Theoretical predictions for $t\bar{t}W$ and $t\bar{t}Z$ production

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- **Higher-order soft-gluon corrections**
- **aN^{3}LO QCD + NLO EW**
- $t\bar{t}W$ **production**
- $t\bar{t}Z$ **production**

QCD@LHC 2024

Soft-gluon corrections

They are important for top-quark processes and they approximate known exact results at NLO and NNLO very well

partonic processes $a(p_a) + b(p_b) \rightarrow t(p_t) + X$

define $s=(p_a+p_b)^2$**,** $t=(p_a-p_t)^2$**,** $u=(p_b-p_t)^2$

For a $2 \rightarrow n$ **process with** $p_a+p_b \rightarrow p_t+p_2+\cdots+p_n$
we define the threshold variable $s_4=s+t+u-m_t^2-(p_2+\cdots+p_n)^2$

Also $s_4=(p_2+\cdots+p_n+p_g)^2-(p_2+\cdots+p_n)^2$ **where extra gluon with** p_g **emitted**

At partonic threshold $p_g \rightarrow 0$ **and thus** $s_4 \rightarrow 0$

Soft corrections $\left[\frac{\ln^k(s_4/m_t^2)}{s_4}\right]_+$ **with** $k \le 2n-1$ **for the order** α_s^n **corrections**

Resum these soft corrections for the double-differential cross section

Soft-gluon corrections

Note that choice of formalism and threshold variable is important

two calculations may not be directly comparable even when at the same formal logarithmic accuracy if different threshold logarithms are used

Here we use single-particle-inclusive (1PI) kinematics and the threshold variable is s_4

[another possibility of threshold variable is the invariant mass of the final state]

Finite-order expansions $\rightarrow$ no prescription needed or used
(this avoids underestimating the size of the corrections)

Approximate NNLO (aNNLO) and approximate N^{3}LO (aN^{3}LO) predictions for cross sections and differential distributions

also add electroweak (EW) corrections at NLO

Thus, derive aN^{3}LO QCD + NLO EW predictions

Soft-gluon Resummation

$$d\sigma_{pp\to tX} = \sum_{a,b} \int dx_a \, dx_b \, \phi_{a/p}(x_a,\mu_F) \, \phi_{b/p}(x_b,\mu_F) \, d\hat{\sigma}_{ab\to tX}(s_4,\mu_F)$$

take Laplace transforms $d\hat{\sigma}_{ab\to tX}(N) = \int (ds_4/s) \, e^{-Ns_4/s} d\hat{\sigma}_{ab\to tX}(s_4)$

and $\tilde{\phi}(N) = \int_0^1 e^{-N(1-x)} \, \phi(x) \, dx$ **with transform variable** N

Then

$$d\tilde{\sigma}_{ab\to tX}(N) = \tilde{\phi}_{a/a}(N_a,\mu_F) \, \tilde{\phi}_{b/b}(N_b,\mu_F) \, d\tilde{\hat{\sigma}}_{ab\to tX}(N,\mu_F)$$

Refactorization for the cross section

$$d\sigma_{ab\to tX}(N) = \tilde{\psi}_{a/a}(N_a,\mu_F) \, \tilde{\psi}_{b/b}(N_b,\mu_F) \, \tilde{J}(N,\mu_F) \, \mathrm{tr} \left\{ H_{ab\to tX}\left(\alpha_s(\mu_R)\right) \, \tilde{S}_{ab\to tX}\left(\frac{\sqrt{s}}{N\mu_F}\right) \right\}$$

$\psi_{a/a}$, $\psi_{b/b}$ $\to$ **collinear emission from incoming partons**

J $\to$ **collinear emission from final-state gluons or massless quarks (if any)**

$H_{ab\to tX}$ **is hard function** $\to$ **short distance**

$S_{ab\to tX}$ **is soft function** $\to$ **noncollinear soft gluons**

Thus

$$d\tilde{\hat{\sigma}}_{ab\to tX}(N) = \frac{\tilde{\psi}_{a/a}(N_a,\mu_F) \, \tilde{\psi}_{b/b}(N_b,\mu_F)}{\tilde{\phi}_{a/a}(N_a,\mu_F) \, \tilde{\phi}_{b/b}(N_b,\mu_F)} \, \tilde{J}(N,\mu_F) \; \mathrm{tr} \left\{ H_{ab\to tX}\left(\alpha_s(\mu_R)\right) \, \tilde{S}_{ab\to tX}\left(\frac{\sqrt{s}}{N\mu_F}\right) \right\}$$

$S_{ab\to tX}$ **satisfies the renormalization group equation**

$$\left(\mu_R \frac{\partial}{\partial \mu_R} + \beta(g_s)\frac{\partial}{\partial g_s}\right) S_{ab\to tX} = -\Gamma^{\dagger}_{S\,ab\to tX}\, S_{ab\to tX} - S_{ab\to tX}\, \Gamma_{S\,ab\to tX}$$

Soft anomalous dimension $\Gamma_{S\,ab\to tX}$ **controls the evolution of the soft function which gives the exponentiation of logarithms of** N

