Precision meets New Physics in the Heavy-Quark Expansion for Lifetimes

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Theoretical Particle Physics (TP1), U Siegen

Heavy quarks: charm and bottom

2024: **50 years of charm quarks!**

Charm quarks are instable and decay through the weak interactions $V - A$

$\Rightarrow$ analogy to the muon decay

$$\Gamma\left(c\right) \approx \frac{G_F^2 m_c^5}{192\pi^3} \approx \mathcal{O}\left(1\,\mathrm{ps}^{-1}\right)$$

Bottom quarks: $m_b \gtrsim 3m_c$.

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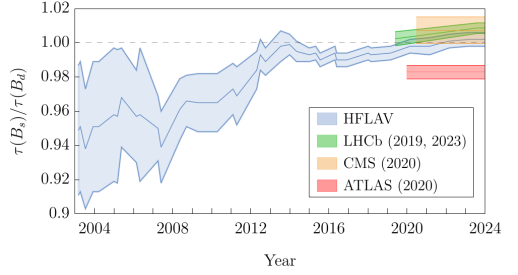
Bottom quarks: $m_b \gtrsim 3m_c$. Does that imply $\Gamma\left(b\right) \gtrsim 250\,\Gamma\left(c\right)$?

No! CKM mechanism to the rescue, $\Gamma\left(b\right) \approx \frac{G_F^2 m_b^5 |V_{cb}|^2}{192\pi^3}$, so

$$\boxed{\Gamma\left(b\right) \sim \Gamma\left(c\right)}$$

But charm and bottom quarks are not free particles. What about their **mesons**?

50 years later



[taken from Albrecht, Bernlochner, Lenz, Rusov, '24]

These days, there is a plethora of heavy-meson data and their lifetimes have been **measured to extremely high precision!**

- $\Gamma(B_d) \approx \Gamma(b)$
- $\tau(B_s)/\tau(B_d) \approx 1$

So $\Gamma(H_Q)$ is **the same** for all hadrons with same single heavy quark Q...

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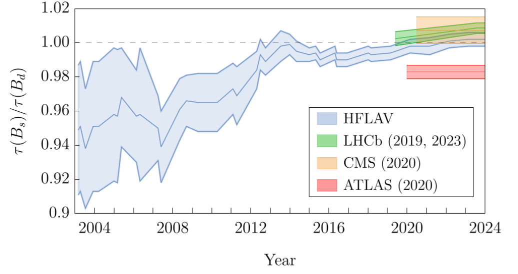

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So $\Gamma(H_Q)$ is **the same** for all hadrons with same single heavy quark Q...

... except they are not the same: $\frac{\tau(B^+)}{\tau(B_d)} = 1.076(4)$, $\frac{\tau(D^+)}{\tau(D^0)} \approx 2.52$ (HFLAV, PDG)

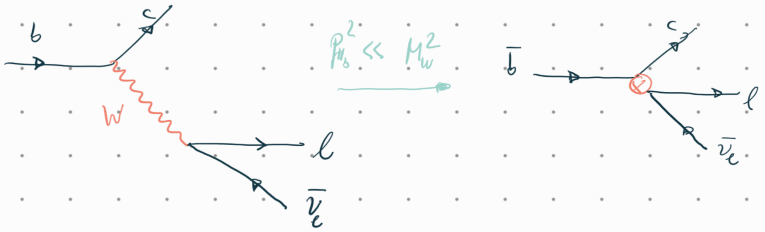
Can we formalise this, and to what extent?

The heavy-quark expansion

Weak b decays: the effective $|\Delta B| = 1$ Hamiltonian

Integrating out the W

- flavour-changing b decays are mediated by W bosons ($M_W \approx 80\,\text{GeV}$)
- decays of hadrons containing a single b quark release momenta of $p^2_{H_b} \sim M^2_{H_b} \approx 5\,\text{GeV}$
- W-boson propagator: $\frac{1}{M_W^2 - p^2_{H_b}} = \frac{1}{M_W^2} + \mathcal{O}\left(\frac{p^2_{H_b}}{M_W^2}\right)$



(Batini/Cerri, PD)

The heavy-quark expansion

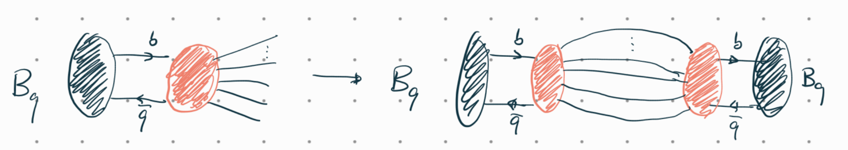
This leaves us with the **effective** $\mathcal{H}^{|\Delta B|=1}$ Hamiltonian, $\mathcal{H}^{|\Delta B|=1} = \sum C_i Q_i$

$$\Gamma\left(H_b\right) = \sum_X \int_{\mathrm{PS}} (2\pi)^4 \, \delta^{(4)}\left(p_{H_b} - p_X\right) \frac{1}{2M_{H_b}} \left| \left\langle X \middle| \mathcal{H}^{|\Delta B|=1} \middle| H_b \right\rangle \right|^2$$

Optical theorem

$$\Gamma\left(H_b\right) = \mathrm{Im}\,\langle H_b | \mathcal{T} | H_b \rangle$$

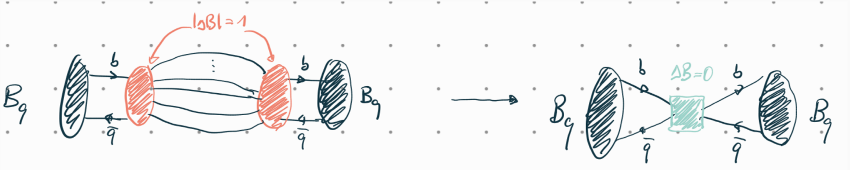
$$\mathcal{T} = \mathrm{i} \int \mathrm{d}^4 x T \left\{ \mathcal{H}^{|\Delta B|=1}(x)\, \mathcal{H}^{|\Delta B|=1}(0) \right\}$$



