Lattice QCD Calculation of Distribution Amplitude for Mesons

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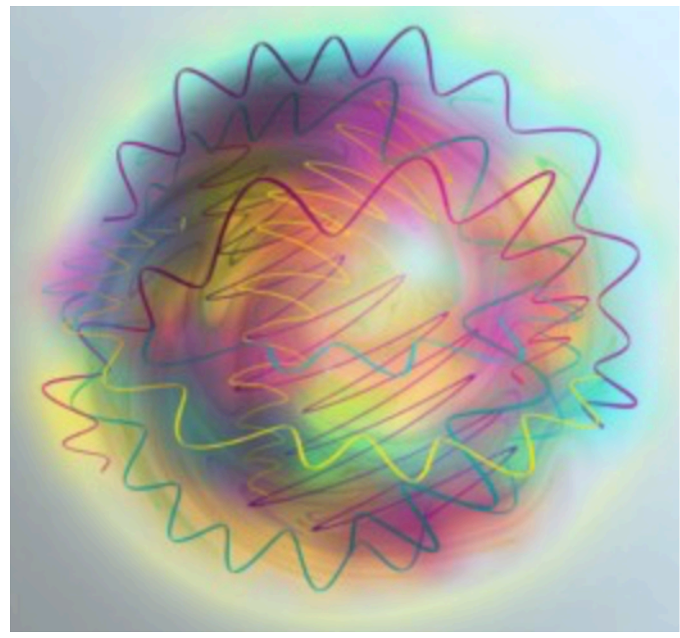
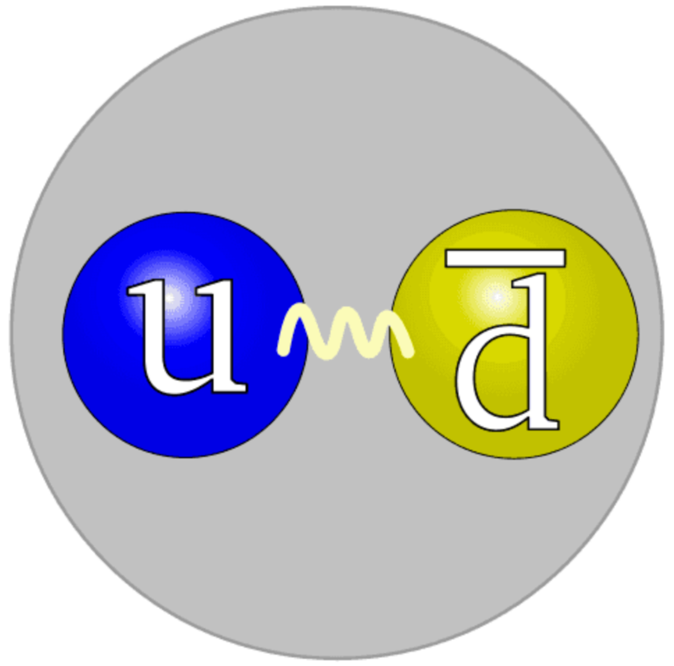
QCD@LHC 2024



Lattice Parton Collaboration

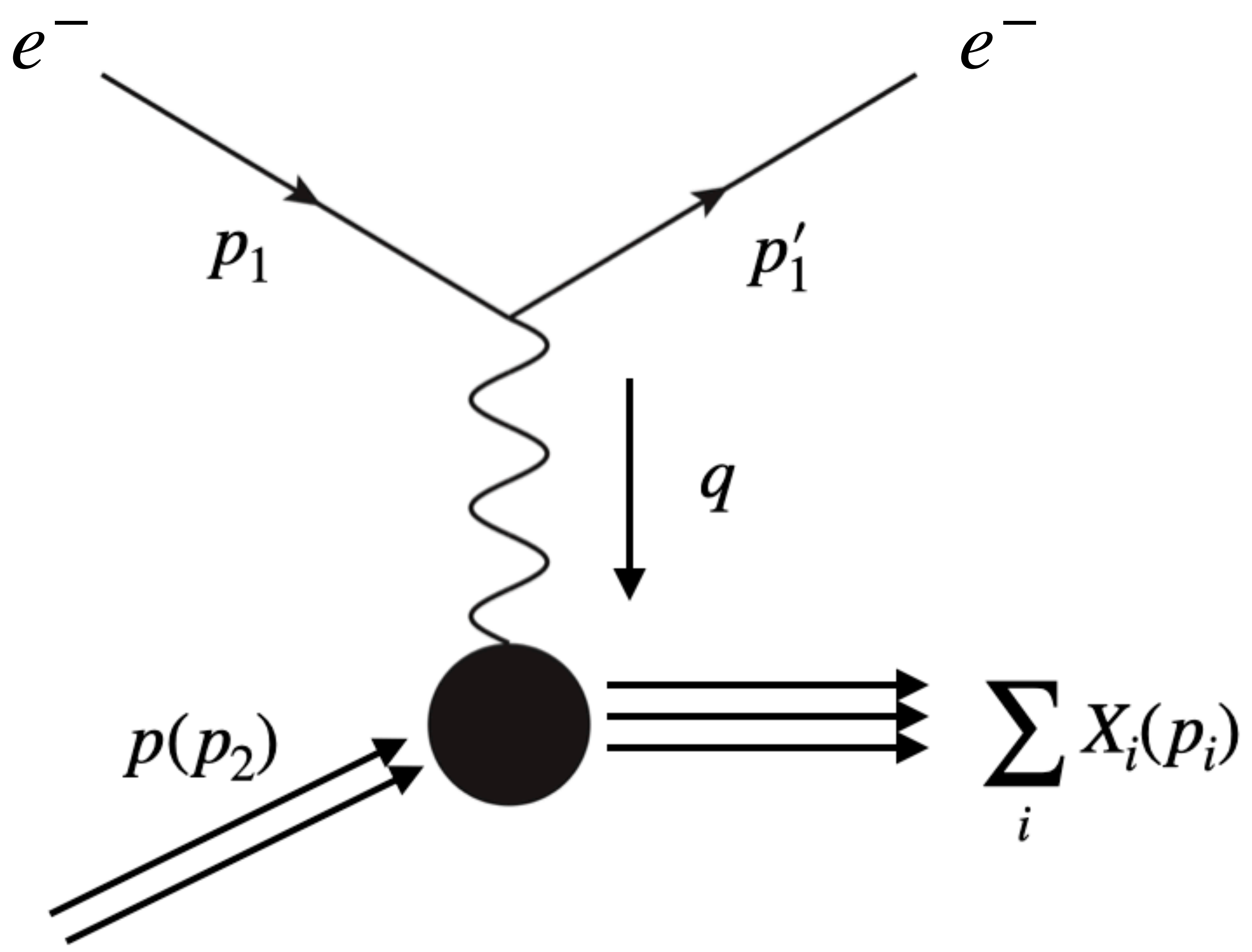
Outline

- Motivation
- Numerical results
- Analysis of uncertainties
- Summary



Motivation: Hadron structure

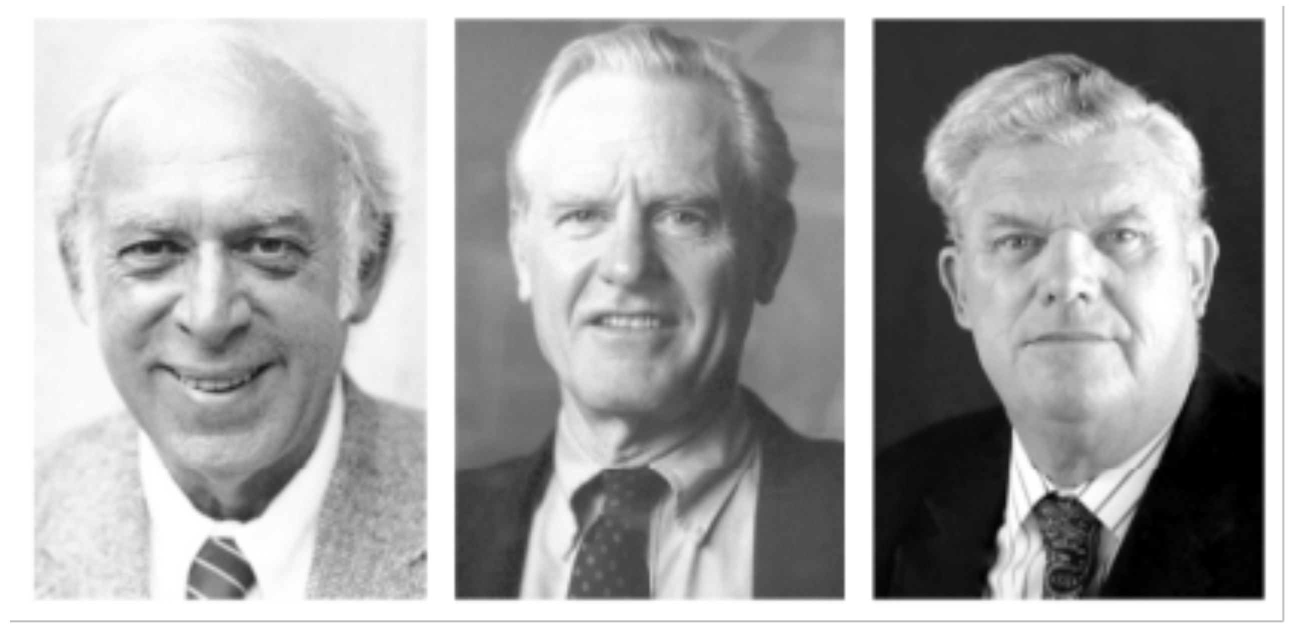
Deep inelastic Scattering process



hadronic part of cross section

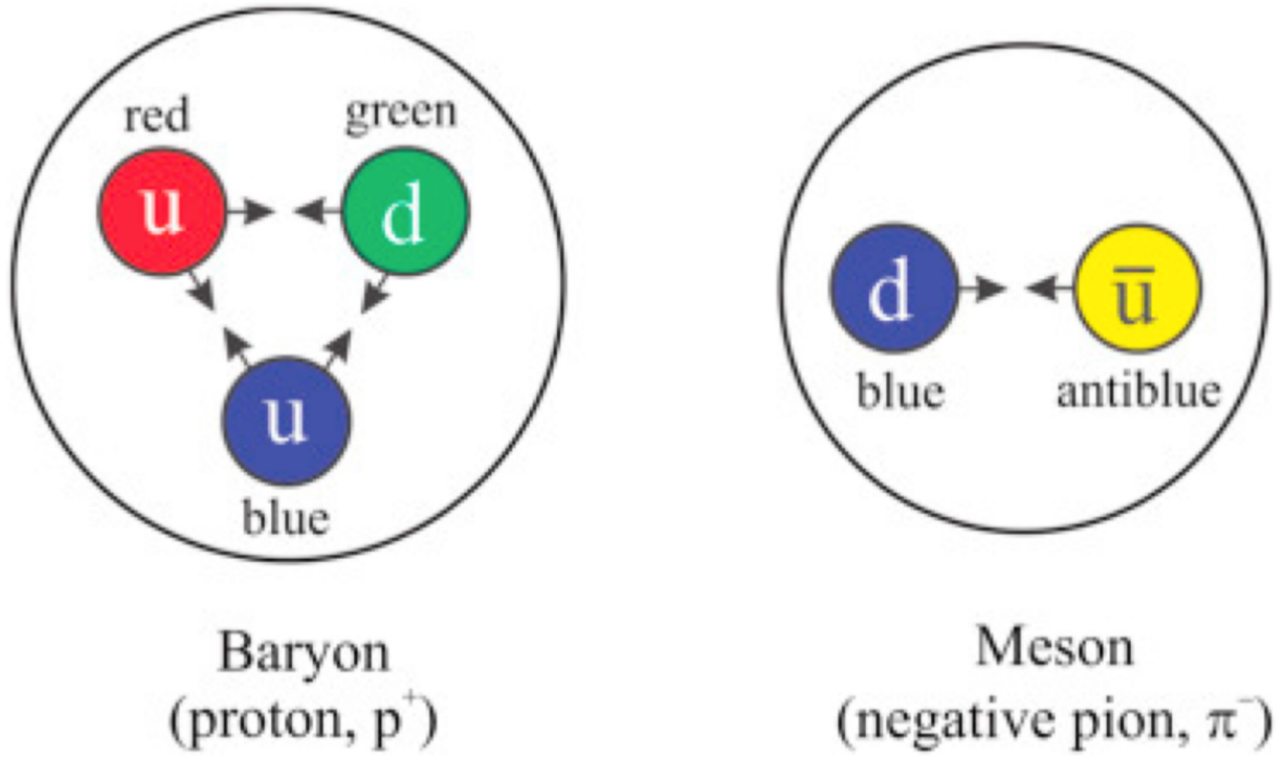
$$\frac{d\sigma}{d\Omega} \propto f(x)$$

