Department of Theoretical Physics

The path to NNLL accurate parton showers

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QCD@LHC 2024

Based on

JHEP 03 (2023) 224 [K. Hamilton, AK, G. P. Salam, L. Scyboz, R. Verheyen]
Phys.Rev.Lett. 131 (2023) 16 [S. Ferrario Ravasio, K. Hamilton, AK, G. P. Salam, L. Scyboz, G. Soyez]
2406.02661 [eid. + M. v. Beekveld, M. Dasgupta, B. K. El-Menoufi, J. Helliwell, P. F. Monni, A. Soto-Ontoso]

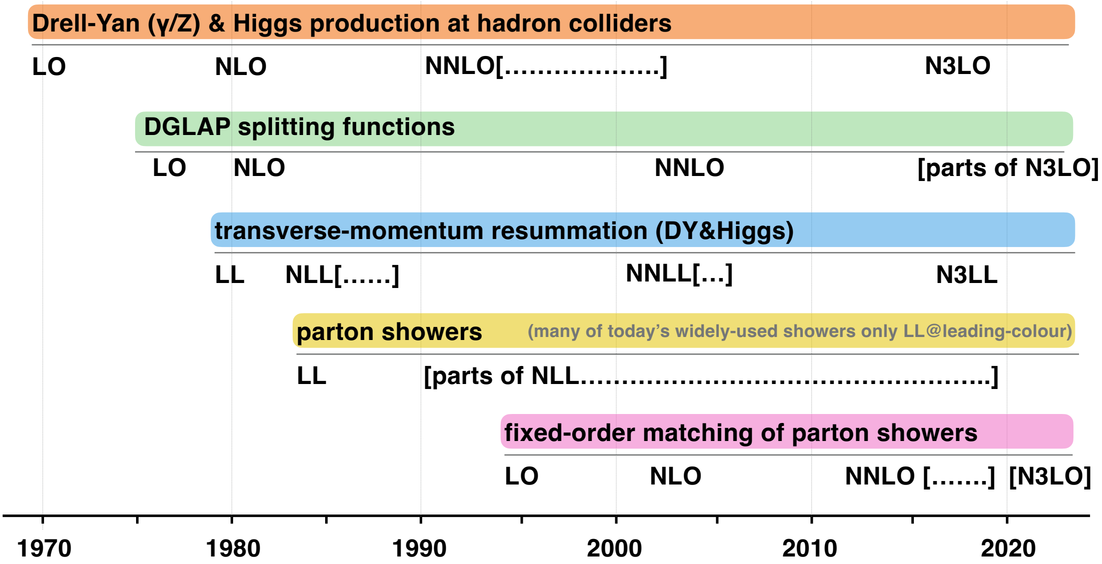
+

using analytic understanding developed in

JHEP 01 (2019) 083 [A. Banfi, B. K. El-Menoufi, P. F. Monni]
JHEP 12 (2021) 158 [M. Dasgupta, B. K. El-Menoufi]
JHEP 05 (2024) 09 [eid. + M. v. Beekveld, J. Helliwell, P. F. Monni]

10^{th} October 2024

selected collider-QCD accuracy milestones



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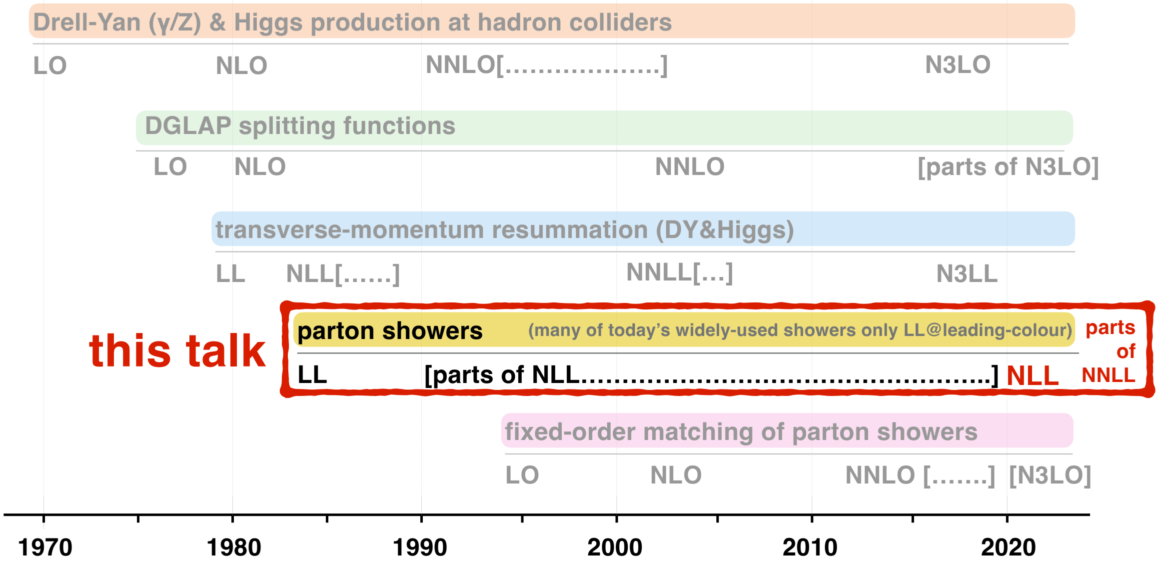

Textbook QFT

Traditionally, finite predictions in QFT can be made through (systematically improvable) perturbation theory

$$\sigma = \sum_{i=1} \alpha_S^i(\mu_R)\sigma^{(i)}(\mu_R)$$

But QFT perturbation theory has many unphysical features:

- UV singularities need renormalization
- IR singularities $\rightarrow$ individual renormalized diagrams not finite, only after summing unresolved (virtual) and resolved (real) diagrams (KLN theorem)
- Perturbative series not positive definite order by order
- Series not convergent in general
- Complexity grows exponentially $\rightarrow$ predictions restricted to low multiplicities

Would like to organise calculation such that it is positive definite and free of singularities $\Rightarrow$ Parton Showers!

