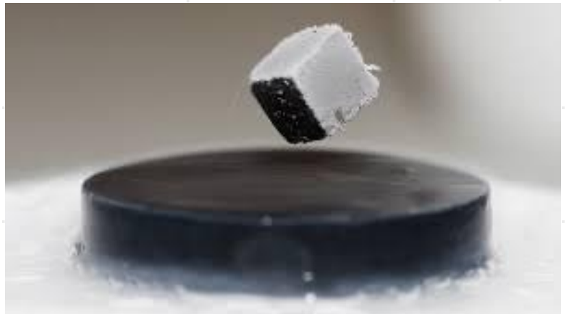
PTQFT

Breakdown of the Meissner effect at the exceptional point



Takano Taira[A]

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➤ Taira, T. (2024). J. Phys. A 57(5), 055001.

Question

How does non-Hermitian Hamiltonian appear ?

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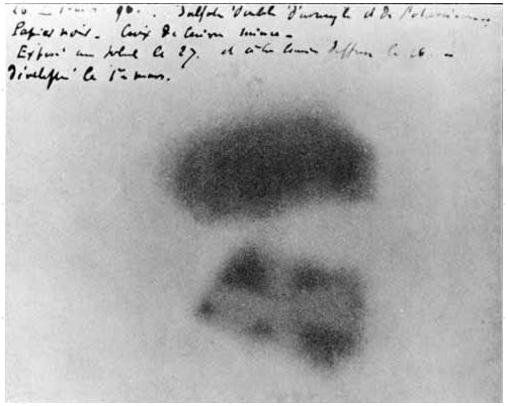
- 1930: Victor Weisskopf, Eugene Wigner



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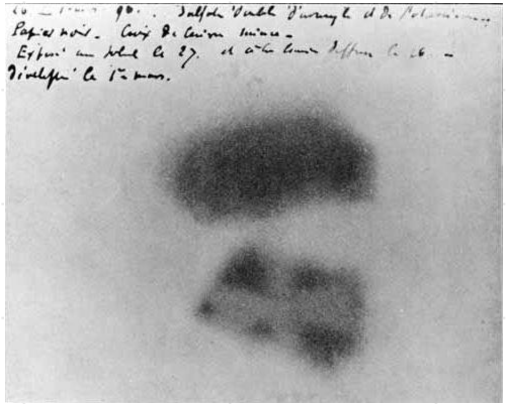

Für die Anregungswahrscheinlichkeit $|a|^2$ des oberen Zustandes ist von vornherein ein exponentieller Abfall zu erwarten. Wir versuchen daher den Ansatz

$$a = e^{-2\pi\Gamma t}. \tag{15a}$$

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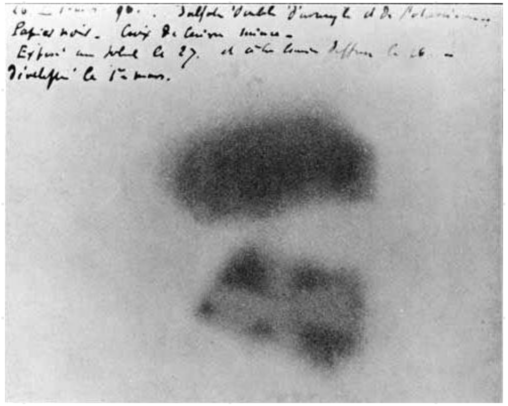

Für die **Anregungswahrscheinlichkeit** $|a|^2$ des oberen Zustandes ist von vornherein ein exponentieller Abfall zu erwarten. Wir versuchen daher den **Ansatz**

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$$a = e^{-2\pi\Gamma t}. \quad (15\,a)$$

$$|\psi(t)\rangle = e^{-i\,H\,t - \pi\Gamma\,t}|\psi(0)\rangle$$

$$H_{eff} = H - i\pi\,\Gamma$$

Question

Für die Anregungswahrscheinlichkeit $|a|^2$ des oberen Zustandes ist von vornherein ein exponentieller Abfall zu erwarten. Wir versuchen daher den Ansatz

$$a = e^{-2\pi \Gamma t}. \tag{15a}$$

$$|\psi(t)\rangle = e^{-i H t - \pi \Gamma t} |\psi(0)\rangle$$

$$H_{eff} = H - i\pi\,\Gamma$$

Decay rate

Non-Hermitian Hamiltonian

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Non-Hermitian Hamiltonian

- 1930: Victor Weisskopf, Eugene Wigner

$$H_{eff} = H - i\pi\,\Gamma$$

$$\vdots$$

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⬇

$$e^{iHt+\pi\Gamma t}|\psi(t)\rangle = |\psi(0)\rangle$$

Question

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⬇

$$e^{i\,H\,t+\,\pi\Gamma\,t}|\psi(t)\rangle = |\psi(0)\rangle$$

Reversible…

But decaying is irreversible…

Question

- 1970: Schulman proposed to use semi-group to implement irreversibility.
- 1971: Williams
- 1971: Horwitz et.al
- 1972: Sinha