Quarks in hadrons



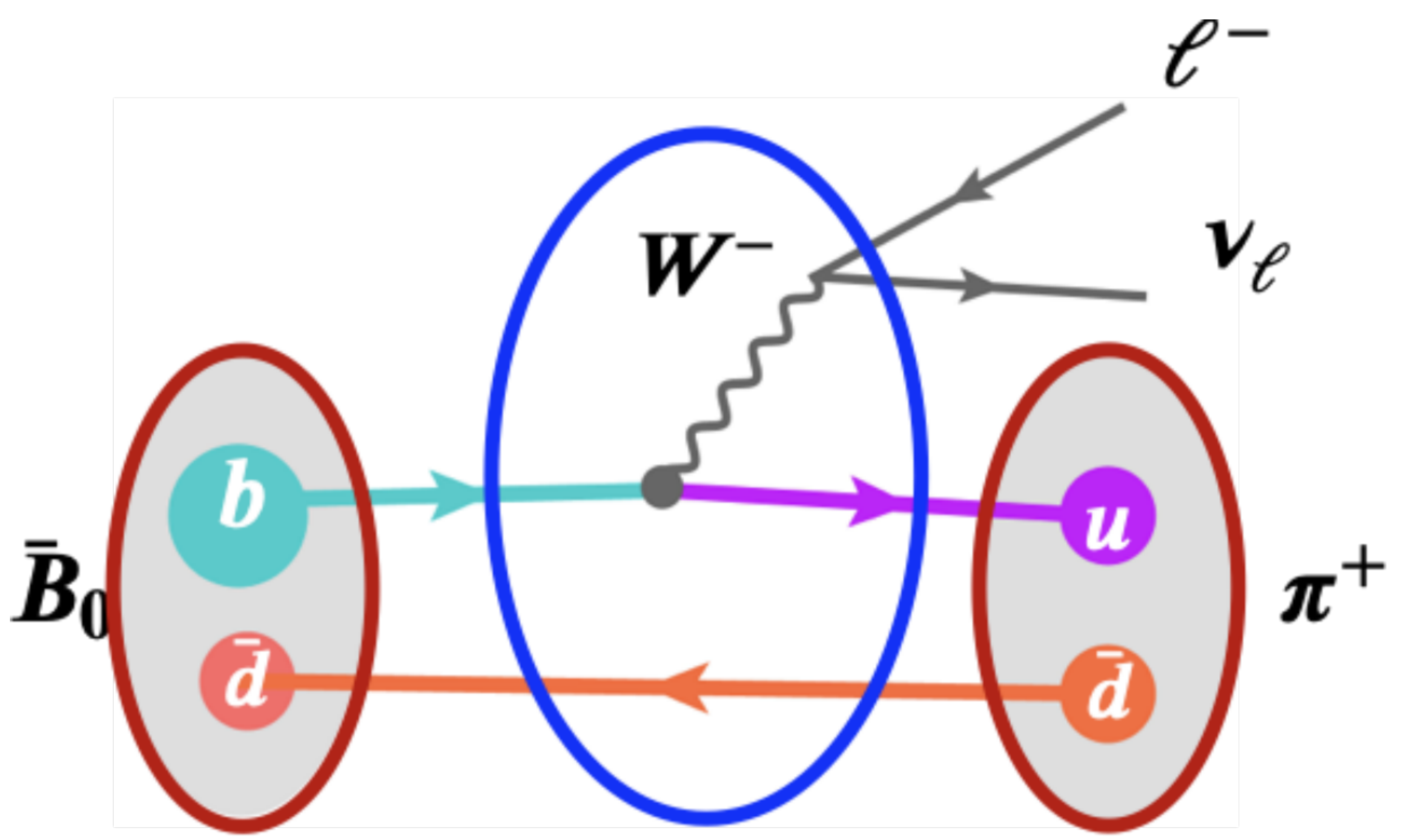
J. Friedman
H. Kendall
R. Taylor
Nobel prize in 1990

pictures



Motivation: Distribution amplitude

DAs are important inputs in hard exclusive processes. Such as $\bar{B}^0 \to \pi^+ + l^- + \nu_l$



decay width

$$iM = \langle \pi^+ l^- \nu_l | \bar{B}^0 \rangle \sim \int [dk] \mathrm{Tr}\left[L(t)H(k_1, k_2, k_3)\Phi_{\bar{B}^0}(x_1)\Phi_{\pi^+}(x_2)\right]$$

At leading-order, DAs represent the coefficients of hadron state expands with Fock states.

$$|h\rangle = \sum_{n,\lambda_i} \int [dx][dk_\perp] \psi_n(x_i, k_{\perp i}, \lambda_i) \prod_{fermions} \frac{u(x_i, k_{\perp i}, \lambda_i)}{\sqrt{x_i}} \prod_{gluons} \frac{\varepsilon(x_i, k_{\perp i}, \lambda_i)}{\sqrt{x_i}} |n\rangle$$

Among these, the simplest one is the pion DA:

$$\int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} \langle 0 | \bar{\psi}_1(0) n \cdot \gamma \gamma_5 U(0, \xi^-) \psi_2(\xi^-) | \pi(p) \rangle = i f_\pi \Phi_\pi(x)$$

Motivation: Distribution amplitude

1980s → 2020s

Asymptotic LCDAs

G.P. Lepage et al., Phys.Lett.B 87 (1979) 359-365

Sum rules

V.L. Chernyak et al., Nucl.Phys.B 204 (1982) 477
V.M. Braun et al., Z.Phys.C 44 (1989) 157
Patricia Ball et al., JHEP 08 (2007) 025

Lattice calculation with OPE

G. Martinelli et al., Phys.Lett.B 196 (1987) 184-190
V.M. Braun et al., Phys.Rev.D 74 (2006) 074501
G.S. Bali et al., JHEP 08 (2019) 065

Quark models

H. Choi et al., Phys.Rev.D 75 (2007) 073016

Dyson-Schwinger equation

F. Gao et al., Phys.Rev.D 90 (2014) 1, 014011
C.D. Roberts et al., Prog.Part.Nucl.Phys. 120 (2021) 103883

Lattice calculation with LaMET

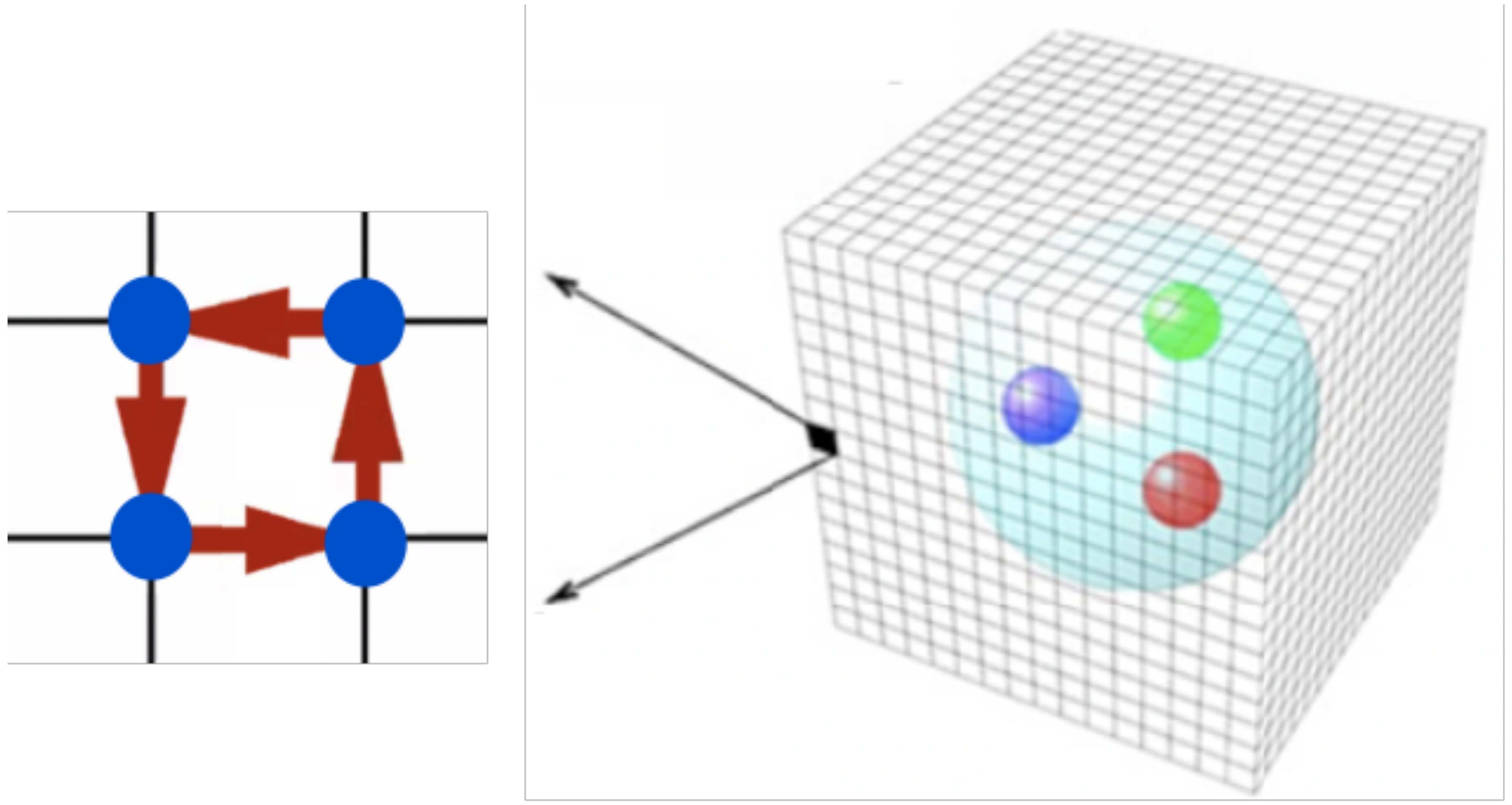
J. Zhang et al., Phys.Rev.D 95 (2017) 9, 094514
R. Zhang et al., Phys.Rev.D 102 (2020) 9, 094519
J. Hua et al., Phys.Rev.Lett. 127 (2021) 6, 062002
J. Hua et al., Phys.Rev.Lett. 129 (2022) 13, 132001

Motivation: Lattice QCD

Numerical simulation in discretized 4D Euclidean space-time;

Lattice QCD action:

$$S_E^{\mathrm{latt}} = -\underbrace{\sum_{\Box} \frac{6}{g^2} \mathrm{Re}\,\mathrm{tr}_N \left(U_{\Box,\mu\nu} \right)}_{\text{gauge action}} - \underbrace{\sum_q \bar{q} \left(D_\mu^{\mathrm{lat}} \gamma_\mu + a m_q \right) q}_{\text{fermion action}}$$



Correlation functions:

$$\langle \mathcal{O}(U,q,\bar{q}) \rangle = \frac{\int [\mathcal{D}U] \prod_q [\mathcal{D}q_q] [\mathcal{D}\bar{q}_q] \, e^{-S_E^{\mathrm{latt}}} \mathcal{O}(U,q,\bar{q})}{\int [\mathcal{D}U] \prod_q [\mathcal{D}q_q] [\mathcal{D}\bar{q}_q] \, e^{-S_E^{\mathrm{latt}}}}$$

$$= \frac{\int [\mathcal{D}U] e^{-S_{\mathrm{glue}}^{\mathrm{latt}}} \prod_q \det \left(D_\mu^{\mathrm{latt}} \gamma_\mu + a m_q \right) \tilde{\mathcal{O}}(U)}{\int [\mathcal{D}U] e^{-S_{\mathrm{glue}}^{\mathrm{latt}}} \prod_q \det \left(D_\mu^{\mathrm{latt}} \gamma_\mu + a m_q \right)}$$

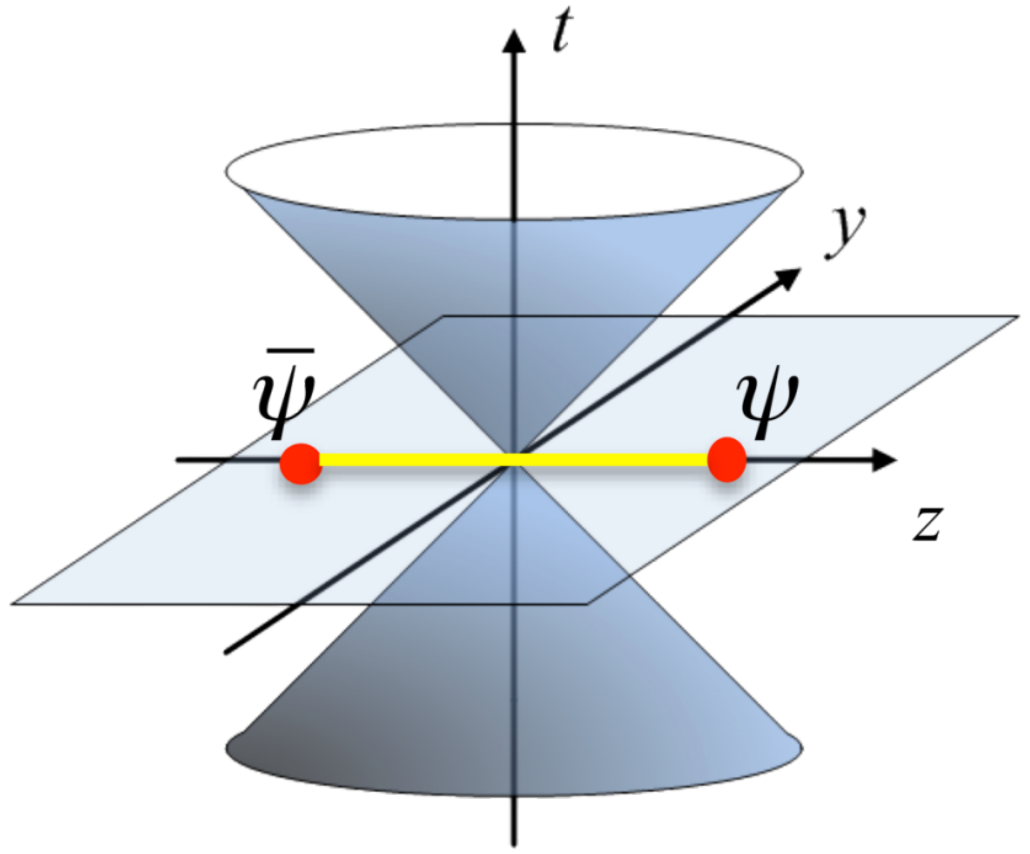
Monte Carlo sampling → configurations → observables