Why are we talking about logarithmic accuracy?

Parton showers evolve hard states from $Q \sim \sqrt{\hat{s}}$ down to $\Lambda \sim 1$ GeV

This evolution generates logarithms of the form $L \sim \ln \frac{Q}{\Lambda} \gg 1$, $(g_X(\alpha_S L) \sim \alpha_S L)$

$$\Sigma(\mathcal{O} < e^{-L}) = \exp\left[-L g_{LL}(\alpha_S L) + g_{NLL}(\alpha_S L) + \alpha_S g_{NNLL}(\alpha_S L) + \ldots\right]$$

		$Q = M_Z$	$Q = 1$ TeV	
$\lvert L g_{LL} \rvert$	$\sim \alpha_S L^2$	2	4	
$\lvert g_{NLL} \rvert$	$\sim \alpha_S L$	0.5	0.6	$\leftarrow \mathcal{O}(100\%)$
$\lvert \alpha_S g_{NNLL} \rvert$	$\sim \alpha_S^2 L$	0.06	0.05	$\leftarrow \mathcal{O}(10\%)$

NNLL crucial to reach percent-level accuracy!

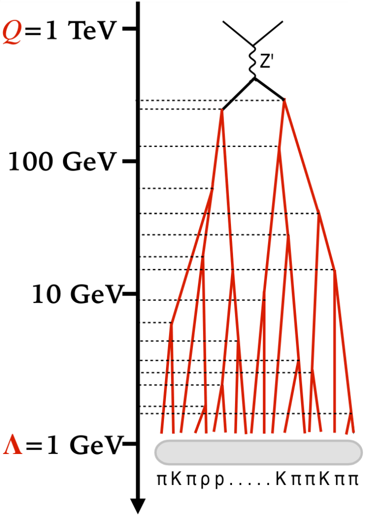

Figure by S. Ferrario Ravasio

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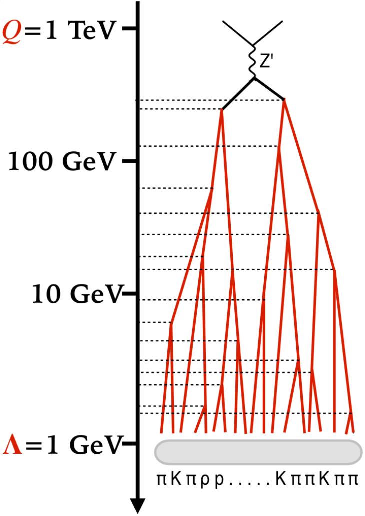


Figure by S. Ferrario Ravasio

Conceptual limitations

- Can we improve systematically from LL → NLL → NNLL → ...?
- How do we incorporate the hard scattering at high orders?

The ubiquitous Parton Shower

Pythia 8

An introduction to PYTHIA 8.2 #1

Torbjörn Sjöstrand (Lund U., Dept. Theor. Phys.), Stefan Ask (Cambridge U.), Jesper R. Christiansen (Lund U., Dept. Theor. Phys.), Richard Corke (Lund U., Dept. Theor. Phys.), Nishita Desai (U. Heidelberg, ITP) et al. (Oct 11, 2014)

Published in: *Comput.Phys.Commun.* 191 (2015) 159-177 • e-Print: 1410.3012 [hep-ph]

pdf links DOI cite 6,423 citations

Herwig 7

Herwig++ Physics and Manual #1

M. Bahr (Karlsruhe U., ITP), S. Gieseke (Karlsruhe U., ITP), M.A. Gigg (Durham U., IPPP), D. Grellscheid (Durham U., IPPP), K. Hamilton (Louvain U.) et al. (Mar, 2008)

Published in: *Eur.Phys.J.C* 58 (2008) 639-707 • e-Print: 0803.0883 [hep-ph]

pdf links DOI cite 3,150 citations

Sherpa

Event generation with SHERPA 1.1 #1

T. Gleisberg (SLAC), Stefan. Hoeche (Zurich U.), F. Krauss (Durham U., IPPP), M. Schonherr (Dresden, Tech. U.), S. Schumann (Edinburgh U.) et al. (Nov, 2008)

Published in: *JHEP* 02 (2009) 007 • e-Print: 0811.4622 [hep-ph]

pdf links DOI cite 3,827 citations

Parton Showers enter one way or another in almost 95% of all ATLAS and CMS analyses. Collider physics would not be the same without them.