Renormalization group evolution → resummation

$$d\tilde{\hat{\sigma}}^{\rm resum}_{ab\to tX}(N) = \exp\left[\sum_{i=a,b} E_i(N_i)\right] \exp\left[\sum_{i=a,b} 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\gamma_{i/i}(N_i)\right] \exp\left[E'(N)\right]$$
$$\times {\rm tr}\left\{H_{ab\to tX}\left(\alpha_s(\sqrt{s})\right) \bar{P}\exp\left[\int_{\sqrt{s}}^{\sqrt{s}/N}\frac{d\mu}{\mu}\,\Gamma^{\dagger}_{S\,ab\to tX}(\alpha_s(\mu))\right] \tilde{S}_{ab\to tX}\left(\alpha_s\left(\frac{\sqrt{s}}{N}\right)\right) P\exp\left[\int_{\sqrt{s}}^{\sqrt{s}/N}\frac{d\mu}{\mu}\,\Gamma_{S\,ab\to tX}(\alpha_s(\mu))\right]\right\}$$

The soft anomalous dimensions Γ_S **and the hard and soft functions are in general matrices in the space of color exchanges in the hard scattering**

Top processes studied - total and differential cross sections

Top pair

$t\bar{t}$ **aN^{3}LO (total; top p_T, y, and double-differential; also A_{FB})**
$t\bar{t}$ **aN^{3}LO + NLO EW (total; top p_T, y)**
$t\bar{t}$ **SMEFT aNNLO (total; top p_T)**

Top-pair+X

$t\bar{t}\gamma$ **aNNLO + NLO EW (total; top p_T, y)**
$t\bar{t}W$ **aN^{3}LO + NLO EW (total; top p_T, y)**
$t\bar{t}Z$ **aN^{3}LO + NLO EW (total; top p_T, y)**

Single top

t**- and** s**-channel aNNLO (total; top p_T) and aN^{3}LO (total)**
tW **aN^{3}LO (total; p_T, y for top and W)**

Single-top+X

tqH **aNNLO (total; top p_T, y)**
$tq\gamma$ **aNNLO (total; top p_T, y)**
tqZ **aNNLO (total; top y)**

Single-top BSM

$t\gamma$, tZ, tZ' **aNNLO (total; top p_T, y)**
tg **aNNLO (total)**
tH^- **aNNLO (total; top p_T, y) and aN^{3}LO (total)**

$t\bar{t}W$ production

in collaboration with Chris Foster

Phys. Lett. B 854, 138708 (2024) [arXiv:2312.00861]

observation of $t\bar{t}W$ events at 7, 8, 13 TeV collisions at the LHC

measurements are significantly higher than theoretical predictions

QCD corrections at NLO are large, $\sim 47\%$ at 13.6 TeV

electroweak corrections are smaller but significant

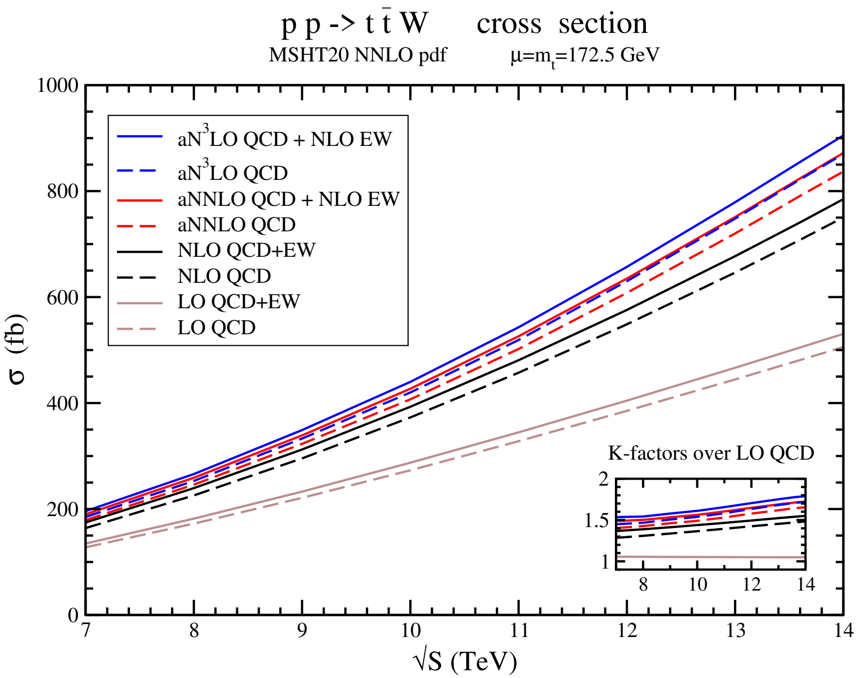
further improvement in theoretical accuracy by the inclusion of higher-order soft-gluon corrections