Where the statistical uncertainties come from

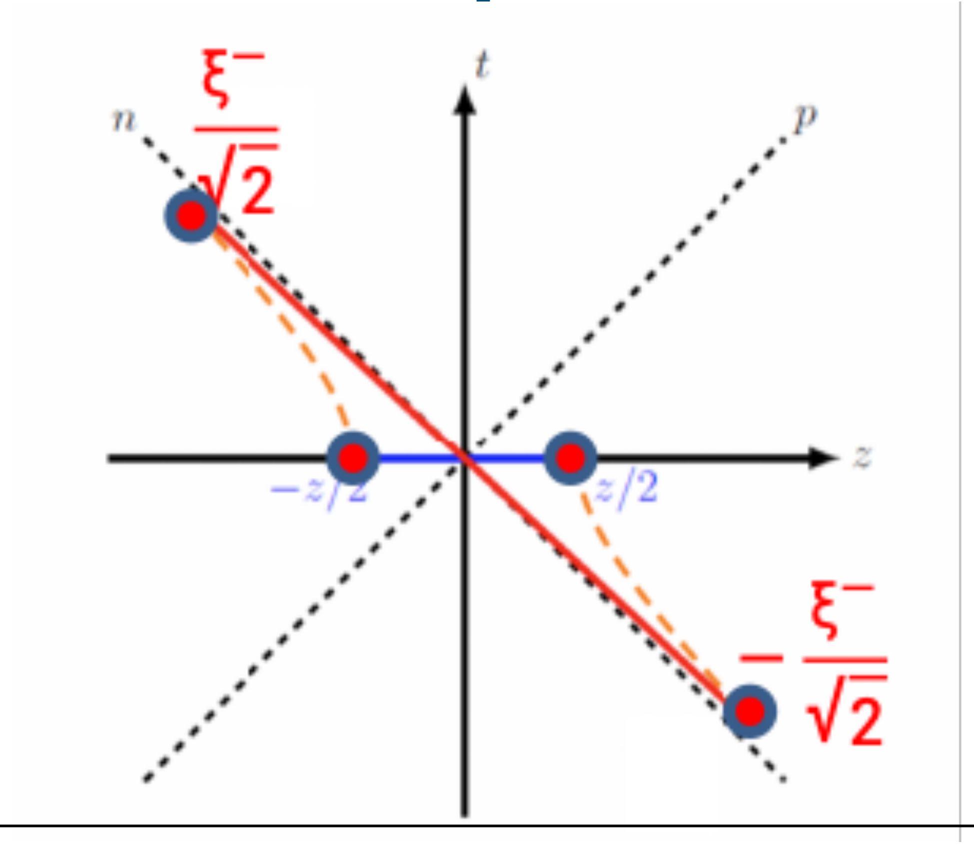
Motivation: LaMET

Equal time correlation

$$\tilde{\psi}(z) \sim \langle 0 | \bar{\psi}(\frac{z}{2}) \Gamma U(\frac{z}{2}, -\frac{z}{2}) \psi(-\frac{z}{2}) | P^z \rangle$$

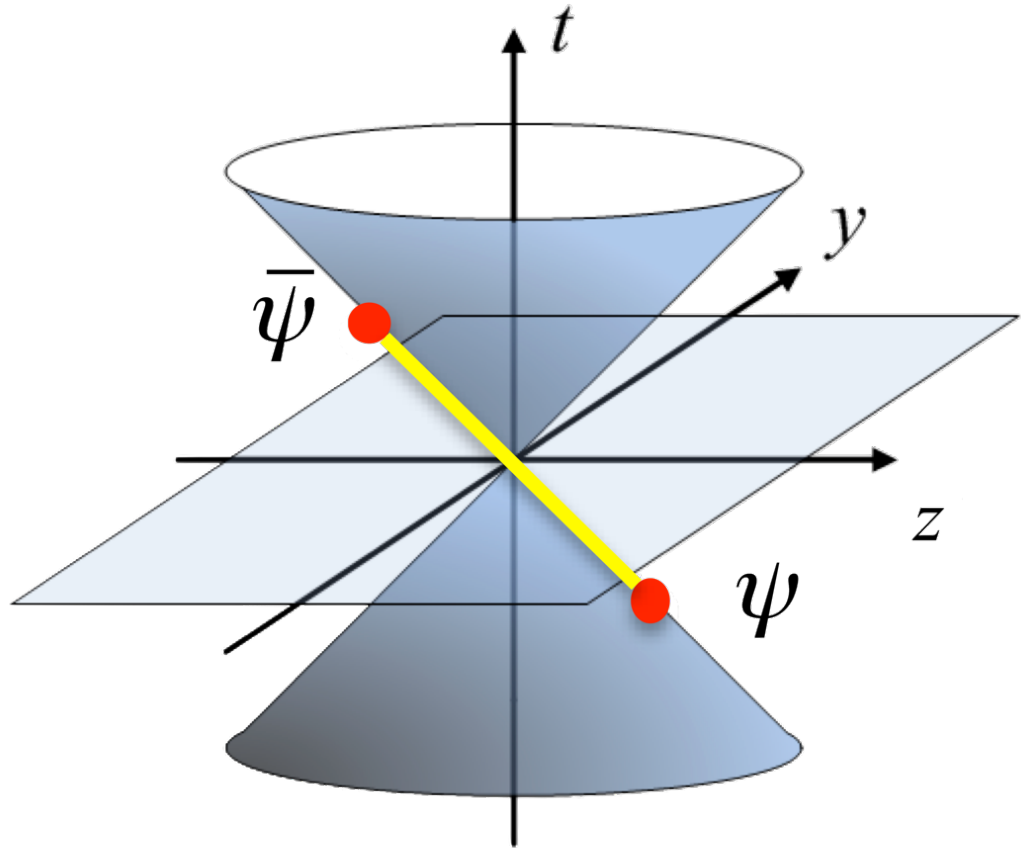


Lorentz boost



X. Ji, Phys.Rev.Lett. 110 (2013) 262002

Matching



Due to the IR structure are only based on states, then the difference between $\psi(x)$ and $\tilde{\psi}(x)$ is only UV structure, which can be perturbatively determined.

Light-cone correlation

$$\psi(x) \sim \langle 0 | \bar{\psi}(\frac{\xi^+}{\sqrt{2}}) \Gamma U(\frac{\xi^+}{\sqrt{2}}, -\frac{\xi^+}{\sqrt{2}}) \psi(-\frac{\xi^+}{\sqrt{2}}) | P^+ \rangle$$

Power suppressed by $\frac{1}{(xP^z)^2}$ **and** $\frac{1}{[(1-x)P^z]^2}$

$$\tilde{\psi}(x, P^z, \mu) = \int_0^1 dy\, C(x, y, \mu, P^z)\, \psi(x, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{(xP^z)^2}, \frac{\Lambda_{QCD}^2}{[(1-x)P^z]^2}\right)$$

Motivation: LaMET

Light-Cone distribution amplitude in LaMET

- Equal time correlation on lattice (quasi-DA):

$$\tilde{h}(z, P^z, \mu, a) = \langle 0 | \bar{\psi}(0) \Gamma W(0,z) \psi(z) | P^z \rangle$$

propagators → 2pt → matrix elements

- Non-perturbative renormalization:

$$\tilde{h}(z, P^z, \mu, a) = Z(z, a) \tilde{h}_B(z, P^z, \mu, a)$$

Wilson line, RI/MOM, self renormalization, hybrid renormalization…

- Fourier transformation:

$$\tilde{\psi}(x, P^z, \mu, a) = \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{-ixzP^z} \tilde{h}(z, P^z, \mu, a)$$

large z extrapolation…

- Matching to the light-cone:

$$\tilde{\psi}(x, P^z, \mu) = \int_0^1 dy\, C(x, y, \mu, P^z)\, \psi(x, \mu)$$

continuum limit, infinite momentum limit…

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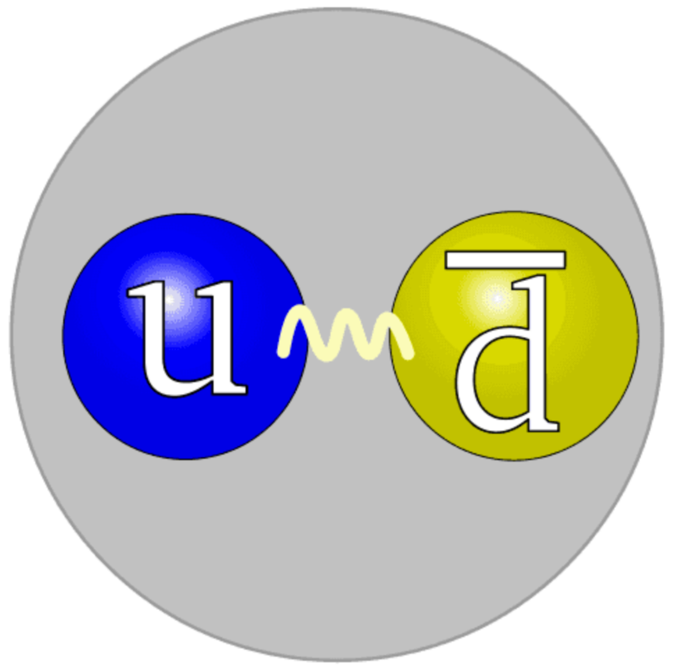


Numerical results: Introduction

Systems

Pseudo scalar meson: π and K

$$\int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle 0 | \bar{\psi}(0) \not{n}\gamma_5 U(0,\xi^-)\psi(\xi^-) | M(P^+) \rangle$$
$$= if_M(P \cdot n)\phi_M(x)$$

J. Hua et al., Phys.Rev.Lett. 129 (2022) 13, 132001

Vetor meson: K^* and ϕ

$$\int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle 0 | \bar{\psi}(0) \not{n} U(0,\xi^-)\psi(\xi^-) | V(P^+) \rangle$$
$$= f_V n \cdot \epsilon \phi_{V,L}(x)$$

$$\int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle 0 | \bar{\psi}(0) \sigma^{+\mu_\perp} U(0,\xi^-)\psi(\xi^-) | V(P^+) \rangle$$
$$= f_V(\epsilon^+ P^{\mu_\perp} - \epsilon^{\mu_\perp} P^+)\phi_{V,T}(x)$$

J. Hua et al., Phys.Rev.Lett. 127 (2021) 6, 062002

Lattice setup (MILC ensembles)

Ensemble	Lattice spacing	Volume	Valence pion mass	Momentum
a12m130	0.12 fm	$48^3 \times 64$	140 MeV	1.29 GeV 1.72 Gev 2.15 GeV
a09m130	0.09 fm	$64^3 \times 96$		
a06m130	0.06 fm	$96^3 \times 192$		

Numerical results: Renormalization

Hybrid renormalization (for vector meson DA)

X. Ji et al., Nucl.Phys.B 964 (2021) 115311

hybrid scheme avoids nonperturbative effects at large z

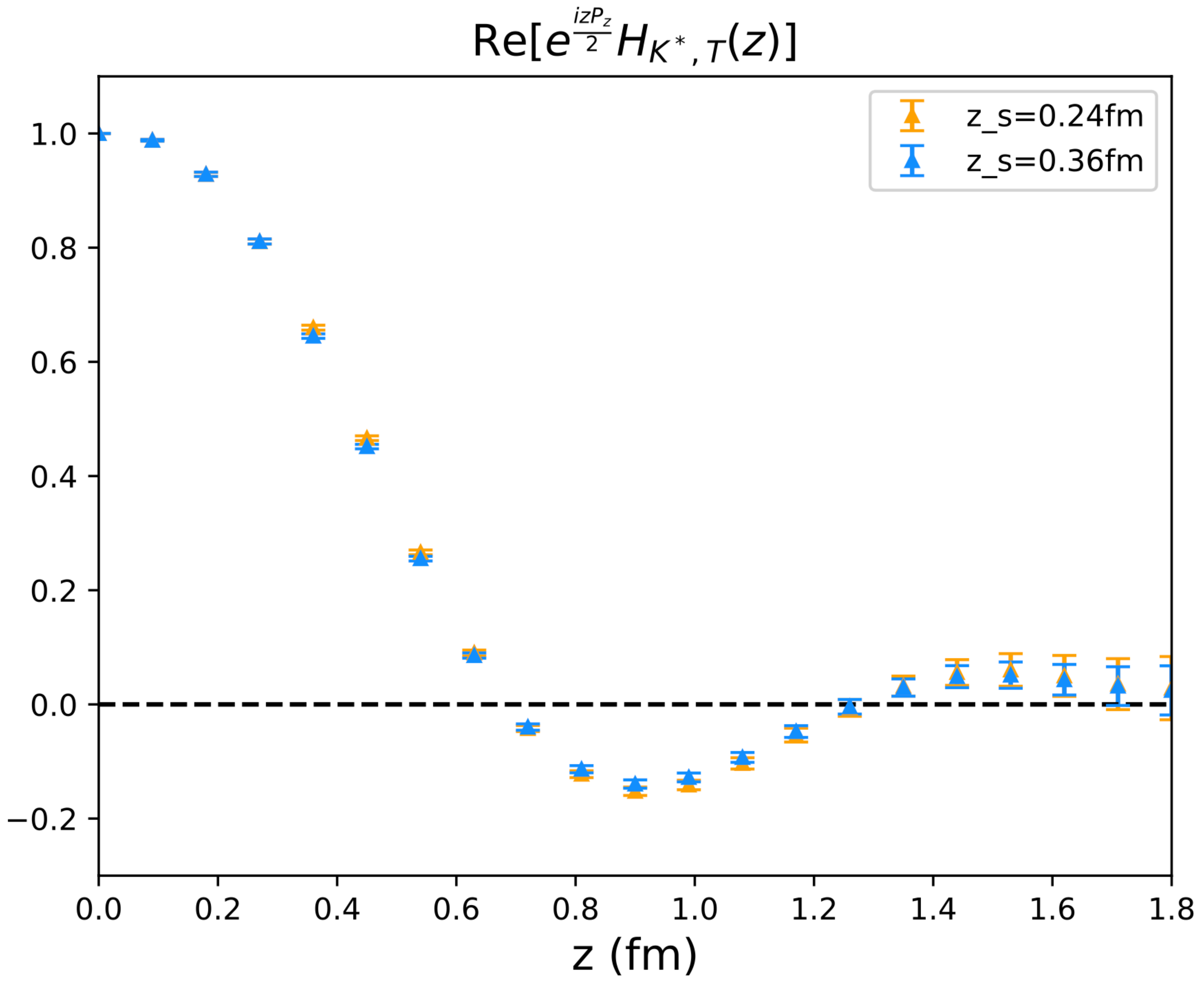
K. Zhang et al., Phys.Rev.Lett. 129 (2022) 8, 082002

$$\tilde{h}(z, P^z, \mu, a) = \begin{cases} \dfrac{\tilde{h}_B(z, P^z, \mu, a)}{Z(z, a)}, & |z| < z_s \\ \tilde{h}_B(z, P^z, \mu, a) e^{-\delta m \cdot z} Z_{hybrid}(z_s, a), & |z| > z_s \end{cases}$$

$Z(z, a)$: RI/MOM renormalization factor

$$Z_{hybrid}(z_s, a) = e^{\delta m z_s} / Z(z_s, a)$$

Choices of z_s



z_s = 0.24 fm, 0.36 fm treated as systematic uncertainty!

