Renormalization-Group Improved Resummation of Super-Leading Logarithms using SCET

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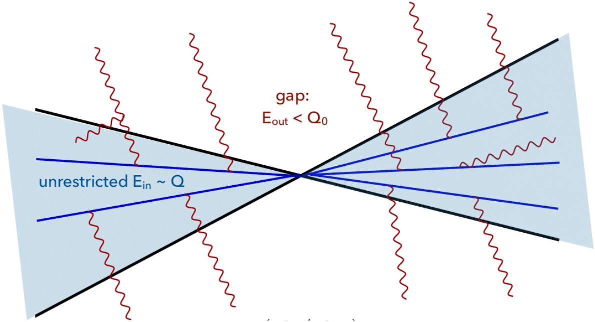
based on

2107.01212
2307.06359

2307.11089 (Glauber series for IS-quarks)
2311.18811 (Glauber series for IS-gluons)
$\rightarrow$ 2405.05305 (RG-improved resummation)
2407.01691 (Resummation at large-N_c)

in collaboration with
Patrick Hager, Matthias Neubert, Michel Stillger, Xiaofeng Xu

Super-leading logarithms in non-global observables

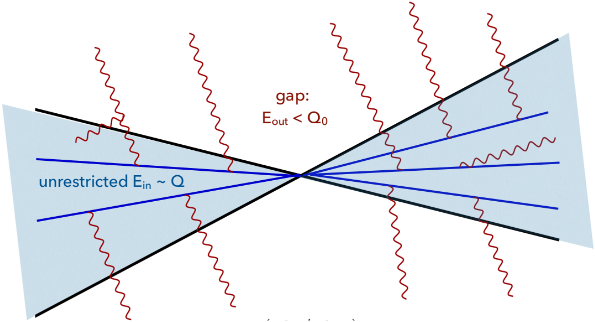


$$L = \ln(Q/Q_0) \gg 1$$

- Perturbative expansion includes "super-leading" logarithms starting at 4 loops

$$\sigma \sim \sigma_{\text{Born}} \times \{1 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \boxed{\alpha_s^4 L^5} + \boxed{\alpha_s^5 L^7} + \dots\}$$

Super-leading logarithms in non-global observables



$$L = \ln(Q/Q_0) \gg 1$$

- A double-logarithmic series starting at 3-loop order:

$$\sigma \sim \sigma_{\text{Born}} \times \{1 + \alpha_s L + \alpha_s^2 L^2 + (\alpha_s \pi^2)(\alpha_s^2 L^3 + \alpha_s^3 L^5 + \dots)\}$$

- $(\alpha_s \pi^2)$ from imaginary part of the large logarithm $\ln(-Q^2/Q_0^2) = 2L - i\pi$.
- Breakdown of color-coherence due to soft (Glauber) gluon exchange between partonic initial-states. [Forshaw, Kyrieleis, Seymour 2006]

The Glauber series

- SLLs require two Glauber phases.
- Not a large logarithm, but associated with large numerical factor of π^2.
- For realistic values, e.g $Q \sim 1$TeV and $Q_0 \sim 40$GeV, find $L \sim \pi$.
- Resum both the SLLs and associated factors of π^2 in the Glauber series.
- Cross section takes the form

$$\sigma^{\text{SLL+G}} \sim \frac{\alpha_s L}{\pi N_c} \sum_{\ell=1}^{\infty} \sum_{n=0}^{\infty} c_{\ell,n}\, w_\pi^\ell\, w^{n+\ell}, \qquad c_{\ell,n} \sim (-1)^{n+\ell}$$