Operator-product expansion (OPE)

Employing the hierarchy $\Lambda_{\mathrm{QCD}} \ll M_{H_b} \approx m_b$ allows for the use of an **OPE**,

$$\mathrm{Im}\,\mathcal{T} = \sum C_n(x)\, Q_n(0)$$



$$\Gamma(H_b) = \Gamma_3 + \Gamma_5 \frac{\langle O_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle O_6 \rangle}{m_b^3} + \ldots + 16\pi^2 \left(\tilde{\Gamma}_6 \frac{\left\langle \tilde{O}_6 \right\rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\left\langle \tilde{O}_7 \right\rangle}{m_b^4} + \ldots \right)$$

we distinguish between *non-spectator effects* and *spectator effects*

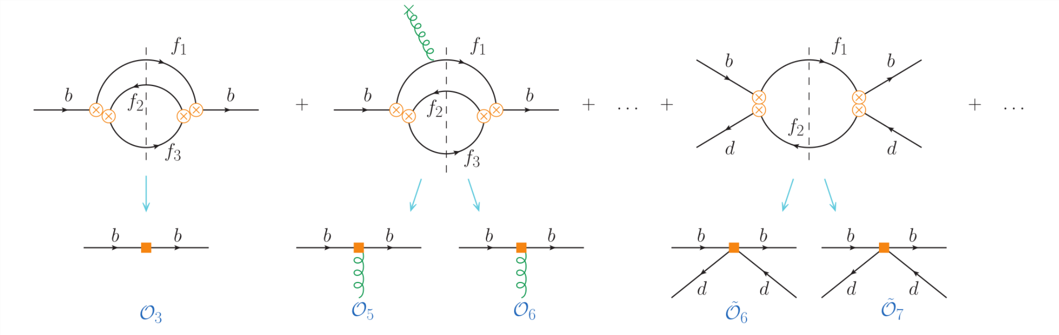
The heavy-quark expansion for lifetimes

$$\Gamma\left(H_b\right) = \Gamma_3 + \Gamma_5 \frac{\langle O_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle O_6 \rangle}{m_b^3} + \ldots + 16\pi^2 \left(\tilde{\Gamma}_6 \frac{\left\langle \tilde{O}_6 \right\rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\left\langle \tilde{O}_7 \right\rangle}{m_b^4} + \ldots \right)$$

- the *Wilson coefficients* are perturbative, $\Gamma_n = \Gamma_n^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_n^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \Gamma_n^{(2)} + \ldots$
- the non-perturbative matrix elements $\langle O_n \rangle \equiv \langle H_b | \mathcal{O}_n | H_b \rangle$ are of $\mathcal{O}\left(\Lambda_{QCD}^{n-3}\right)$
- the free quark decay width,

$$\Gamma_3 \sim \frac{G_F^2 m_b^5}{192\pi^3} \cdot \sum_{\text{decay channels}} V_{\text{CKM}} \cdot f_{\text{PS}} \left(\frac{m_c^2}{m_b^2}, \frac{m_\tau^2}{m_b^2} \right)$$

Total decay rates in the HQE



[taken from Lenz, Piscopo, Rusov, 2020]

$\mathcal{O}_3$: $\quad \mathcal{O}_3 = \bar{b}b$

$\mathcal{O}_5$: $\quad \mathcal{O}_{\rm kin} = \bar{b}_v\,(\mathrm{i}D_\mu)\,(\mathrm{i}D^\mu)\,b_v \qquad \mathcal{O}_{\rm mag} = \bar{b}_v\,(\mathrm{i}D_\mu)\,(\mathrm{i}D_\nu)\,(-\mathrm{i}\sigma^{\mu\nu})\,b_v$

$\mathcal{O}_6$: $\quad \mathcal{O}_{\rm LS} = \bar{b}_v\,(\mathrm{i}D_\mu)\,(\mathrm{i}v\cdot D)\,(\mathrm{i}D_\nu)\,(-\mathrm{i}\sigma^{\mu\nu})\,b_v \qquad \mathcal{O}_{\rho_D} = \bar{b}_v\,(\mathrm{i}D_\mu)\,(\mathrm{i}v\cdot D)\,(\mathrm{i}D^\mu)\,b_v$

Hadronic matrix elements: two-quark operators

Hadronic matrix elements of non-spectator operators

$$\frac{\langle B_q|\bar{b}b|B_q\rangle}{2M_{B_q}} = 1 - \frac{\mu_\pi^2(B_q) - \mu_G^2(B_q)}{2M_{B_q}} + \mathcal{O}\left(\frac{1}{m_b^5}\right)$$

$$2m_{B_q}\mu_\pi^2(B_q) = -\langle B_q|\mathcal{O}_{\mathrm{kin}}|B_q\rangle$$

$$2m_{B_q}\mu_G^2(B_q) = \langle B_q|\mathcal{O}_{\mathrm{mag}}|B_q\rangle \qquad \text{chromomagnetic operator}$$

$$2m_{B_q}\rho_{\mathrm{LS}}^3(B_q) = \langle B_q|\mathrm{O}_{\mathrm{LS}}|B_q\rangle$$

$$2m_{B_q}\rho_D^3(B_q) = \langle B_q|\mathrm{O}_{\rho_D}|B_q\rangle \qquad \text{Darwin term}$$

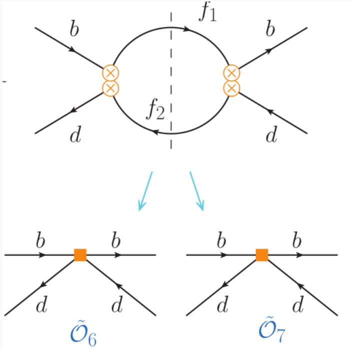
Including only the non-spectator operators, we have $\Gamma\left(B^0\right) \approx \Gamma\left(B^+\right) \approx \Gamma\left(B_s\right)$ to excellent approximation