Numerical results: Renormalization

Self renormalization (for pseudo scalar meson DA)

Y. Huo et al., Nucl.Phys.B 969 (2021) 115443

1. It can match to continuum scheme at short distance;
2. It is universal across hadrons and fermion actions;
3. avoids nonperturbative effects at large z;

K. Zhang et al., Phys.Rev.Lett. 129 (2022) 8, 082002

$$\tilde{h}^{R}(z) = \tilde{h}^{B}(z,a)/Z^{\text{self}}(z,a)$$

$$Z^{\text{self}}(z,a) = \exp\left\{ \frac{kz}{a\ln[a\Lambda_{QCD}]} + m_0 z + f(z)a + \frac{3C_F}{b_0}\ln\left[\frac{\ln[1/(a\Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]}\right] + \ln\left[1 + \frac{d}{\ln(a\Lambda_{QCD})})\right]\right\}$$

systematic uncertainty

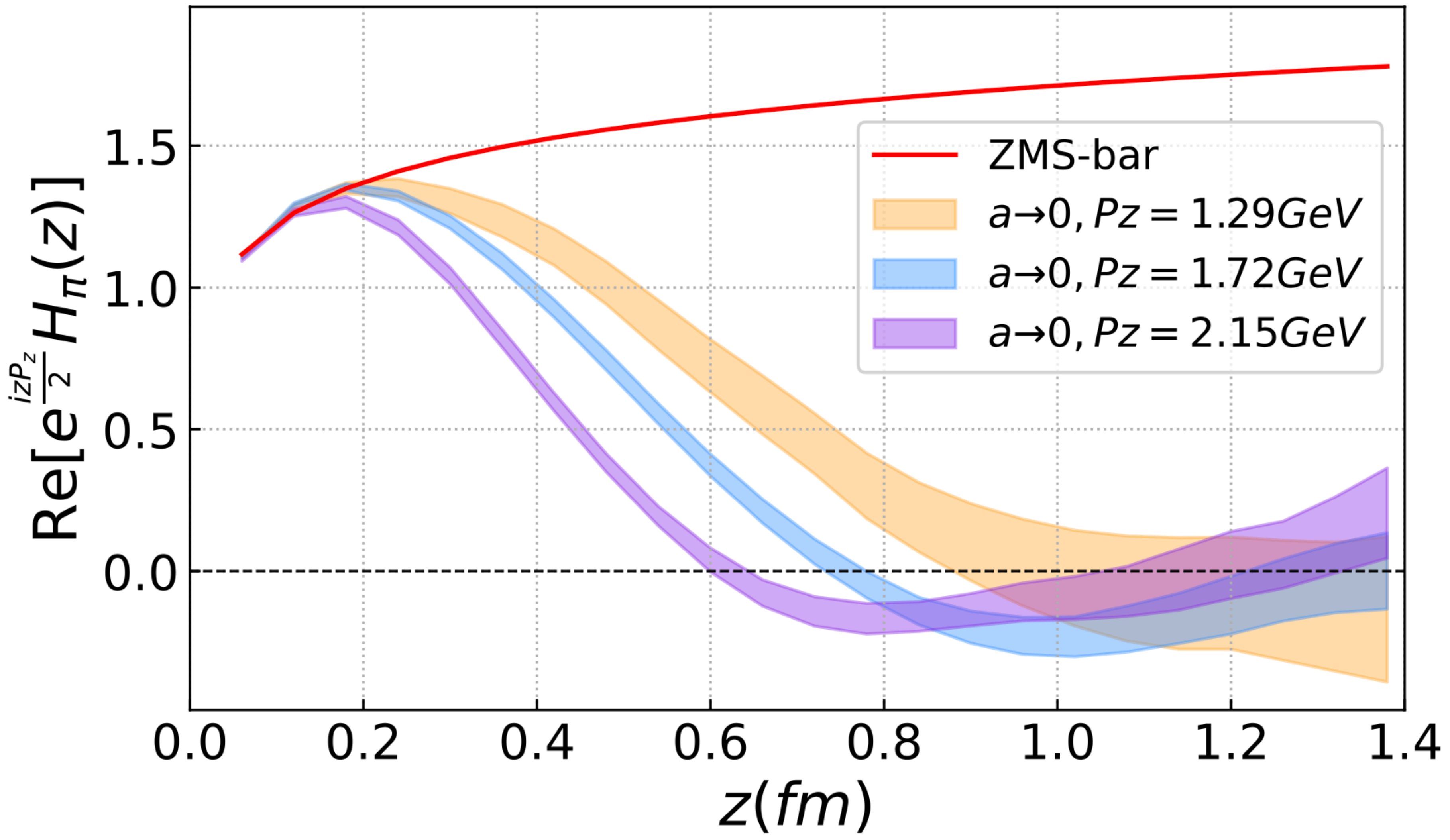
Short distance: $\phi_m^{\overline{\text{MS}},1-\text{loop}}(z,\mu) = \tilde{h}_m^{B}(z,a)/Z^{\text{self}}(z,a)$

perturbative results

apply in long distance

$$\tilde{h}_m^{R}(z) = \frac{\tilde{h}_m^{B}(z,a)}{Z^{\text{self}}(z,a)}$$

comparison of lattice and perturbative



Numerical results: Extrapolation

quasi-DA $\tilde{h}^{\mathrm{R}}(z)$ is within finite range $|z| \leq z_{max}$

while Fourier transformation needs $\int_{-\infty}^{\infty} dz$

then extrapolation for z is needed:

$$\tilde{\phi}(x) \sim \phi(x) \sim x^a(1-x)^b$$

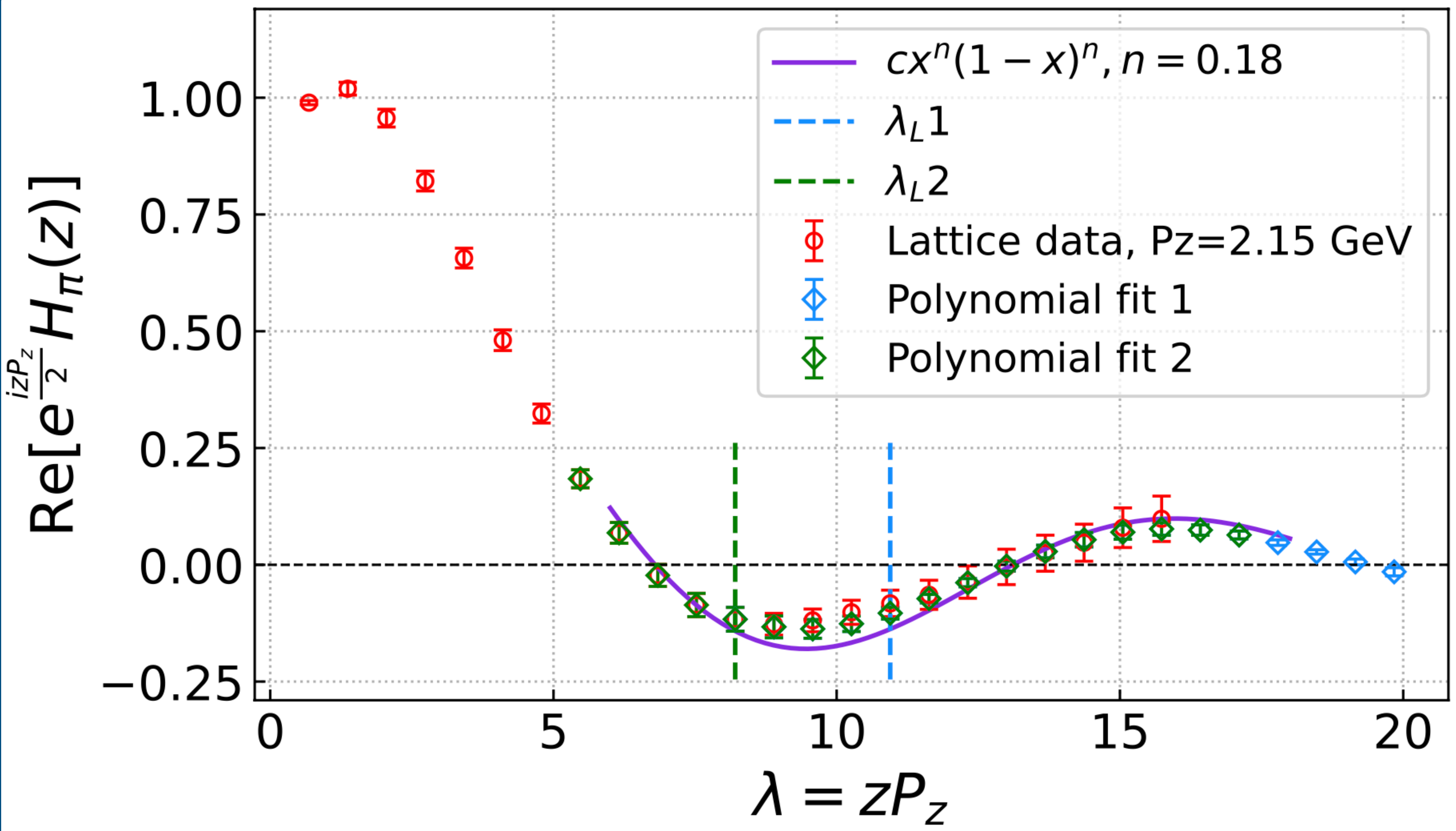
↓ inverse Fourier transformation

$$\tilde{h}^{\mathrm{R}}(\lambda = zP^z) = \left[\frac{c_1}{(i\lambda)^a} + e^{-i\lambda}\frac{c_2}{(-i\lambda)^b}\right] e^{-\lambda/\lambda_0}$$

finite momentum

X. Ji et al., Nucl.Phys.B 964 (2021) 115311

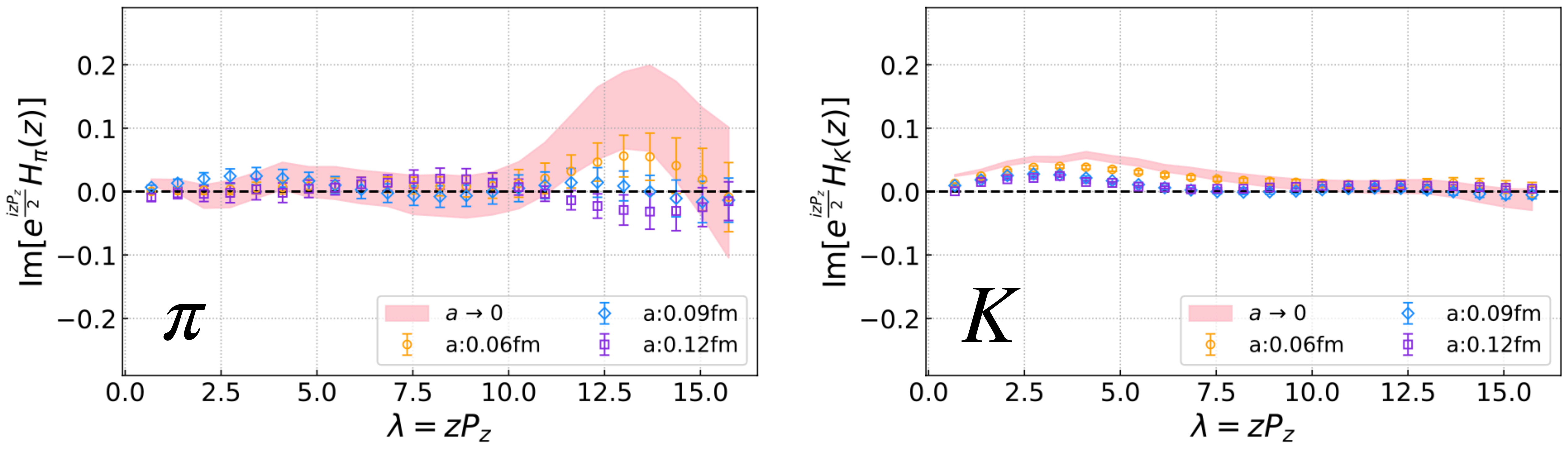
quasi-DA $\tilde{h}^{\mathrm{R}}(z)$ for pion



systematics: extrapolation region

Numerical results: quasi-DA

Symmetries for quasi-DA $\tilde{h}^{\mathrm{R}}(z)$



Fourier transformation: $\tilde{f}(x) = \int_{-\infty}^{\infty} P^z dz e^{ixzP^z} \tilde{h}^R(z)$

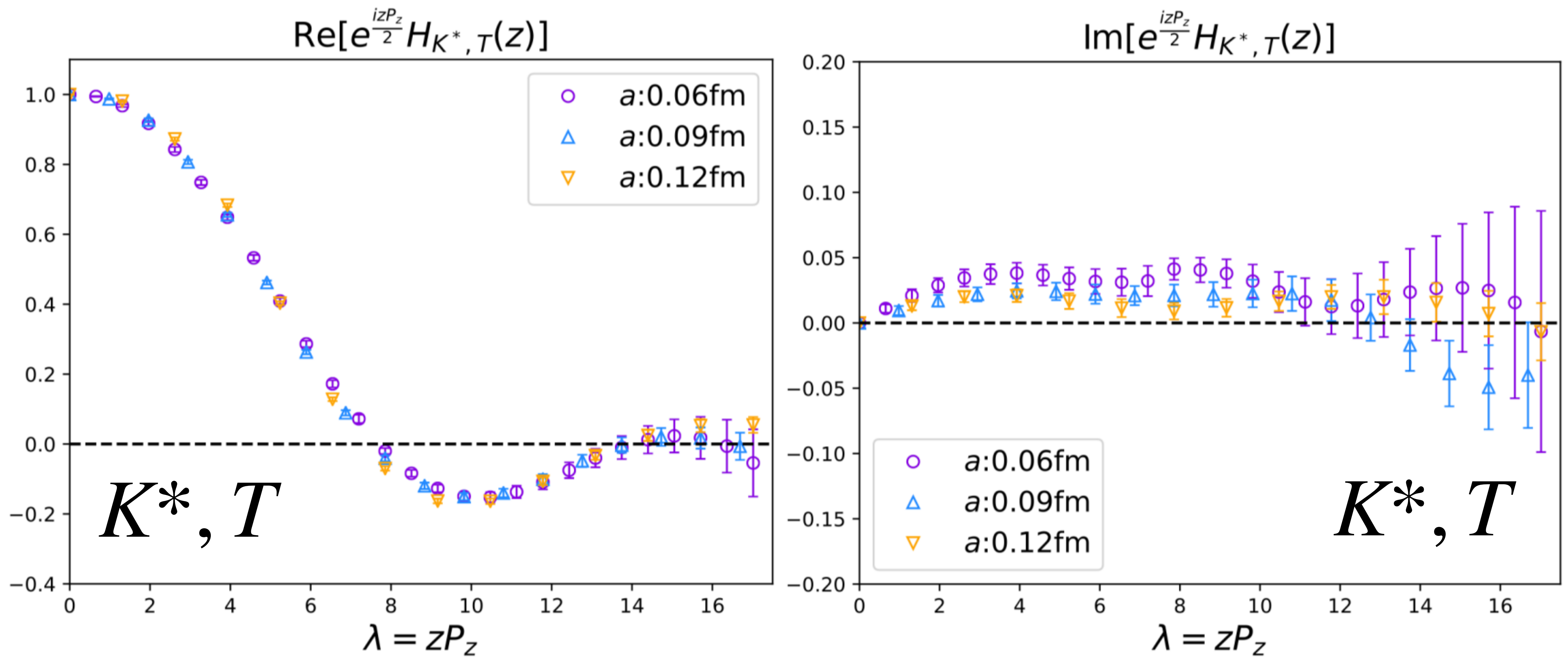
$$\tilde{f}(x) = \int_{-\infty}^{\infty} P^z dz e^{i(x-\frac{1}{2})zP^z} e^{\frac{izP^z}{2}} \tilde{h}^R(z)$$

↓ when $\tilde{h}^{\mathrm{R}}(z)$ is real

$$\tilde{f}(x) = \int_{-\infty}^{\infty} P^z dz \cos\left[\left(x - \frac{1}{2}\right) zP^z\right] e^{\frac{izP^z}{2}} \tilde{h}^R(z)$$

The imaginary part is not zero, which leads to the asymmetry for u/d and s quarks.