CMS [2312.16343]

Current status on parton showers

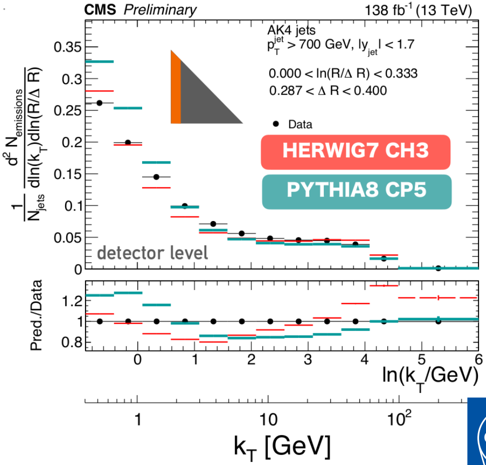
- The most widely-used event generators, Pythia, Herwig, and Sherpa, are all formally limited to LL
- Overall they do a good job at the LHC, but places where big differences are seen

→ very differential phase space regions of jets are associated with 10 – 30% differences

- Feeds into many analysis, becoming even more important to get right as machine learning will learn wrong features

Lund Plane

(Recent CMS results; also ATLAS & ALICE)



The NLL revolution

The PanScales collaboration has lead the effort to go beyond LL.

core principle for NLL showers:

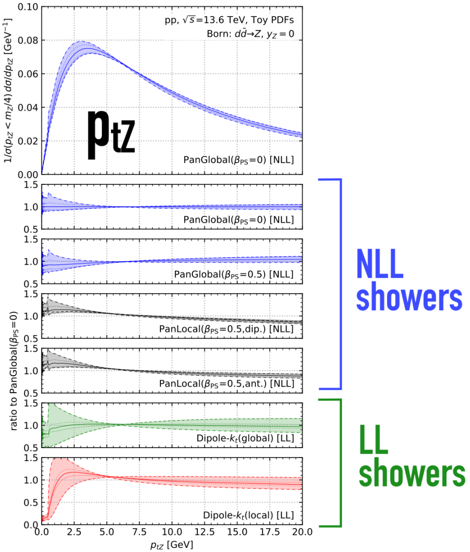
QCD factorisation $\Rightarrow$ Parton showers must correctly reproduce QCD matrix elements in single soft/collinear limits, where QFT amplitudes factorise

This principle is violated by most standard showers!

Other work

NLL also achieved by other groups: Alaric Herren, Höche, Krauss, Reichelt, Schoenherr [2208.06057], [2404.14360], Apollo Preuss [2403.19452], Deductor Nagy, Soper [2011.04773], and Forshaw-Holguin-Plätzer [2003.06400]. Two latter with additional novelty due to amplitude evolution.

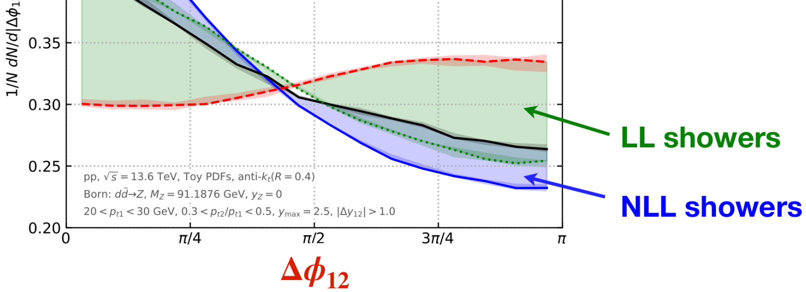
PanScales [2207.09467]

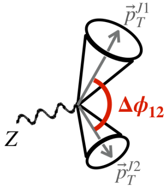


Part of advantage in LL→NLL is reduction in residual scale uncertainties for inclusive quantities like the transverse momentum of the Z

PanScales [2207.09467]

$m_{\ell\ell} = m_Z$

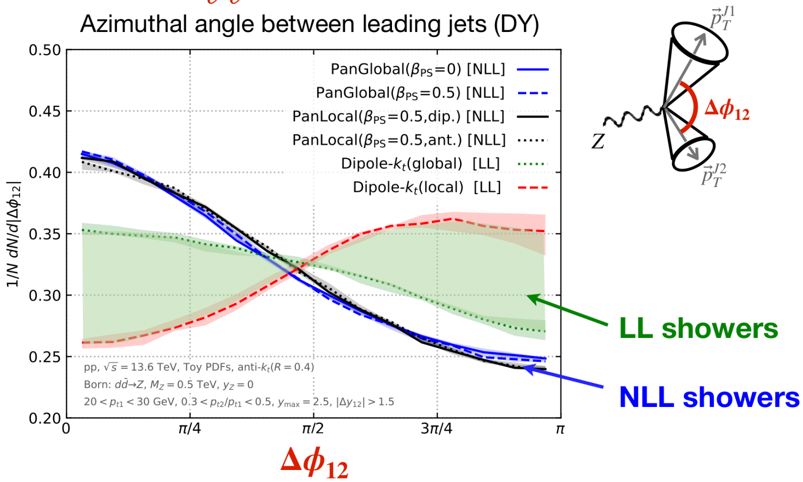




Larger shape differences can be observed in more exclusive observables

PanScales [2207.09467]

$m_{\ell\ell} = 500$ GeV



Larger shape differences can be observed in more exclusive observables

further enhanced at large scales!