Question

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- 1971: Williams
- 1971: Horwitz et.al
- 1972: Sinha
- 1976 May: Gorini Kossakowski Sudarshan N-level
- 1976: Lindblad derived general form of equation solved by semi-group

Reminder

$$|\psi(t)\rangle$$

Reminder

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$$\rho(t) = |\psi(t)\rangle\langle\psi(t)|$$

Reminder

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$$\rho(t) = |\psi(t)\rangle\langle\psi(t)|$$

$$\rho(t) = \sum_n p_{nm}(t)|n\rangle\langle m|$$

Non-Hermitian Hamiltonian

- 1930: Victor Weisskopf, Eugene Wigner

- 1976: Gorini, Kossakowski, Sudarshan and Lindblad

Non-Hermitian Hamiltonian

- 1930: Victor Weisskopf, Eugene Wigner

$$H_{eff} = H - i\pi\,\Gamma$$

$$\frac{d\rho(t)}{dt} = -i\left(H_{eff}\rho(t) - \rho(t)H_{eff}^{\dagger}\right)$$

- 1976: Gorini, Kossakowski, Sudarshan and Lindblad

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$$\frac{d\rho(t)}{dt} = -i\left(H_{eff}\rho(t) - \rho(t)H_{eff}^{\dagger}\right) + 2\gamma\sum_{n} L_n\rho(t)L_n^{\dagger}$$

Non-Hermitian Hamiltonian

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- 1976: Gorini, Kossakowski, Sudarshan and Lindblad

$$\frac{d\rho(t)}{dt} = -i\,\mathcal{L}[\rho(t)]$$

Non-Hermitian

Non-Hermitian Hamiltonian

- 1930: Victor Weisskopf, Eugene Wigner

$$H_{eff} = H - i\pi\,\Gamma \qquad N \times N$$

- 1976: Gorini, Kossakowski, Sudarshan and Lindblad

$$\mathcal{L}[\rho] = -i\left(H_{eff}\rho(t) - \rho(t)H_{eff}^{\dagger}\right) + 2\gamma \sum_{n} L_n \rho(t) L_n^{\dagger} \qquad N^2 \times N^2$$

Non-Hermitian Hamiltonian

Assumption

- 1930: Victor Weisskopf, Eugene Wigner

$$H_{eff} = H - i\pi\,\Gamma \qquad N \times N$$

- 1976: Gorini, Kossakowski, Sudarshan and Lindblad

$$\mathcal{L}[\rho] = -i\left(H_{eff}\rho(t) - \rho(t)H_{eff}^{\dagger}\right) + 2\gamma\sum_{n} L_n\rho(t)L_n^{\dagger} \qquad N^2 \times N^2$$

Non-Hermitian Hamiltonian

Markovian approximation

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Non-Hermitian Hamiltonian

Markovian approximation

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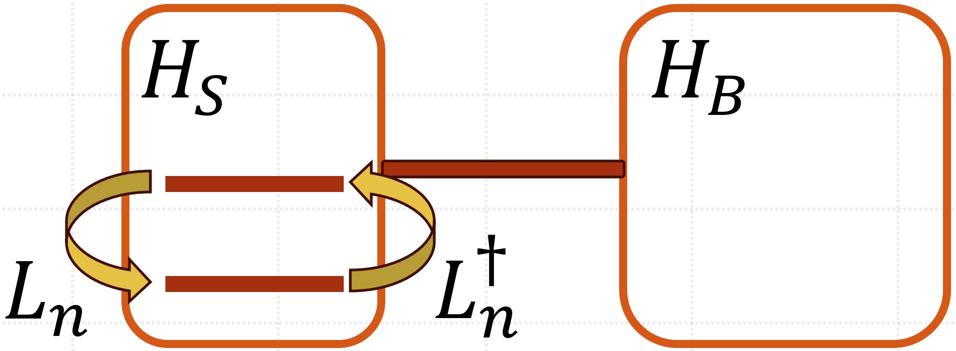
- 2024: Taira, Hatano, Nishino Non-Hermitian and non-linear Hamiltonian with Feshbach formalism. https://arxiv.org/abs/2406.17436

Set up

- 1976: Gorini, Kossakowski, Sudarshan and Lindblad

$$\mathcal{L}[\rho] = -i\left(H_{eff}\rho(t) - \rho(t)H_{eff}^{\dagger}\right) + 2\gamma\sum_{n} L_n\rho(t)L_n^{\dagger}$$