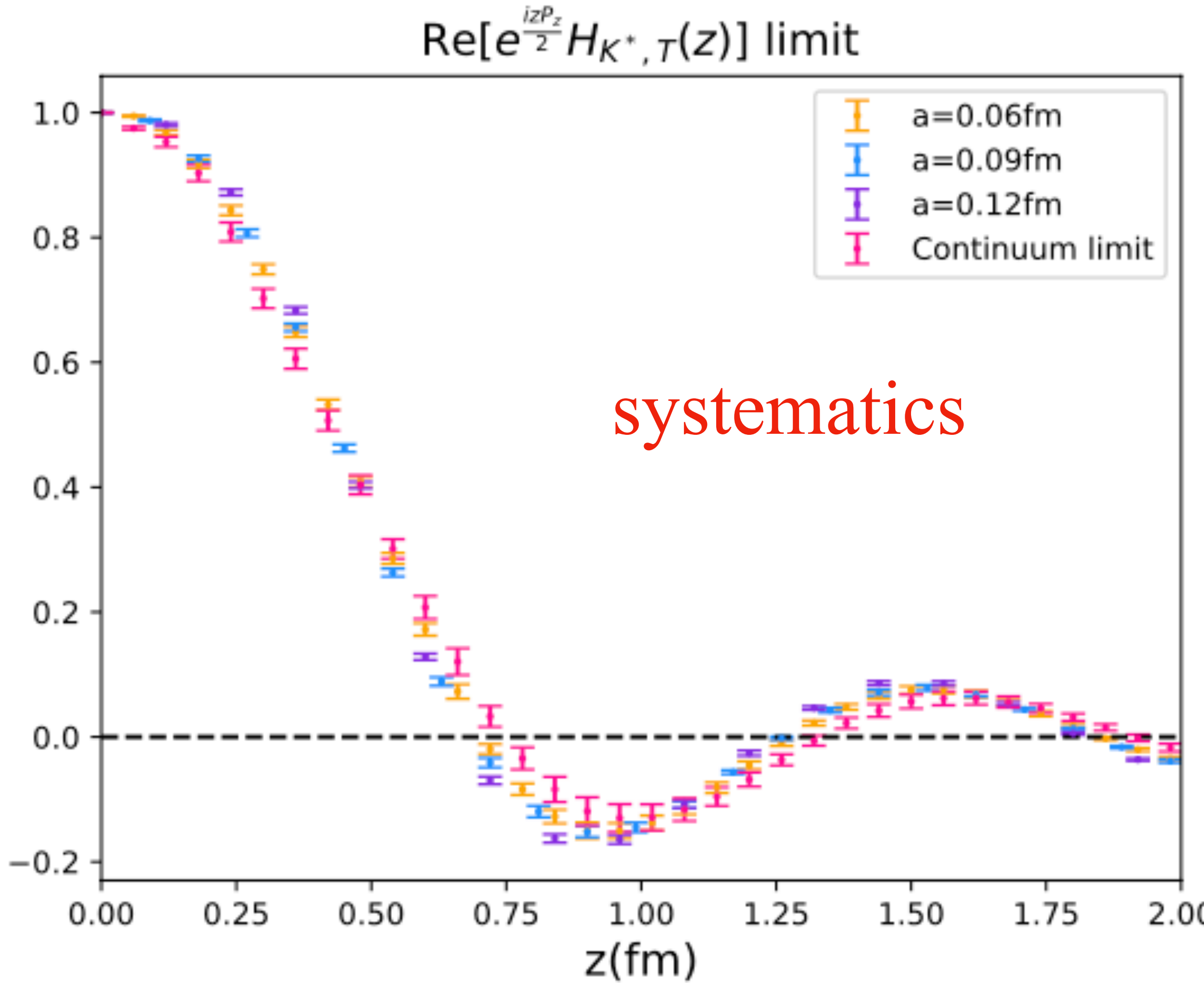
Symmetries for quasi-DA $\tilde{h}^{\mathrm{R}}(z)$



continuum limit in co. sapce

$$\tilde{\psi}(a) = \tilde{\psi}(a \to 0) + c_1 a + \mathcal{O}(a^2)$$

$$\sigma_{sys} = |\tilde{\psi}(a \to 0) - \tilde{\psi}(a = 0.06 \text{ fm})|$$



Numerical results: matching

Collinear factorization of quasi-DA: $\tilde{\psi}(x, P^z, \mu) = \int_0^1 dy\, C(x, y, P^z, \mu)\, \psi(y) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{(yP^z)^2}, \frac{\Lambda_{QCD}^2}{((1-y)P^z)^2}\right)$

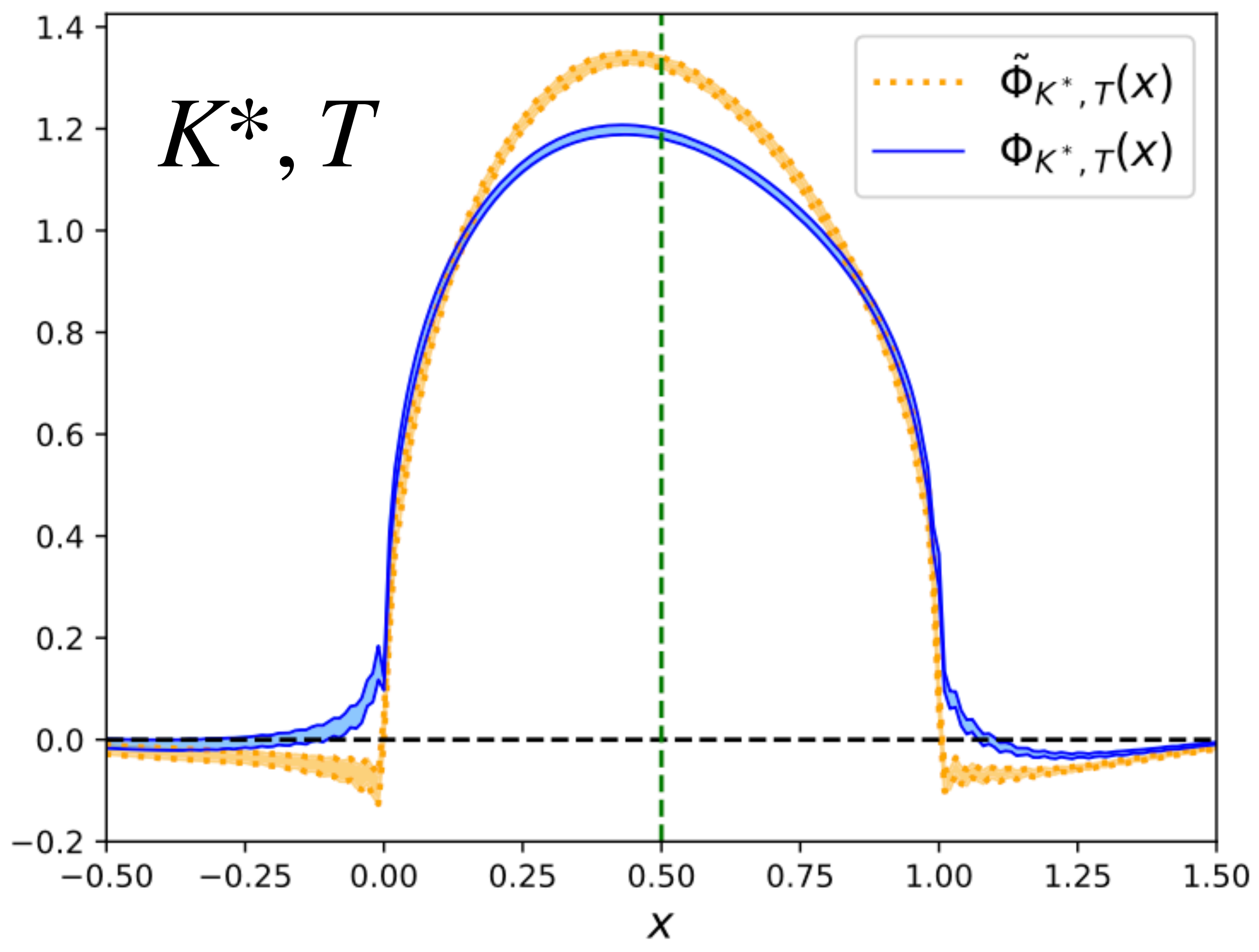
Matching in hybrid scheme

$$C^{(1)}_{hybrid} = C^{(1)}_{RI/MOM} + \int dy' \int \frac{P^z dz}{2\pi}\left[e^{i(1-y')z_s P^z_R} - e^{i(1-y')zP^z_R}\right]\tilde{q}^{(1)}(y')\theta(|z| > z_s)$$

Y. Liu et al., Phys.Rev.D 99 (2019) 9, 094036

Technical operation

1. extend the range for x: when 0<x<1: $C(x, y) = C_{hybrid}(x, y)$; when x<0 or x>1: $C(x, y) = \delta(x - y)$
2. inverse matching by inv. matrix

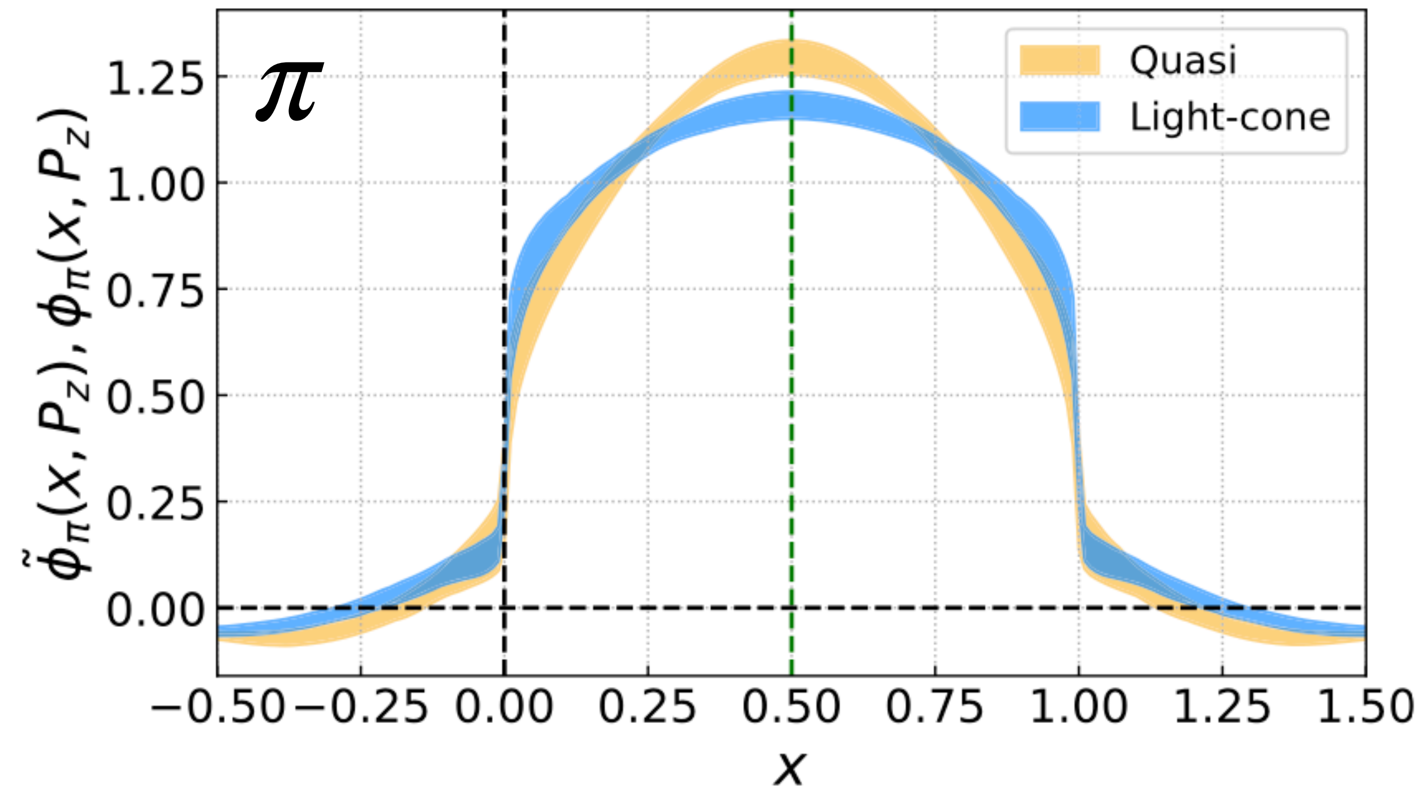


Matching in self renormalization scheme

$$C_{self} = \delta(x - y) + \left[C_B^{(1)}(x, y, \frac{P^z}{\mu})\right]_+ + \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{2|x - y|}\right)_+$$

$$C^{(1)}_{RI/MOM} = \delta(x - y) + C_B^{(1)}(x, y, \frac{P^z}{\mu}) + C_{CT}(x, y, r, \frac{P^z}{P^Z_R})$$

