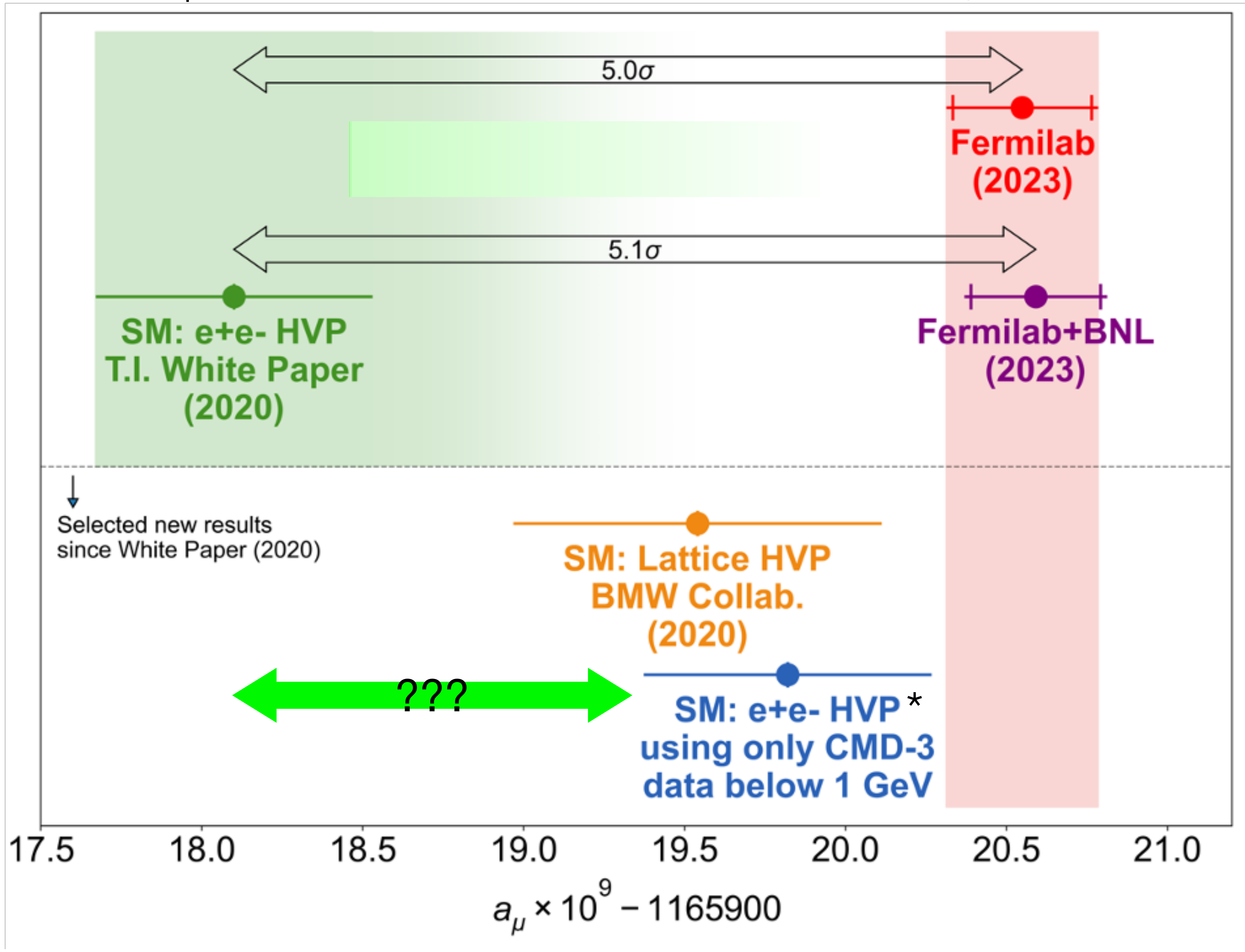
Experiment vs SM theory

- 2021



- 2023

adapted from J. Mott @ Scientific Seminar, 10 Aug 2023



*A. Keshavarzi @ Lattice 2023

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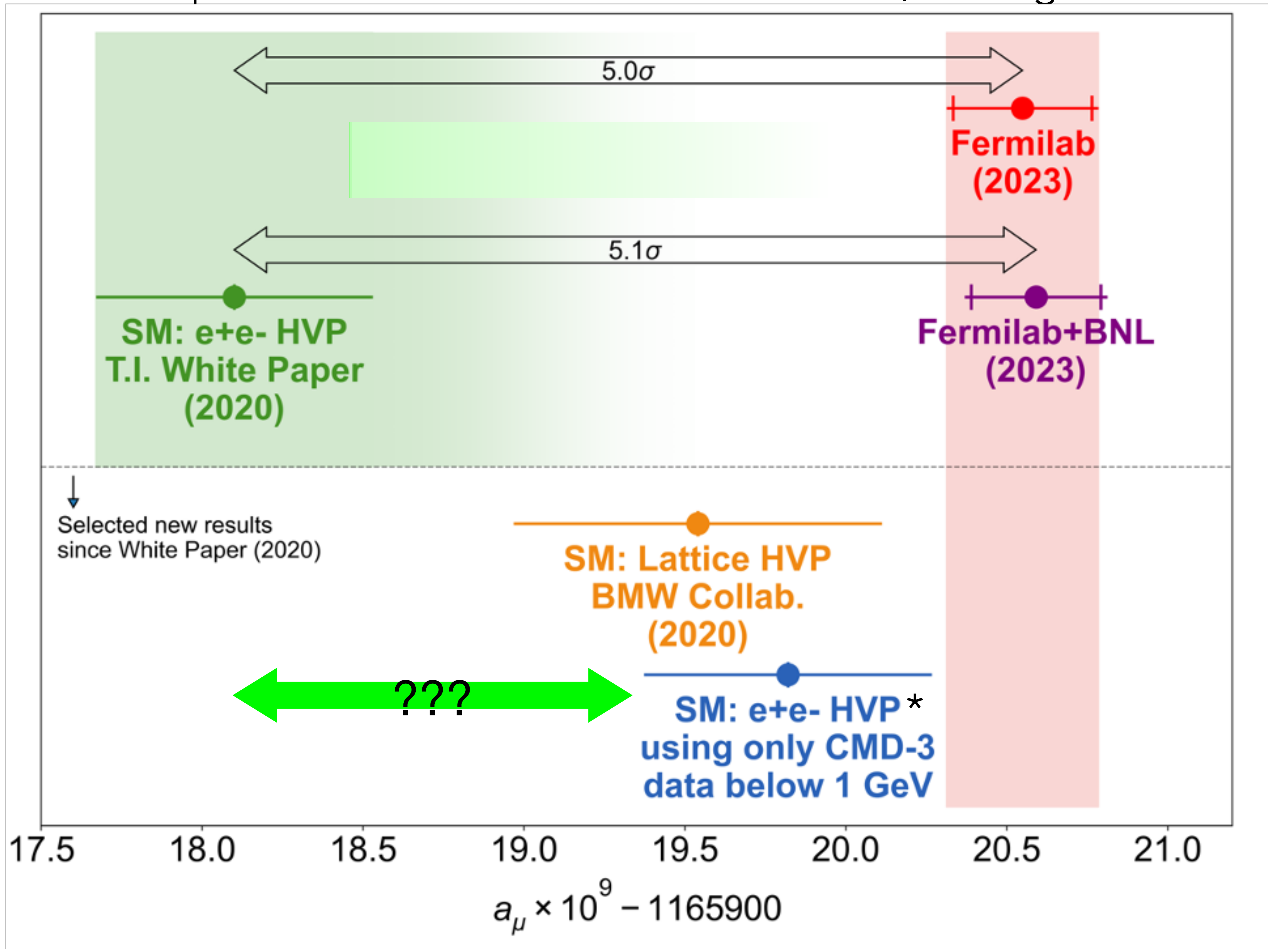
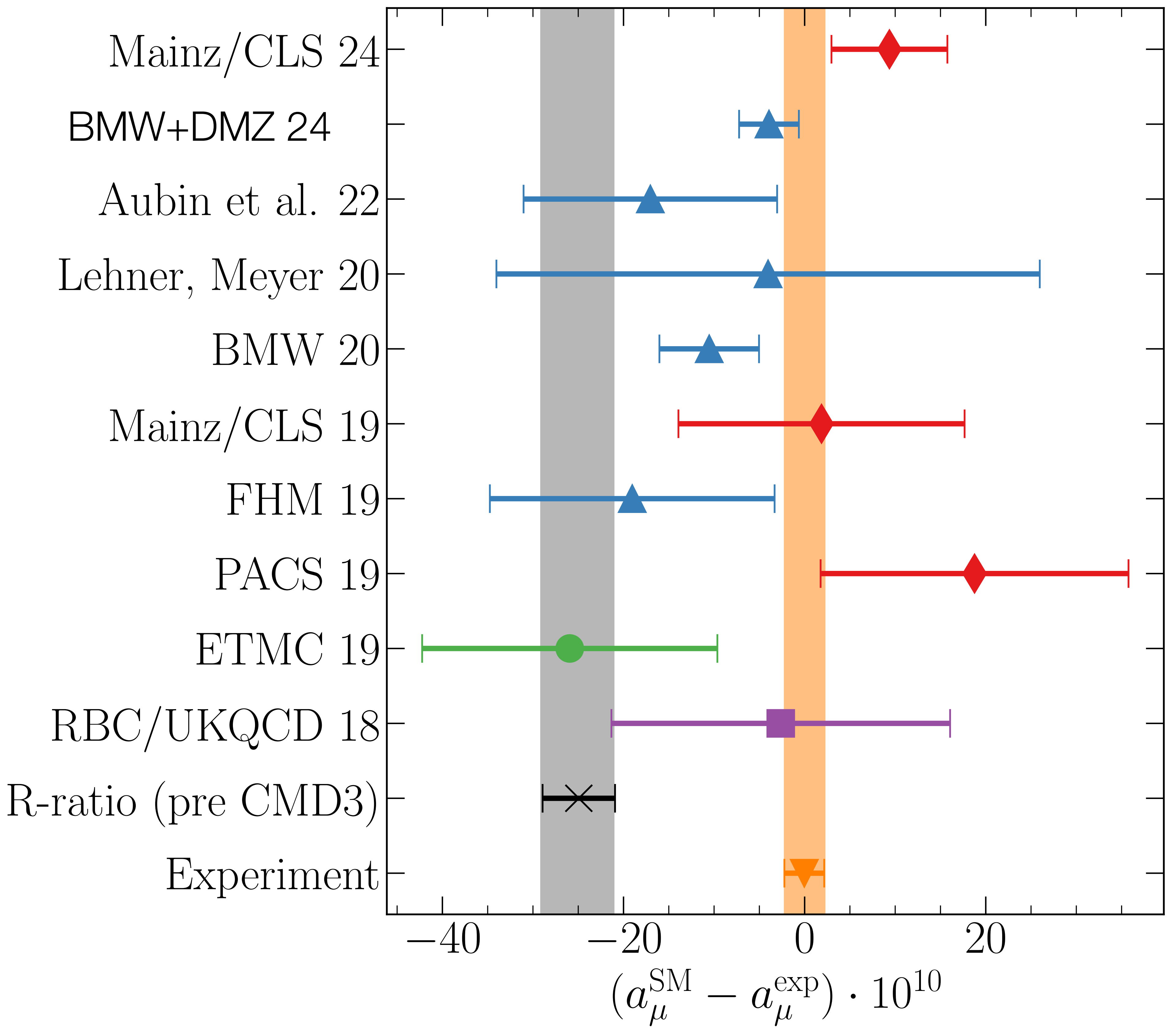


*A. Keshavarzi @ Lattice 2023

- 2024

New LQCD results (all using blind analyses):
BMW+DMZ 24 [arXiv:2407.10913]: LQCD+R-ratio (hybrid)
RBC/UKQCD: Lehner@Lattice 2024
Mainz: Kuberski @ KEK 2024
FNAL/HPQCD/MILC: exp. fall 2024

S. Kuberski @ KEK 2024



Overview of the experiments

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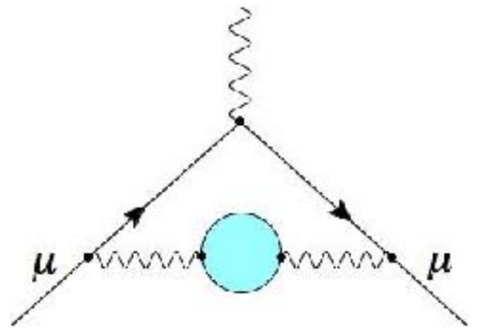
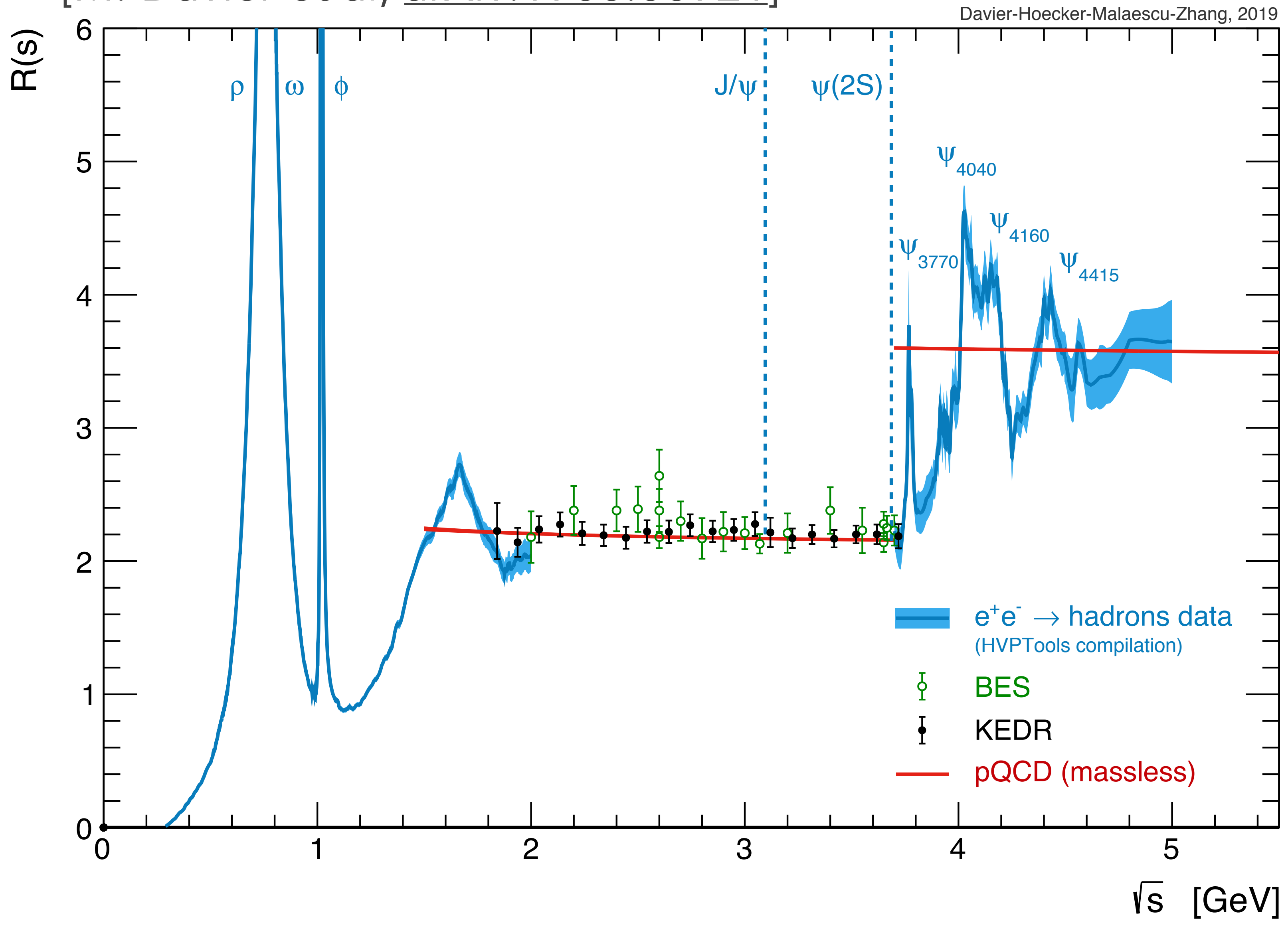


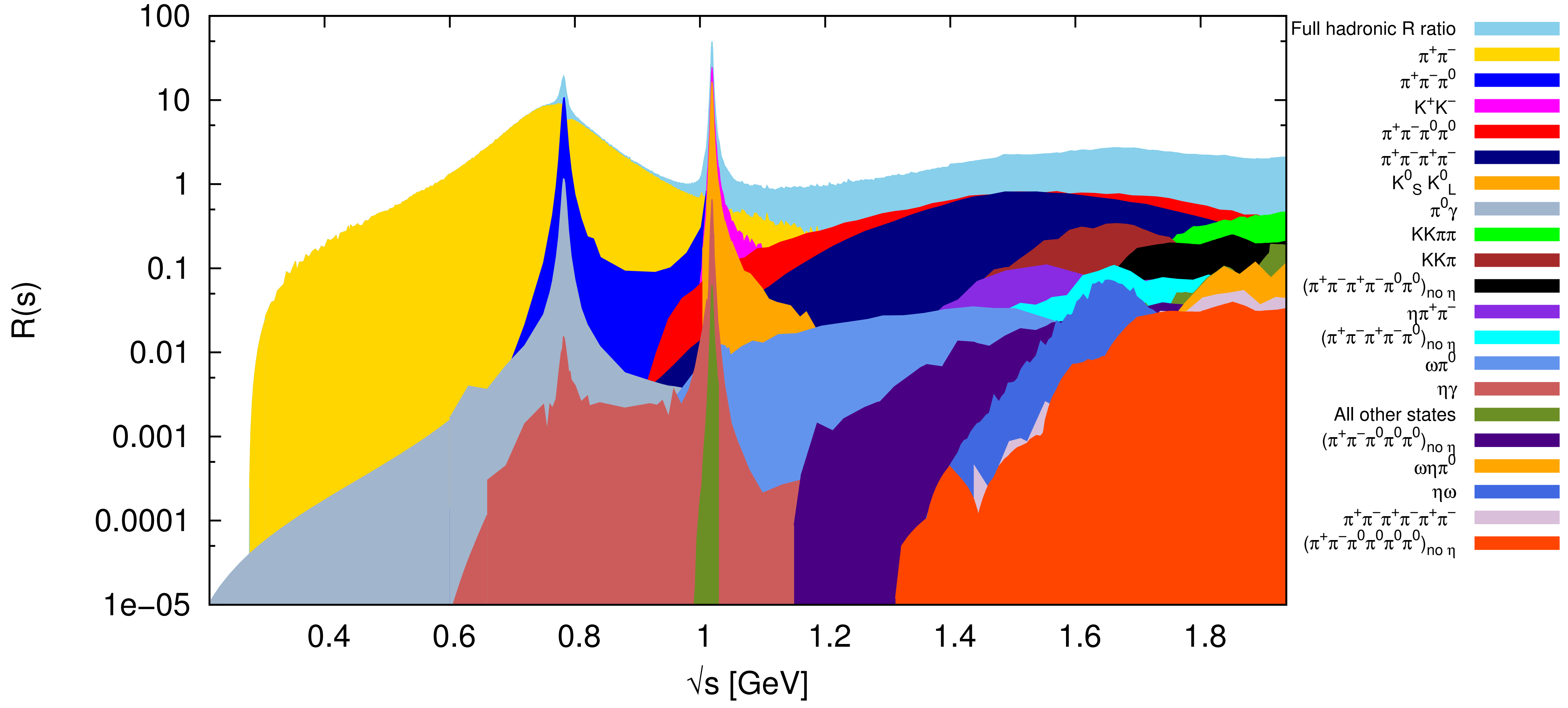
HVP: data-driven

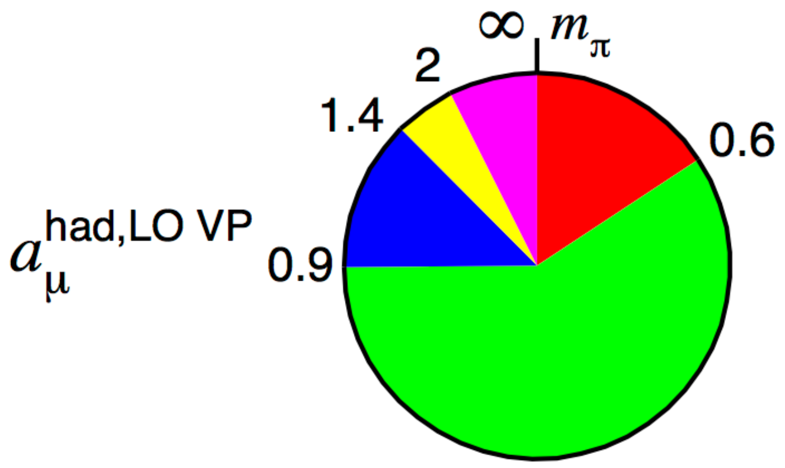
$$a_\mu^{\mathrm{HVP,LO}} = \frac{m_\mu^2}{12\pi^3}\int ds \frac{\hat{K}(s)}{s}\sigma_{\mathrm{exp}}(s)$$

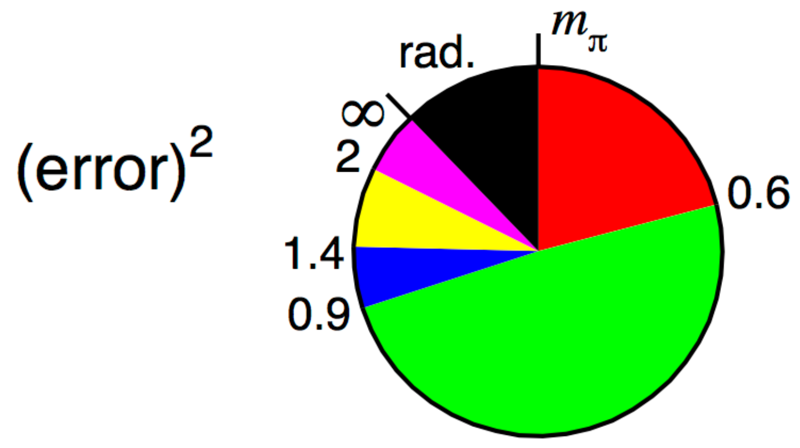
[M. Davier et al, arXiv:1908.00921]



[A. Keshavarzi et al, arXiv:1802.02995]







- $\sigma_{\mathrm{had}}(s)$ defined to include real & virtual photons
- direct integration method: no modelling of $\sigma_{\mathrm{had}}(s)$, summing up contributions from all hadronic channels
- total hadronic cross section $\sigma_{\mathrm{had}}(s)$ from > 100 data sets in 35+ channels summed up to $\sqrt{s} \sim 2\,\mathrm{GeV}$
- $\sqrt{s} > 2$ GeV: inclusive data + pQCD + narrow resonances
- two independent compilations (DHMZ, KNT)

Tensions (of up to 3σ) between data sets:
➠ conservative procedure to include differences in error estimate

HVP: data-driven

Since 2020:

- 20+ papers with new experimental measurements for $\sigma_{\rm had}(s)$, for example: $\pi\pi$, $\pi\pi\pi$, $K\bar{K}\pi$, $\eta\pi\gamma$, 4π, $\pi\pi\pi\eta$, …
- all ~ consistent with previous results

HVP: data-driven

Since 2020:
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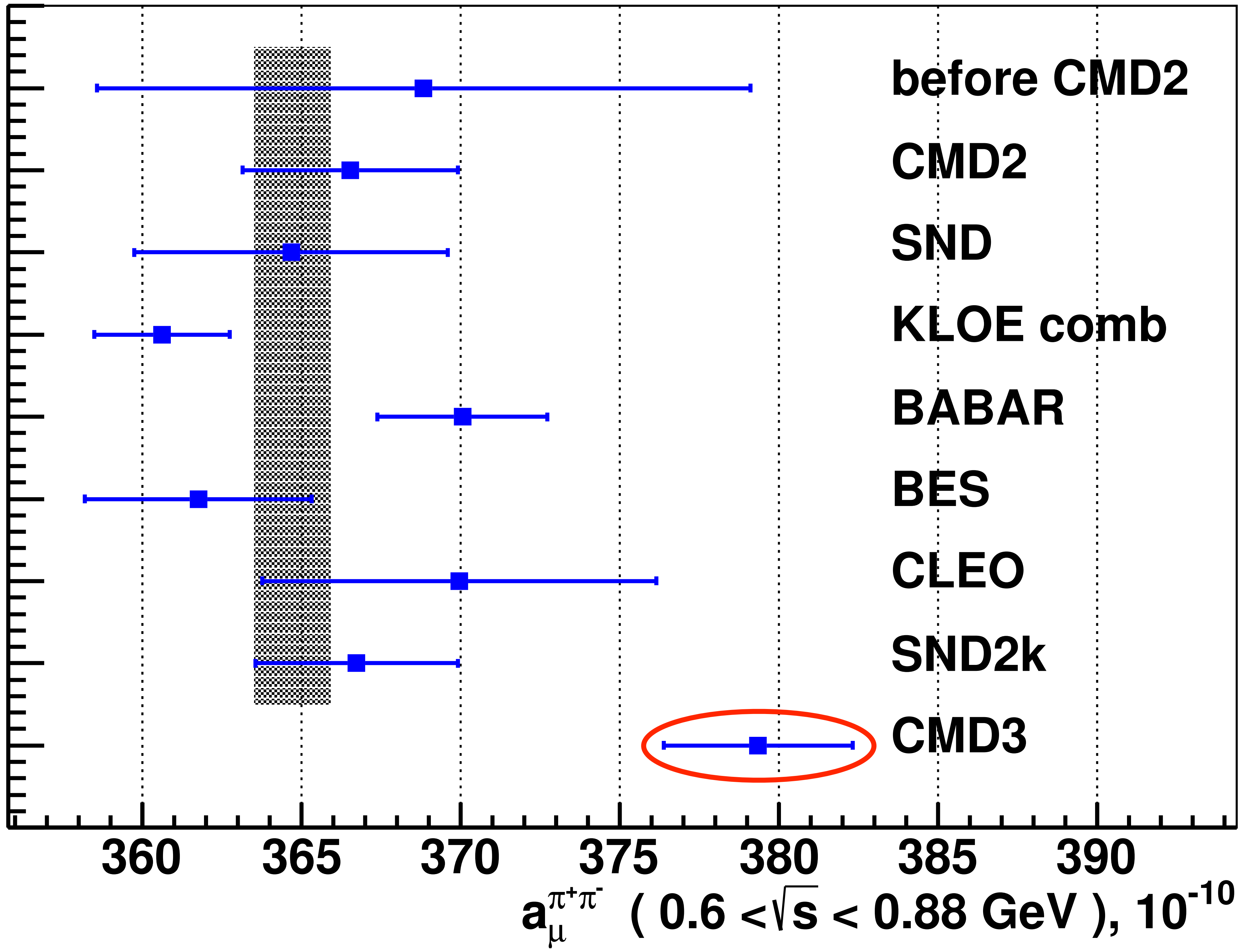
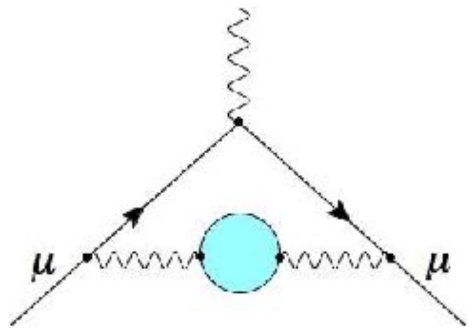
Feb 2023: from CMD-3 [F. Ignatov et al, arXiv:2302.08834, PRD 2024]



A new puzzle!
- discrepancies between experiments now $\gtrsim (3-5)\,\sigma$ need to be understood/resolved
- (virtual) scientific seminar + discussion panel on CMD-3 measurement March 27 (8:00 –11:00 am US CDT)
- 2nd CMD-3 discussion meeting (20 July 2023)
- Discussions are continuing….