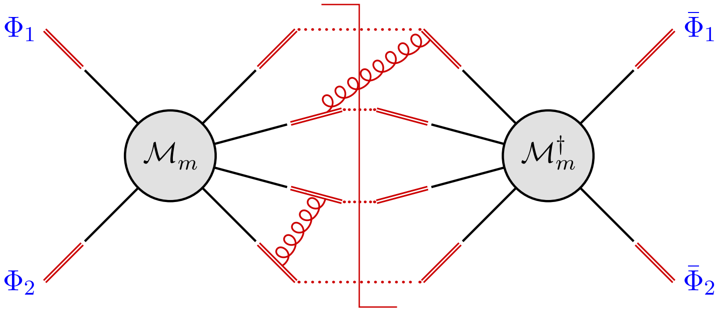
with $\mathcal{O}(1)$ expansion parameters $w_\pi = \frac{N_c \alpha_s}{\pi} \pi^2$ and $w = \frac{N_c \alpha_s}{\pi} L^2$

Factorisation Theorem

For gap-between-jets cross sections

$$\sigma_{2\to M}(Q,Q_0) = \int dx_1 dx_2 \sum_{m=2+M}^{\infty} \left\langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,x_1,x_2,\mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\},Q_0,x_1,x_2,\mu) \right\rangle$$

[Becher, Neubert, Shao (2021); Becher, Neubert, Rothen, Shao (2015,2016)]



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[Becher, Neubert, Shao (2021); Becher, Neubert, Rothen, Shao (2015,2016)]

- Hard function in color space

$$\boldsymbol{\mathcal{H}}_m(\{\underline{n}\}) = \int dE_m \, |\mathcal{M}_m(\{\underline{p}\})\rangle\langle\mathcal{M}_m(\{\underline{p}\})|$$

  where energy integrals contain the phase-space constraints

- Renormalisation-group equation:

$$\frac{d}{d\ln\mu}\boldsymbol{\mathcal{H}}_l(\{\underline{n}\},Q,\mu) = -\sum_{m\le l} \boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu) \star \boldsymbol{\Gamma}^H_{ml}(\{\underline{n}\},Q,\mu)$$

Resummation of super-leading logarithms

- Evaluate factorisation theorem at low scale $\mu_s \sim Q_0$:
- Low-energy matrix element reduces to PDFs

$$\boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, x_1, x_2, \mu_s) = f_1(x_1) f_2(x_2)\, \mathbf{1} + \mathcal{O}(\alpha_s)$$

- Evolve hard function from $\mu_h \sim Q$ to $\mu_s \sim Q_0$

$$\boldsymbol{\mathcal{H}}_m(\mu_s) = \sum_{l \le m} \boldsymbol{\mathcal{H}}_l(\mu_h) \star \mathbf{P} \exp\left[\int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu}\, \boldsymbol{\Gamma}^H(\mu)\right]_{lm}$$

- Task: need to manage the $\mathbf{P}$-ordered evolution operator.

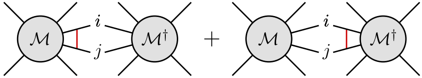
Anomalous Dimension

- One-loop anomalous dimension contains virtual and real contributions

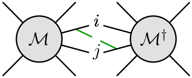
$$\boldsymbol{\Gamma}^H(\{\underline{n}\}, s, \mu) = \frac{\alpha_s}{4\pi} \begin{pmatrix} \boldsymbol{V}_{2+M} & \boldsymbol{R}_{2+M} & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_{2+M+1} & \boldsymbol{R}_{2+M+1} & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_{2+M+2} & \boldsymbol{R}_{2+M+2} & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_{2+M+3} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- Action on hard functions:

$$\boldsymbol{\mathcal{H}}_m \, \overline{\boldsymbol{V}}_m = \sum_{(ij)}$$



$$\boldsymbol{\mathcal{H}}_m \, \overline{\boldsymbol{R}}_m = \sum_{(ij)}$$



- Real-emission contributions extend the color space!

