Five-parton scattering in the high-energy limit

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based on results from [arXiv:2311.09870]: B. Agarwal, FB, F. Devoto, G. Gambuti, A. von Manteuffel and L. Tancredi

+ ongoing work in collaboration with: F. Caola, F. Devoto and G. Gambuti

Outline of the talk

Warm up: 2→2

- gluon reggeisation in QCD
- Regge-pole factorisation at NLL and violation at NNLL (multi-reggeon exchanges)

Multi-Regge Kinematics (MRK): 2→3

- Central-emission vertex
- Regge-pole factorisation at NLL + violation at NNLL

Two-loop 5pt QCD amplitudes in MRK and EFT for MR

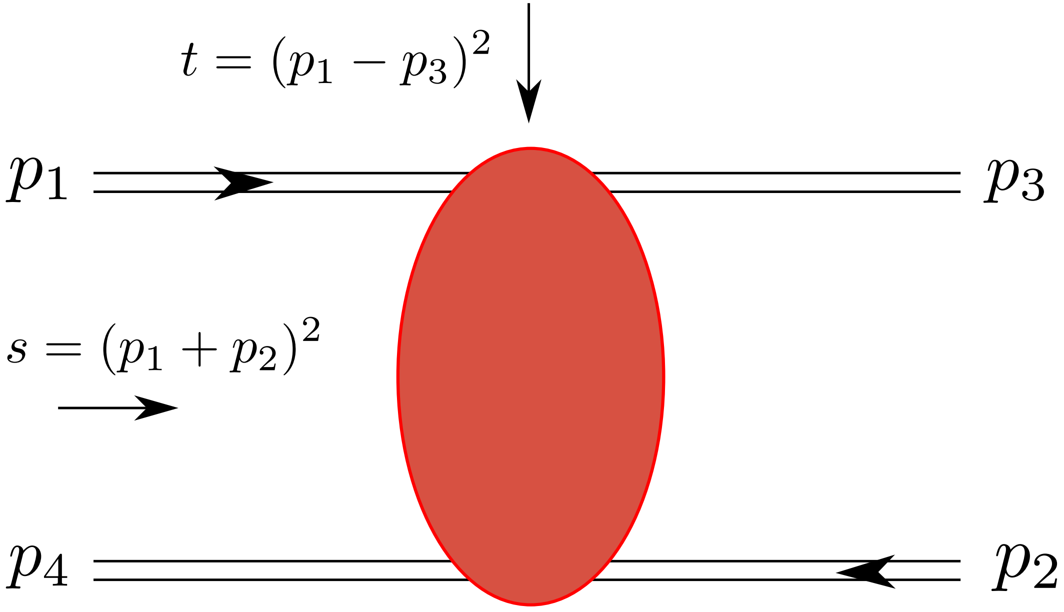
- expansion of two-loop full colour QCD results
- quick description of Wilson lines approach + rapidity evolution

Results and checks

- Regge-pole factorisation at NNLL (universality)

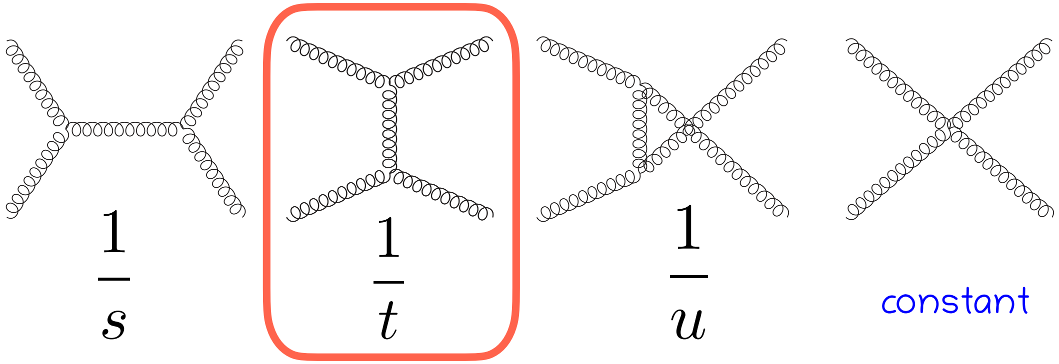
The high-energy limit, aka Regge kinematics

Kinematics: $s \sim |u| \gg -t \qquad x = -t/s$

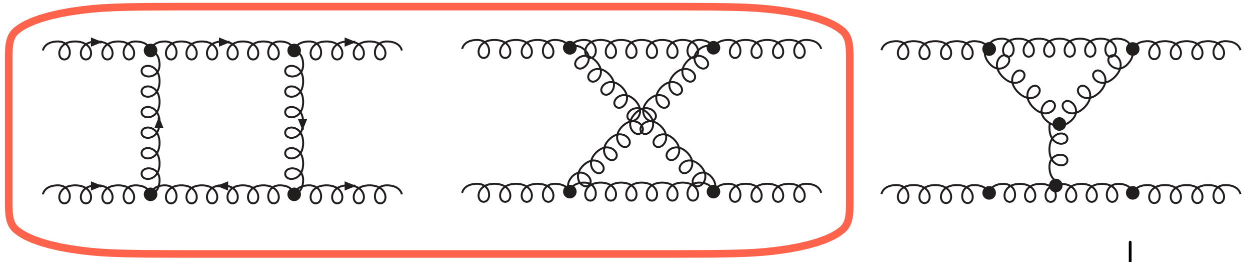


Large rapidity gap: $y \sim \frac{1}{2} \log \frac{s}{-t}$

four gluons at tree-level:

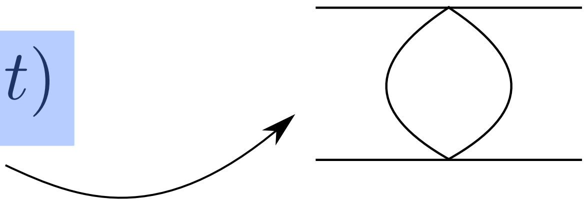


Loop corrections are more interesting:



$+\ldots$

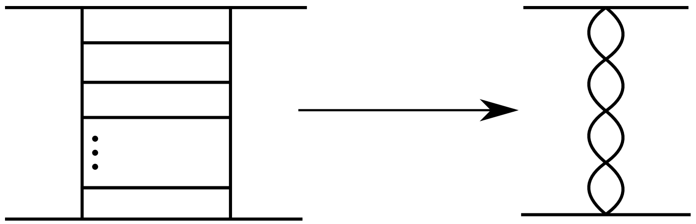
$\mathcal{A} \sim \frac{\alpha_s}{\pi} \frac{\ln x}{x} \tau_g(t)$



Gluon reggeisation and LL factorisation

To all-orders: the structure is repeated. Dominant contribution from (generalised) ladder diagrams

$$\text{LL} \sim \left(\frac{\alpha_s}{2\pi} \ln x\right)^n$$



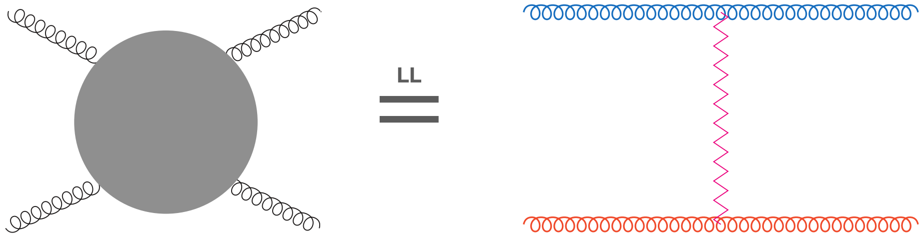
NB: This picture is schematic, in QCD exchange of generalised ladders