HVP: data-driven

Updates presented at the 7th Muon g-2 TI workshop @ KEK

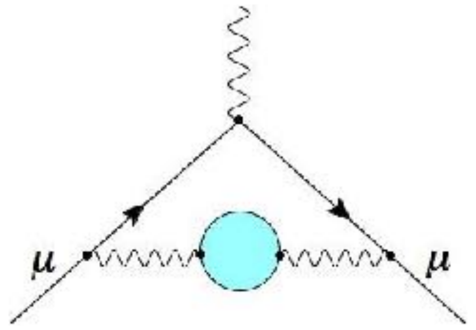
Ongoing work on experimental inputs:

- BaBar: ongoing analysis of large data set (not included before) in $\pi\pi$ channel, also $\pi\pi\pi$, other channels
- CMD3: ongoing analyses, comparisons with CMD-2 procedures, new data expected
- KLOE: ongoing analysis of large data set in $\pi\pi$ channel (not included before), other channels
- SND: new results for $\pi\pi$ channel, other channels in progress
- BESIII: new results in 2021 for $\pi\pi$ channel, continued analysis also for $\pi\pi\pi$, other channels
- Belle II: first results for $\pi\pi\pi$ in 2024, ramping up $\pi\pi$ analysis
  Better ultimate statistics than BaBar or KLOE; similar or better systematics for low-energy cross sections

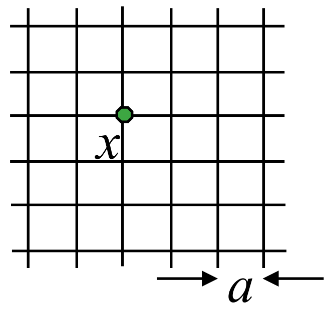
Ongoing work on theoretical aspects:

- better treatment of structure dependent radiative corrections (NLO) in $\pi\pi$ and $\pi\pi\pi$ channels
- new dispersive treatment [Colangelo at al, arXiv:2207.03495]
- Developing NNLO Monte Carlo generators (STRONG 2020 workshop https://agenda.infn.it/event/28089/)

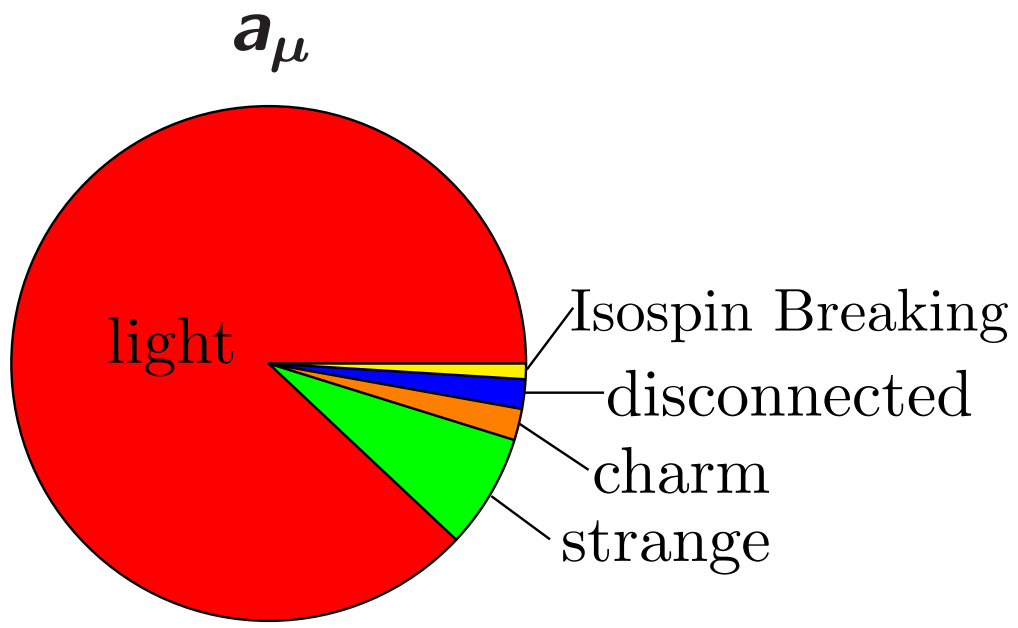
- including τ decay data: requires nonperturbative evaluation of IB correction [M. Bruno et al, arXiv:1811.00508]



Lattice HVP: Introduction



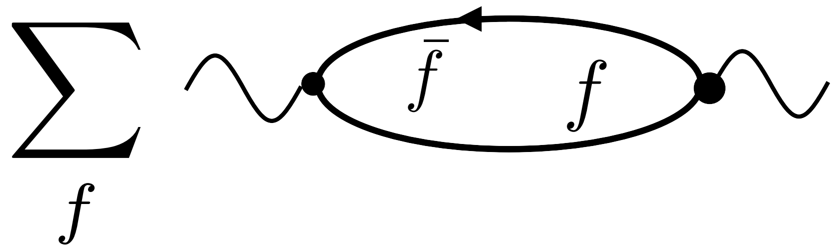
Calculate $a_\mu^{\rm HVP}$ in Lattice QCD: $$a_\mu^{\rm HVP,LO} = \sum_f a_{\mu,f}^{\rm HVP,LO} + a_{\mu,\rm disc}^{\rm HVP,LO}$$



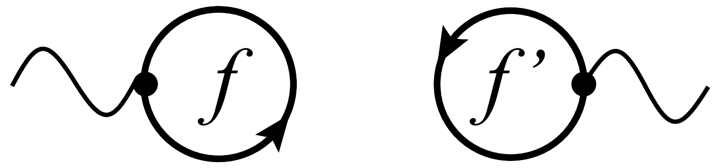
- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)

  Note: almost always $m_u = m_d$

$f = ud, s, c, b$

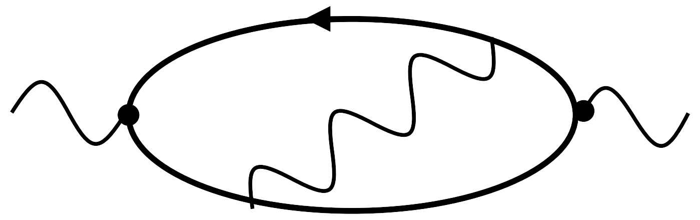


- light-quark connected contribution: $a_\mu^{\rm HVP,LO}(ud)$ ~90% of total
- *s*,*c*,*b*-quark contributions $a_\mu^{\rm HVP,LO}(s,c,b)$ ~8%, 2%, 0.05% of total

\+ 

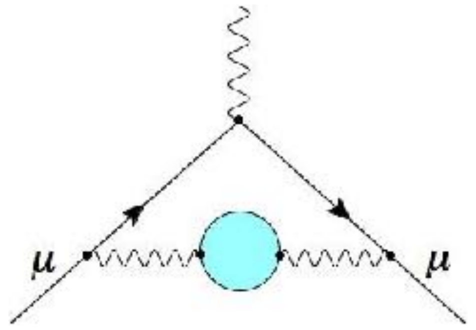
- disconnected contribution: $a_{\mu,\rm disc}^{\rm HVP,LO}$ ~2% of total

- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:

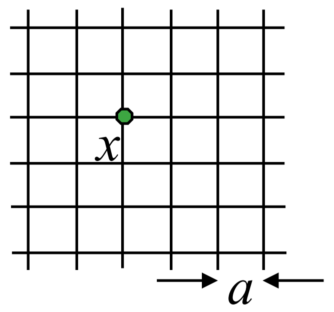
\+  + …

- Isospinbreaking (QED + $m_u \neq m_d$) corrections: $\delta a_\mu^{\rm HVP,LO}$ ~1% of total

$$a_\mu^{\rm HVP,LO} = a_\mu^{\rm HVP,LO}(ud) + a_\mu^{\rm HVP,LO}(s) + a_\mu^{\rm HVP,LO}(c) + a_{\mu,\rm disc}^{\rm HVP,LO} + \delta a_\mu^{\rm HVP,LO}$$

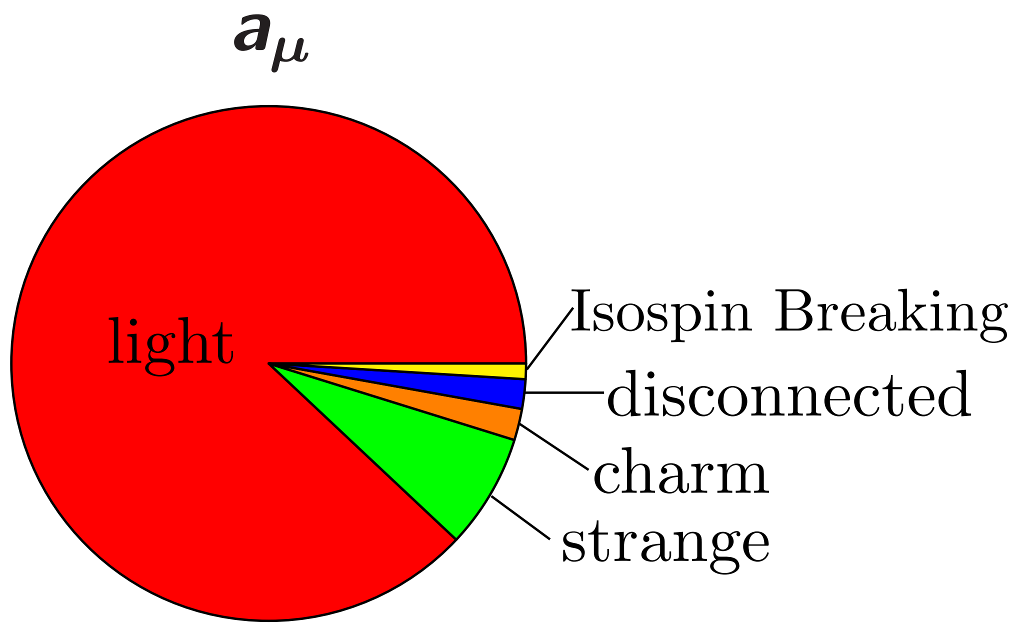


Lattice HVP: challenges



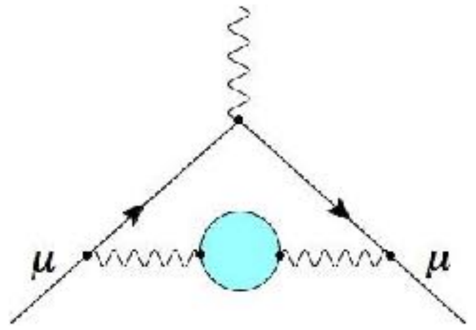
Calculate a_μ^{HVP} in Lattice QCD: $$a_\mu^{\mathrm{HVP,LO}} = \sum_f a_{\mu,f}^{\mathrm{HVP,LO}} + a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}}$$

$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{\mathrm{HVP,LO}}(ud) + a_\mu^{\mathrm{HVP,LO}}(s) + a_\mu^{\mathrm{HVP,LO}}(c) + a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}} + \delta a_\mu^{\mathrm{HVP,LO}}$$

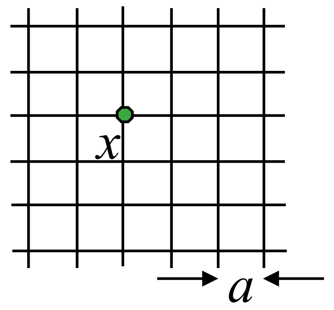


- $a_\mu^{\mathrm{HVP,LO}}$ needed with $< 0.5\,\%$ precision
- subpercent statistical precision:
  exponentially growing noise-to-signal in $C(t)$ as $t \to \infty$
  affects light-quark contributions
- sizable finite volume effects
- sensitivity to scale setting uncertainty
- control discretization effects
- quark-disconnected diagrams: control noise
- include isospin-breaking effects
  Separation of $a_\mu^{\mathrm{HVP,LO}}$ into $a_\mu^{\mathrm{HVP,LO}}(ud)$ and $\delta a_\mu^{\mathrm{HVP,LO}}$ is scheme dependent.

- light-quark connected contribution:
  $a_\mu^{\mathrm{HVP,LO}}(ud)$ ~90% of total
- *s,c,b*-quark contributions
  $a_\mu^{\mathrm{HVP,LO}}(s,c,b)$ ~8%, 2%, 0.05% of total
- disconnected contribution:
  $a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}}$ ~2% of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
  $\delta a_\mu^{\mathrm{HVP,LO}}$ ~1% of total



Windows in Euclidean time



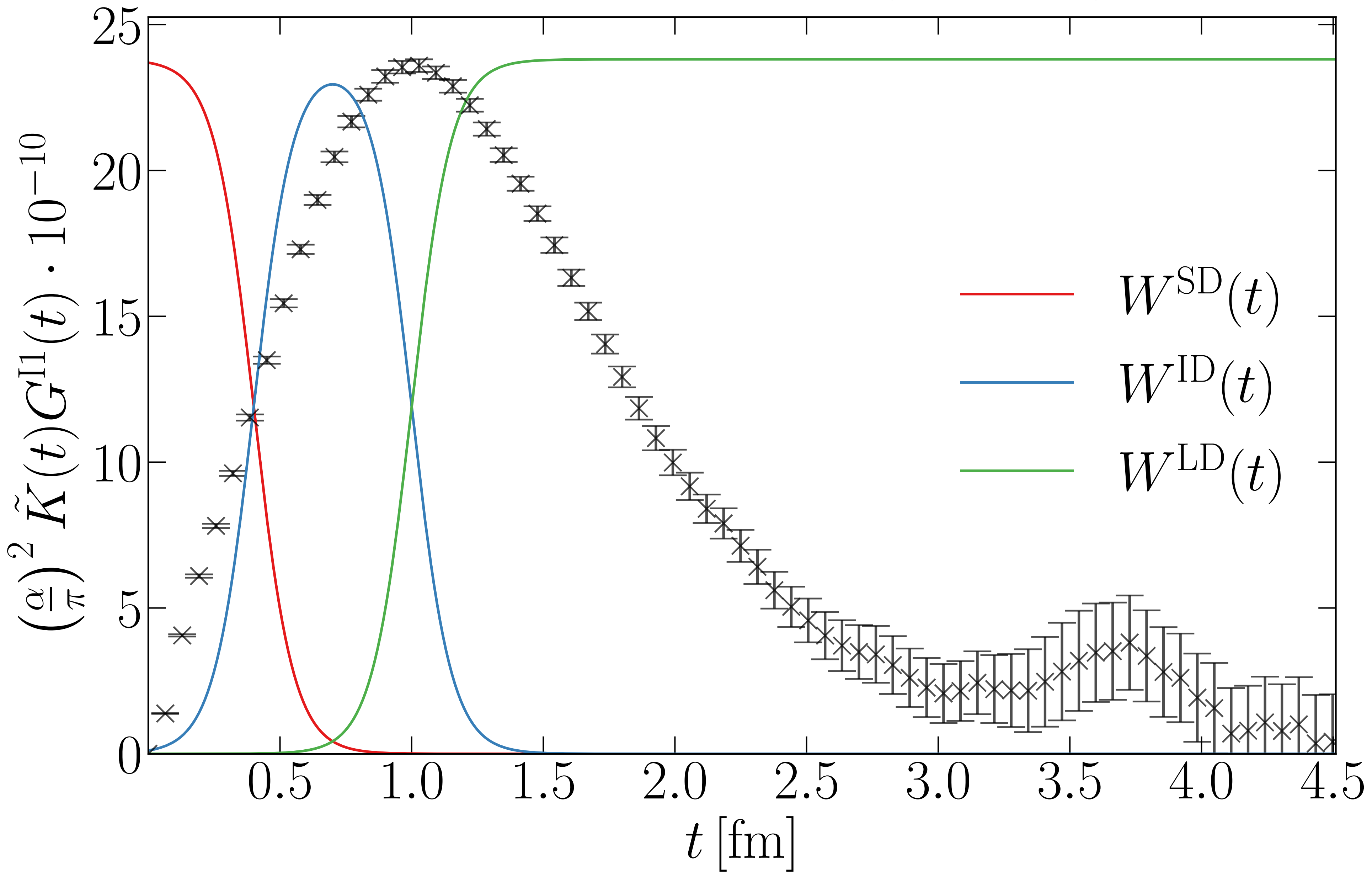
$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

- Use windows in Euclidean time to consider the different time regions separately
  [T. Blum et al, arXiv:1801.07224, 2018 PRL]

$t_0 = 0.4\,\mathrm{fm}, t_1 = 1.0\,\mathrm{fm}$

Short Distance (SD)	$t: 0 \to t_0$
Intermediate (W)	$t: t_0 \to t_1$
Long Distance (LD)	$t: t_1 \to \infty$

S. Kuberski @ Lattice 2023



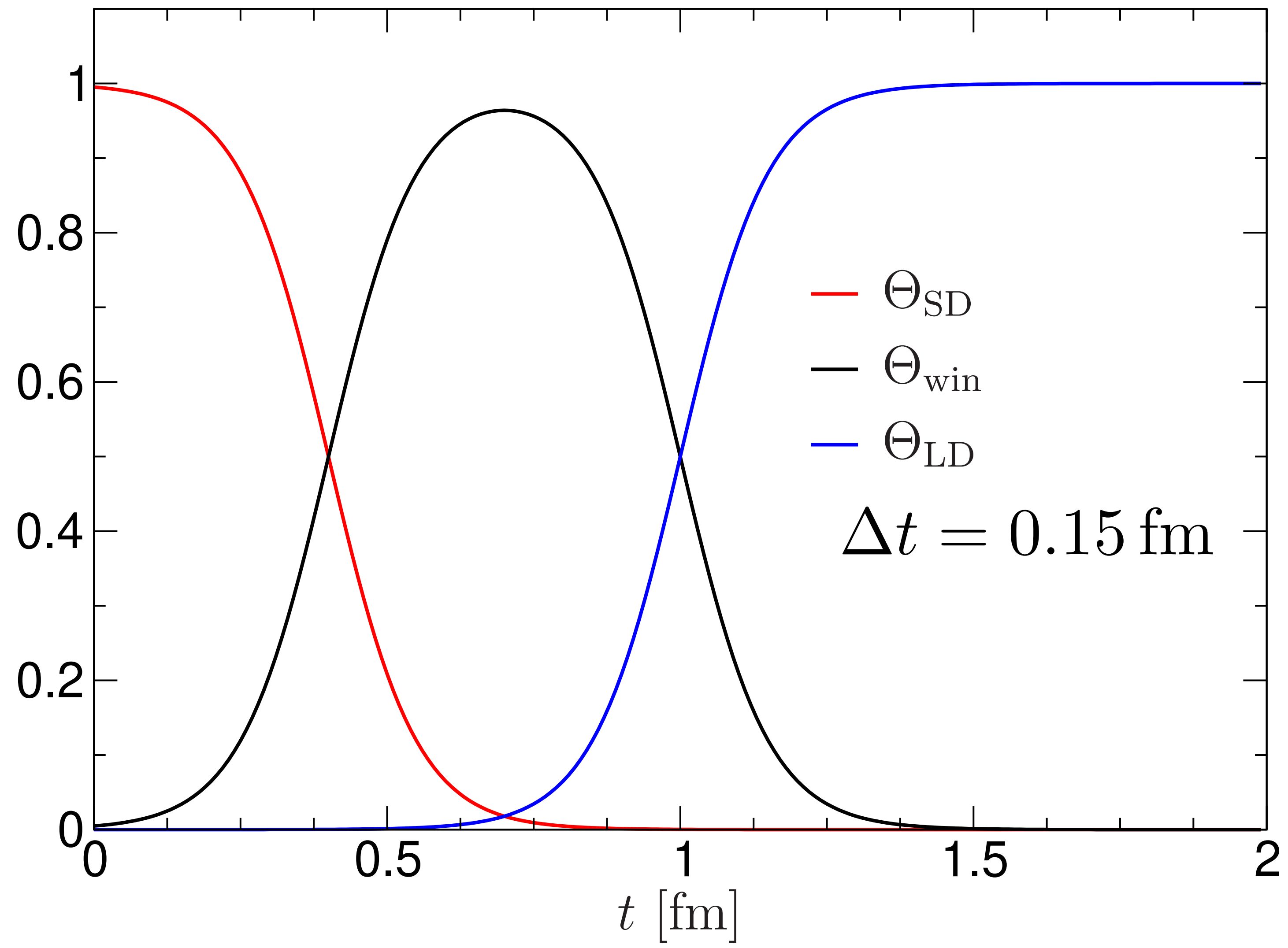
- disentangle systematics/statistics from long distance/FV and discretization effects
- intermediate window: easy to compute in lattice QCD; compare to disperse approach
- Internal cross check: compute each window separately (in continuum, infinite volume limits,…) and combine: $a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$

cross section inputs to windows observables

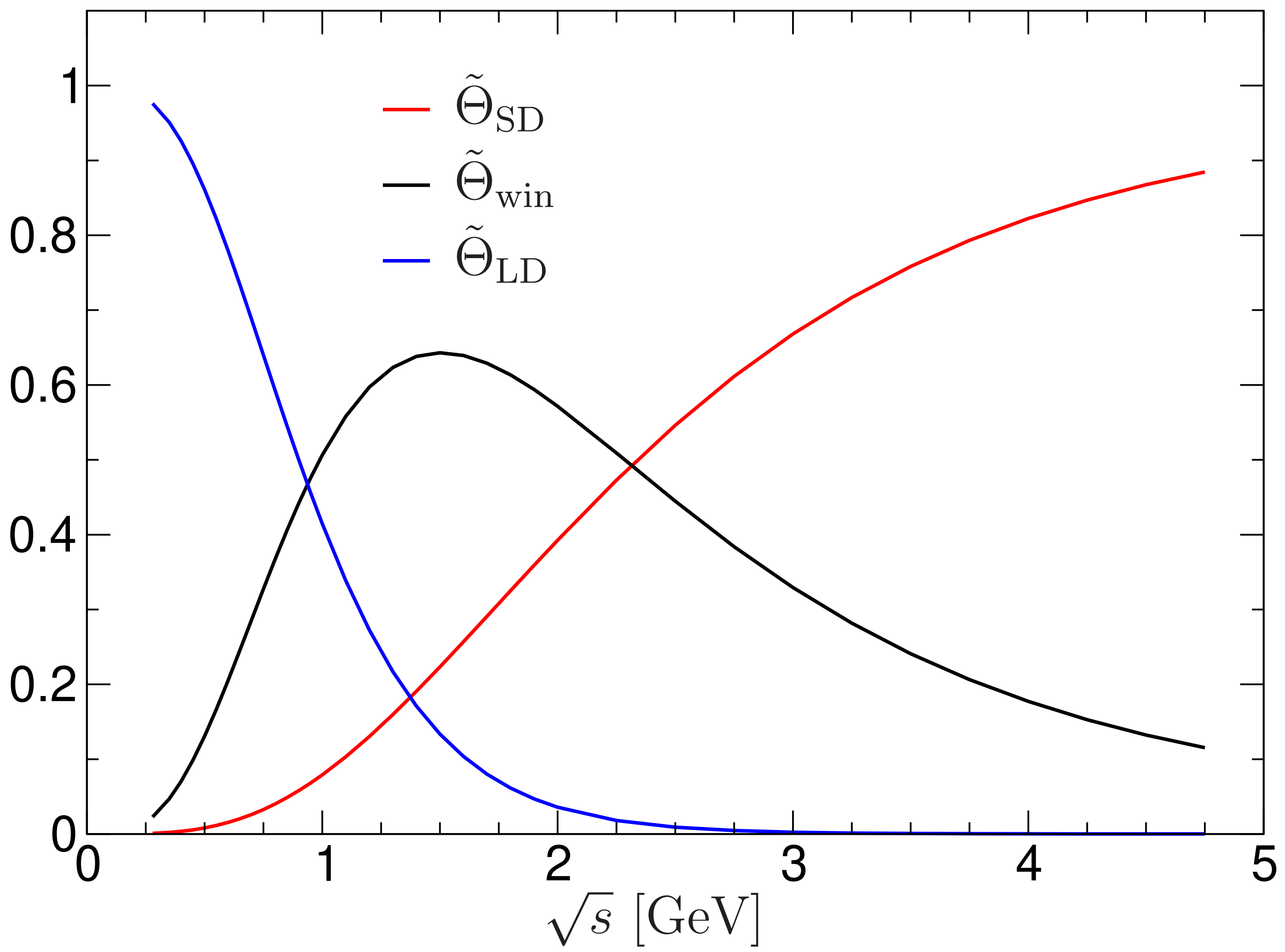
C. Davies @ Lattice 2024

Comparing data-driven and lattice HVP results

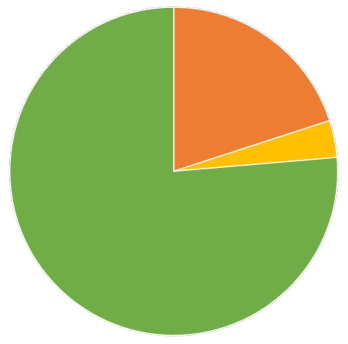
Colangelo et al
2205.12963



Mapping of window effects ↔

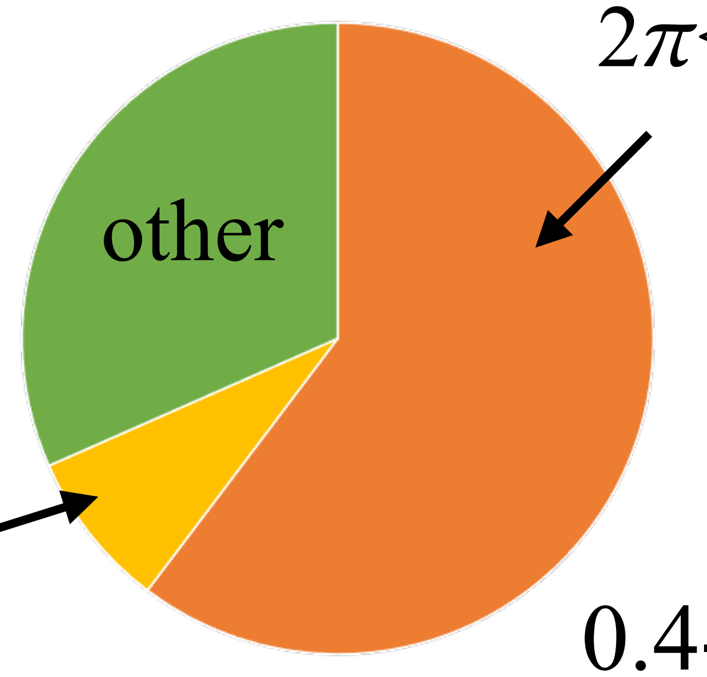


short-distance (SD)



0-0.4 fm

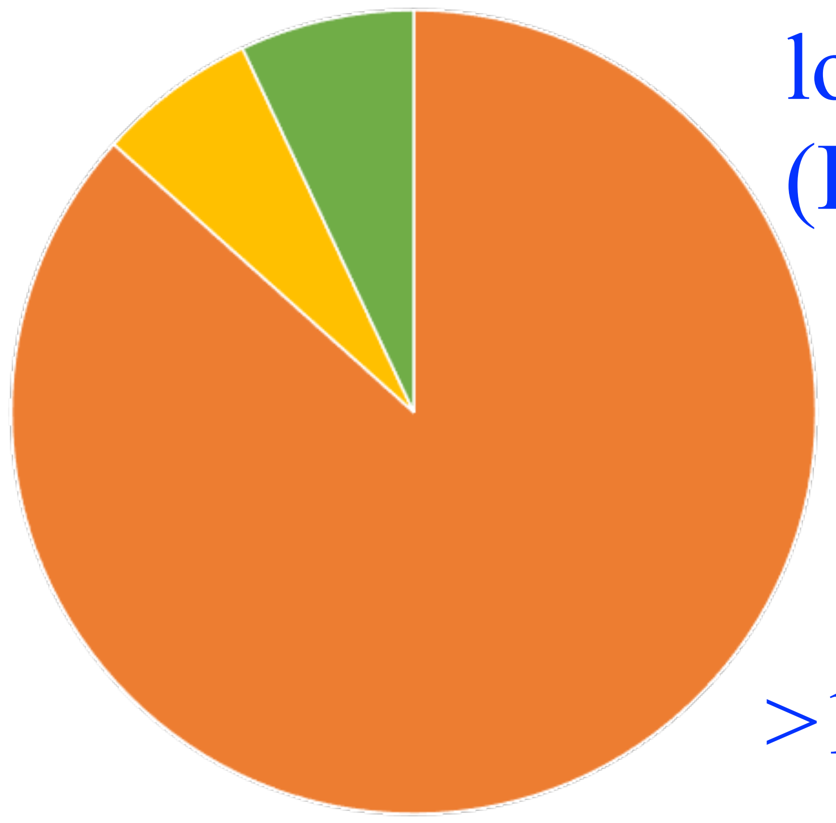
intermediate-distance



2π< 1 GeV

3π< 1.8 GeV

0.4-1.0 fm

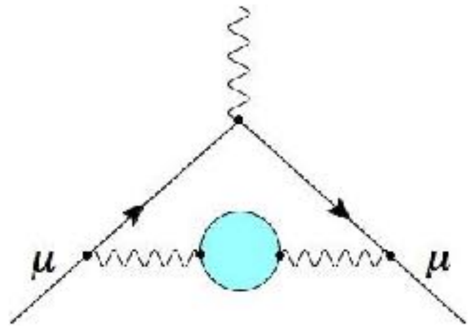


long-distance (LD)

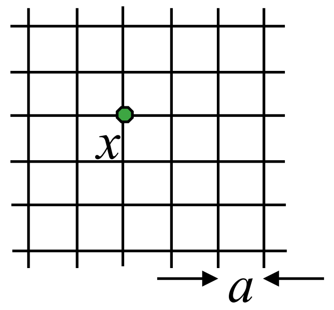
Data-driven contributions

>1.0 fm

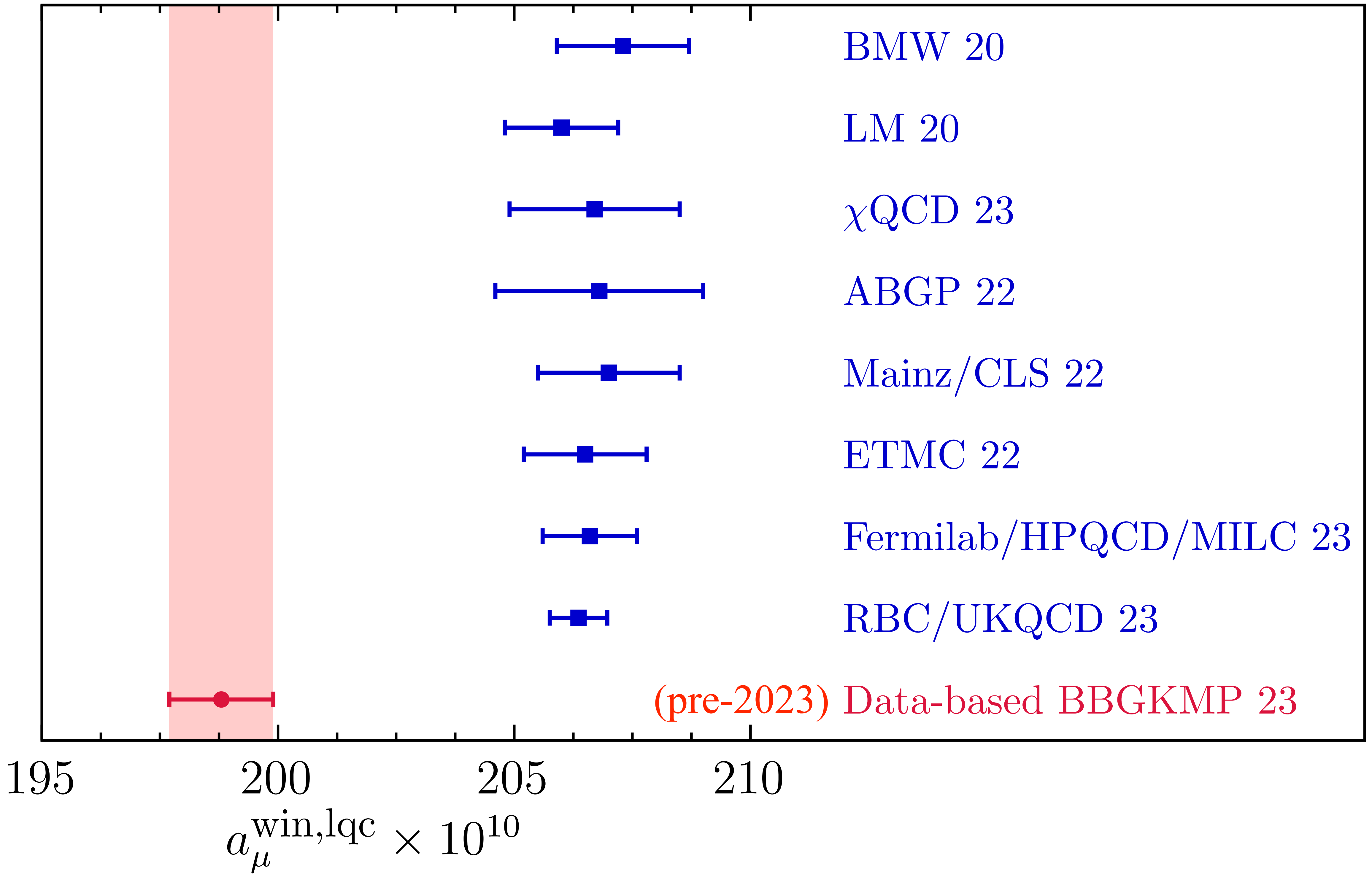
18



Lattice HVP: results for W



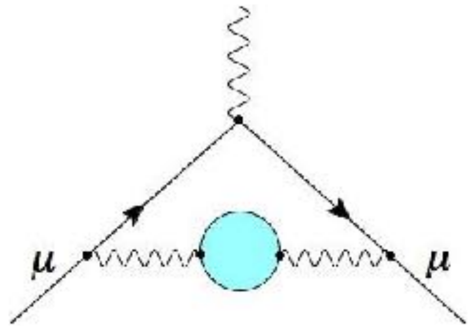
- new results in 2022-2024 for intermediate and short-distance windows.
- blind analyses by all lattice groups for results from 2023+
- focus on light-quark connected contribution to intermediate window:



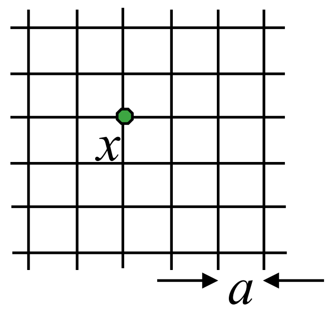
$\sim 5\sigma$ tension between LQCD and (pre-2023) data-driven evaluations for intermediate window

- dispersive evaluation of light-quark connected contribution [G. Benton, et al, arXiv:2306.16808]*

*based on disp. results for IB [Hoferichter et al, arXiv:2208.08993]



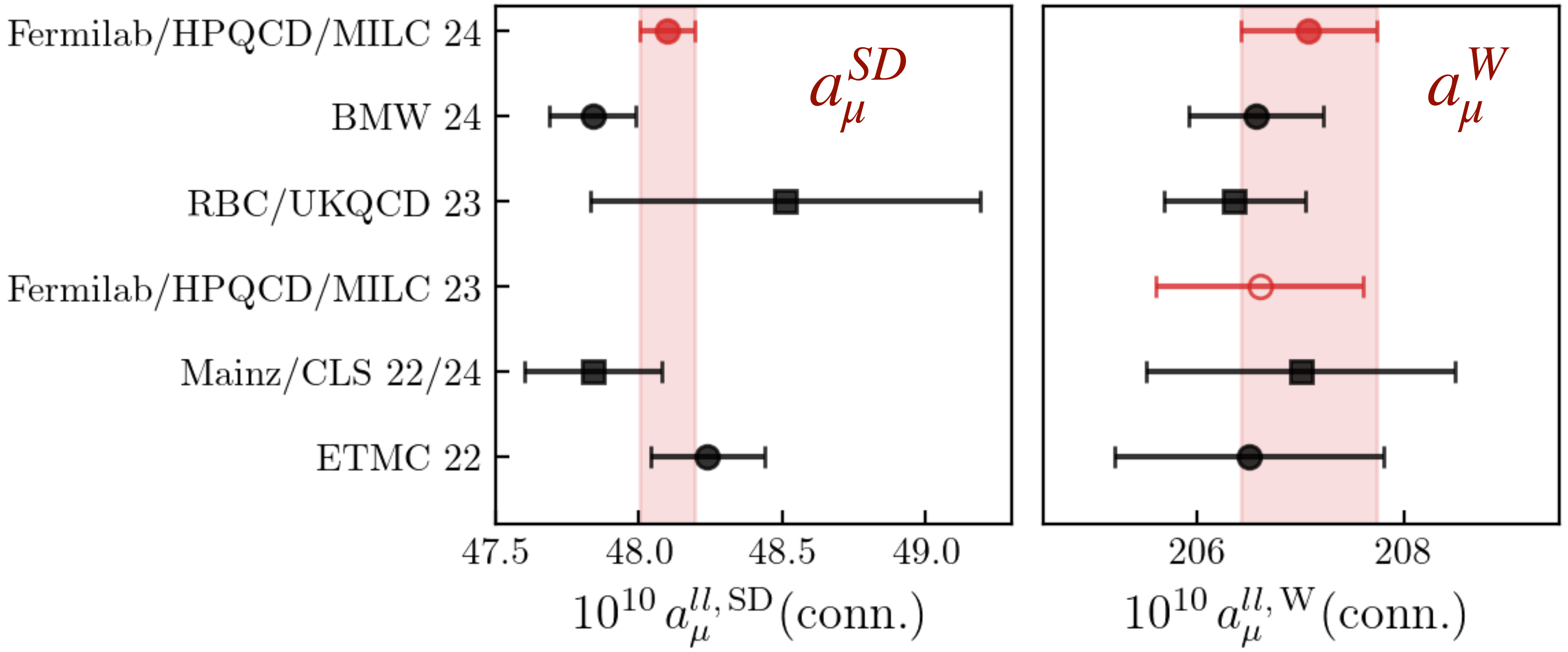
Lattice HVP: windows



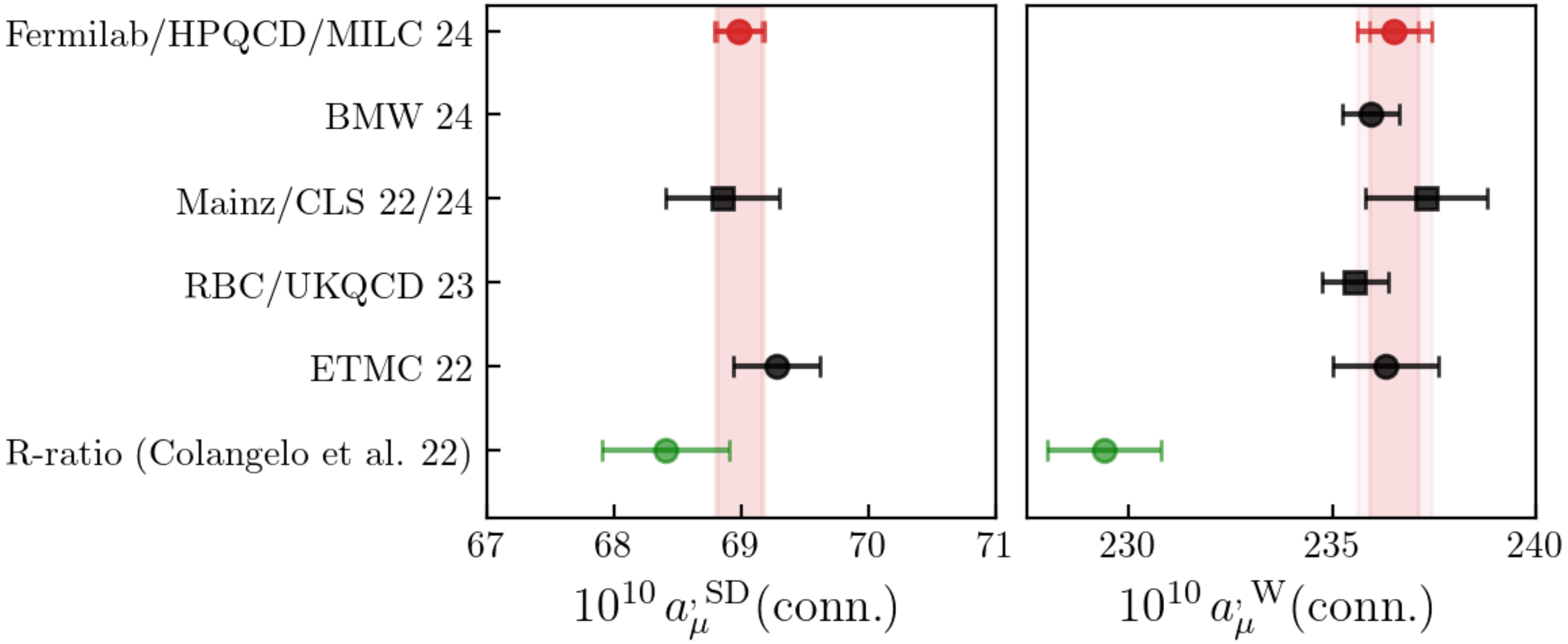
update: Fermilab/HPQCD/MILC 2024

S. Lahert talk @ KEK workshop:
Unblinded results for (all) contributions to a_μ^{SD} and a_μ^{W} (including correlations).

- light-quark connected

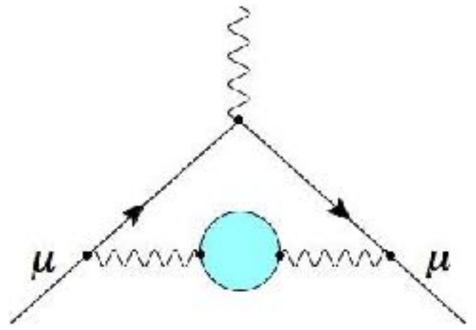


- total (u, d, s, c) + disc + IB

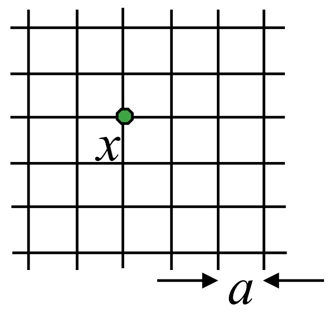


- W:
  - Consistent w/ all previous determinations
  - Leading uncertainty: scale setting (w_0 fm).
- SD: Good agreement with other groups.

➠ appendix for updates from BMW 2024 for W, SD windows



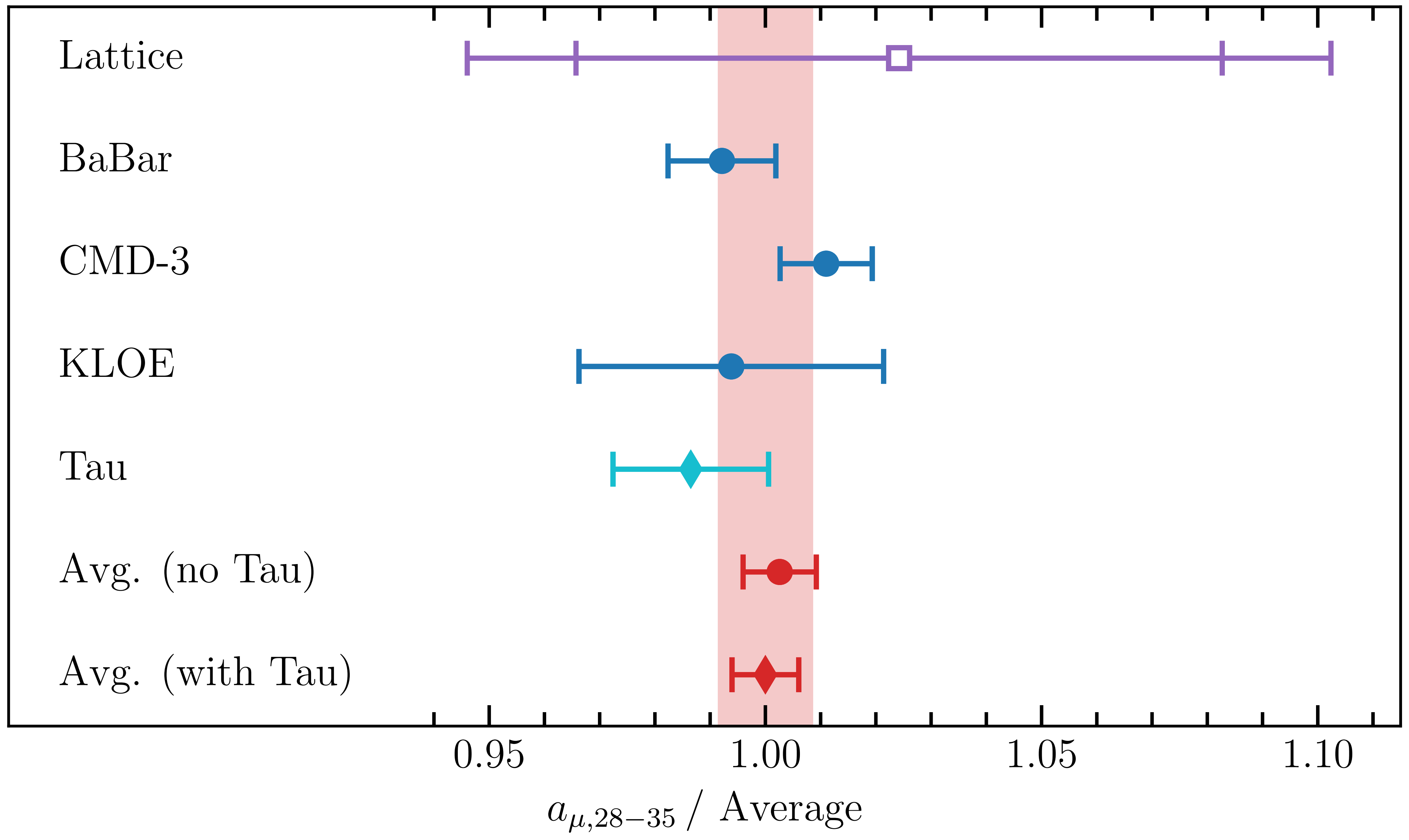
Lattice HVP: long-distance window



update: BMW+DMZ 2024

- BMW+DMZ 24 [A. Boccaletti et al, arXiv:2407.10913]

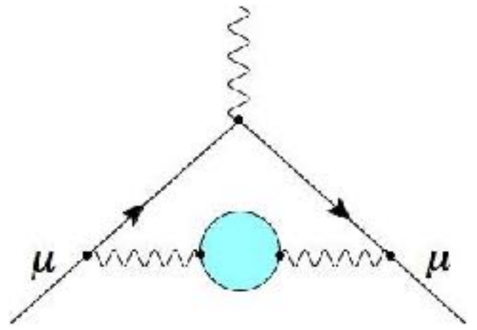
- statistical/systematic errors at long distances, $t \gtrsim 2.5$ fm still large:



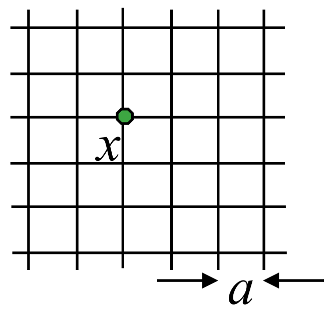
Laurent Lellouch @ Lattice @ CERN:

- Partial tail $a_{\mu,28\text{-}35}^{\text{LO-HVP}}$ for comparison with lattice dominated by cross section below ρ peak: $\sim 70\%$ for $\sqrt{s} \leq 0.63\,\text{GeV}$

- cross section measurements compatible for $\sqrt{s} < 0.5$ GeV



Lattice HVP: long-distance window

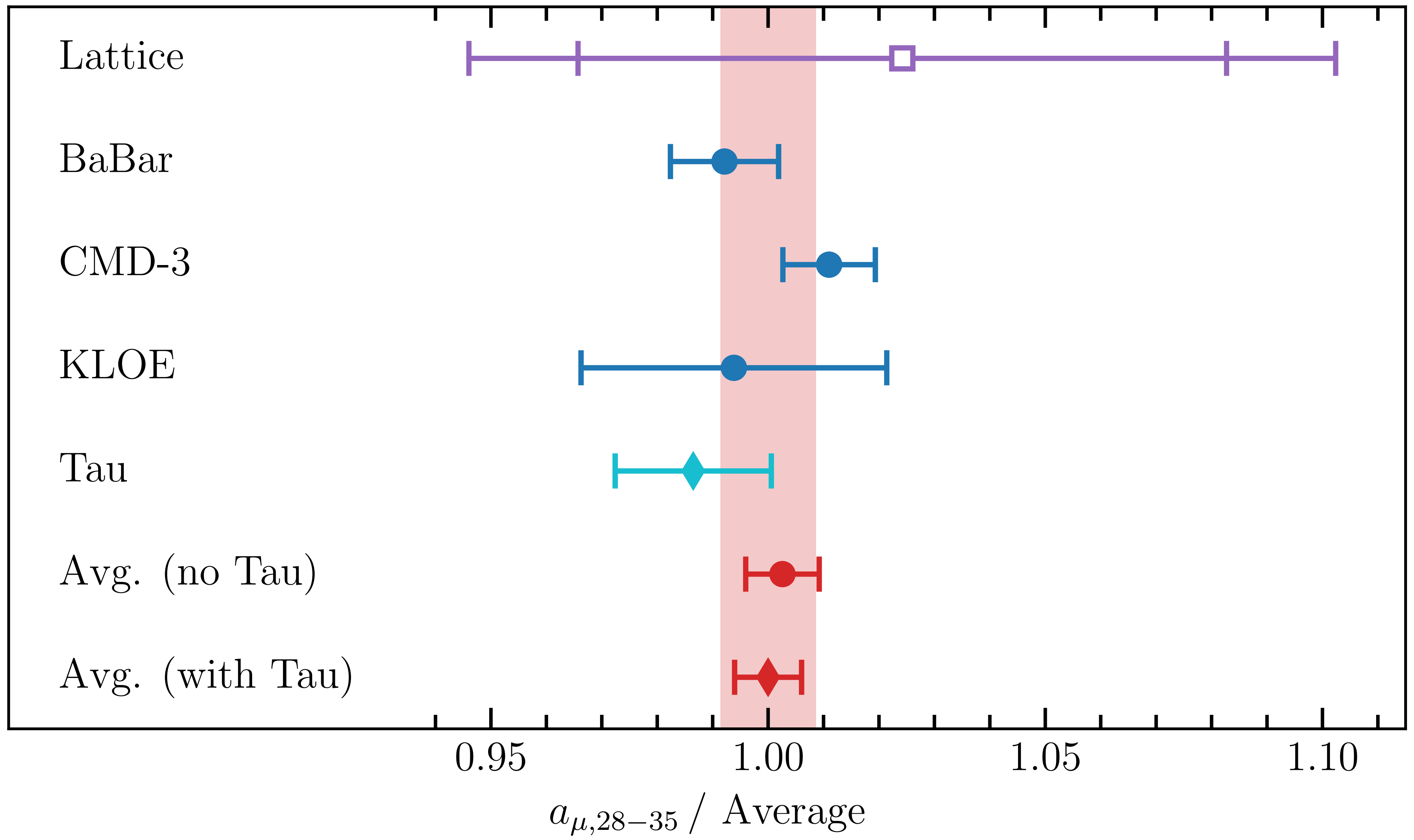


update: BMW+DMZ 2024

- BMW+DMZ 24 [A. Boccaletti et al, arXiv:2407.10913]

- statistical/systematic errors at long distances, $t \gtrsim 2.5$ fm still large:

[Laurent Lellouch Lattice @ CERN]



- All data-driven result agree very well
- Weighted average taken w/ and w/out τ: χ^2dof $= 1.1$ for both
- Final number: average w/ τ, PDG factor, and systematic = full difference τ/no-τ added linearly

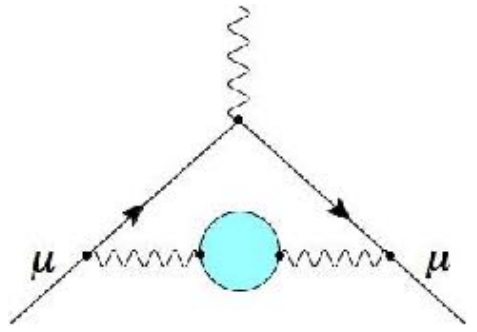
$$a_{\mu,28\text{-}35}^{\text{LO-HVP}} = 18.12(11)(5)[16]$$

- Excellent agreement w/ lattice, but uncertainty reduced by factor ~ 15
- Final number: average w/ τ, and systematic = full difference τ/no-τ added linearly

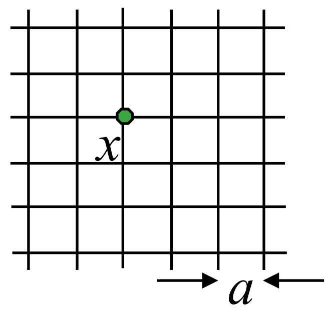
$$a_{\mu,28\text{-}\infty}^{\text{LO-HVP}} = 27.59(17)(9)[26]$$

- Only $\lesssim 5\%$ of final result for a_μ
- Contributes $\sim 65\%$ to total squared uncertainty uncertainty improvement: $5.5 \to 3.3$

➠ **hybrid evaluation:** combine lattice QCD calculation of one-sided window [Davies at at, arXiv:2207.04765] $a_\mu(t_1 = 2.8\text{ fm})$ with data-driven evaluation of long tail, $t > 2.8$ fm.



Lattice HVP: long-distance window

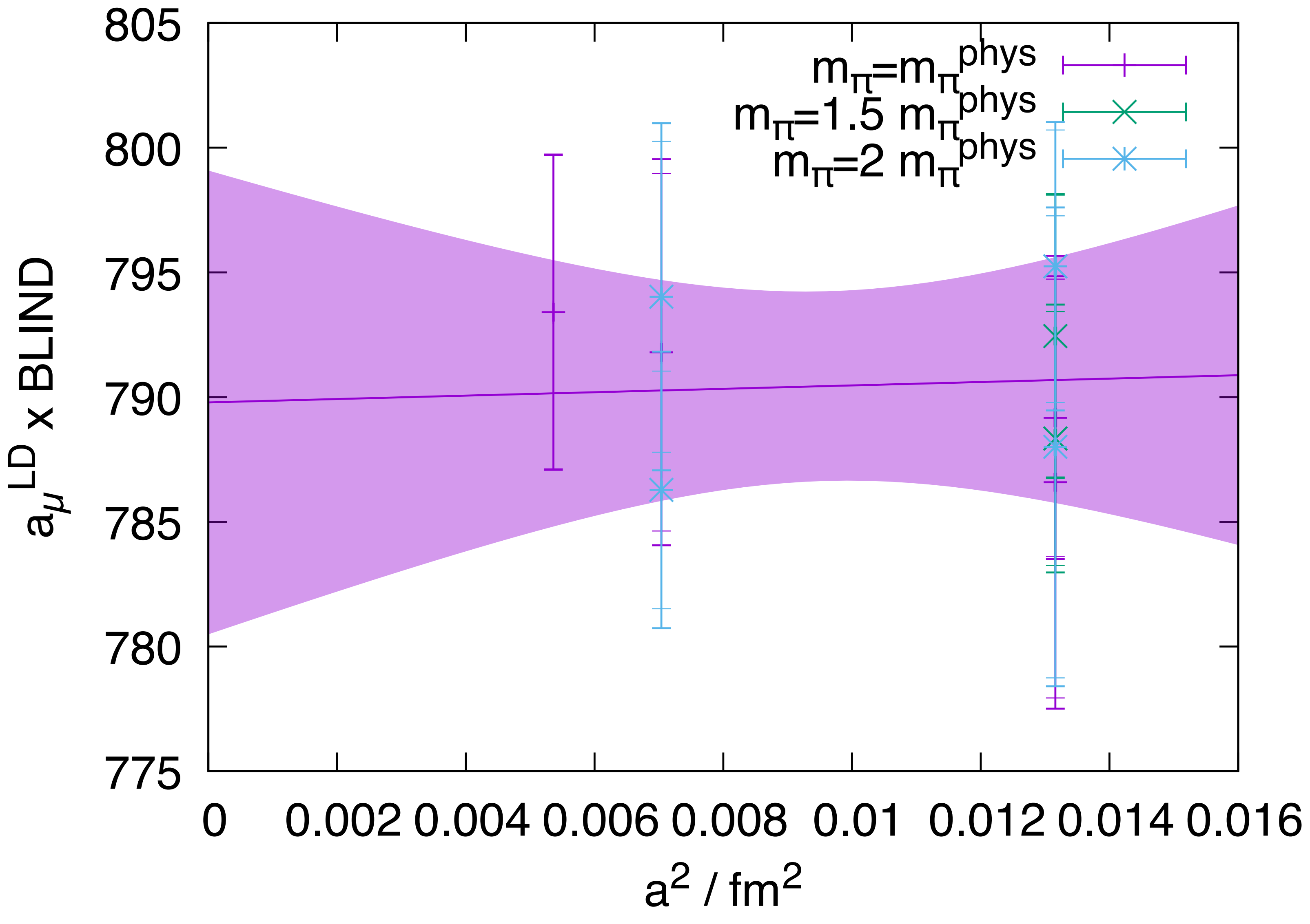


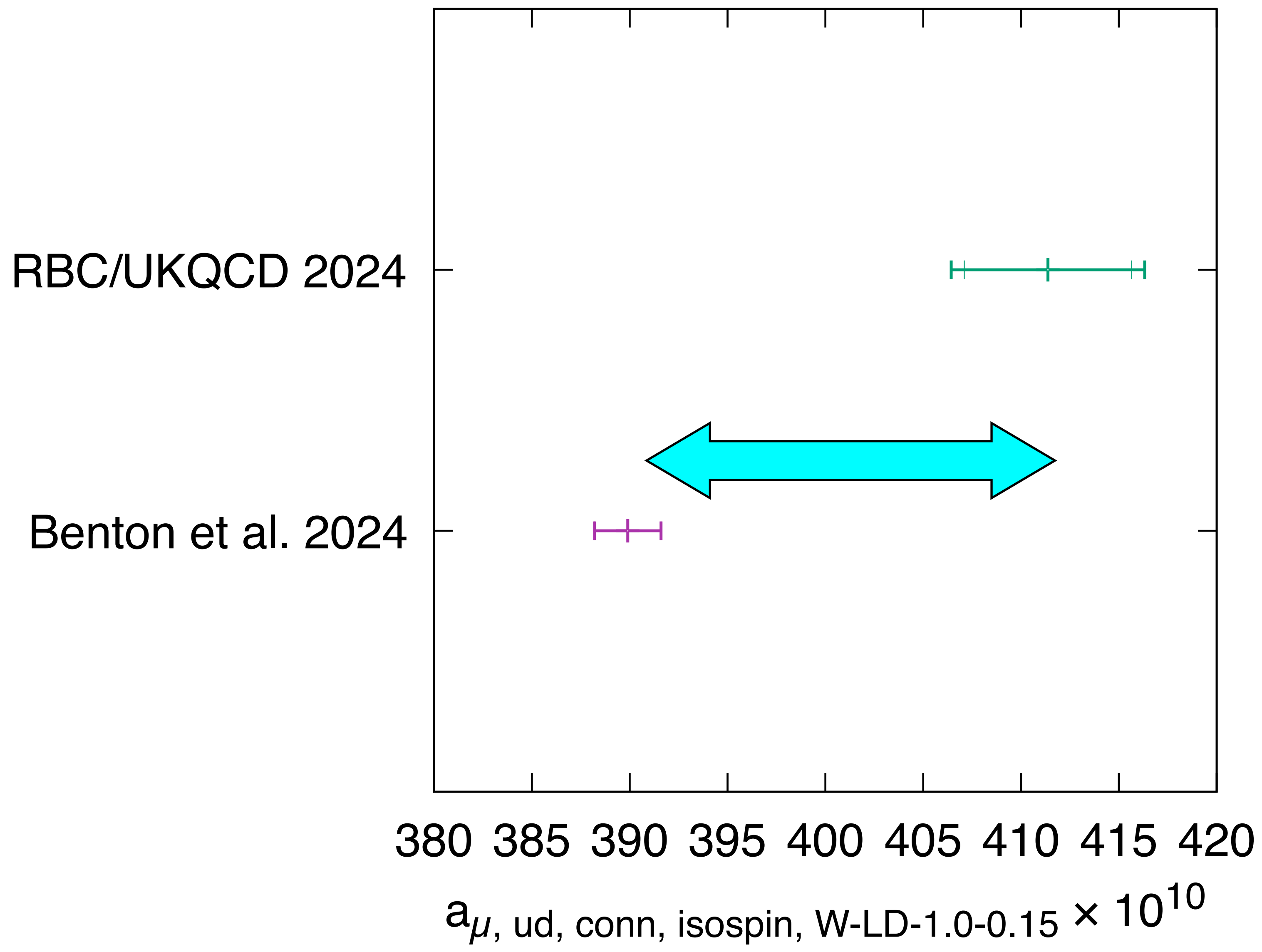
update: RBC/UKQCD 2024

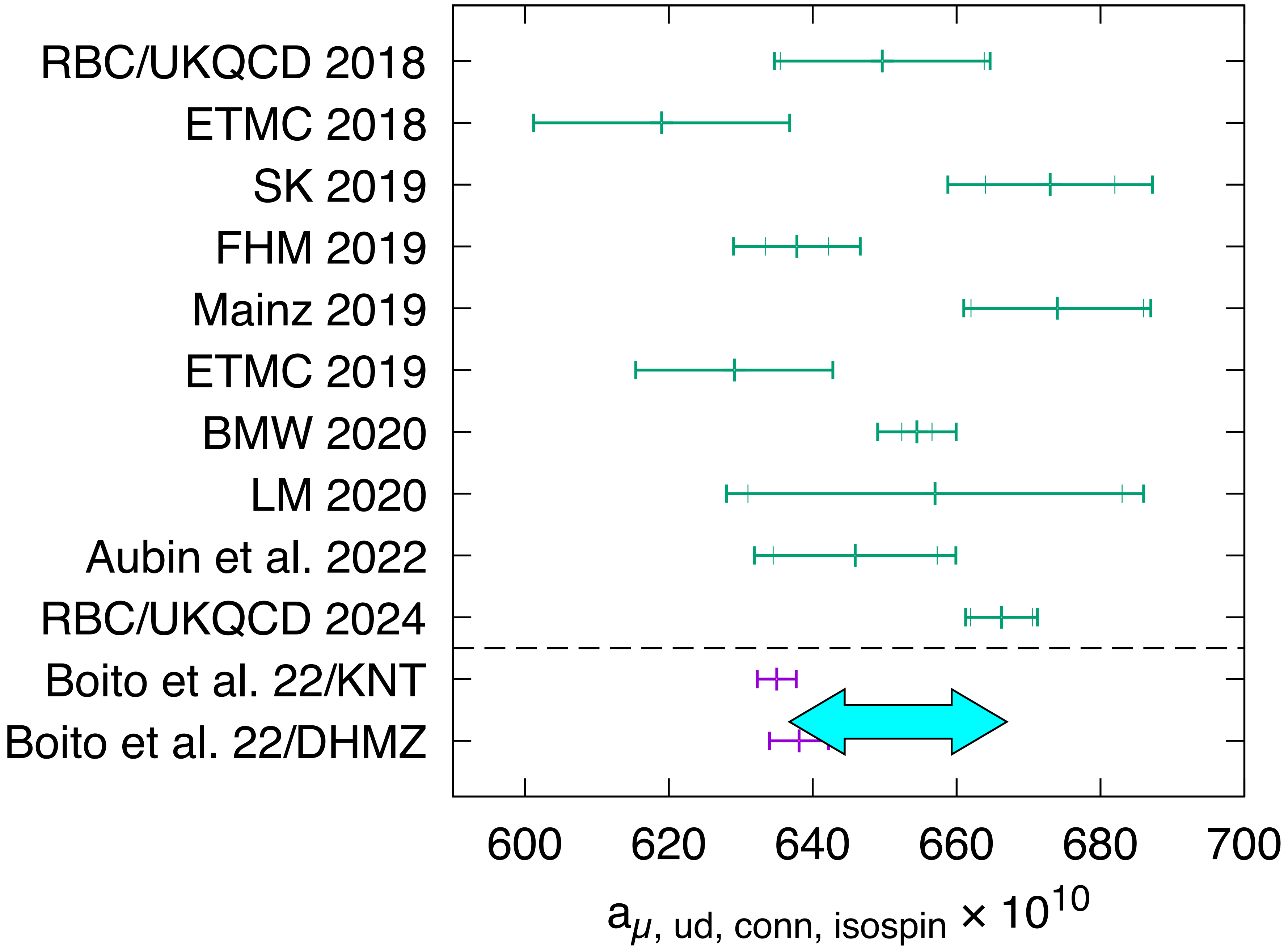
- long-distance window a_μ^{LD} and full a_μ^{ll}(conn.)