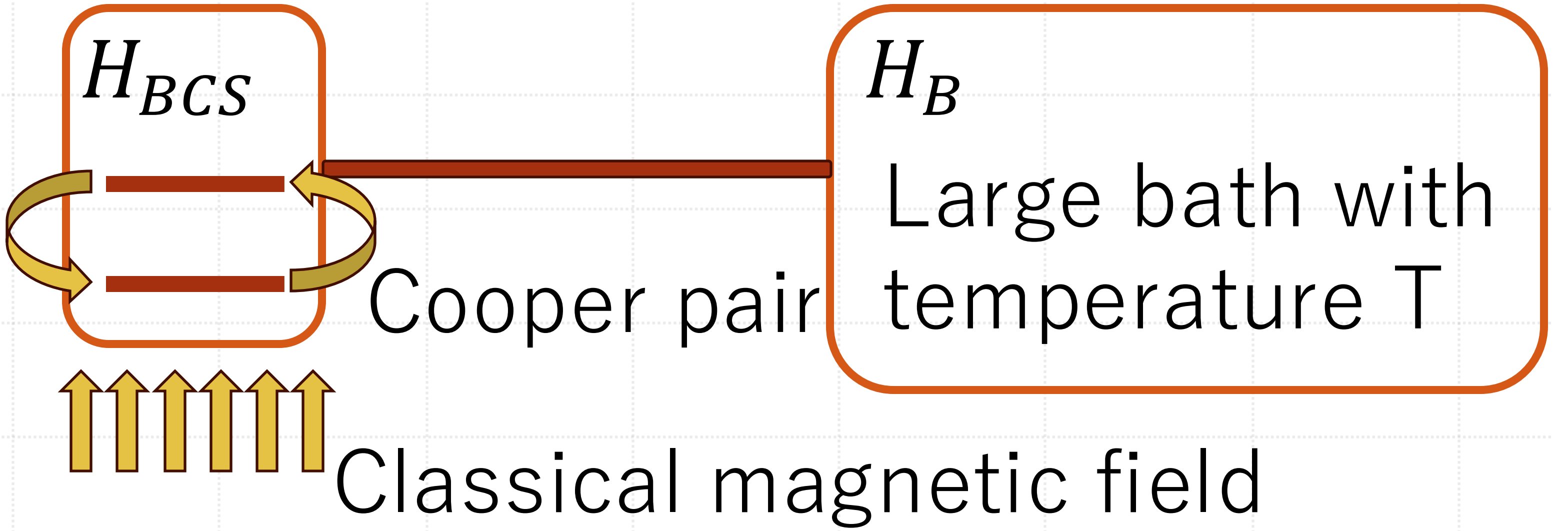


$$H_{eff} = H_S - i\sum_{n} L_n^{\dagger}L_n$$

Set up

- 2023: Taira, T. (2024). J. Phys. A 57(5), 055001.

$$\mathcal{L}[\rho] = -i\left(H_{eff}\rho(t) - \rho(t)H_{eff}^{\dagger}\right) + 2\gamma\sum_{n} L_n\rho(t)L_n^{\dagger}$$



Model

- 2023: Taira, T. (2024). J. Phys. A 57(5), 055001.

$$H_{\mathrm{Tot}} = H_{\mathrm{BCS}}[c_1^\dagger, c_1] + H_{\mathrm{Int}}[c_1^\dagger, c_2, c_1^\dagger, c_2] + H_{\mathrm{NH}}[c_2^\dagger, c_2] \quad (1.1)$$

$$H_{\mathrm{BCS}} = \int d^3r \sum_{\sigma=\uparrow,\downarrow} c_{1\sigma}^\dagger(\vec{r}) \left[-\frac{1}{2m_1}\left(\nabla - ie\vec{A}\right)^2 - \mu_1 \right] c_{1\sigma}(\vec{r}) - g c_{1\uparrow}^\dagger(\vec{r}) c_{1\downarrow}^\dagger(\vec{r}) c_{1\downarrow}(\vec{r}) c_{1\uparrow}(\vec{r}), \quad (1.2)$$

$$H_{\mathrm{NH}} = \int d^3r \sum_{\sigma=\uparrow,\downarrow} c_{2\sigma}^\dagger(\vec{r}) \left[-\frac{1}{2m_2}\left(\nabla - ie\vec{A}\right)^2 - \mu_2 \right] c_{2\sigma}(\vec{r}) \quad (1.3)$$

$$H_{\mathrm{int}} = -\mu \int d^3r \; c_{1\uparrow}^\dagger c_{1\downarrow}^\dagger c_{2\uparrow} c_{2\downarrow} + c_{2\uparrow}^\dagger c_{2\downarrow}^\dagger c_{1\uparrow} c_{1\downarrow}. \quad (1.4)$$