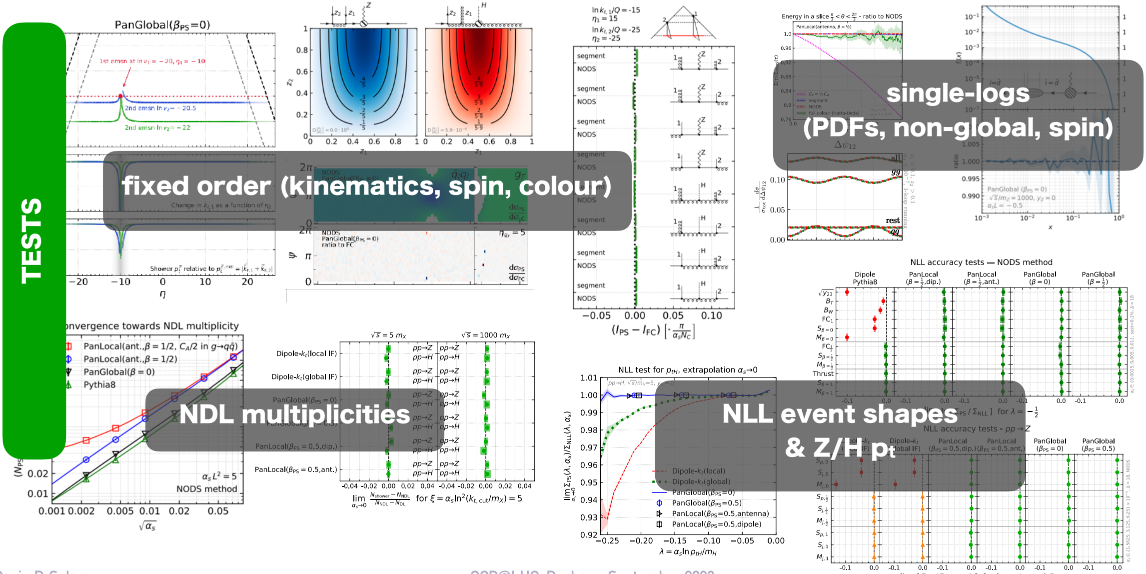
PanLocal	PanGlobal	Colour	Spin
$k_t\sqrt{\theta}$ ordered	k_t or $k_t\sqrt{\theta}$ ordered	nested ordered double soft (NODS)	for correct azimuthal structure in collinear and soft→collinear
Recoil $\perp$: local, $+$: local, $-$: local	**Recoil** $\perp$: global, $+$: local, $-$: local	Designed to ensure LL are full colour (also gets many NLL at full colour)	[Collins-Knowles extended to soft sector]
Dipole partition event CoM	**Dipole partition** event CoM		

e^+e^-: Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez [2002.11114]; pp (w/spin+colour): van Beekveld, Ferrario Ravasio, Salam, Soto-Ontoso, Soyez, Verheyen [2205.02237]; + pp tests: eid. + Hamilton [2207.09467]; DIS+VBF: van Beekveld, Ferrario Ravasio [2305.08645]

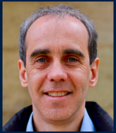
Hamilton, Medves, Salam, Scyboz, Soyez [2011.10054]

AK, Salam, Scyboz, Verheyen [2103.16526], eid. + Hamilton [2111.01161]

a selection of the logarithmic accuracy tests



Oxford


Gavin Salam


Nicolas Schalch


Silvia Zanoli

Monash


Ludovic Scyboz


Basem El-Menoufi


Jack Helliwell

CERN


AK


Pier Monni


Silvia Ferrario Ravasio

UCL


Keith Hamilton

Manchester


Mrinal Dasgupta

NIKHEF


Melissa van Beekveld

IPhT


Gregory Soyez

UGR


Alba Soto-Ontoso

PanScales current members

A project to bring logarithmic understanding and accuracy to parton showers

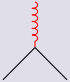
Going beyond NLL

Already made significant progress, in a few years, taking NLL $\to$ NNLL.

core additional principle for NNLL showers:

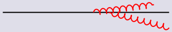
achieve QCD factorisation also for commensurate scale "pair" of emissions $\Rightarrow$

1 hard emission



double-soft emissions

triple-collinear emissions



- leading-order α_s matching $\to$ Hamilton, AK, Salam, Scyboz, Verheyen [2301.09645]
- double-soft emissions $\to$ Ferrario Ravasio, Hamilton, AK, Salam, Scyboz, Soyez [2307.11142]
- parts of triple-collinear $\to$ Dasgupta, El-Menoufi [2109.07496], eid. + van Beekveld, Helliwell, Monni [2307.15734], eid. + AK [2402.05170], PanScales [2406.02661]

Analytic structure beyond NLL

Taking an event shape, $\mathcal{O}$, to be less than some value $e^{-|L|}$ we have at NNLL (focusing for now on e^+e^- only)

$$\Sigma(\mathcal{O} < e^{-|L|}) = (1 + \alpha_s C_1 + \dots) \exp\left[\frac{1}{\alpha_s} g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots\right] \quad (1)$$

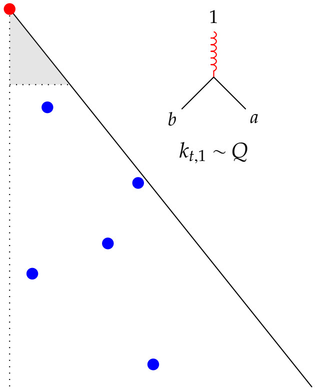
where g_1 accounts for LL terms, g_2 for NLL terms, and g_3 and C_1 for NNLL terms[1]. Whereas an analytic resummation in principle retains only the terms that are put in (i.e. g_1 and g_2 at NLL) the shower will instead generate spurious higher order terms