Unblinded results in BMW20 isospin-symmetric world

C. Lehner @ Lattice 2024





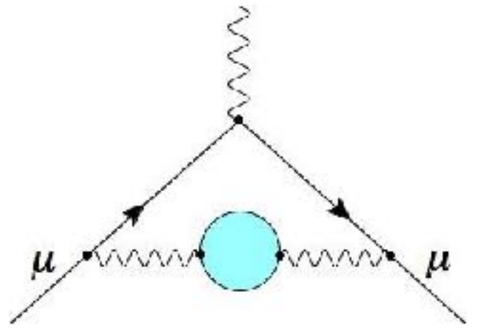


- "BMW20 world": fixed $w_0 = 0.1716$ fm scale uncertainty not included
- paper in progress

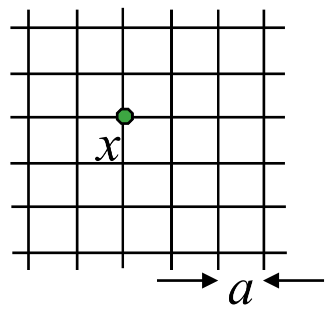
Result for $a_\mu^{\text{iso lqc}}$ with 7.5/1000 precision.

$$a_\mu^{\text{LD iso lqc}} = 411.4(4.3)_{\text{stat.}}(2.3)_{\text{syst.}} \times 10^{-10}\,,$$

$$a_\mu^{\text{iso lqc}} = 666.2(4.3)_{\text{stat.}}(2.5)_{\text{syst.}} \times 10^{-10}\,.$$



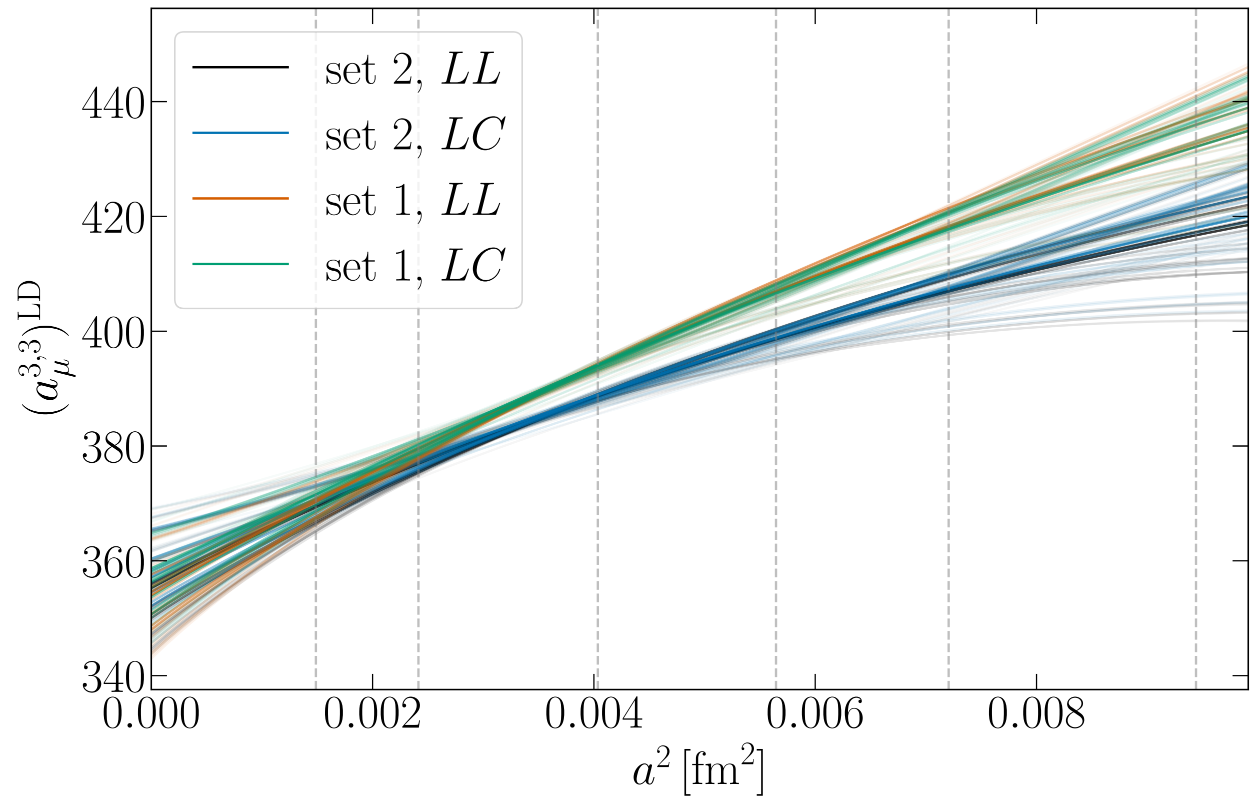
Lattice HVP: long-distance window



update: Mainz 2024

S. Kuberski @ KEK 2024

$(a_\mu^{\mathrm{hvp}})^{\mathrm{LD}}$ in the isovector channel: cutoff dependence



- Dependence of $(a_\mu^{3,3})^{\mathrm{LD}}$ on a^2 at physical quark masses.
- Four sets of data (colors) differ by $\mathrm{O}(a^2)$.
- Each line represents a fit in the model average.
- Include terms à la $[\alpha_s(1/a)]^{0.395}\, a^2$ [Husung, 2401.04303].

$(a_\mu^{\mathrm{hvp}})^{\mathrm{LD}}$: Status and outlook

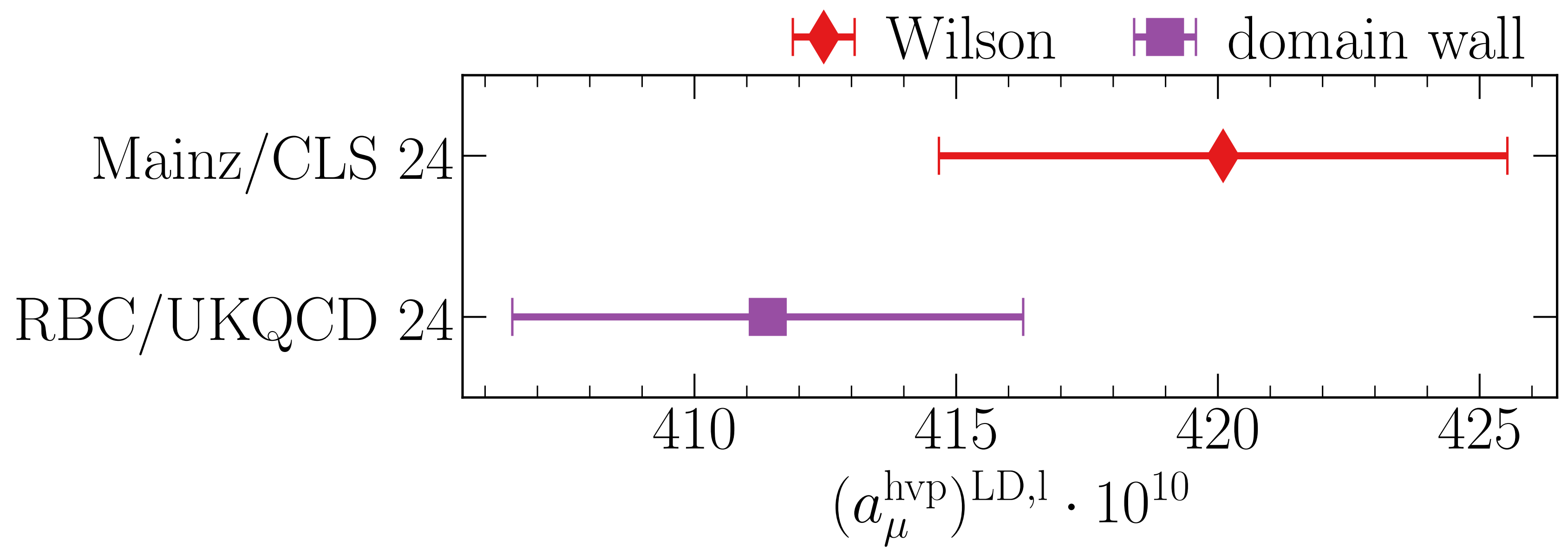
Achievements

- High statistical precision at m_π^{phys} and excellent control of the m_π dependence.
- Large span of lattice spacings to control the continuum extrapolation.
- Compute full isoQCD $(a_\mu^{\mathrm{hvp}})^{\mathrm{LD}}$ to 1.3% precision (statistics dominated).

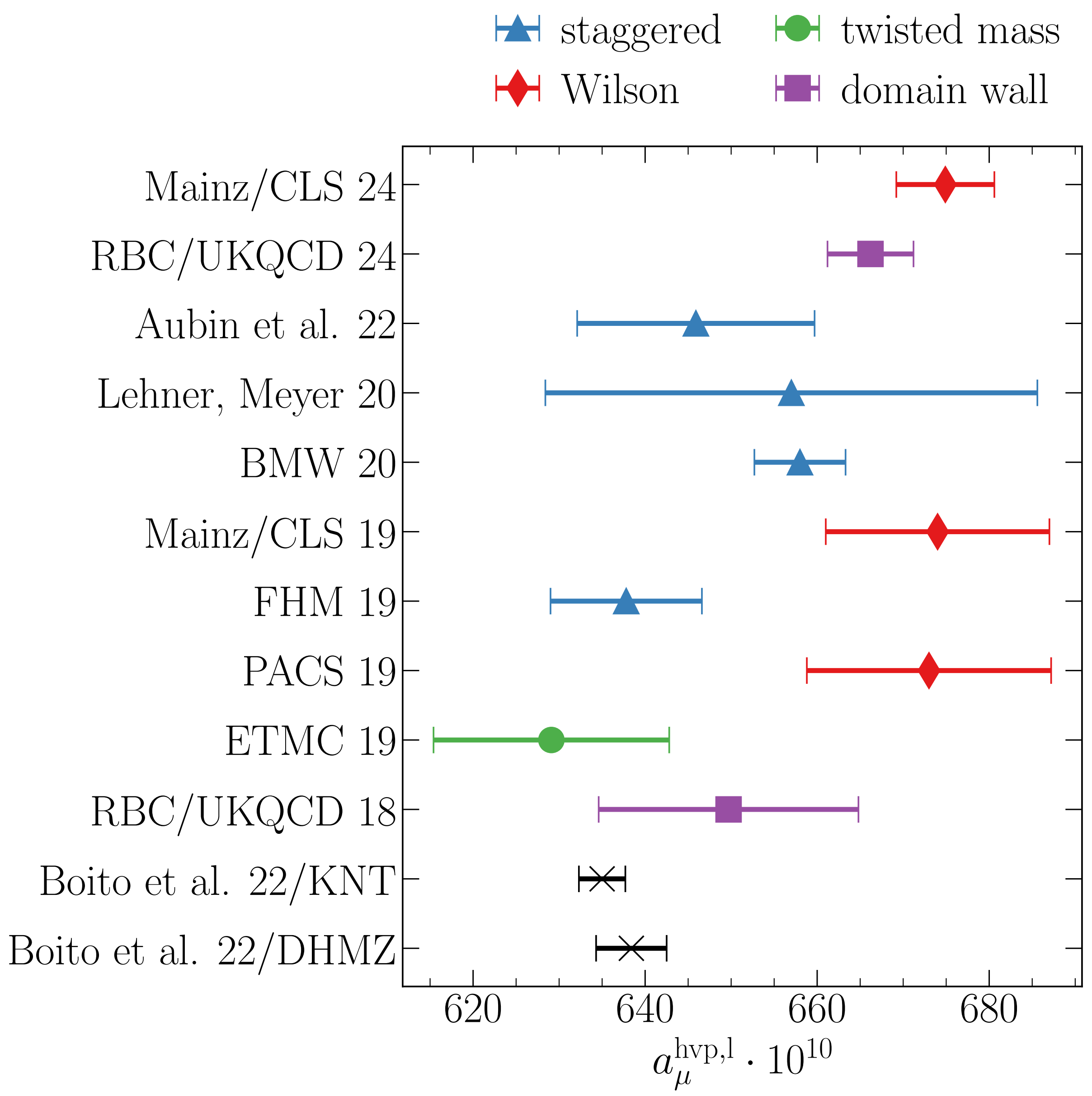
Outlook

- More data at fine lattice spacing and m_π^{phys} is being computed.
- Strong scale dependence in the long-distance regime:
  - We observe a strong scheme dependence: due to differences in the scale setting?
  - The global status of gradient flow scales is unsatisfactory [FLAG23].

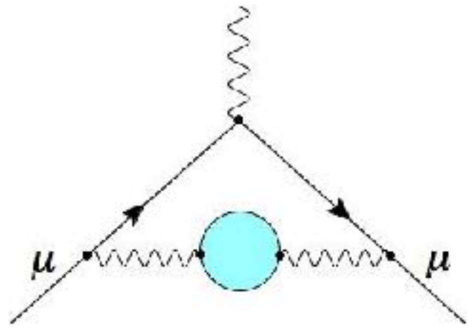
Comparison LD window (light):



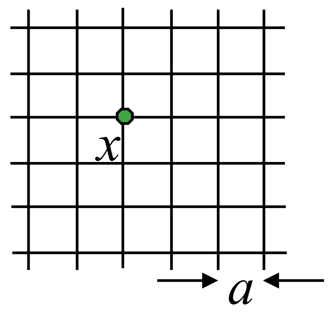
Comparison for full light-quark connected:



- Compute contributions to a_μ^{hvp} in isoQCD (Mainz world) by combinations with $(a_\mu^{\mathrm{hvp}})^{\mathrm{SD}}$ and $(a_\mu^{\mathrm{hvp}})^{\mathrm{ID}}$.
- We (will) publish the derivatives w.r.t. the input that defines our scheme. See [Portelli] for a comparison of schemes.
- $a_\mu^{\mathrm{hvp,l}}$ determined to 0.8% precision
- Excellent compatibility of Mainz/CLS 19 with Mainz/CLS 24.

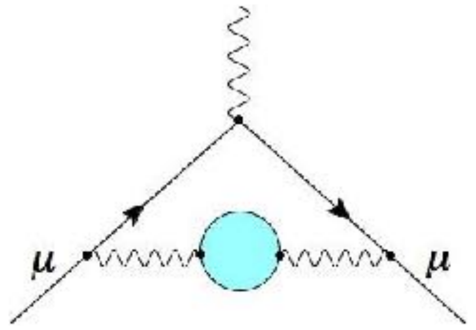


Lattice HVP: outlook

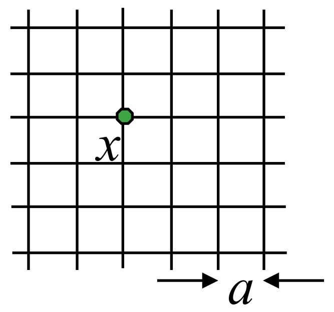


For total HVP and long-distance window:

- expect unblinded lattice results from FNAL/HPQCD/MILC (fall 2024) and ETM (~2025)
  - ➠ consolidated (?) lattice HVP for White Paper 2



Lattice HVP: outlook

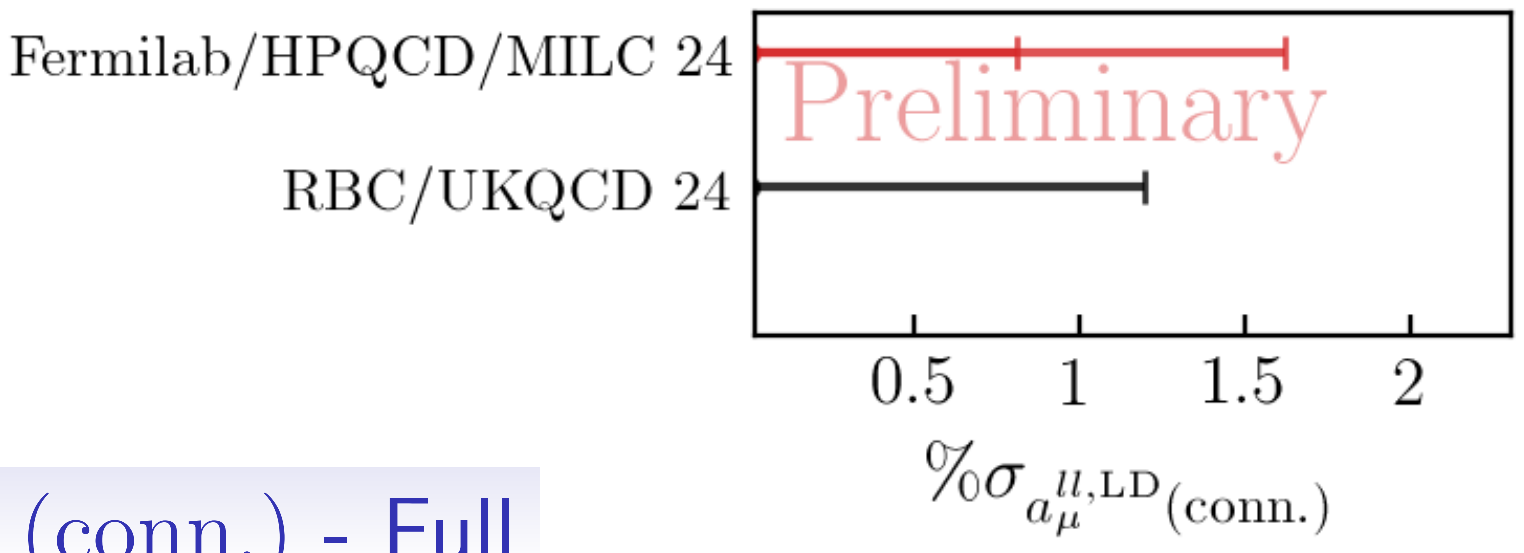


update: Fermilab/HPQCD/MILC 2024

M. Lynch @ Lattice 2024
S. Lahert @ KEK 2024

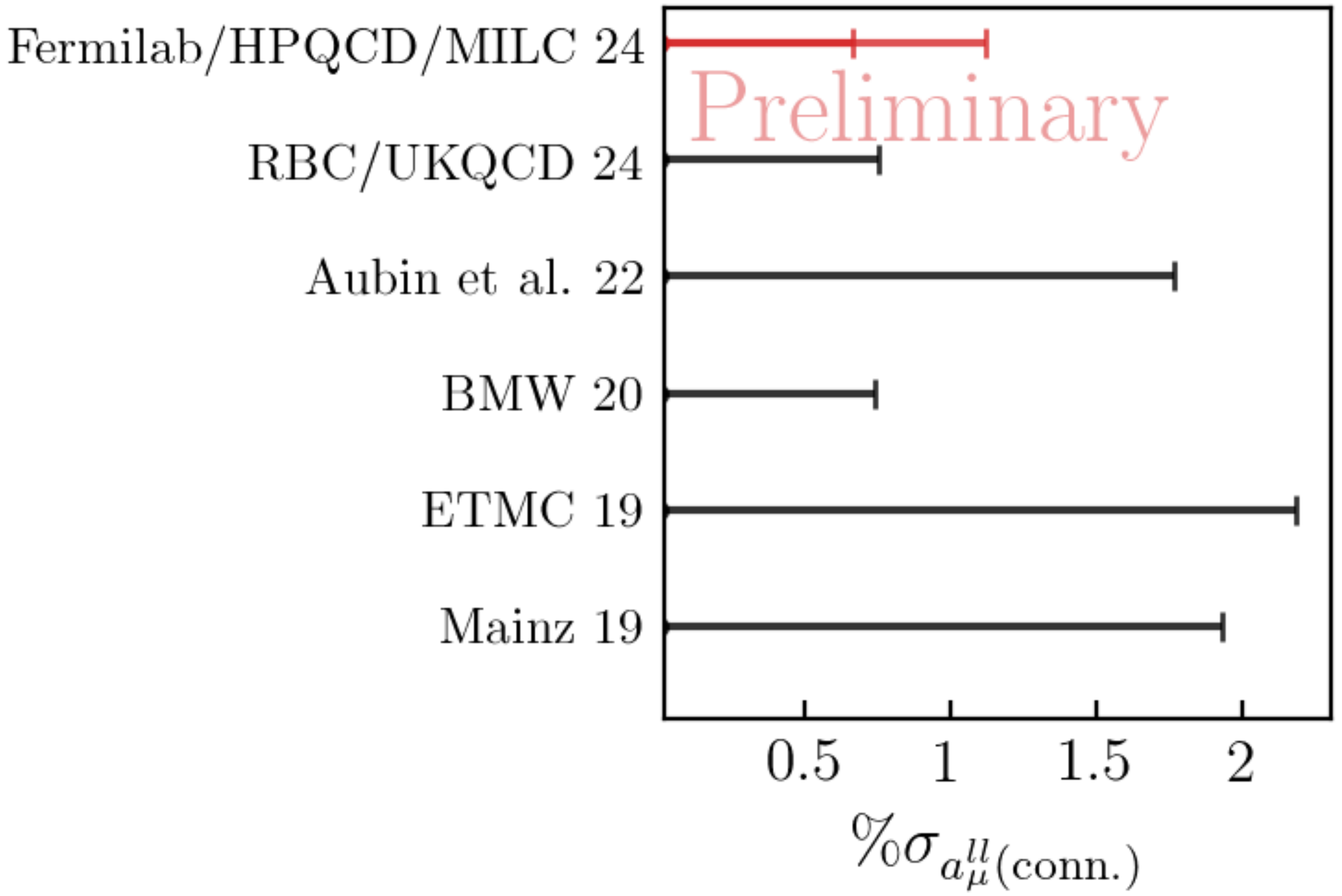
- long-distance window a_μ^{LD} and full a_μ^{ll}(conn.)

a_μ^{ll} (conn.) - LD window



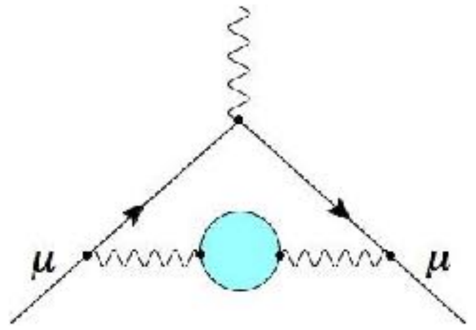
- Inner error w/o scale setting (w_0 fm) uncertainty
- Scale setting is now dominant error contributor.

a_μ^{ll} (conn.) - Full

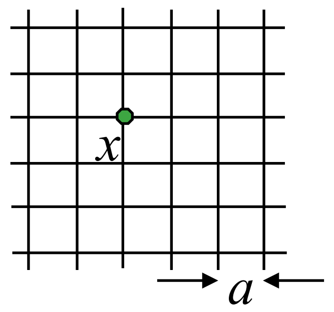


We expect further improvements in stat., sys. uncertainties from...

- Generation of correlator data at a lattice spacing of 0.04 fm is in progress.
- Improved scale setting via M_Ω.
- Joint fit analysis with multiple vector current discretizations
- Direct finite volume study: $L \sim 5.5$ fm $\to L \sim 11$ fm (at $a = 0.09$ fm) to replace EFT-based FV error estimates.
- Calculation of two-pion contributions to vector-current correlation functions at finer lattice spacings.



Lattice HVP: full window

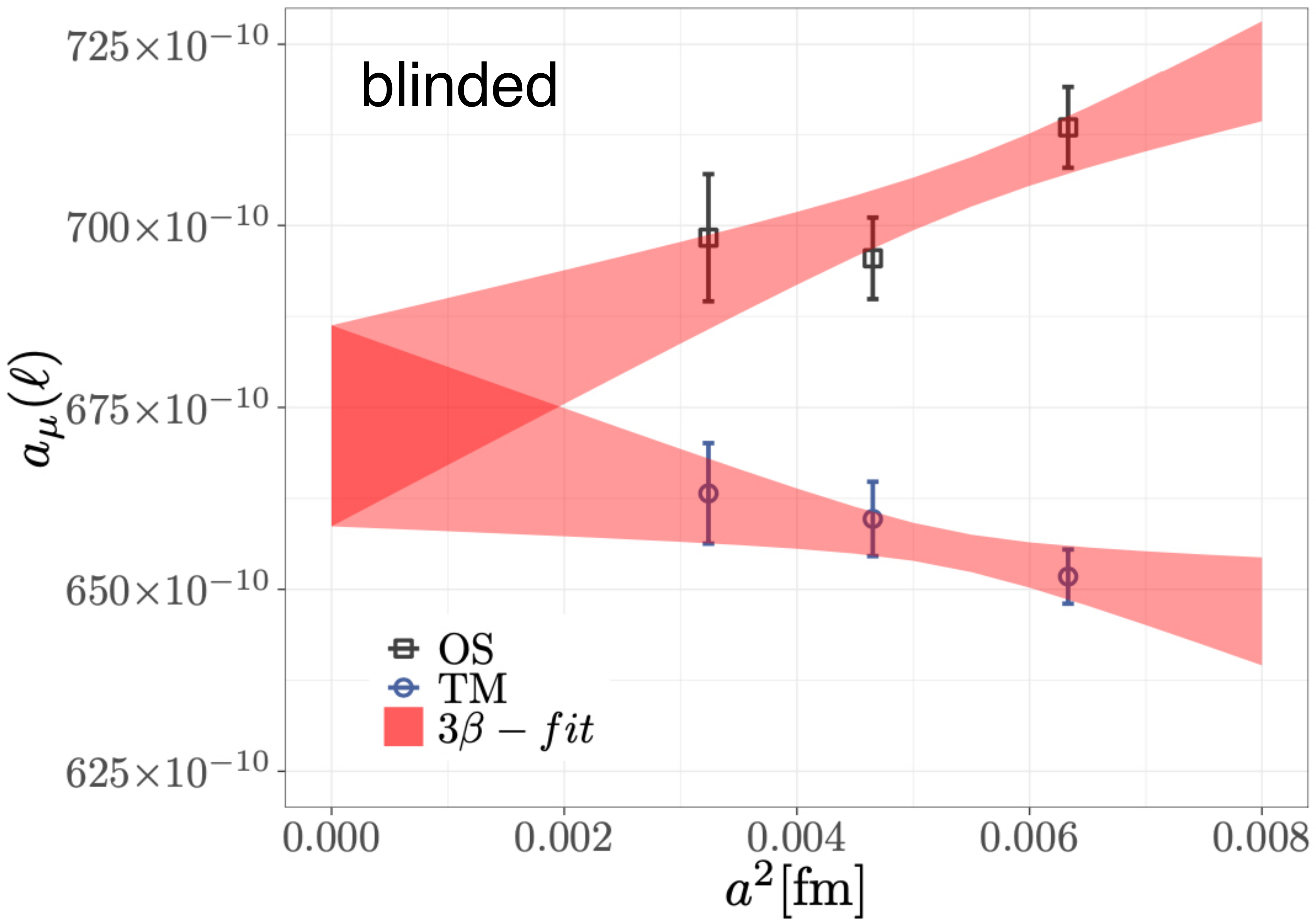


update: ETMC 2024

M. Garofalo @ Lattice 2024

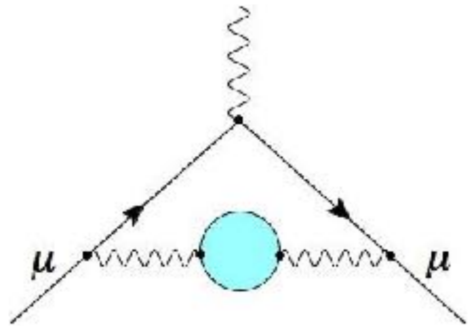
U. Wenger @ KEK 2024

ℓ-quark connected [Preliminary]

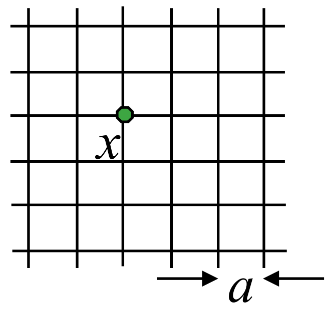


- Data interpolated to $L = 5.46$ fm:
  - ⇒ using GS/MLLGS approach
- Correction to isoQCD point:
  - ⇒ calculation in progress
- Significant reduction in uncertainty possible:
  - ⇒ additional final lattice spacing
  - ⇒ higher statistics in progress
  - ⇒ EFT model for TM lattice artefacts ($\rho\pi\gamma$-model)

also prelim. results for strange & charm



Lattice HVP: outlook



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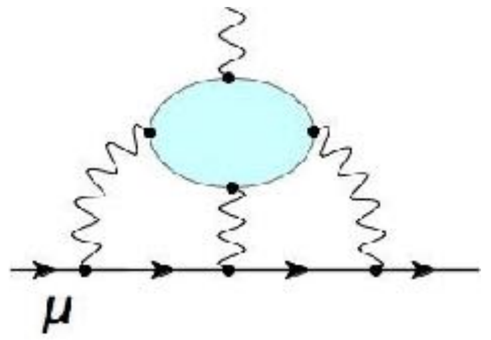
- expect unblinded lattice results from FNAL/HPQCD/MILC (fall 2024) and ETM (~2025)
  ➠ consolidated (?) lattice HVP for White Paper 2
- Including $\pi\pi$ states for refined long-distance computation (Mainz, RBC/UKQCD, FNAL/MILC)
- smaller lattice spacings to test continuum extrapolations crucial

