

# GENEVA: Colour singlet + 1jet Event Generation

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Davide Napoletano, QCD@LHC '24, Freiburg 08/10/24



# Intro

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- **GENEVA: fully differential event generator at NNLO + PS accuracy**

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- **In practice, many ingredients are required and many possible technical difficulties may arise in doing so**

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- **GENEVA: fully differential event generator at NNLO + PS accuracy**

**In principle completely agnostic on the final state!**

- **In practice, many ingredients are required and many possible technical difficulties may arise in doing so**

**Which comprises the work of many more people than just a single speaker!**

# GENEVA: **Towards** colour singlet + 1jet Event Generation

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# Intro: Geneva

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$$\begin{aligned}\sigma(X) = & \int d\Phi_0 \frac{d\sigma_0^{\text{MC}}}{d\Phi_0}(r_0^{\text{cut}}) M_X(\Phi_0) \\ & + \int d\Phi_1 \frac{d\sigma_1^{\text{MC}}}{d\Phi_1}(r_0 > r_0^{\text{cut}}; r_1^{\text{cut}}) M_X(\Phi_1) \\ & + \int d\Phi_2 \frac{d\sigma_{\geq 2}^{\text{MC}}}{d\Phi_2}(r_0 > r_0^{\text{cut}}, r_1 > r_1^{\text{cut}}) M_X(\Phi_2)\end{aligned}$$

- **Split NNLO observables into contributions you can compute**

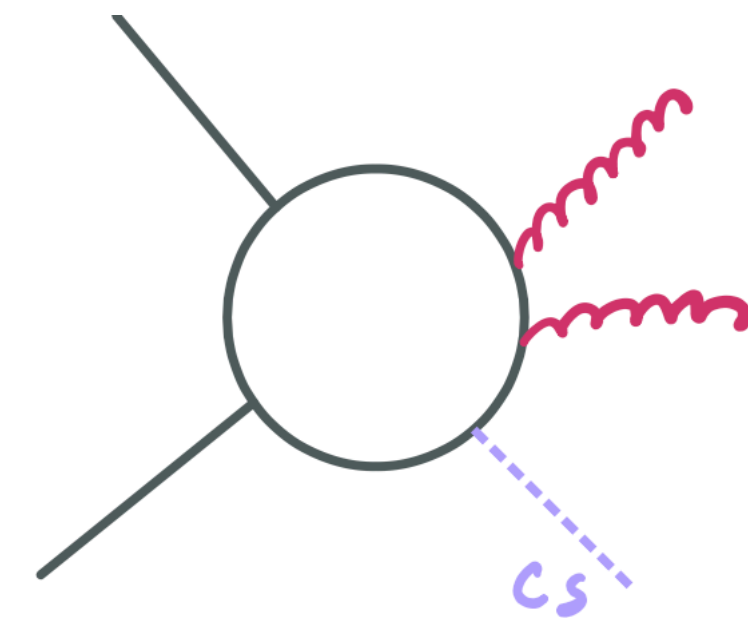
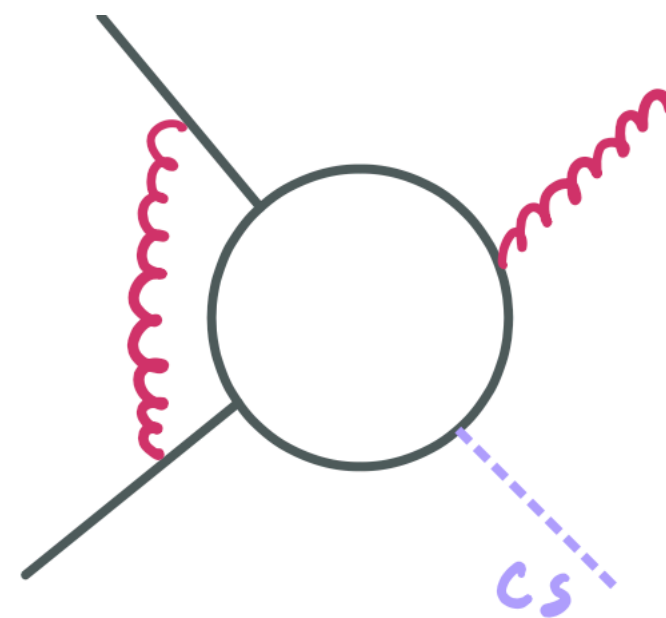
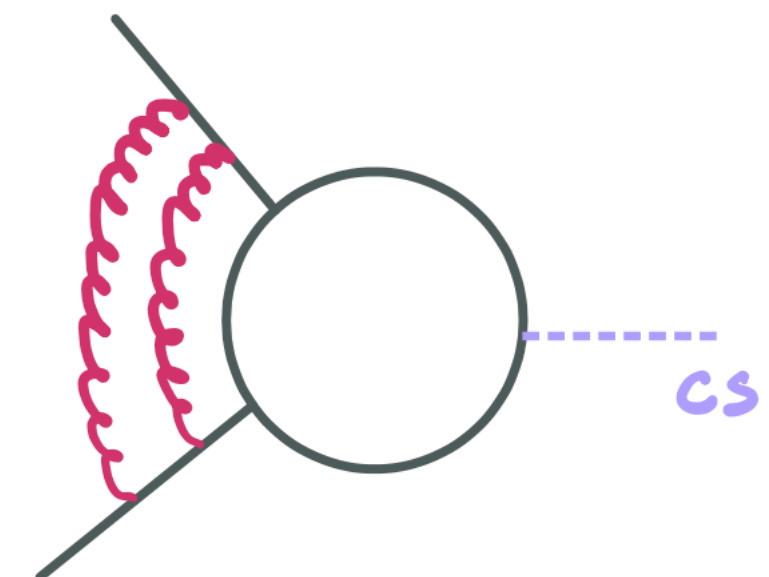
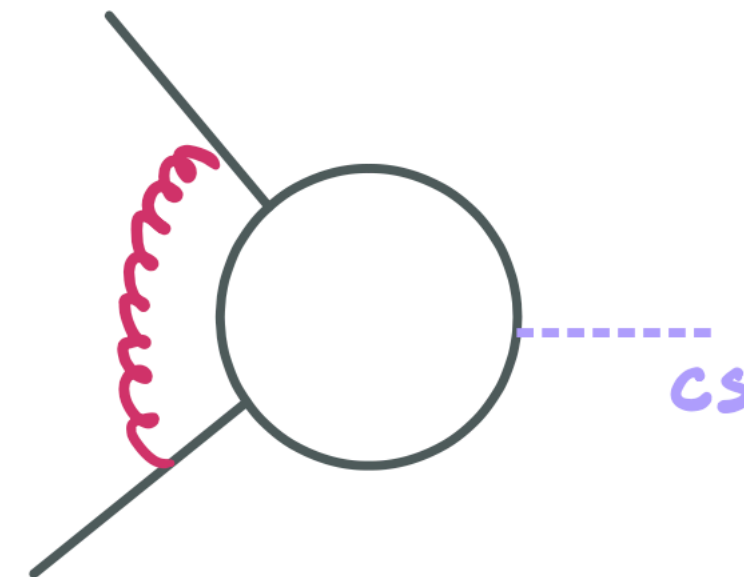
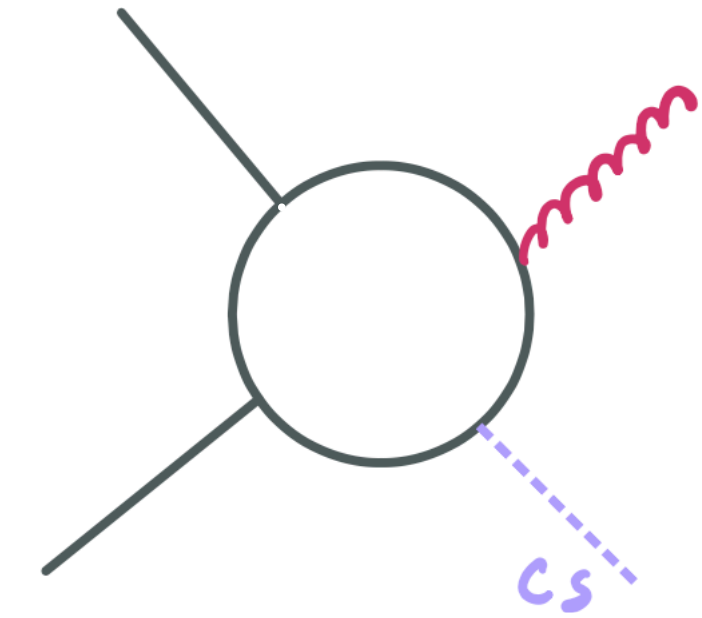
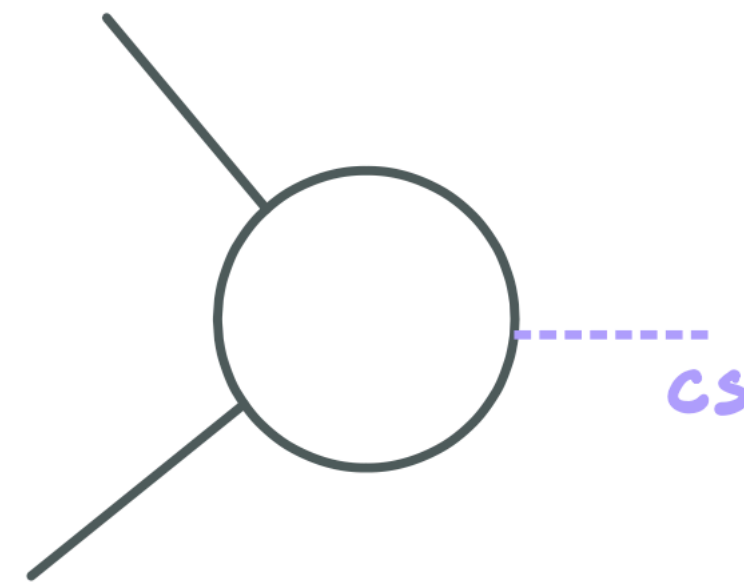
- **Better if you know the resummation of  $r_i$  to high accuracy!**

# Introduction

$$\sigma(X) = \int d\Phi_0 \frac{d\sigma_0^{\text{MC}}}{d\Phi_0}(r_0^{\text{cut}}) M_X(\Phi_0)$$

$$+ \int d\Phi_1 \frac{d\sigma_1^{\text{MC}}}{d\Phi_1}(r_0 > r_0^{\text{cut}}; r_1^{\text{cut}}) M_X(\Phi_1)$$

$$+ \int d\Phi_2 \frac{d\sigma_{\geq 2}^{\text{MC}}}{d\Phi_2}(r_0 > r_0^{\text{cut}}, r_1 > r_1^{\text{cut}}) M_X(\Phi_2)$$

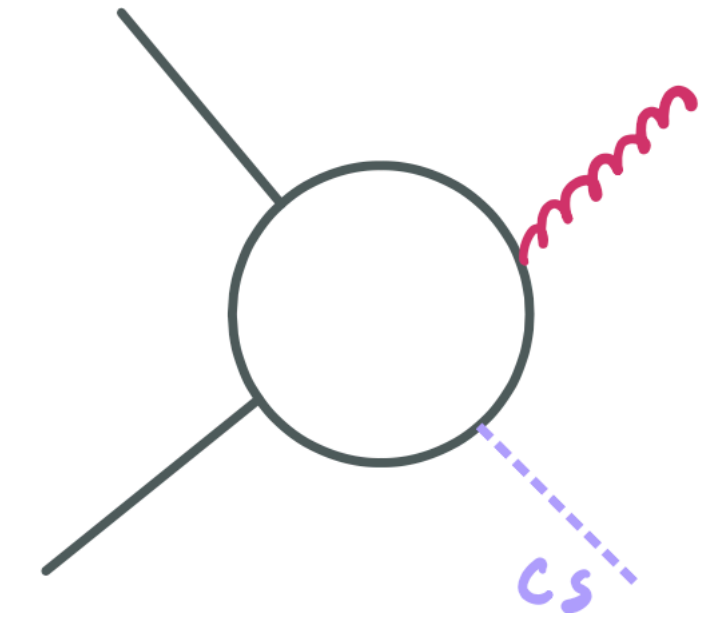
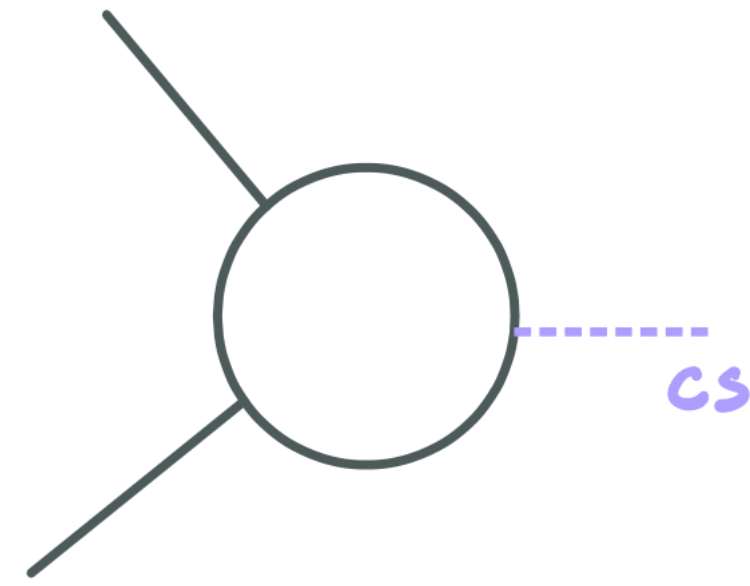


# Introduction

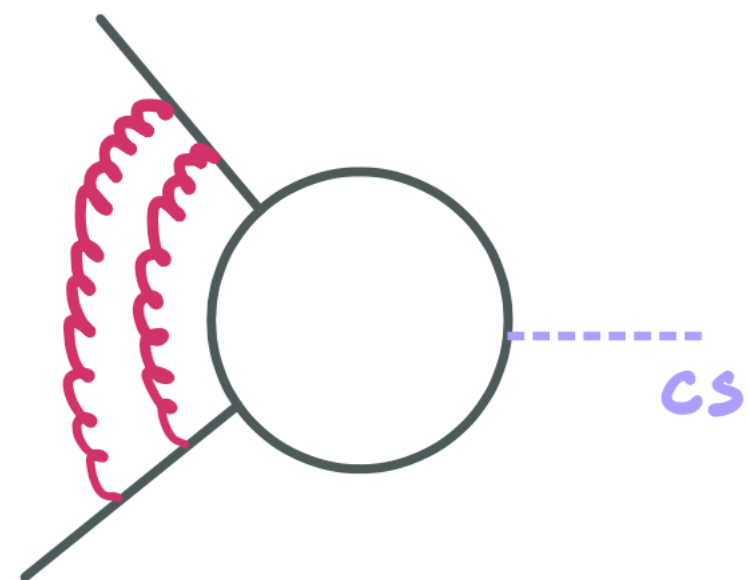
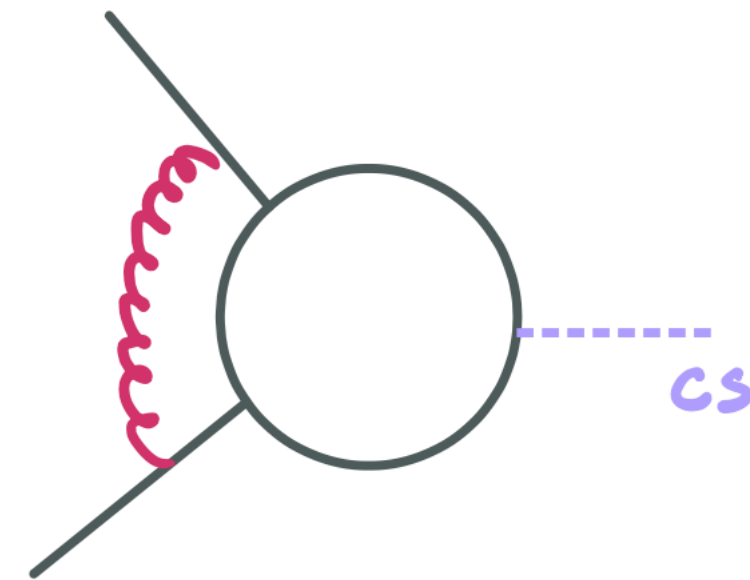
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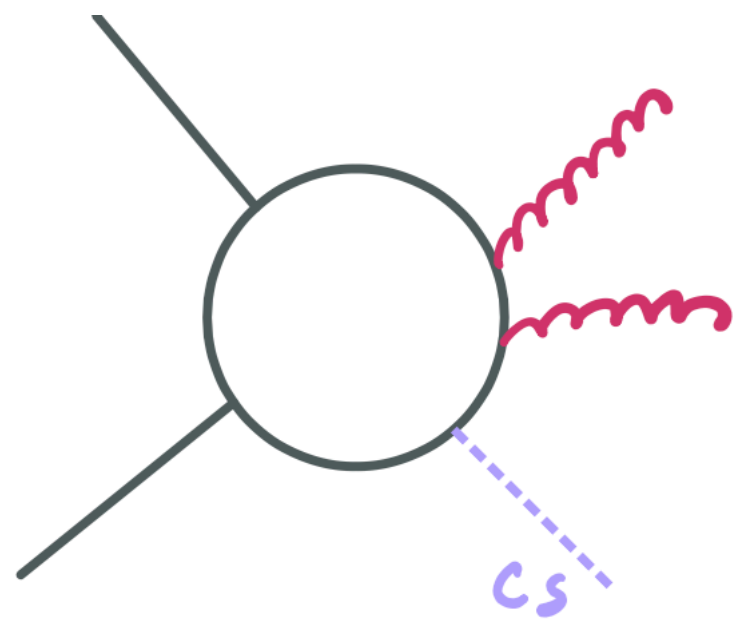
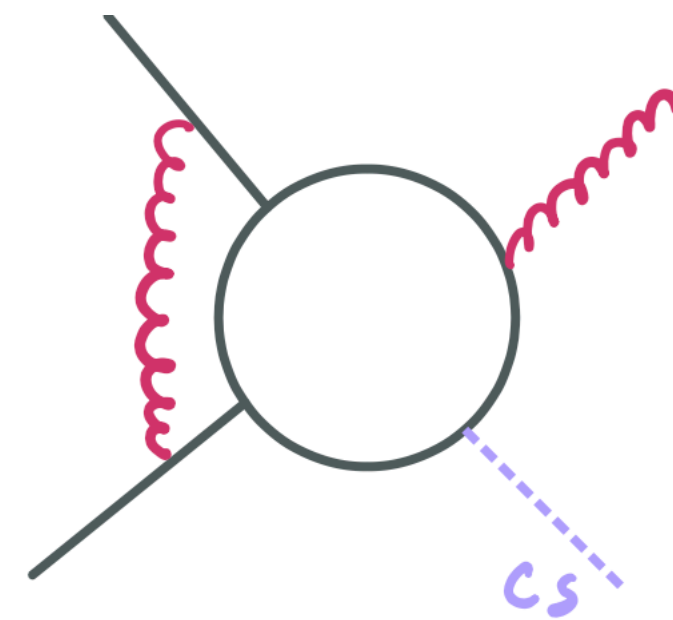
$\Phi_0$  Event