Exponentiation

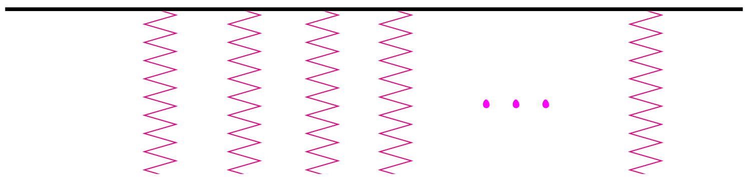
$$\frac{s}{-t} \to \frac{s}{-t}\left(\frac{s}{-t}\right)^{C_A \alpha_s \tau_g}$$

τ_g: regge trajectory

Gluon "reggeisation" at LL



Reggeon ~ QCD reggeised gluon



effectively, theory with "reggeons" as dof

Amplitude factorisation at LL:

$$\mathcal{A}(s,t) \simeq \mathcal{A}^0(s,t)\left(\frac{s}{-t}\right)^{C_A \alpha_s \tau_g(t)}$$

Factorisation ~ universality:
same form holds regardless of the partonic nature of projectiles (gg, qg, qq scattering)

Regge-pole factorisation at NLL

For discussion see e.g.
[Caron-Huot, Gardi, Vernazza 1701.05241]

Regge-pole factorisation still holds at NLL
[Fadin, Lipatov hep-ph/9802290]

$$\text{LL} \sim \left(\frac{\alpha_s}{2\pi}\ln x\right)^n$$

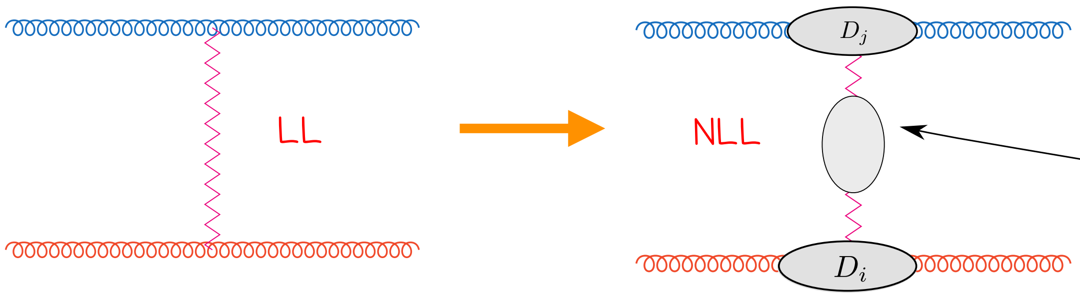
$$\text{NLL} \sim \frac{\alpha_s}{2\pi}\left(\frac{\alpha_s}{2\pi}\ln x\right)^n$$

$$\mathcal{A}(s,t) = \frac{\mathcal{A}(s,t)-\mathcal{A}(u,t)}{2} + \frac{\mathcal{A}(s,t)+\mathcal{A}(u,t)}{2}$$

$\mathcal{A}^{(-)}(s,t)$ $\qquad$ $\mathcal{A}^{(+)}(s,t)$

u = -s -t
in Regge kinematics u ~ -s

$$L = \ln\frac{s}{-t} - i\frac{\pi}{2}$$



$$\mathcal{A}_(s,t) \simeq \mathcal{A}^0(s,t)\left(\frac{s}{-t}\right)^{C_A\alpha_s\tau_g(t)}$$

$$\mathcal{A}^{(-)}(s,t) \simeq \mathcal{A}^0(s,t)\,D_i(t)D_j(t)\left(\frac{s}{-t}\right)^{C_A\alpha_s\tau_g(t,\alpha_s)}$$

impact factors

depend on the
partonic species

radiative corrections to
the regge trajectory

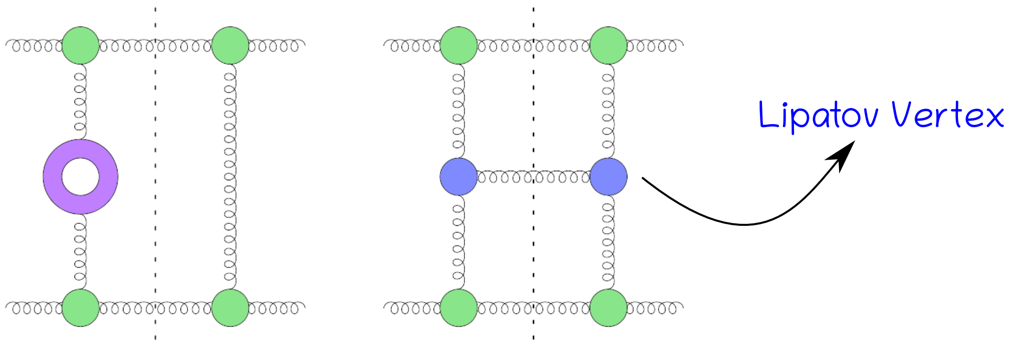
Ingredients:

- two-loop corrections to regge trajectory
- one-loop corrections to impact factors

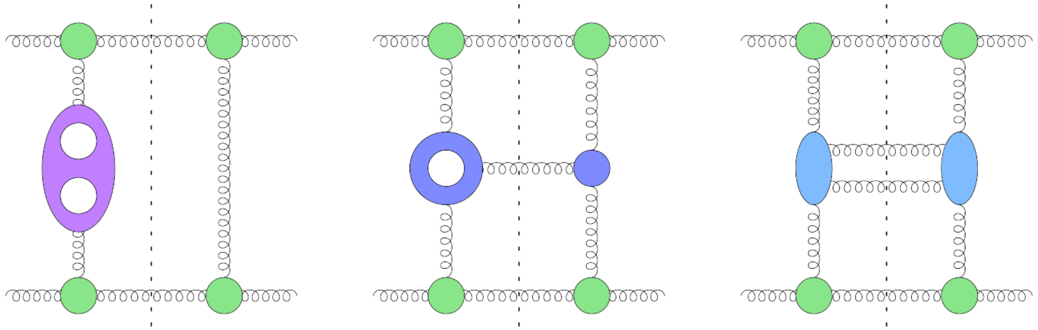
BFKL evolution equation (ingredients beyond NLL)

Consider dijet production at large rapidity gap, i.e. $|\Delta y| >> 1$, $\Delta y \sim \ln(s/-t)$

LL



NLL



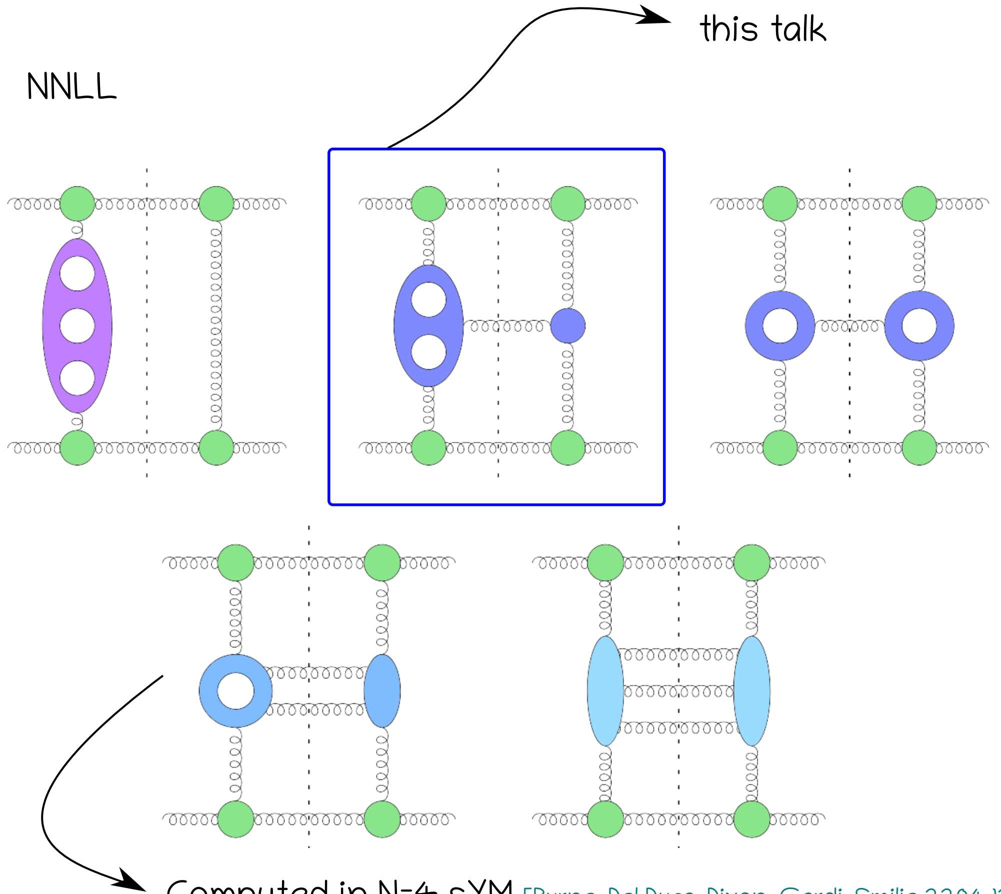
Artwork courtesy of:

[Byrne, Del Duca, Dixon, Gardi, Smilie 2204.12459]

BFKL equation resums large logarithms $\ln(s/-t)$:

BFKL kernel known up to NLL, thus resummation up to NLL

NNLL



Computed in N=4 sYM [Byrne, Del Duca, Dixon, Gardi, Smilie 2204.12459] [Byrne 2312.15051]

Result in QCD missing

Breaking of Regge-pole factorisation at NNLL

Simple factorisation is violated at NNLL [Del Duca, Glover hep-ph/0109028, Del Duca, Falcioni, Magnea, Vernazza 1409.8330]

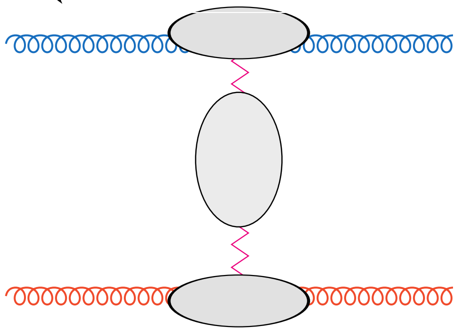
$$\text{LL} \sim \left(\frac{\alpha_s}{2\pi}\ln x\right)^n$$

$$\text{NLL} \sim \frac{\alpha_s}{2\pi}\left(\frac{\alpha_s}{2\pi}\ln x\right)^n$$

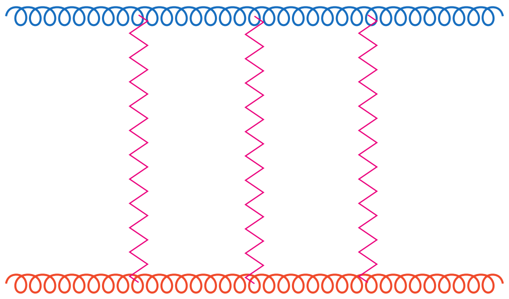
$$\text{NNLL} \sim \left(\frac{\alpha_s}{2\pi}\right)^2\left(\frac{\alpha_s}{2\pi}\ln x\right)^n$$

$$\mathcal{A}(s,t) \overset{\text{NNLL}}{\simeq} \mathcal{A}^0(s,t)\, D_i(t) D_j(t) \left(\frac{s}{-t}\right)^{C_A \alpha_s \tau_g(t,\alpha_s)} + \text{Multi-Reggeon Exchange}$$

Single Reggeon exchange



Regge poles vs Regge cuts



responsible for violation of factorisation

However:
after removing the cut contamination
Regge-pole factorisation is recovered

How to disentangle poles/cuts:

- Wilson-line approach [Balitsky/JIMWLK + Caron-Huot 1309.6521, Caron-Huot, Gardi, Vernazza 1701.05241]
- NNLL is understood via the exchange of 1 or 3 reggeons
- Non-planar part of multi-reggeon exchange → Regge cut
  the planar part of multi-Reggeon → Regge pole [Falcioni et al 2112.11098]

Multi-Regge Kinematics (MRK)

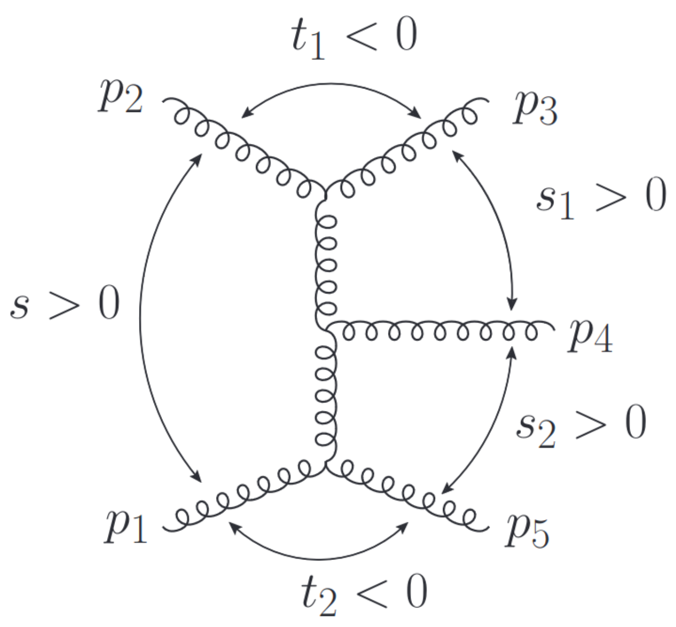
A,A' and B,B' same partonic flavour, $g(p_4)$ centrally-emitted gluon

$$A^{(h_A)}(p_1)\; B^{(h_B)}(p_2) \to B'^{(h_{B'})}(p_3)\; g^{(h_g)}(p_4)\; A'^{(h_{A'})}(p_5)$$

p_1 and p_2 define two light-cone components >> transverse ones

$$q^\mu = q^+ n_1^\mu + q^- n_2^\mu + q_\perp\, n^\mu + \bar{q}_\perp\, \bar{n}^\mu$$

Dynamics decouples for longitudinal and transverse degrees of freedom



[Chicherin, Henn, Caron-Huot, Zhang, Zoia 2003.03120]

Definition of MRK:
strong rapidity ordering at commensurate transverse components << longitudinal

$$p_5^+ \gg p_4^+ \gg p_3^+\,, \quad p_3^- \gg p_4^- \gg p_5^- \qquad p_4^\pm \sim |p_{3,\perp}| \sim |p_{4,\perp}| \sim |p_{5,\perp}|$$

Introduce scaling parameter x for longitudinal and transverse components (MRK limit for x→0)

$$p_1^+, p_5^+, p_2^-, p_3^- \sim 1/x \qquad p_4^+, p_4^- \sim 1 \qquad p_2^+, p_3^+ \sim x$$

Independent kinematic invariants (MRK parametrisation)

$$\boldsymbol{s} = \{s_{12}, s_{12}, s_{23}, s_{45}, s_{51}\}$$

[Chicherin, Henn, Caron-Huot, Zhang, Zoia 2003.03120]

$$s_{12} = \frac{s}{x^2}, \quad s_{23} = -\frac{s_1 s_2}{s} z\bar{z}, \quad s_{34} = \frac{s_1}{x},$$
$$s_{45} = \frac{s_2}{x}, \quad s_{51} = -\frac{s_1 s_2}{s}(1-z)(1-\bar{z})$$