➠ Slides from Lattice 2024 conference

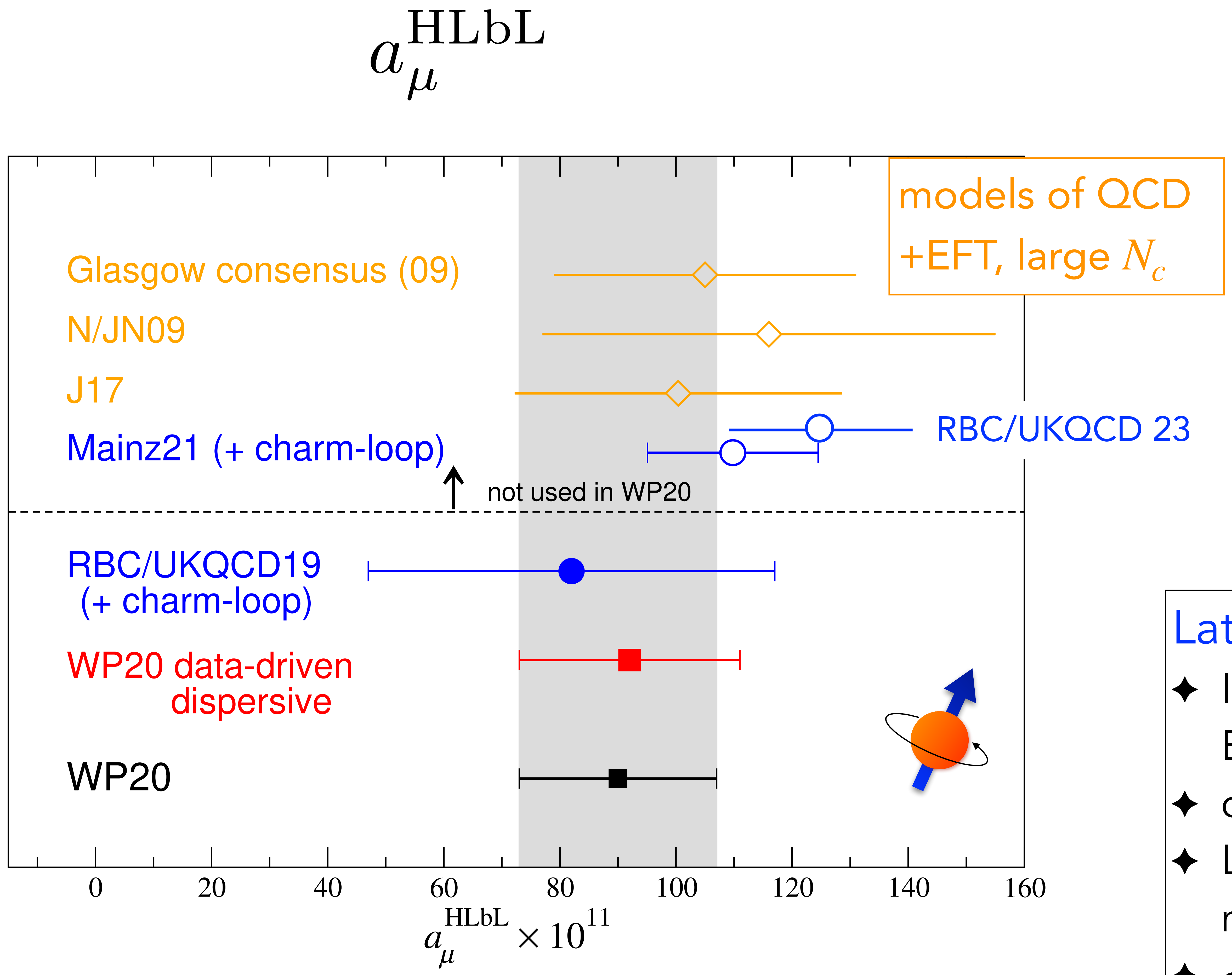
➠ Slides from Muon g-2 TI workshop @ KEK

More windows:

- Use linear combinations of finer windows to locate the tension (if it persists) in $\sqrt{s}$ [Colangelo et al, arXiv:12963]
- One-sided windows (excluding the long-distance region $t \gtrsim 2.5\ \mathrm{fm}$) to test data-driven evaluations [Davies at at, arXiv:2207.04765]



Hadronic Light-by-Light: Summary



Dispersive approach:

[Colangelo at al, 2014; Pauk & Vanderhaegen 2014; …]

- ✦ model independent
- ✦ significantly more complicated than for HVP
- ✦ provides a framework for data-driven evaluations
- ✦ can also use lattice results as inputs
- ✦ current 20% uncertainty ⇒ ~10% feasible

Lattice QCD+QED:

- ✦ Independent calculations by four groups (Mainz, RBC/UKQCD, ETM, BMW)
- ✦ consistent with each other and with previous calculations
- ✦ Lattice groups are continuing to improve their calculations, adding more statistics, lattice spacings, physical mass ensembles
- ✦ ongoing LQCD calculations of π, η, η' transition form factors to determine pseudo scalar pole contributions [Mainz, ETMC, BMW, RBC/UKQCD]

Summary

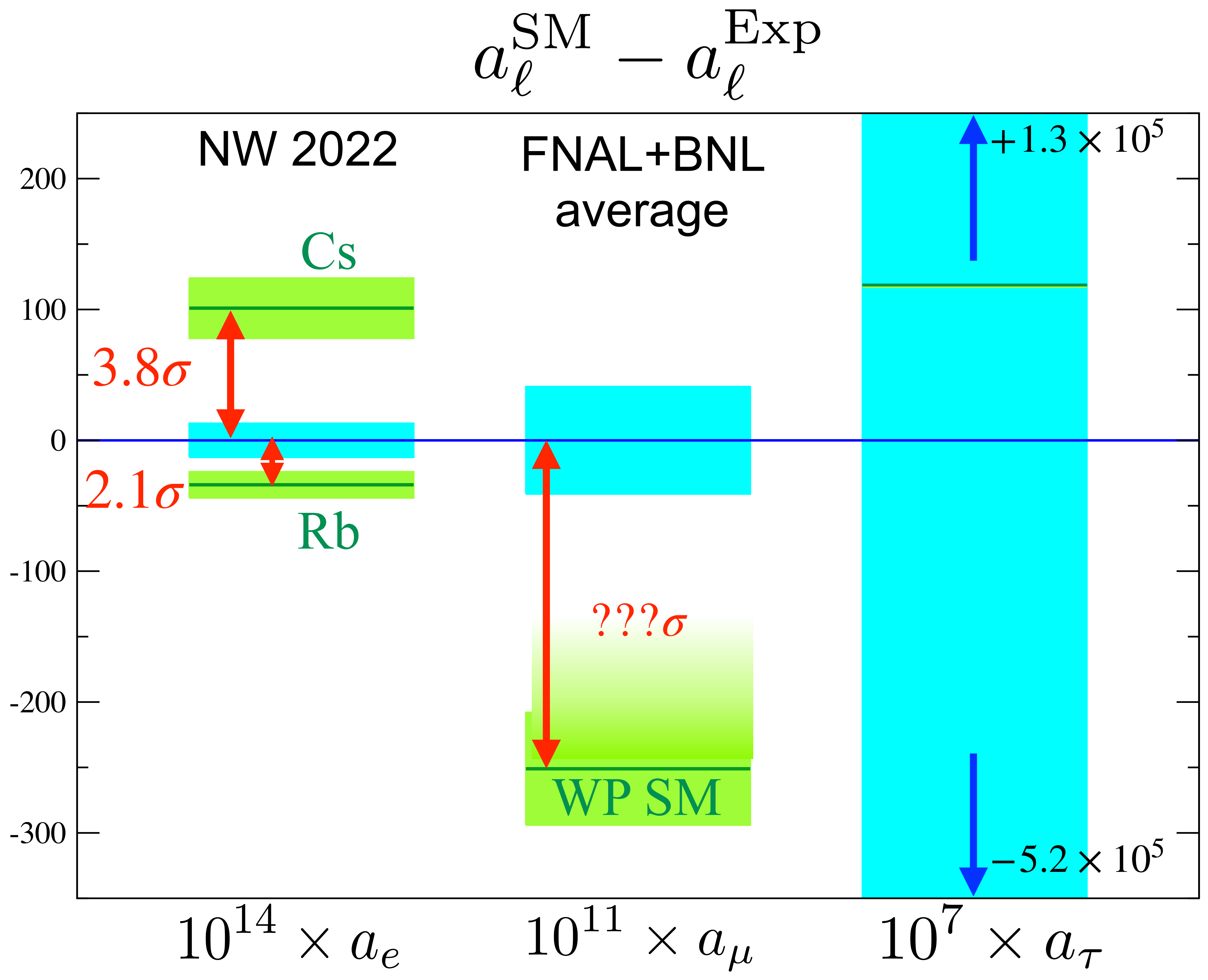
- ☆ consistent results from independent, precise LQCD calculations for light-quark connected contribution to **intermediate window** $a_\mu^{\rm W}$ (~ 1/3 of $a_\mu^{\rm HVP,LO}$) ➠ $\sim 5\,\sigma$ tension with (pre-2023) data-driven results
- ☆ 2 new LQCD results for long-distance contribution with $\sim 0.8\,\%$ precision, more coming soon!
  ➠ check consolidation, develop lattice HVP average for White Paper 2
- ☆ Programs and plans in place for:
  - data-driven HVP:
    new analyses from BaBar, KLOE, CMD3, SND, Belle II,…. will shed light on current discrepancies
    improved treatment of structure dependent radiative corrections (NLO) in $\pi\pi$ and $\pi\pi\pi$ channels
  - lattice HVP: **if no tensions** between independent lattice results, $\sim 0.5\,\%$ possible
  - dispersive HLbL and lattice HLbL: no puzzles, steady progress, $\sim 10\,\%$
- ☆ Need to understand tensions between data-driven and lattice HVP if they persist
- ☆ including τ decay data in data-driven approach:
  requires nonperturbative evaluation of IB correction [M. Bruno et al, arXiv:1811.00508]

➠ continued coordination by Theory Initiative: 2nd WP in progress

Beyond the SM possibilities

- a_μ is loop-induced, conserves CP & flavor, flips chirality.
- Generically expect: $a_\mu^{\mathrm{NP}} \sim a_\mu^{\mathrm{EW}} \times \dfrac{M_W^2}{\Lambda^2} \times \text{couplings}$
  - ➠ the difference between exp-WP20 is large: $\Delta a_\mu = 249\,(48) \times 10^{-11} \; > \; a_\mu(\mathrm{EW})$
- Will likely be different if using (consolidated) lattice HVP average.
- Tensions between data-driven and lattice HVP results:
  - Can new physics hide in the low-energy $\sigma(e^+e^- \to \pi\pi)$ cross section?
    ➠ No [Luzio, et al, arXiv:2112.08312]
  - New boson at ~ 1GeV decays into $\mu^+\mu^-, e^+e^-$, affects $\sigma(e^+e^- \to \pi\pi)$ indirectly [L. Darmé et al, arXiv:2112.09139]
  - Neutral, long-lived hadrons, heretofore undetected?
    [Farrar, arXiv:2206.13460]
  - Z′ at < 1 GeV, coupling to 1st gen matter particles
    [Coyle, Wagner, arXiv:2305.02354]

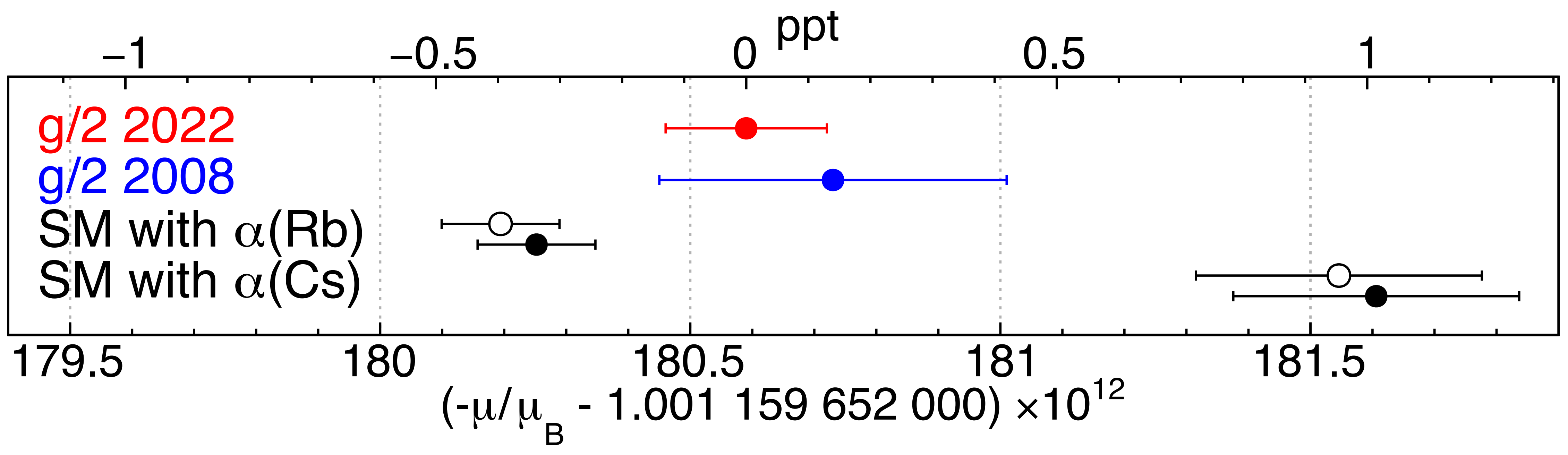
Lepton moments summary



Sensitivity to heavy new physics: $a_\ell^{\rm NP} \sim \frac{m_\ell^2}{\Lambda^2}$

$(m_\mu/m_e)^2 \sim 4\times10^4$

[X. Fan et al (NW group), arXiv:2209.13084]



Cs: α from Berkeley group [Parker et al, Science 360, 6385 (2018)]

Rb: α from Paris group [Morel et al, Nature 588, 61–65(2020)]

Outlook

- Experimental program beyond 2025:
  - J-PARC: Muon g-2/EDM
  - CERN: MUonE
  - Fermilab: future muon campus experiments?
  - Belle II, BESIII, Novosibirsk,…
  - Chiral Belle (?)
- Data-driven/dispersive program beyond 2025:
  - development of NNLO MC generators
  - for HLbL, improved experimental/lattice inputs together with further development of dispersive approach
- MUonE will provide a space-like determination of HVP
- Lattice QCD beyond 2025:
  - access to future computational resources (coming Exascale) will enable improvements of all errors (statistical and systematic)
  - concurrent development of better methods and algorithms (gauge-field sampling, noise reduction) will accelerate progress
  - **beyond g-2: a rich program relevant for all areas of HEP**

Outlook

m_{ud}, m_u/m_d, m_s ⟸ $a_\mu^{\text{HVP LO}}$ a_μ^{HLbL} d_n MEs for light nuclei

m_c, m_b $\langle \bar{B}_q^0 | \mathcal{O}_i^{\Delta B=2} | B_q^0 \rangle$ PDFs, GPDs, TMDs,..

$\alpha_{\overline{MS}}(m_z)$ $\langle NN | O_i | NN \rangle$

$\langle \bar{D}^0 | \mathcal{O}_i^{\Delta C=2} | D^0 \rangle$ $\Lambda_b \to (p, \Lambda_c, \Lambda)\, \ell\nu$ $D_s \to \ell\nu\gamma$ $B \to \pi\pi\, \ell\nu$ ...

$\hat{B}_K, \ldots$ nucleon form factors, .. $K^+ \to \ell^+\nu\,(\gamma)$ $B \to X_c\ell\nu$, other inclusive decay rates, ...

$f_{B_{(s)}}$, ... g_A, g_T, g_S $\pi\pi \to \rho$

$\pi \to \ell^+\ell^-$, $K \to \gamma\gamma$

$f^{K\to\pi}(q^2)$ $f_{+,0,T}^{B\to D}(q^2), \ldots$ $\langle \pi\pi_{(I=0)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$ $K^+ \to \pi^+\ell^+\ell^-$ ...

$f_{K^\pm}$ $f_{+,0,T}^{B\to\pi}(q^2), \ldots$ $\langle \pi\pi_{(I=2)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$ ΔM_K, ϵ_K $K^+ \to \pi^+\nu\bar{\nu}$

Complexity →

✓ LQCD flagship results

First complete LQCD results, large(ish) errors

First results, physical params, incomplete systematics

new methods, pilot projects, unphysical kinematics

new ideas, first studies





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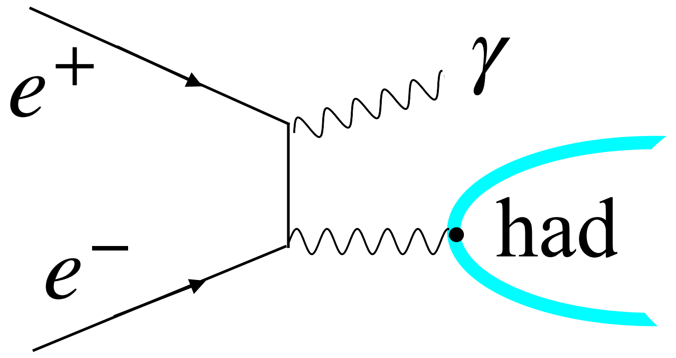
Appendix

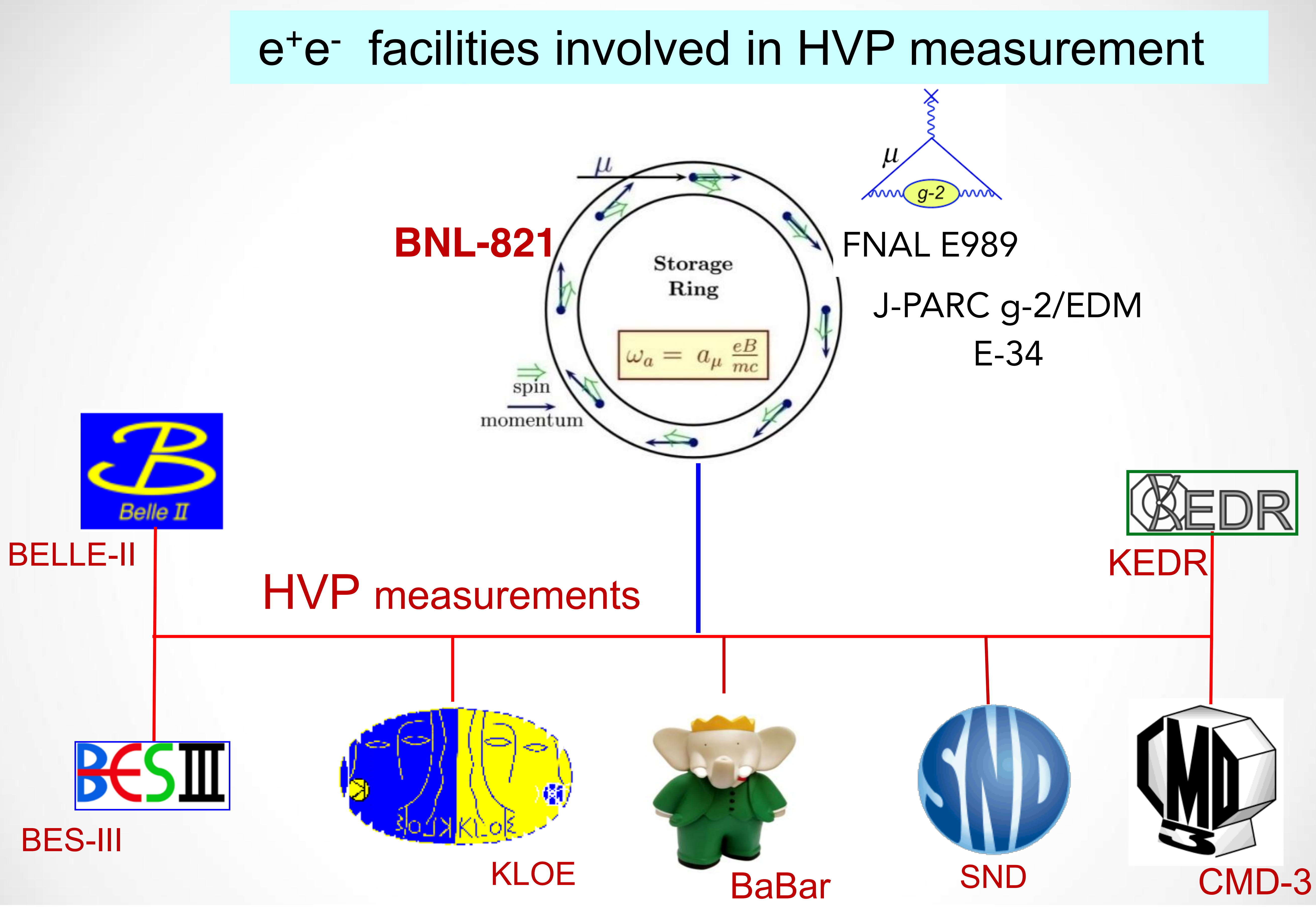
Experimental Inputs to HVP

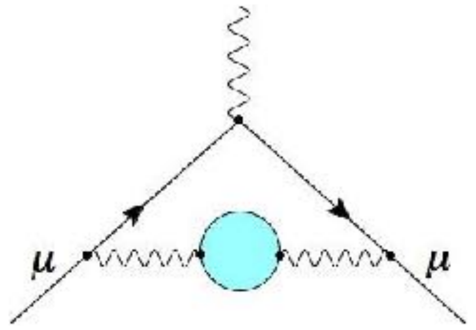
- ☆ two exp. approaches
  - ``Direct scan": change CM energy of e^+e^- beams
  - ``Radiative Return":
    with fixed e^+e^- CM energy, select events with initial state radiation (ISR)



- ☆ complemented by:
  - MC generators for $\sigma_{\mathrm{had}}(s)$ (e.g. PHOKARA)
  - detailed studies of radiative corrections (now known through NLO)

S. Serednyakov (for SND) @ HVP KEK workshop





HVP: data-driven

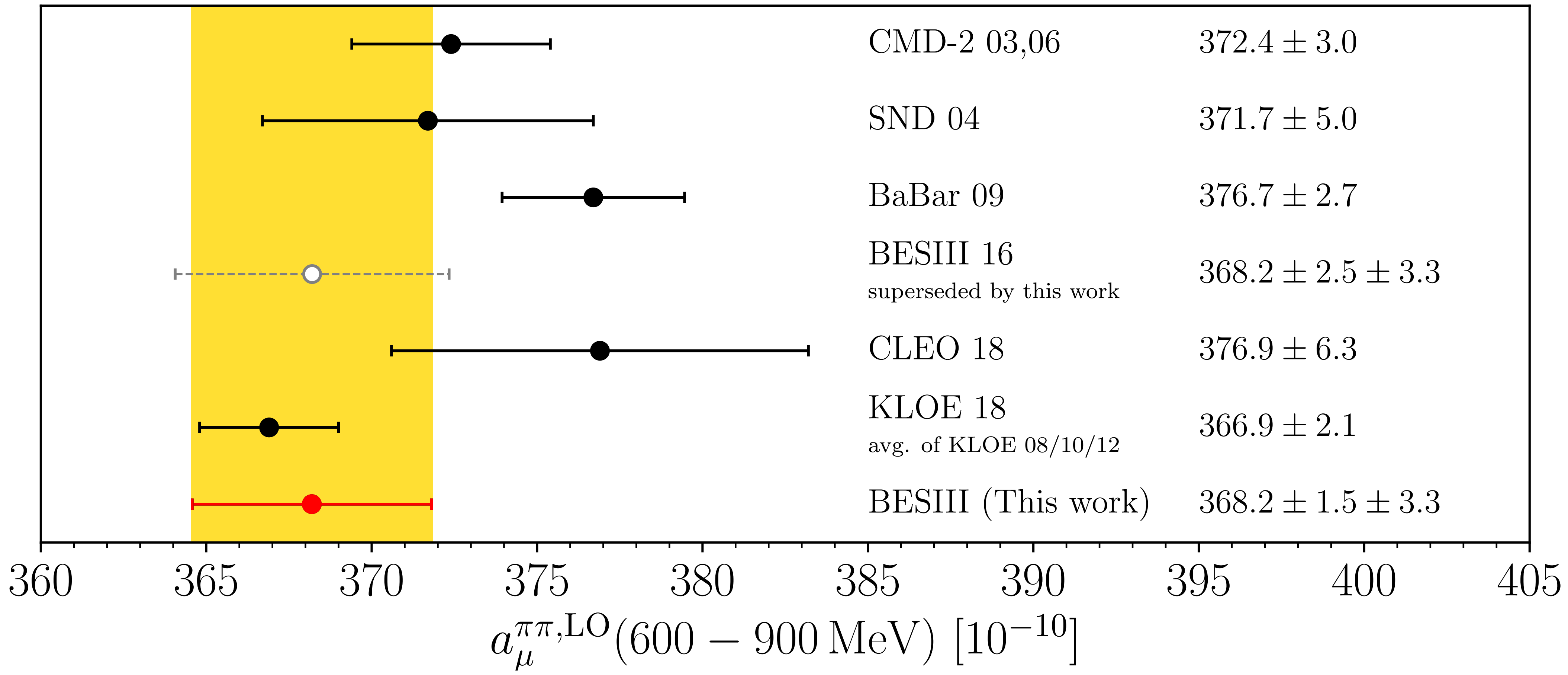
2020 White Paper [T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166.]

Conservative merging procedure to obtain a realistic assessment of the underlying uncertainties:

- account for tensions between data sets
- account for differences in methodologies for compilation of experimental inputs and treatment of correlations between systematic errors
- include results using constraints from unitarity & analyticity in $\pi\pi$ and $\pi\pi\pi$ channels
  [Colangelo et al, 2018; Anantharayan et al, 2018; Davier et al, 2019; Hoferichter et al, 2019]
- Full NLO radiative corrections [Campanario et al, 2019]

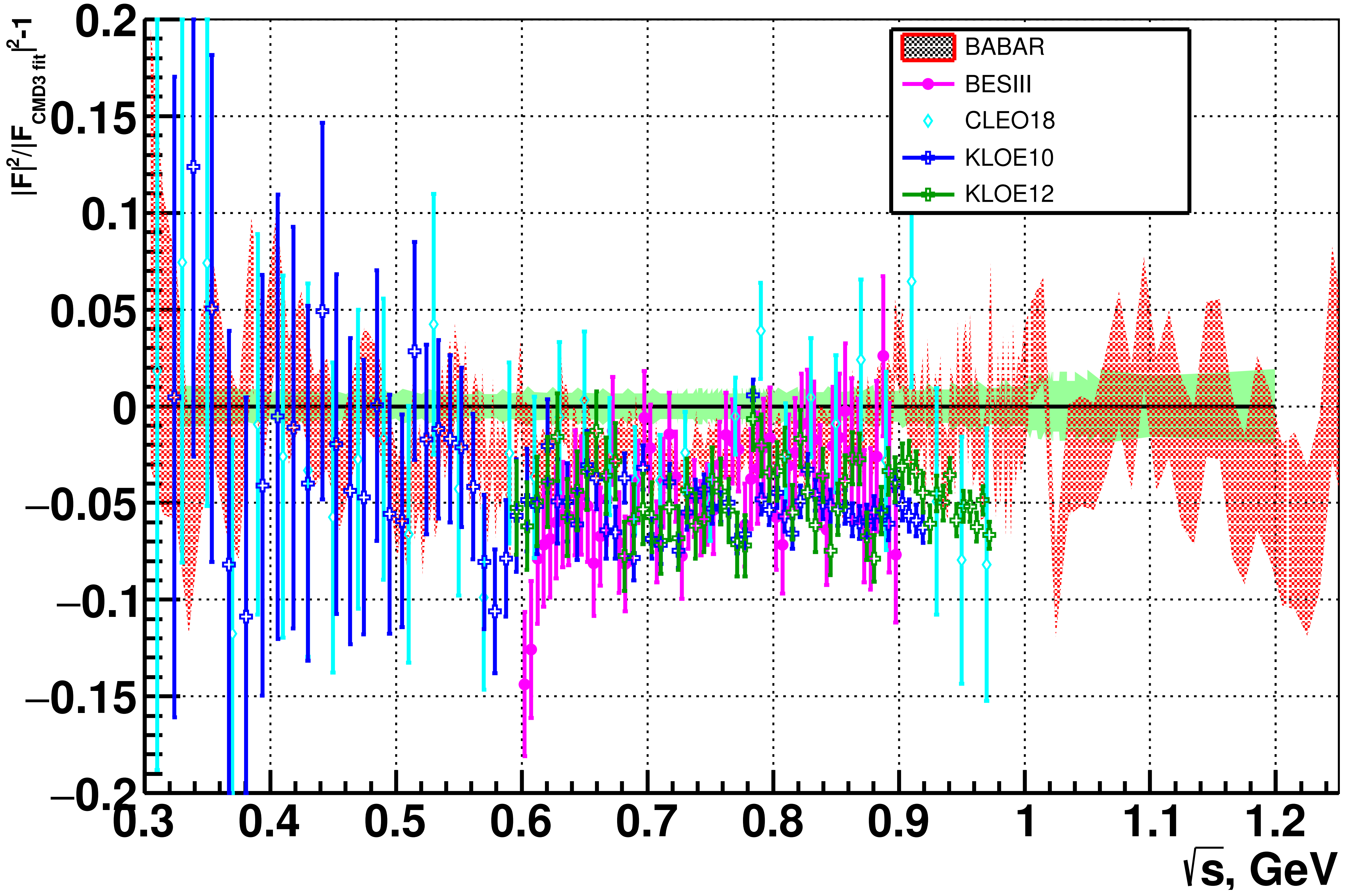
$$a_\mu^{\rm HVP,LO} = 693.1\,(2.8)_{\rm exp}\,(0.7)_{\rm DV+pQCD}\,(2.8)_{\rm BaBar-KLOE} \times 10^{-10}$$
$$= 693.1\,(4.0) \times 10^{-10}$$

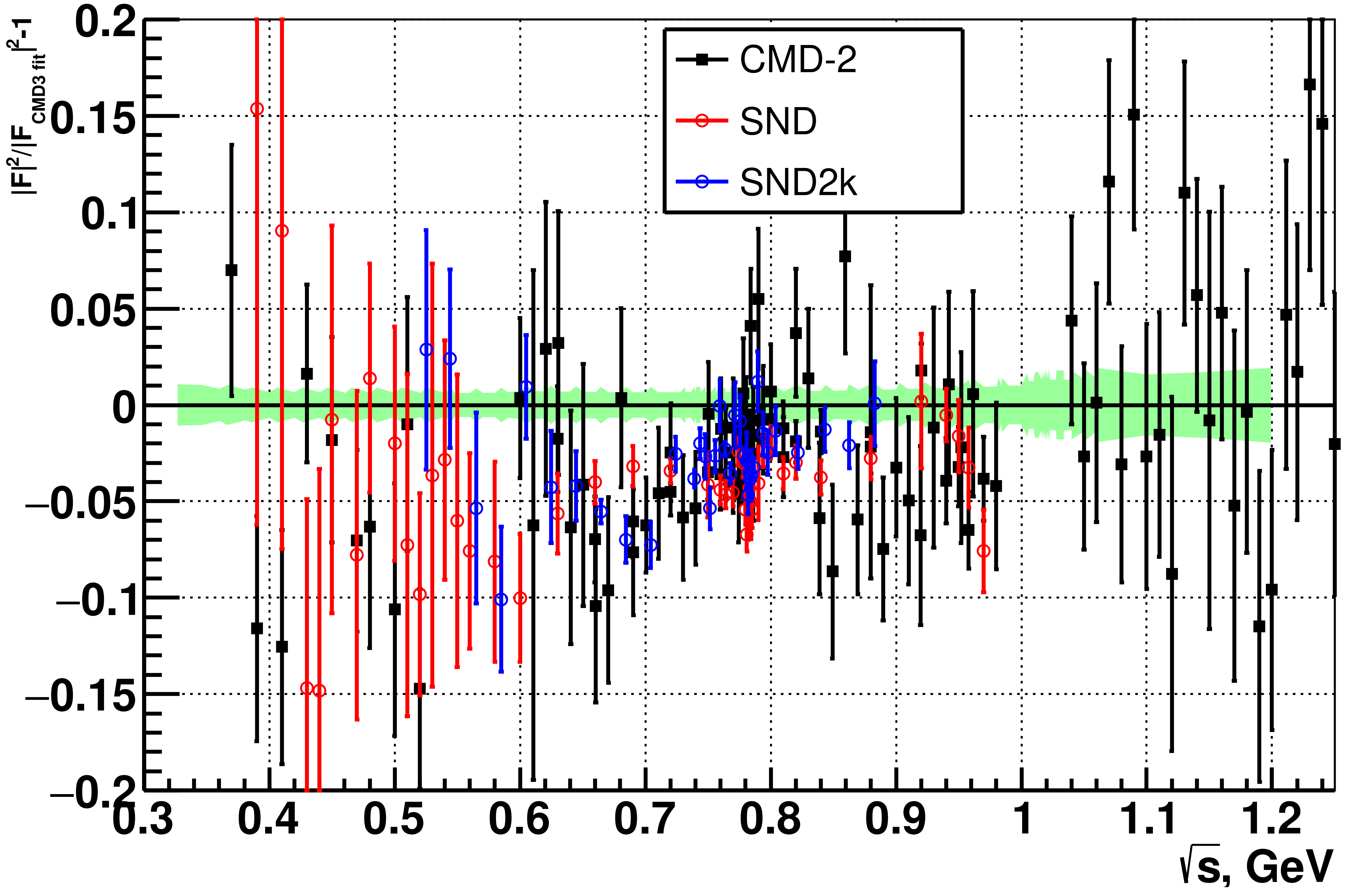
[M. Ablikim et al (BES III), arXiv:2009.05011]