Anomalous Dimension

- Decompose into soft emission, Glauber phase and collinear emission:

$$\boldsymbol{\Gamma}^H = \Gamma^C + \gamma_{\text{cusp}}(\alpha_s) \left(\boldsymbol{\Gamma}^c \ln \frac{\mu^2}{\mu_h^2} + \boldsymbol{V}^G \right) + \frac{\alpha_s}{4\pi} \overline{\boldsymbol{\Gamma}} + \mathcal{O}(\alpha_s^2)$$

- where

$$\boldsymbol{\Gamma}^c = \sum_{i=1,2} \left[C_i \, \mathbf{1} - \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{i,R} \, \delta(n_k - n_i) \right],$$

$$\boldsymbol{V}^G = -2i\pi \left(\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R} \right),$$

$$\overline{\boldsymbol{\Gamma}} = 2 \sum_{(ij)} (\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R}) \int \frac{d\Omega(n_k)}{4\pi} \, \overline{W}_{ij}^k - 4 \sum_{(ij)} \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{j,R} \, \overline{W}_{ij}^k \, \Theta_{\text{hard}}(n_k)$$

Anomalous Dimension

- Decompose into soft emission, Glauber phase and collinear emission:

$$\boldsymbol{\Gamma}^H = \Gamma^C + \gamma_{\text{cusp}}(\alpha_s)\left(\boldsymbol{\Gamma}^c \ln\frac{\mu^2}{\mu_h^2} + \boldsymbol{V}^G\right) + \frac{\alpha_s}{4\pi}\,\overline{\boldsymbol{\Gamma}} + \mathcal{O}(\alpha_s^2)$$

- Important properties:

$$\left[\boldsymbol{\Gamma}^c, \overline{\boldsymbol{\Gamma}}\right] = 0\,, \quad \langle \boldsymbol{\mathcal{H}}_m\, \boldsymbol{\Gamma}^c \otimes \mathbf{1}\rangle = 0\,, \quad \langle \boldsymbol{\mathcal{H}}_m\, \boldsymbol{V}^G \otimes \mathbf{1}\rangle = 0$$

(color-coherence) (collinear safety) (cyclicity of trace)

- rightmost structure in color-trace must be $\boldsymbol{V}^G\overline{\boldsymbol{\Gamma}}$.

SLLs in RG-Improved PT

- keep log-enhanced soft-collinear piece in exponential

$$\boldsymbol{U}_{\rm SLL}(\{\underline{n}\},\mu_h,\mu_s) = \int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\int_{\mu_s}^{\mu_1}\frac{d\mu_2}{\mu_2}\int_{\mu_s}^{\mu_2}\frac{d\mu_3}{\mu_3}$$
$$\times\, \boldsymbol{U}_c(\mu_h,\mu_1)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_1)\big)\,\boldsymbol{V}^G\,\boldsymbol{U}_c(\mu_1,\mu_2)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_2)\big)\,\boldsymbol{V}^G\,\frac{\alpha_s(\mu_3)}{4\pi}\,\overline{\boldsymbol{\Gamma}}$$

- Sudakov-like evolution operator resums double-log's

$$\boldsymbol{U}_c(\mu_i,\mu_j) = \exp\left[\boldsymbol{\Gamma}^c\int_{\mu_j}^{\mu_i}\frac{d\mu}{\mu}\,\gamma_{\rm cusp}\big(\alpha_s(\mu)\big)\ln\frac{\mu^2}{\mu_h^2}\right].$$

- One scale integral for each insertion of $\boldsymbol{V}^G$ and $\overline{\boldsymbol{\Gamma}}$.
- Running coupling implemented.