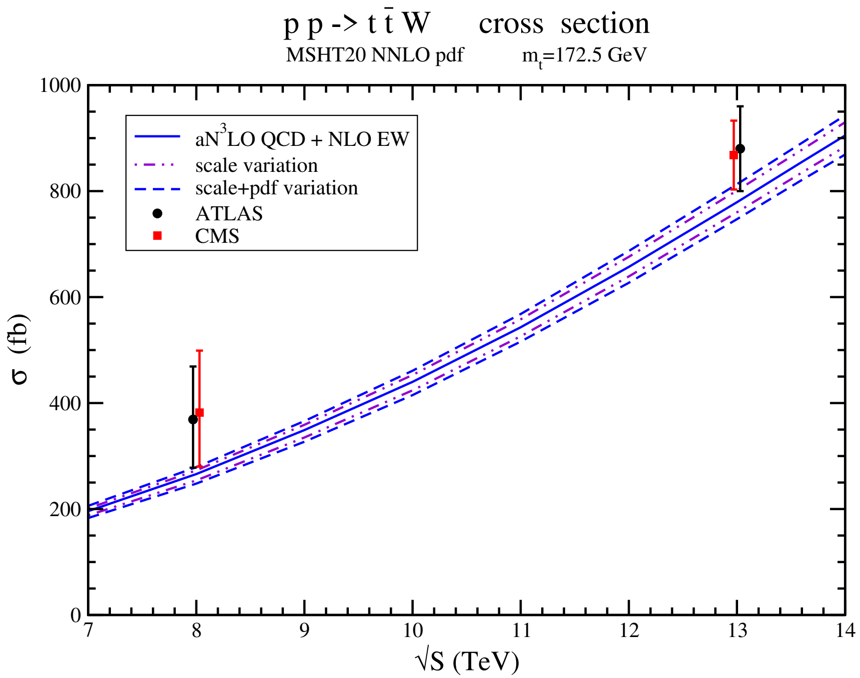
NLO expansions closely approximate exact NLO results for total cross sections and top-quark p_T and rapidity distributions

NNLO expansions (aNNLO) are consistent with (partial) NNLO results for total cross sections

aN^{3}LO QCD + NLO EW is state of the art

Cross sections for $t\bar{t}W$ production





large K-factors

improved agreement with data at aN^{3}LO QCD + NLO EW

$t\bar{t}W$ cross sections

$t\bar{t}W$ cross sections in pp collisions at the LHC					
σ in fb	7 TeV	8 TeV	13 TeV	13.6 TeV	14 TeV
LO QCD	128^{+39}_{-28}	172^{+51}_{-36}	445^{+114}_{-84}	481^{+121}_{-90}	506^{+126}_{-94}
LO QCD+EW	135^{+41}_{-29}	182^{+53}_{-38}	467^{+119}_{-88}	505^{+127}_{-94}	531^{+132}_{-98}
NLO QCD	164^{+13}_{-17}	226^{+20}_{-23}	646^{+83}_{-74}	708^{+94}_{-82}	750^{+101}_{-88}
NLO QCD+EW	175^{+12}_{-17}	239^{+19}_{-23}	677^{+80}_{-74}	741^{+90}_{-82}	785^{+97}_{-88}
aNNLO QCD	179^{+6}_{-10}	246^{+9}_{-15}	720^{+29}_{-43}	791^{+32}_{-47}	837^{+34}_{-50}
aNNLO QCD + NLO EW	190^{+6}_{-10}	259^{+9}_{-15}	751^{+27}_{-43}	824^{+29}_{-47}	872^{+31}_{-50}
aN^{3}LO QCD	185^{+5}_{-8}	253^{+7}_{-12}	748^{+24}_{-19}	822^{+26}_{-20}	870^{+28}_{-21}
aN^{3}LO QCD + NLO EW	196^{+5}_{-8}	266^{+7}_{-12}	779^{+22}_{-19}	855^{+23}_{-20}	905^{+25}_{-21}

At 13.6 TeV

NLO QCD corrections → 47%

aNNLO QCD corrections → 17%

aN^{3}LO QCD corrections → 6%

electroweak NLO corrections → 7%

Total aN^{3}LO QCD+NLO EW cross section is 78% bigger than LO QCD

$t\bar{t}W^+$ and $t\bar{t}W^-$ cross sections

$t\bar{t}W^+$ and $t\bar{t}W^-$ cross sections in pp collisions at the LHC				
σ in fb	$t\bar{t}W^+$ 13 TeV	$t\bar{t}W^+$ 13.6 TeV	$t\bar{t}W^-$ 13 TeV	$t\bar{t}W^-$ 13.6 TeV
LO QCD	299^{+77}_{-57}	322^{+82}_{-60}	146^{+37}_{-28}	159^{+40}_{-30}
LO QCD+EW	313^{+80}_{-59}	337^{+85}_{-63}	154^{+39}_{-29}	168^{+42}_{-31}
NLO QCD	431^{+54}_{-49}	470^{+61}_{-54}	215^{+29}_{-25}	238^{+33}_{-28}
NLO QCD+EW	450^{+51}_{-48}	490^{+58}_{-53}	227^{+28}_{-25}	251^{+32}_{-28}
aNNLO QCD	480^{+19}_{-28}	525^{+21}_{-31}	240^{+10}_{-15}	266^{+11}_{-16}
aNNLO QCD + NLO EW	499^{+17}_{-28}	545^{+19}_{-31}	252^{+10}_{-15}	279^{+10}_{-16}
aN^{3}LO QCD	498^{+16}_{-12}	545^{+17}_{-13}	250^{+8}_{-7}	277^{+9}_{-7}
aN^{3}LO QCD + NLO EW	517^{+14}_{-12}	565^{+15}_{-13}	262^{+8}_{-7}	290^{+8}_{-7}