Signature and colour in MRK

Only well-defined signature amplitude have pole/cut contributions:

$$\mathcal{A}^{(\sigma_a,\sigma_b)}(p_1,p_2,p_3,p_4,p_5) = \mathcal{A}(p_1,p_2,p_3,p_4,p_5) + \sigma_a\mathcal{A}(p_5,p_2,p_3,p_4,p_1) + \sigma_b\mathcal{A}(p_1,p_3,p_2,p_4,p_5) + \sigma_a\sigma_b\mathcal{A}(p_5,p_3,p_2,p_4,p_1)$$

A(-,-) odd,odd

A(-,+) odd,even

A(+,-) even, odd

A(+,+) even, even

A(-,-) receives pole and cuts contributions → idea: subtract cuts from A(-,-)

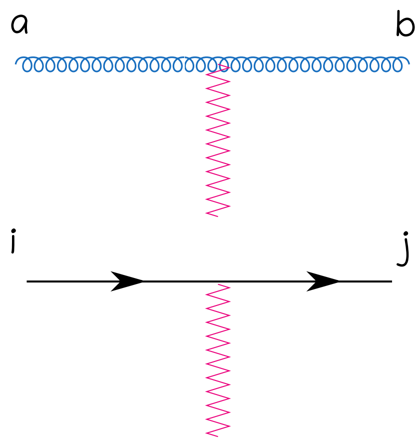
A(-,+), A(+,-) and A(+,+) receive only cuts contributions

Colour: choose a basis which respects the symmetry properties

$$(\mathbf{T}_1+\mathbf{T}_5)^2 \quad (\mathbf{T}_2+\mathbf{T}_3)^2$$

t-channel exchanges in irreducible representations of SU(N) (i.e. eigenstates of colour insertions)

label colour element (r_1,r_2)



$$8 \otimes 8 = \mathbf{0} + \mathbf{1} + \mathbf{8}_a + \mathbf{8}_s + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{27}$$

imaginary symmetric

real antisymmetric

only amplitude (8a,8a) receives pole contribution

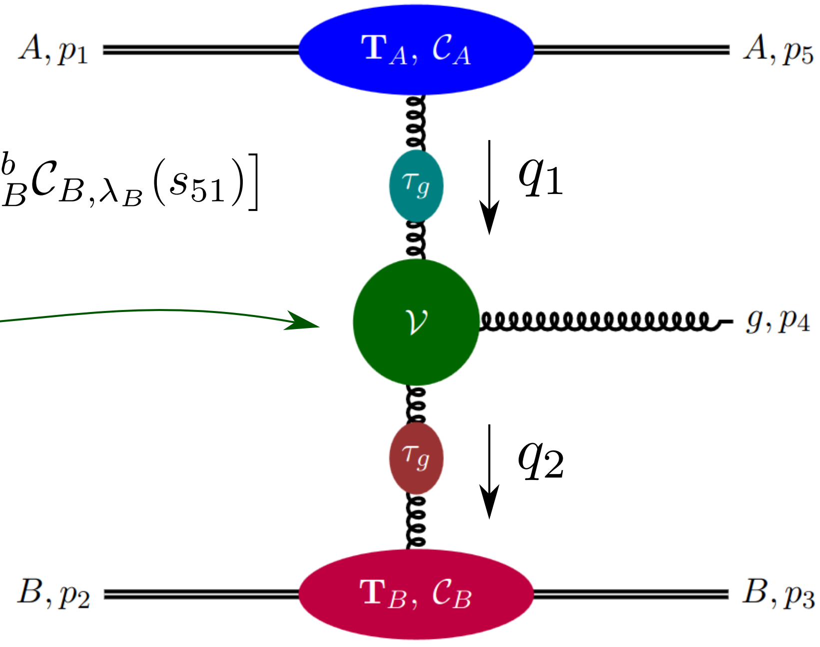
$$3 \otimes \bar{3} = \mathbf{1} + \mathbf{8}$$

the symmetry of the colour element dictates the pole/cut contribution

Regge-pole factorisation in 2→3 at NLL

Regge-pole factorisation for 2→3 process still holds at NLL for A(-,-)

$$\mathcal{A}^{AB}_{\lambda}(\mathbf{s}) = s_{12}\left[\mathbf{T}^a_A \mathcal{C}_{A,\lambda_A}(s_{51})\right] \frac{\mathcal{R}(s_{45},s_{51})}{s_{51}} \left[f^{aba_4}\, \mathcal{V}_{\lambda_g}(k_\perp, \mathbf{q}_1, \mathbf{q}_2)\right] \frac{\mathcal{R}(s_{34},s_{23})}{s_{23}} \left[\mathbf{T}^b_B \mathcal{C}_{B,\lambda_B}(s_{51})\right]$$



central-emission vertex (CEV) aka Lipatov's effective vertex

reggeon-reggeon-gluon (RRG) vertex

$$\mathcal{V}^{(0)}_{\lambda_g=+1} = \frac{q^*_{1,\perp} q_{2,\perp}}{k_\perp}$$

$$\mathcal{R}(s,t) = \frac{1}{2}\left[\left(\frac{s}{-t}\right)^{\alpha_s N_c \tau_g(t)} + \left(\frac{-s}{-t}\right)^{\alpha_s N_c \tau_g(t)}\right]$$

State-of-the-art:

- Known at 1-loop since long [Del Duca, Schmidt hep-ph/9810215]
- Recently computed at 1-loop to $O(\varepsilon^2)$ [Fadin, Fucilla, Papa 2302.09868]

Direct calculation: unitarity-cuts structure in MRK + diagrammatic approach

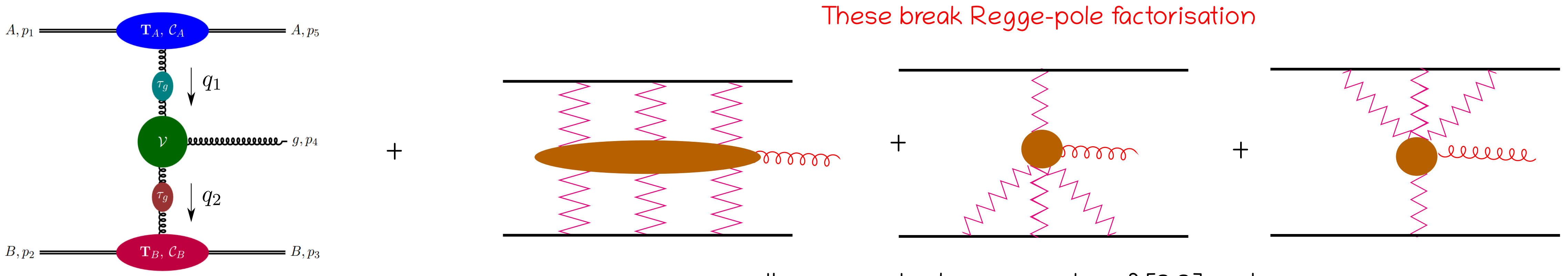
Extracted from 1-loop 5pt amplitudes

Regge-pole factorisation at NLL: amplitudes in MRK and subract 2-loop regge trajectory and impact factors

Regge-pole + Multi-Reggeon (MR) exchanges at NNLL

Regge-pole factorisation broken at NNLL for A(-,-):

$$\mathcal{A}_{\lambda}^{AB}(\mathbf{s}) = s_{12}\,[\mathbf{T}_A^a \mathcal{C}_{A,\lambda_A}(s_{51})]\,\frac{\mathcal{R}(s_{45},s_{51})}{s_{51}}\,[f^{aba_4}\,\mathcal{V}_{\lambda_g}(k_\perp,\mathbf{q}_1,\mathbf{q}_2)]\,\frac{\mathcal{R}(s_{34},s_{23})}{s_{23}}\,[\mathbf{T}_B^b \mathcal{C}_{B,\lambda_B}(s_{51})] + \text{Multi-Reggeon exchanges}$$



they respect colour symmetry of [8,8] exchange

Our goal: recover/show Regge-pole factorisation at NNLL + extract 2-loop CEV

Our strategy:

1. Expand 2-loop five-pt QCD amplitudes in MRK [Agarwal, FB, Devoto, Gambuti, von Manteuffel,Tancredi 2311.09870]
2. Use an effective theory that allows for the calculation/prediction of MR exchanges [Caron-Huot 1309.6521]

✔ 3-loop Regge-Trajectory
[Caola et al 2112.11097, Falcioni et al 2112.11098]

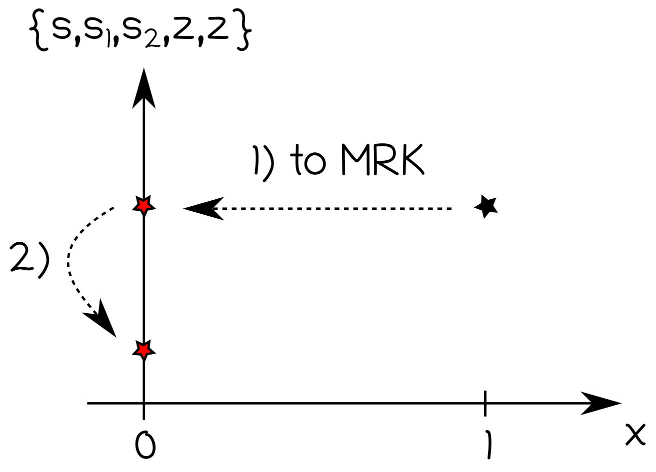
✔ 2-loop impact factors
[taken from Caola et al 2112.11097]

Expansion of two-loop amplitudes in MRK

- Amplitude is a product of rational (s_{ij}) and transcendental functions (pentagon functions)
- $\{s_{12}, s_{23}, s_{34}, s_{45}, s_{51}\} \to (s, s_1, s_2, z, \bar{z}) + x\}$ x is a scaling: x~1 "physical point", x→0 MRK
- Expanding rational functions in x straightforward but tedious (large polynomials + "spurious poles")

  Final result: keep only leading power (LP) in x, i.e. x^0 ($1/x^2$ factorised in LO)

  Intermediate expansions: x^{-k}, with $k \leq 2$ → need to cancel against trans. functions
  → require pentagon functs to NNLP
- Rotate amplitudes to suitable basis ("trace basis" → "MRK basis")



Pentagon functions:

- Solve differential equations in x as generalised power series (from diff. eqs. for canonical integrals) [Chicherin, Sotnikov 2009.07803]

$$f^{(w)}(\vec{s};x) = \sum_{n=0} \sum_{m=1}^{w} f^{(w)}_{mn}(\vec{s}) x^n \ln^m x$$

- Only thing needed is the "boundary" condition", i.e. the LP term x^0

  Solution for all 5-pt MIs (planar and non-planar) at LP → to any arbitrary order, up to w=4

Checked against
numerical evaluation in QP
up to N^4LP (excellent agreement)

Expansion of pentagon functions in MRK

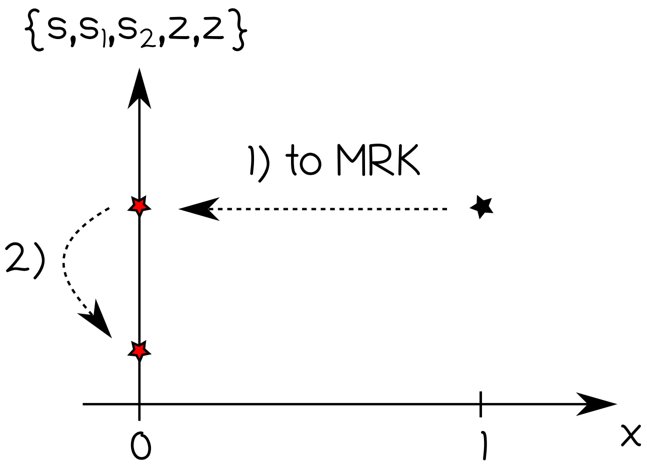
$$\mathrm{d}I_i(\vec{s}) = \epsilon\,\mathrm{d}A_{ij}(\vec{s})I_j(\vec{s}) \qquad \mathrm{d}A_{ij}(\vec{s}) = \sum_{n=1} a_{ij}^{n}\mathrm{d}\log(W_n)$$

$$W_n \to W_n(x)$$ [Caron-Huot, Chicherin, Henn, Zhang, Zoia 2003.03120]

For fixed $\{s_{ij}\} \sim y$, one gets a 1-d differential equation in x

$$\begin{cases} \dfrac{\partial}{\partial x}\vec{f}(x,y,\epsilon) = \epsilon A_x(x,y)\vec{f}(x,y,\epsilon) \\ \dfrac{\partial}{\partial y}\vec{f}(x,y,\epsilon) = \epsilon A_y(x,y)\vec{f}(x,y,\epsilon) \end{cases}$$

$$A_x(x,y) = \frac{A_0}{x} + \sum_{k\geq 0} x^k A_{k+1}(y)$$



$$\vec{f}(x,y,\epsilon) = x^{\epsilon A_0}\,\mathbb{P}\exp\left[\epsilon\int_{y_0}^{y} A_y(0,y')dy'\right]\vec{g}_0(\epsilon)$$

pentagon functions at LP

Nice property of MRK at LP:
Gram-determinant a perfect square
all square-roots in letters rationalized

$$\Delta = \epsilon_5^2 \underset{x\to 0}{\sim} \frac{s_1^2 s_2^2 (z-\bar{z})^2}{x^4} + \mathcal{O}\left(\frac{1}{x^3}\right)$$

$$\{x\}\,,$$
$$\left\{\frac{s_1 s_2}{s}\right\}\,,$$
$$\{s_1, s_2, s_1 - s_2, s_1 + s_2\}\,,$$
$$\{z, \bar{z}, 1-z, 1-\bar{z}, z-\bar{z}, 1-z-\bar{z}\}$$

Alphabet in MRK much simpler than in full kinematics (35→12 letters)

B-JIMWLK rapidity evolution

At the core: use Balitsky-JIMWLK rapidity evolution equation + shockwave formalism [Caron-Huot 1309.6521, Caron-Huot, Gardi, Vernazza 1701.05241]

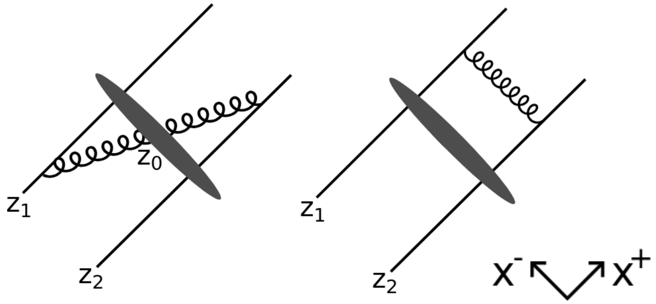
$$\mathcal{A} \simeq \langle T\left\{\mathcal{O}_1(\eta_1)\mathcal{O}_2(\eta_2)\ldots\mathcal{O}_n(\eta_n)\right\}\rangle$$

$O_i(\eta_i)$ are (composite) operators at rapidities $\eta_1 >> \eta_2 >> \ldots >> \eta_n$

Starting point: represent fast-moving particles via infinite Wilson lines

"compute scattering between Wilson lines"

$$U_r(\mathbf{z}) \equiv \mathbf{P}\exp\left\{ig_s\int_{-\infty}^{+\infty} dx^+ A^a_+(x^+, x^-=0, \mathbf{z})T^a_r\right\}$$



a) $O_1(\eta_1)$ represented as a product of Wilson lines then b) evolve this product from $\eta_1 \to \eta_2$