Color basis for Quark-initiated Processes

- 5 color structures: (20 for gg, 14 for qg)

$$\boldsymbol{X}_1 = \sum_{j>2} J_j \, i \, f^{abc} \, \boldsymbol{T}_1^a \boldsymbol{T}_2^b \, \boldsymbol{T}_j^c \,, \qquad \boldsymbol{X}_4 = \frac{1}{N_c} \, J_{12} \, \boldsymbol{T}_1 \cdot \boldsymbol{T}_2 \,,$$

$$\boldsymbol{X}_2 = \sum_{j>2} J_j \, (\sigma_1 - \sigma_2) \, d^{abc} \, \boldsymbol{T}_1^a \boldsymbol{T}_2^b \boldsymbol{T}_j^c \,, \qquad \boldsymbol{X}_5 = J_{12} \, \boldsymbol{1} \,.$$

$$\boldsymbol{X}_3 = \frac{1}{N_c} \sum_{j>2} J_j \, (\boldsymbol{T}_1 - \boldsymbol{T}_2) \cdot \boldsymbol{T}_j \,,$$

- Matrix representation of elements of anomalous dimension

$$\boldsymbol{\Gamma}^c \to N_c \, \mathbb{\Gamma}^c \,, \quad \boldsymbol{V}^G \to i\pi N_c \, \mathbb{V}^G \,, \quad \boldsymbol{V}^G \, \overline{\boldsymbol{\Gamma}} \otimes \boldsymbol{1} \to 16 i\pi \, \boldsymbol{X}_1 = 16 i\pi \, \boldsymbol{X}^T \, \varsigma$$

$$\mathbb{\Gamma}^c = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -\frac{C_F}{N_c} & 0 & 0 \end{pmatrix} , \qquad \mathbb{V}^G = \begin{pmatrix} 0 & -2\,\delta_{q\bar{q}} & \frac{N_c^2-4}{N_c^2} & \frac{4}{N_c^2} & 0 & 0 \\ -\frac{1}{2} & 0 & & 0 & 0 & 0 \\ 1 & 0 & & 0 & 0 & 0 \\ 0 & 0 & & 0 & 0 & 0 \\ 0 & 0 & & 0 & 0 & 0 \end{pmatrix}$$

Representation of the evolution operator

- $\boldsymbol{\Gamma}^c$ appears inside the Sudakov-like matrix exponential

$$\mathbb{U}_c(\mu_i,\mu_j) = \exp\left[N_c\,\mathbb{\Gamma}^c \int_{\mu_j}^{\mu_i} \frac{d\mu}{\mu}\,\gamma_{\rm cusp}\big(\alpha_s(\mu)\big)\ln\frac{\mu^2}{\mu_h^2}\right].$$

- The exponential can be performed to yield

$$\mathbb{U}_c(\mu_i,\mu_j) = \begin{pmatrix} U_c(1;\mu_i,\mu_j) & 0 & 0 & 0 & 0 \\ 0 & U_c(1;\mu_i,\mu_j) & 0 & 0 & 0 \\ 0 & 0 & U_c(\frac{1}{2};\mu_i,\mu_j) & 0 & 0 \\ 0 & 0 & 2\big[U_c(\frac{1}{2};\mu_i,\mu_j) - U_c(1;\mu_i,\mu_j)\big] & U_c(1;\mu_i,\mu_j) & 0 \\ 0 & 0 & \frac{2\,C_F}{N_c}\big[1 - U_c(\frac{1}{2};\mu_i,\mu_j)\big] & 0 & 1 \end{pmatrix}$$

with the scalar Sudakov factor

$$U_c(v;\mu_i,\mu_j) = \exp\left[v\,N_c \int_{\mu_j}^{\mu_i} \frac{d\mu}{\mu}\,\gamma_{\rm cusp}\big(\alpha_s(\mu)\big)\ln\frac{\mu^2}{\mu_h^2}\right]$$

LO in RG-improved perturbation theory

- The cross section can be expressed as a matrix product

$$\hat{\sigma}_{2\to M}^{\mathrm{SLL+G}}(Q_0) = \sum_{l=1}^{\infty} \left\langle \boldsymbol{\mathcal{H}}_{2\to M}(\mu_h)\, \boldsymbol{X}^T \right\rangle \mathbb{U}_{\mathrm{SLL}}^{(l)}(\mu_h, \mu_s)\; \varsigma$$