$\Phi_1$  Event



$\Phi_2$  Event

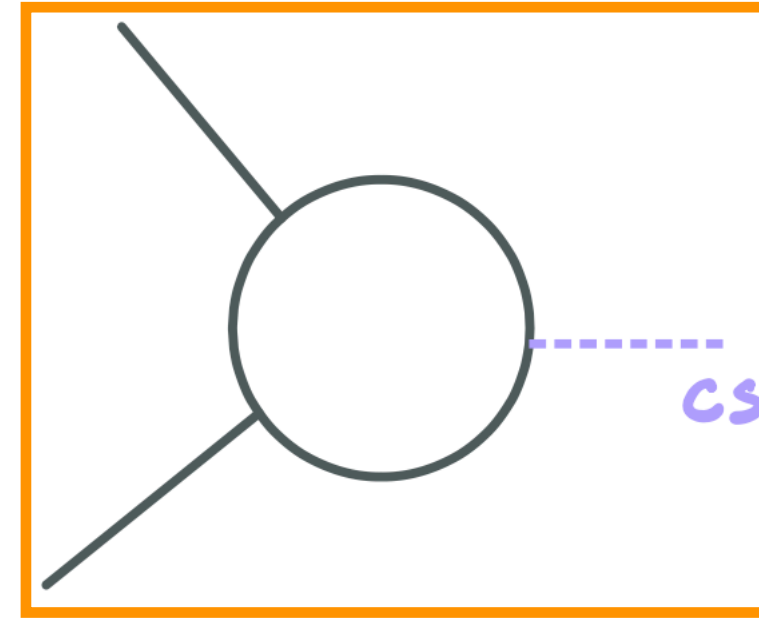




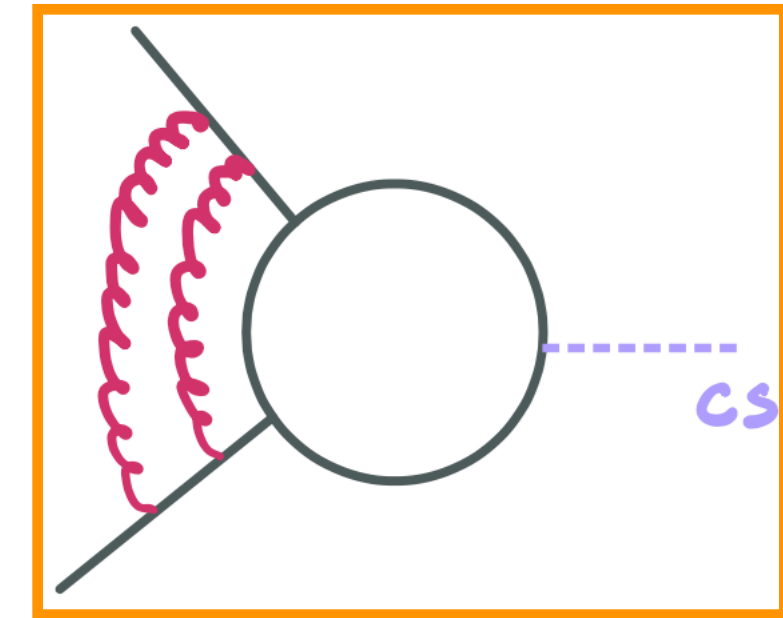
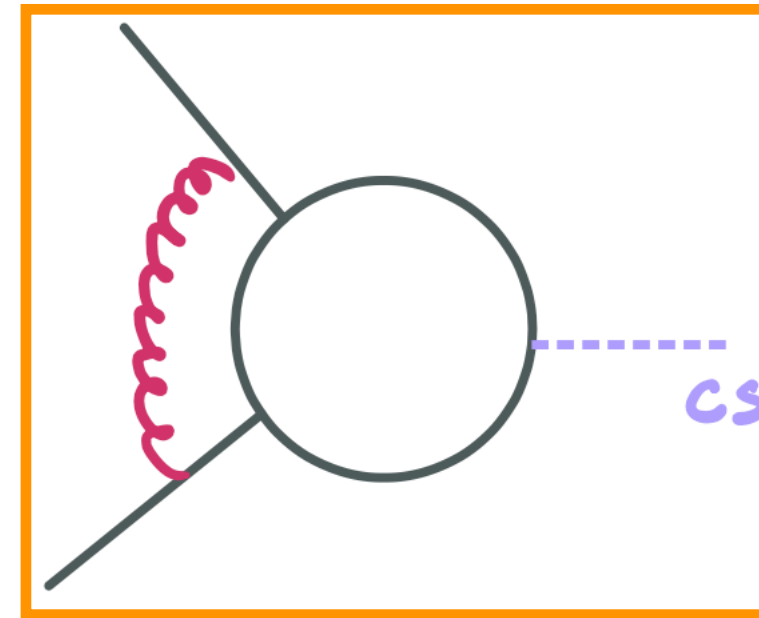
# Introduction



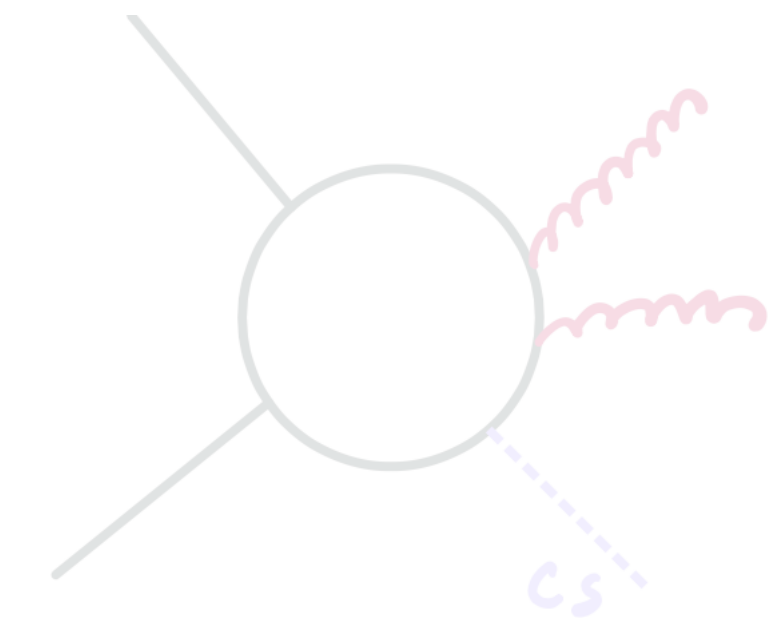
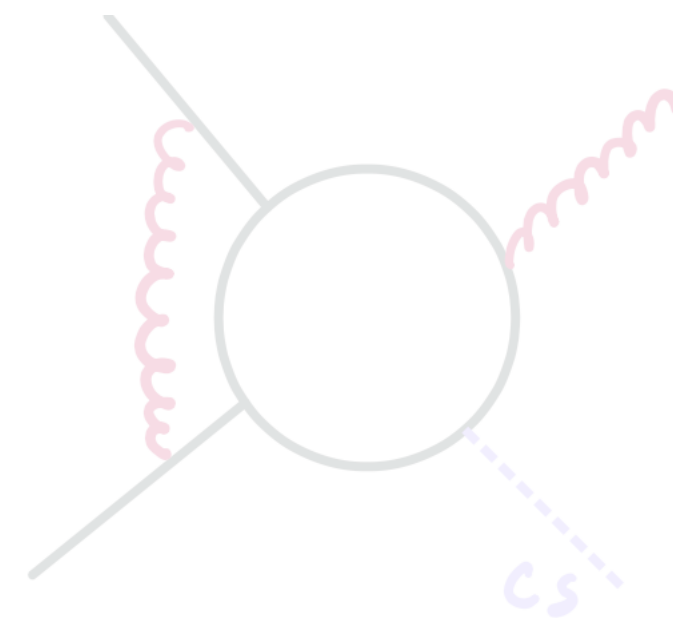
$\Phi_0$  Event



$\Phi_1$  Event

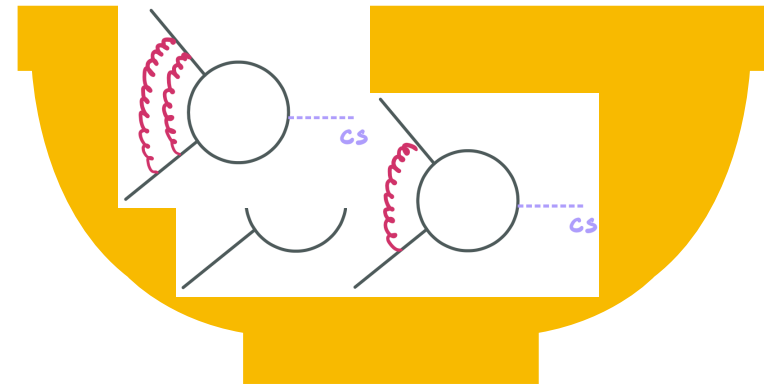


$\Phi_2$  Event

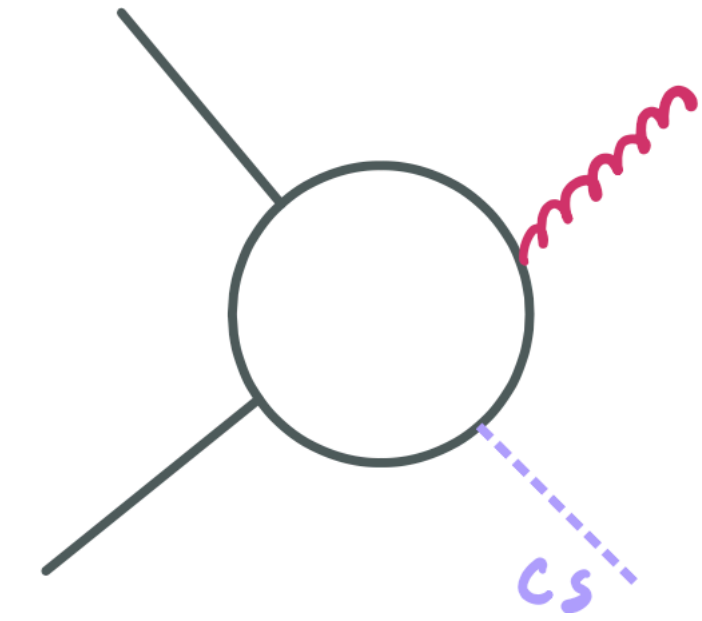


# Introduction

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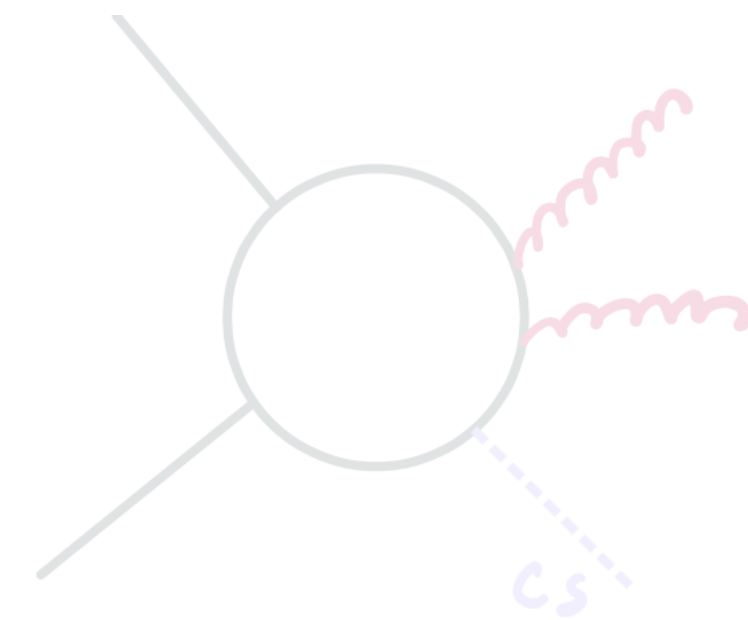
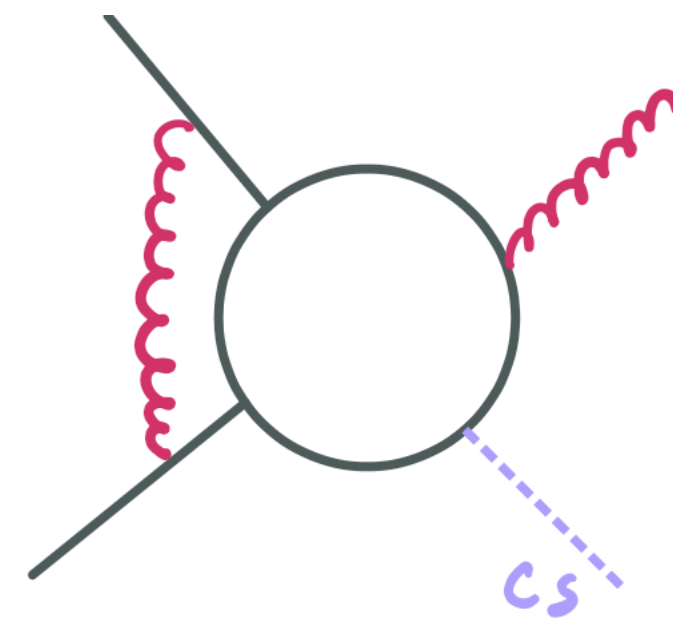
$\Phi_0$  Event



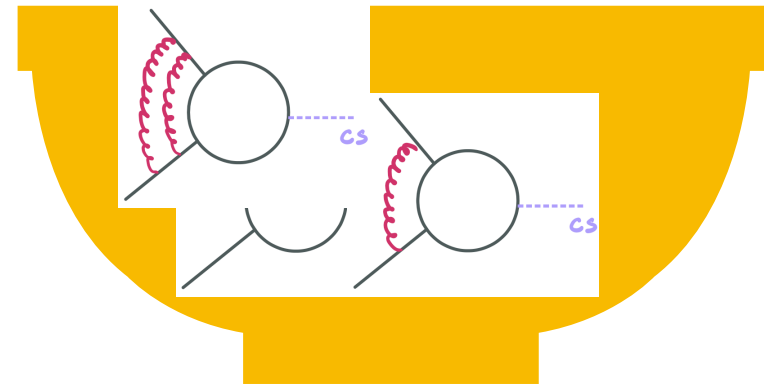
$\Phi_1$  Event



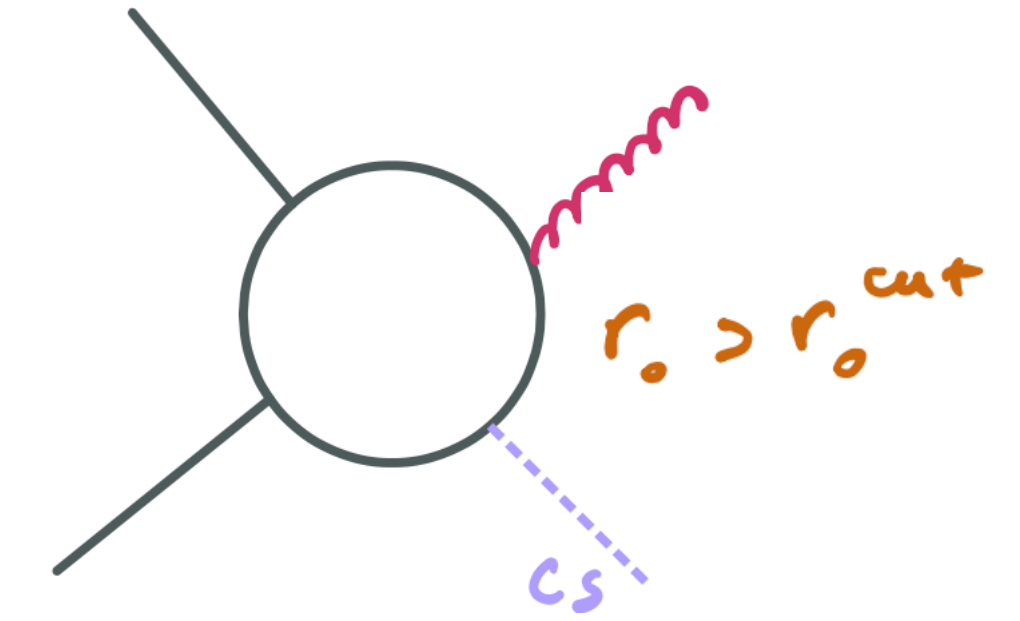
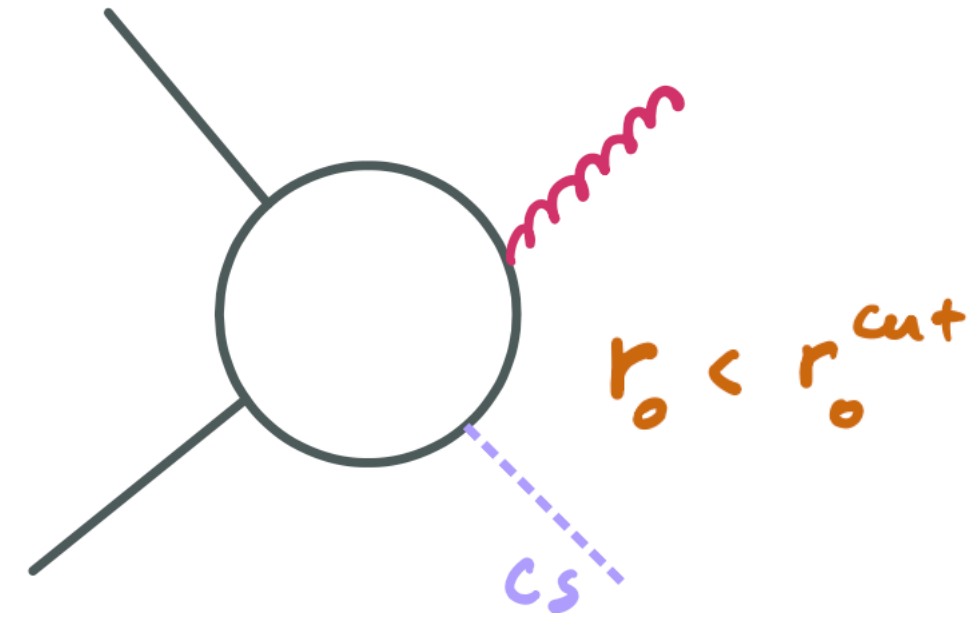
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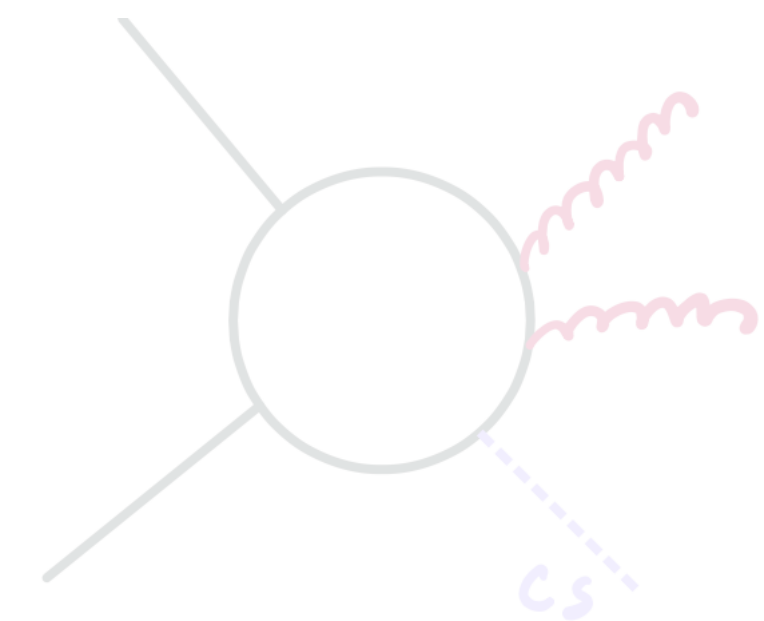
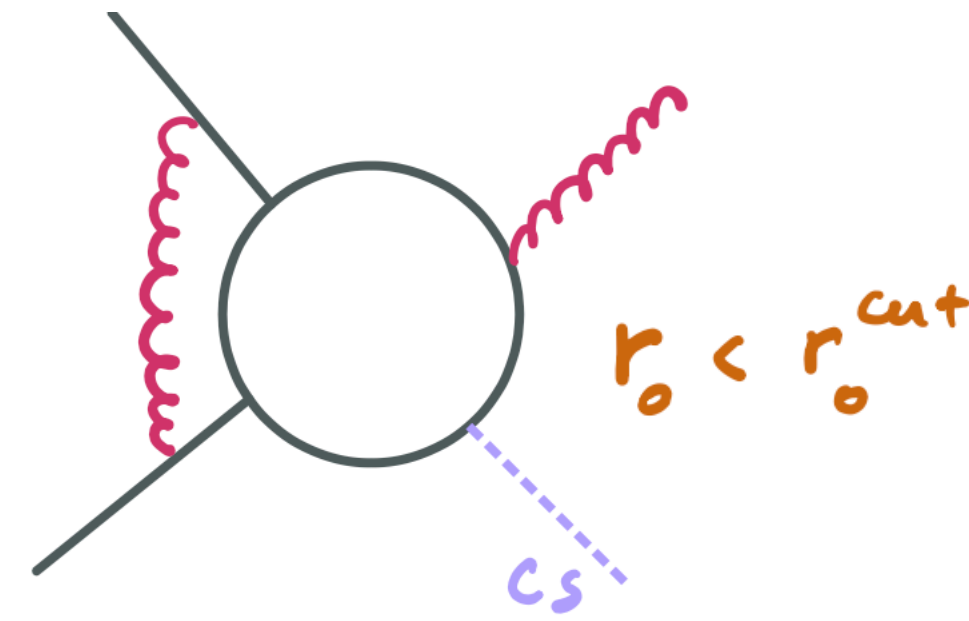
# Introduction