Balitsky-JIMWLK evolution equation

$$-\frac{d}{d\eta}U(\mathbf{z}_1)\ldots U(\mathbf{z}_n) = H\,U(\mathbf{z}_1)\ldots U(\mathbf{z}_n)$$

$$H = \frac{\alpha_s}{2\pi^2}\frac{\Gamma^2(1-\epsilon)}{\pi^{-2\epsilon}}\int[d\mathbf{z}_0][d\mathbf{z}_i][d\mathbf{z}_j]\frac{\mathbf{z}_{0i}\cdot\mathbf{z}_{0j}}{\left[\mathbf{z}_{0i}^2\mathbf{z}_{0j}^2\right]^{1-\epsilon}}\times$$
$$\left\{\left[T^a_{i,L}T^a_{j,L} + (L\leftrightarrow R)\right] - U^{ab}_{\rm adj}(z_0)\left[T^a_{i,L}T^b_{j,R} + (i\leftrightarrow j)\right]\right\}$$

non-linear evolution

c) OPE for operator products of the type [Caron-Huot 1309.6521]

$$[U_{\eta_2}\otimes\cdots\otimes U_{\eta_2}](\mathbf{p})a^{a_4}_\lambda(p_4)$$

$$U(\mathbf{p})a^a_\lambda(p_4) \sim -2g_s\int[d\mathbf{z}_1][d\mathbf{z}_2]e^{-i\mathbf{p}\cdot\mathbf{z}_1 - i\mathbf{p}_4\cdot\mathbf{z}_2}\left[U^{ab}_{\rm adj}(\mathbf{z}_2)\hat{T}^b_{1,R} - \hat{T}^a_{1,L}\right]U(\mathbf{z}_1)\times$$
$$\times\int[d\mathbf{k}]e^{i\mathbf{k}\cdot(\mathbf{z}_2-\mathbf{z}_1)}\frac{\varepsilon_\lambda\cdot\mathbf{k}}{\mathbf{k}^2}$$

we will need it with one Wilson line only

W field ~ "Reggeon": linearisation + perturbative expansion

"Reggeon field", linearisation of B/JIMWLK evolution equation and perturbative expansion

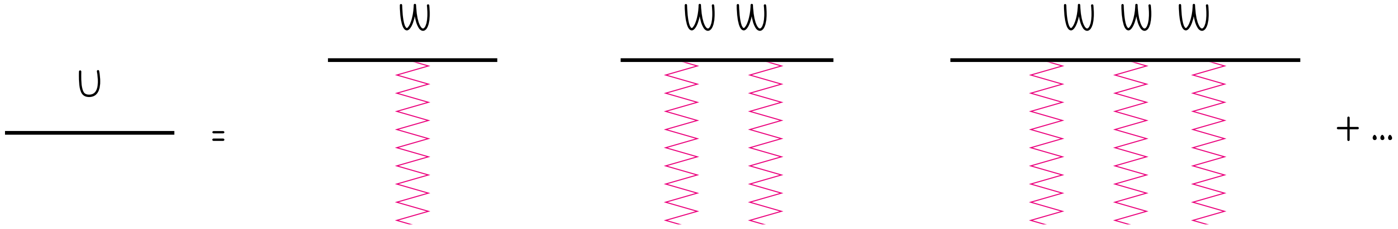
Consider weak-field limit, U close to identity, write:

$$U_r(\mathbf{z}) \equiv \exp\left\{ i g_s T_r^a W_r^a(\mathbf{z}) \right\}$$

expand perturbatively and work with Ws fields
(these are not reggeons)

Reformulate rapidity evolution + OPE in terms of W fields (+ Fourier transform in transverse momentum space)

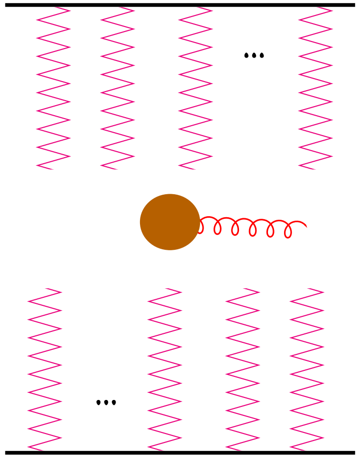
(pictorially) an external fast-moving particle expanded as:



(just a representation these are not "Feynman diagrams")

Single W field is an eigenstate of the linear rapidity evolution Hamiltonian (eigenvalue = Regge trajectory)

$$H_{W\to W} W^a(\eta,\mathbf{p}) = \alpha_s \tau_g^{(1)}(\mathbf{p}) W^a(\eta,\mathbf{p})$$



evolve state η_1 with WW...W

central gluon at η_2 + OPE ~ products of Ws

evolved to η_3 → Wick contractions of Ws define an inner product <W(q) W(p)>

OPE: W fields and central gluon

Sketch: single Reggeon OPE

$$W(\mathbf{p})^b\, a^a_\lambda(q) \sim$$

$$2g_s [\![W]\!]^{ab}(\mathbf{q}+\mathbf{p})\left[\frac{\varepsilon_\lambda\cdot\mathbf{p}}{\mathbf{p}^2}+\frac{\varepsilon_\lambda\cdot\mathbf{q}}{\mathbf{q}^2}\right]$$

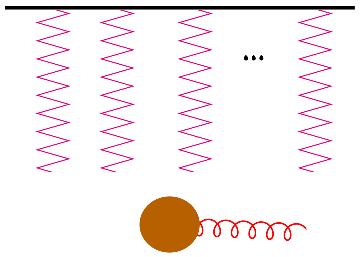
$$+\, i g_s^2 \int [d\mathbf{k}_1][\![W(\mathbf{q}+\mathbf{p}-\mathbf{k}_1)W(\mathbf{k}_1)]\!]^{ab}\left[\frac{\varepsilon_\lambda\cdot\mathbf{p}}{\mathbf{p}^2}+\frac{\varepsilon_\lambda\cdot(\mathbf{k}_1-\mathbf{p})}{(\mathbf{k}_1-\mathbf{p})^2}\right]$$

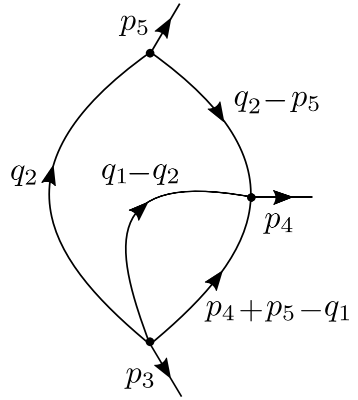
1 → 2

$$+\, g_s^3 \int [d\mathbf{k}_1][d\mathbf{k}_2][\![W(\mathbf{q}+\mathbf{p}-\mathbf{k}_1)W(\mathbf{k}_1-\mathbf{k}_2)W(\mathbf{k}_2)]\!]^{ab}$$
$$\times\left[\frac{1}{6}\left(\frac{\varepsilon_\lambda\cdot(\mathbf{k}_1-\mathbf{p})}{(\mathbf{k}_1-\mathbf{p})^2}\right)-\frac{1}{2}\left(\frac{\varepsilon_\lambda\cdot(\mathbf{k}_2-\mathbf{p})}{(\mathbf{k}_2-\mathbf{p})^2}\right)-\frac{1}{3}\left(\frac{\varepsilon_\lambda\cdot\mathbf{p}}{\mathbf{p}^2}\right)\right]$$

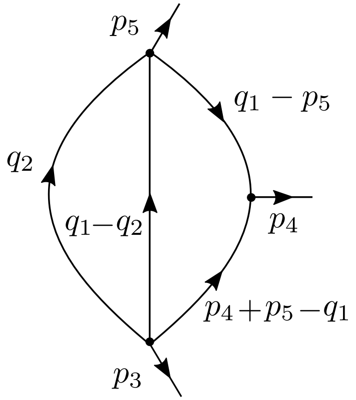
1 → 3

IMPORTANT:
ultimately we are interested in contributions to odd-odd amplitude





1 → 3



3 → 3

double Reggeon OPE

$$[W^b\otimes W^c](\mathbf{p})\, a^a_\lambda(q)\sim 2g_s\int[d\mathbf{k}_1][\![W(\mathbf{q}+\mathbf{p}-\mathbf{k}_1)]\!]^{ab}W^c(\mathbf{k}_1)\times$$
$$\times\left[\frac{\varepsilon_\lambda\cdot\mathbf{q}}{\mathbf{q}^2}-\frac{\varepsilon_\lambda\cdot(\mathbf{k}_1-\mathbf{p})}{(\mathbf{k}_1-\mathbf{p})^2}\right]+(b\leftrightarrow c)$$