with the evolution operator

$$\mathbb{U}_{\mathrm{SLL}}^{(l)}(\mu_h, \mu_s) = 16\,(i\pi)^l\, N_c^{l-1} \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \dots \int_{\mu_s}^{\mu_l} \frac{d\mu_{l+1}}{\mu_{l+1}}\, \mathbb{U}_c(\mu_h, \mu_1)$$
$$\times \left[\prod_{i=1}^{l-1} \gamma_{\mathrm{cusp}}(\alpha_s(\mu_i))\, \mathbb{V}^G\, \mathbb{U}_c(\mu_i, \mu_{i+1})\right] \gamma_{\mathrm{cusp}}(\alpha_s(\mu_l))\, \frac{\alpha_s(\mu_{l+1})}{4\pi}$$

- Two-loop approx. in exponent ($x_i = \alpha_s(\mu_i)/\alpha_s(\mu_h)$), one-loop elsewhere

$$U_c(v; \mu_i, \mu_j) = \exp\left\{ \frac{\gamma_0\, v N_c}{2\beta_0^2} \left[\frac{4\pi}{\alpha_s(\mu_h)} \left(\frac{1}{x_i} - \frac{1}{x_j} - \ln\frac{x_j}{x_i} \right) + \left(\frac{\gamma_1}{\gamma_0} - \frac{\beta_1}{\beta_0} \right) \left(x_i - x_j + \ln\frac{x_j}{x_i} \right) + \frac{\beta_1}{2\beta_0} \left(\ln^2 x_j - \ln^2 x_i \right) \right] \right\},$$

LO in RG-improved perturbation theory

- Process-independent evolution functions with two $\boldsymbol{V}^G$ insertions:

$$\mathbb{U}_{\rm SLL}^{(2)}(\mu_h,\mu_s)\,\varsigma = -\frac{32\pi^2}{\beta_0^3}\,N_c\int_1^{x_s}\frac{dx_2}{x_2}\,\ln\frac{x_s}{x_2}\int_1^{x_2}\frac{dx_1}{x_1}$$

$$\times\begin{pmatrix} 0 \\ -\frac{1}{2}\,U_c(1;\mu_h,\mu_2) \\ U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2) \\ 2\left[U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2)-U_c(1;\mu_h,\mu_2)\right] \\ \frac{2C_F}{N_c}\left[U_c(1;\mu_1,\mu_2)-U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2)\right]\end{pmatrix}$$

- contains products of evolution factors with different eigenvalues
- Two remaining scale integrals required to resum SLLs

Asymptotic Behaviour

- Asymptotic expansion for $\alpha_s L \sim 1, \alpha_s L^2 \gg 1$ (restricting to a fixed coupling)

$$\mathbb{U}^{(2)}_{\rm SLL}(\mu_h,\mu_s)\,\varsigma = -\frac{2\pi^2}{3}\,N_c\left(\frac{\alpha_s}{\pi}\,L_s\right)^3 \begin{pmatrix} 0 \\ -\frac{1}{2}\,\Sigma(1,1;w) \\ \Sigma(\frac{1}{2},1;w) \\ 2\left[\Sigma(\frac{1}{2},1;w)-\Sigma(1,1;w)\right] \\ \frac{2C_F}{N_c}\left[\Sigma(0,1;w)-\Sigma(\frac{1}{2},1;w)\right] \end{pmatrix}$$

- Expansions extracted from a method-of-regions analysis $\quad (w = \frac{N_c\alpha_s}{\pi}L^2)$

$$\Sigma(1,1;w) = \frac{3}{w} - \frac{3\sqrt{\pi}}{2\,w^{3/2}} + \mathcal{O}(e^{-w})\,,$$

$$\Sigma(\tfrac{1}{2},1;w) = \frac{3\sqrt{2}\,\ln(1+\sqrt{2})}{w} - \frac{3\sqrt{\pi}}{\sqrt{2}\,w^{3/2}} + \mathcal{O}(e^{-\frac{w}{2}})\,,$$

$$\Sigma(0,1;w) = \frac{3}{2}\,\frac{\ln(4w)+\gamma_E-2}{w} + \frac{3}{4\,w^2} + \mathcal{O}(w^{-3})\,.$$

