Calculation of generalized parton distributions on the lattice and their uncertainties

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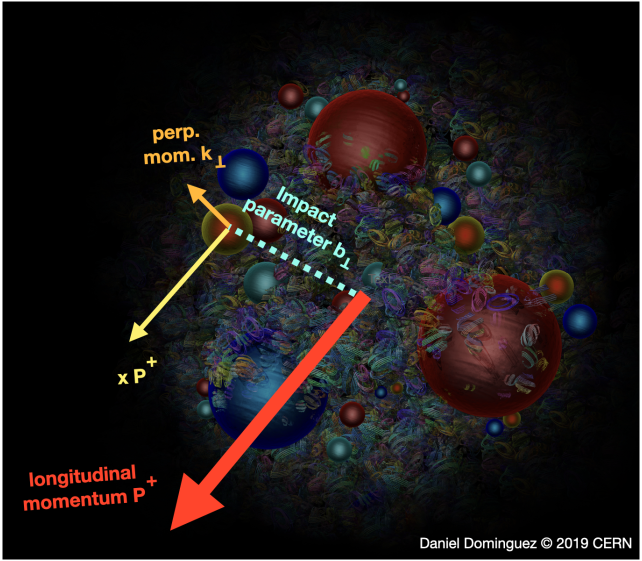
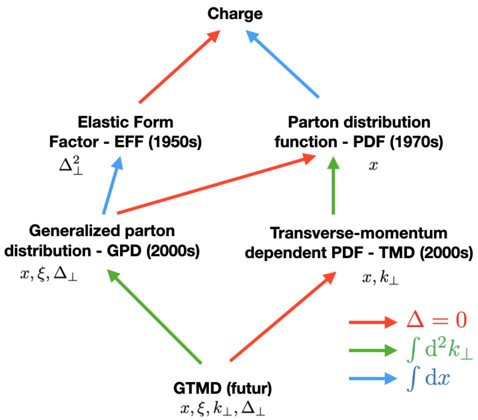
Overview

1. **GPDs: physics case and phenomenological challenges**
2. Uncertainties of lattice GPD calculations examplified by the Hadstruc calculation
3. Perspectives

Parton distributions





[Lorcé, Pasquini, Vanderhaeghen, 2011]

Unpolarized PDF:

$$f(x) = \frac{1}{2}\int_{-\infty}^{\infty} \frac{\mathrm{d}z^-}{2\pi} e^{-ixP^+z^-} \left\langle P \middle| \bar{\psi}^q\left(-\frac{z}{2}\right)\gamma^+ W\left(-\frac{z}{2},\frac{z}{2}\right)\psi^q\left(\frac{z}{2}\right) \middle| P \right\rangle \Big|_{z_\perp=0,\ z^+=0}$$

Unpolarized GPDs **[Müller et al, 1994]**, **[Ji, 1996]**, **[Radyushkin, 1996]**

$$\frac{1}{2}\int_{-\infty}^{\infty} \frac{\mathrm{d}z^-}{2\pi} e^{-ixP^+z^-} \left\langle P_2 \middle| \bar{\psi}^q\left(-\frac{z}{2}\right)\gamma^+ W\left(-\frac{z}{2},\frac{z}{2}\right)\psi^q\left(\frac{z}{2}\right) \middle| P_1 \right\rangle \Big|_{z_\perp=0,\ z^+=0}$$

$$= \frac{1}{2P^+}\bar{u}(P_2)\left(H^q(x,\xi,t)\gamma^+ + E^q(x,\xi,t)\frac{i\sigma^{+\mu}\Delta_\mu}{2M}\right)u(P_1)$$

$$\Delta = P_2 - P_1\,, \quad t = \Delta^2\,, \quad \xi = \frac{P_1^+ - P_2^+}{P_1^+ + P_2^+} = -\frac{\Delta^+}{2P^+}$$

Important for later: light-like separations dominant in the Bjorken limit of the hadronic tensor

$$W^{\mu\nu} \propto \sum_X |\mathcal{M}(\gamma^* P \to X)|^2 \propto \int d^4z\, e^{iq.z}\langle P|J^\mu(z)J^\nu(0)|P\rangle \text{ dominated by } z^2 \leq \mathcal{O}(1/Q^2)$$

- **Hadron tomography** [Burkardt, 2003]:

$$I(x,\mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp\cdot\Delta_\perp} H^q(x,\xi=0,t=-\Delta_\perp^2)$$

- **Proton's spin decomposition** [Ji, 1996]:

$$\frac{1}{2} = \sum_q \frac{1}{2}\int_{-1}^{1} dx\, x\left[H^q + E^q\right]\Big|_{t=0} + \frac{1}{2}\int_{-1}^{1} dx\left[H^g + E^g\right]\Big|_{t=0}$$

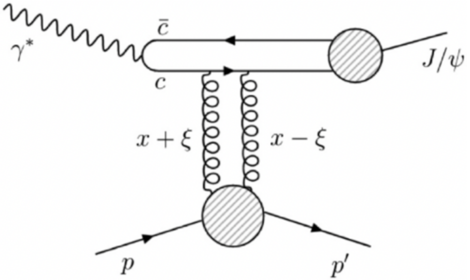
- **Gravitational form factors** [Polyakov, 2003], [Lorcé et al, 2017]: radial energy / pressure

$$\langle P_2|T_a^{\mu\nu}|P_1\rangle = \bar{u}(P_2)\Bigg\{\frac{P^\mu P^\nu}{M} A_a(t) + \frac{\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C_a(t) + M\eta^{\mu\nu}\bar{C}_a(t)$$
$$+ \frac{P^{\{\mu}i\sigma^{\nu\}\rho}\Delta_\rho}{4M}\left[A_a(t) + B_a(t)\right] + \frac{P^{[\mu}i\sigma^{\nu]\rho}\Delta_\rho}{4M} D_a^{GFF}(t)\Bigg\} u(P_1)$$

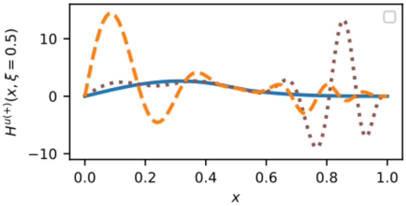
$$\int_{-1}^{1} \mathrm{d}x\, x\, H^q(x,\xi,t,\mu^2) = A_q(t,\mu^2) + 4\xi^2 C_q(t,\mu^2)$$

But at the LHC?