Method

- 2023: Taira, T. (2024). J. Phys. A 57(5), 055001.

$$H_{\mathrm{Tot}}[c_i, c_i^\dagger] = H_{\mathrm{Tot}}[c_i, c_i^\dagger]^\dagger$$

$$\mathcal{L}[\rho] = -i\left(H_{eff}\rho(t) - \rho(t)H_{eff}^\dagger\right) + 2\gamma\sum_n L_n\rho(t)L_n^\dagger$$

$$H_{eff} = H_S - i\sum_n L_n^\dagger L_n$$

Jump term
i.e. Only decay

Method

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$$H_{\mathrm{Tot}}[c_i, c_i^\dagger] = H_{\mathrm{Tot}}[c_i, c_i^\dagger]^\dagger$$

Cooper pair

$L_n = c_{n\downarrow} c_{n\uparrow}$

$$H_{eff} = H_S - i \sum_n L_n^\dagger L_n$$

Method

- 2023: Taira, T. (2024). J. Phys. A 57(5), 055001.

$$H_{\text{eff}}[c_i, c_i^\dagger] \neq H_{\text{eff}}[c_i, c_i^\dagger]^\dagger$$

$$c_n|\psi_n\rangle = \psi_n|\psi_n\rangle$$

$$S_{\text{eff}}[\psi_i, \psi_i^\dagger]$$

Method

- 2023: Taira, T. (2024). J. Phys. A 57(5), 055001.

$$S_{eff}[\psi_i, \psi_i^{\dagger}]$$

Non-Hermitian
Mean field theory

?

$$\tilde{S}[\psi_i, \psi_i^{\dagger} \Delta_i, \bar{\Delta}_i]$$

See appendix

Method

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$$\tilde{S}[\psi_i, \psi_i^\dagger \Delta_i, \bar{\Delta}_i]$$

Integrate out in partition function

$$S_{\mathrm{eff}}[\Delta_i, \bar{\Delta}_i]$$

See appendix

Complex field-theoretic action

- 2023: Taira, T. (2024). J. Phys. A 57(5), 055001.

$$
\begin{aligned}
S_{\text{eff}} \;=\; & -\operatorname{Tr}\log\begin{pmatrix} i\omega_n + \epsilon^{(1)}_{\vec{k}} & \Delta_1 + \frac{\mu}{i\gamma}\Delta_2 \\ \overline{\Delta}_1 + \frac{\mu}{i\gamma}\overline{\Delta}_2 & i\omega_n - \epsilon^{(1)}_{\vec{k}} \end{pmatrix} \\
& -\operatorname{Tr}\log\begin{pmatrix} i\omega_n + \epsilon^{(2)}_{\vec{k}} & \Delta_2 + \frac{\mu}{g_1}\Delta_1 \\ \overline{\Delta}_2 + \frac{\mu}{g_1}\overline{\Delta}_1 & i\omega_n - \epsilon^{(2)}_{\vec{k}} \end{pmatrix} \\
& + \frac{\mu}{i\gamma g}(\overline{\Delta}_1\Delta_2 + \overline{\Delta}_2\Delta_1) + \frac{1}{g}\overline{\Delta}_1\Delta_1 + \frac{1}{i\gamma}\overline{\Delta}_2\Delta_2,
\end{aligned}
\tag{5}
$$

See appendix

Complex field-theoretic action

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$$
\begin{aligned}
S_{\text{eff}} = \int d^3r\ & \alpha_1 \nabla_i \overline{\Delta}_1 \nabla_i \Delta_1 \\
& + \left(r_1 - \frac{1}{g} + r_2 \delta^2 \right) \overline{\Delta}_1 \Delta_1 + u_1 \left(\overline{\Delta}_1 \Delta_1 \right)^2 \\
& + \alpha_2 \nabla_i \overline{\Delta}_2 \nabla_i \Delta_2 + \left(r_2 - \frac{1}{g} \frac{\epsilon}{i\delta} \right) \overline{\Delta}_2 \Delta_2 \\
& + \left(-i\epsilon r_1 + \delta \left\{ r_2 - \frac{1}{g} \frac{\epsilon}{i\delta} \right\} \right) \left(\overline{\Delta}_1 \Delta_2 + \overline{\Delta}_2 \Delta_1 \right),
\end{aligned}
\qquad (6)
$$