Y. Liu et al., Phys.Rev.D 99 (2019) 9, 094036



Matching in coordinate space

$$H^R(z, \lambda, \mu_R) = \int_0^1 \theta(1 - x - y)\, C(x, y, z^2, \mu_R, \mu)\, h^R_m(x, y, \lambda, \mu)$$

$$h^R_m(x, y, \lambda, \mu) = \int_0^1 du\, e^{iu(x-1)\lambda - i(1-u)y\lambda}\psi(u, \mu)$$

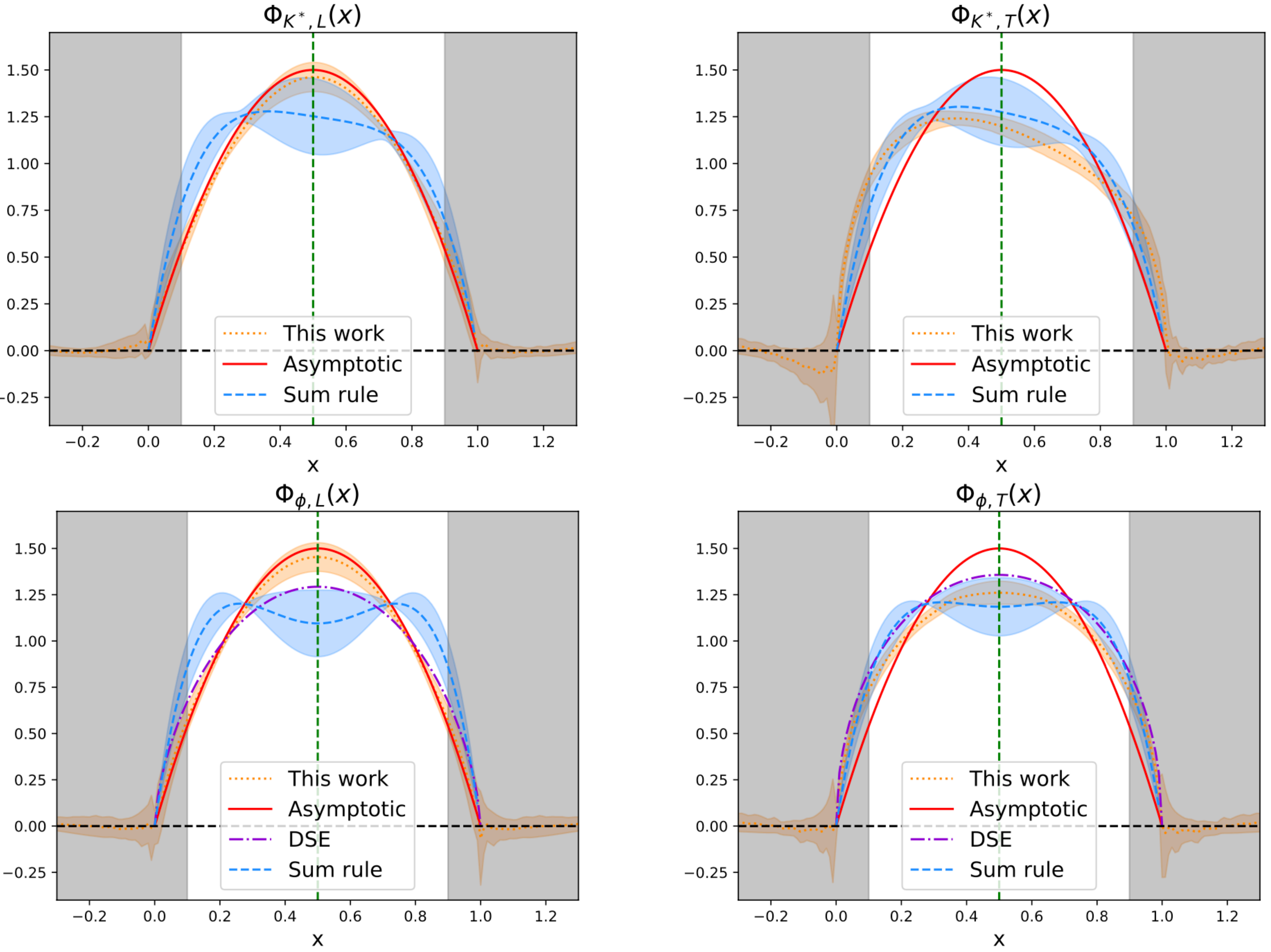
Determined from this work.

Numerical results: final results

vector meson LCDAs in x space

the infinity momentum limit: $\phi(x, P_z) = \phi(x, P_z \to \infty) + \frac{c_2(x)}{P_z^2}$ is adopted

longitudinal polarized — transverse polarized



Gegenbauer moments

$$\phi(x) = 6x(1-x)\left[1 + \sum_{n}^{\infty} a_n C_n^{3/2}(2x-1)\right]$$

first few moments

Gegenbauer moments	a_1	a_2	a_4
K^*, L	-0.005(07)(07)	0.015(10)(08)	0.013(09)(09)
K^*, T	-0.074(06)(07)	0.181(07)(12)	0.064(07)(06)
ϕ, L	−−	0.018(09)(09)	0.007(10)(20)
ϕ, T	−−	0.128(03)(21)	0.044(04)(08)

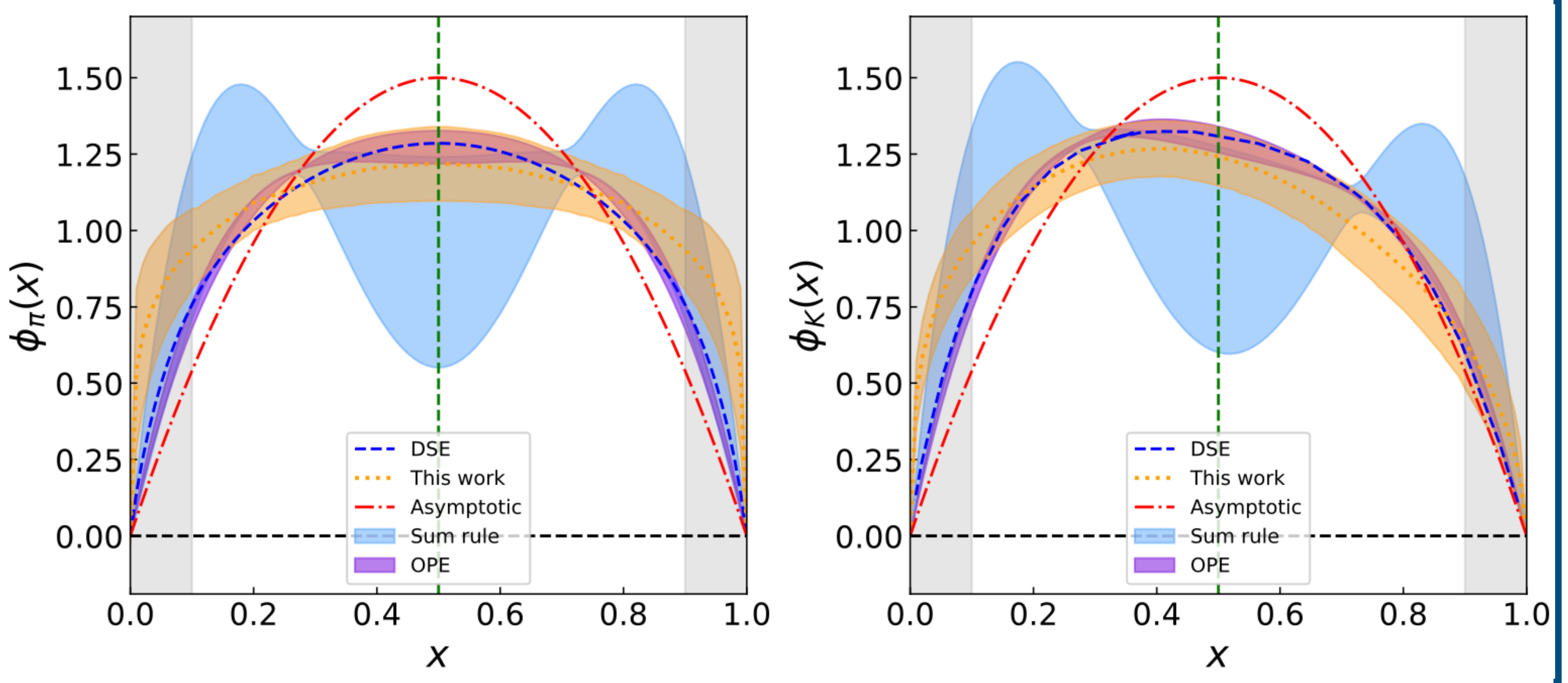
statistic uncertainties — systematic uncertainties

renormalization
large λ extrapolation

Numerical results: final results

pion and kaon LCDAs in x space

the infinity momentum limit: $\phi(x, P_z) = \phi(x, P_z \to \infty) + \frac{c_2(x)}{P_z^2}$ is adopted



comparison with others

	DSE	6x(1-x)	QCD sum rule	OPE
This work	close	not close	hard to say	close

Gegenbauer moments

$$\phi(x) = 6x(1-x)\left[1 + \sum_n^{\infty} a_n C_n^{3/2}(2x-1)\right]$$

first few moments

	a_1	a_2	a_3	a_4
π	—	0.258(70)(52)	—	0.122(46)(31)
K	-0.108(14)(51)	0.170(14)(44)	-0.043(06)(22)	0.073(08)(21)

V.M. Braun et al., Phys.Rev.D 74 (2006) 074501

Moments from this work is consist with OPE results in 2006, but disagrees with results in 2019

G.S. Bali et al., JHEP 08 (2019) 065

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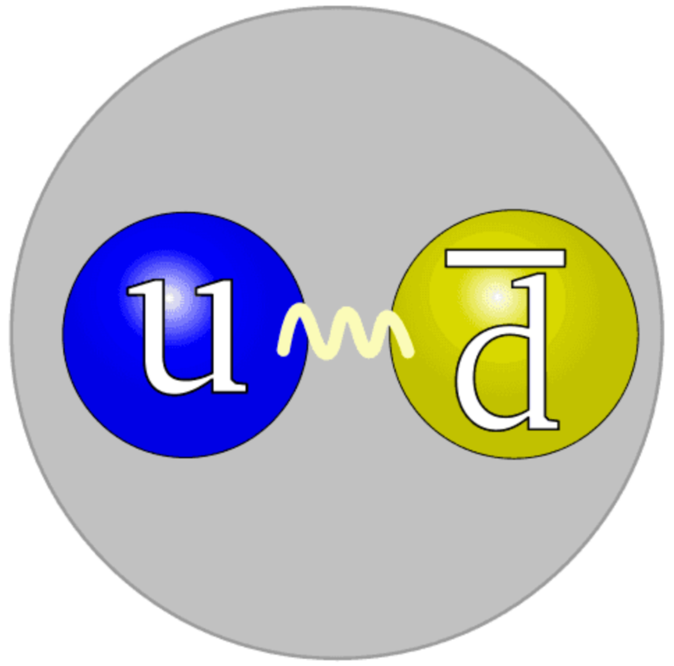


Analysis of uncertainties: systematics

processes of DA calculation

Renormalization: renormalization scale and dividing point

Hybrid renormalization

$$\tilde{h}(z, P^z, \mu, a) = \begin{cases} \dfrac{\tilde{h}_B(z, P^z, \mu, a)}{Z(z, a)}, & |z| < z_s \\ \tilde{h}_B(z, P^z, \mu, a) e^{-\delta m \cdot z} Z_{hybrid}(z_s, a), & |z| > z_s \end{cases}$$

dividing point (z_s)

renormalization scale (μ)

Self renormalization

dividing point (z_s)

$$\tilde{h}^{\mathrm{R}}(z) = \tilde{h}^{\mathrm{B}}(z, a)/Z^{\mathrm{self}}(z, z_s, a)$$

renormalization scale (μ)

Short distance: $\phi_m^{\overline{\mathrm{MS}},1-\mathrm{loop}}(z, \mu) = \tilde{h}_m^{\mathrm{B}}(z, a)/Z^{\mathrm{self}}(z, a)$

↓ apply in long distance

$$\tilde{h}_m^{\mathrm{R}}(z) = \frac{\tilde{h}_m^{\mathrm{B}}(z, a)}{Z^{\mathrm{self}}(z, a)}$$

Large λ extrapolation: polynomial decay terms and range for fittings.

$$\tilde{h}^{\mathrm{R}}(\lambda = zP^z) = \left[\frac{c_1}{(i\lambda)^a} + e^{-i\lambda}\frac{c_2}{(-i\lambda)^b}\right] e^{-\lambda/\lambda_0}$$

polynomial decay

Approaching to continuum limit: difference between fitting results and results at a=0.06 fm.

$$\tilde{\psi}(a) = \tilde{\psi}(a \to 0) + c_1 a + \mathcal{O}(a^2)$$

Approaching to infinite momentum limit: difference between fitting results and results at largest P^z.