- GPDs involved in **factorization of exclusive processes** such as the J/ψ photoproduction in UPC at the LHC.
- Formally very sensitive to the **gluon GPD** down to $x_B \sim 10^{-7}$.
- Often overlooked in favor of the **gluon PDF** in the analysis, but increasing attention on the skewness ξ dependence **[Flett, Jones, Martin, Ryskin, Teubner]**



Current bulk of experimental characterization from JLab in the deeply virtual Compton scattering process (DVCS). In both processes, a deconvolution problem **[Bertone, HD, Mezrag, Moutarde, Sznajder, PRD 103 (2021) 11]**



$$\mathcal{H}^q(\xi, t, Q^2) = \int_{-1}^{1} \frac{\mathrm{d}x}{\xi}\, T^q\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s\right) H^q(x, \xi, t, \mu^2)$$

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Parton distributions in lattice QCD

Lattice QCD computes the Euclidean QCD path integral in a discretized space-time. so no light-cone parton distribution. But, among other proposals:

- **Local operators** (low-order Mellin moments only due to power-divergent mixing when approaching the continuum limit)
- **Large-momentum effective theory (LaMET)** [Ji, 2013]: use a large hadron boost $P \to \infty$ to approach the light-like separation
- **Short-distance factorization** [Radyushkin, 2017]: OPE of non-local space-like operators in the limit $z^2 \to 0$

$$\langle P|\bar{\psi}(z)\gamma^\mu W(z,0)\psi(0)|P\rangle = P^\mu \mathcal{M}_\nu(\nu = P\cdot z, z^2) + z^\mu \mathcal{N}_\nu(\nu = P\cdot z, z^2)$$

$$\text{Ioffe time } \nu = \text{FT of } x: \quad \mathcal{Q}(\nu,\mu^2) = \int_{-1}^{1} dx\, e^{-i\nu x} f(x,\mu^2)$$

$$\mathcal{M}_\nu(\nu, z^2) = \int_{-1}^{1} d\alpha\, C(\alpha, \mu^2 z^2)\mathcal{Q}(\alpha\nu, \mu^2) + \mathcal{O}(z^2\Lambda_{QCD}^2)$$

Uncertainties of lattice calculations

Traditional lattice uncertainties (discretization errors, finite volume effects, excited state contamination, unphysical pion mass, statistical gauge noise)

Specific uncertainties

1. reconstruction of the x-dependence: inverse problem due to the finite range in P and z
2. power corrections: we do not isolate the pure leading-twist component
3. perturbative uncertainty: uncertainty in the Wilson coefficient relating the z^2 pseudo-PDF to the $\overline{MS}$ PDF

There are connections between those:

1. and 2. $\rightarrow$ matter of limit in the kinematic domain and their order

2. and 3. $\rightarrow$ connection of the renormalon ambiguity of perturbation theory and the higher twist contributions

GPD matrix element **[Bhattacharya et al, PRD 106 (2022) 11]**:

$$\langle P_2|\bar{\psi}\left(-\frac{z}{2}\right)\gamma^\mu\psi\left(\frac{z}{2}\right)|P_1\rangle = \bar{u}(P_2)\Big[\gamma^\mu A_1 + z^\mu A_2 + \sigma^{\mu\nu}z_\nu A_3 + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}A_4 + \frac{\Delta^\mu}{2m}A_5 + \frac{i\sigma^{\alpha\beta}z_\alpha\Delta_\beta}{2m}\left(P^\mu A_6 + \Delta^\mu A_7 + z^\mu A_8\right)\Big]u(P_1)$$

Identify the terms that survive in the light-cone limit:

$$H(\nu,\xi,t,z^2) = \lim_{z^2\to 0} A_1 - \xi A_5$$
$$E(\nu,\xi,t,z^2) = \lim_{z^2\to 0} A_4 + \nu A_6 - 2\xi\nu A_7 + \xi A_5$$

Match to the light-cone limit in the short-distance factorization for GPDs **[Radyushkin, 2019]**:

$$\begin{pmatrix} H \\ E \end{pmatrix}(\nu,\xi,t,\mu^2) = \int_{-1}^{1} d\alpha C(\alpha,\xi\nu,\mu^2 z^2)\begin{pmatrix} H \\ E \end{pmatrix}(\alpha\nu,\xi,t,z^2) + \text{power corrections}$$