See appendix

Complex field-theoretic action

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$$\Delta_1, \Delta_2, \Delta_1^\dagger, \overline{\Delta_2}$$

$$\Delta := \begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} \qquad \bar{\Delta} := \begin{pmatrix} \Delta_1^\dagger \\ \overline{\Delta_2} \end{pmatrix}$$

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$$S_{eff} = \int d^3r \,(\vec{\nabla} - e\,\vec{A})\bar{\Delta}(\vec{\nabla} - e\,\vec{A})\Delta$$

$$+\bar{\Delta}M\Delta - u_1\left(\Delta_1^\dagger \Delta_1\right)^2$$

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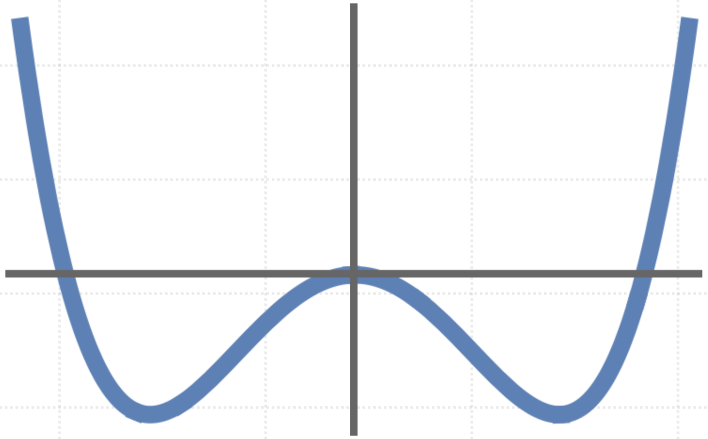

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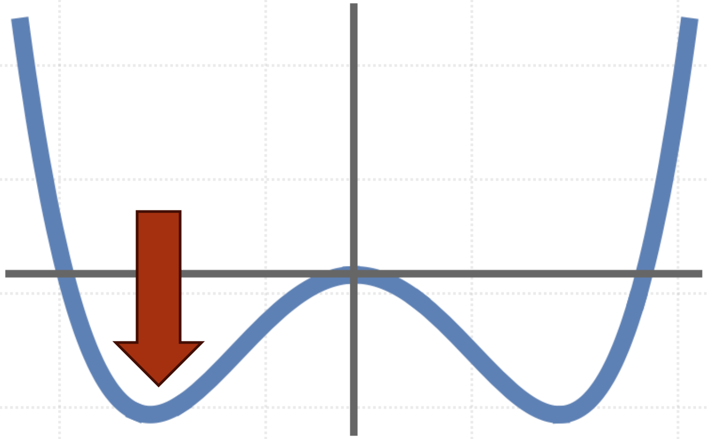

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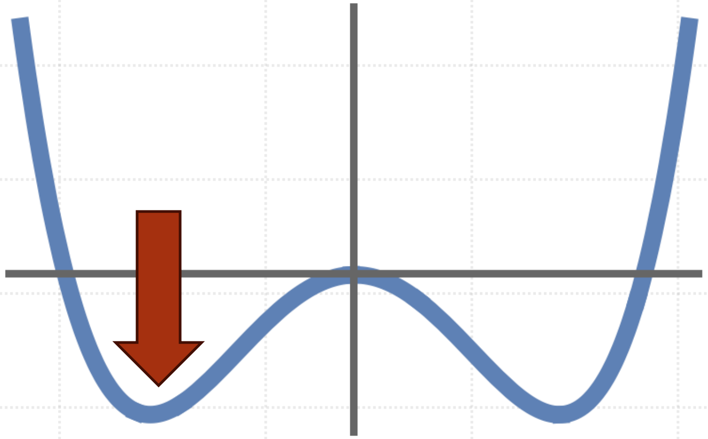
$$+\bar{\Delta} M_{eff} \Delta + \cdots$$

Non-Hermitian mass matrix



See appendix

Meissner effect

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See appendix

Meissner effect

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$$\nabla \times \left.\frac{\delta S_{eff}}{\delta A_i}\right|_{A=A_0} = 0 \quad \Rightarrow \quad \nabla \times \vec{B} = \overline{\Delta_{vac}}\Delta_{vac}\vec{B}$$

$$\vec{B} = \begin{pmatrix} B_x \\ 0 \\ 0 \end{pmatrix}$$

See appendix

Meissner effect

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$$B_x = e^{-\sqrt{\overline{\Delta_{vac}}\Delta_{vac}}\ x}$$