2 → 2

triple Reggeon OPE

$$[W^b\otimes W^c\otimes W^d](\mathbf{p})\, a^a_\lambda(q)\sim 2g_s\int[d\mathbf{k}_1][d\mathbf{k}_2][\![W(\mathbf{q}+\mathbf{p}-\mathbf{k}_1)]\!]^{ab}W^c(\mathbf{k}_1-\mathbf{k}_2)W^d(\mathbf{k}_2)\times$$

3 → 3

$$\times\left[\frac{\varepsilon_\lambda\cdot\mathbf{q}}{\mathbf{q}^2}-\frac{\varepsilon_\lambda\cdot(\mathbf{k}_1-\mathbf{p})}{(\mathbf{k}_1-\mathbf{p})^2}\right]+(b\leftrightarrow c)+(b\leftrightarrow d)$$

Checks at 1-loop: Regge-pole + multi-Reggeon contributions

Regge-pole:

no MR exchanges in [8,8] at 1-loop (Regge-pole factorisation)

1-loop amplitudes available to $O(\varepsilon^2)$: extract CEV to same order

(after appropriate adjustments)
full agreement with recent calculation [Fadin, Fucilla, Papa 2302.09868]

✓ checks expansion of the amplitude at 1-loop

Regge-cuts:

Wilson-line approach predicts cuts of *all-amplitude* @1-loop

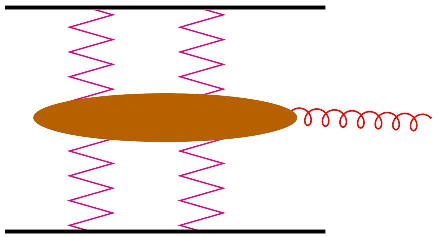
all colour structures → all signatures/symmetries

$$\mathbf{T}^{\pm}_{ij} = \mathbf{T}_i \pm \mathbf{T}_j$$

acting on tree-level

2→2 A(+,+)

$\mathbf{T}^{-}_{15} \cdot \mathbf{T}^{-}_{23}$



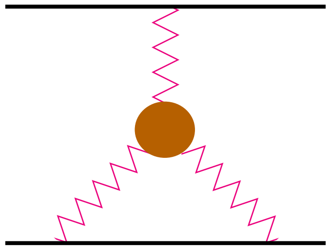
e.g.

gg: [8s,8s], [27,27], [0,0]

qg: $[8,8s]_a$

1→2 A(-,+)

$\mathbf{T}^{+}_{15} \cdot \mathbf{T}^{-}_{23}$



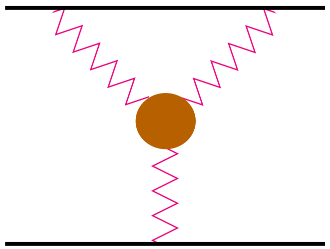
e.g.

gg: [8a,8s], [8a,1], [8a,27]

qg: [8,1], [8,27], [8,0]

2→1 A(-,+)

$\mathbf{T}^{-}_{15} \cdot \mathbf{T}^{+}_{23}$



full agreement with amplitude

✓ checks expansion in the effective theory

Checks at 2-loop: N=4 and multi-Reggeon contributions

Check vs N=4

extract finite remainder (IR subtraction) of QCD amplitudes

calculation in N=4 SYM [Caron-Huot, Chicherin, Henn, Zoia 2003.03120]

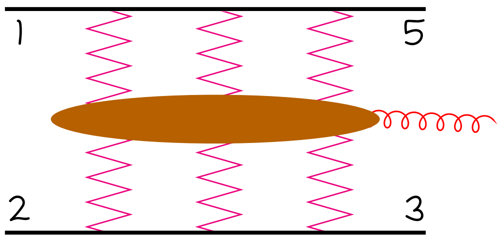
leading transcendentaliy QCD
full agreement with N=4

✓ checks expansion of the amplitude at 2-loop

Multi-reggeon contributions: 3→3, 1→3, 3→1

contributions in odd-odd colour structures (not [8a,8a]): isolated check on MR exchanges

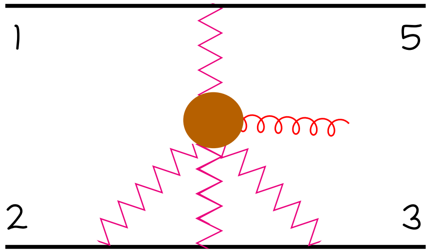
3→3 A(-,-)



e.g.

gg: [10+$\overline{10}$,10+$\overline{10}$]

1→3 A(-,-)



e.g.

gg: [8a,10+$\overline{10}$]

qg: [8,10+$\overline{10}$]

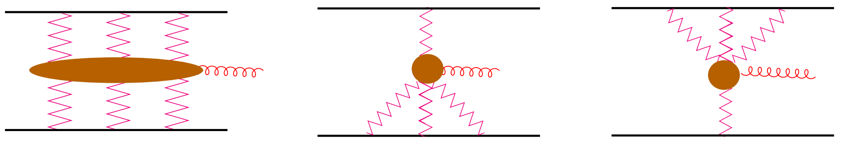
$$\mathcal{A}^{2l,\mathrm{MR}}_{qg,[10+\overline{10}]} = \left(\frac{\alpha_s}{4\pi}\right)^2 (i\pi)^2 \left(\frac{\mu^2}{k_\perp^2}\right)^{2\epsilon} \mathcal{R}(z,\bar{z},\epsilon)$$

$$\mathcal{R}(z,\bar{z},\epsilon) = N_c \Bigg(\frac{-1}{2\epsilon^2} + \frac{\log(z\bar{z}) - 2\log((1-z)(1-\bar{z}))}{\epsilon} + \frac{\zeta_2}{2} - 6iD_2(z,\bar{z}) + \frac{1}{2}\log^2((1-z)(1-\bar{z})) - \log(z\bar{z}) + \log^2(z\bar{z})\log((1-z)(1-\bar{z})) \Bigg)$$

full agreement with amplitude

✓ checks expansion in the effective theory

Results for multi-Reggeon exchange in [8,8]



$$\mathcal{A}^{2l,\mathrm{MR}}_{ab,[8,8]} = \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{i\pi}{3}\right)^2 \left(\frac{\mu^2}{k_\perp^2}\right)^{2\epsilon} \left[\mathcal{F}_{\mathrm{LC}}(z,\bar{z},\epsilon) + \mathcal{F}_{ab}(z,\bar{z},\epsilon)\right]$$

$$\mathcal{F}_{\mathrm{LC}}(z,\bar{z},\epsilon) = N_c^2 \left(\frac{2}{\epsilon^2} - \frac{\log(z\bar{z}) + \log((1-z)(1-\bar{z}))}{\epsilon} + 6iD_2(z,\bar{z}) - 2\zeta_2 + \frac{5}{2}\log^2(z\bar{z}) + \frac{5}{2}\log^2((1-z)(1-\bar{z})) - \log(z\bar{z})\log((1-z)(1-\bar{z})) \right)$$