Extracting each amplitude A_k requires to measure matrix elements with various combinations of helicity and gamma structure (kinematic matrix inversion)

polynomiality of moments of GPDs:

$$\int_{-1}^{1} dx\, x^{n-1} H(x,\xi,t) = \sum_{k=0 \text{ even}}^{n-1} A_{n,k}(t)\,\xi^k + \mathrm{mod}(n+1,2)\, C_n(t)\xi^n$$

$$A_{1,0}(t) = F_1(t) \quad \text{(elastic form factor)}$$

Analysis of D-term can be separated from the rest of the GPD. Hence small Ioffe-time behavior without D-term:

$$H(\nu,\xi,t) = \int dx\, e^{-ix\nu} H(x,\xi,t)$$

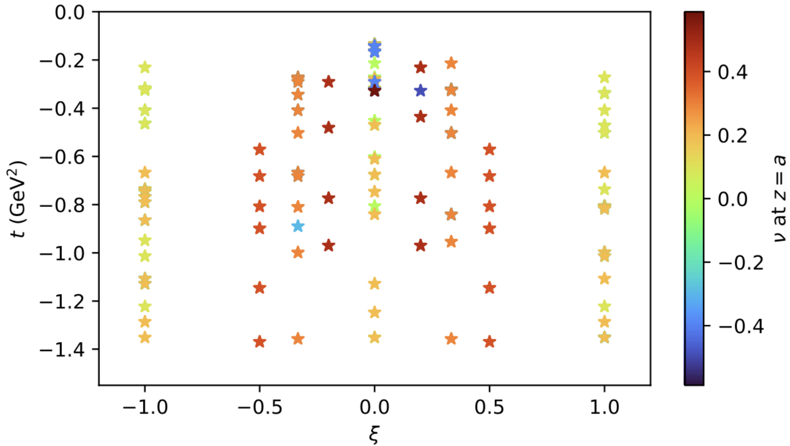
$$= F_1(t) - i\nu A_{2,0}(t) - \frac{\nu^2}{2}[A_{3,0}(t) + \xi^2 A_{3,2}(t)] + \ldots + \text{power corrections}$$

With momenta up to 1.4 GeV used in this study, we have signal up to $A_{4,0}$ and $A_{4,2}$.

$$\text{Dipole fit:} \quad A_{n,k}(t) = A_{n,k}(t=0)\left(1 - \frac{t}{\Lambda^2_{n,k}}\right)^{-2}$$

[Hadstruc collaboration, JHEP 08 (2024) 162]

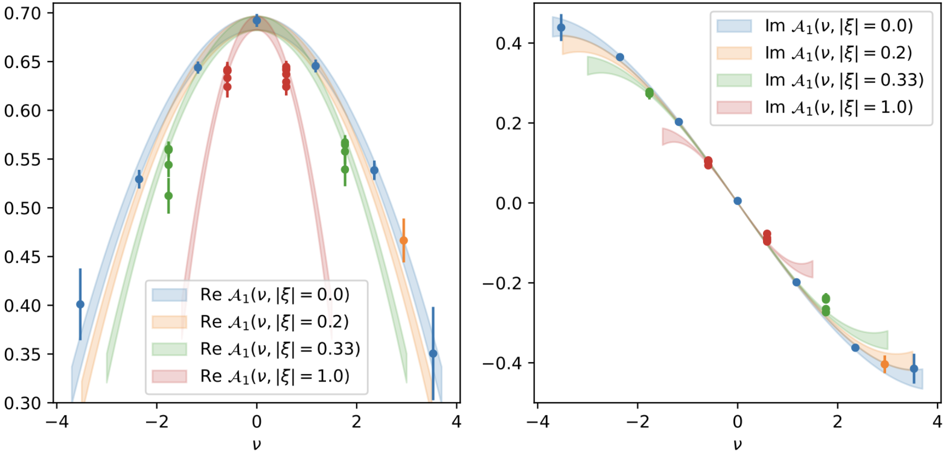
ID	a (fm)	m_π (MeV)	β	$m_\pi L$	$L^3 \times N_T$	N_{cfg}	N_{srcs}	rk $(\mathcal{D})$
a094m358	0.094(1)	358(3)	6.3	5.4	$32^3 \times 64$	348	4	64



186 pairs $(\vec{p}_f, \vec{p}_i)$

Final analysis forms 12 bins in t with a median number of 12 pairs $(\vec{p}_f, \vec{p}_i)$ inside, many being related by symmetry properties to attempt to reduce noise. In the end, we have typically between 4 and 7 independent measurements to extract 3 quantities. Many are quite cheap to compute!

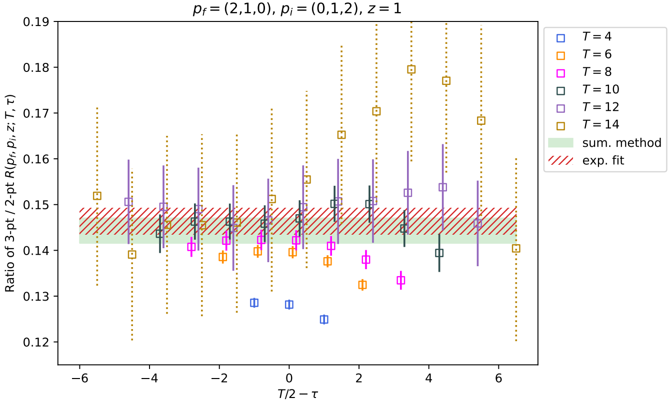
Fixed t and $z = 0.56$ fm, varying $P_{1,2}$ (therefore ν and ξ) - slight excited state contamination?