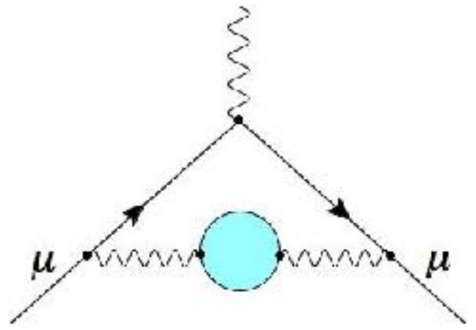
cross section comparisons

[CMD-3, F. Ignatov et al, arXiv:2302.08834, PRD2024]

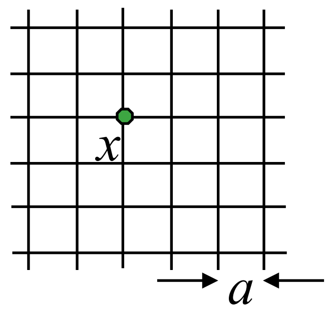




- For $\sqrt{s} \lesssim 0.6$ GeV: good consistency between cross section measurements
- For 0.6 GeV $\lesssim \sqrt{s} \lesssim 1$ GeV: significant differences between measurements



Lattice HVP: Introduction



Leading order HVP contribution: $a_\mu^{\rm HVP,LO} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)$

[B. Lautrup, A. Peterman, E. de Rafael, Phys. Rep 1972; E. de Rafael, Phys. Let. B 1994; T. Blum, PRL 2002]

- Calculate $a_\mu^{\rm HVP,LO}$ in Lattice QCD

  Start with correlation function of EM currents: $C(t) = \frac{1}{3}\sum_{i,x}\langle j_i^{\rm EM}(x,t) j_i^{\rm EM}(0,0)\rangle$ $\quad j_\mu^{\rm EM} = \sum_f q_f\, \bar\psi_f(x,t)\,\gamma_\mu\,\psi_f(x,t)$ $\quad f = u, d, s, c, \ldots$

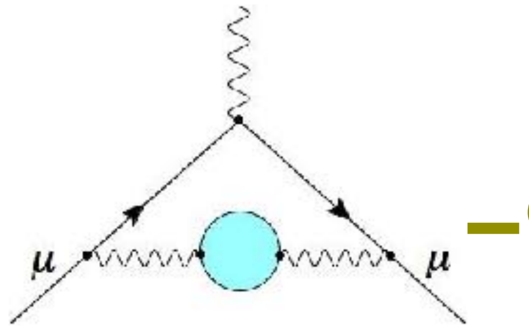
  Fourier transform yields $\hat{\Pi}(Q^2) = 4\pi^2 \int_0^\infty dt\, C(t)\left[t^2 - \frac{4}{Q^2}\sin^2\left(\frac{Qt}{2}\right)\right]$

  [D. Bernecker and H. Meyer, arXiv:1107.4388, EPJA 2011]

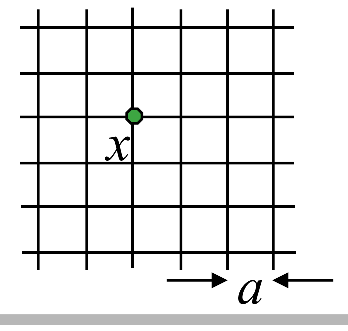
  so that $a_\mu^{\rm HVP,LO}$ can be obtained as an integral over Euclidean time, aka time momentum representation (TMR):

$$a_\mu^{\rm HVP,LO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2\, w(Q^2)\, \hat{\Pi}(Q^2) = 4\,\alpha^2 \int_0^\infty dt\, C(t) \int_0^\infty dQ^2 w(Q^2)\left[t^2 - \frac{4}{Q^2}\sin^2\left(\frac{Qt}{2}\right)\right]$$

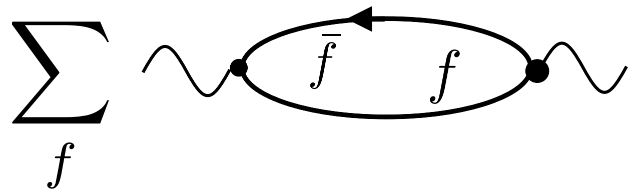
$$\Rightarrow \quad a_\mu^{\rm HVP,LO} = 4\,\alpha^2 \int_0^\infty dt\, C(t)\, \tilde{w}(t)$$



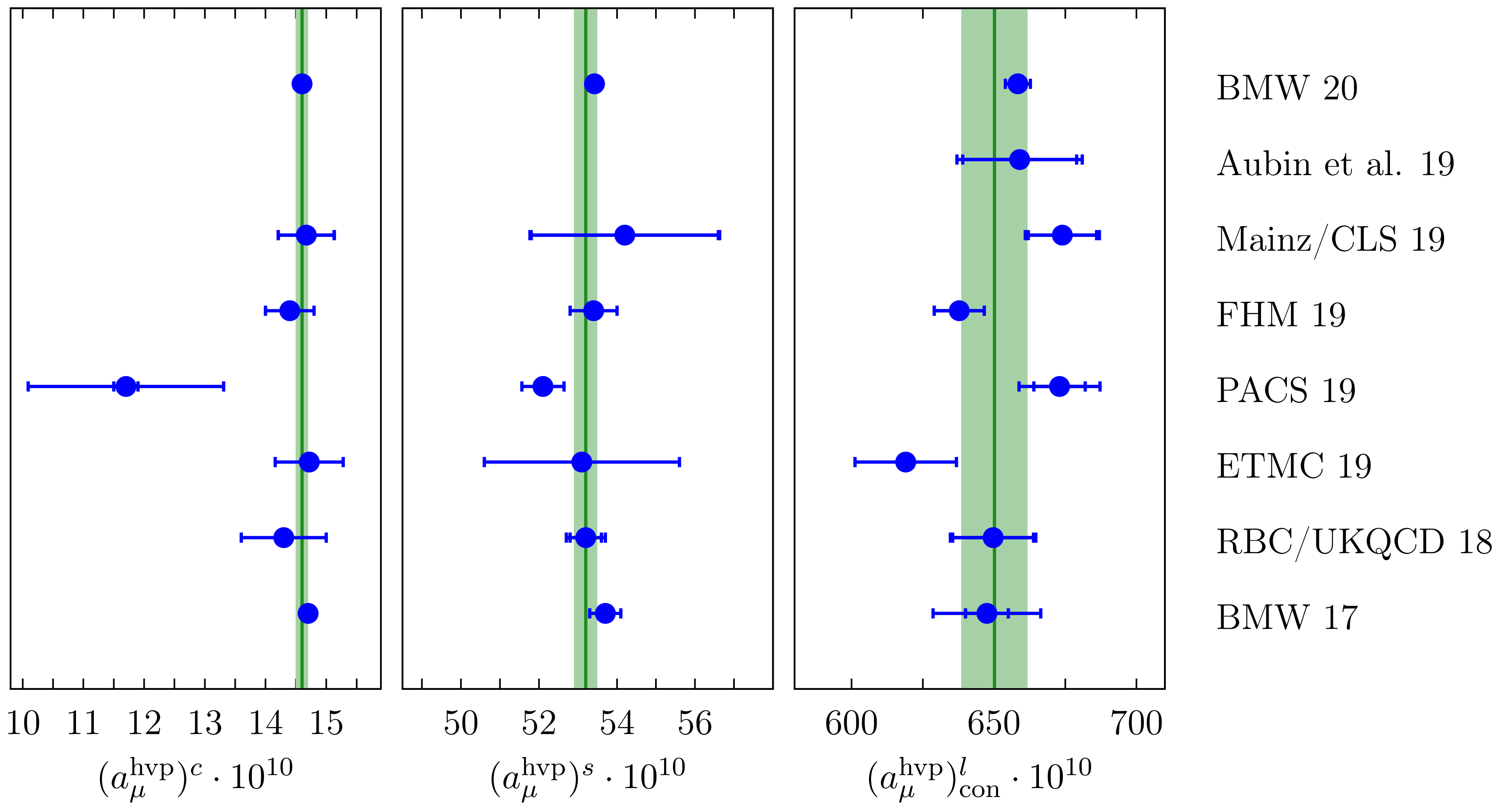
Lattice HVP: summary of contributions

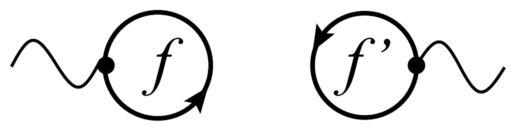


H. Wittig @ Lattice HVP workshop

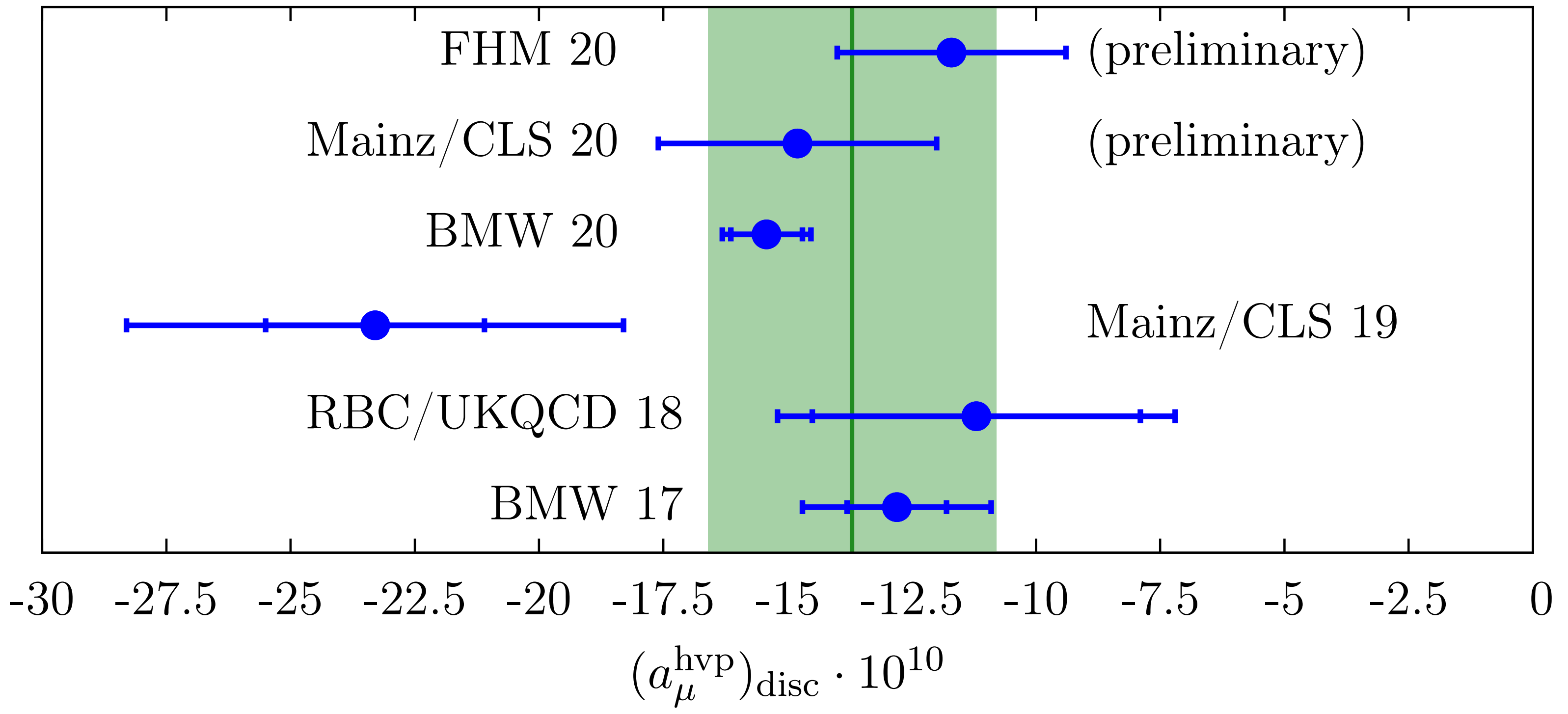


- Charm, strange contributions already well determined.
- Mild tensions for light contribution

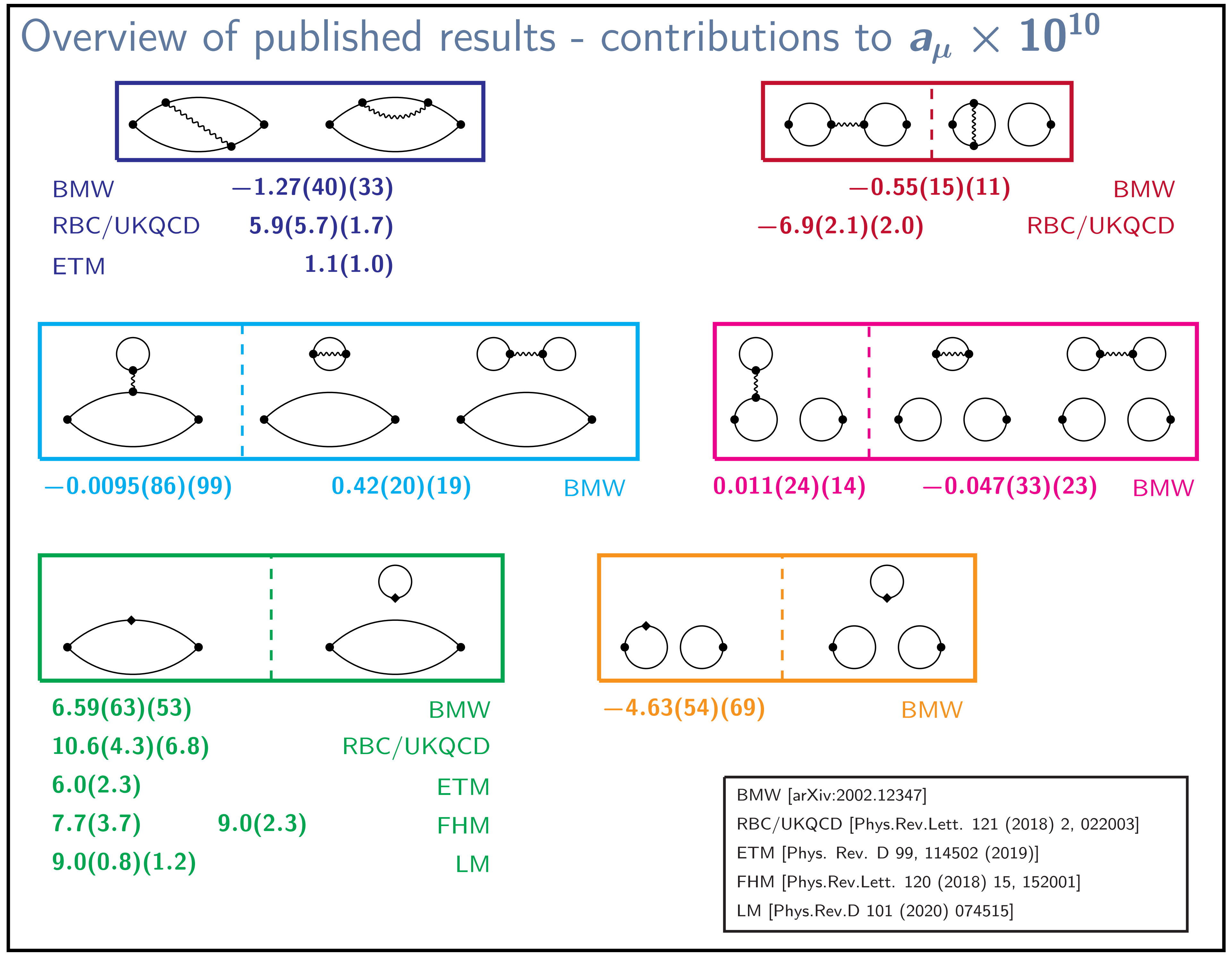




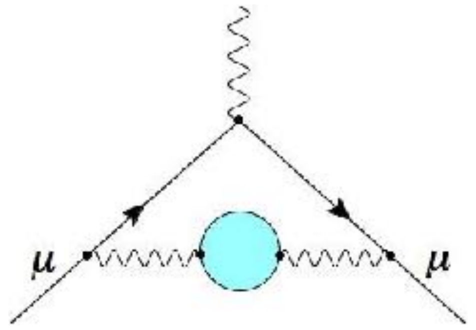
Consistent results with increasing precision



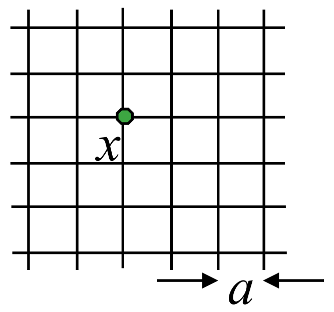
V. Gülpers @ Lattice HVP workshop



- Some tensions between lattice results for individual contributions.
- Large cancellations between individual contributions: $\delta a_\mu^{\rm IB} \lesssim 1\,\%$



Lattice HVP: long-distance tail



$$C(t) = \frac{1}{3}\sum_{i,x}\langle j_i^{\rm EM}(x,t)\, j_i^{\rm EM}(0,0)\rangle$$

- Start with spectral decomposition: $C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$

✦ bounding method:

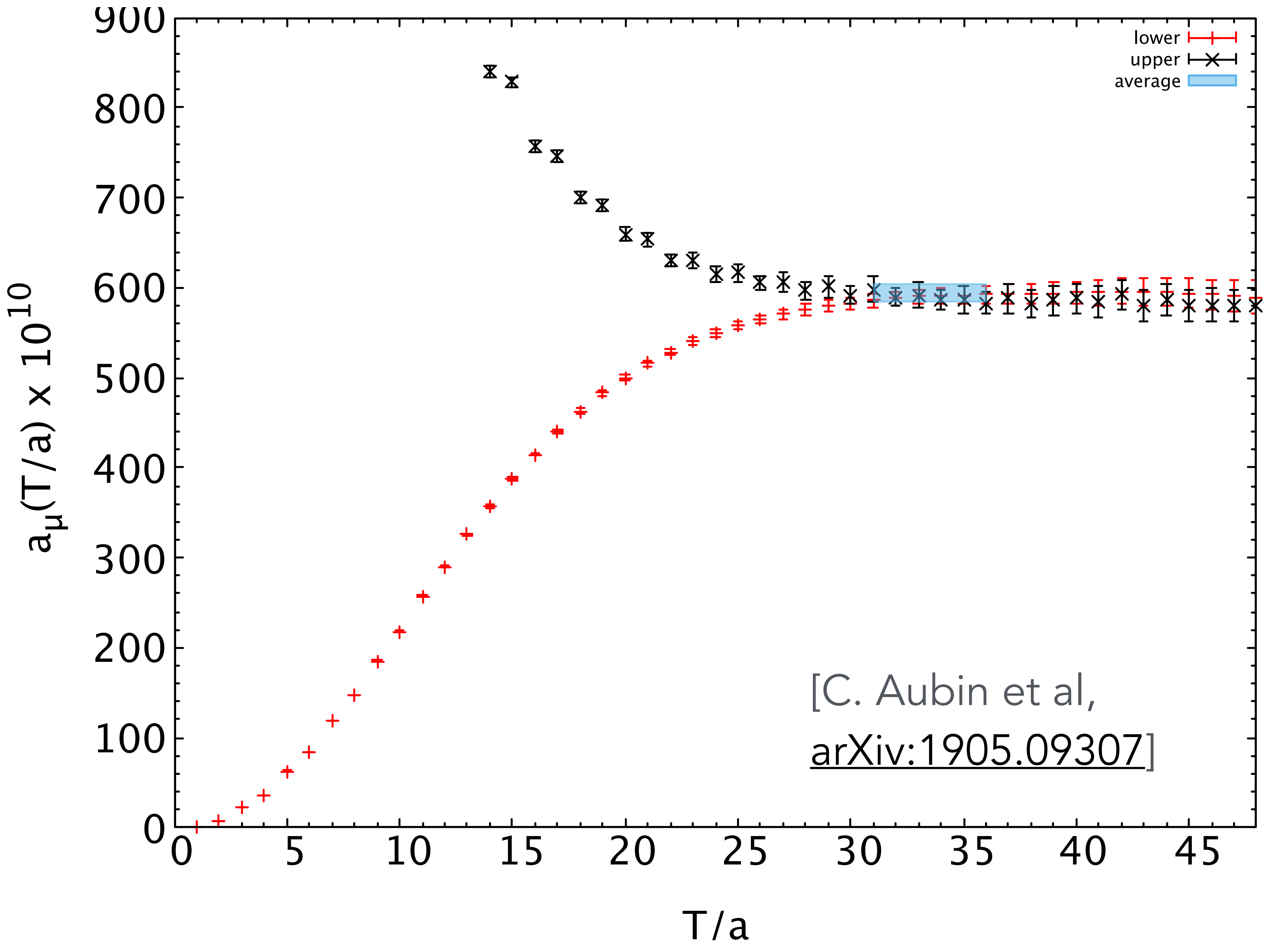
[Borsanyi et al, PRL 2018, Blum et al, PRL 2018]

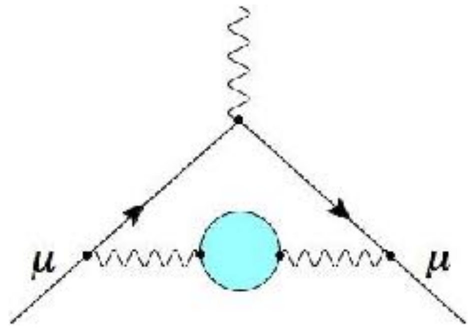
for $t > t_c$: $0 \leq C(t_c)\, e^{-\bar{E}_{t_c}(t-t_c)} \leq C(t) \leq C(t_c)\, e^{-E_0(t-t_c)}$

$\bar{E}_{t_c}$: effective mass of $C(t)$ at t_c

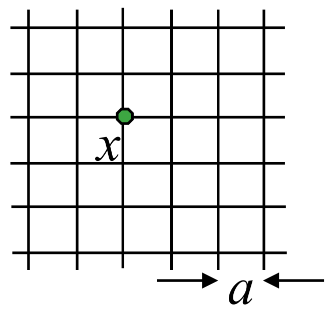
E_0: ground state energy

replace $G(t > t_c)$ with upper and lower bound, vary t_c





Lattice HVP: long-distance tail (again)

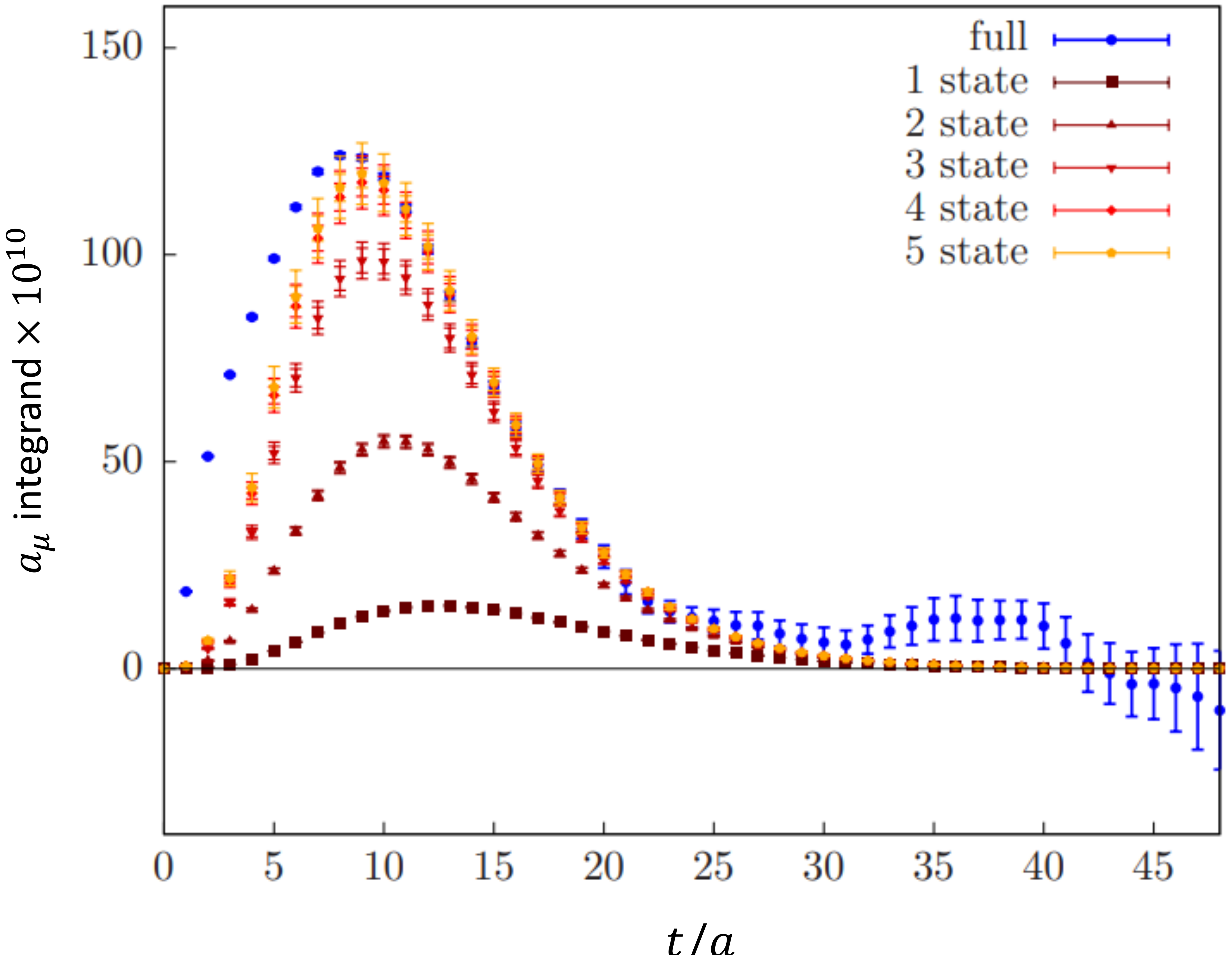


$$C(t) = \frac{1}{3}\sum_{i,x}\langle j_i^{\rm EM}(x,t)\, j_i^{\rm EM}(0,0)\rangle$$

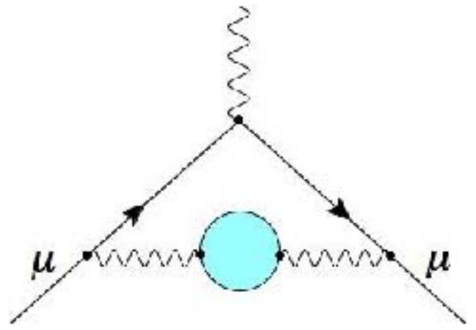
- Start with spectral decomposition: $C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$
  - ✦ obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states
  - ✦ use to reconstruct $C(t > t_c)$
  - ✦ can be used to improve bounding method:

    $$C(t) \rightarrow C(t) - \sum_{n=0}^{N} A_n^2\, e^{-E_n t}$$

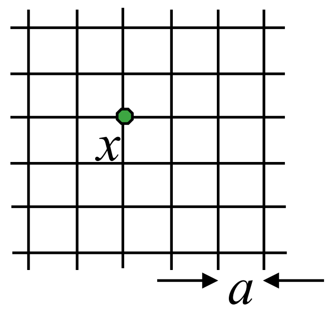
    use E_{N+1} in upper bound
  - ✦ yields big reduction in stat. errors (compared with bounding method)