Leading-colour universal → originating from planar contributions → Regge-pole contribution

$$\mathcal{F}_{qg}(z,\bar{z},\epsilon) = \frac{27}{\epsilon^2} + \frac{1}{\epsilon}\Big(54\log((1-z)(1-\bar{z})) - 36\log(z\bar{z})\Big) + 216iD_2(z,\bar{z}) - 27\zeta_2 + 45\log^2(z\bar{z}) - 36\log(z\bar{z})\log((1-z)(1-\bar{z}))$$

$$\mathcal{F}_{gg}(z,\bar{z},\epsilon) = \frac{72}{\epsilon^2} - \frac{36}{\epsilon}\Big(\log((1-z)(1-\bar{z})) + \log(z\bar{z})\Big) + 216iD_2(z,\bar{z}) - 72\zeta_2 + 90\log^2(z\bar{z}) + 90\log^2((1-z)(1-\bar{z})) - 36\log(z\bar{z})\log((1-z)(1-\bar{z}))$$

SLC: process dependent

$$D_2(z,\bar{z}) = -i\left(\frac{\log(z\bar{z})}{2}\left(\log(1-z) - \log(1-\bar{z})\right) + \mathrm{Li}_2(z) - \mathrm{Li}_2(\bar{z})\right)$$

Check against independent calculation [Abreu, Falcioni, Gardi, de Laurentis, Milloy, Vernazza, see PoS LL2024 (2024) 085]

Universality and Regge-pole factorisation at NNLL

MR contributions (3→3, 3→1 and 1→3) in [8,8]: leading-colour universal + sub-leading colour

part of the Regge pole

subtract these from [8,8] amplitude

Defines our scheme (for now)

Assume NLL-like Regge-pole factorisation:
trajectories + impact factors + CEV

(NEW)
2-loop
central-emission vertex

Universality: same CEV for all scattering channels (gg, qg, qq)

Regge-pole factorisation at NNLL

Features:

- ✓ SLC from MR contributions subtract "all" SLC that would enter the CEV: left with N_c^2, $N_f N_c$, N_f^2 and N_f/N_c (SLC but universal)
- ✗ Worked with UV renorm. amplitudes so far:
  spurious letter z-zb present in GPLs (associated with Gram)
  this should vanish

not exactly a surprise
one should look at
hard-function
(IR subtracted)

IR subtraction and finite CEV [preliminary]

how to subtract IR diverges at amplitude level is well understood [Catani 9802439, Becher, Neubert 0903.1126, Del Duca et al 1109.3581]

$$\mathbf{A}(\epsilon,\{p\},\mu) = \mathbf{Z}_{IR}(\epsilon,\{p\},\mu_{IR},\mu)\ \mathbf{H}(\epsilon,\{p\},\mu_{IR},\mu) \qquad \mathbf{Z}_{IR}(\epsilon,\{p\},\mu) = \mathbb{P}\exp\left[\int_\mu^\infty \frac{d\mu'}{\mu'}\mathbf{\Gamma}_{IR}(\{p\},\mu')\right]$$

we would like to define objects that are individually IR finite
(trajectory, impact factors, CEV)

$$\mathbf{\Gamma}_{IR}(\{p\},\mu) = \gamma_K(\alpha_s)\sum_{\substack{i,j=1\\ i>j}}^n \mathbf{T}_i\cdot\mathbf{T}_j \log\left(\frac{\mu^2}{-s_{ij}-i\delta}\right) + \sum_{i=1}^n \gamma_i(\alpha_s)$$

Expand Γ_{IR} in MRK limit and reorganise (freedom in Regge-fact. scale, here $\tau_A = s_{51}$, $\tau_B = s_{23}$)

$$\begin{aligned}\mathbf{\Gamma}_{IR} = &\left(\gamma_K C_A \ln\frac{-s_{51}}{\mu^2} + 2\gamma_A\right) + \left(\gamma_K C_B \ln\frac{-s_{23}}{\mu^2} + 2\gamma_B\right) + \\ &+\gamma_K\left[\left(\ln\frac{s_{45}}{s_{51}} - \frac{i\pi}{2}\right)(\mathbf{T}_+^{15})^2 + \left(\ln\frac{s_{34}}{s_{23}} - \frac{i\pi}{2}\right)(\mathbf{T}_+^{23})^2\right] \\ &+\frac{\gamma_K}{2}\left[-C_4\ln\frac{\mu^2}{|\mathbf{p}_4|^2} + \ln\frac{-s_{51}}{|\mathbf{p}_4|^2}(\mathbf{T}_+^{15})^2 + \ln\frac{-s_{23}}{|\mathbf{p}_4|^2}(\mathbf{T}_+^{23})^2 - i\pi\,\mathbf{T}_+^{15}\mathbf{T}_+^{23}\right] + \gamma_4 \\ &+\frac{\gamma_K}{2}\times i\pi\left(\mathbf{T}_+^{15}\mathbf{T}_-^{23} + \mathbf{T}_-^{15}\mathbf{T}_-^{23} + \mathbf{T}_-^{15}\mathbf{T}_+^{23}\right)\end{aligned}$$

impact factors

Regge trajectories

CEV

Regge cuts

Suggests an IR subtraction operation for the CEV (gory details not fixed yet)

- in IR finite CEV no spurious letters
- transcendental weight drop
  weight 4 = product of simple logs!
- highest (genuine) weight = 3, i.e. Li_3
- result expressible via single-valued MPLs

very symilar scenario to N=4 [Caron-Huot 2003.03120]

Summary and outlook

- QCD amplitudes in the high-energy limit exhibit remarkable structures: very interesting physics laboratory
- Regge-pole factorisation violated by multi-Reggeon (MR) exchanges starting at NNLL
- use recent results for full colour QCD 5-point scattering amplitudes to investigate MRK@2loops
- use EFT (rapidity evolution + B/JIMWLK) to predict multi-Reggeon contributions to the odd-odd amplitude
- Subtract MR from [8a,8a] and show Regge-pole factorisation at NNLL (universal CEV for all partonic channels)
- Expansion of the amplitude in QCD: leading transcendentality matches N=4, can extract vertex in both theories
- Remarkable simplifications in IR subtracted CEV (very nice analytic structure)

TODO:

- Refine the IR subtraction operation: guideline on how to define CEV (?)

Future:

- multi-Reggeon contributions in other signatures at 2loop, A(-,+), A(+,-) and A(+,+)
- try to investigate the Regge-cuts/multi-reggeon contributions from "direct calculation"

Backup

Regge pole and signatures

Following
[Caron-Huot, Gardi, Vernazza 1701.05241]

Expand the amplitudes in terms of the signature symmetric log:

$$L = \ln \frac{s}{-t} - i\frac{\pi}{2}$$

$$\mathcal{A}(s,t) = \underbrace{\frac{\mathcal{A}(s,t) - \mathcal{A}(u,t)}{2}}_{\mathcal{A}^{(-)}(s,t)} + \underbrace{\frac{\mathcal{A}(s,t) + \mathcal{A}(u,t)}{2}}_{\mathcal{A}^{(+)}(s,t)}$$

u = -s t
in Regge kinematics u ~ -s

Mellin transform:

$$\mathcal{A}^{(-)} = \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{dj}{\sin(\pi j)} \sin\left(\frac{\pi j}{2}\right) a_j^{(-)} e^{jL}$$

real, antisymmetric

$$\mathcal{A}^{(+)} = i \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{dj}{\sin(\pi j)} \cos\left(\frac{\pi j}{2}\right) a_j^{(+)} e^{jL}$$

imaginary, symmetric

colour + kinematics
will have to respect this symmetries

→ Even(+) or Odd(-)
number of Reggeons

Simplest behaviour: pole in the j complex plane

$$a_j^{(-)}(t) \simeq \frac{1}{j - 1 - \alpha(t)}$$

Reggeisation in odd amplitude
from pole contributoin

$$\mathcal{A}^{(-)}(s,t)|_{\text{Regge pole}} = \frac{\pi}{\sin\left(\frac{\pi\alpha(t)}{2}\right)} \frac{s}{t} e^{L\alpha(t)} + \text{sub-leading}$$