the $t\bar{t}W^+$ cross sections are larger than for $t\bar{t}W^-$

but the corrections are slightly bigger for $t\bar{t}W^-$

Comparison with 8 and 13 TeV CMS and ATLAS data

NLO and even aNNLO results are not sufficient

we need aN^{3}LO corrections to describe the data

At 8 TeV, measurements from

CMS: 382^{+117}_{-102} **fb**

and from

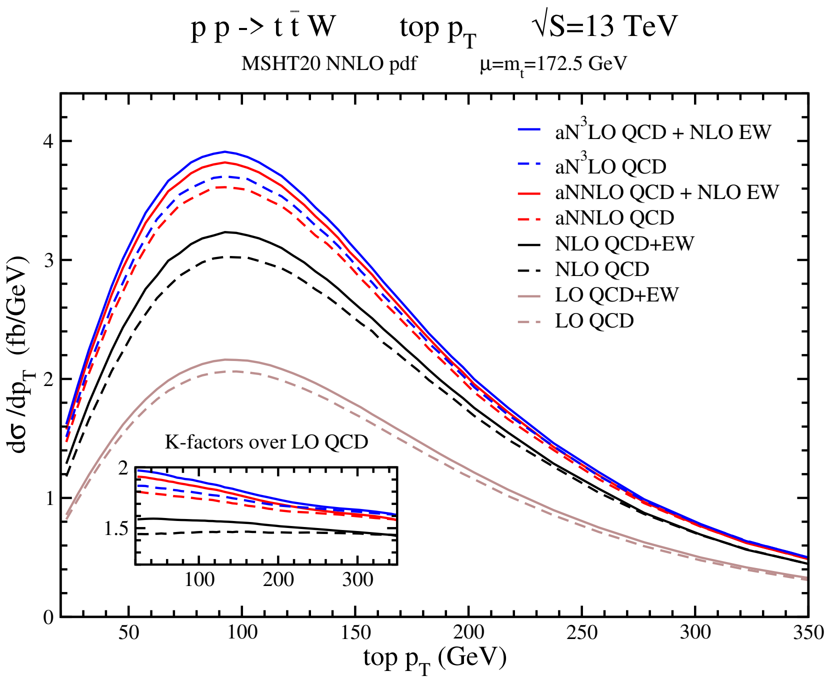
ATLAS: 369^{+100}_{-91} **fb**

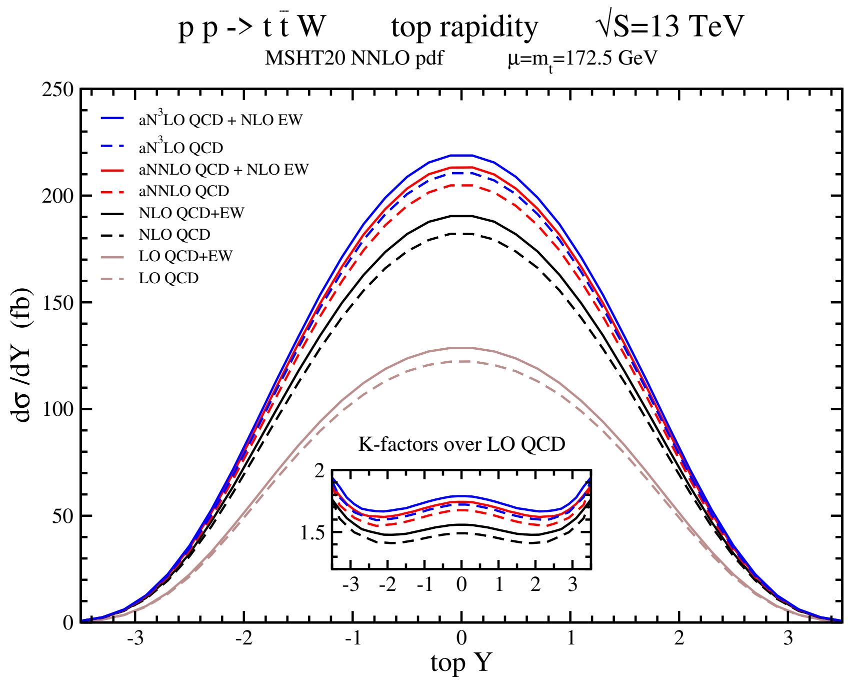
Theoretical prediction is
aN^{3}LO QCD + NLO EW: $266^{+7}_{-12}{}^{+6}_{-6}$ **fb**

At 13 TeV, CMS finds 868 ± 65 **fb with** $t\bar{t}W^+$ 553 ± 42 **fb and** $t\bar{t}W^-$ 343 ± 36 **fb**
while ATLAS finds 880 ± 80 **fb with** $t\bar{t}W^+$ 583 ± 58 **fb and** $t\bar{t}W^-$ 296 ± 40 **fb**

Theoretical prediction is
aN^{3}LO QCD + NLO EW: $779^{+22}_{-19}{}^{+12}_{-13}$ **fb with** $t\bar{t}W^+$ $517^{+14}_{-12}{}^{+8}_{-9}$ **fb and** $t\bar{t}W^-$ $262^{+8}_{-7}{}^{+4}_{-4}$ **fb**