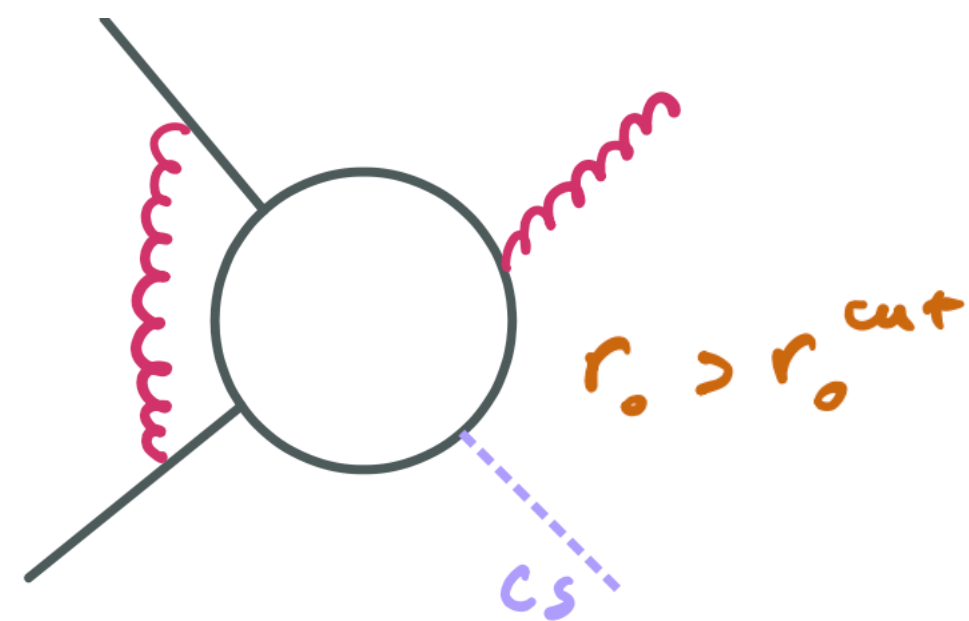
$\Phi_0$  Event



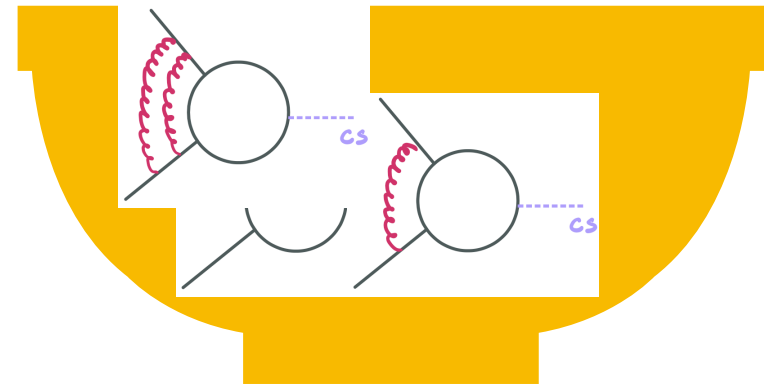
$\Phi_1$  Event



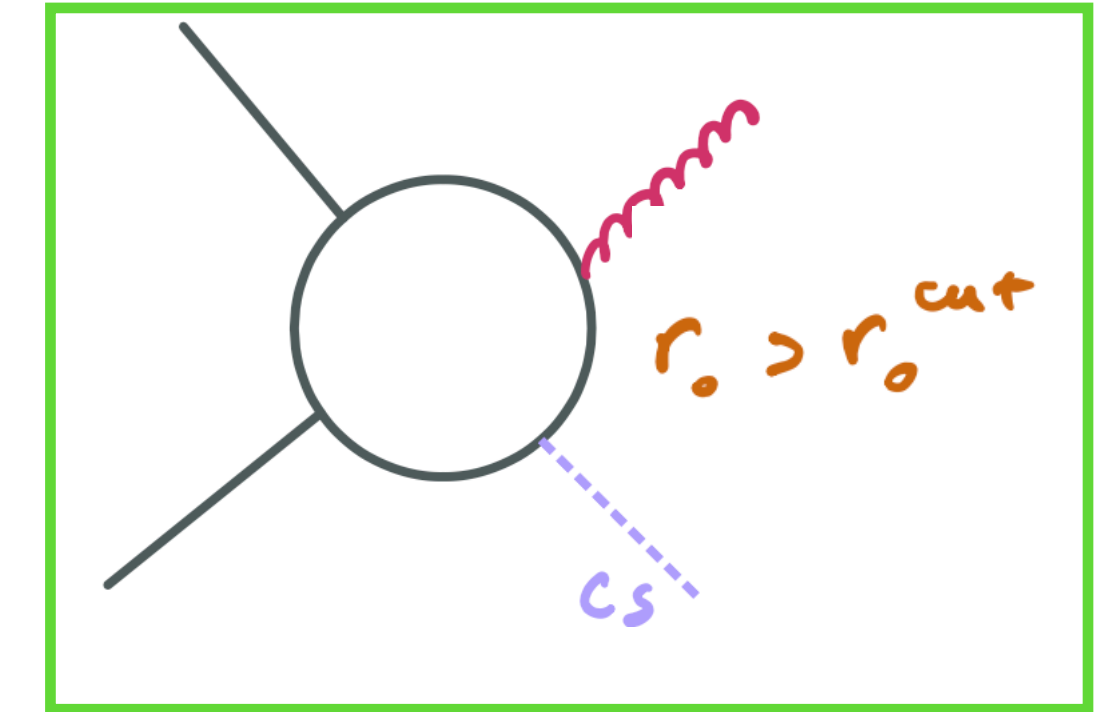
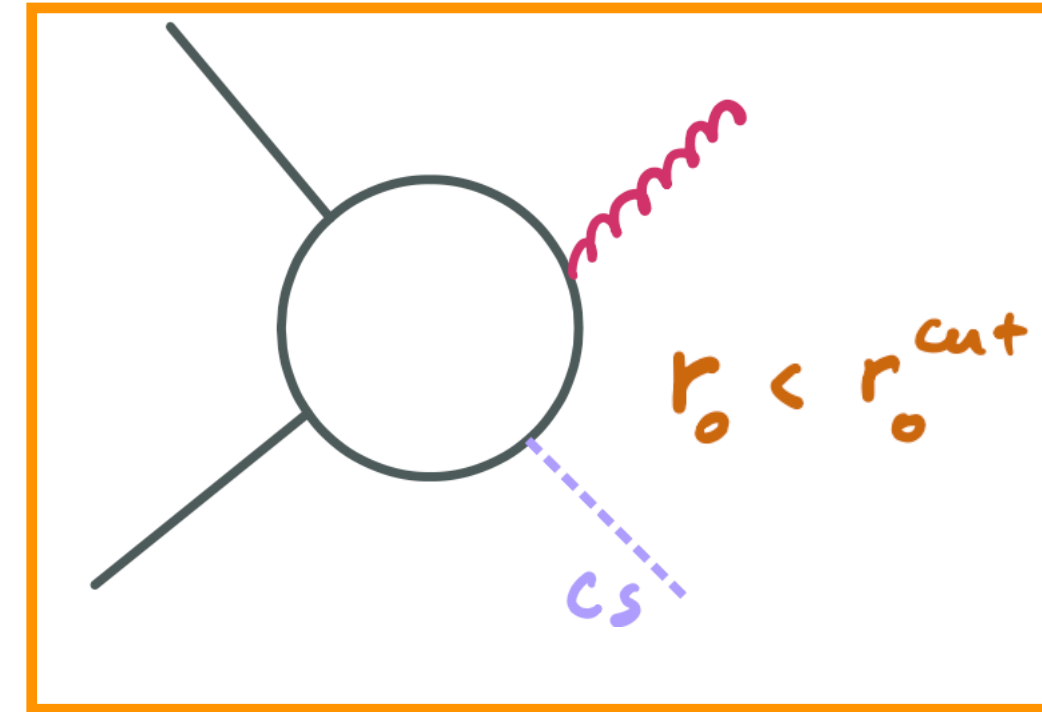
$\Phi_2$  Event



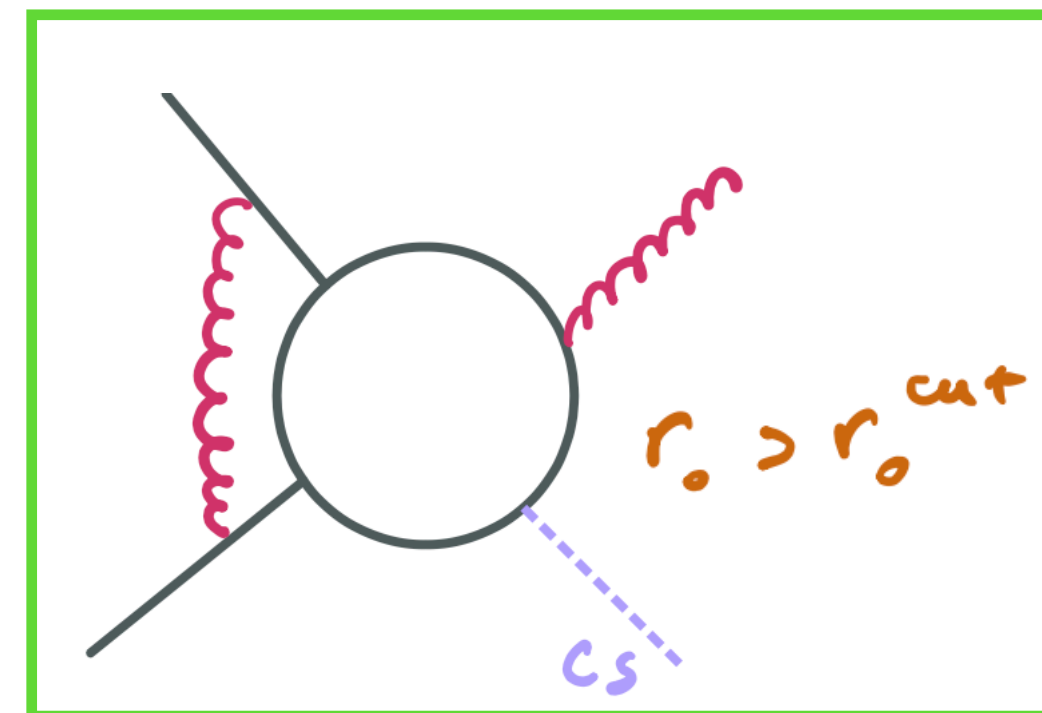
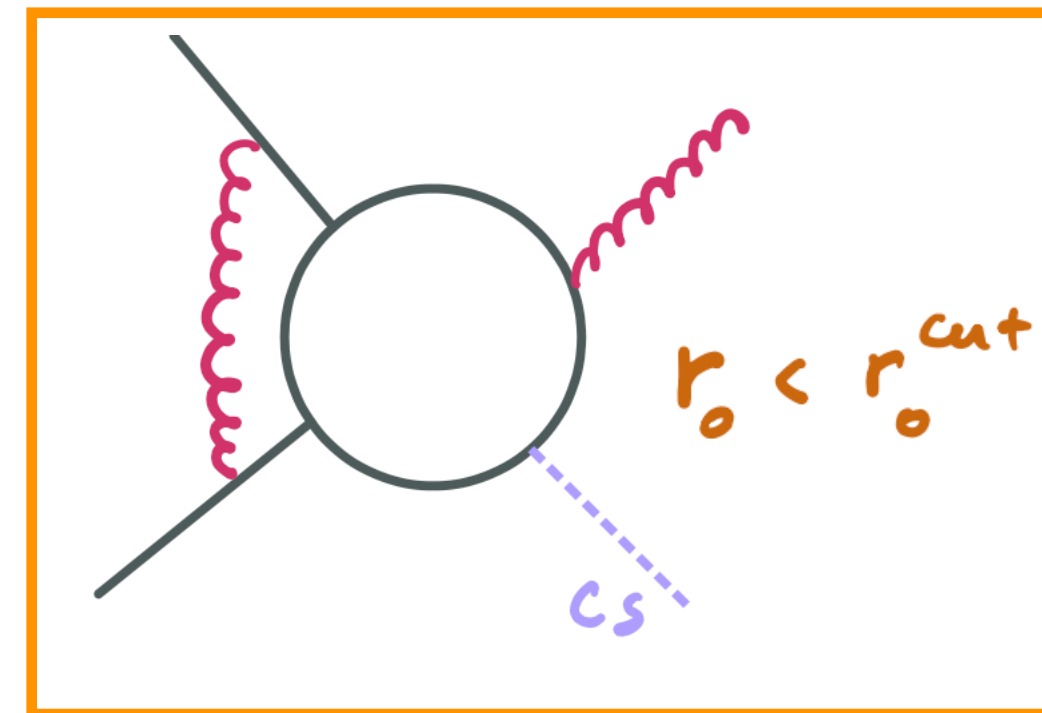
# Introduction