- parametric suppression for higher Glauber phases

$$\Sigma(v^{(1)},\dots,v^{(l)};w) \sim w^{-l/2}$$

Large N_c

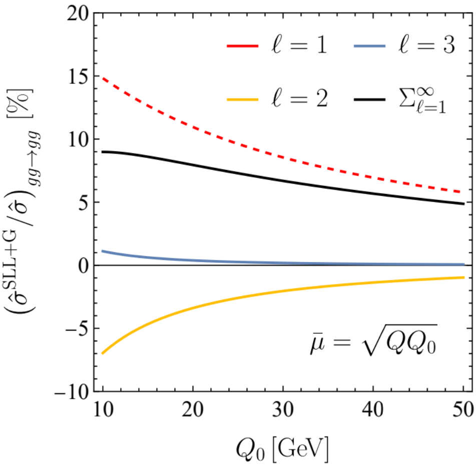
- Matrix structures simplify, such that all Glauber exchanges can be resummed in terms of a two-fold scale integral

$$\sum_{\ell=1}^{\infty} \mathbb{U}_{\text{SLL}}^{(2\ell-1)}(\mu_h, \mu_s)\, \varsigma = \frac{16i\pi}{\beta_0^2} \int_1^{x_s} \frac{dx_2}{x_2} \ln\frac{x_s}{x_2}\, U_c(1;\mu_h,\mu_2) \left[1 - 2\delta_{q\bar{q}} \sin^2\left(\frac{\pi N_c}{\beta_0} \ln x_2\right)\right] \varsigma\,,$$

$$\sum_{\ell=1}^{\infty} \mathbb{U}_{\text{SLL}}^{(2\ell)}(\mu_h, \mu_s)\, \varsigma = \frac{16\pi}{\beta_0^2} \frac{2\pi N_c}{\beta_0} \int_1^{x_s} \frac{dx_2}{x_2} \ln\frac{x_s}{x_2} \int_1^{x_2} \frac{dx_1}{x_1} \left[1 - 2\delta_{q\bar{q}} \sin^2\left(\frac{\pi N_c}{\beta_0} \ln\frac{x_2}{x_1}\right)\right]$$

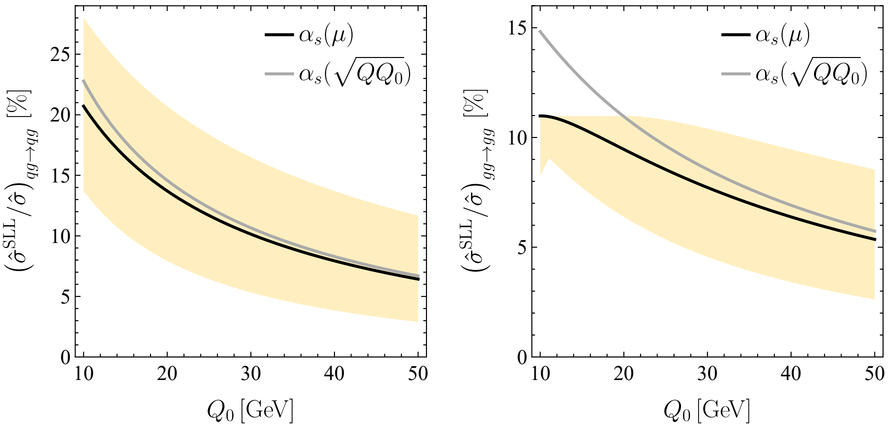
$$\times \begin{pmatrix} 0 \\ \frac{1}{2} U_c(1;\mu_h,\mu_2) \\ -U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2) \\ 2\left[U_c(1;\mu_h,\mu_2) - U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2)\right] \\ \frac{2C_F}{N_c}\left[U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2) - U_c(1;\mu_1,\mu_2)\right] \end{pmatrix}$$

Numerical Estimates



- $Q = 1\text{TeV}$ and rapidity gap $\Delta Y = 2$.