Top-quark p_T and rapidity distributions in $t\bar{t}W$ production at 13 TeV

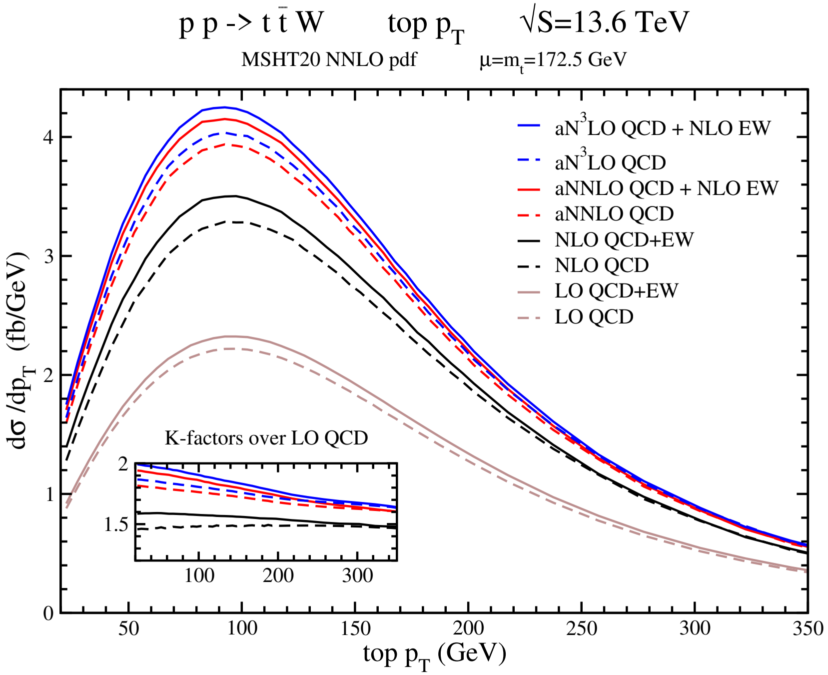


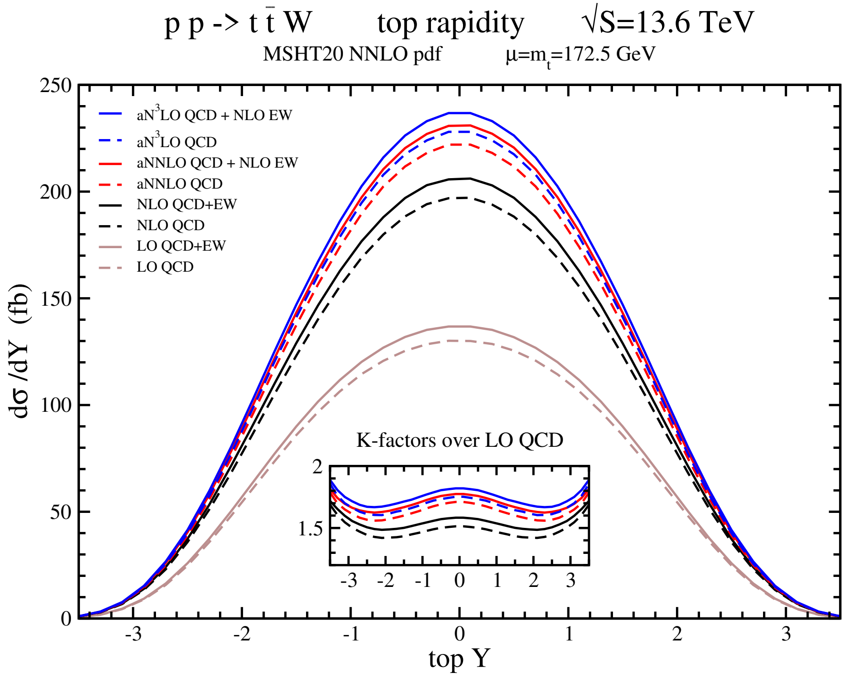


K-factors decrease at larger top p_T

K-factors increase at larger rapidities

Top-quark p_T and rapidity distributions in $t\bar{t}W$ at 13.6 TeV





K-factors decrease at larger top p_T

K-factors increase at larger rapidities

$t\bar{t}Z$ production

in collaboration with Chris Foster, arXiv:2410.01214

observation of $t\bar{t}Z$ events at 7, 8, 13 TeV collisions at the LHC

important for measuring coupling of the top quark to the Z boson

QCD corrections at NLO are large, $\sim 32\%$ at 13.6 TeV

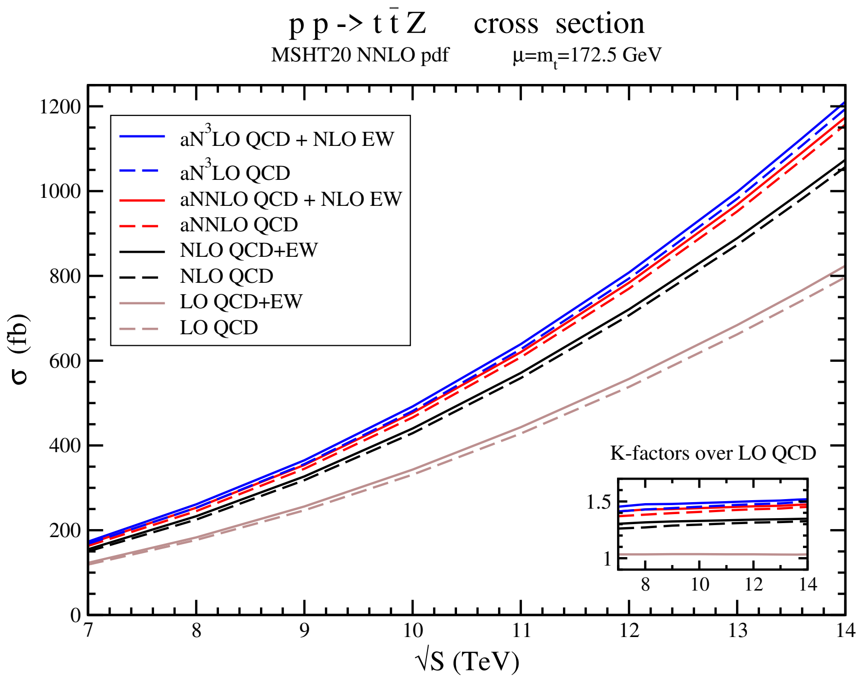
electroweak corrections are smaller but significant

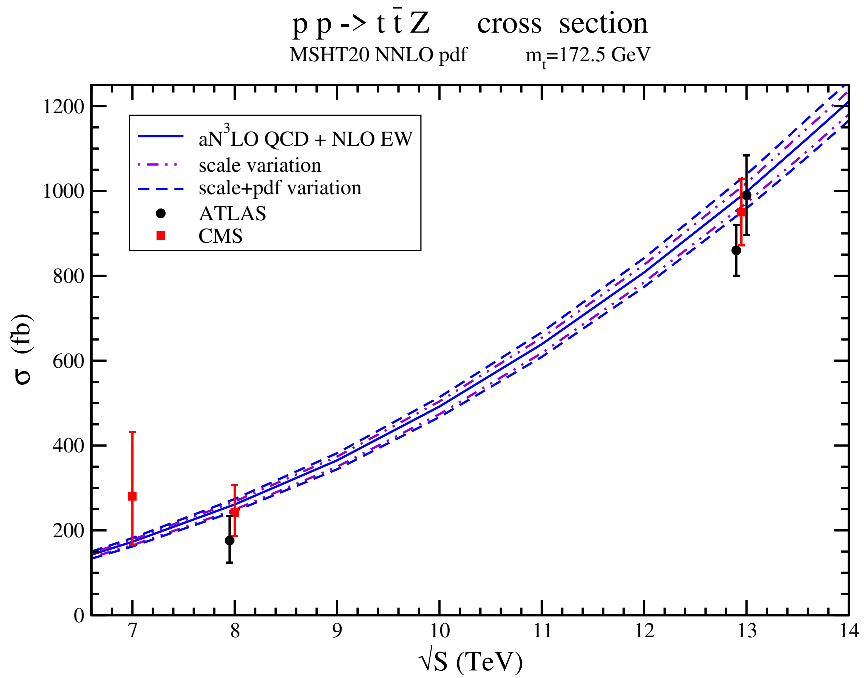
further improvement in theoretical accuracy by the inclusion of higher-order soft-gluon corrections

NLO expansions closely approximate exact NLO results for total cross sections and top-quark p_T and rapidity distributions