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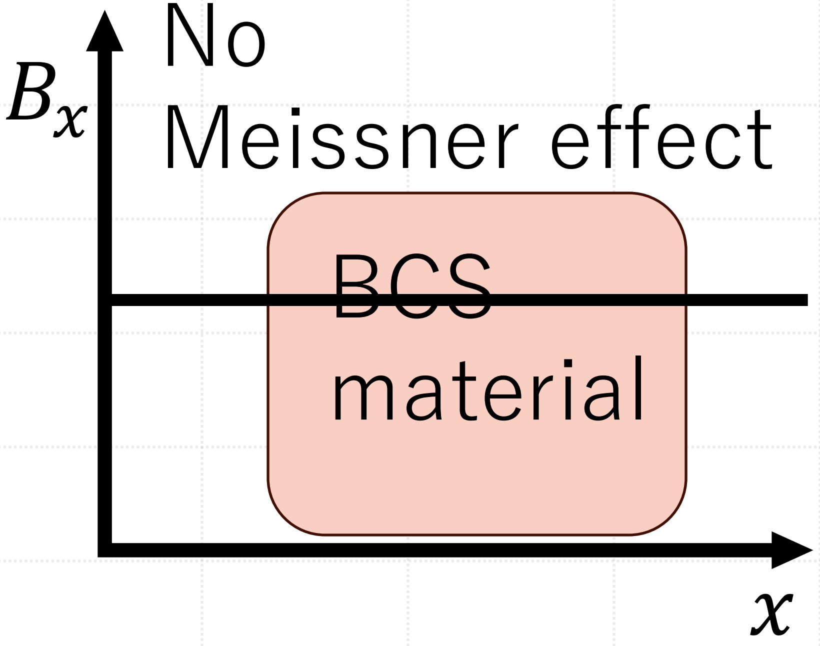

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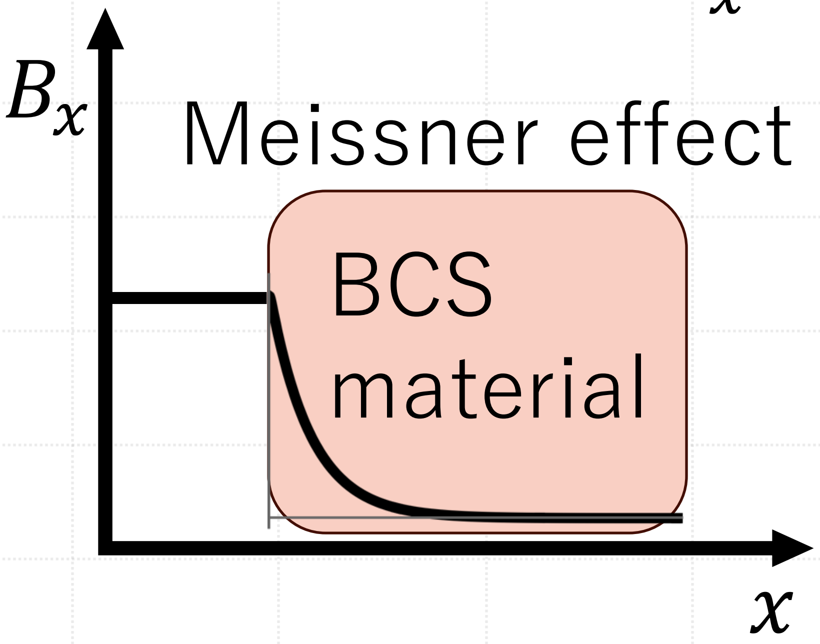