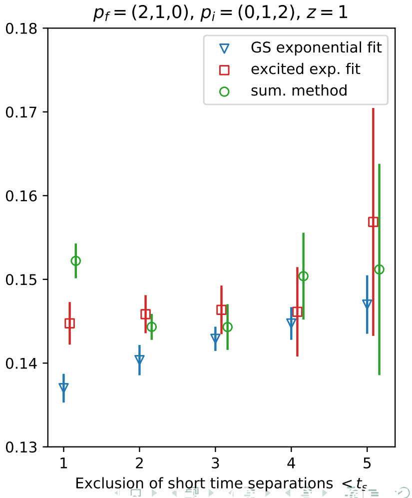


$$H(\nu, \xi, t) = F_1(t) - i\nu A_{2,0}(t) - \frac{\nu^2}{2}[A_{3,0}(t) + \xi^2 A_{3,2}(t)] + \ldots + \text{power corrections}$$

Excited state contamination

Pedestrian approach: cuts in Euclidean time





Pion mass = 0.36 GeV - Proton mass = 1.12 GeV

No continuum limit - signs of discretization errors / light-cone uncertainty

Matching at 2 GeV with leading logarithmic accuracy

Value at t = 0

GPD H^{u-d}		GPD E^{u-d}	
$A_{1,0}$: 0.974^{+12}_{-5}		$B_{1,0}$: 3.40^{+7}_{-1}	
$A_{2,0}$: 0.206^{+2}_{-6}		$B_{2,0}$: 0.370^{+9}_{-24}	
$A_{3,0}$: 0.064^{+2}_{-6}	$A_{3,2}$: 0.39^{+11}_{-3}	$B_{3,0}$: 0.063^{+24}_{-8}	$B_{3,2}$: 1.1^{+4}_{-8}
$A_{4,0}$: 0.065^{+5}_{-19}	$A_{4,2}$: 0.5^{+3}_{-3}	$B_{4,0}$: 0.06^{+16}_{-2}	$B_{4,2}$: > 1.1

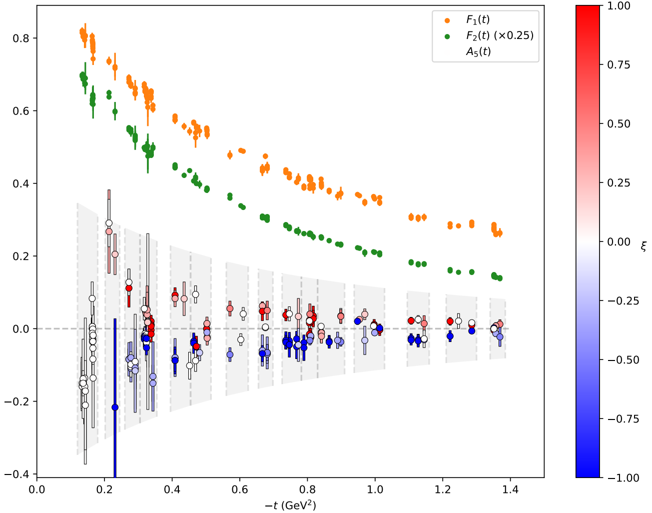
Dipole mass (GeV)

GPD H^{u-d}		GPD E^{u-d}	
$A_{1,0}$: 1.255^{+3}_{-29}		$B_{1,0}$: 0.987^{+2}_{-6}	
$A_{2,0}$: 1.83^{+9}_{-3}		$B_{2,0}$: 1.39^{+11}_{-5}	
$A_{3,0}$: 2.3^{+2}_{-5}	$A_{3,2}$: 1.10^{+7}_{-11}	$B_{3,0}$: 2.2^{+36}_{-5}	$B_{3,2}$: 0.78^{+77}_{-9}
$A_{4,0}$: > 3.5	$A_{4,2}$: > 0.9	$B_{4,0}$: > 0.6	$B_{4,2}$: 0.5^{+5}_{-2}

D-term^{u-d}

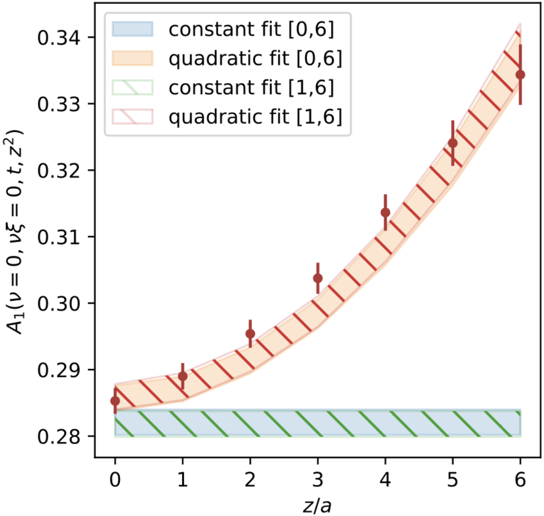
	Value at t = 0	Dipole mass (GeV)
C_2	0.025^{+8}_{-8}	> 2.2

$$\langle p'|\bar{\psi}^q\gamma^\mu\psi^q|p\rangle\Big|_{z=0} = \bar{u}(p')\left[F_1^q(t)\gamma^\mu + F_2^q(t)\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m} + A_5^q(t)\frac{\Delta^\mu}{2m}\right]u(p)$$

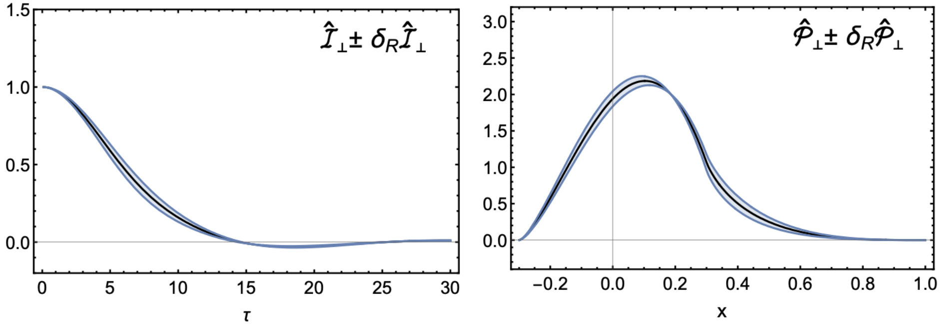


probable sign of lattice discretization in A_5 + enhanced sensitivity to excited state contamination

If $p_{f,z} = p_{i,z} = 0$, then $\nu = 0$ and $\nu\xi = 0$, so we have non-local data with signal only of the EFF + whatever parasitic contribution. But are these **power corrections (higher twists)** or **lattice discretization errors**? $A_5(z=0)$ has quite certainly discretization errors.