aN^{3}LO QCD + NLO electroweak is state of the art

Cross sections for $t\bar{t}Z$ production





large K-factors

good agreement with data at aN^{3}LO QCD+NLO EW

$t\bar{t}Z$ cross sections

$t\bar{t}Z$ cross sections in pp collisions at the LHC					
σ in fb	7 TeV	8 TeV	13 TeV	13.6 TeV	14 TeV
LO QCD	119^{+44}_{-30}	177^{+64}_{-44}	662^{+218}_{-152}	742^{+242}_{-169}	797^{+258}_{-181}
LO QCD+EW	123^{+46}_{-31}	183^{+67}_{-45}	684^{+226}_{-157}	766^{+250}_{-175}	824^{+267}_{-187}
NLO QCD	150^{+9}_{-16}	225^{+15}_{-24}	873^{+69}_{-93}	982^{+79}_{-104}	1057^{+85}_{-112}
NLO QCD+EW	155^{+8}_{-16}	233^{+13}_{-24}	889^{+61}_{-90}	999^{+70}_{-101}	1074^{+74}_{-107}
aNNLO QCD	163^{+7}_{-10}	245^{+10}_{-15}	952^{+29}_{-48}	1074^{+33}_{-54}	1157^{+35}_{-58}
aNNLO QCD + NLO EW	168^{+6}_{-10}	253^{+9}_{-15}	968^{+25}_{-46}	1091^{+29}_{-51}	1174^{+30}_{-54}
aN^{3}LO QCD	168^{+5}_{-8}	253^{+8}_{-12}	982^{+25}_{-28}	1108^{+28}_{-32}	1194^{+30}_{-34}
aN^{3}LO QCD + NLO EW	173^{+5}_{-8}	261^{+7}_{-12}	998^{+21}_{-26}	1125^{+24}_{-30}	1211^{+25}_{-30}