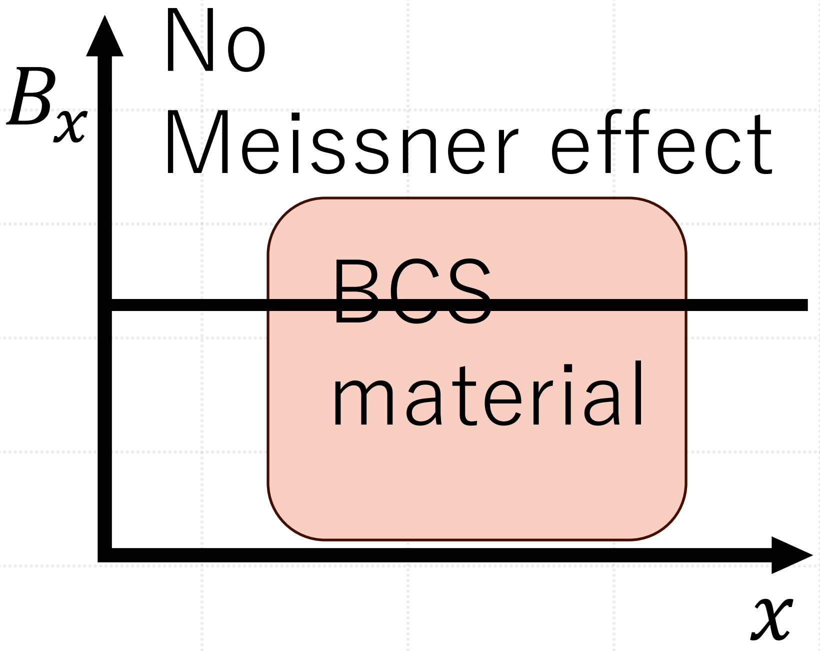

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Meissner effect

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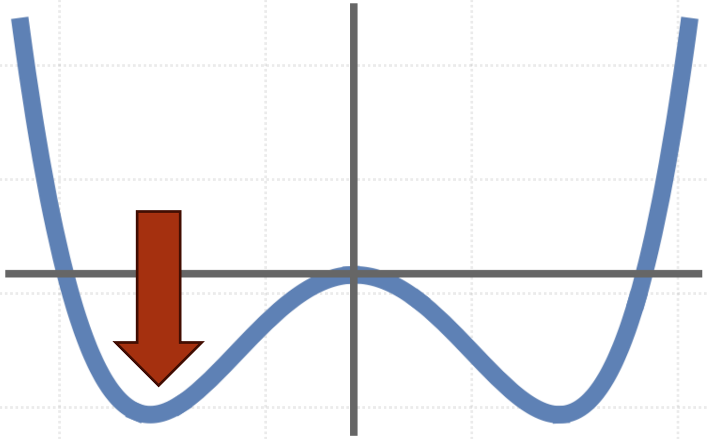
$$S_{eff} = \int d^3r\, (\vec{\nabla} - e\,\vec{A})\bar{\Delta}(\vec{\nabla} - e\,\vec{A})\Delta$$

$$+\bar{\Delta}M_{eff}\Delta + \cdots$$

Non-Hermitian mass matrix



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Set up

- ➢ 2023: Taira, T. (2024). J. Phys. A 57(5), 055001.

$$\frac{\delta S_{eff}}{\delta \Delta} = 0 \Rightarrow M_{eff}\Delta_{vac} = 0$$ Right eigenvector

$$\frac{\delta S_{eff}}{\delta \overline{\Delta}} = 0 \Rightarrow \overline{\Delta_{vac}} M_{eff} = 0$$ Left eigenvector

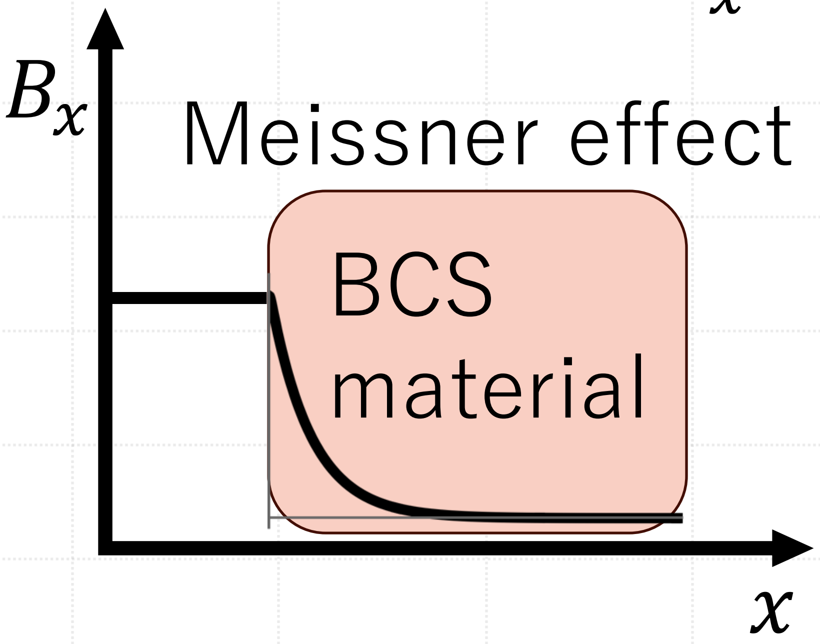
$\overline{\Delta_{vac}}\Delta_{vac} = 0$ at the exceptional point!!