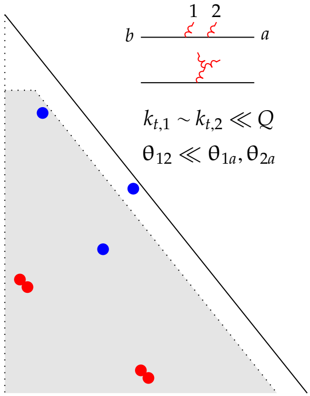
$$\Sigma(\mathcal{O} < e^{-|L|}) = (1 + \alpha_s \tilde{C}_1 + \dots) \exp\left[\frac{1}{\alpha_s} g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s \tilde{g}_3(\alpha_s L) + \dots\right] \quad (2)$$

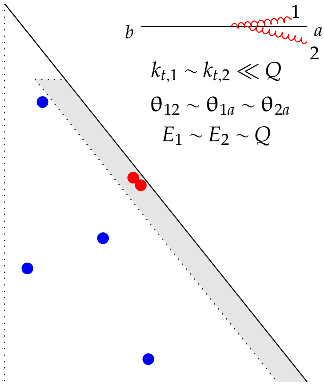
When thinking about going beyond NLL we need to address two things: 1) what are the necessary analytic ingredients from resummation and 2) how do we compensate the NNLL terms already present in the shower?

[1] In the language of q_T resummation A_1 is responsible for LL terms, A_2 and B_1 for NLL terms and A_3 and B_2 for NNLL terms (together with the hard coefficient function $C_1(z)$).

Lund plane picture







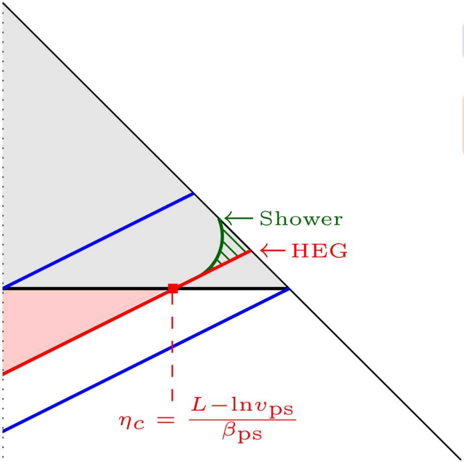
hard matching → α_S correct for first emission

double-soft → get any pair of soft commen- -surate energy/angle right

triple-collinear → account for genuine $2 \to 4$ collinear splittings

Hamilton, AK, Salam, Scyboz, Verheyen [2301.09645]

Match without breaking NLL



Standard matching → don't break fixed-order!

Log-aware matching → first step in improving the shower log accuracy!

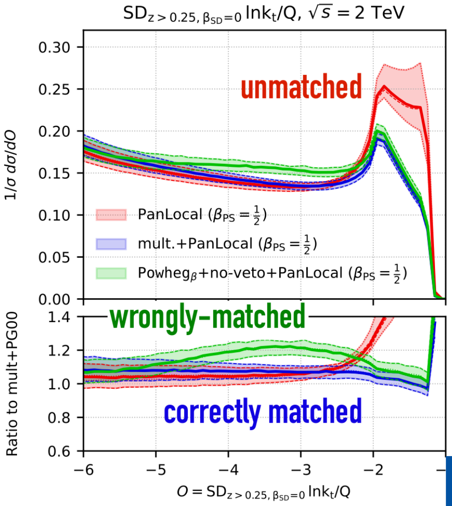
- Existing matching schemes not necessarily suited.
- Main concern related to kinematic mismatch between shower and hardest emission generator. This issue has been studied in the past Corke, Sjöstrand [1003.2384] but logarithmic understanding is new.
- Further subtelty in how shower partitions $g \to gg$ splitting function

Hamilton, AK, Salam, Scyboz, Verheyen [2301.09645]

Phenomenological impact

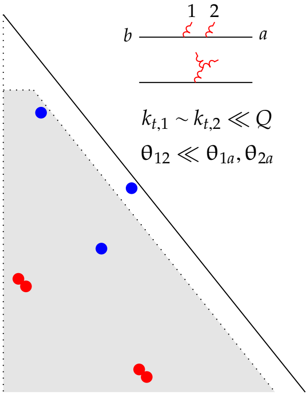
- Contour mismatch by area $\alpha\Delta$ leads to breaking of NLL and exponentiation
- Correct matching on the other hand augments the shower from NLL to NLL+NNDL for event shapes.
- Impact of NLL breaking terms vary - for SoftDrop they have a big impact due to the single-logarithmic nature of the observable. In particular the breaking manifests as terms with super-leading logs

$$\partial_L \Sigma_{SD}(L) = \bar{\alpha} c\, e^{\bar{\alpha} c L - \bar{\alpha}\Delta} - 2\bar{\alpha} L e^{-\bar{\alpha} L^2}\left(1 - e^{-\bar{\alpha}\Delta}\right)$$



Ferrario Ravasio, Hamilton, AK, Salam, Scyboz, Soyez [2307.11142]

Include double-soft corrections



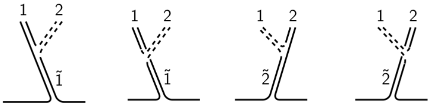
Double-soft corrections necessary for general NNLL accuracy → sufficient for large classes of observables