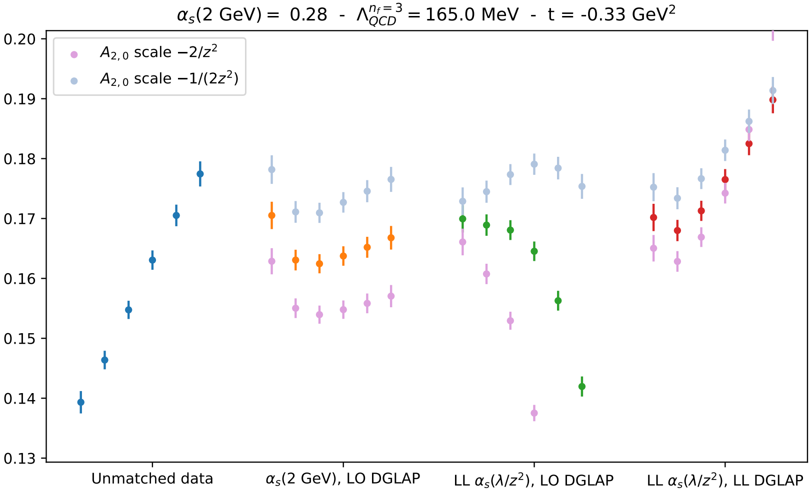
Even with a space-like separation of 1 fm, the power-corrections at $\xi = 0.3$ might be very small (model-dependent estimate of non-perturbative higher-twist contributions through renormalon ambiguity) **[Braun, Koller, Schoenleber, 2024]**:



Possible interpretation: higher-twist contribution largely independent on the external momentum, and suppressed by the ratio method

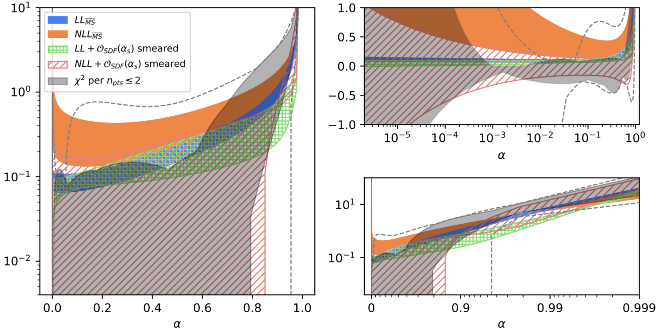
What we checked: using $z = 0.38$ fm or $z = 0.56$ fm does not change the results in any meaningful way. But it is not nearly as paradoxical as it sounds considering the degree of correlation of those points and the current uncertainties.

Perturbative matching uncertainty: only relevant for a few bins of the GFF at this stage

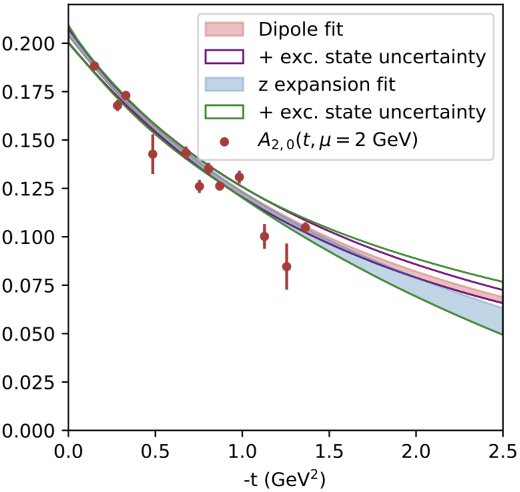


$a = 0.094$ fm, so range of scales from $z = a \sim 2$ GeV to $z = 6a \sim 350$ MeV.

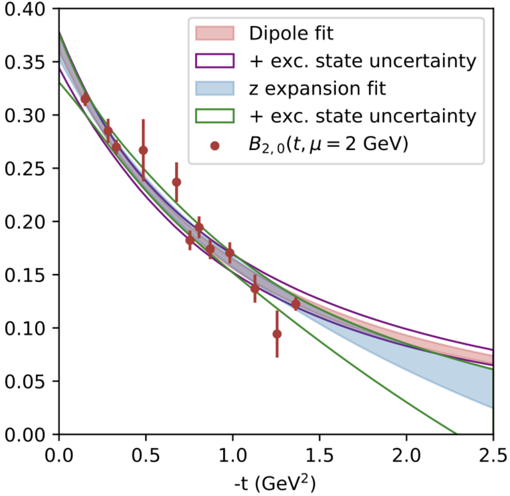
- This begs the question of the use of perturbation theory when $1/z^2 \sim \Lambda^2_{QCD}$. If one restricts oneself to $z \sim 0.3$ fm, with the largest momenta currently achievable of ~ 3 GeV, gives Ioffe time up to 4.5 at most.
- Why not let data inform the breaking of factorization, and obtain evolution operators on the lattice? PDF anomalous dimensions in quenched lattice QCD **[Guagnelli, Jansen, Petronzio, 1998]**
- Comparison of a "data-driven" evolution kernel vs 1-loop and 2-loop pQCD