Spectator effects: four-quark operators

At dimension 6 spectator operators of the form $\bar{b}\Gamma q\,\bar{q}\Gamma' b$ become possible.

$$Q_1^q = \bar{b}\gamma_\mu(1-\gamma_5)\,q\,\bar{q}\gamma^\mu(1-\gamma_5)\,b\,, \qquad Q_2^q = \bar{b}(1-\gamma_5)\,q\,\bar{q}(1+\gamma_5)\,b\,,$$

$$T_1^q = \bar{b}\gamma_\mu(1-\gamma_5)\,T^a q\,\bar{q}\gamma^\mu(1-\gamma_5)\,T^a b\,, \qquad T_2^q = \bar{b}(1-\gamma_5)\,T^a q\,\bar{q}(1+\gamma_5)\,T^a b$$



Hadronic matrix elements of spectator operators

for lifetime ratios: $Q_i \equiv Q_i^u - Q_i^d$, $T_i \equiv T_i^u - T_i^d$

$$\langle Q_i(\mu)\rangle = f_{B_q}^2 M_{B_q}^2 B_i(\mu)\,, \qquad B_i(\mu) \approx 1$$

$$\langle T_i(\mu)\rangle \approx f_{B_q}^2 M_{B_q}^2 \epsilon_i(\mu)\,, \qquad \epsilon_i(\mu)\ \text{much smaller}$$

Improving HQE predictions: generalities

Improving predictions within the HQE

- Perturbative: higher-order α_s corrections to $\Gamma_i, \tilde{\Gamma}_i$
- $\Lambda_{\mathrm{QCD}}/m_b$: include higher-dimensional operators from the OPE
- Non-perturbative: more precise matrix element determinations (lattice, sum rules)

Status of the HQE for meson lifetimes

Status: Early 2024

experiment: HFLAV, PDG

$$\Gamma\left(B^+\right) = 0.611(2)\,\text{ps}^{-1}$$
$$\Gamma\left(B_d\right) = 0.658(2)\,\text{ps}^{-1}$$
$$\Gamma\left(B_s\right) = 0.657(2)\,\text{ps}^{-1}$$
$$\frac{\tau\left(B^+\right)}{\tau\left(B_d\right)} = 1.076(4)$$
$$\frac{\tau\left(B_s\right)}{\tau\left(B_d\right)} = 1.002(4)$$

theory: Albrecht, Bernlochner, Lenz, Rusov, '24, Lenz, Piscopo, Rusov, '22

$$\Gamma\left(B^+\right) = \left(0.58^{+0.11}_{-0.07}\right)\mathrm{ps}^{-1}$$
$$\Gamma\left(B_d\right) = \left(0.63^{+0.11}_{-0.07}\right)\mathrm{ps}^{-1}$$
$$\Gamma\left(B_s\right) = \left(0.63^{+0.11}_{-0.07}\right)\mathrm{ps}^{-1}$$
$$\frac{\tau\left(B^+\right)}{\tau\left(B_d\right)} = 1.086(22)$$
$$\frac{\tau\left(B_s\right)}{\tau\left(B_d\right)} = 1.003(6)\ \text{small Darwin, small}\ \cancel{SU(3)_F}$$

Short-distance coefficients, early 2024

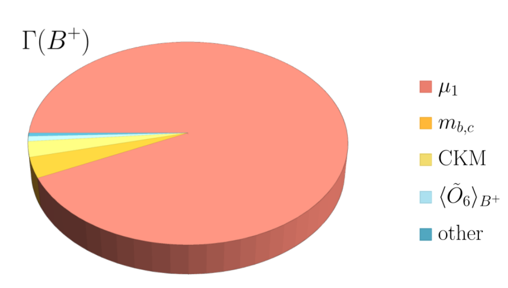
semileptonic decays:

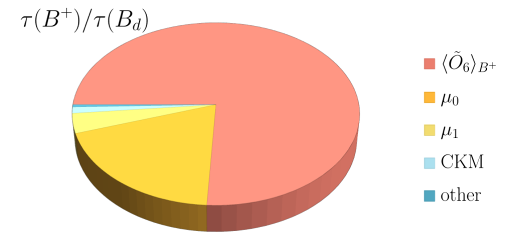
- $\Gamma_3^{(3)}$: Fael, Schönwald, Steinhauser, '21, Czakon, Czarnecki, Dowling, '21
- $\Gamma_5^{(1)}$: Alberti, Gambino, Nandi, '14, Mannel, Pivovarov, Rosenthal, '15
- $\Gamma_6^{(1)}$: Mannel, Moreno, Pivovarov, '22, Moreno, '22
- $\tilde{\Gamma}_6^{(1)}$: Lenz, Rauh, '13
- $\Gamma_7^{(0)}$: Dassinger, Mannel, Turczyk, '07
- $\Gamma_8^{(0)}$: Mannel, Turczyk, Uraltsev, '10

nonleptonic decays:

- $\Gamma_3^{(2)}$ (incomplete): Czarnecki, Slusarczyk, Tkachov, '06
- $\Gamma_5^{(1)}$ (massless): Mannel, Moreno, Pivovarov, '23
- $\Gamma_6^{(0)}$: Lenz, Piscopo, Rusov, '20, Mannel, Moreno, Pivovarov, '20, Moreno, '21
- $\tilde{\Gamma}_6^{(1)}$: Beneke, Buchalla, Greub, Lenz, Nierste, '02, Franco, Lubicz, Mescia, Tarantino, '02

Theoretical uncertainties, early 2024





[taken from Albrecht, Bernlochner, Lenz, Rusov, '24]

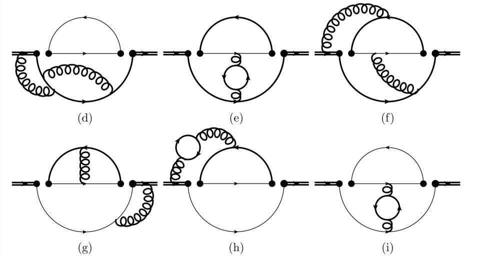
- total decay rates are dominated by the scale uncertainty μ_1 of Γ_3
- for $\tau(B^+)/\tau(B_d)$, the current HQET sum rules determination is still the dominant source of uncertainty $\Rightarrow$ lattice results on the way

What has happened in 2024?