At 13.6 TeV

- **NLO QCD corrections $\rightarrow$ 32%**
- **aNNLO QCD corrections $\rightarrow$ 12%**
- **aN^{3}LO QCD corrections $\rightarrow$ 5%**
- **electroweak NLO corrections $\rightarrow$ 2%**

Total aN^{3}LO QCD+NLO EW cross section is 52% bigger than LO QCD

Comparison with 7, 8, and 13 TeV LHC data

At 7 TeV, measurement from

CMS: $0.28^{+0.14+0.06}_{-0.11-0.03}$ **pb**

At 8 TeV, measurements from

ATLAS: 176^{+58}_{-52} **fb**

and from

CMS: 242^{+65}_{-55} **fb**

At 13 TeV, measurements from

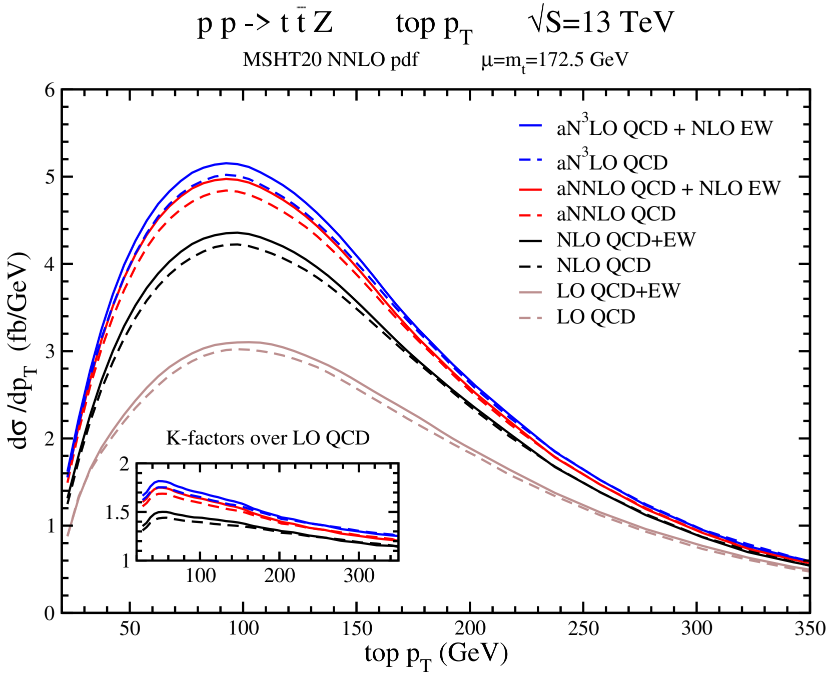
CMS: $0.95 \pm 0.05 \pm 0.06$ **pb**

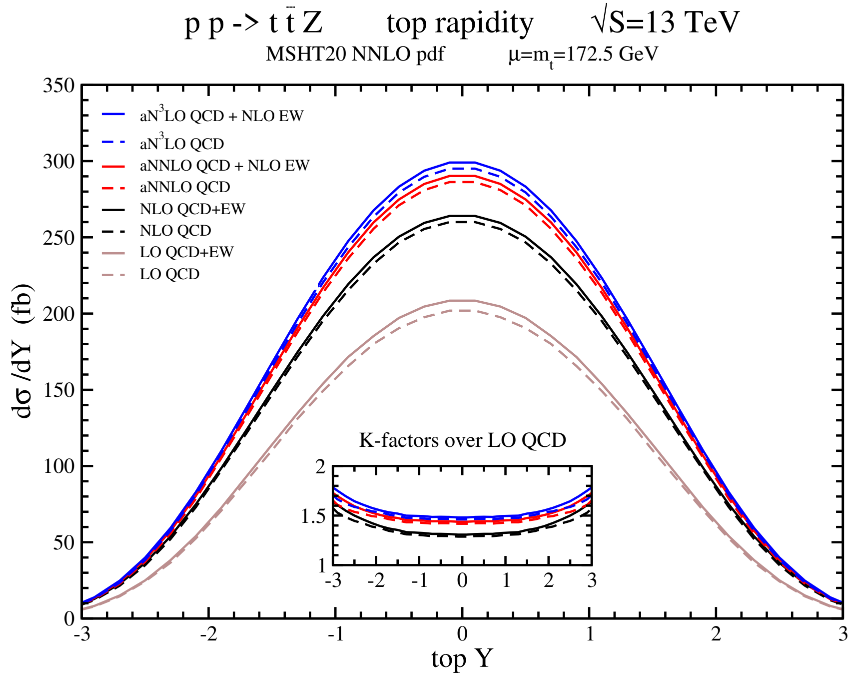
and from

ATLAS: $0.99 \pm 0.05 \pm 0.08$ **pb and** $0.86 \pm 0.04 \pm 0.04$ **pb**

aN^{3}LO QCD + NLO EW $t\bar{t}Z$ cross section in pp collisions at the LHC					
σ in fb	7 TeV	8 TeV	13 TeV	13.6 TeV	14 TeV
MSHT20 NNLO pdf	173^{+5+4}_{-8-3}	$261^{+7\ +6}_{-12-4}$	998^{+21+20}_{-26-13}	1125^{+24+22}_{-30-14}	1211^{+25+23}_{-30-15}
MSHT20 aN^{3}LO pdf	170^{+5+4}_{-8-4}	$255^{+7\ +6}_{-12-6}$	974^{+20+19}_{-25-21}	1096^{+23+22}_{-29-23}	1182^{+24+23}_{-29-25}
NNPDF4.0 aN^{3}LO pdf	165^{+5+2}_{-7-2}	$248^{+7\ +3}_{-11-3}$	962^{+20+6}_{-25-6}	1083^{+23+7}_{-29-7}	1170^{+24+7}_{-29-7}