$\Phi_0$  Event

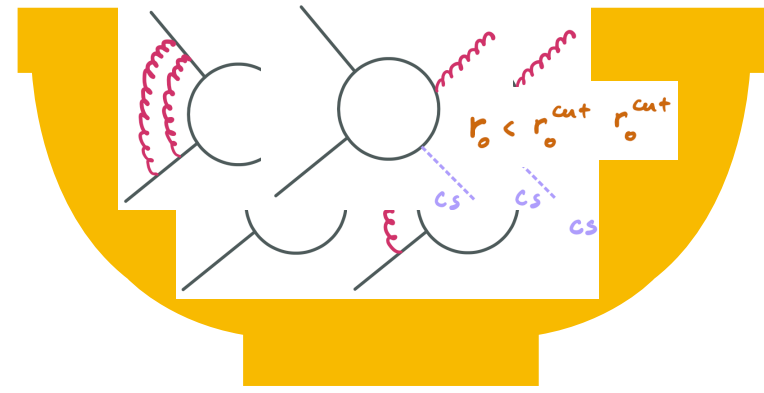


$\Phi_1$  Event

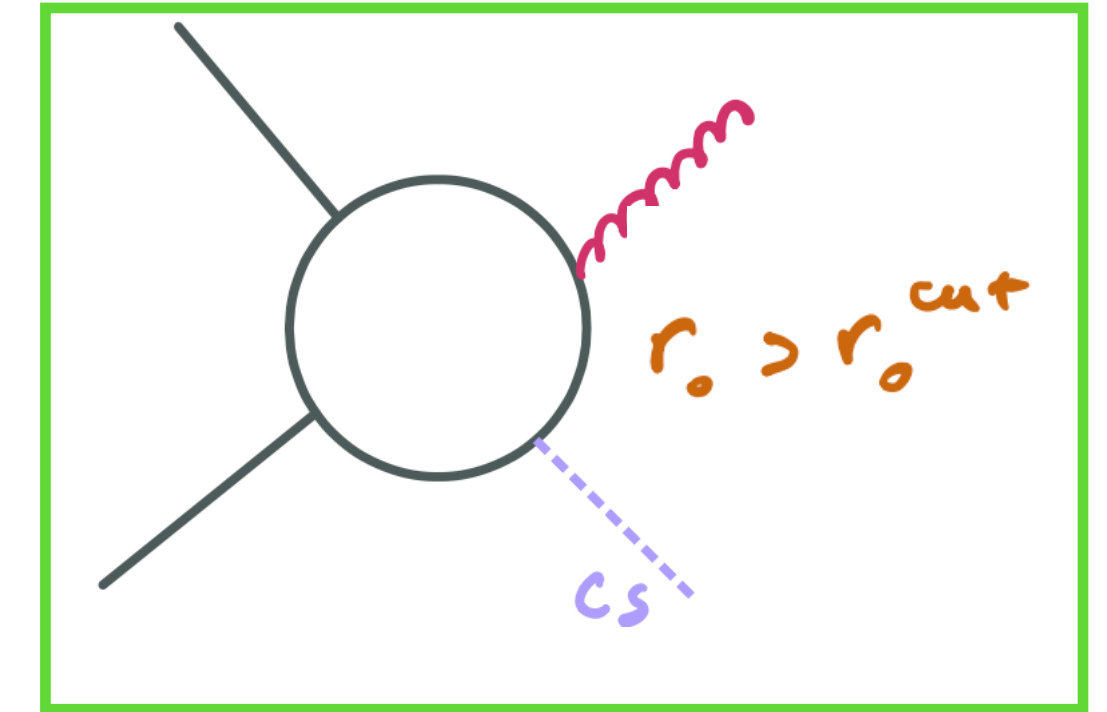


$\Phi_2$  Event

# Introduction



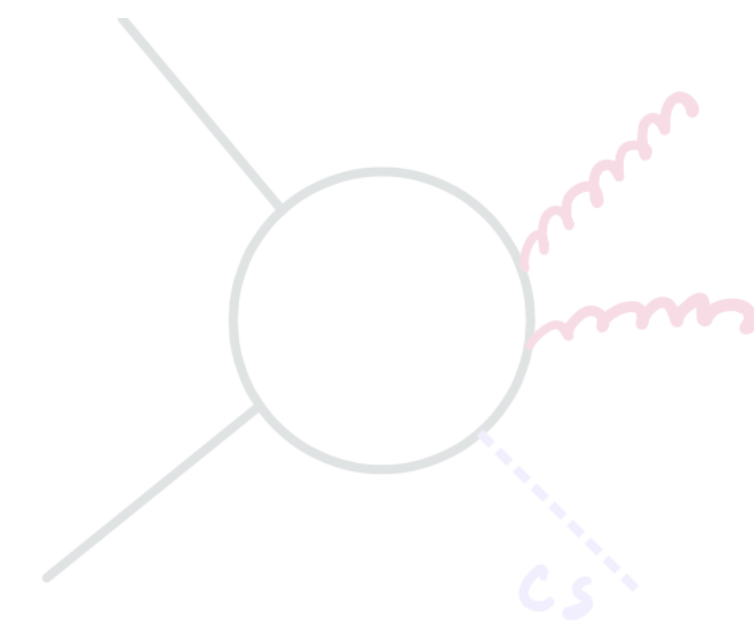
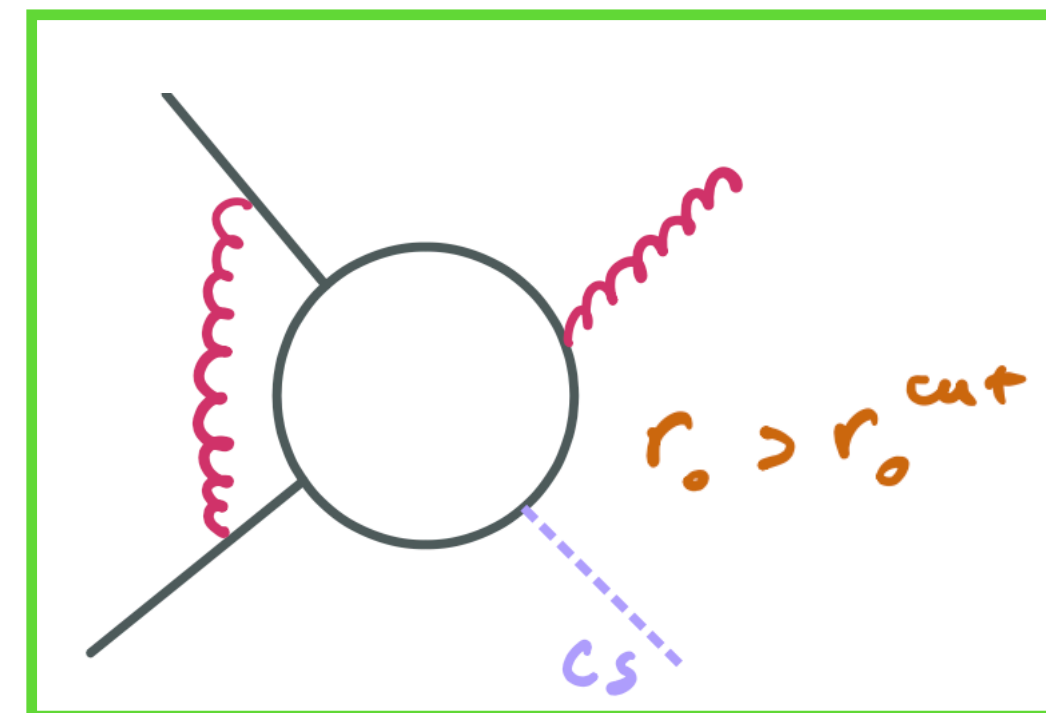
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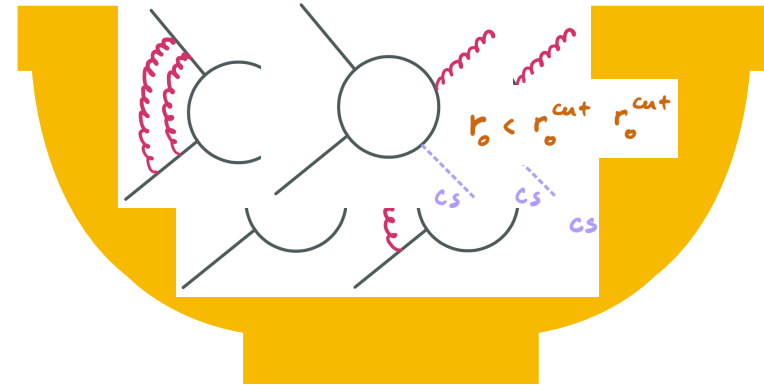
$\Phi_1$  Event



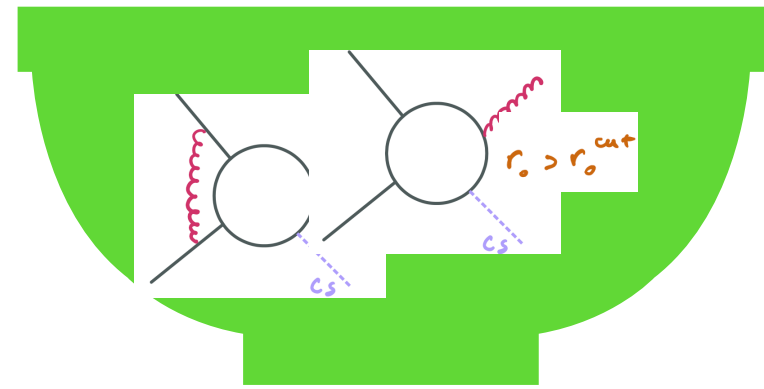
$\Phi_2$  Event



# Introduction



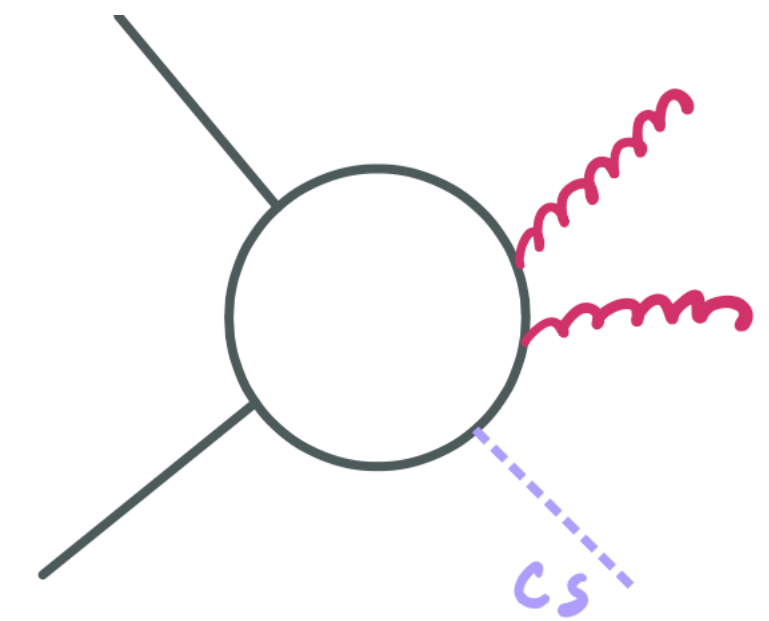
$\Phi_0$  Event



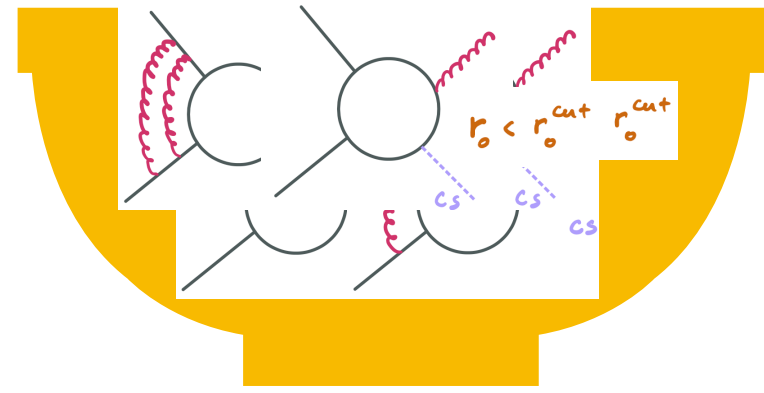
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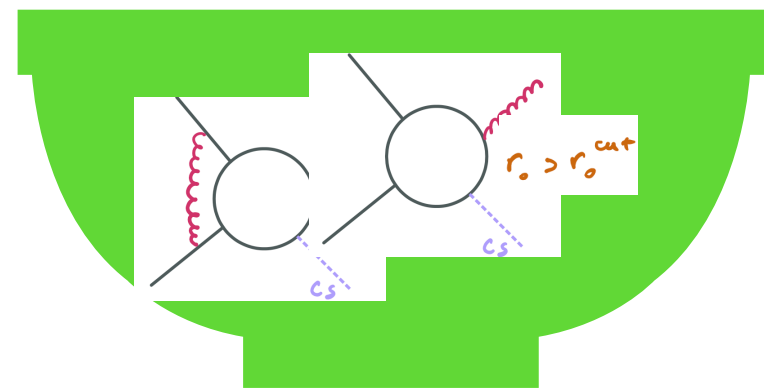
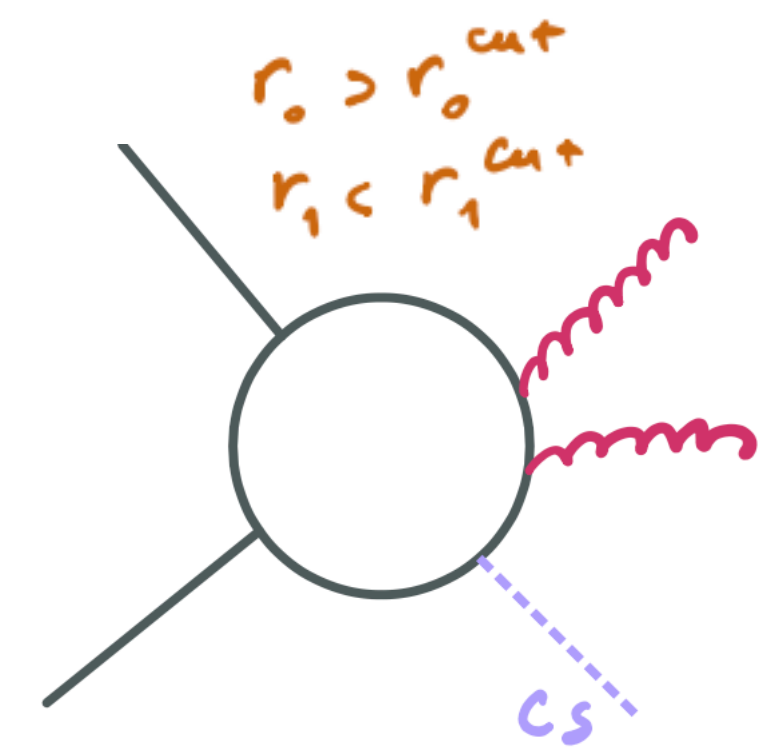
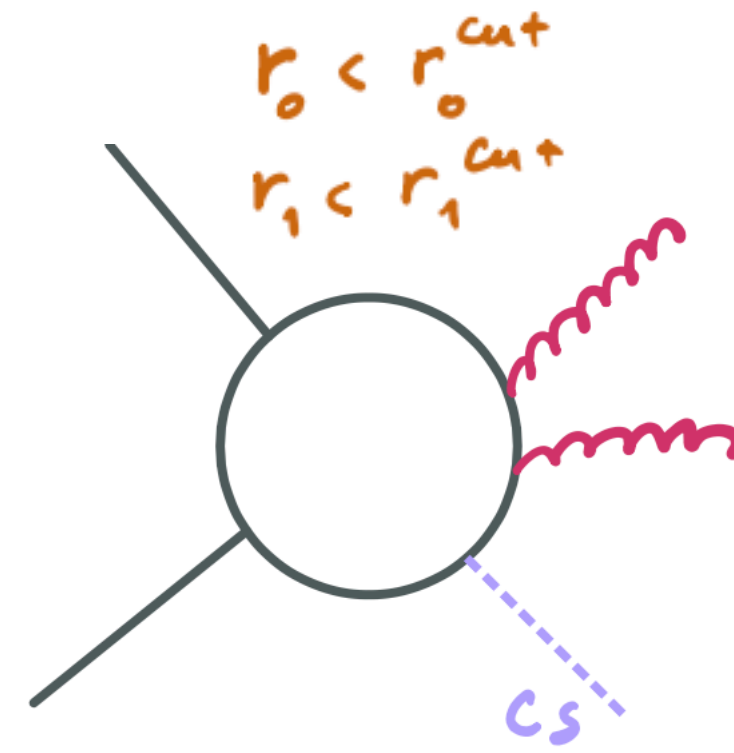
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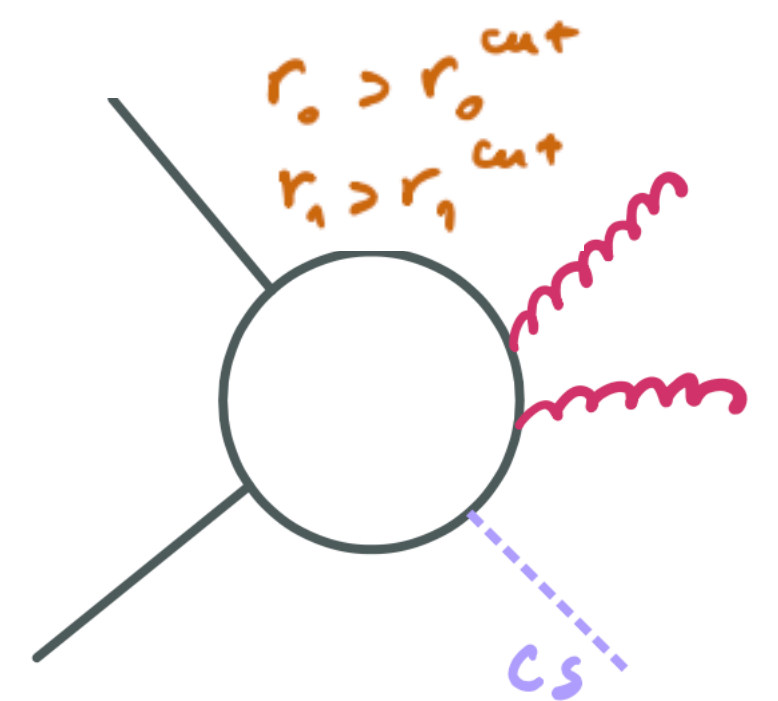
# Introduction



$\Phi_0$  Event

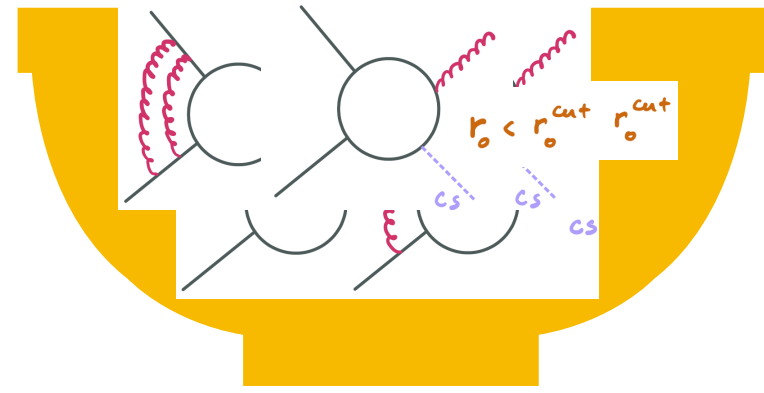


$\Phi_1$  Event

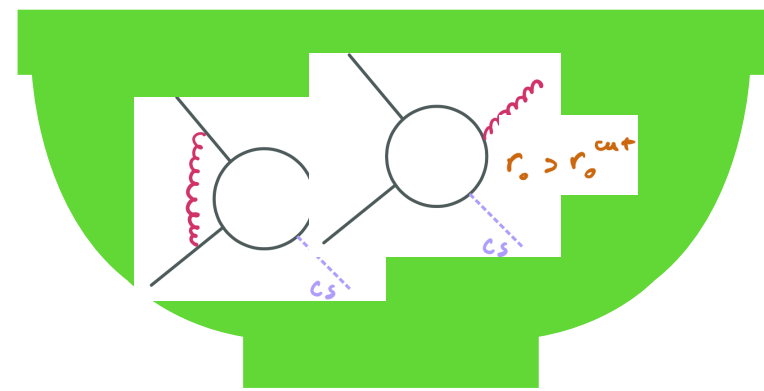
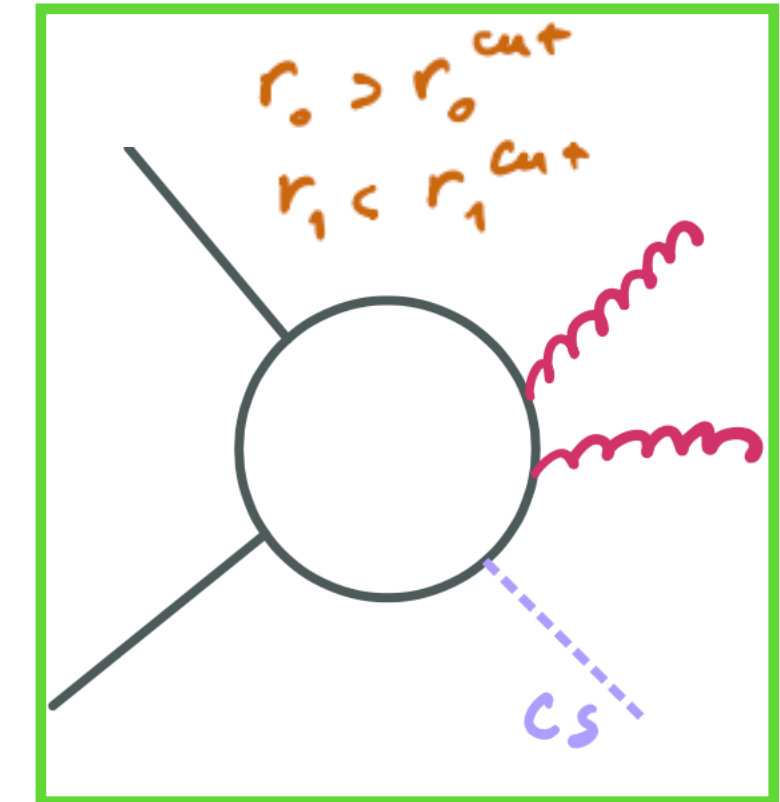
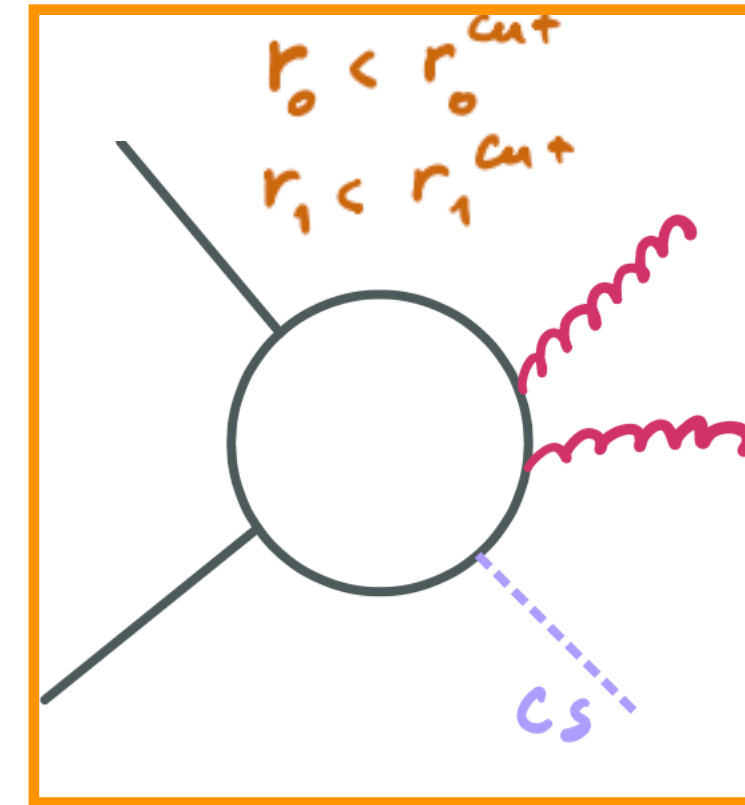