NNLO corrections to hadronic b-quark decays (aka news from Karlsruhe)

The dominant uncertainty by varying μ_1 can be reduced with the calculation of higher perturbative corrections to the free b-quark decay

$\Rightarrow$ **NNLO QCD** Egner, Fael, Schönwald, Steinhauser, ’24



[taken from Egner, Fael, Schönwald, Steinhauser, ’24]

- higher evanescent operators

$$
\begin{aligned}
E_1^{(1),q_1q_2q_3} &= (\bar{q}_1^\alpha \gamma^{\mu_1\mu_2\mu_3} P_L b^\beta)(\bar{q}_2^\beta \gamma_{\mu_1\mu_2\mu_3} P_L q_3^\alpha) - (16 - 4\epsilon + A_2\epsilon^2) O_1^{q_1q_2q_3}, \\
E_2^{(1),q_1q_2q_3} &= (\bar{q}_1^\alpha \gamma^{\mu_1\mu_2\mu_3} P_L b^\alpha)(\bar{q}_2^\beta \gamma_{\mu_1\mu_2\mu_3} P_L q_3^\beta) - (16 - 4\epsilon + A_2\epsilon^2) O_2^{q_1q_2q_3}, \\
E_1^{(2),q_1q_2q_3} &= (\bar{q}_1^\alpha \gamma^{\mu_1\mu_2\mu_3\mu_4\mu_5} P_L b^\beta)(\bar{q}_2^\beta \gamma_{\mu_1\mu_2\mu_3\mu_4\mu_5} P_L q_3^\alpha) - (256 - 224\epsilon + B_1\epsilon^2) O_1^{q_1q_2q_3}, \\
E_2^{(2),q_1q_2q_3} &= (\bar{q}_1^\alpha \gamma^{\mu_1\mu_2\mu_3\mu_4\mu_5} P_L b^\alpha)(\bar{q}_2^\beta \gamma_{\mu_1\mu_2\mu_3\mu_4\mu_5} P_L q_3^\beta) - (256 - 224\epsilon + B_2\epsilon^2) O_2^{q_1q_2q_3},
\end{aligned}
$$

- master integrals computed using the “expand and match” approach

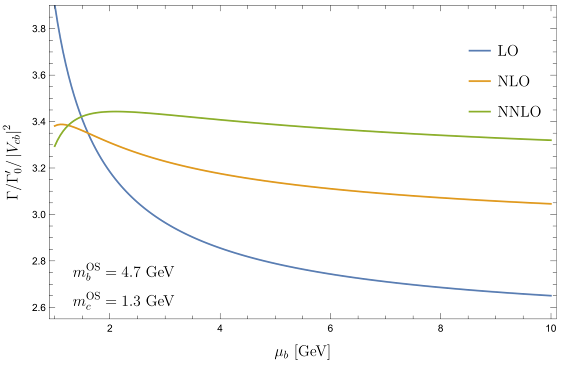
$$
I_i(\rho, \epsilon) = \sum_{j=-4}^{\epsilon_{\max}} \sum_{m=0}^{j+4} \sum_{n=n_{\min}}^{n_{\max}} c_{i,j,m,n} \epsilon^j (\rho - \rho_0)^{n/2} \log^m(\rho - \rho_0)
$$

NNLO corrections to hadronic b-quark decays

Complexity:

- $\mathcal{O}(1300)$ diagrams per hadronic decay channel ⇒ hundreds of four-loop MIs
- expansion points depend on the decay channel

This necessitates a **high degree of automatisation!**

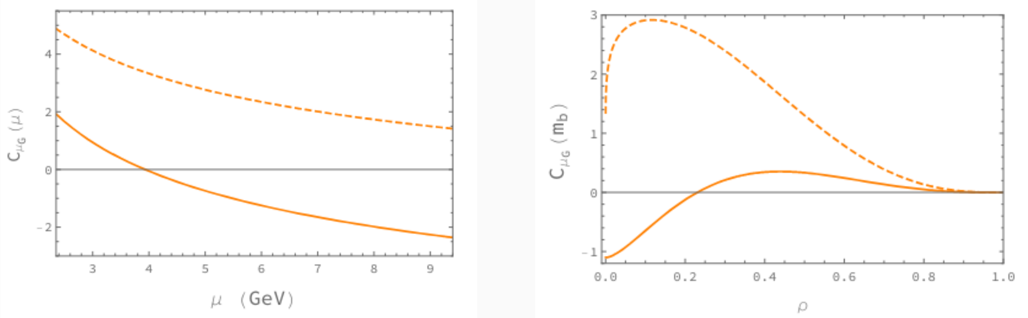


[taken from Egner, Fael, Schönwald, Steinhauser, ’24]

Extensive phenomenological analysis to appear **soon** ⇒ Stay tuned!

NLO corrections to $1/m_b^2$

- $\mathcal{O}(\alpha_s)$ to $b \to c\bar{u}d$ at $1/m_b^2$, including m_c dependence Mannel, Moreno, Pivovarov, '24
- the NLO results do not have the same strong cancellation as the LO results
- 2 % enhancement of the total $b \to c\bar{u}d$ decay rate



[$\rho = m_c^2/m_b^2$, taken from Mannel, Moreno, Pivovarov, '24]

Further room for future improvement of the SM predictions

Future improvement of the SM prediction?