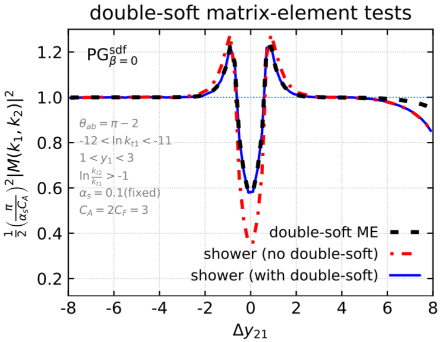
Achieves NNDL ($\alpha_s^n L^{2n-2}$) for multiplicities and NSL ($\alpha_s^n L^{n-1}$) for non-global observables (at leading colour)

We implement through multiplicative matrix element correction, care needed to get correct NLO normalisation (interplay with K_{CMW})

Ferrario Ravasio, Hamilton, AK, Salam, Scyboz, Soyez [2307.11142]

The double-soft ME





- Any two-emission configuration in a dipole-shower comes with a number of histories
- We accept any such configuration with the true ME divided by the shower's effective double-soft ME summed over all histories that could have lead to that configuration
- NB: Efficiency depends on shower overestimate!

$$P_{\mathrm{accept}} = \frac{|M^2_{\mathrm{DS}}|}{\sum_h |M^2_{\mathrm{shower},h}|}$$

Ferrario Ravasio, Hamilton, AK, Salam, Scyboz, Soyez [2307.11142]

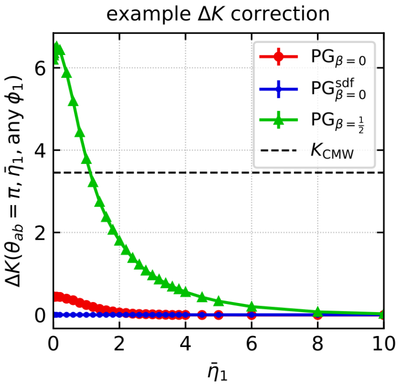
...and associated virtuals!

- Shower needs to reproduce both real and virtual contributions!
- Virtuals are always included through the Sudakov veto and an effective coupling (CMW-coupling)

$$\alpha_s \to \alpha_s + \alpha_s^2 K_1/2\pi$$

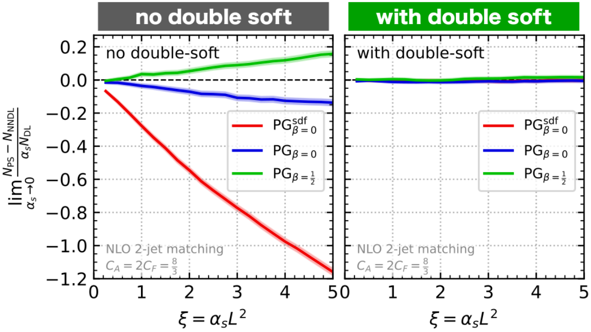
- Shower-recoil effectively modifies the Sudakov/coupling → needs compensating term ΔK_1

$$\Delta K_1 = \int d\Phi^{(\mathrm{PS})}_{12/\tilde{1}} |M^{(\mathrm{PS})}_{12/\tilde{1}}|^2 - \int d\Phi^{(\mathrm{PS})}_{12/\tilde{1}_{sc}} |M^{(\mathrm{PS})}_{12/\tilde{1}_{sc}}|^2.$$



Ferrario Ravasio, Hamilton, AK, Salam, Scyboz, Soyez [2307.11142]

Lund Multiplicities at NNDL ($\alpha_s^n L^{2n-2}$)



$$\lim_{\alpha_S \to 0} \frac{N_{(PS)} - N_{NNDL}}{\alpha_S N_{DL}}\bigg|_{\text{fixed } \alpha_S L^2}$$

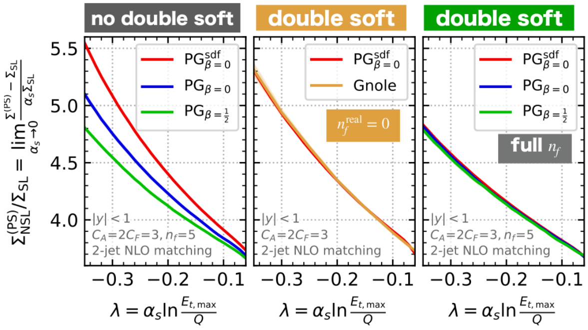
Reference NNDL analytic result from Medves, Soto-Ontoso, Soyez [2205.02861]

Showers without double-soft corrections show clear differences from reference (and each other).

Adding the double-soft corrections brings NNDL agreement.

Ferrario Ravasio, Hamilton, AK, Salam, Scyboz, Soyez [2307.11142]

Energy in a slice at NSL ($\alpha_s^n L^{n-1}$)



$$\lim_{\alpha_S \to 0} \frac{\Sigma^{(PS)} - \Sigma_{SL}}{\alpha_S}\bigg|_{\text{fixed } \alpha_S L}$$

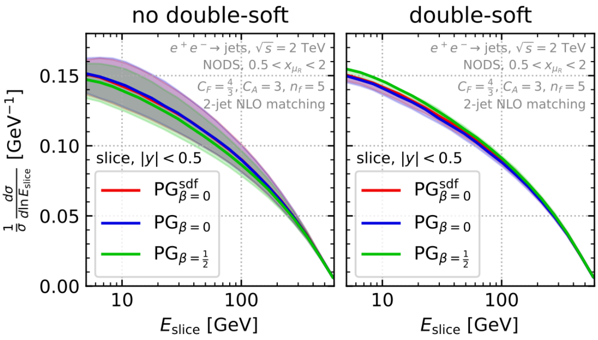
Reference NSL from Gnole Banfi, Dreyer, Monni [2111.02413] (see also Becher, Schalch, Xu [2307.02283]).