Numerical Estimates



- $Q = 1\text{TeV}$ and rapidity gap $\Delta Y = 2$.
- Error band from scale variation $\mu_s \in (\frac{Q_0}{2}, 2Q_0)$.

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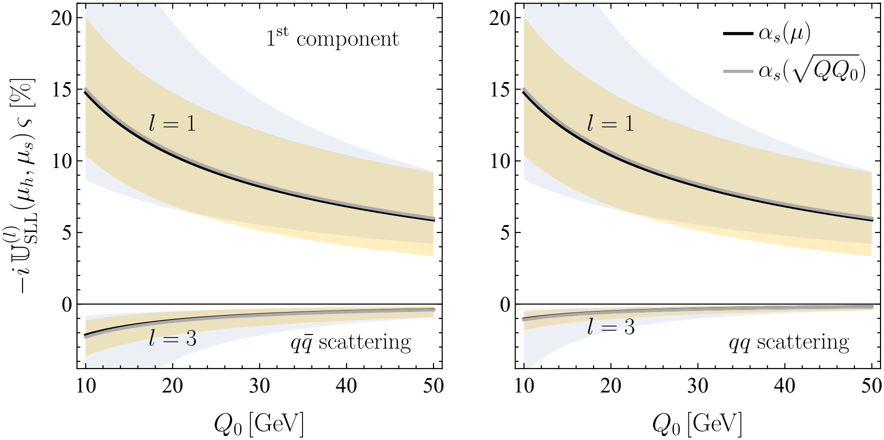


- $Q = 1$TeV and rapidity gap $\Delta Y = 2$.
- Error band from scale variation $\mu_s \in (\frac{Q_0}{2}, 2Q_0)$.

Conclusion

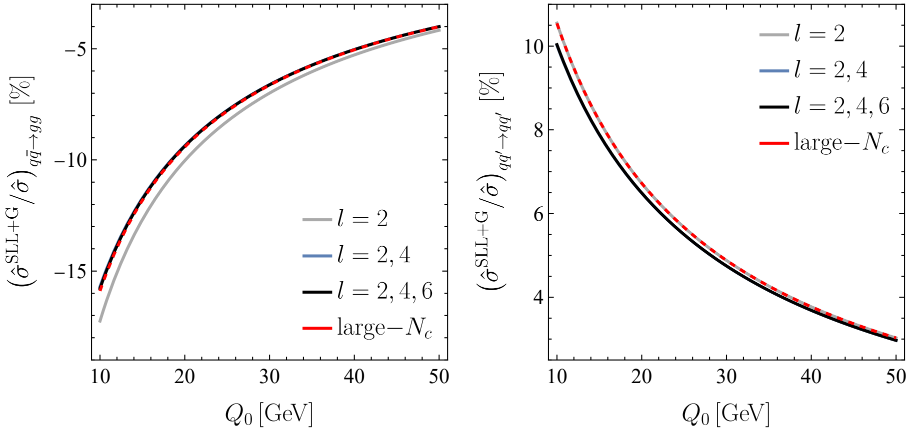
- Resummation of SLLs lifted to standard language of RG-improved PT
  - SLLs are integrals over Sudakov factors
  - Estimate of pert. uncertainties through scale variation
- Asymptotic scaling shows parametric suppression of higher Glauber exchanges
- Closed-form expression for entire Glauber series at large N_c

Outlook:

- How large is the impact beyond the partonic level?
- How to push the formalism to include double (multiple) soft emissions?
- Factorization properties of low-energy matrix element?
  - Is double-logarithmic evolution/SLLs compatible with PDF factorization?
  - "Factorization restoration through Glauber gluons" [Becher et al. '24]

Backup-Slides

Large N_c comparison



Large N_c comparison