- $b \to c\bar{c}s$ @ NLO QCD @ $\frac{1}{m_b^2}$
- hadronic contributions to the Darwin term $(1/m_b^3)$ @ NLO QCD
- lattice determination of $\left\langle \tilde{O}_6 \right\rangle$
- $\tilde{\Gamma}_6$ $(1/m_b^3)$ @ NNLO QCD
- matrix elements of dimension-7 operators beyond the vacuum insertion approximation

New physics effects in the HQE

New physics in lifetimes

Standard Model:

$$\Gamma\left(B_q\right) = \Gamma_b + \delta\Gamma^{\mathrm{SM}}_{B_q} \qquad \Rightarrow \qquad \frac{\tau\left(B_q\right)}{\tau\left(B_{q'}\right)} = 1 + \tau\left(B_q\right)\left[\delta\Gamma^{\mathrm{SM}}_{B_{q'}} - \delta\Gamma^{\mathrm{SM}}_{B_q}\right]$$

Including some generic BSM effects: $\Gamma\left(B_q\right) = \Gamma_b + \delta\Gamma^{\mathrm{SM}}_{B_q} + \delta\Gamma^{\mathrm{BSM}}_{B_q}$

$$\begin{aligned}\frac{\tau\left(B_q\right)}{\tau\left(B_{q'}\right)} &= 1 + \tau\left(B_q\right)\left[\delta\Gamma^{\mathrm{SM}}_{B_{q'}} - \delta\Gamma^{\mathrm{SM}}_{B_q}\right] + \tau\left(B_q\right)\left[\delta\Gamma^{\mathrm{BSM}}_{B_{q'}} - \delta\Gamma^{\mathrm{BSM}}_{B_q}\right] \\ &= 1 + \tau\left(B_q\right)\left[\delta\Gamma^{\mathrm{SM}}_{B_{q'}} - \delta\Gamma^{\mathrm{SM}}_{B_q}\right] + \left[\mathcal{B}\left(B_{q'} \xrightarrow{\text{BSM}} X\right)\frac{\tau\left(B_q\right)}{\tau\left(B_{q'}\right)} - \mathcal{B}\left(B_q \xrightarrow{\text{BSM}} Y\right)\right]\end{aligned}$$

BSM contributions to the dimension-6 spectator effects

(in collaboration with M. Black, A. Lenz, Z. Wüthrich)

- Spectator effects at dimension-6 are the main reason for $\Gamma(B_d) \neq \Gamma(B^+)$
- NP in the $|\Delta B| = 1$ Hamiltonian would give rise to additional $|\Delta B| = 0$ operators in the HQE $\Rightarrow$ **What are their matrix elements?**
- We want to compute them via heavy-quark effective theory (HQET) sum rules

Why sum rules (and not lattice QCD)?...

... because sum rules are an independent systematic approach, computationally cheaper and rather flexible

... but they are intrinsically rather imprecise and it is non-trivial to systematically refine them

Bag parameters

Vacuum saturation/insertion approximation (VSA/VIA)

Take the operator $Q(\mu) = 4\bar{b}\gamma_\mu P_L q\,\bar{q}\gamma^\mu P_L b$ and $\mathbb{1} = \sum_X |X\rangle\langle X|$, then

$$\begin{aligned}\langle B|Q|B\rangle &= 4\sum_X \langle B|\bar{b}\gamma_\mu P_L q|X\rangle\langle X|\bar{q}\gamma^\mu P_L b|B\rangle \\ &= 4\langle B|\bar{b}\gamma_\mu P_L q|0\rangle\langle 0|\bar{q}\gamma^\mu P_L b|B\rangle + 4\sum_{|X\rangle\neq|0\rangle}\langle B|\bar{b}\gamma_\mu P_L q|X\rangle\langle X|\bar{q}\gamma^\mu P_L b|B\rangle \\ &= f_B^2 M_B^2 + 4\sum_{|X\rangle\neq|0\rangle}\langle B|\bar{b}\gamma_\mu P_L q|X\rangle\langle X|\bar{q}\gamma^\mu P_L b|B\rangle \equiv f_B^2 M_B^2 B(\mu)\end{aligned}$$

The bag parameter $B(\mu)$ incorporates the deviations from the VIA!

QCD: $\langle 0|\bar{b}\gamma_\mu\gamma_5 q|B(p)\rangle = -\mathrm{i} f_B p_\mu$, HQET: $\langle 0|\bar{h}\gamma_\mu\gamma_5 q|B(v)\rangle = -\mathrm{i}F(\mu)\,v_\mu$

HQET sum rules for bag parameters

Consider the three-point correlator for an HQET operator $\tilde{Q}$

$$K_{\tilde{Q}}(\omega_1, \omega_2) = \int \mathrm{d}^d x_1 \mathrm{d}^d x_2 \mathrm{e}^{\mathrm{i}p_1 \cdot x_1 - \mathrm{i}p_2 \cdot x_2} \left\langle 0 \middle| T\left\{\tilde{j}^\dagger(x_2)\,\tilde{Q}(0)\,\tilde{j}(x_1)\right\} \middle| 0 \right\rangle$$

with $\tilde{j}_5 \equiv \bar{h}\gamma_5 q$ and $\omega_{1,2} = p_{1,2} \cdot v$

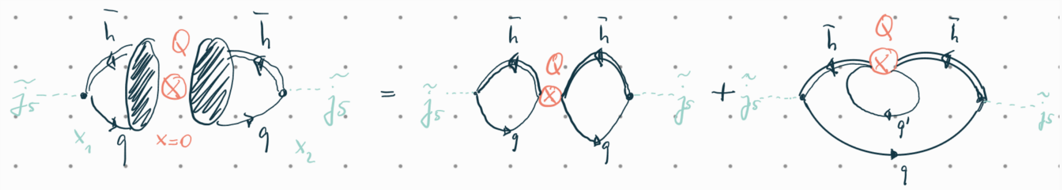
- K_Q is analytic in $\omega_{1,2}$ except for discontinuities at positive real $\omega_{1,2} \Rightarrow$ **dispersion relation**

$$K_{\tilde{Q}}(\omega_1, \omega_2) = \int_0^\infty \mathrm{d}\eta_1 \mathrm{d}\eta_2 \frac{\tilde{\rho}(\omega_1, \omega_2)}{(\eta_1 - \omega_1)(\eta_2 - \omega_2)} + \text{ subtraction terms}$$