Top-quark p_T and rapidity distributions in $t\bar{t}Z$ production at 13 TeV

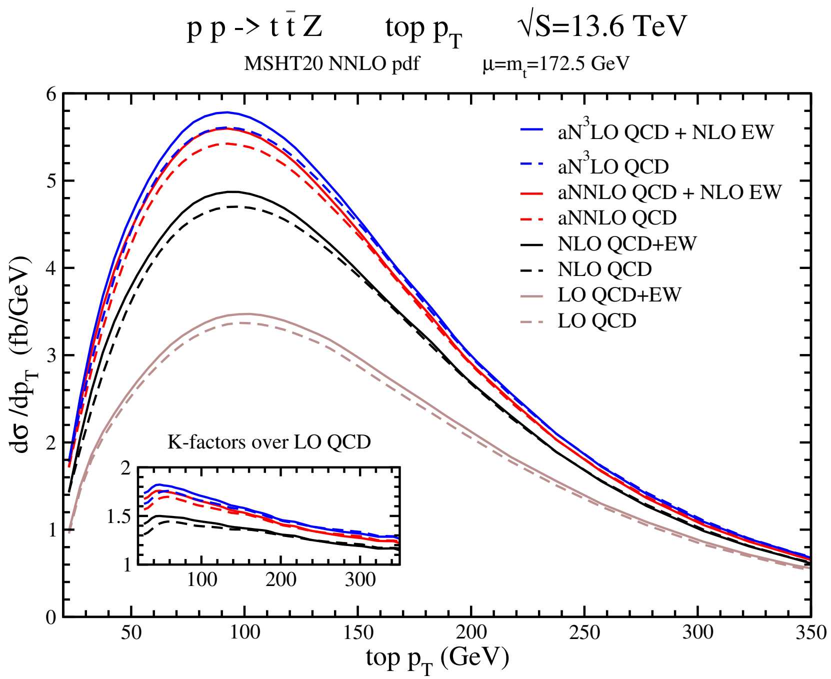


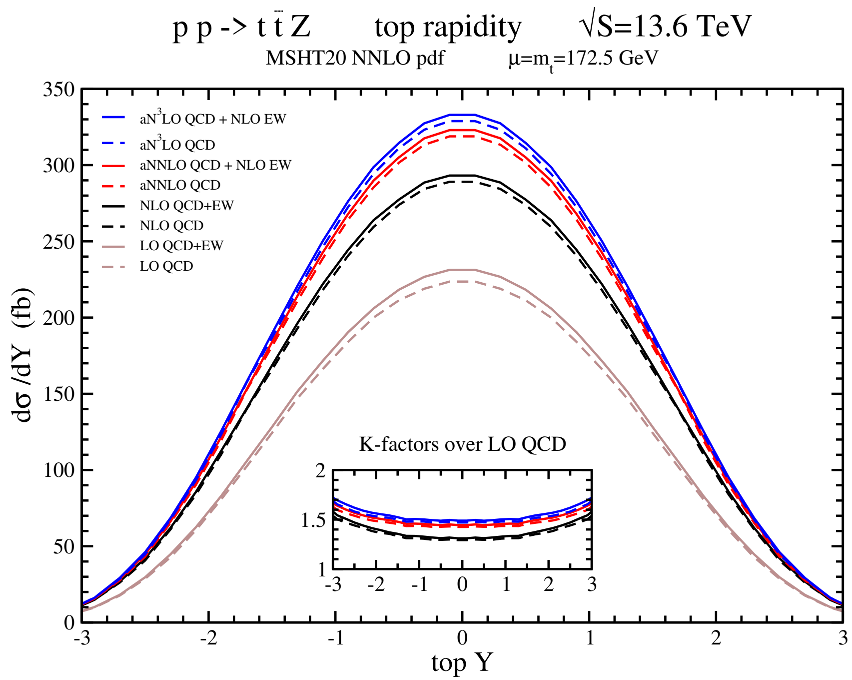


K-factors decrease at larger top p_T

K-factors increase at larger rapidities

Top-quark p_T and rapidity distributions in $t\bar{t}Z$ at 13.6 TeV





K-factors decrease at larger top p_T

K-factors increase at larger rapidities

Summary

- **higher-order corrections for top-quark production processes**
- **soft-gluon resummation and aNNLO, aN^{3}LO expansions**
- $t\bar{t}W$ **production**
- $t\bar{t}Z$ **production**
- **aN^{3}LO QCD + NLO EW predictions**
- **results for total cross sections and top-quark p_T and rapidity distributions**
- **higher-order corrections further enhance and improve the theoretical predictions**
- **good agreement with LHC data**