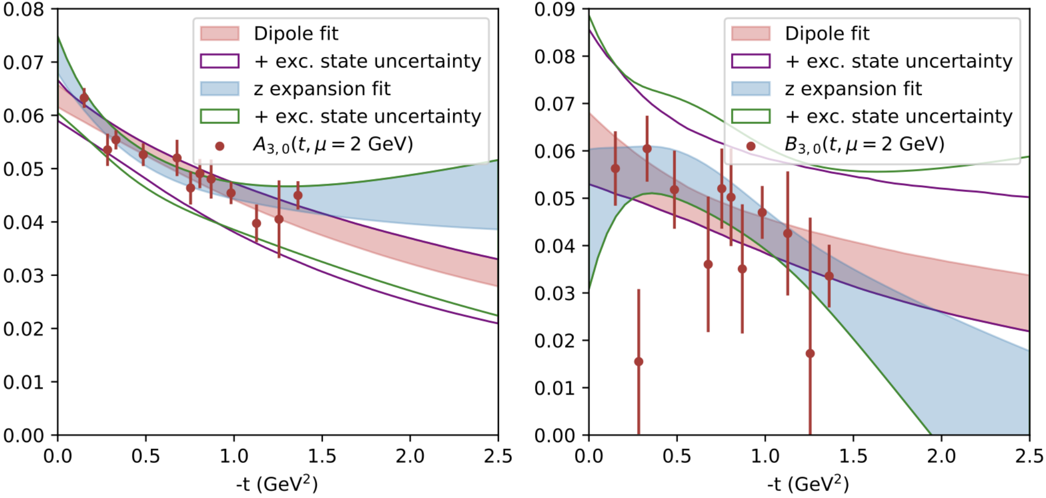
[HD, Karpie, Monahan, Orginos, Zafeiropoulos, JHEP 04 (2024) 061]



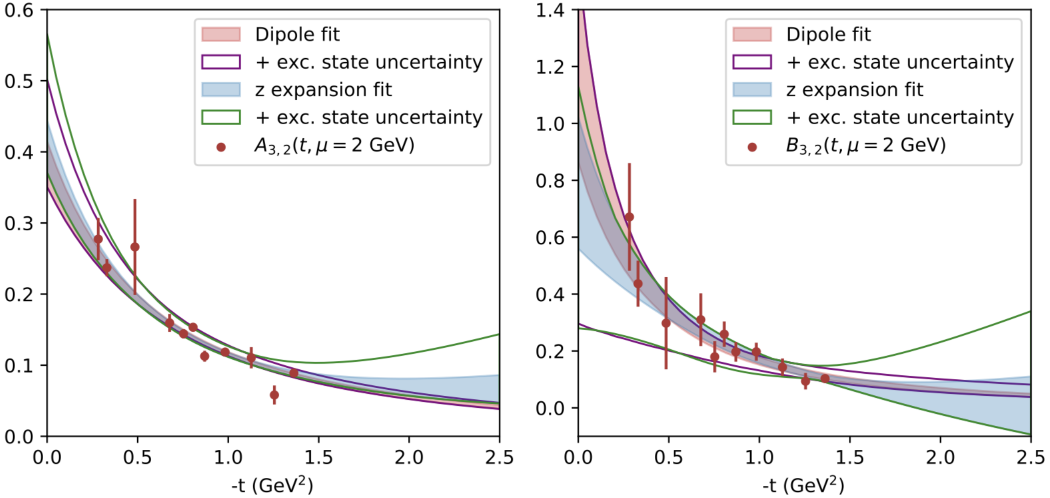
Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$A_{2,0}(t, \mu = 2\ \text{GeV})$
0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
0.0
0.5
1.0
1.5
2.0
2.5
-t (GeV^2)



Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$B_{2,0}(t, \mu = 2\ \text{GeV})$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.0
0.5
1.0
1.5
2.0
2.5
-t (GeV^2)

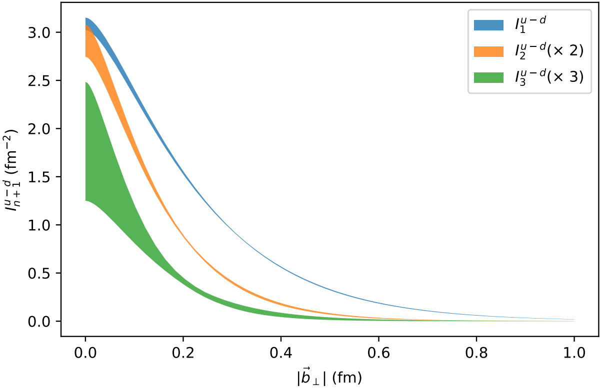


Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$A_{3,0}(t, \mu = 2\text{ GeV})$
$B_{3,0}(t, \mu = 2\text{ GeV})$
-t (GeV^2)
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.0
0.5
1.0
1.5
2.0
2.5


Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$A_{3,2}(t, \mu = 2$ GeV)
-t (GeV^2)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.0
0.5
1.0
1.5
2.0
2.5
Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$B_{3,2}(t, \mu = 2$ GeV)
-t (GeV^2)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Impact parameter distribution: number density of unpolarized quarks in an unpolarized proton with mom. fraction x and radial distance to the center of longitudinal momentum $\vec{b}_\perp$: **model dependence!**

$$I(x,\vec{b}_\perp) = \int \frac{\mathrm{d}^2\vec{\Delta}_\perp}{(2\pi)^2}\, e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} H(x,\xi=0,t=-\vec{\Delta}_\perp^2)$$