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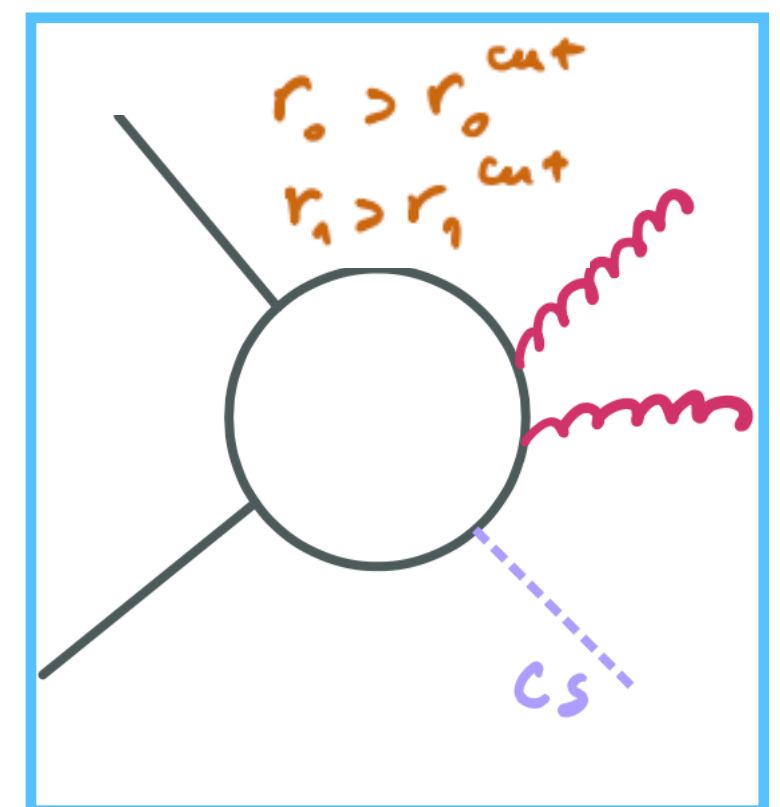
# Introduction



$\Phi_0$  Event



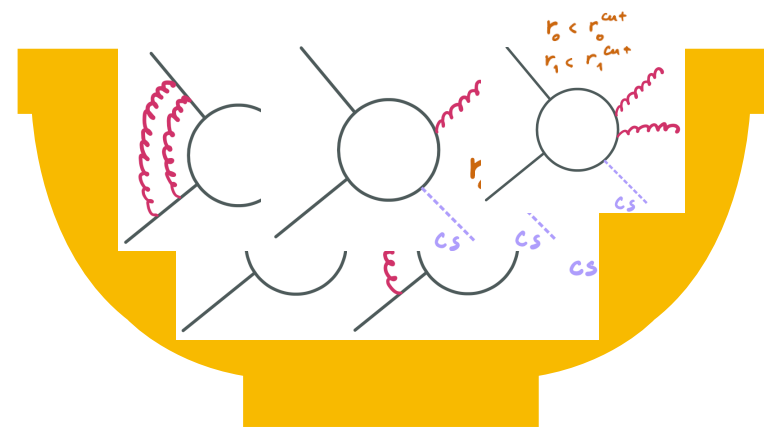
$\Phi_1$  Event



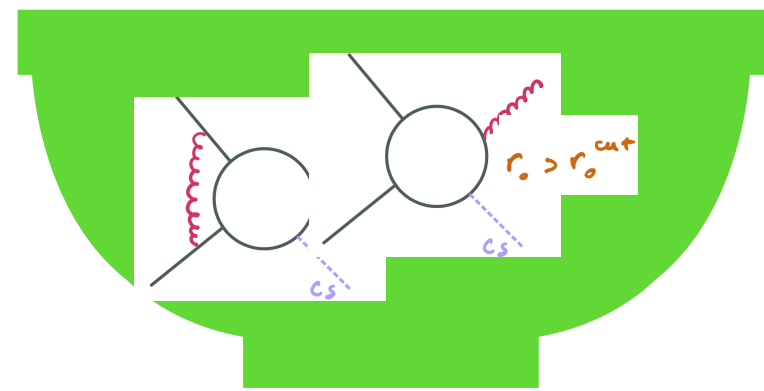
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# Introduction



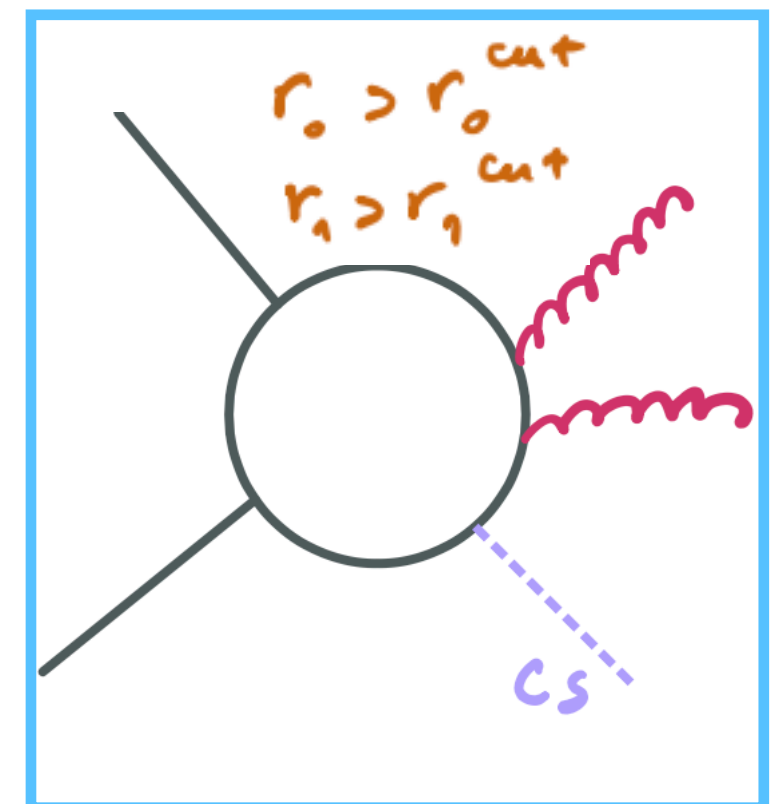
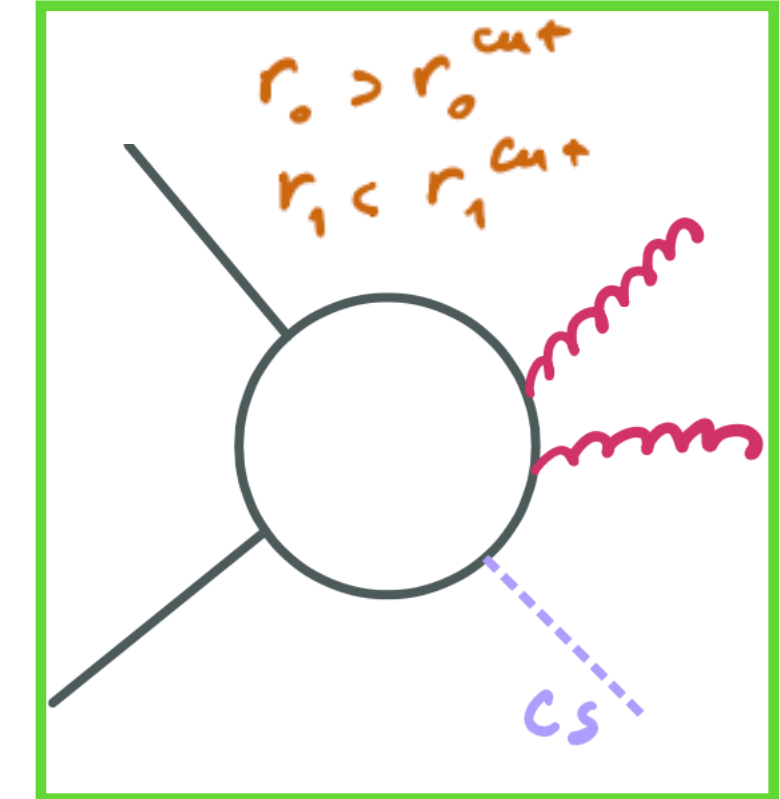
$\Phi_0$  Event



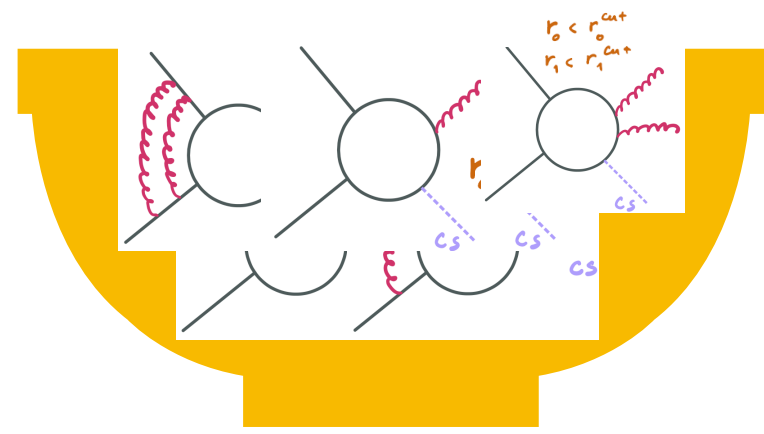
$\Phi_1$  Event



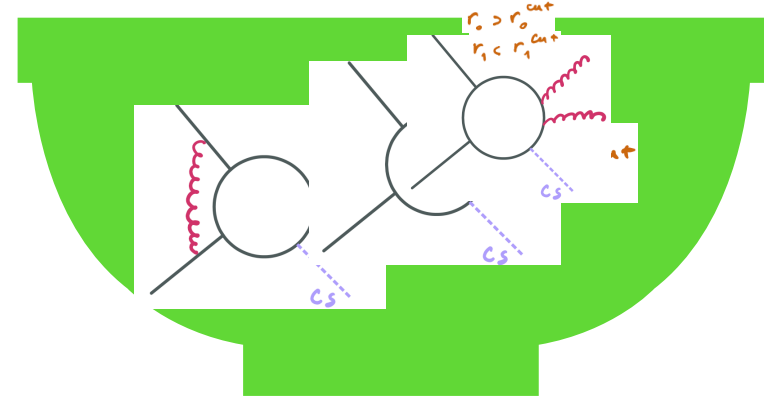
$\Phi_2$  Event



# Introduction



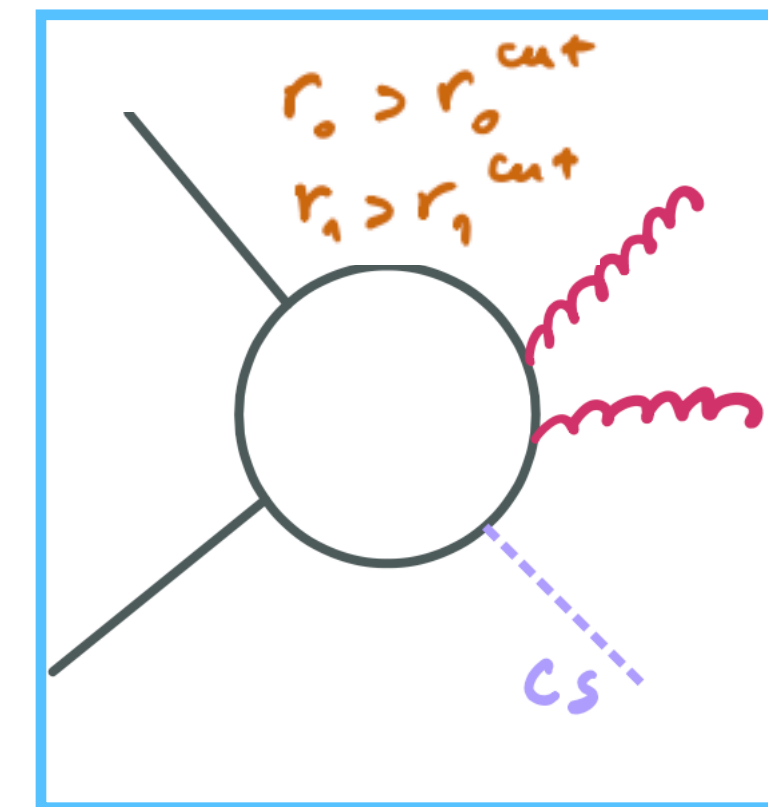
$\Phi_0$  Event



$\Phi_1$  Event

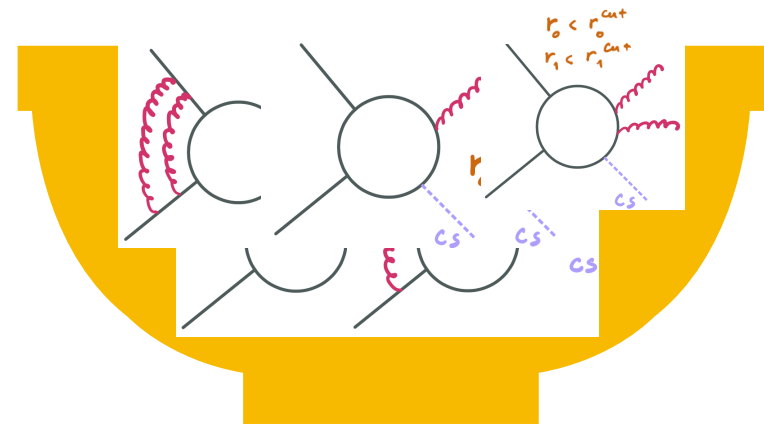


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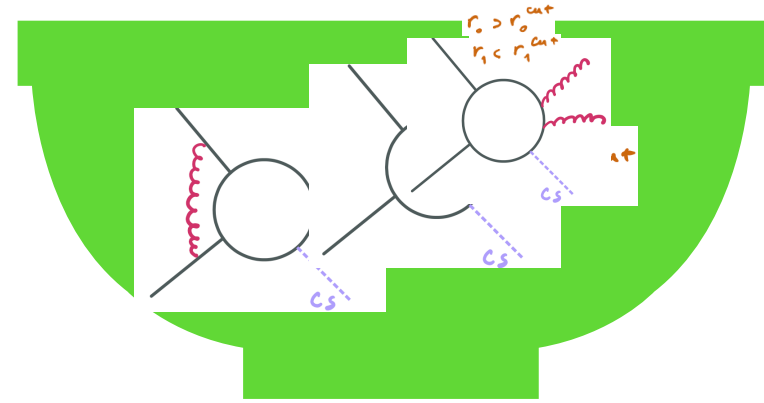


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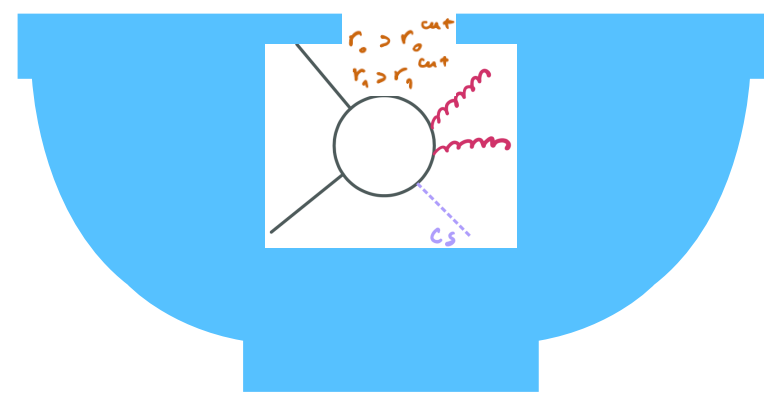
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$\Phi_0$  Event

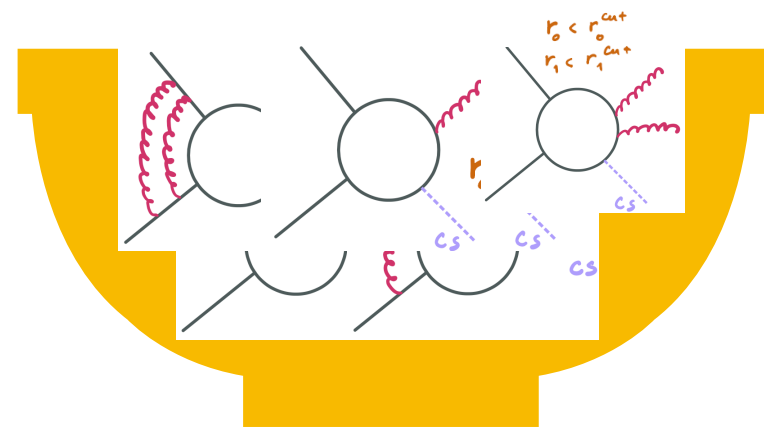


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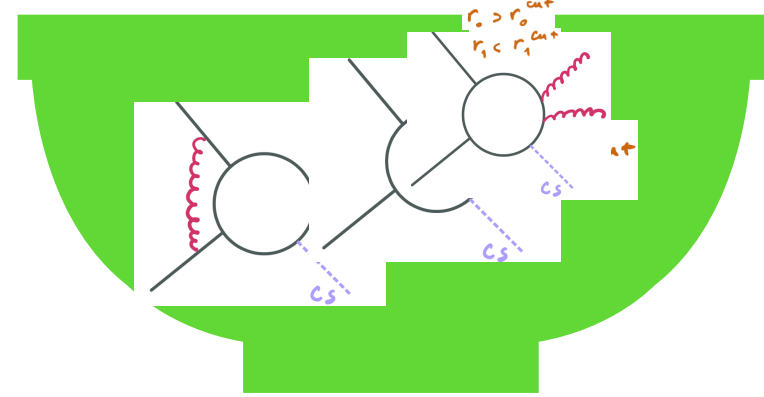


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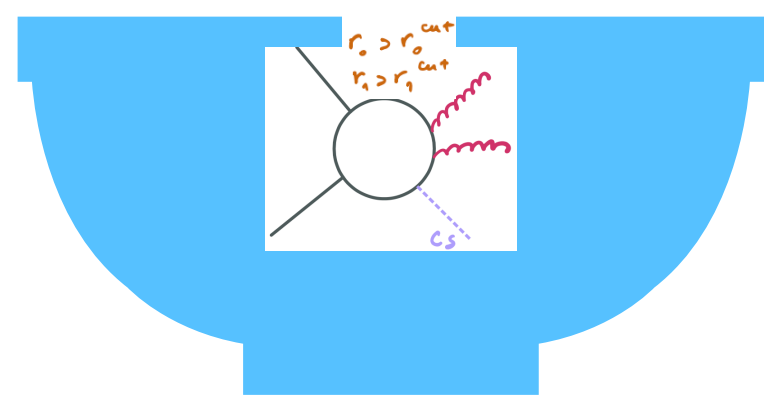
# Introduction