We did this test semi-blind: only compared to Gnole after we had agreement between the three PanGlobal variants.

We have NSL agreement with Gnole (using $n_f^{\text{real}} = 0$) and agreement between all showers with full-n_f dependence (first calculation of this kind as a by-product!)

Ferrario Ravasio, Hamilton, AK, Salam, Scyboz, Soyez [2307.11142]

What about pheno?



- We studied energy flow between two hard (1 TeV) jets as a preliminary pheno case
- The three PanGlobal variants are remarkably close without double-soft corrections, but have large uncertainties
- With double-soft corrections we see a small shift in central values but a significant reduction in uncertainties.

Compute triple-collinear ingredients

Double-soft corrections are not sufficient to reach NNLL accuracy for event shapes. Need triple-collinear ingredients (cf. Dasgupta, El-Menoufi [2109.07496], eid. + van Beekveld, Helliwell, Monni [2307.15734], eid. + AK [2402.05170] for work in this direction)

However, with the inclusion of real double-soft emissions, only the Sudakov form factor needs to be modified to reach NNLL, i.e. we do not need the fully differential triple-collinear structure (hot off the press: van Beekveld, Dasgupta, El-Menoufi, Helliwell, Monni, Salam [2409.08316])

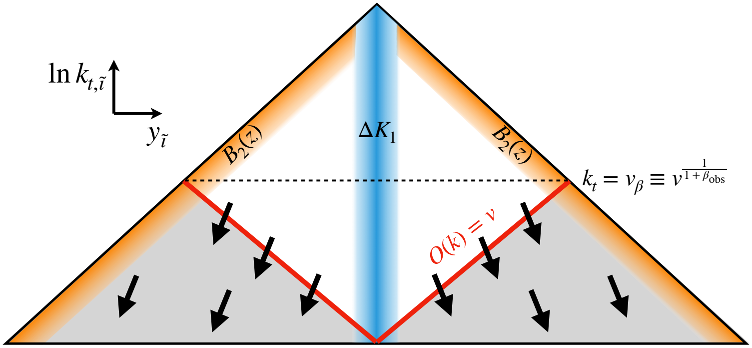
Taking

$$\alpha_{\mathrm{eff}} = \alpha_s \left[1 + \frac{\alpha_s}{2\pi} \left(K_1 + \Delta K_1(y) + B_2(z) \right) + \frac{\alpha_s^2}{4\pi^2} K_2 \right]$$

there are two pieces missing - B_2 which is of triple-collinear origin [2109.07496], [2307.15734] and K_2 (A_3) which is known Banfi, El-Menoufi, Monni [1807.11487], Catani, De Florian, Grazzini [1904.10365]

NB: NLL showers generate spurious $\tilde{B}_2$ and $\tilde{K}_2$ $\rightarrow$ must be compensated by ΔB_2 and ΔK_2

An intuitive picture



Recoil induces a drift of emissions in the Lund plane. Main novelty here is numerical compensation. Shower but not observable dependent!

Relation between shower and resummation ingredients

It is fairly straightforward to see that at NNLL we only depend on ΔK_1 and B_2 through their respective integrals

$$\Delta K_1^{\text{int}} \equiv \int_{-\infty}^{\infty} dy\, \Delta K_1(y)\,, \; B_2^{\text{int}} \equiv \int_0^1 dz \frac{P_{gq}(z)}{2C_F} B_2(z).$$

These (and K_2) can be related to the drifts in y ($\langle \Delta_y \rangle$), $\ln z$ ($\langle \Delta_{\ln z} \rangle$), and $\ln k_t$ ($\langle \Delta_{\ln k_t} \rangle$) and analytical resummation through

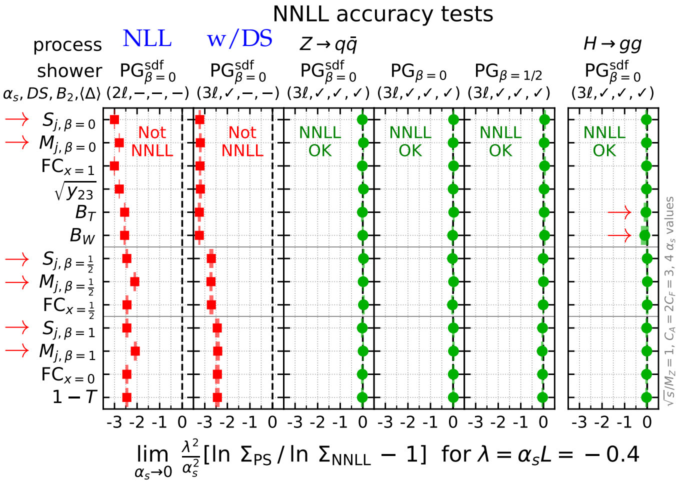
$$\Delta K_1^{\text{int,PS}} = 2\langle \Delta_y \rangle, \quad B_2^{\text{int,PS}} = B_2^{\text{int,NLO}} - \underbrace{\langle \Delta_{\ln z} \rangle}_{=\Delta B_2}, \quad K_2^{\text{PS}} = K_2^{\text{resum}} - \underbrace{4\beta_0 \langle \Delta_{\ln k_t} \rangle}_{=\Delta K_2}.$$

Using these relations and taking $B_2^{\text{int,NLO}}$ from [2109.07496], [2307.15734] and K_2^{resum} from [1807.11487] one can prove that our showers are NNLL accurate for event-shape observables

.