$$\phi(x, P_z) = \phi(x, P_z \to \infty) + \frac{c_2(x)}{P_z^2}$$

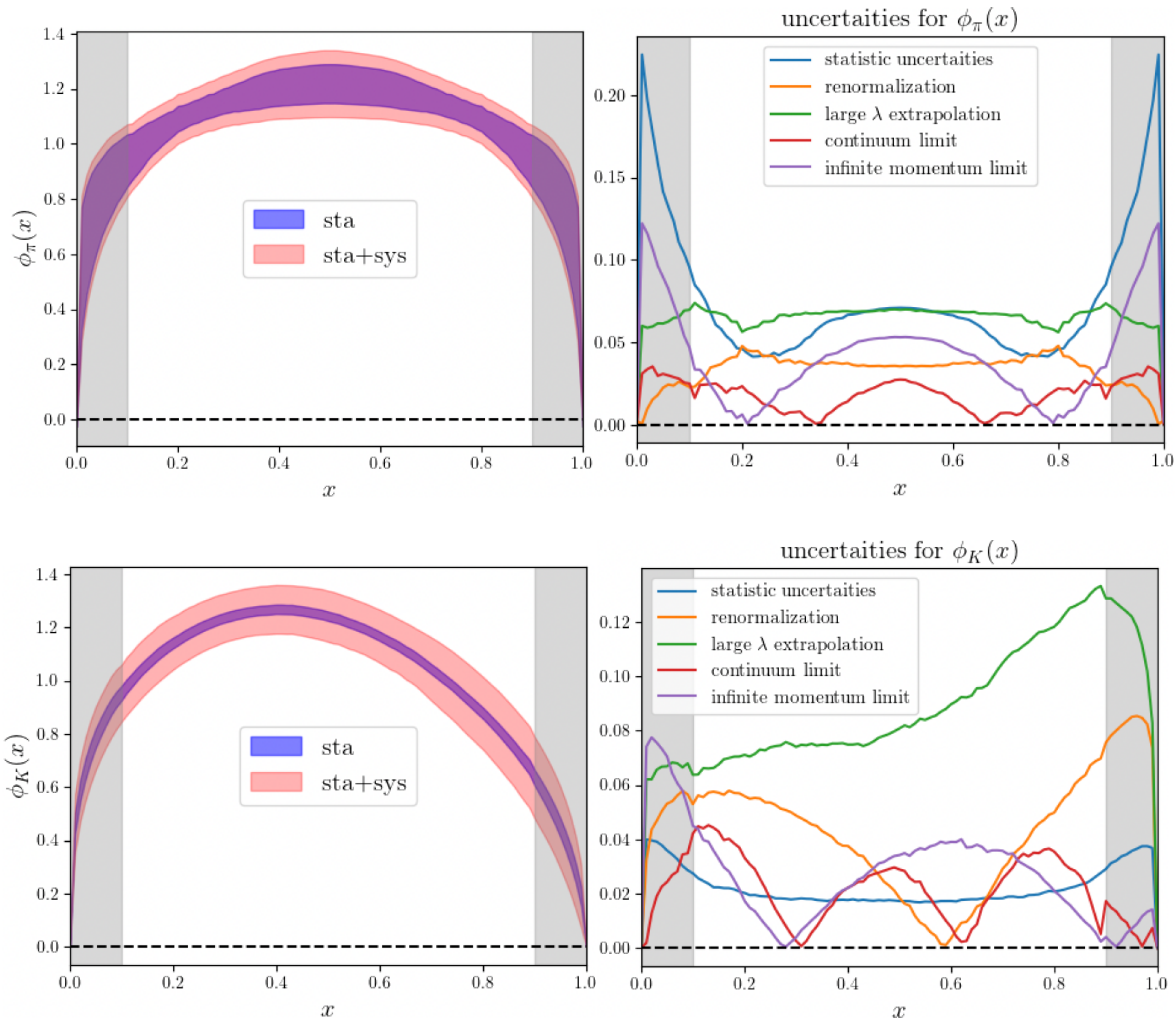
Analysis of uncertainties: systematics

pion and kaon LCDA

value = central value(statistic)(renormalization)(large λ)(continuum limit)(infinite momentum limit)

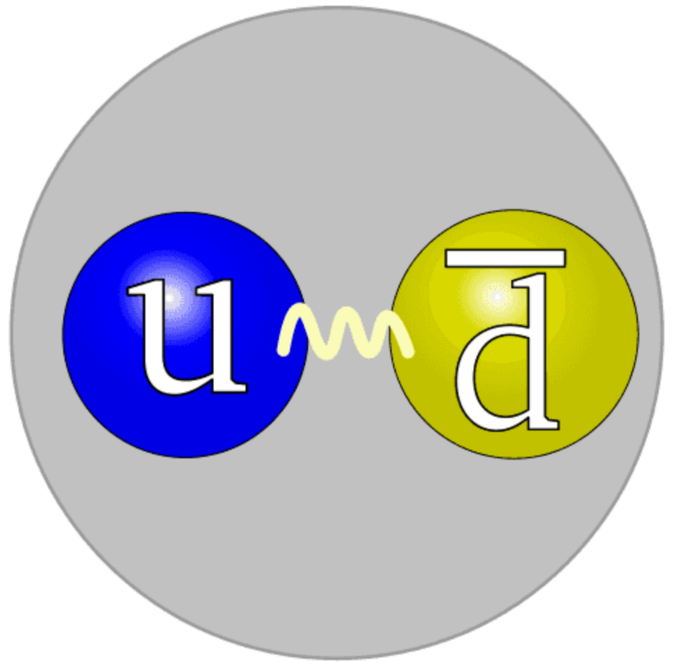
x	π	K
0.05	0.81(14)(09)(03)(06)(02)	0.78(04)(07)(02)(07)(05)
0.10	0.94(10)(05)(02)(07)(02)	0.95(03)(05)(04)(06)(05)
0.15	1.02(06)(02)(02)(07)(04)	1.06(02)(03)(04)(07)(06)
0.20	1.09(05)(01)(02)(06)(05)	1.14(02)(02)(03)(07)(06)
0.25	1.13(04)(01)(01)(06)(04)	1.20(02)(01)(02)(07)(05)
0.30	1.16(05)(03)(01)(07)(04)	1.24(02)(01)(01)(07)(05)
0.35	1.19(06)(04)(01)(07)(04)	1.26(02)(01)(01)(07)(04)
0.40	1.20(07)(05)(01)(07)(04)	1.27(02)(02)(02)(07)(04)
0.45	1.21(07)(05)(02)(07)(04)	1.26(02)(03)(03)(08)(03)
0.50	1.22(07)(05)(03)(07)(04)	1.24(02)(03)(03)(08)(02)
0.55	1.21(07)(05)(02)(07)(04)	1.21(02)(04)(02)(08)(01)
0.60	1.20(07)(05)(01)(07)(04)	1.17(02)(04)(01)(09)(01)
0.65	1.19(06)(04)(01)(07)(04)	1.11(02)(04)(01)(10)(02)
0.70	1.16(05)(03)(01)(07)(04)	1.04(02)(04)(03)(10)(03)
0.75	1.13(04)(01)(01)(06)(04)	0.97(02)(03)(03)(11)(04)
0.80	1.09(05)(01)(02)(06)(05)	0.88(02)(02)(04)(12)(05)
0.85	1.02(06)(02)(03)(07)(04)	0.77(02)(01)(03)(13)(06)
0.90	0.94(10)(04)(02)(07)(02)	0.64(03)(01)(02)(13)(08)
0.95	0.81(14)(09)(03)(06)(02)	0.45(04)(01)(01)(12)(09)

uncertainties in plots



Outline

- Motivation
- Numerical results
- Analysis of uncertainties
- Summary



Summary

- Precise knowledge of meson LCDAs are important for understanding various exclusive processes.
- LaMET and Lattice QCD now allow us to do ab initio calculations of these meson DAs and make a comparison with experimental measurements.
- Improved renormalization schemes are adopted to avoid problems in RI/MOM.
- Several extrapolation strategies including large λ, continuum limit, and infinite momentum limit, have been proposed to increase the accuracy of results.

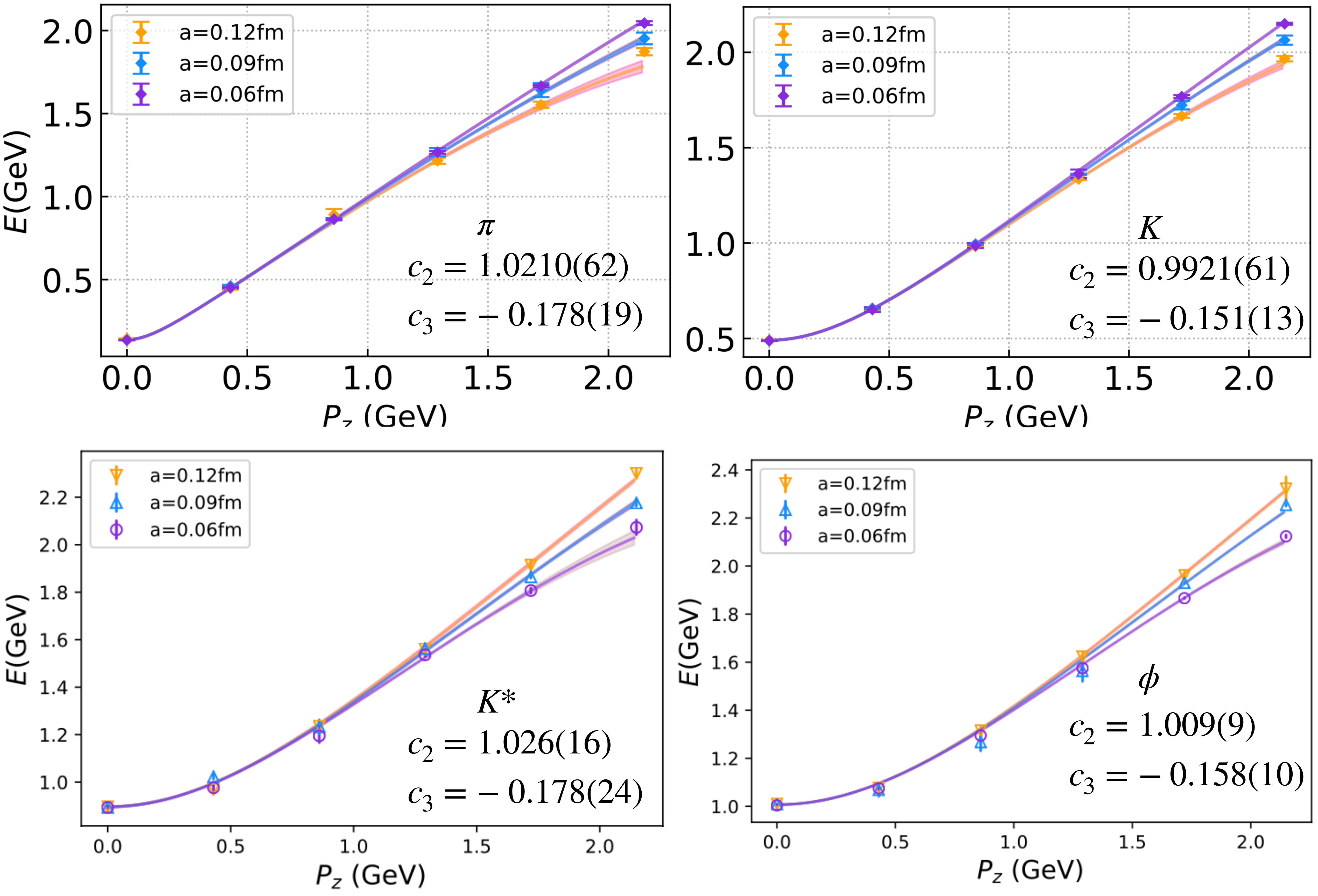
Thank you!

Backup slides

Numerical results: Introduction

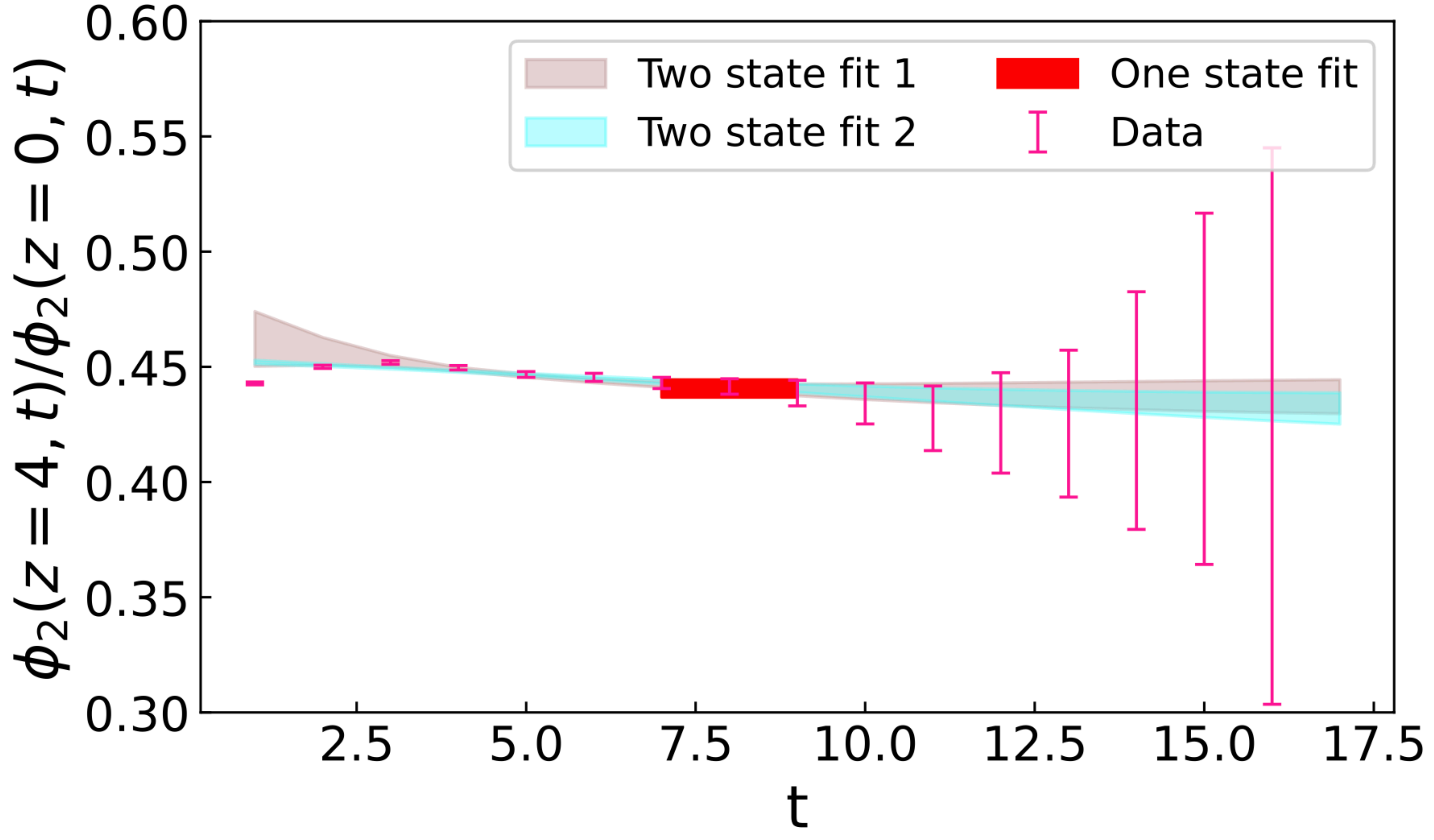
dispersion relation

$$E^2 = m^2 + c_2(P^z)^2 + c_3(P^z)^4 a^2$$



Fit for 2pt

$$\frac{C_2^m(z, \vec{P}, t)}{C_2^m(z=0, \vec{P}, t)} = \frac{H_m^B(z)(1 + c_m(z)e^{-\Delta E t})}{(1 + c_m(0)e^{-\Delta E t})}$$