$\Phi_0$  Event

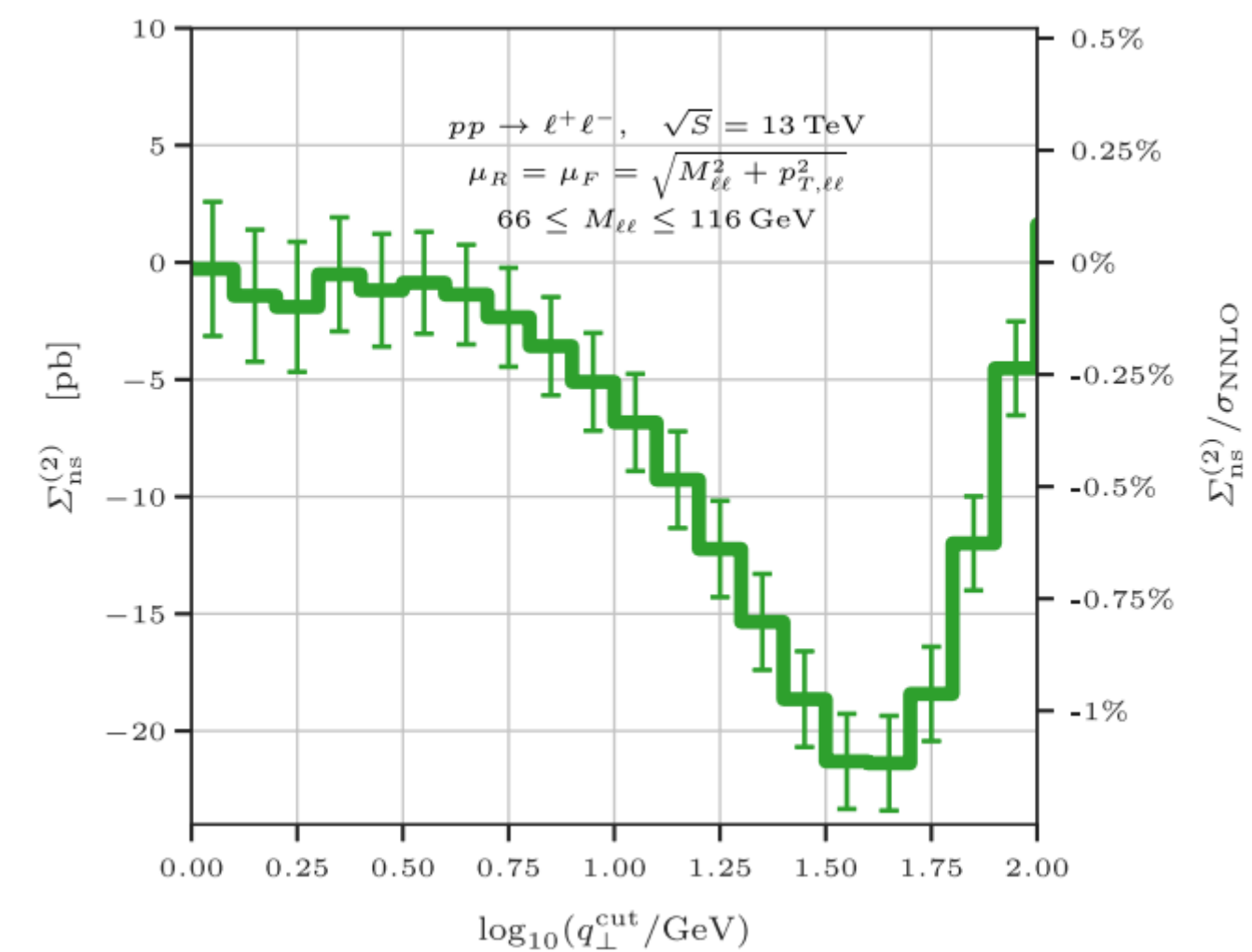


$\Phi_1$  Event

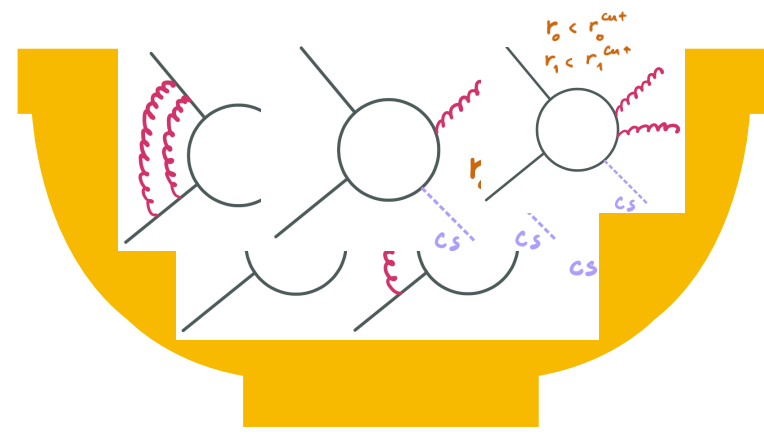


$\Phi_2$  Event

- Exchange IR singularities with logs of resolution variable
- However, need to lower cut as much as possible

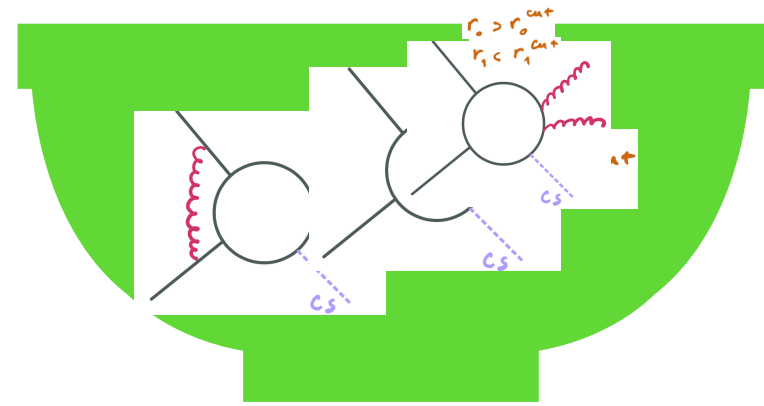


# Introduction

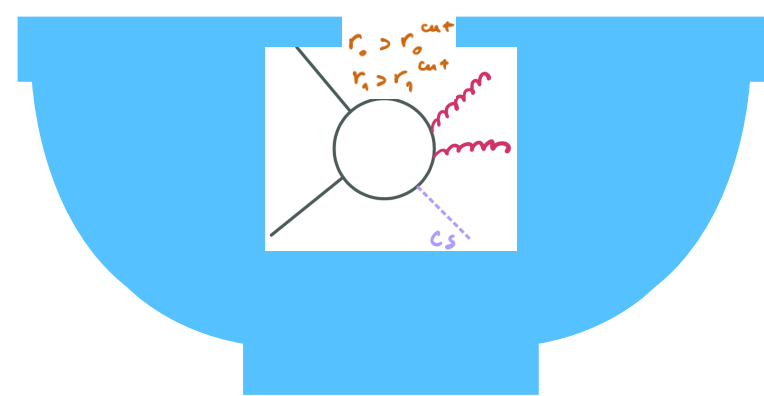


$\Phi_0$  Event

=> Resummation to the rescue!



$\Phi_1$  Event



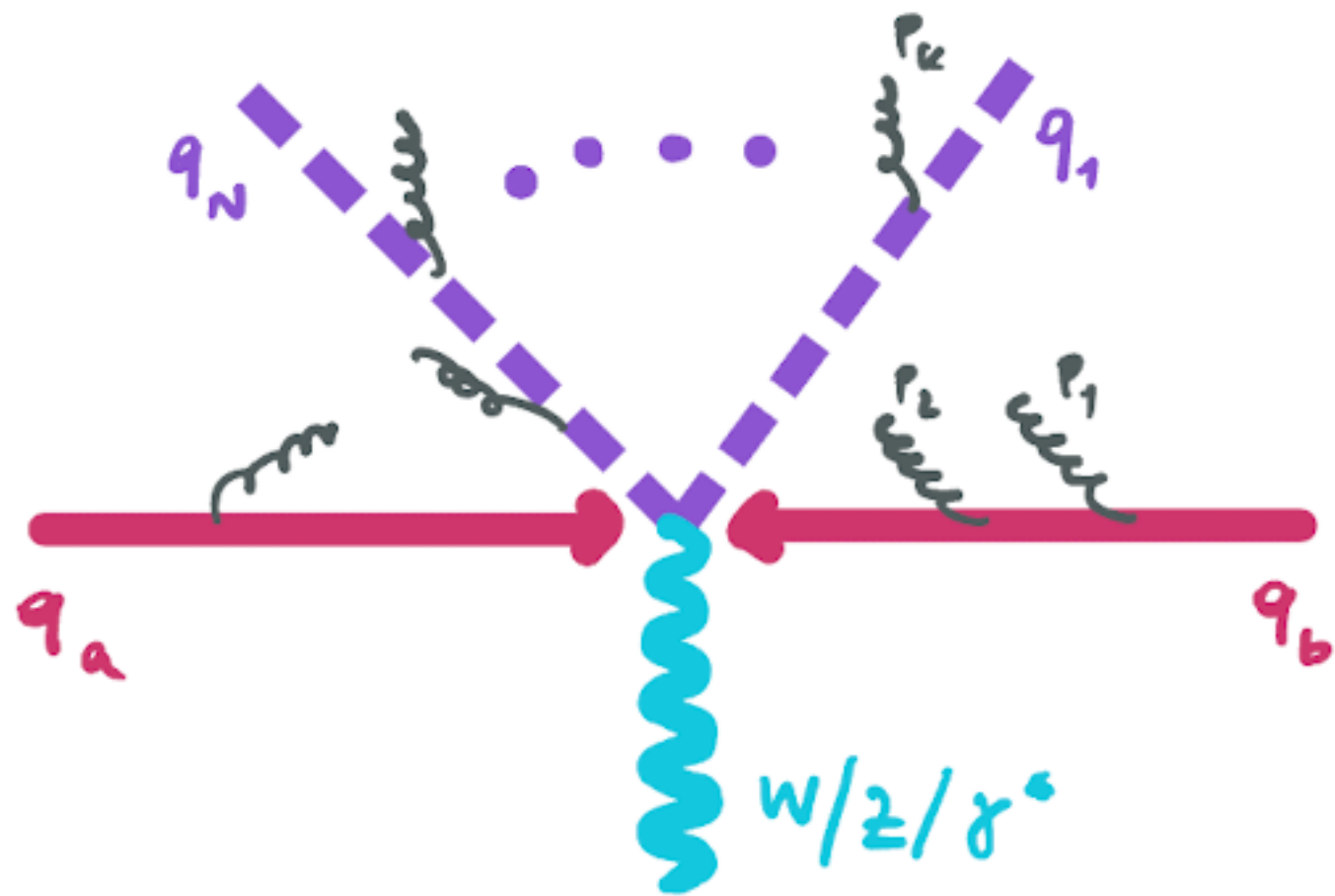
$\Phi_2$  Event

$$\frac{d\sigma}{d\Phi dr} = \frac{d\sigma^{\text{NNLL}'}}{d\Phi dr} - \frac{d\sigma^{\text{resExp}}}{d\Phi dr} + \frac{d\sigma^{\text{FO}}}{d\Phi dr}$$

# Intro: N-Jettiness

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- Historically only variable used in GENEVA (now also  $p_T$ )
- Geometric measure to tell how N-Jett like an event is:



$$\tau_N = \sum_{k=1} \min \left( 2p_k \cdot \hat{q}_a, 2p_k \cdot \hat{q}_b, 2p_k \cdot \hat{q}_1, \dots, 2p_k \cdot \hat{q}_N \right)$$

# Introduction

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- **Ingredients:**

- \* **NNLO FO events for V+j**

- \* **NNLL' resummation formula for  $\mathcal{T}_1$  with three coloured partons**

- \* **(optional) NNL resummation for  $\mathcal{T}_2$**

- \* **A suitable matching procedure to the shower**

# NNLO Validation

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•  $\mathcal{T}_1$  - slicing:

$$\Theta^{\delta\text{NNLO}_1}(\phi_1) = \left. \frac{d\sigma^{\text{N}^3\text{LL}}}{d\phi_1}(z_1^{\text{cut}}) \right|_{\sigma(\alpha_s^3)} \Theta(\phi_1) + \int_{z_1^{\text{cut}}}^{z_1^{\text{max}}} \frac{d\phi_2}{d\phi_1} \frac{d\sigma^{\delta\text{NNLO}_2}}{d\phi_2} \Theta(\phi_{2,3})$$



# NNLO Validation

- $\mathcal{T}_1$  - slicing:

$$\Theta^{\delta\text{NNLO}_1}(\phi_1) = \underbrace{\frac{d\sigma^{\text{N}^3\text{LL}}}{d\phi_1}(z_1^{\text{cut}})}_{\text{cumulant expanded}} \Big|_{\theta(\alpha_s^3)} \theta(\phi_1) + \int_{\tau_1^{\text{cut}}}^{\tau_1^{\text{max}}} \frac{d\phi_2}{d\phi_1} \frac{d\sigma^{\delta\text{NLO}_2}}{d\phi_2} \theta(\phi_{2,3})$$

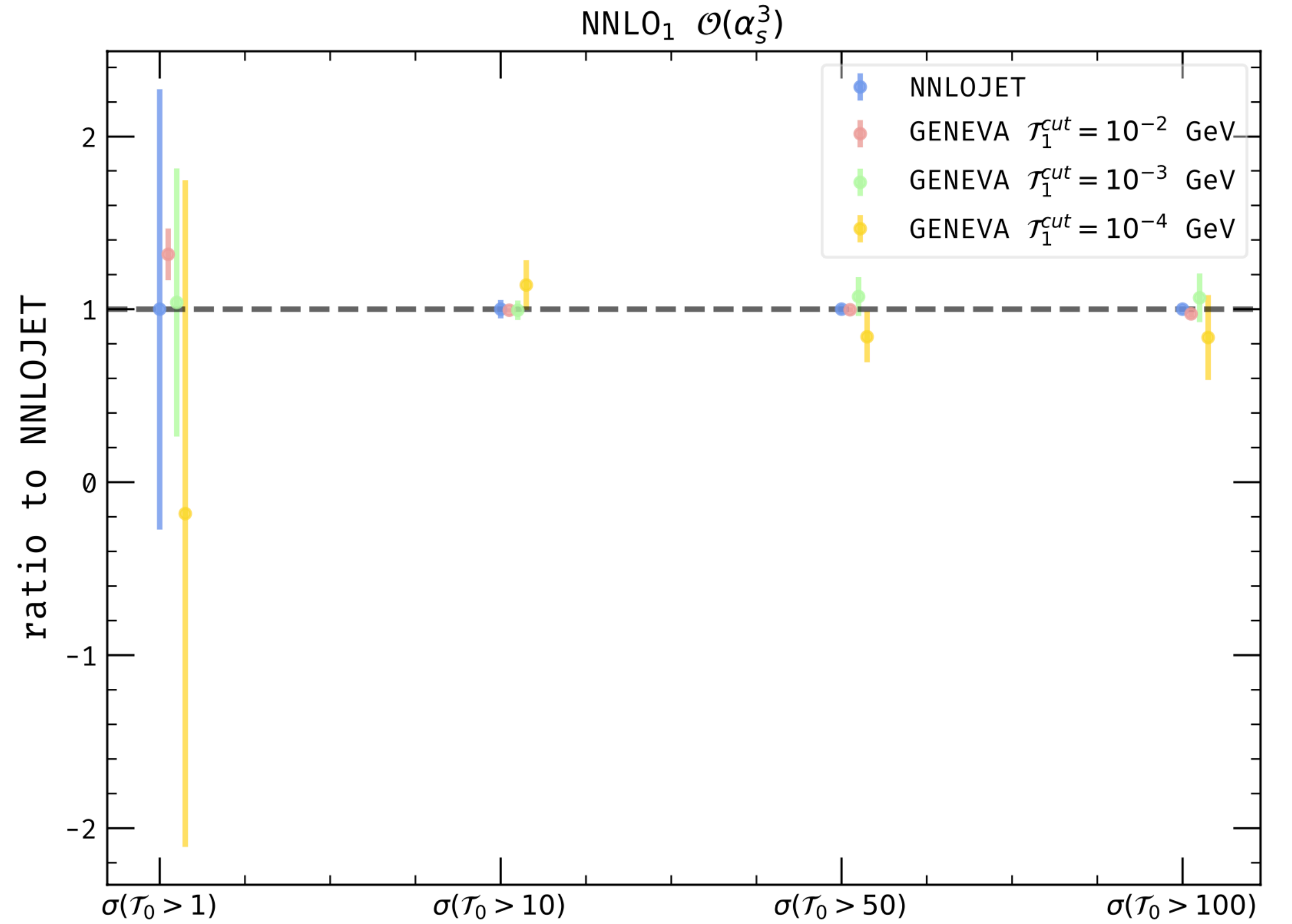
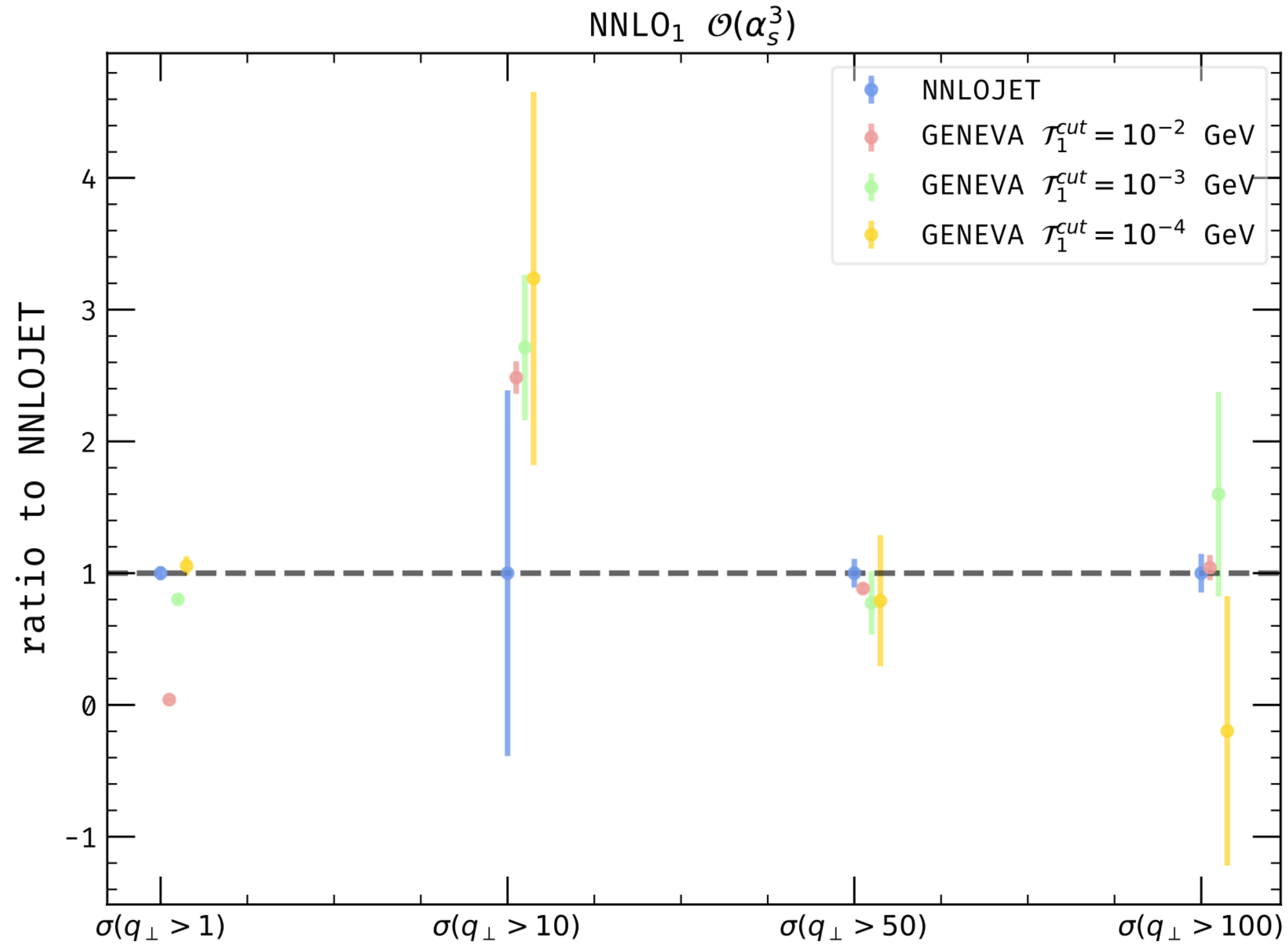
$\downarrow$  NLO with FKS