PanScales [2406.02661]

NNLL numerical tests

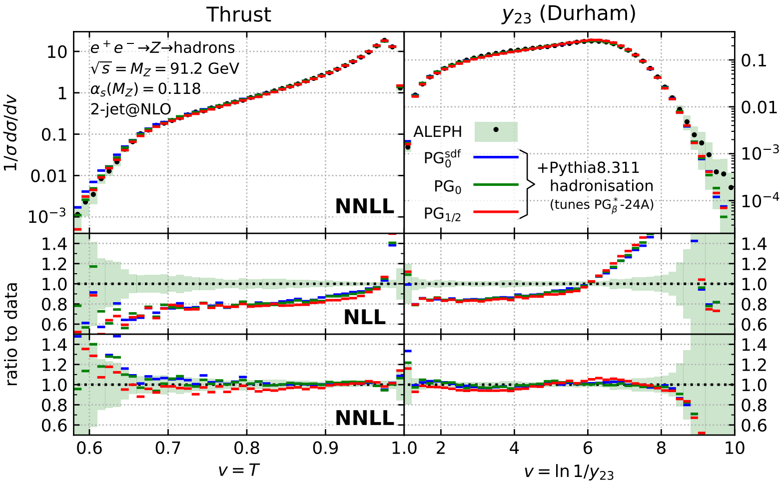


→: New analytic results, not available in literature

With no NNLL improvements, the coefficient of NNLL difference is significant, $\mathcal{O}(2-3)$, indicating importance of getting NNLL right

After inclusion of shifts and B_2 and K_2 we have perfect agreement

Impact of NNLL

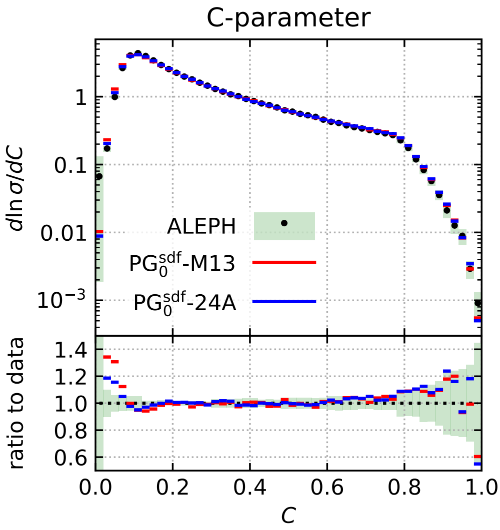


Long-standing tension between LEP data and Pythia8 unless using an anomalously large value of $\alpha_S(M_Z) = 0.137$ Skands, Carrazza, Rojo [1404.5630]

Inclusion of NNLL brings large corrections wrt NLL. Agreement with data achieved without anomalously large value of α_S

PanScales [2406.02661]

What about tuning?

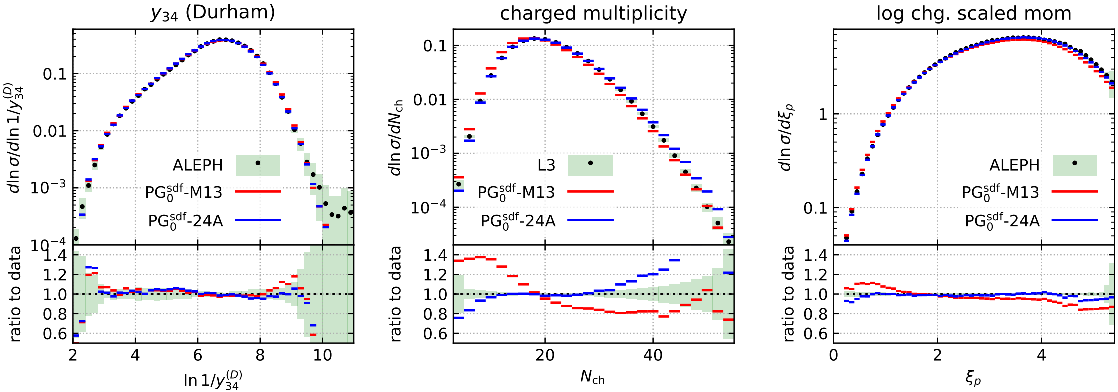


Improved agreement with data across a large range of event shapes

We start from the Monash tune (see ref. above) but fix $\alpha_S(M_Z) = 0.118$ (M13)

Full tuning exercise still to be done, but very little impact on infrared safe observables!

What about tuning?



Impact of tune very minor on infrared safe observables, even those that are only NLL accurate

Impact on unsafe observables much larger, bringing good agreement with ALEPH data.

Conclusions and outlook

NNLL parton showers have arrived!

Full phenomenological impact still to be studied but encouraging results observed in e^+e^-

Our code, including the NNLL improvements discussed here from v0.2, can be obtained from `https://gitlab.com/panscales/panscales-0.X`.

Next steps are to extend to hadron collisions...