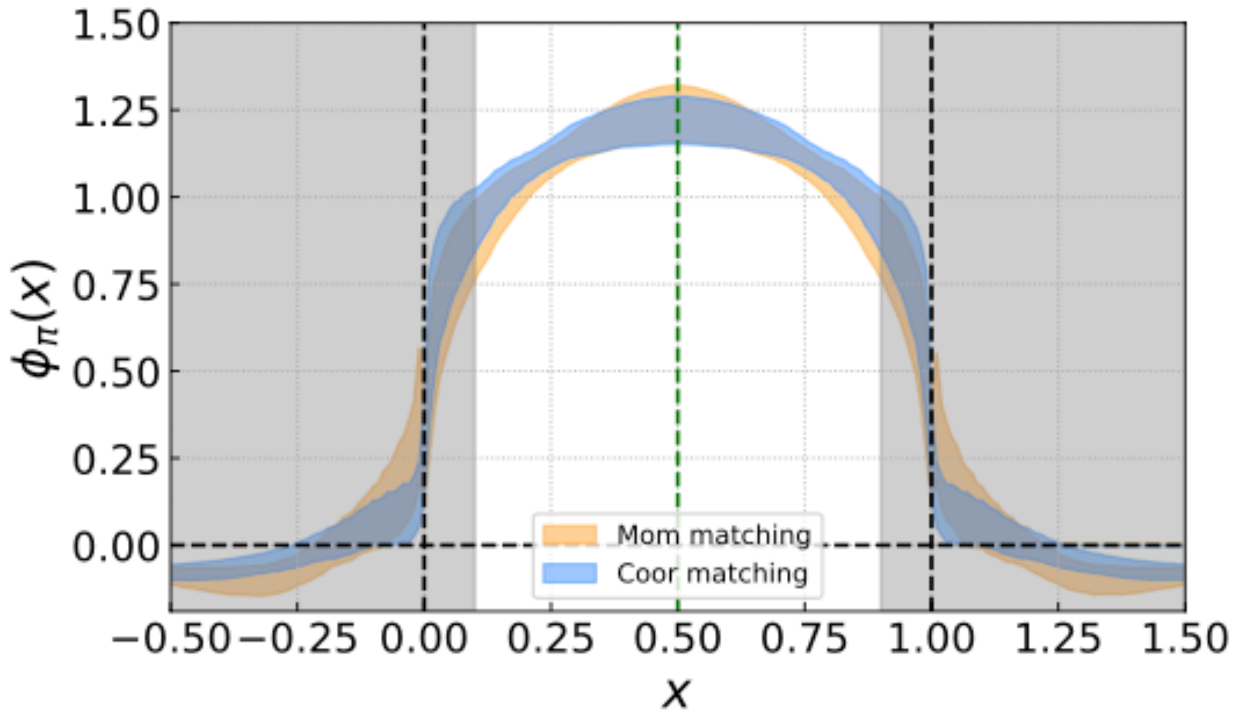
one-state fit is more stable and conservative!

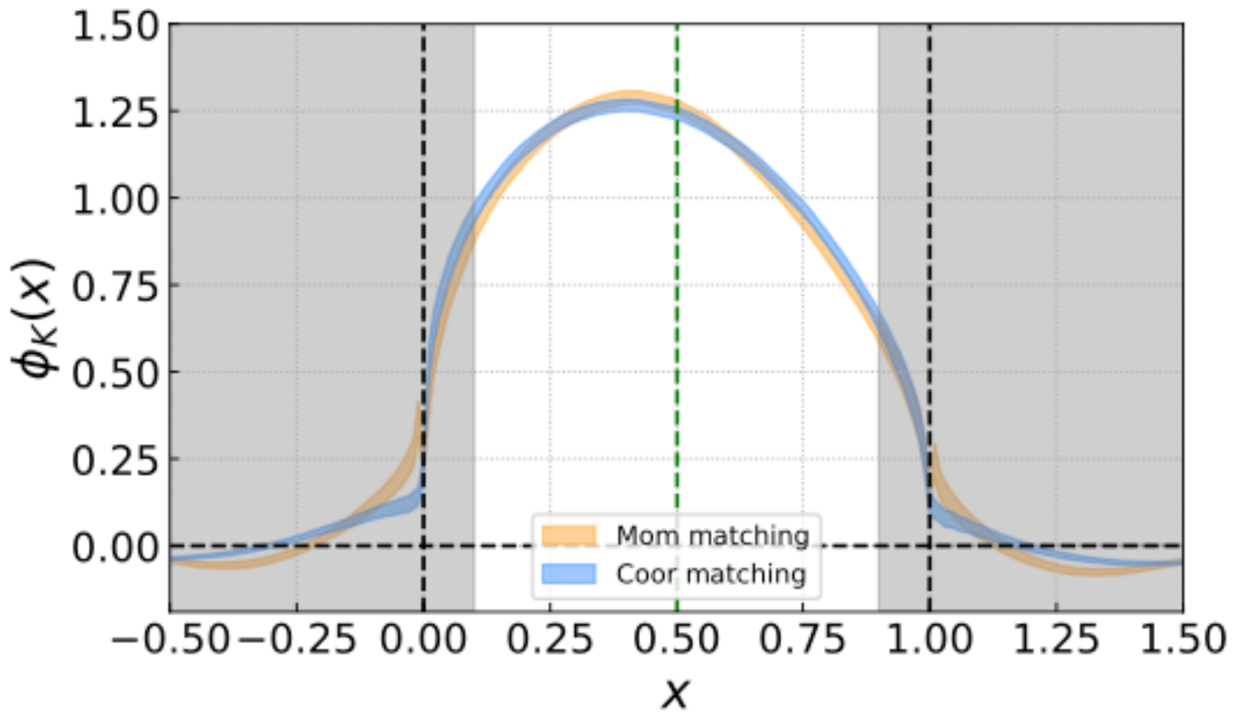
Numerical results: two techniques

co. matching v.s. mo. matching

quasi-DA in co. $\xrightarrow{matching}$ LCDA in co. $\xrightarrow{\text{F.T.}}$ LCDA in mo.

quasi-DA in co. $\xrightarrow{\text{F.T.}}$ quasi-DA in mo. $\xrightarrow{matching}$ LCDA in mo.





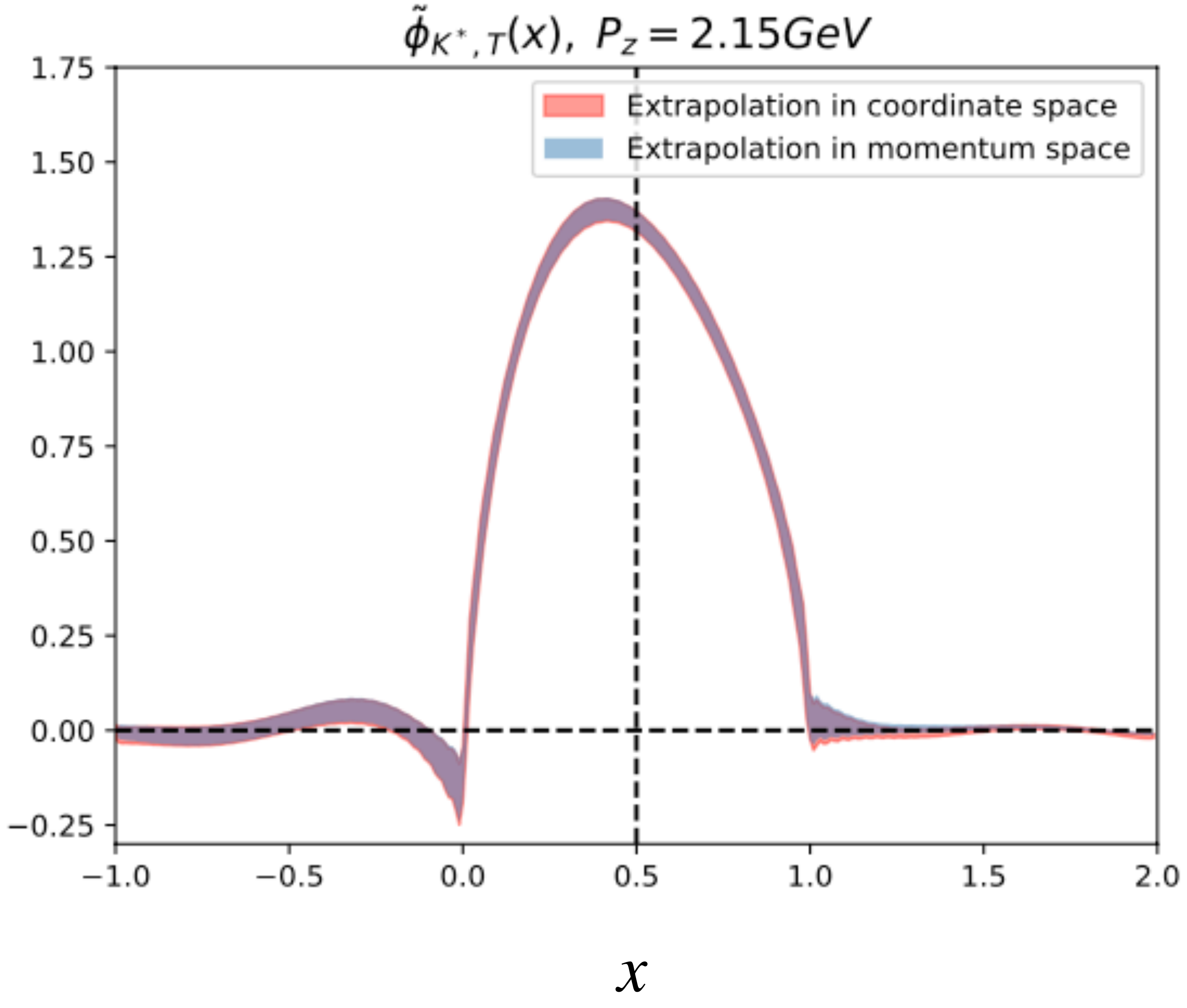
- consistent within uncertainties
- advantage of co. matching: large λ extrapolation could be performed for LCDA in co. after matching
- shaded regions suffer much power correction effects as terms of:

$$\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{(yP^z)^2}, \frac{\Lambda_{QCD}^2}{((1-y)P^z)^2}\right)$$

continuum limit

quasi-DA in co. $\xrightarrow{a \to 0}$ quasi-DA in co. $\xrightarrow{\text{F.T.}}$ quasi-DA in mo.

quasi-DA in co. $\xrightarrow{\text{F.T.}}$ quasi-DA in mo. $\xrightarrow{a \to 0}$ quasi-DA in mo.

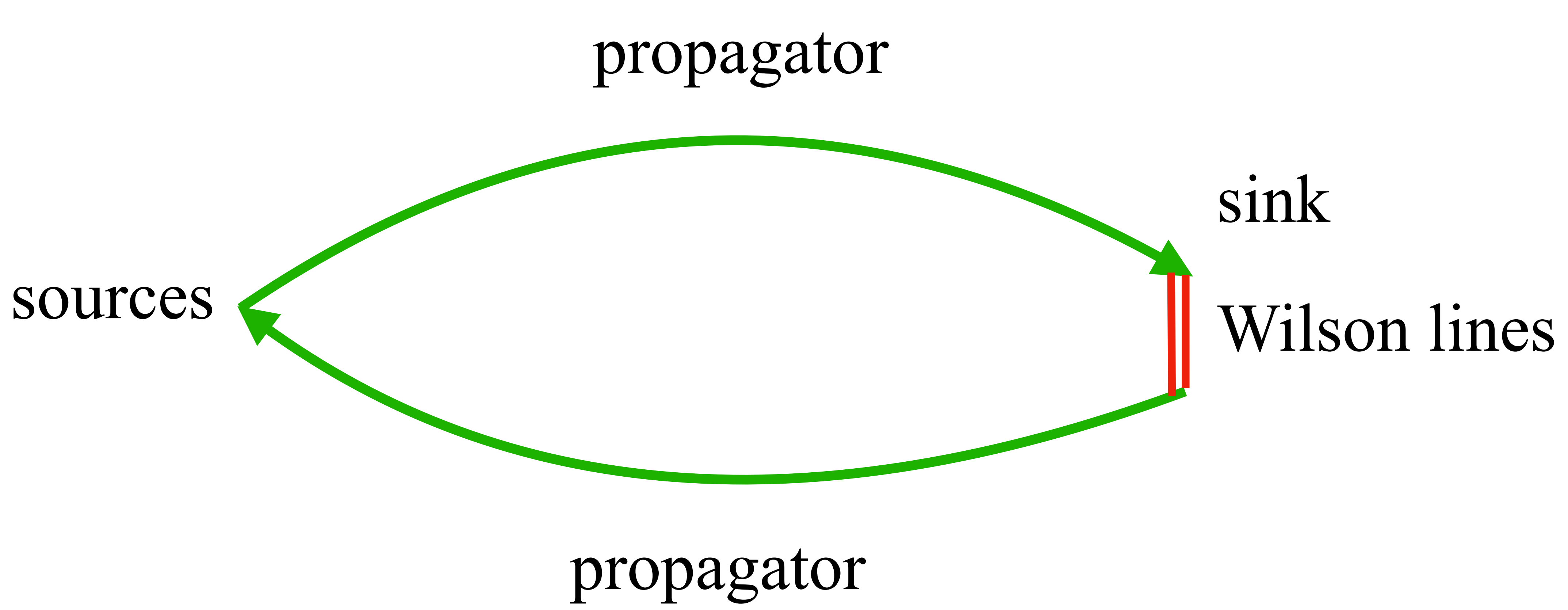


Analysis of uncertainties: statistics

processes of DA calculation

Propagators: smeared point source to wall sink. Average cases at different source locations.

nonlocal two point function



Wilson lines: according to the symmetry $\tilde{h}(z) = \tilde{h}^*(-z)$, one could perform average +z and -z to increase the statistics.

Renormalization: sample by sample.

Ground state fit for 2pt: bootstrap resampling is employed, and we do one fitting on each sample and keep central value as results. It remains correlations in data.

Large λ extrapolation, approaching continuum limit and infinite momentum limit: one fitting of asymptotic form on each example.

The statistic uncertainties come from bootstrap samples!