# NNLO Validation

•  $\mathcal{T}_1$  - slicing:

$$\theta^{\delta\text{NNLO}_1}(\phi_1) = \underbrace{\frac{d\sigma^{\text{N}^3\text{LL}}}{d\phi_1}(\tau_1^{\text{cut}})}_{\text{cumulant expanded}} \bigg|_{\mathcal{O}(\alpha_s^3)} \theta(\phi_1) + \int_{\tau_1^{\text{cut}}}^{\tau_1^{\text{max}}} \frac{d\phi_2}{d\phi_1} \frac{d\sigma^{\text{NNLO}_2}}{d\phi_2} \theta(\phi_{2,3})$$

NLO with FKS



# NNLO Validation

## • $\mathcal{T}_1$ - Subtraction:

$$\theta^{\text{NNLO}_1}(\phi_1) = \left. \frac{d\sigma^{\text{N}^3\text{LL}}}{d\phi_1}(\tau_1^{\text{cut}}) \right|_{\mathcal{O}(\alpha_s^3)} \theta(\phi_1) + \int_{\tau_1^{\text{cut}}}^{\tau_1^{\text{max}}} \frac{d\phi_2}{d\phi_1} \frac{d\sigma^{\text{NNLO}_2}}{d\phi_2} \theta(\phi_{2,3}) + \int_{\tau_1^\delta}^{\tau_1^{\text{cut}}} \frac{d\phi_2}{d\phi_1} \left[ \frac{d\sigma^{\text{NNLO}_2}}{d\phi_2} \theta(\phi_{2,3}) - \left. \frac{d\sigma^{\text{N}^3\text{LL}}}{d\phi_1 d\tau_1} \right|_{\mathcal{O}(\alpha_s^3)} \theta(\phi_1) \mathcal{P}(\phi_2) \right]$$

Non-singular

## • $\mathcal{T}_1$ - Dynamical Subtraction:

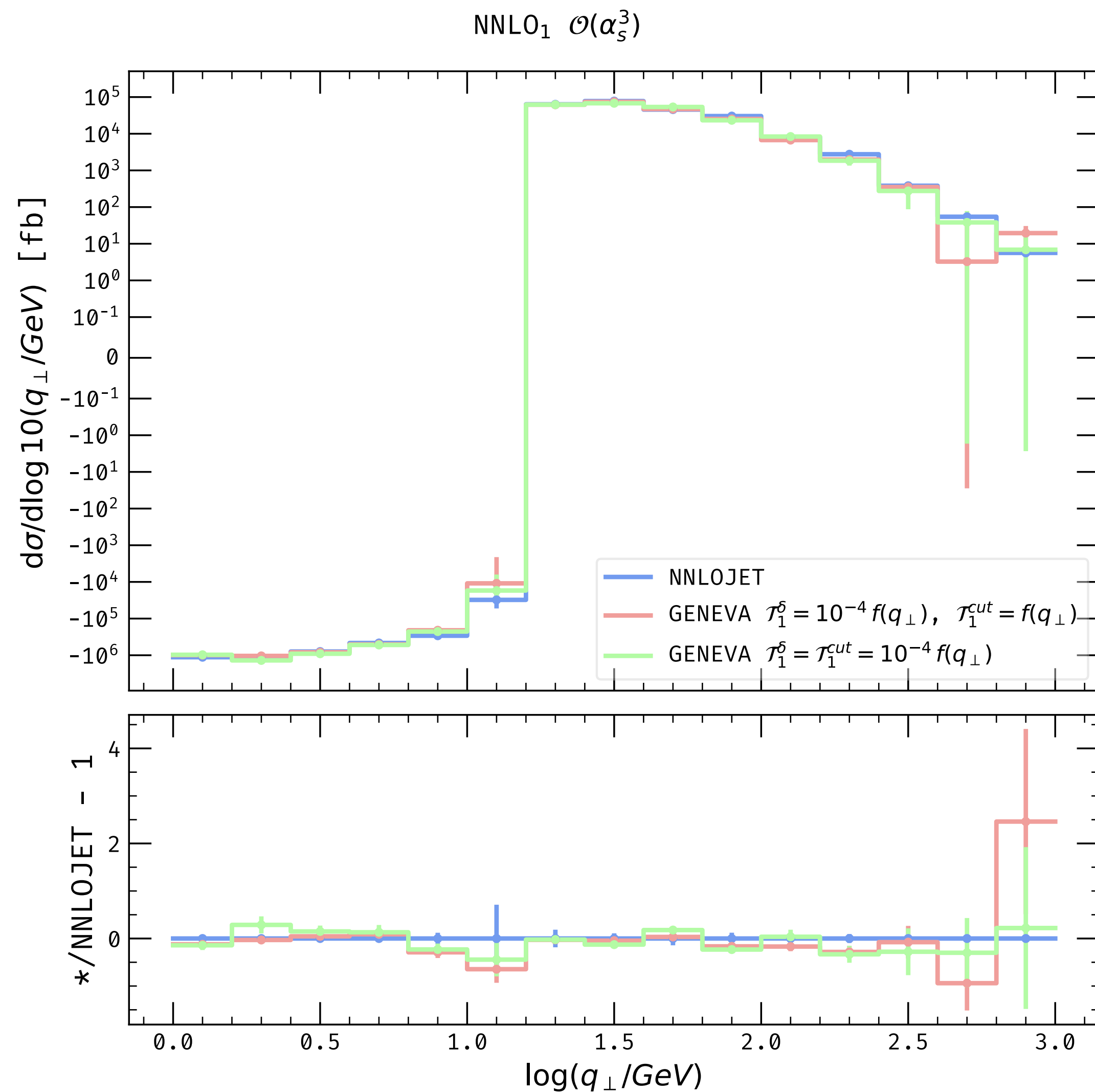
$$\mathcal{T}_1^{\text{cut}} \equiv \mathcal{T}_1^{\text{cut}}(q_\perp)$$

# NNLO Validation

## • $\mathcal{T}_1$ - Dynamical Subtraction:

$$\theta^{\text{NNLO}_1}(\phi_1) = \underbrace{\frac{d\sigma^{\text{N}^3\text{LL}}}{d\phi_1}(\tau_1^{\text{cut}})}_{\text{Cumulant expanded}} \Big|_{\mathcal{O}(\alpha_s^3)} \theta(\phi_1) + \int_{\tau_1^{\text{cut}}}^{\tau_1^{\text{max}}} \frac{d\phi_2}{d\phi_1} \frac{d\sigma^{\text{SNLO}_2}}{d\phi_2} \theta(\phi_{2,3})$$

$\int_{\tau_1^{\text{cut}}}^{\tau_1^{\text{max}}} \frac{d\phi_2}{d\phi_1} \left[ \frac{d\sigma^{\text{SNLO}_2}}{d\phi_2} \theta(\phi_{2,3}) - \frac{d\sigma^{\text{N}^3\text{LL}}}{d\phi_1 d\tau_1} \Big|_{\mathcal{O}(\alpha_s^3)} \theta(\phi_1) \mathcal{P}(\phi_2) \right]$ 
  
*Non-singular*



# Event Generation!

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- **Ingredients:**

- \* **NNLO FO events for V+j**



- \* **NNLL' resummation formula for  $\mathcal{T}_1$  with three coloured partons**

- \* **(optional) NNL resummation for  $\mathcal{T}_2$**

- \* **A suitable matching procedure to the shower**

$$\frac{d\sigma^{N^3LL}}{d\Phi_1 d\tau_1} = \sum_k H_k(\mu_H) B_k(x_{a_1}, \mu_B) B_k(x_{b_1}, \mu_B) J(\mu_J) S(\mu_S)$$

$$\frac{d\sigma^{N^3LL}}{d\Phi_1 d\tau_1} = \sum_k H_k(\mu_H) B_k(x_{a1}, \mu_B) B_k(x_{b1}, \mu_B) J(\mu_J) S(\mu_S)$$

● **Hard Function and hard evolution**

[Gehrmann-Tancredi, Becher-Neubert]

● **Beam Functions**

[Mistlberger et al, Becher-Bell, Gaunt et al]

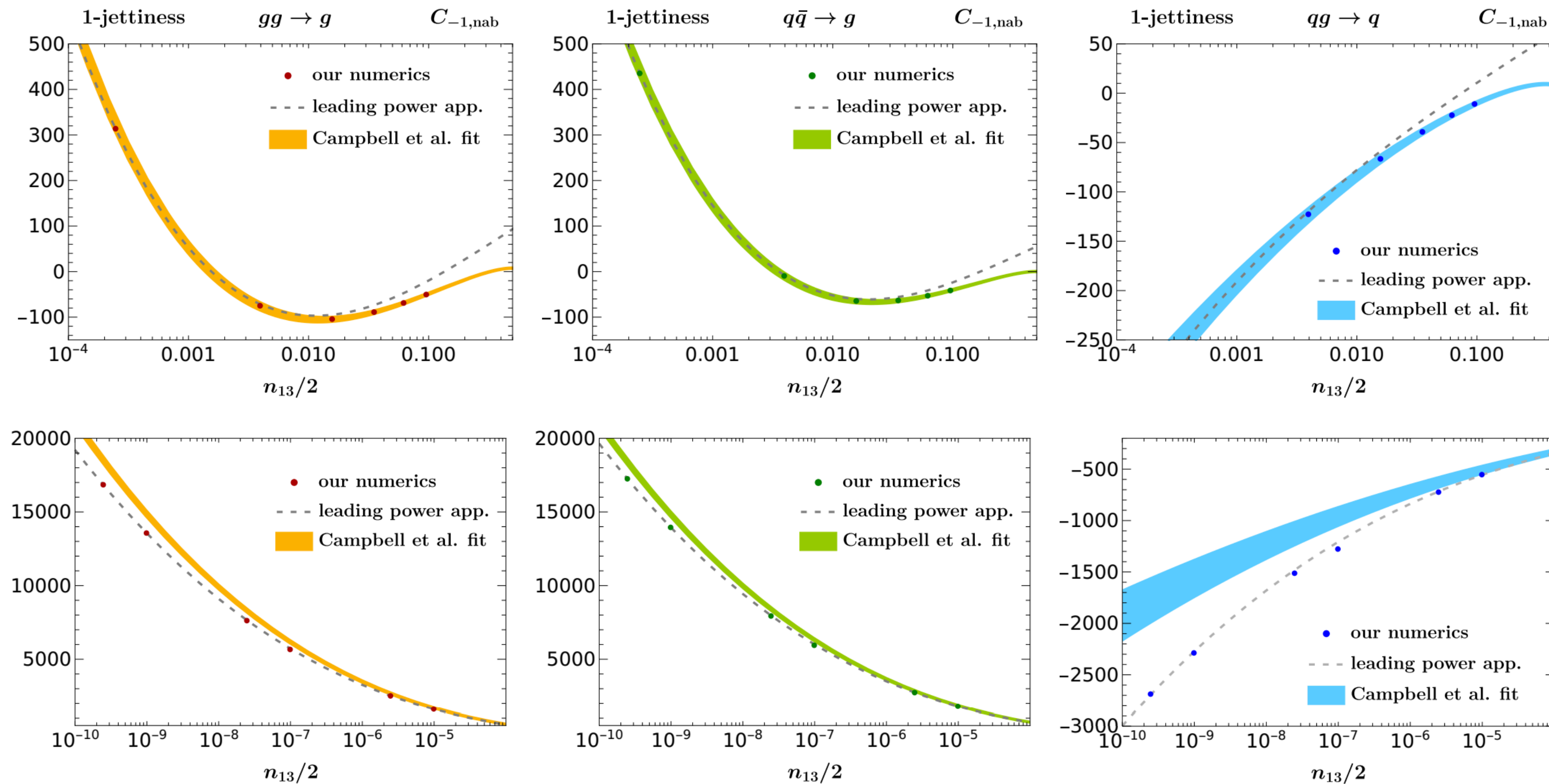
● **Jet Functions**

● **Soft Function and Evolution**



$$\frac{d\sigma^{N^3LL}}{d\phi_i d\tau_i} = \sum_k H_k(\mu_H) B_k(x_{a_i}, \mu_B) B_k(x_{b_i}, \mu_B) J(\mu_J) S(\mu_S)$$

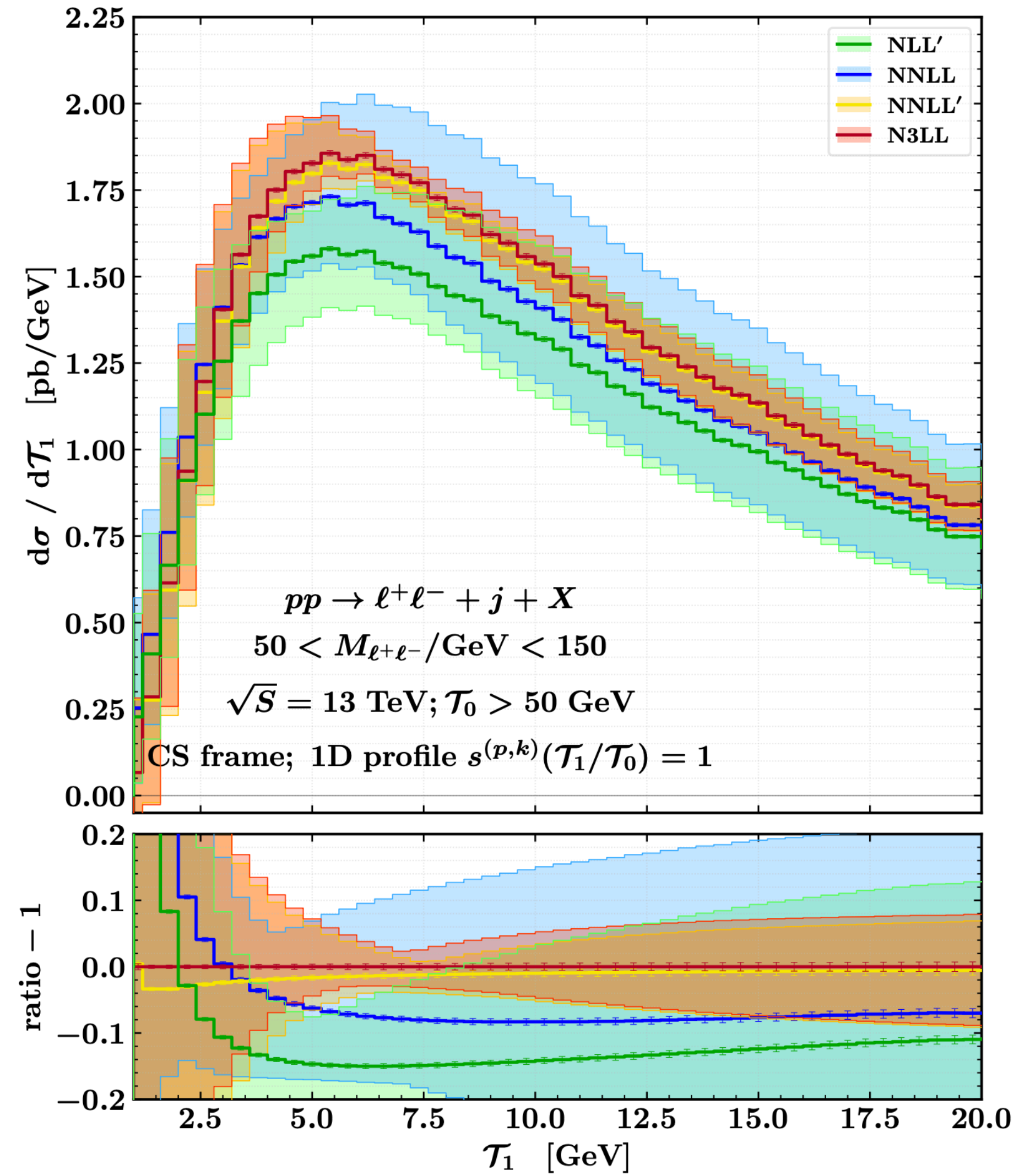
## Soft Function and Evolution: Boundary conditions from SoftServe



**Non-trivial kinematical dependence/better control of small angles**

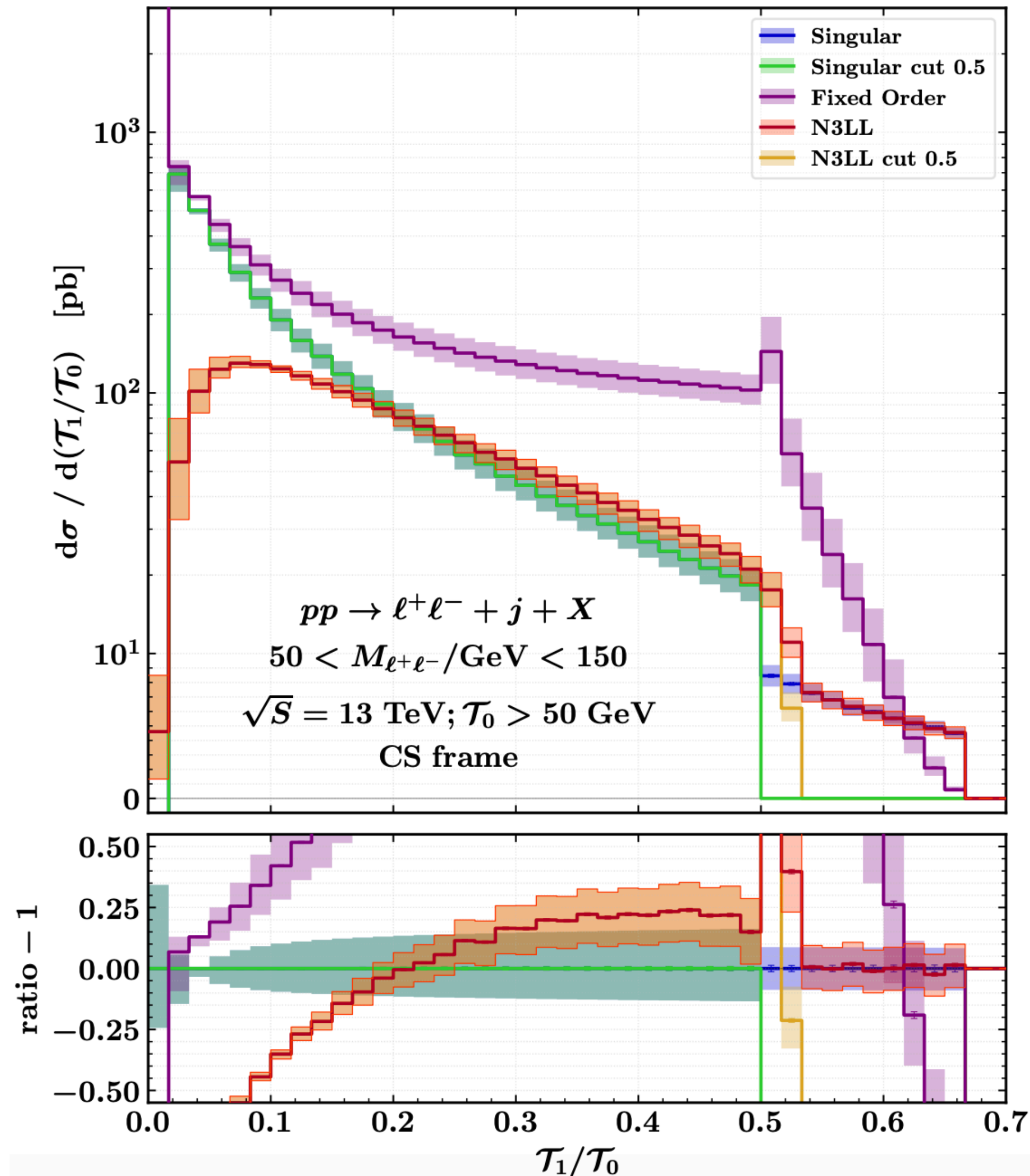


$$\frac{d\sigma^{\text{N}^3\text{LL}}}{d\Phi_1 d\mathcal{T}_1} = \sum_{\kappa} H_{\kappa}(\Phi_1, \mu_H) B_{\kappa}(x_a, \mathcal{T}_1, \mu_B) B_{\kappa}(x_b, \mathcal{T}_1, \mu_B) J_{\kappa}(\mathcal{T}_1, \mu_J) S_{\kappa}(\mathcal{T}_1, \mu_S)$$