- at large negative $\omega_{1,2}$ compute $K_{\tilde{Q}}$ through an OPE:

$$K_{\tilde{Q}}^{\mathrm{OPE}}(\omega_1, \omega_2) = K_{\tilde{Q}}^{\mathrm{pert}}(\omega_1, \omega_2) + K_{\tilde{Q}}^{\langle \bar{q}q \rangle}(\omega_1, \omega_2)\langle \bar{q}q \rangle + K_{\tilde{Q}}^{\langle \alpha_s G^2 \rangle}(\omega_1, \omega_2)\langle \alpha_s G^2 \rangle + \ldots$$

Bag parameter: perturbative part



- non-factorisable diagrams contribute to $B(\mu) - 1$
- the "eye diagrams" mix into lower-dimensional operators (difficult)
- for decay rate differences (ratios) we can focus on the left class of diagrams (with a gluon exchanged)

Lifetimes: BSM operators

$|\Delta B| = 0$ operators (HQET)

$$\tilde{O}_1^q = \tilde{Q}_1^q \equiv 4\bar{b}\gamma_\mu P_L q\, \bar{q}\gamma^\mu P_L b\,, \qquad \tilde{O}_2^q = \tilde{Q}_2^q \equiv 4\bar{b}P_L q\, \bar{q}P_R b\,,$$

$$\tilde{O}_3^q = \tilde{T}_1^q \equiv 4\bar{b}\gamma_\mu P_L T^a q\, \bar{q}\gamma^\mu P_L T^a b\,, \qquad \tilde{O}_4^q = \tilde{T}_2^q \equiv 4\bar{b}P_L T^a q\, \bar{q}P_R T^a b\,,$$

$$\tilde{O}_5^q \equiv 4\bar{b}\gamma_\mu P_L q\, \bar{q}\gamma^\mu P_R b\,, \qquad \tilde{O}_6^q \equiv 4\bar{b}P_L q\, \bar{q}P_L b\,,$$

$$\tilde{O}_7^q \equiv 4\bar{b}\gamma_\mu P_L T^a q\, \bar{q}\gamma^\mu P_R T^a b\,, \qquad \tilde{O}_8^q \equiv 4\bar{b}P_L T^a q\, \bar{q}P_L T^a b$$

Operators $\tilde{O}_{9,(10)} = 4\bar{h}\sigma_{\mu\nu}P_L(T^a)q\,\bar{q}\sigma^{\mu\nu}P_L(T^a)h$ are not independent in HQET!

(Lenz, Müller, Piscopo, Rusov, '23])

renormalised correlator:

$$\boxed{K_{\tilde{Q}_i}^{(1)} = K_{\tilde{Q}_i}^{(1),\mathrm{bare}} + \frac{1}{2\epsilon}\left[\left(2\tilde{\gamma}_{\tilde{j}}^{(0)}\delta_{ij} + \tilde{\gamma}_{\tilde{Q}_i\tilde{Q}_j}^{(0)}\right)K_{\tilde{Q}_j}^{(0)} + \tilde{\gamma}_{\tilde{Q}_i\tilde{E}_j}^{(0)}K_{\tilde{E}_j}^{(0)}\right]}$$

Current status:

- HQET Feynman rules, IBP reduction, master integrals ✓[Grozin, Lee, ’09]
  $\Rightarrow$ bare correlation function ✓
- double discontinuity ✓
- γ_5?
- operator mixing computed, final renormalisation: WIP
- comparison to the first SM HQET SR determination: WIP [Kirk, Lenz, Rauh, ’17]
- sign of $\tilde{\epsilon}_1$?

Summary

Recent progress, in particular higher-order perturbative calculations, have made the HQE an increasingly powerful framework, not only for lifetime ratios, **but also for absolute lifetimes!**

- future NLO corrections to $1/m_b^2$ $(c\bar{c}s)$, $1/m_b^3$ (NL) operators will further increase its power
- first lattice (and more precise HQET SR) determinations of the dimension-6 and -7 matrix elements will reduce uncertainties of lifetime ratios
- these recent advances enable us to use the HQE as a probe of new physics these days

Backup

If $b(x)$ is the QCD b-quark field, then $b(x) \equiv \mathrm{e}^{-\mathrm{i} m_b v_\mu x^\mu} b_v(x)$, where $p_b^\mu = m_b v^\mu + \mathrm{i} D^\mu$, such that b_v carries "momentum" $p_b^\mu - m_b v^\mu$

Hadronic matrix elements, numerically:

- $\mu_G^2(B_q) = 0.332(62)\,\mathrm{GeV}^2$ (semileptonic fits), D mesons: similar size (spectroscopy)
- $\mu_{\pi^2}(B_q) = 0.465(68)\,\mathrm{GeV}^2$ (semileptonic fits), uncertainties in D system somewhat larger
- $\rho_D^3(B_q) = 0.170(38)\,\mathrm{GeV}^3$ (semileptonic fits); D mesons: various different estimates