J. Mckeon @ Lattice 2024 (RBC/UKQCD)
See also:
[Bruno et al, RBC/UKQCD, arXiv:1910.11745]



Lattice HVP: long-distance tail

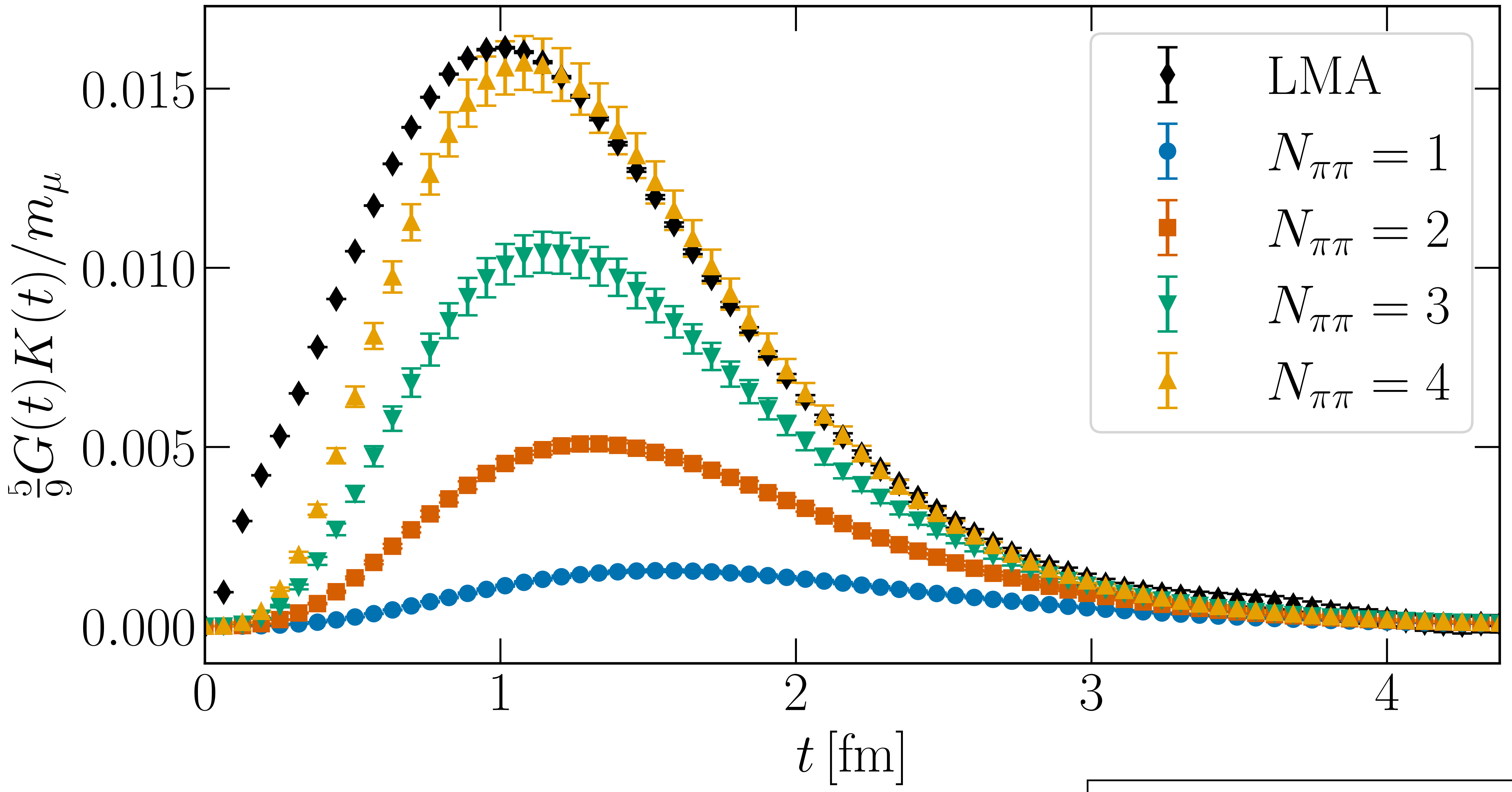


$$C(t) = \frac{1}{3}\sum_{i,x}\langle j_i^{\rm EM}(x,t)\, j_i^{\rm EM}(0,0)\rangle$$

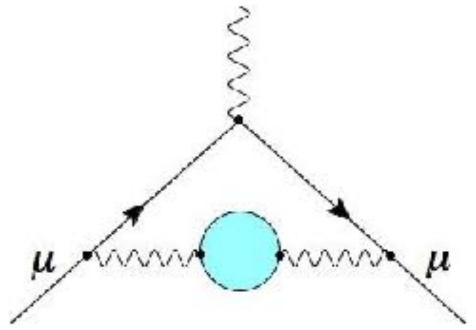
- Start with spectral decomposition: $C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$
  - ✦ obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states
  - ✦ use to reconstruct $C(t > t_c)$
  - ✦ can be used to improve bounding method:

    $$C(t) \to C(t) - \sum_{n=0}^{N} A_n^2\, e^{-E_n t}$$

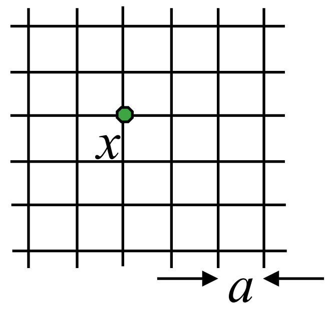
    use E_{N+1} in upper bound
  - ✦ yields $\sim \times 2$ reduction in stat. errors



S. Kuberski & N. Miller @ Lattice 2024 (Mainz)
See also: A. Gerardin et al, PRD 2019



Lattice HVP: long-distance tail

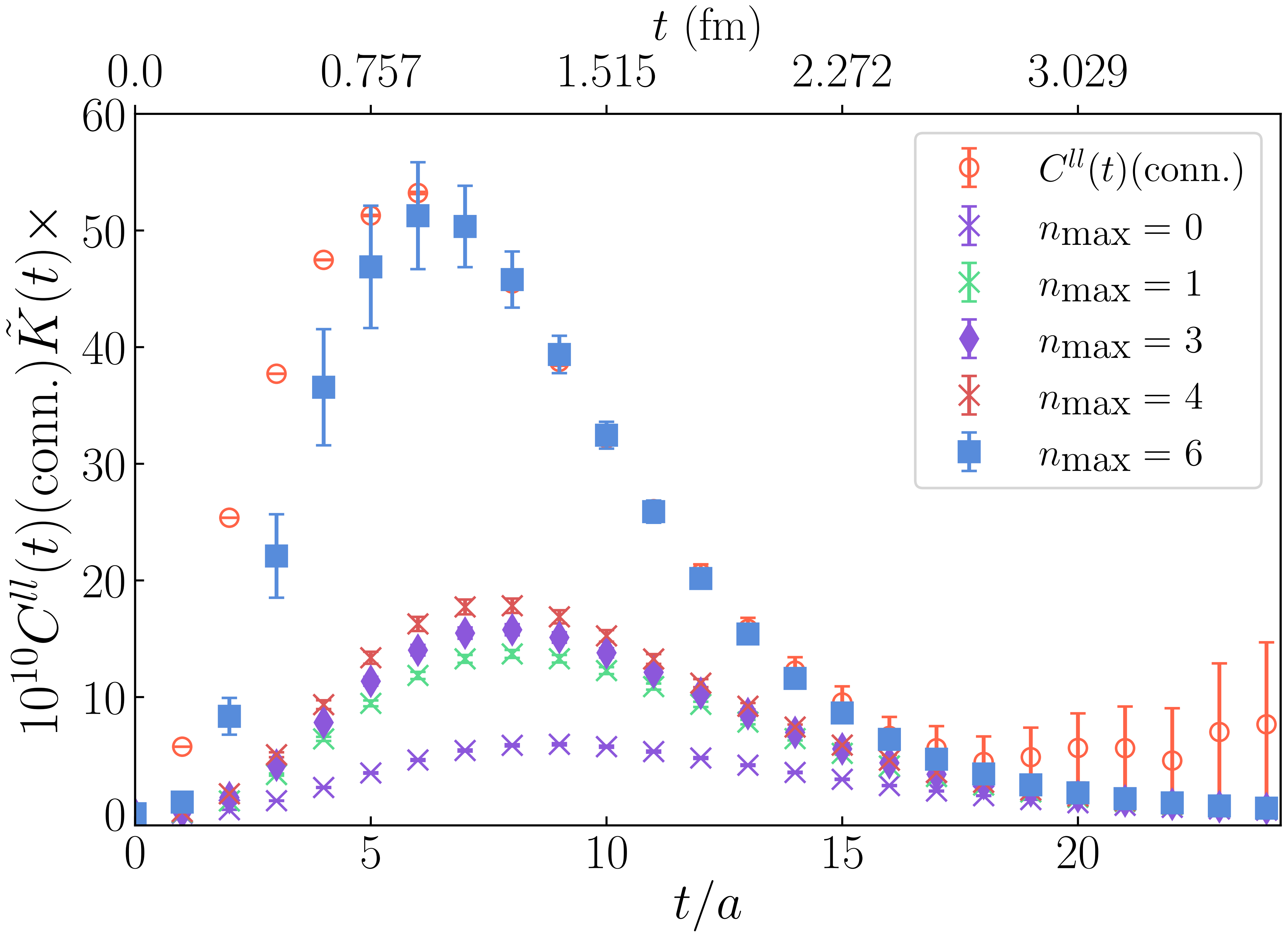


$$C(t) = \frac{1}{3}\sum_{i,x}\langle j_i^{\rm EM}(x,t)\, j_i^{\rm EM}(0,0)\rangle$$

- Start with spectral decomposition: $C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$
  - ✦ obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states
  - ✦ use to reconstruct $C(t > t_c)$
  - ✦ can be used to improve bounding method:

    $$C(t) \to C(t) - \sum_{n=0}^{N} A_n^2 e^{-E_n t}$$

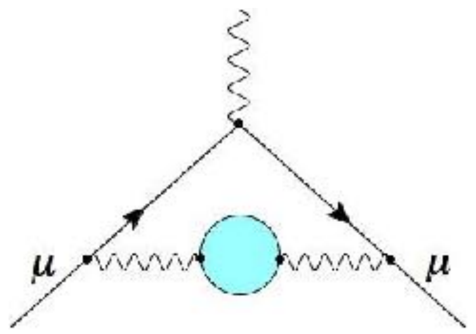
    use E_{N+1} in upper bound
  - ✦ yields $\sim \times 2.5$ reduction in stat. errors (compared with bounding method)



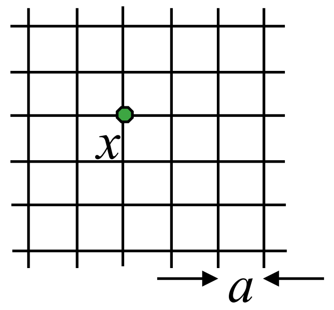
[Lahert, et al, arXiv:2409.00756]
see also:
[Lahert et al, arXiv:2112.11647]

First LQCD calculation with staggered multi-pion operators

see also:
[Frech for BMW @ Lattice 2024]



Long-distance tail (ud)

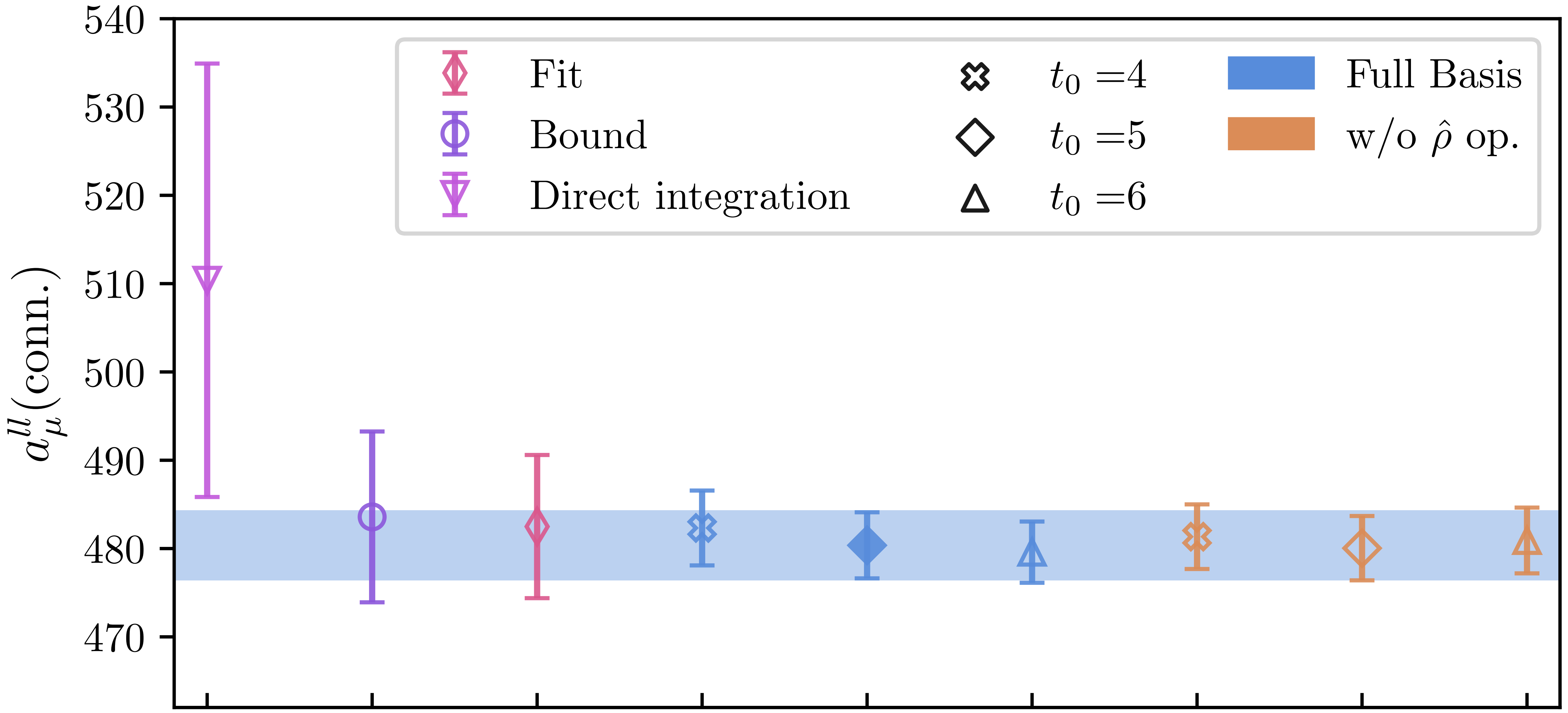


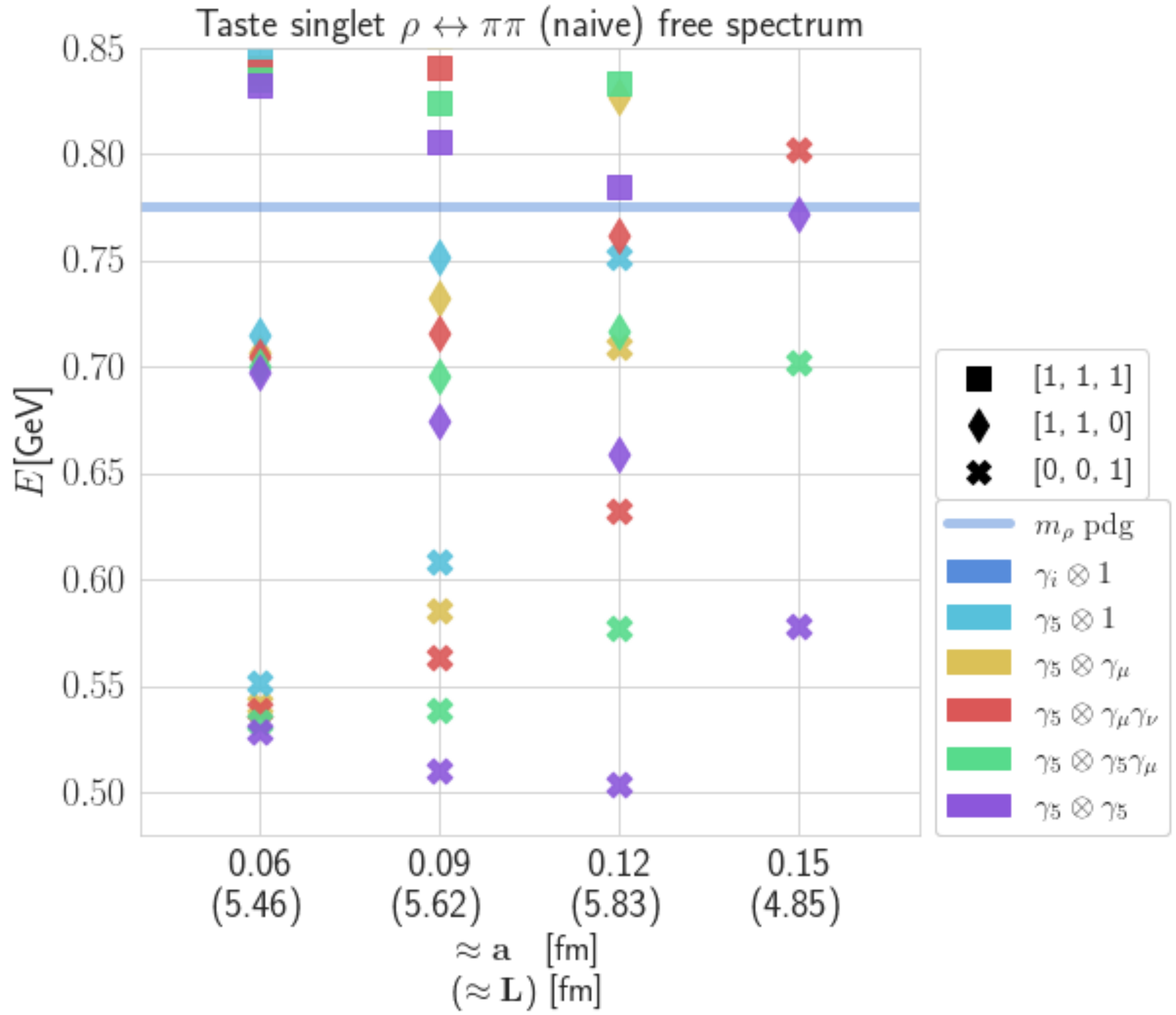
[Shaun Lahert et al, arXiv: 2409.00756]

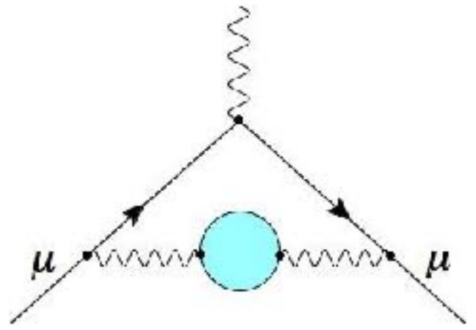
$$C(t) = \frac{1}{3}\sum_{i,x}\langle j_i^{\mathrm{EM}}(x,t)\, j_i^{\mathrm{EM}}(0,0)\rangle$$

- Spectral reconstruction: $C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$
  - ✦ obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states
  - ✦ First LQCD calculation with staggered multi-pion operators
  - ✦ Construct matrix of correlators (2,3,4-point functions)
  - ✦ Use GEVP to obtain energies and amplitudes for $\pi\pi$ states
  - ✦ Reconstruct vector-current correlator

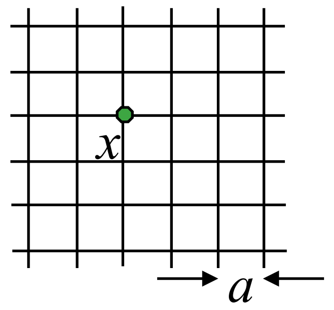
$$\mathbf{C}(t) = \begin{pmatrix} C(t)_{J,\tilde{J}\to J,\tilde{J}} & C(t)_{J,\tilde{J}\to\pi\pi} \\ C(t)_{\pi\pi\to J,\tilde{J}} & C(t)_{\pi\pi\to\pi\pi} \end{pmatrix}$$







Lattice HVP: results

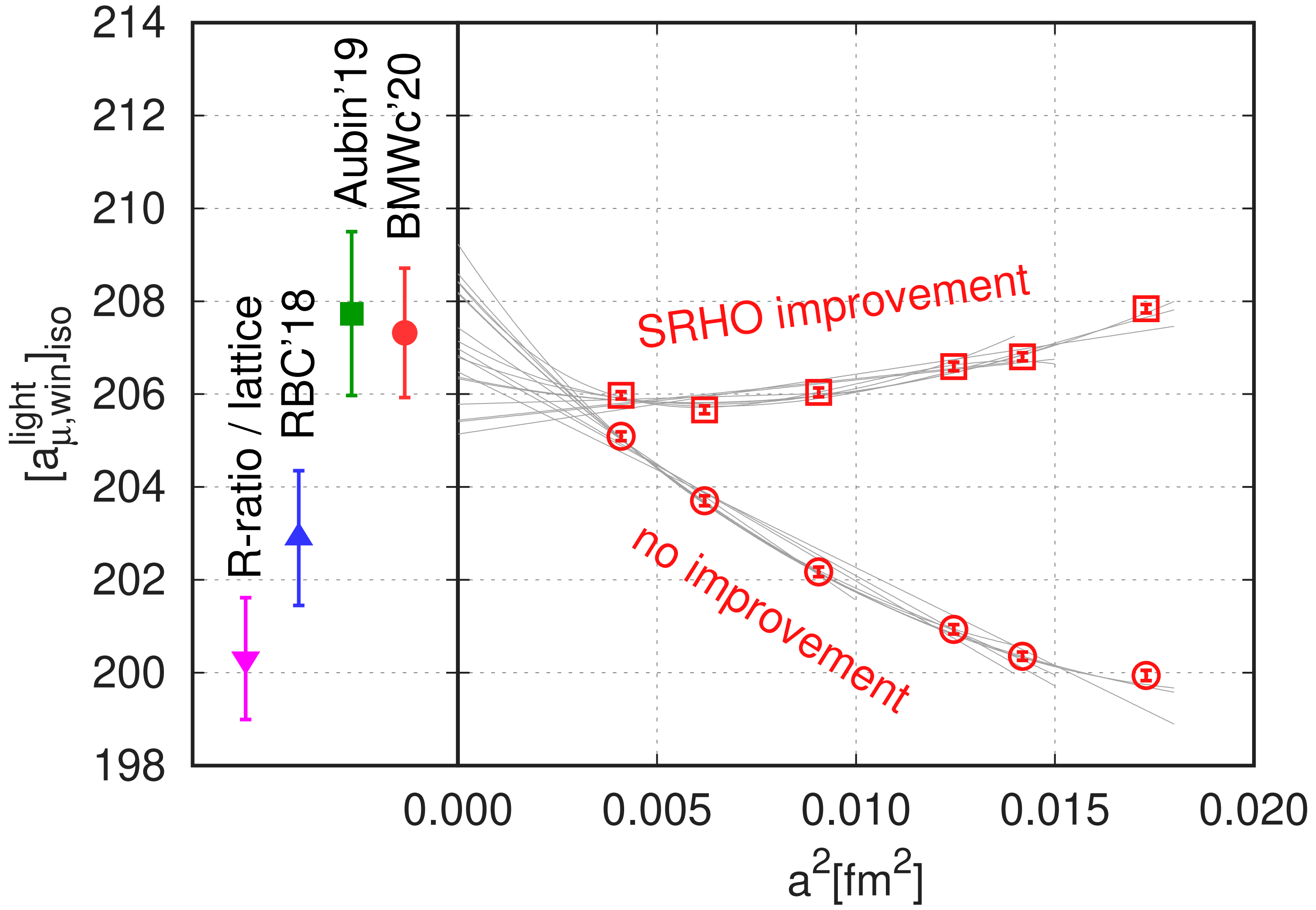


In 2020 WP:

- Lattice HVP average at $2.6\,\%$ total uncertainty: $a_\mu^{\mathrm{HVP,LO}} = 711.6\,(18.4) \times 10^{10}$
- BMW 20 [Sz. Borsanyi et al, arXiv:2002.12347, 2021 Nature] first LQCD calculation with sub-percent ($0.8\,\%$) error in tension with data-driven HVP (2.1σ)
- Further tensions for intermediate window:

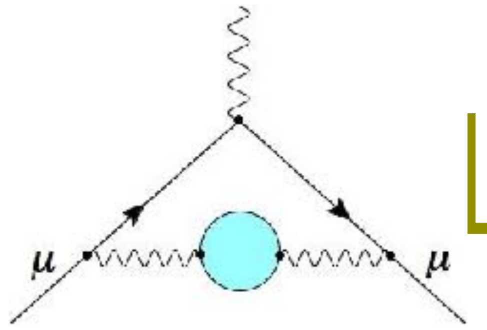
  -3.7σ tension with data-driven evaluation

  -2.2σ tension with RBC/UKQCD18

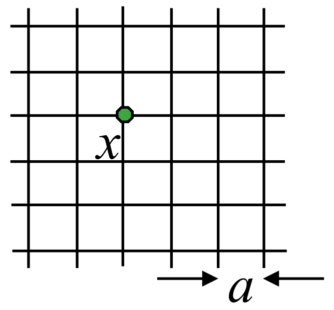


Staggered fermions:

- taste-breaking effects (which yield taste splittings) are significant (sometimes dominant) source of discretization errors
- possible to use EFT schemes (ChPT, Chiral Model, MLLGS) to correct for taste-splitting effects before taking continuum extrapolation: continuum limit should not be affected



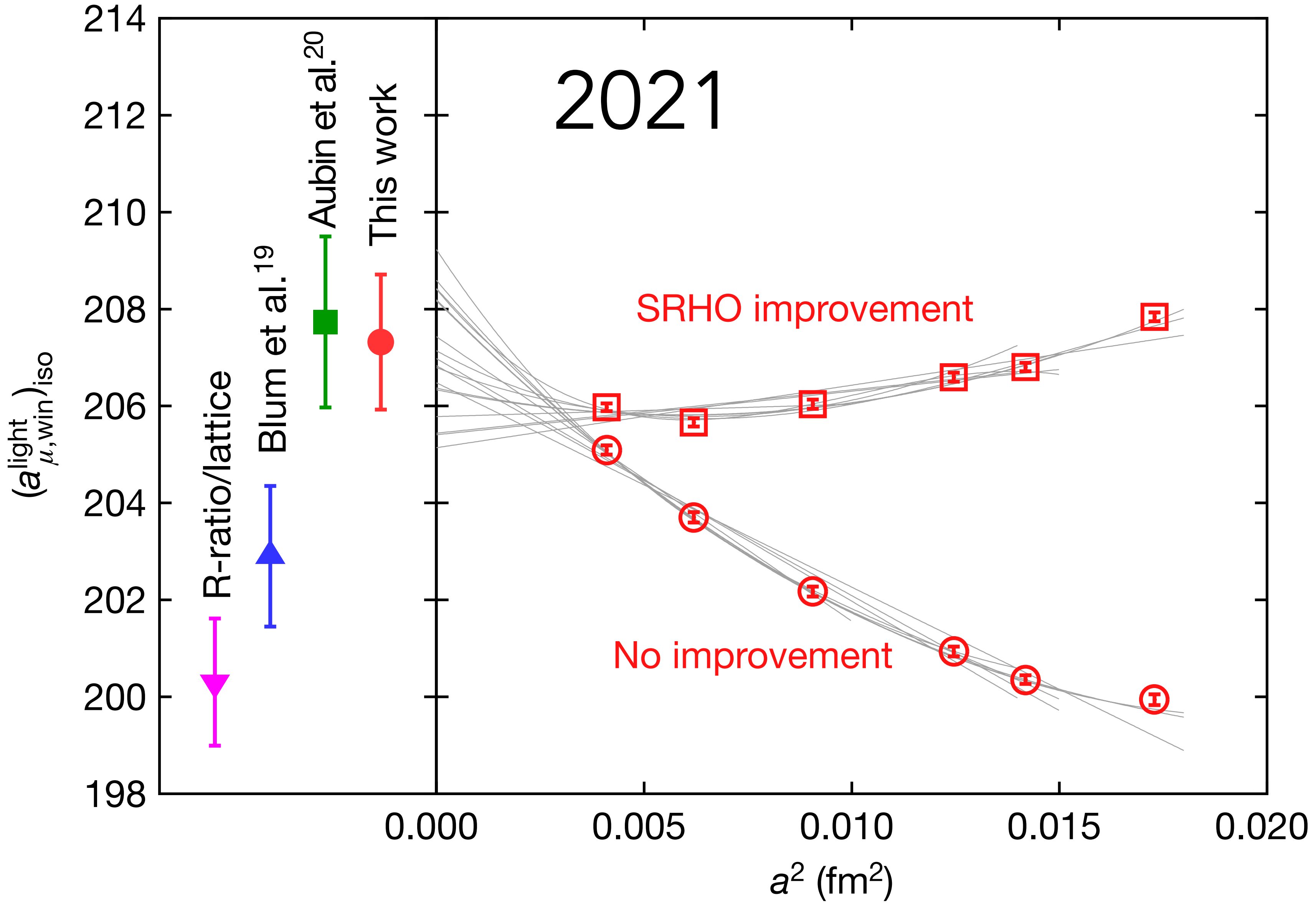
Lattice HVP: intermediate window (W)