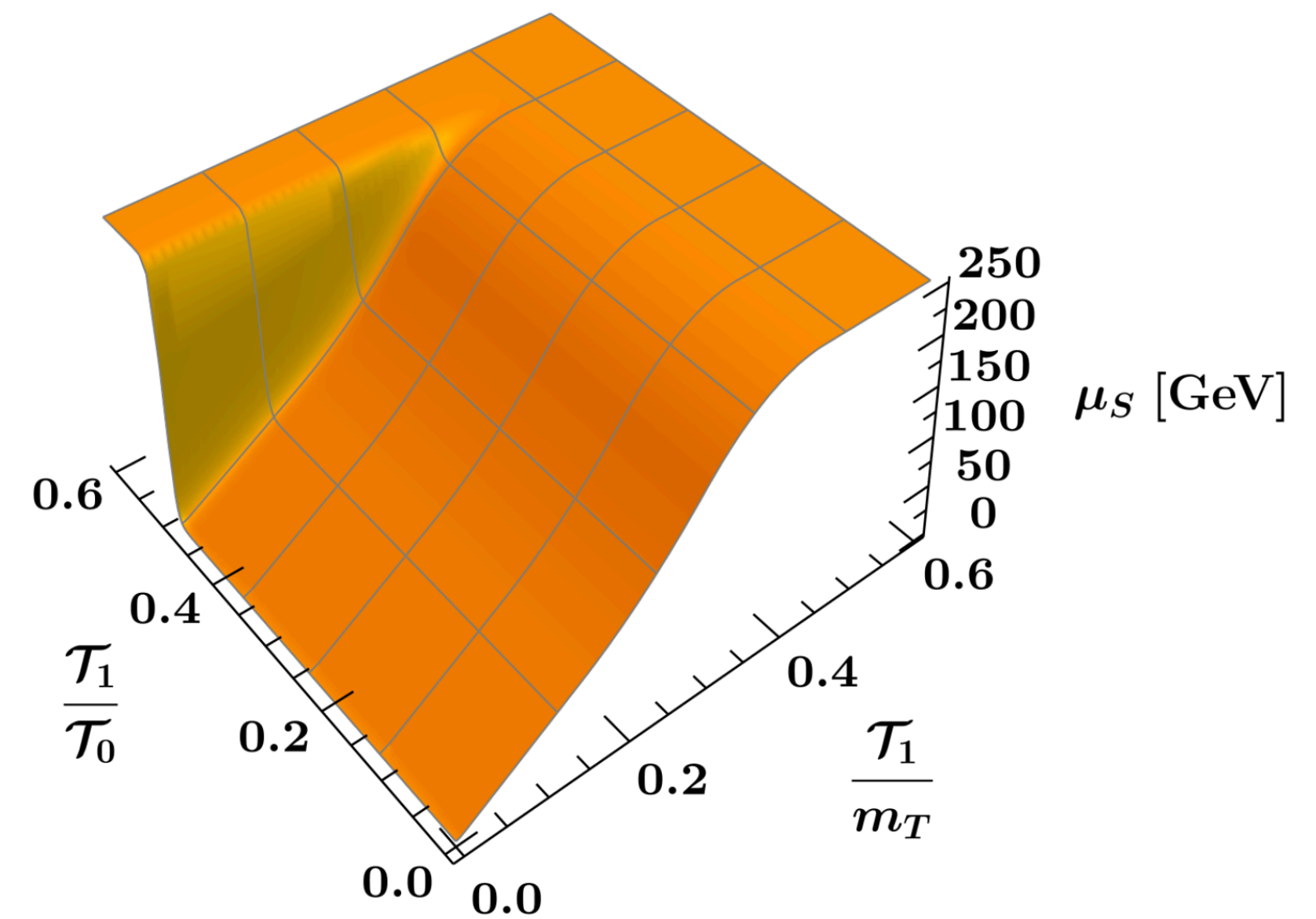


$$\frac{d\sigma^{N^3LL}}{d\Phi_1 d\mathcal{T}_1} = \sum_{\kappa} H_{\kappa}(\Phi_1, \mu_H) B_{\kappa}(x_a, \mathcal{T}_1, \mu_B) B_{\kappa}(x_b, \mathcal{T}_1, \mu_B) J_{\kappa}(\mathcal{T}_1, \mu_J) S_{\kappa}(\mathcal{T}_1, \mu_S)$$

• Need to impose kinematic constraint



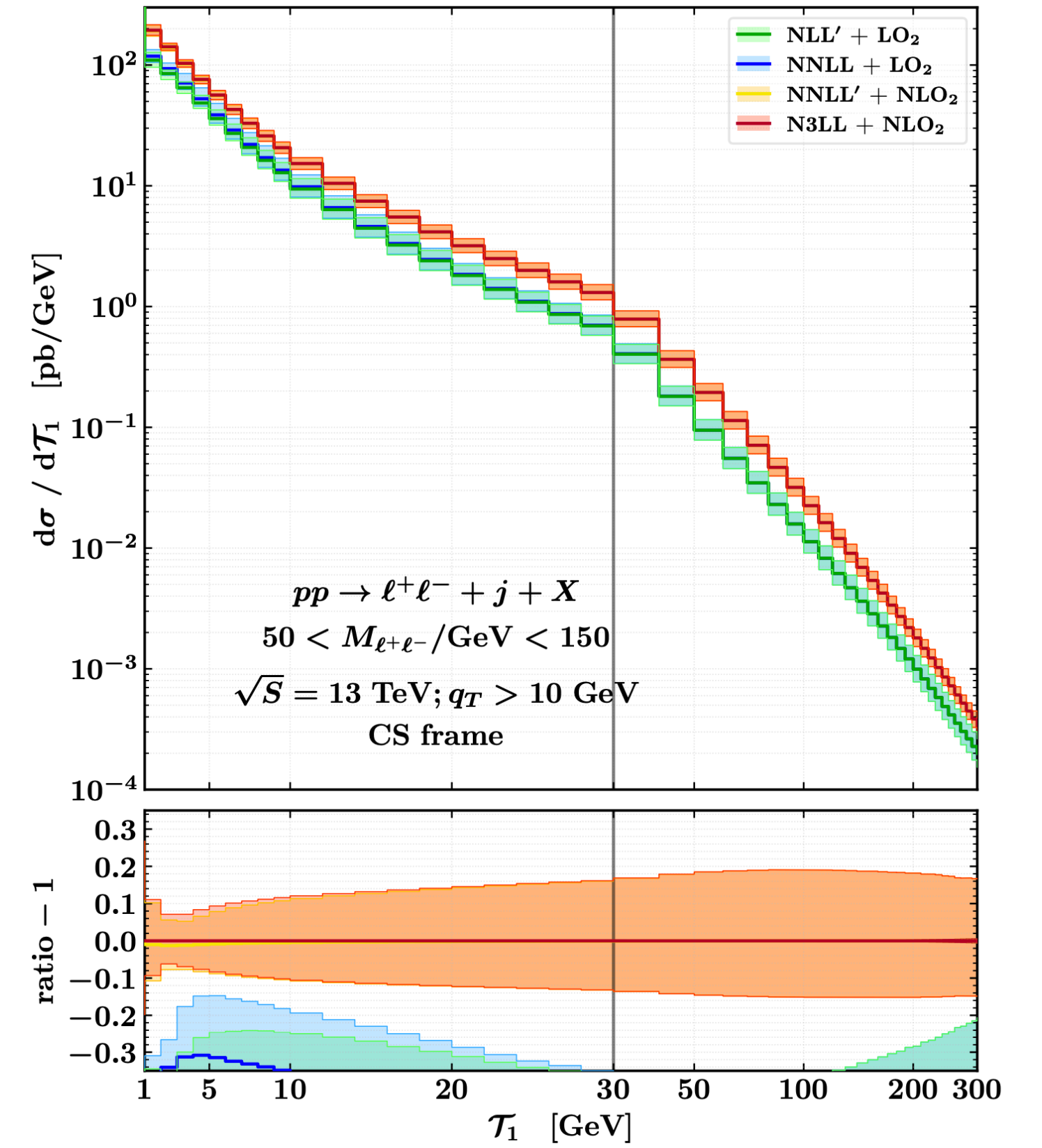
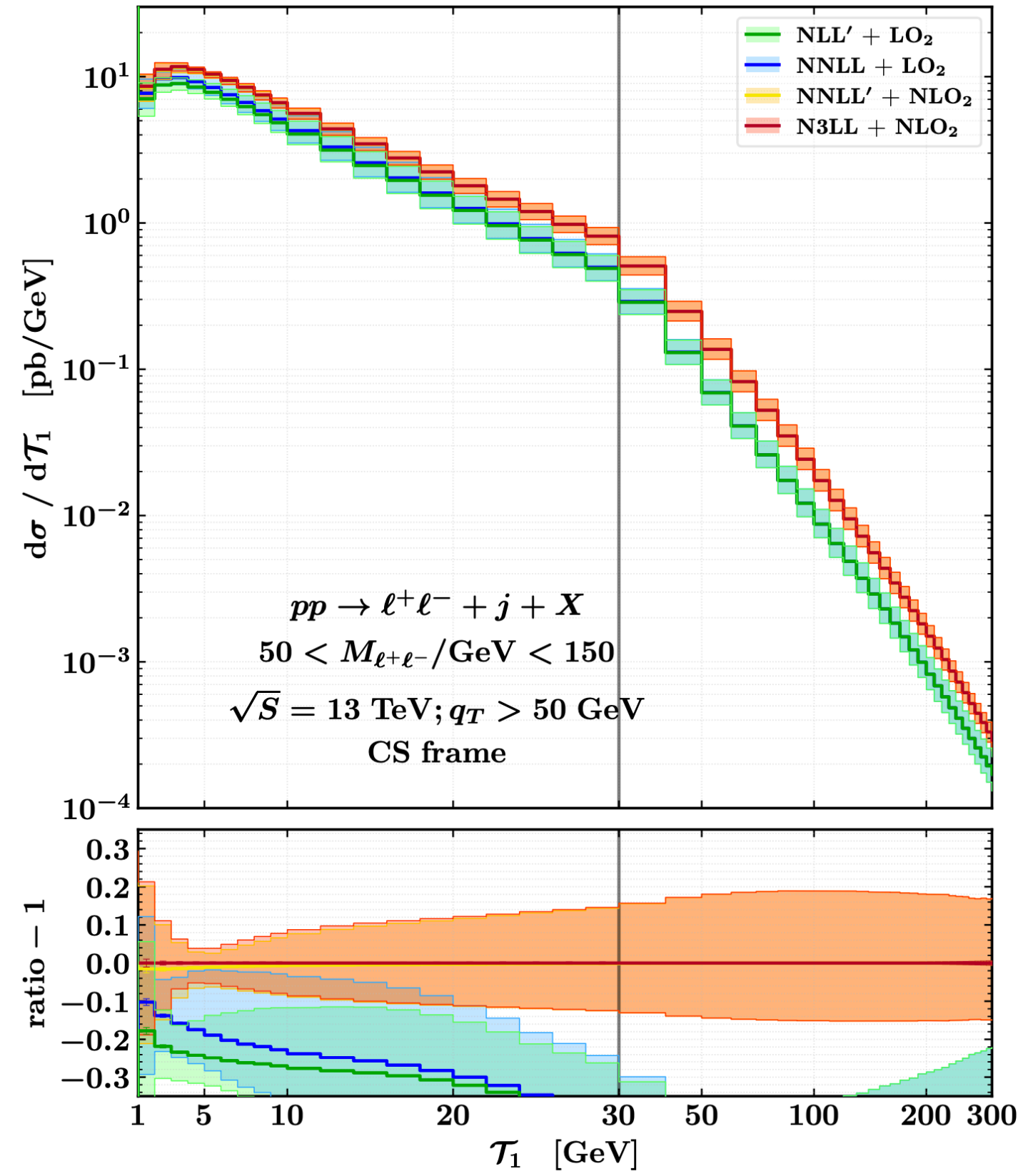
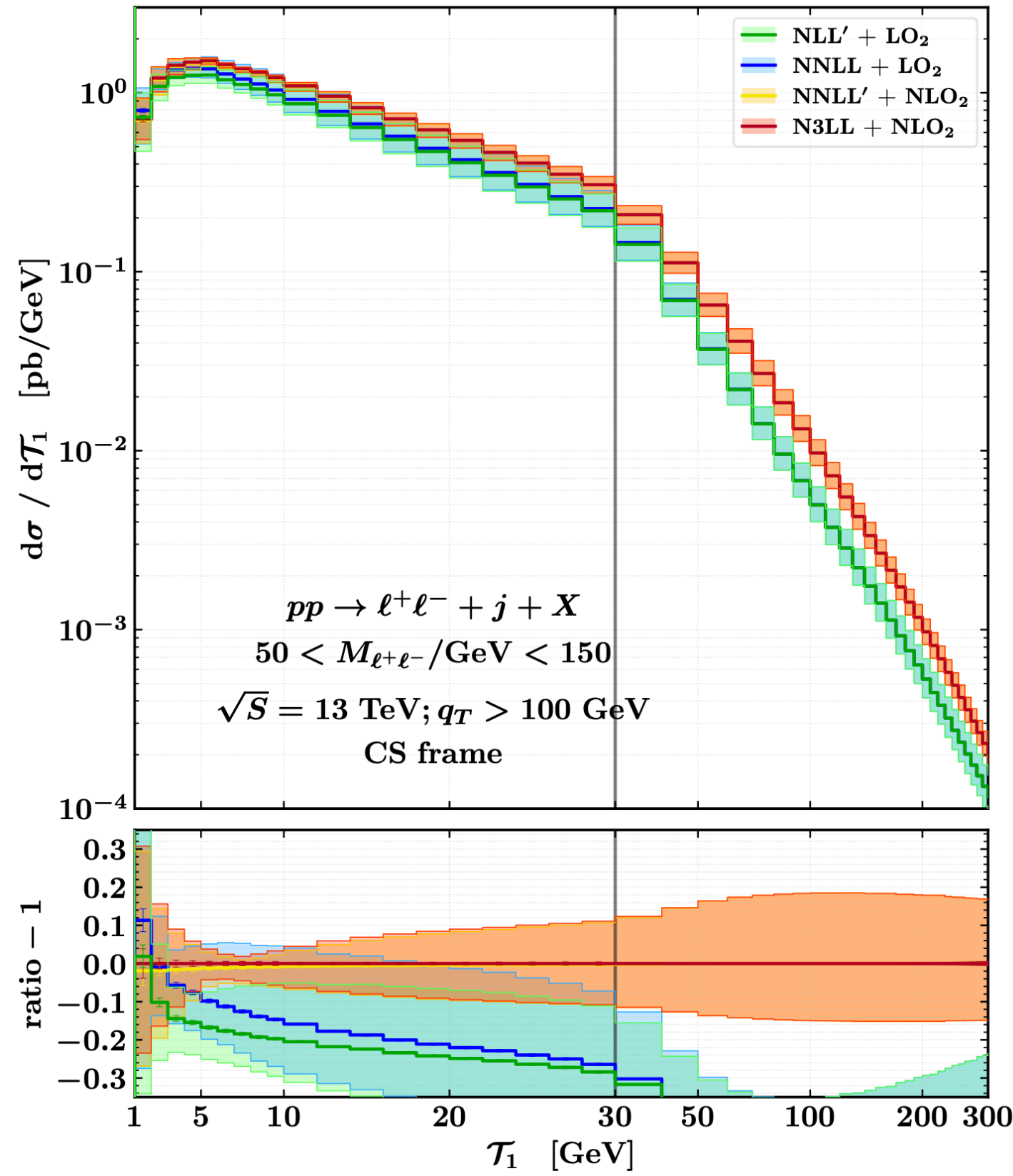
$$\frac{\mathcal{T}_1(\Phi_N)}{\mathcal{T}_0(\Phi_N)} \leq \frac{N-1}{N} = \begin{cases} 1/2, & N=2 \\ 2/3, & N=3 \end{cases}$$





# Matching FO and Resummation

$$\frac{d\sigma}{d\Phi dr} = \frac{d\sigma^{\text{resum}}}{d\Phi dr} - \frac{d\sigma^{\text{resExp}}}{d\Phi dr} + \frac{d\sigma^{\text{FO}}}{d\Phi dr}$$



• Sizeable contribution from non-singular for  $q_{\perp} \rightarrow 0$

# Resummation

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- **Ingredients:**

- \* **NNLO FO events for V+j** ✓

- \* **NNLL' resummation formula for  $\mathcal{T}_1$  with three coloured partons** ✓

- \* **(optional) NNL resummation for  $\mathcal{T}_2$**

- \* **A suitable matching procedure to the shower**

# Missing ingredients

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\* **Mapping for NLO<sub>2</sub> phase-space points:**  $\Phi_2^{\mathcal{T}_{1,q_\perp,\lambda_q}} \rightarrow \Phi_3^{\mathcal{T}_{1,q_\perp,\lambda_q}}$

**(required for subtraction and spreading of resummation)**




\* **Sudakov suppression of  $\Phi_2 \rightarrow \Phi_3$  events,  $\mathcal{T}_2$  resummation**

**(required for shower matching)**

# Conclusions

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## • Ingredients:

- \* NNLO FO events for V+j 
- \* NNLL' resummation formula for  $\mathcal{T}_1$  with three coloured partons 
- \* (optional) NNL resummation for  $\mathcal{T}_2$  
- \* A suitable matching procedure to the shower 