Hadronic matrix elements, 4-quark operators (SM)

$$\tilde{B}_1(1.5\text{ GeV}) = 1.000\,^{+0.020}_{-0.020} = 1.000\,^{+0.000}_{-0.000}(\overline{\Lambda})\,^{+0.020}_{-0.020}(\text{intr.})\,^{+0.002}_{-0.002}(\text{cond.})\,^{+0.000}_{-0.001}(\mu_\rho),$$

$$\tilde{B}_2(1.5\text{ GeV}) = 1.000\,^{+0.020}_{-0.020} = 1.000\,^{+0.000}_{-0.000}(\overline{\Lambda})\,^{+0.020}_{-0.020}(\text{intr.})\,^{+0.002}_{-0.002}(\text{cond.})\,^{+0.000}_{-0.001}(\mu_\rho),$$

$$\tilde{\epsilon}_1(1.5\text{ GeV}) = -0.016\,^{+0.021}_{-0.022} = -0.016\,^{+0.007}_{-0.008}(\overline{\Lambda})\,^{+0.020}_{-0.020}(\text{intr.})\,^{+0.003}_{-0.003}(\text{cond.})\,^{+0.003}_{-0.003}(\mu_\rho),$$

$$\tilde{\epsilon}_2(1.5\text{ GeV}) = 0.004\,^{+0.022}_{-0.022} = 0.004\,^{+0.007}_{-0.008}(\overline{\Lambda})\,^{+0.020}_{-0.020}(\text{intr.})\,^{+0.004}_{-0.004}(\text{cond.})\,^{+0.002}_{-0.002}(\mu_\rho).$$

$$\overline{B}_1(\mu = \overline{m}_b(\overline{m}_b)) = 1.028\,^{+0.064}_{-0.056} = 1.028\,^{+0.019}_{-0.019}(\text{sum rule})\,^{+0.061}_{-0.053}(\text{matching}),$$

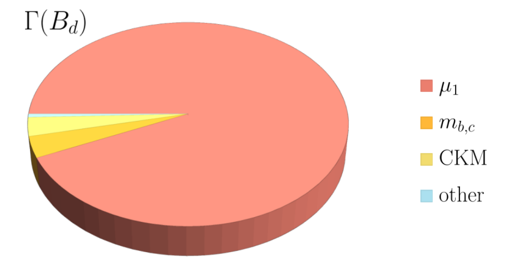
$$\overline{B}_2(\mu = \overline{m}_b(\overline{m}_b)) = 0.988\,^{+0.087}_{-0.079} = 0.988\,^{+0.020}_{-0.020}(\text{sum rule})\,^{+0.085}_{-0.077}(\text{matching}),$$

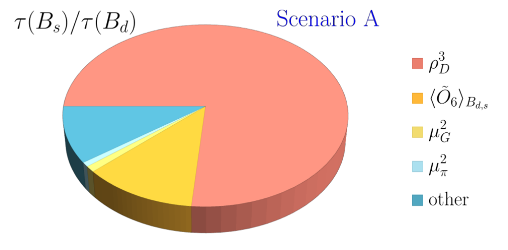
$$\overline{\epsilon}_1(\mu = \overline{m}_b(\overline{m}_b)) = -0.107\,^{+0.028}_{-0.029} = -0.107\,^{+0.023}_{-0.024}(\text{sum rule})\,^{+0.015}_{-0.017}(\text{matching}),$$

$$\overline{\epsilon}_2(\mu = \overline{m}_b(\overline{m}_b)) = -0.033\,^{+0.021}_{-0.021} = -0.033\,^{+0.018}_{-0.018}(\text{sum rule})\,^{+0.011}_{-0.011}(\text{matching}).$$

[Kirk, Lenz, Rauh, '17]

Uncertainties, early 2024





[taken from Albrecht, Bernlochner, Lenz, Rusov, '24]

HQET sum rules for lifetimes: evanescent operators

In HQET, the $\sigma_{\mu\nu}$ operators are not independent (Lenz, Müller, Piscopo, Rusov, '23]):

$$\bar{h}\sigma_{\mu\nu}q\,\bar{q}\sigma^{\mu\nu}h = -2\left[\bar{h}q\,\bar{q}h - \bar{h}\gamma_\mu q\,\bar{q}\gamma^\mu h + \bar{h}\gamma_5 q\,\bar{q}\gamma_5 h + \bar{h}\gamma_\mu\gamma_5 q\,\bar{q}\gamma^\mu\gamma_5 h\right] + \mathcal{O}\left(\frac{1}{m_b}\right) .$$

evanescent operators:

$$\begin{aligned}
\tilde{E}_1^q &\equiv \bar{h}\gamma_{\mu\nu\rho}\,(1-\gamma_5)\,q\,\bar{q}\gamma^{\rho\nu\mu}\,(1-\gamma_5)\,h \quad - (4 + a_1\epsilon)\,\tilde{O}_1^q\,,\\
\tilde{E}_2^q &\equiv \bar{h}\gamma_{\mu\nu}\,(1-\gamma_5)\,q\,\bar{q}\gamma^{\nu\mu}\,(1+\gamma_5)\,h \quad - (4 + a_2\epsilon)\,\tilde{O}_2^q\,,\\
\tilde{E}_3^q &\equiv \bar{h}\gamma_{\mu\nu\rho}\,(1-\gamma_5)\,T^a q\,\bar{q}\gamma^{\rho\nu\mu}\,(1-\gamma_5)\,T^a h \quad - (4 + a_1\epsilon)\,\tilde{O}_3^q\,,\\
\tilde{E}_4^q &\equiv \bar{h}\gamma_{\mu\nu}\,(1-\gamma_5)\,T^a q\,\bar{q}\gamma^{\nu\mu}\,(1+\gamma_5)\,T^a h \quad - (4 + a_2\epsilon)\,\tilde{O}_4^q\,,\\
\tilde{E}_5^q &\equiv \bar{h}\gamma_{\mu\nu\rho}\,(1-\gamma_5)\,q\,\bar{q}\gamma^{\rho\nu\mu}\,(1+\gamma_5)\,h \quad - (16 + a_3\epsilon)\,\tilde{O}_5^q\,,\\
\tilde{E}_6^q &\equiv \bar{h}\gamma_{\mu\nu\rho}\,(1-\gamma_5)\,T^a q\,\bar{q}\gamma^{\rho\nu\mu}\,(1+\gamma_5)\,T^a h \quad - (16 + a_3\epsilon)\,\tilde{O}_7^q\,,
\end{aligned}$$