Ioffe-time GPDs are great GPDs! [Braun, Gornicki, Mankiewicz, 1995]

x-dependent GPDs

non analytical form

triangular for x > xi (only needs larger momentum fractions)

needs all x range for x < xi

$$\frac{d}{d\ln(\mu^2)}H^q(x,\xi,\mu^2) = \int_0^1 \frac{dy}{y} C\left(\frac{x}{y},\frac{\xi}{x},\alpha_s(\mu^2)\right) H^q\left(y,\xi,\mu^2\right)$$

$$C\left(\alpha,\frac{\xi}{x},\alpha_s(\mu^2)\right) = \delta(1-\alpha) + \frac{\alpha_s(\mu^2)C_F}{2\pi}\left\{\theta(1-\alpha)\left[\left(\frac{1+\alpha^2}{1-\alpha}\right)_+ + \mathcal{O}(\xi^2)\right] + \theta(x\leq\xi)\left[-\left(\frac{1}{1-y}\right)_{++} + \ldots\right]\right\}$$

Ioffe-time GPDs

simple analytical form

triangular (only needs smaller Ioffe time range)

$$\frac{d}{d\ln(\mu^2)}H^q(\nu,\xi,\mu^2) = \int_0^1 d\alpha\, K(\alpha,\xi\nu,\alpha_s(\mu^2))H^q(\alpha\nu,\xi,\mu^2)$$

$$K(\alpha,\xi\nu,\alpha_s(\mu^2)) = \delta(1-\alpha) + \frac{\alpha_s(\mu^2)C_F}{2\pi}\left[\left(\frac{2\alpha}{1-\alpha}\right)_+ \cos(\bar\alpha\xi\nu) + \frac{\sin(\bar\alpha\xi\nu)}{\xi\nu} - \frac{\delta(1-\alpha)}{2}\right]$$

Conformal moments

simple analytical form

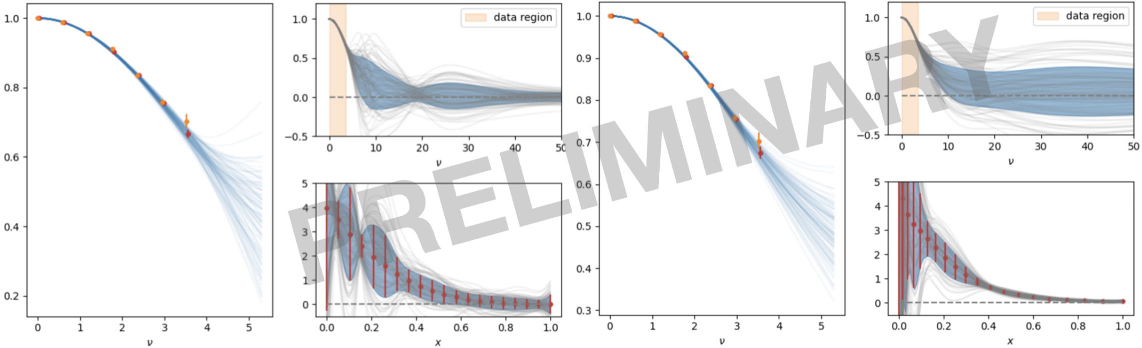
diagonal (each moment evolves independently)

$$O_n(\xi,\mu^2) \propto \int_{-1}^1 dx\, C_n^{(3/2)}\left(\frac{x}{\xi}\right) H^q(x,\xi,\mu^2)$$

$$O_n(\xi,\mu^2) = O_n(\xi,\mu_0^2)\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\gamma_n/(2\pi\beta_0)}$$

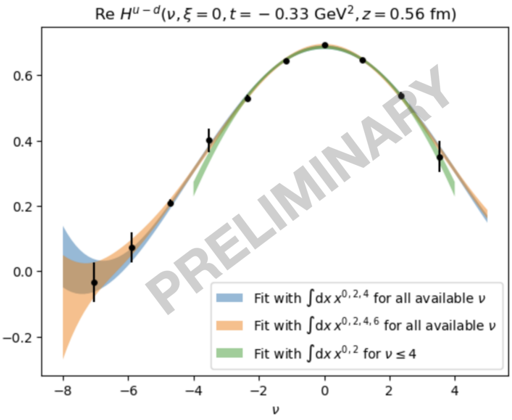
But

if you desire x-dependence, beware of the regularization / reconstruction framework dependence:

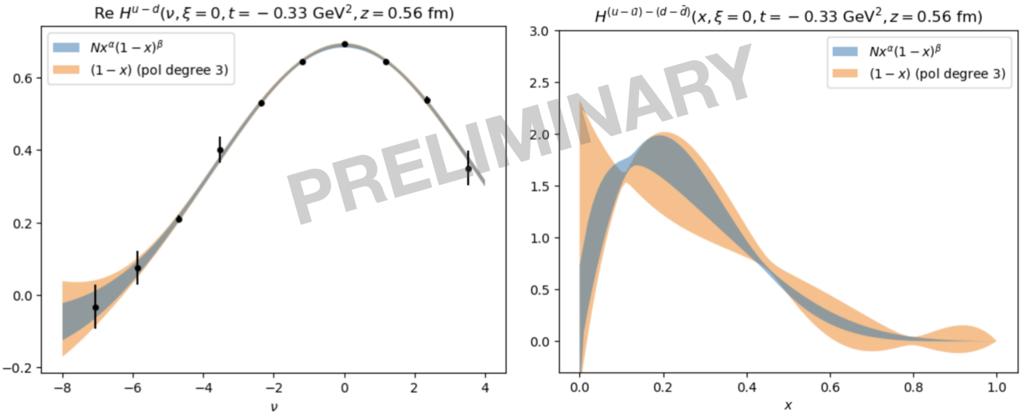


$z = 0.27$ fm, $P_z = 2.7$ GeV $\rightarrow \nu = 3.7$

Results in differences of order of magnitude in the precision in x while reproducing accurately the data region.