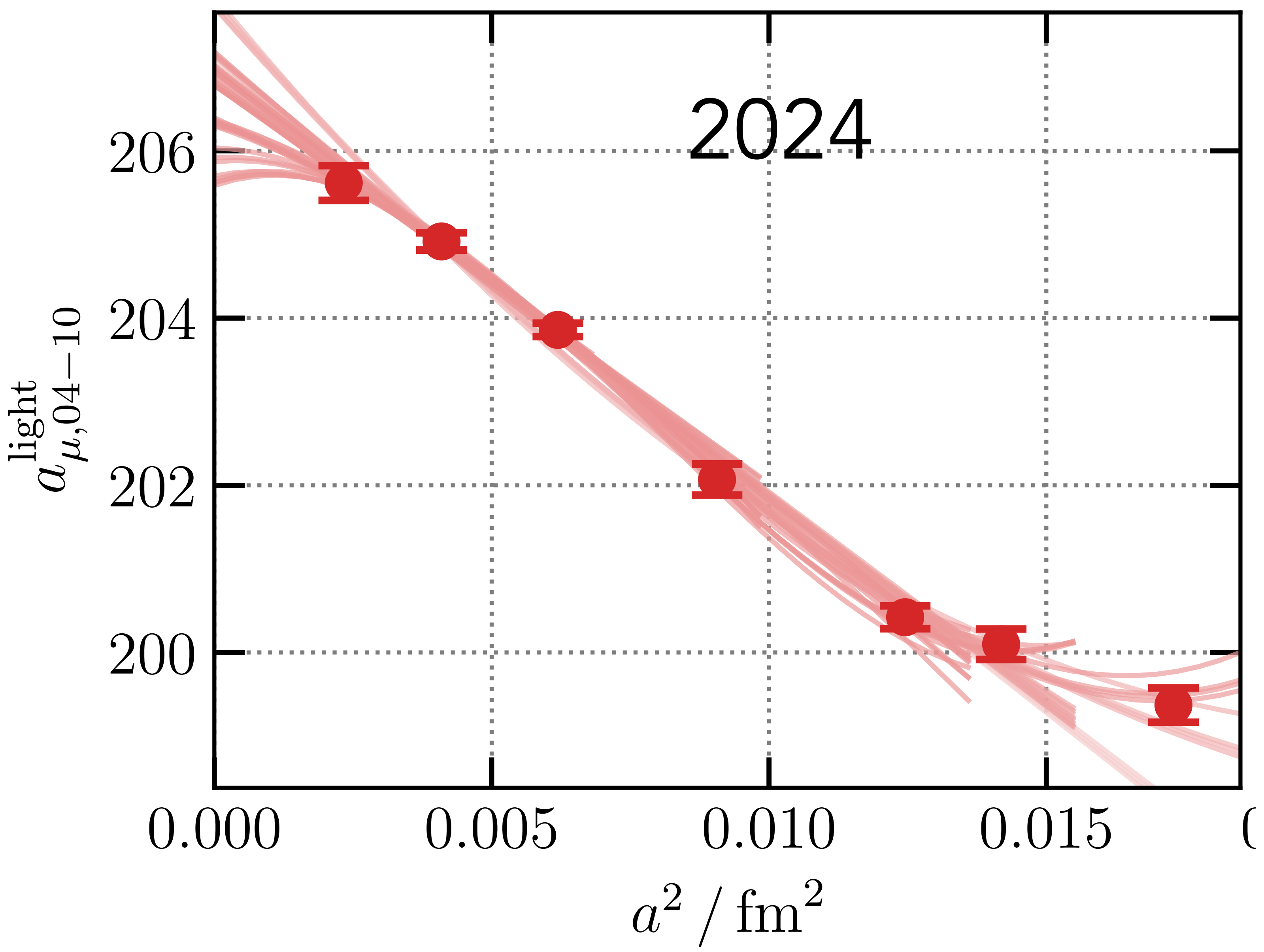
update: BMW+DMZ 2024

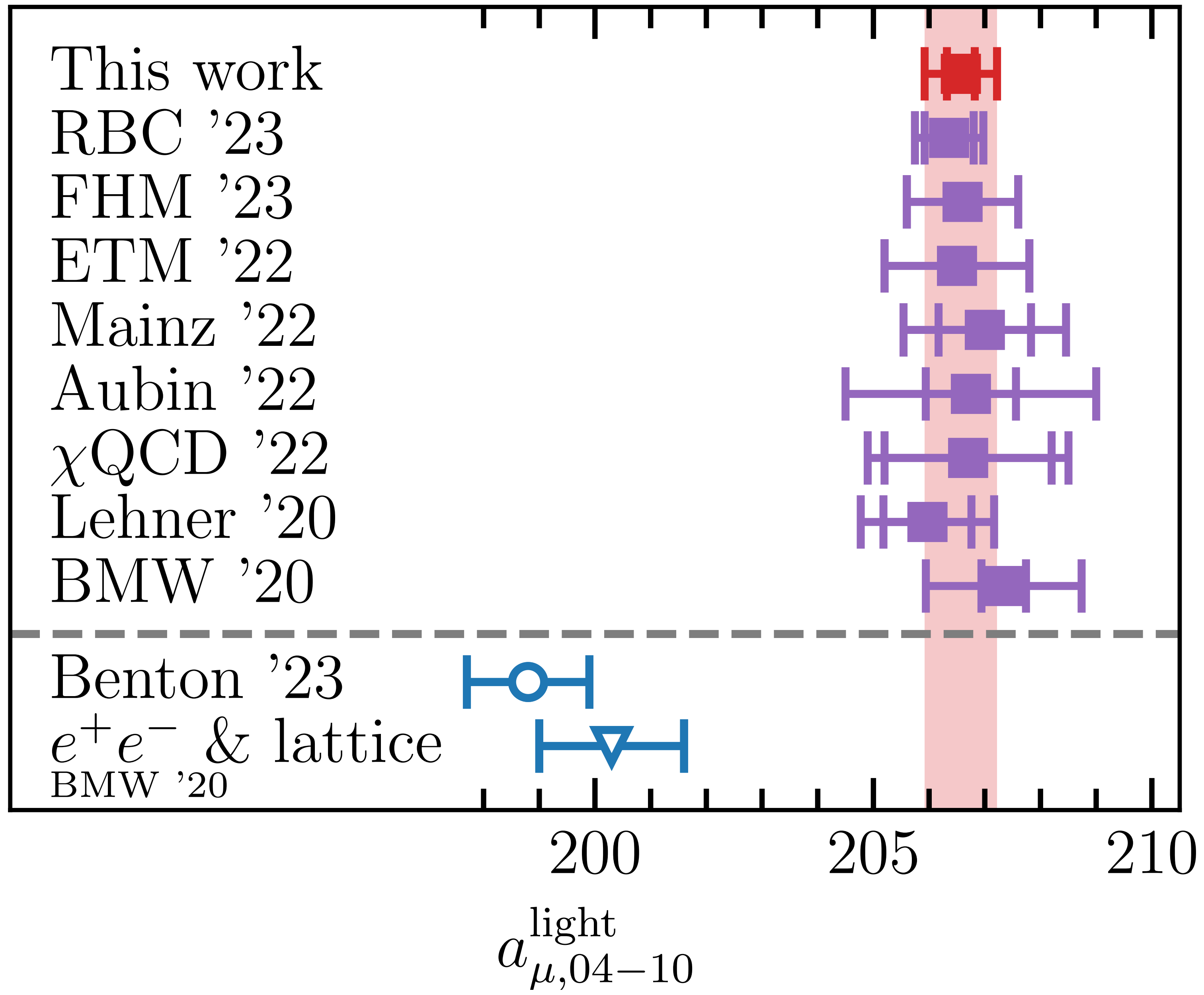
- intermediate window, a_μ^W

- BMW 20 [Sz. Borsanyi et al, arXiv:2002.12347, 2021 Nature]



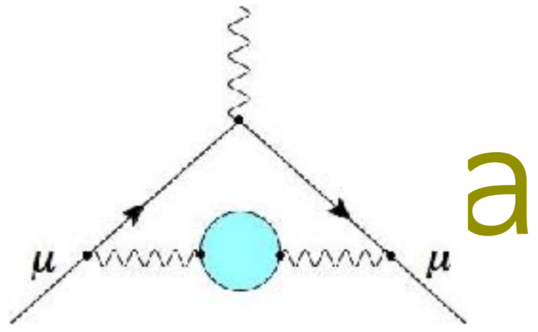
- BMW-DMZ 24 [A. Boccaletti et al, arXiv:2407.10913]



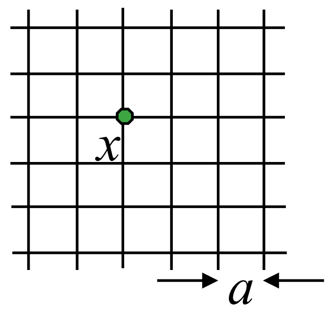


- Improvement:
  correct lattice data for discretization effects due to taste-splittings before taking continuum limit.

Continuum extrapolations obtained only from data not corrected for taste-splittings.



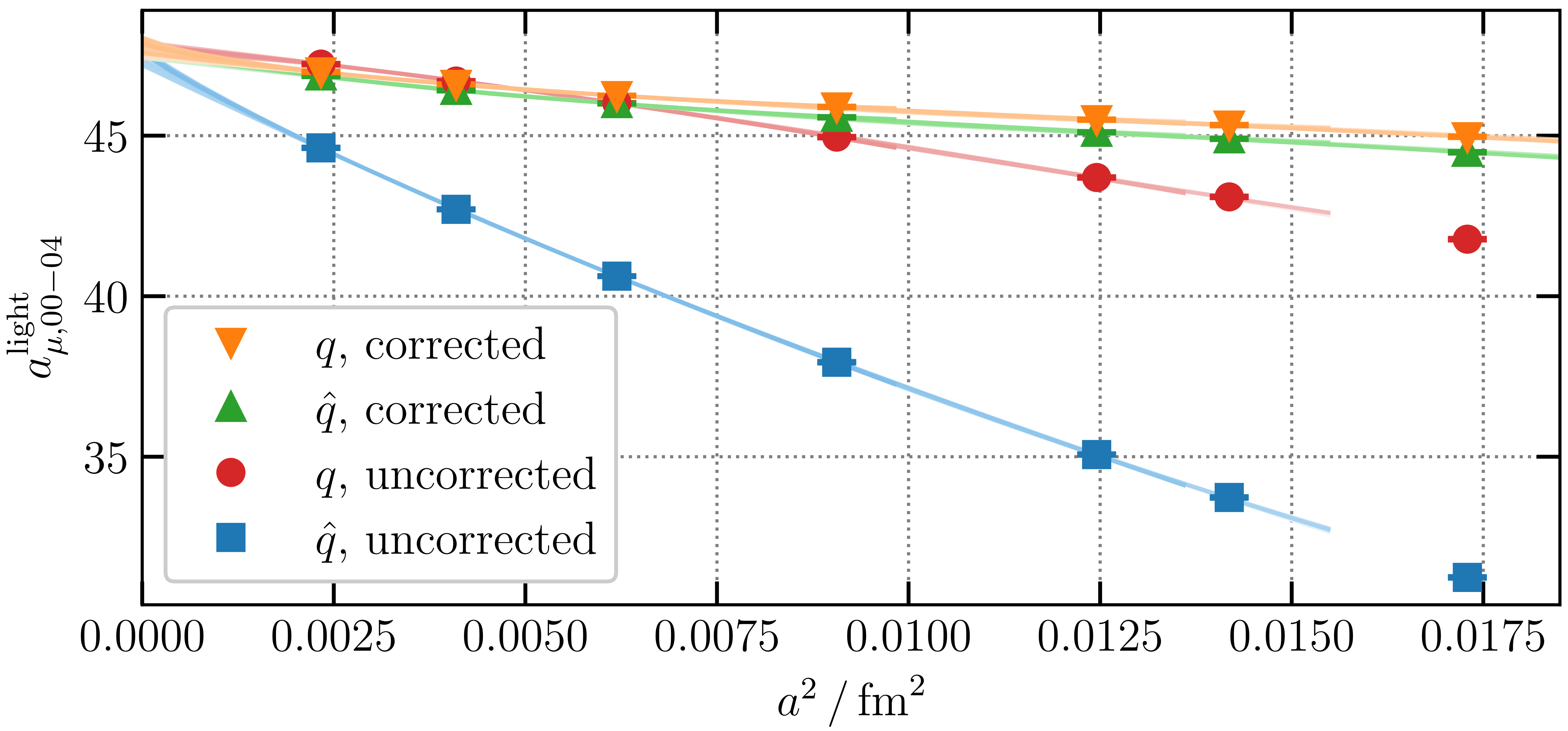
attice HVP: short-distance window (SD)



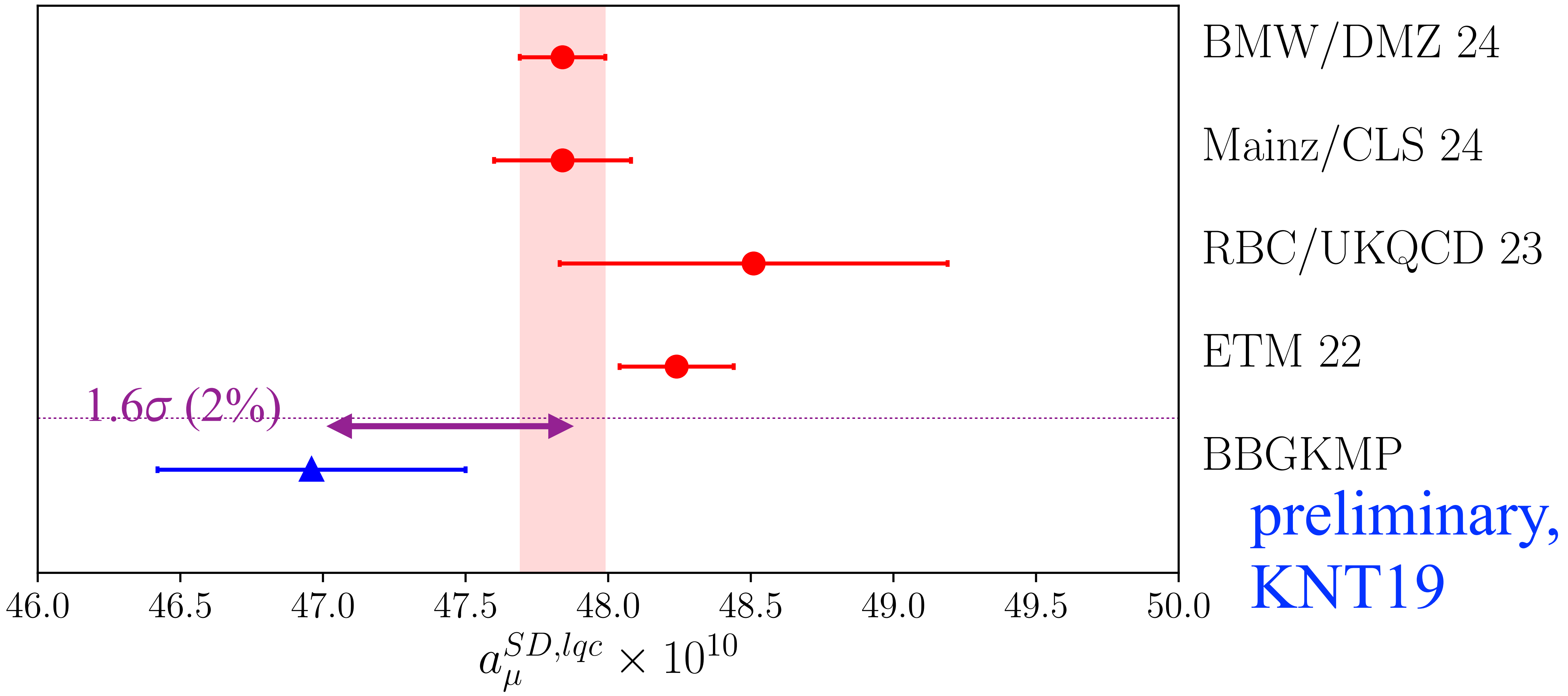
update: BMW+DMZ 2024

- short-distance window, a_μ^{SD}

- BMW 24 [A. Boccaletti et al, arXiv:2407.10913]



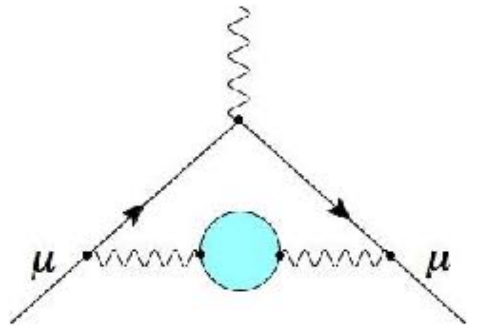
C. Davies @ Lattice 2024



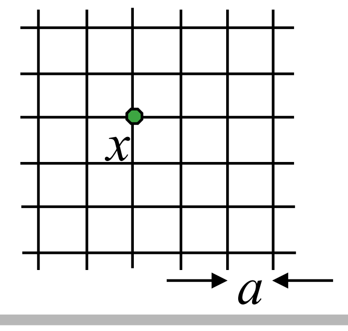
- corrected: remove log-enhanced discretization effects at tree-level

- $\hat{q}$: $\left[t^2 - \frac{4}{(aQ)^2}\sin^2\left(\frac{aQt}{2}\right)\right] \to \left[t^2 - \frac{4}{(a\hat{Q})^2}\sin^2\left(\frac{aQt}{2}\right)\right]$

small tension in SD with pre-2023 data-driven evaluation

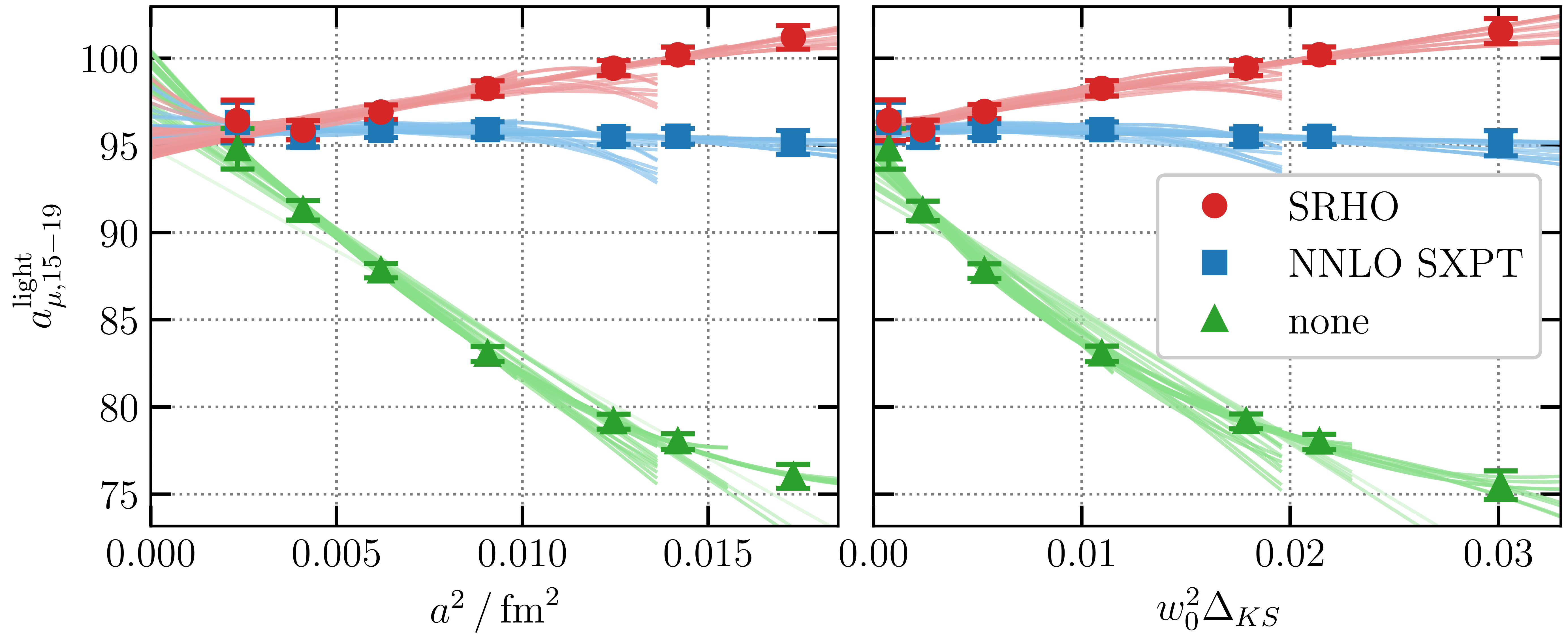


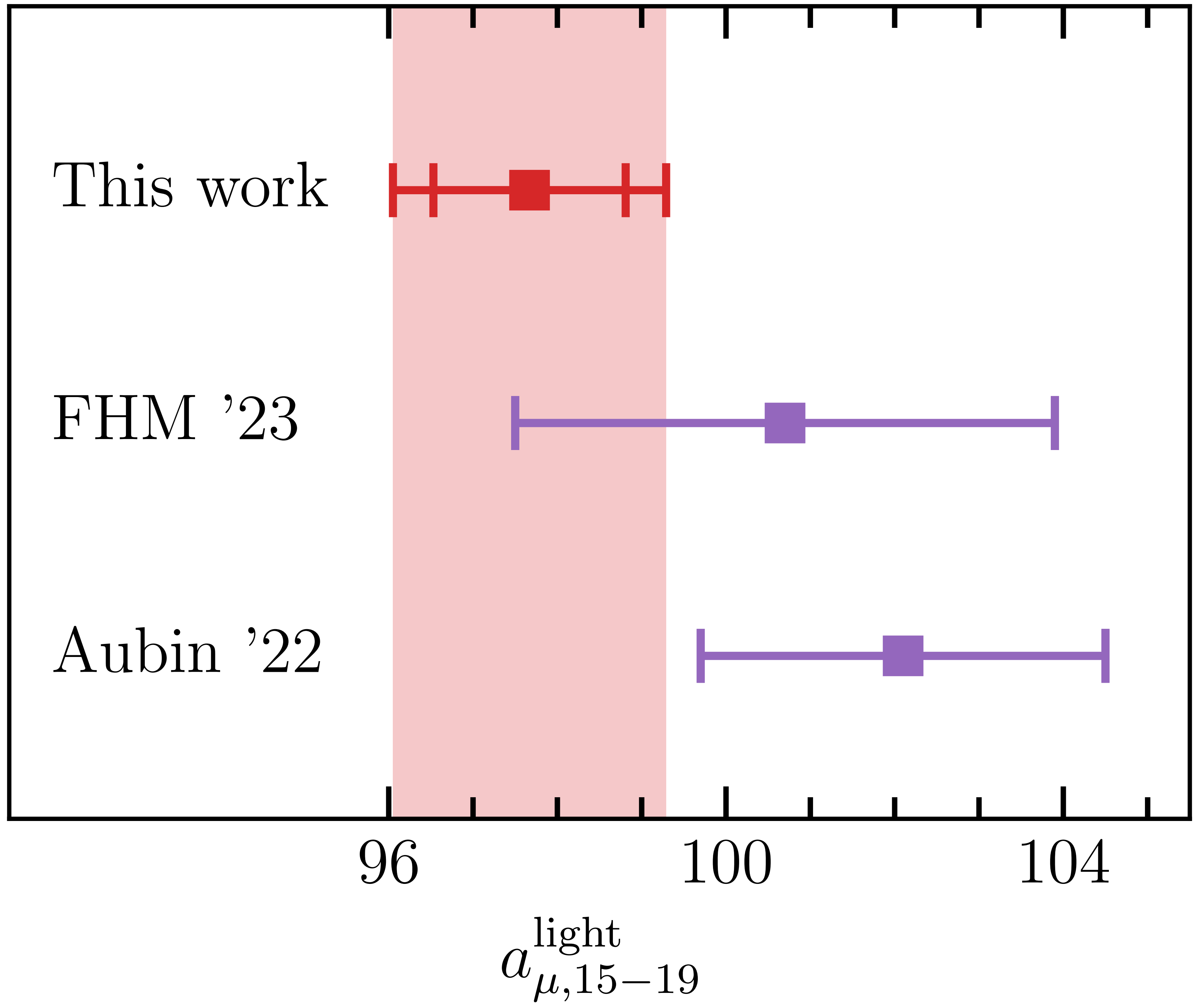
Lattice HVP: 2nd window (W2)



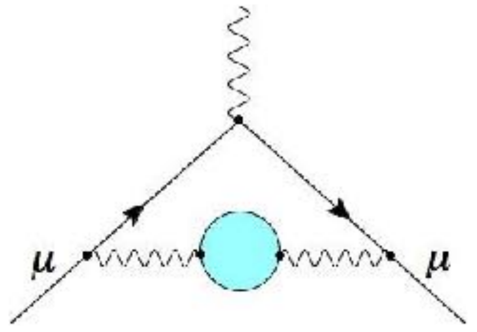
update: BMW+DMZ 2024

- [A. Boccaletti et al, arXiv:2407.10913]

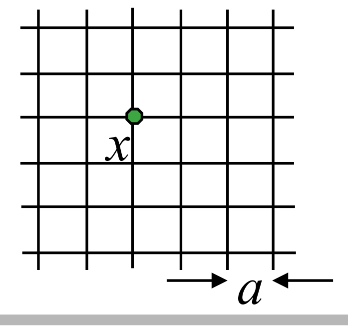




Continuum extrapolations of data with ``No improvement" (green) excluded from model average.



Lattice HVP: windows



update: Fermilab/HPQCD/MILC 2024

FNAL/HPQCD/MILC @ Lattice 2024:

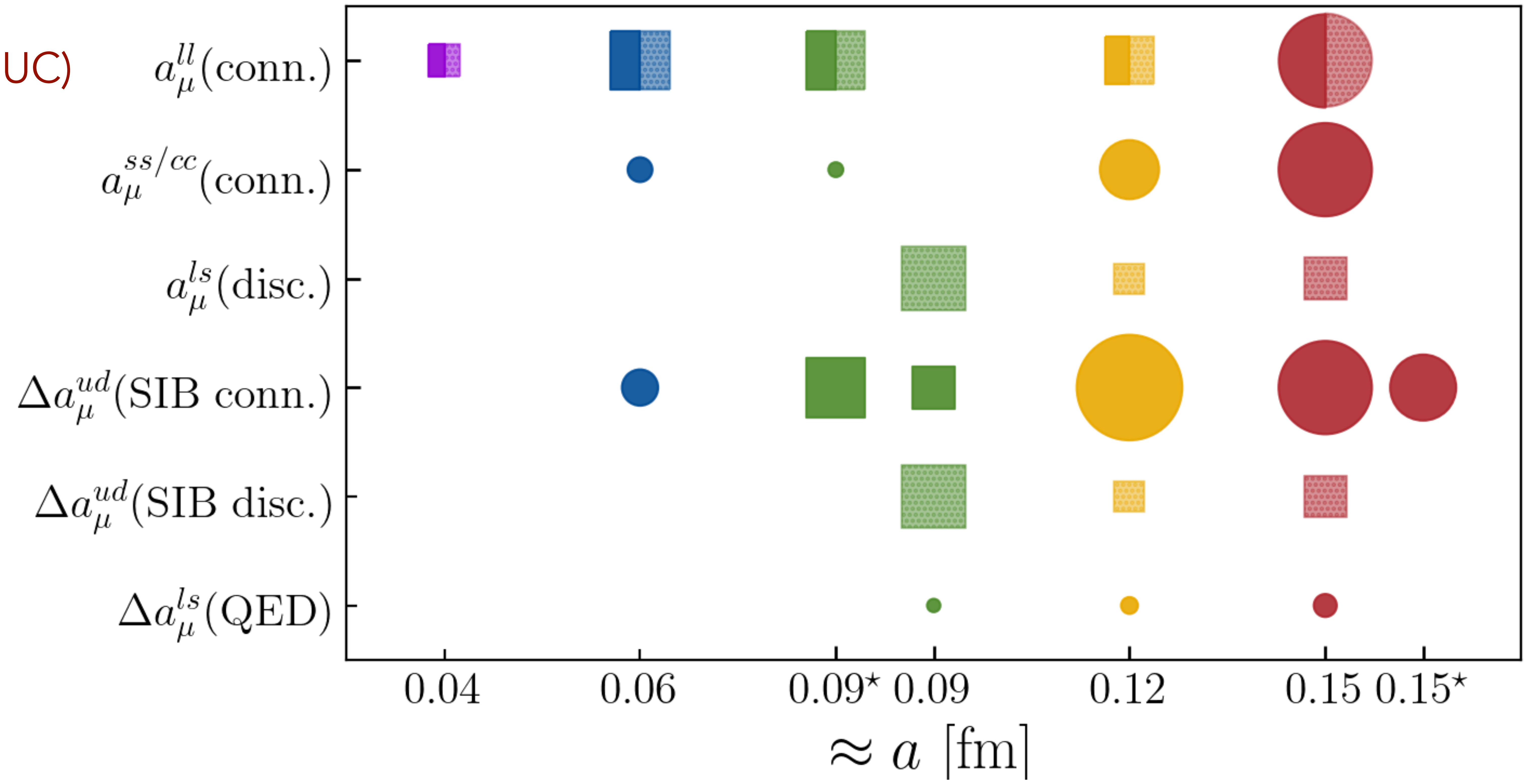
Shaun Lahert (Utah) & Michael Lynch (UIUC)

Shaun Lahert (Utah)

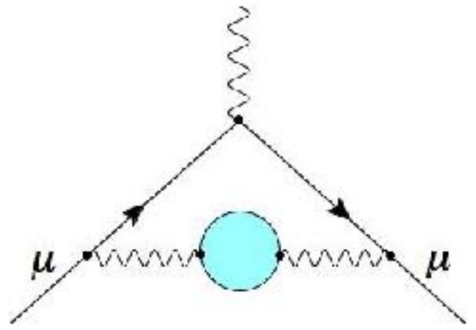
David Clarke (Utah)

Jake Sitison (U Colorado)

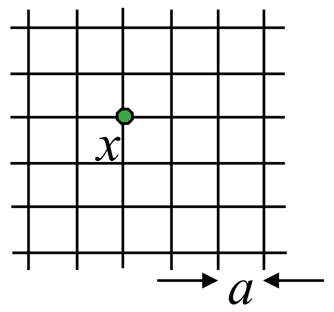
Craig McNeile (Plymouth)



- Solid color (local current) hatched (one-link)
- Squares: low-mode improved.
- Size ∼ statistics



Lattice HVP: long-distance window

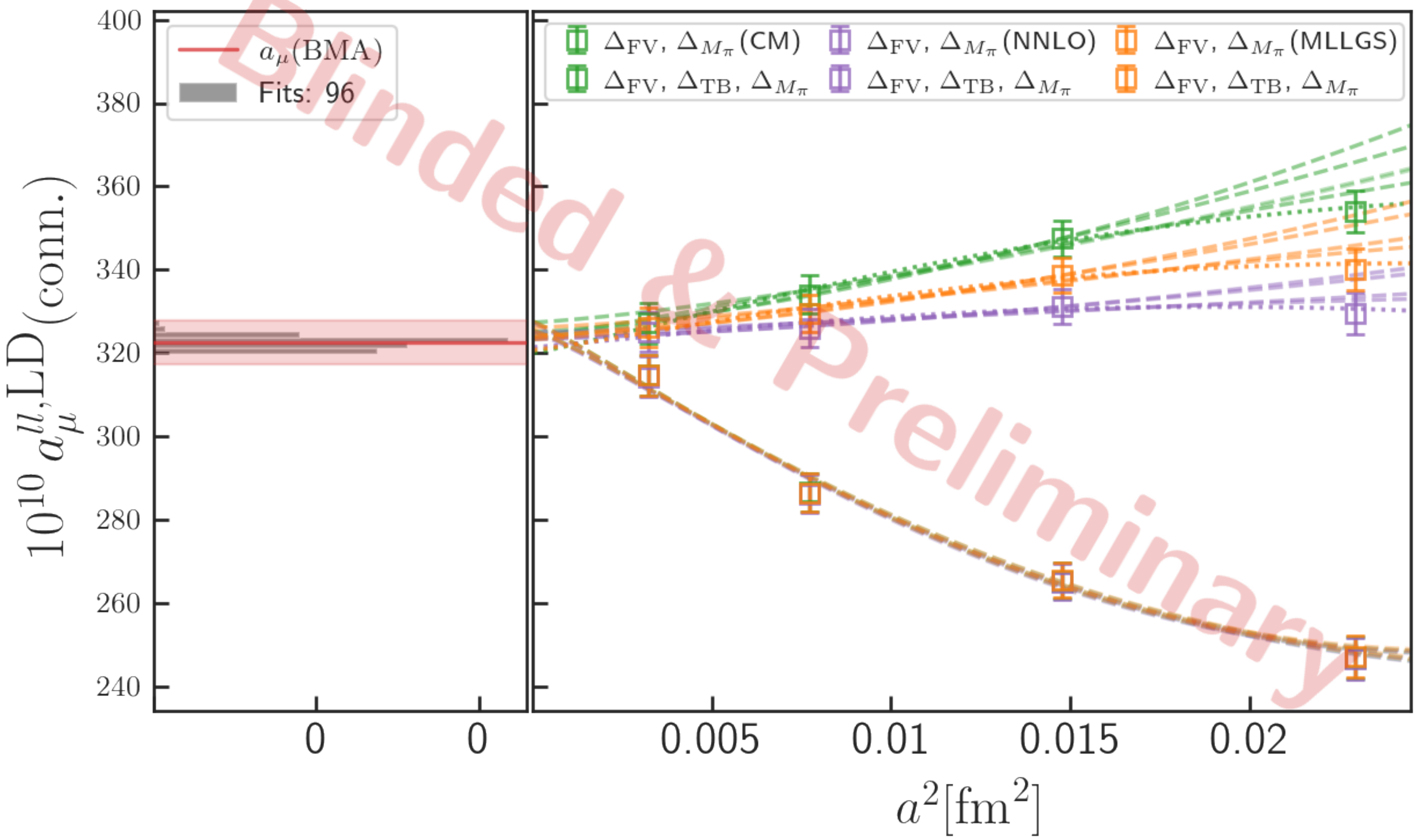


update: Fermilab/HPQCD/MILC 2024

Michael Lynch @ Lattice 2024

- long-distance window a_μ^{LD} and full a_μ^{ll}(conn.)

a_μ^{ll} (conn.) - LD window



a_μ^{ll} (conn.) - Full