D-meson lifetimes

D-meson lifetimes (PDG)

$$\tau\left(D^0\right) = 0.4103(10)\,\text{ps}$$
$$\tau\left(D^+\right) = 1.033(5)\,\text{ps}$$
$$\tau\left(D_s^+\right) = 0.5012(22)\,\text{ps}$$
$$\frac{\tau\left(D^+\right)}{\tau\left(D^0\right)} \approx 2.52$$
$$\frac{\tau\left(D_s^+\right)}{\tau\left(D^0\right)} \approx 1.22$$

b-baryon lifetimes (HFLAV, (PDG))

$$\tau\left(\Lambda_b = udb\right) = 1.471(9)\,\text{ps}$$
$$\tau\left(\Xi_b^+ = uub\right) = 1.572(40)\,\text{ps}$$
$$\tau\left(\Xi_b^0 = usb\right) = 1.480(30)\,\text{ps}$$
$$\tau\left(\Omega_b^- = ssb\right) = \left(1.64^{+0.18}_{-0.17}\right)\,\text{ps}$$
$$\frac{\tau\left(\Lambda_b\right)}{\tau\left(B_d\right)} = 0.964 \pm 0.007$$

theory: $\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.955 \pm 0.014$ [GLMNPR, '23]

Spectator effects: topologies

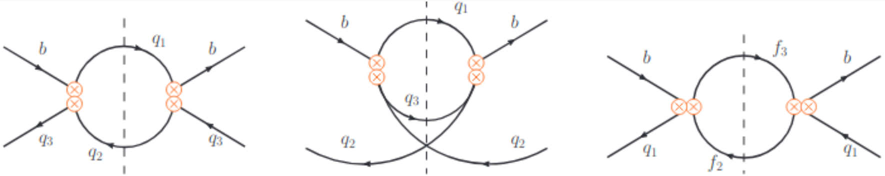


Fig. 5 Diagrams describing the weak exchange (left), Pauli interference (middle) and weak annihilation (right) topologies, at LO-QCD. Here, $q_{1,2} = c, u$, $q_3 = d, s$, $f_1 = c, u, \ell$, and $f_2 = d, s, \nu_\ell$, with $\ell = e, \mu, \tau$.

[taken from Albrecht, Bernlochner, Lenz, Rusov, '24]

Sum rules

$$K^{\text{had}}_{\tilde{Q}}(\omega_1,\omega_2) = \int_0^\infty d\eta_1 d\eta_2 \, \frac{\rho^{\text{had}}_{\tilde{Q}}(\eta_1,\eta_2)}{(\eta_1-\omega_1)(\eta_2-\omega_2)} + [\text{subtraction terms}]\,,$$

$$\rho^{\text{had}}_{\tilde{Q}}(\omega_1,\omega_2) = F^2(\mu)\langle\tilde{Q}(\mu)\rangle\delta(\omega_1-\overline{\Lambda})\delta(\omega_2-\overline{\Lambda}) + \rho^{\text{cont}}_{\tilde{Q}}(\omega_1,\omega_2). \tag{28}$$

We use a double Borel transformation with respect to $\omega_{1,2}$ to remove the contribution from the integration over the circle at infinity and to suppress the sensitivity to the continuum part $\rho^{\text{cont}}_{\tilde{Q}}$ of the spectral function, which yields the sum rule

$$\int_0^\infty d\omega_1 d\omega_2 e^{-\frac{\omega_1}{t_1}-\frac{\omega_2}{t_2}}\rho^{\text{OPE}}_{\tilde{Q}}(\omega_1,\omega_2) = \int_0^\infty d\omega_1 d\omega_2 e^{-\frac{\omega_1}{t_1}-\frac{\omega_2}{t_2}}\rho^{\text{had}}_{\tilde{Q}}(\omega_1,\omega_2). \tag{29}$$

[Kirk, Lenz, Rauh, '17]

Sum rules

$$\Delta B_{\tilde{Q}_i} = \frac{1}{A_{\tilde{Q}_i} F(\mu)^4} \int_0^{\omega_c} d\omega_1 d\omega_2 e^{\frac{\overline{\Lambda}-\omega_1}{t_1}+\frac{\overline{\Lambda}-\omega_2}{t_2}} \Delta\rho_{\tilde{Q}_i}(\omega_1,\omega_2)$$

$$w_{\tilde{Q}_i}(\omega_1,\omega_2) = \frac{\Delta\rho^{\text{pert}}_{\tilde{Q}_i}(\omega_1,\omega_2)}{\rho^{\text{pert}}_{\Pi}(\omega_1)\rho^{\text{pert}}_{\Pi}(\omega_2)} = \frac{C_F}{N_c}\frac{\alpha_s}{4\pi} r_{\tilde{Q}_i}(x, L_\omega),$$

we can remove the integration in (43) altogether and find the simple result

$$\Delta B^{\text{pert}}_{\tilde{Q}_i}(\mu_\rho) = \frac{C_F}{N_c A_{\tilde{Q}_i}} \frac{\alpha_s(\mu_\rho)}{4\pi} r_{\tilde{Q}_i}\left(1, \log\frac{\mu_\rho^2}{4\overline{\Lambda}^2}\right).$$

[Kirk, Lenz, Rauh, ’17]