	2 moments	3 moments	4 moments
$A_{1,0}$	0.686 ± 0.005	0.687 ± 0.005	0.689 ± 0.006
$A_{2,0}$	0.054 ± 0.003	0.056 ± 0.002	0.062 ± 0.005
$A_{4,0}$		0.007 ± 0.001	0.011 ± 0.004
$A_{6,0}$			0.002 ± 0.002


Re $H^{u-d}(\nu, \xi = 0, t = -0.33\ \text{GeV}^2, z = 0.56\ \text{fm})$
$Nx^\alpha(1-x)^\beta$
$(1-x)$ (pol degree 3)
0.6
0.4
0.2
0.0
−0.2
−8
−6
−4
−2
0
2
4
ν
PRELIMINARY
$H^{(u-\bar{u})-(d-\bar{d})}(x, \xi = 0, t = -0.33\ \text{GeV}^2, z = 0.56\ \text{fm})$
$Nx^\alpha(1-x)^\beta$
$(1-x)$ (pol degree 3)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.2
0.4
0.6
0.8
1.0
x

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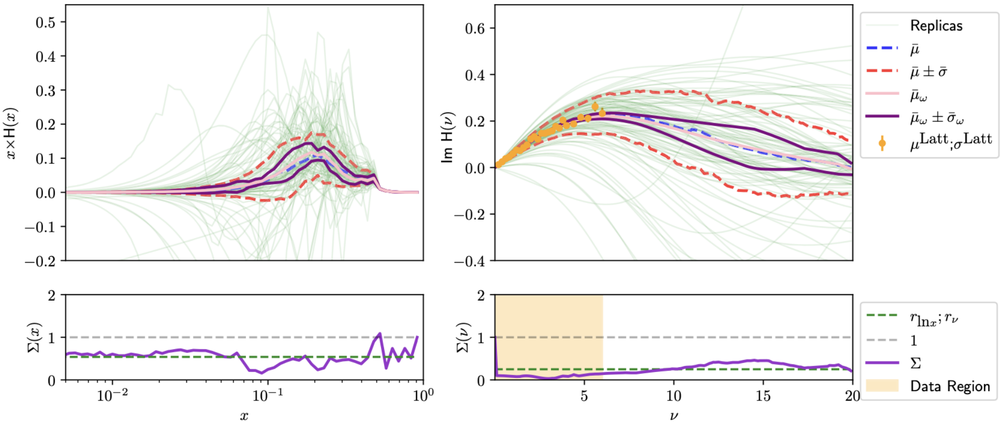
3. **Perspectives**

A joint lattice - experimental phenomenology

Modest example: using a model whose uncertainty is purely theoretical deconvolution uncertainty (not experimental) and pseudo-lattice data **[Riberdy, HD, Mezrag, Sznajder, 2023]**

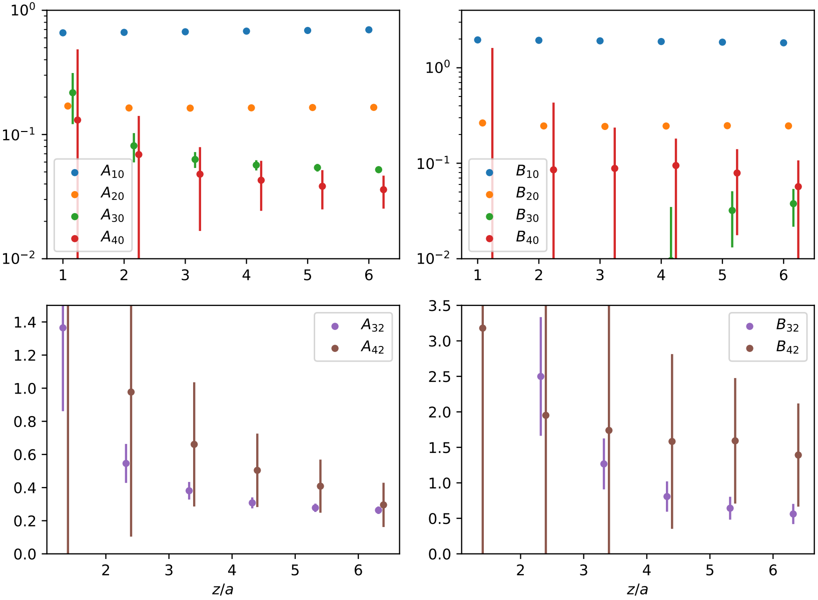
$\xi = 0.5$



Conclusion

- We have a framework with excellent signal and very large kinematic coverage for GPDs
- There is not one source of uncertainty that will be spared to us:
  - Clear plausible sign of lattice discretization error in the EFF $A_5(t)$ and the anomalous running with scale of the usual EFFs
  - Plausible sign of increasing excited state contamination when momentum beyond 1 GeV that we have drowned in statistical noise so far.
  - Status of perturbative and power correction uncertainties undecided within the current precision.
  - Model dependence in the extrapolation to large t to construct impact parameter pictures, the reconstruction of the x-dependence (deserves a lot more discussion) and the potential correction of pion mass effects far from the physical pion mass.
- We have a lot of work at hand, stay tuned!

Thank you for your attention!



10^0
10^{-1}
10^{-2}
A_{10}
A_{20}
A_{30}
A_{40}
B_{10}
B_{20}
B_{30}
B_{40}
A_{32}
A_{42}
B_{32}
B_{42}
z/a
z/a