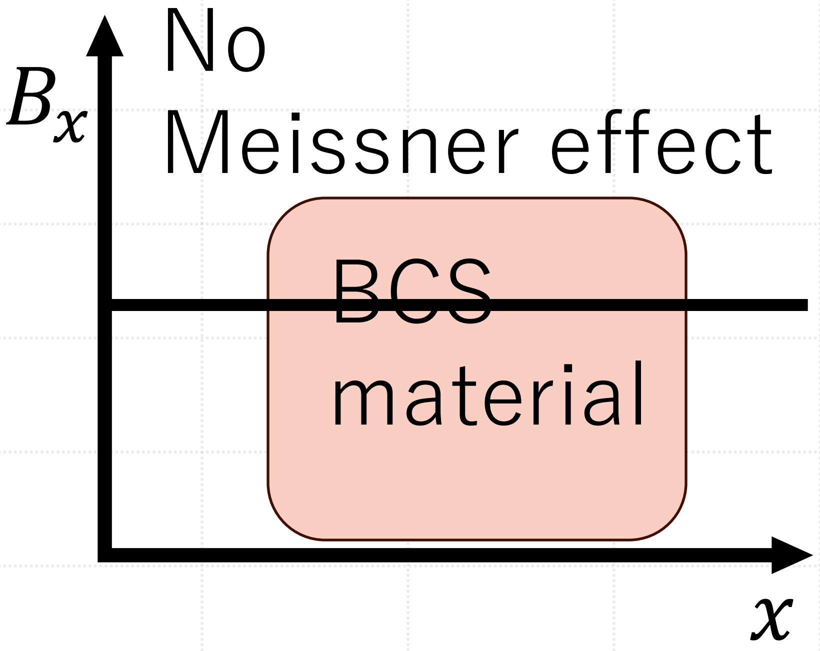
See appendix

Set up

- 2023: Taira, T. (2024). J. Phys. A 57(5), 055001.

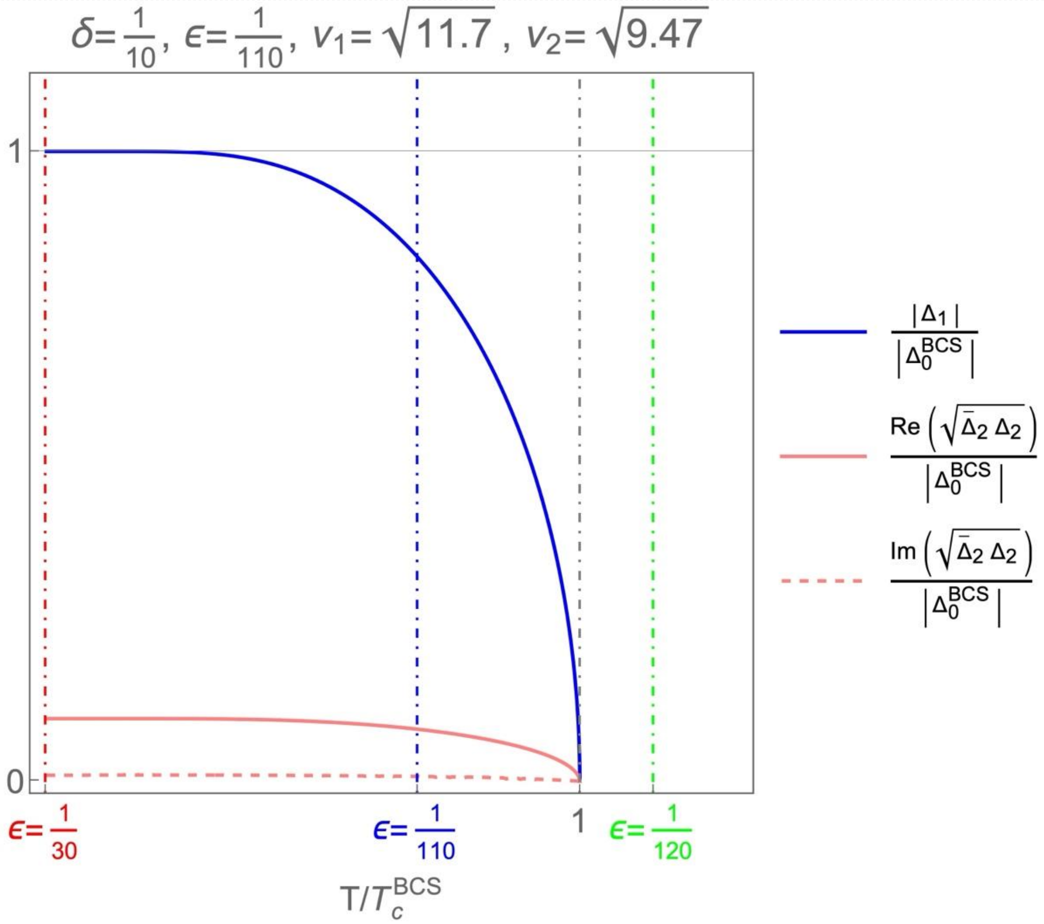
$$B_x = e^{-\sqrt{\overline{\Delta_{vac}}\Delta_{vac}}\; x}$$





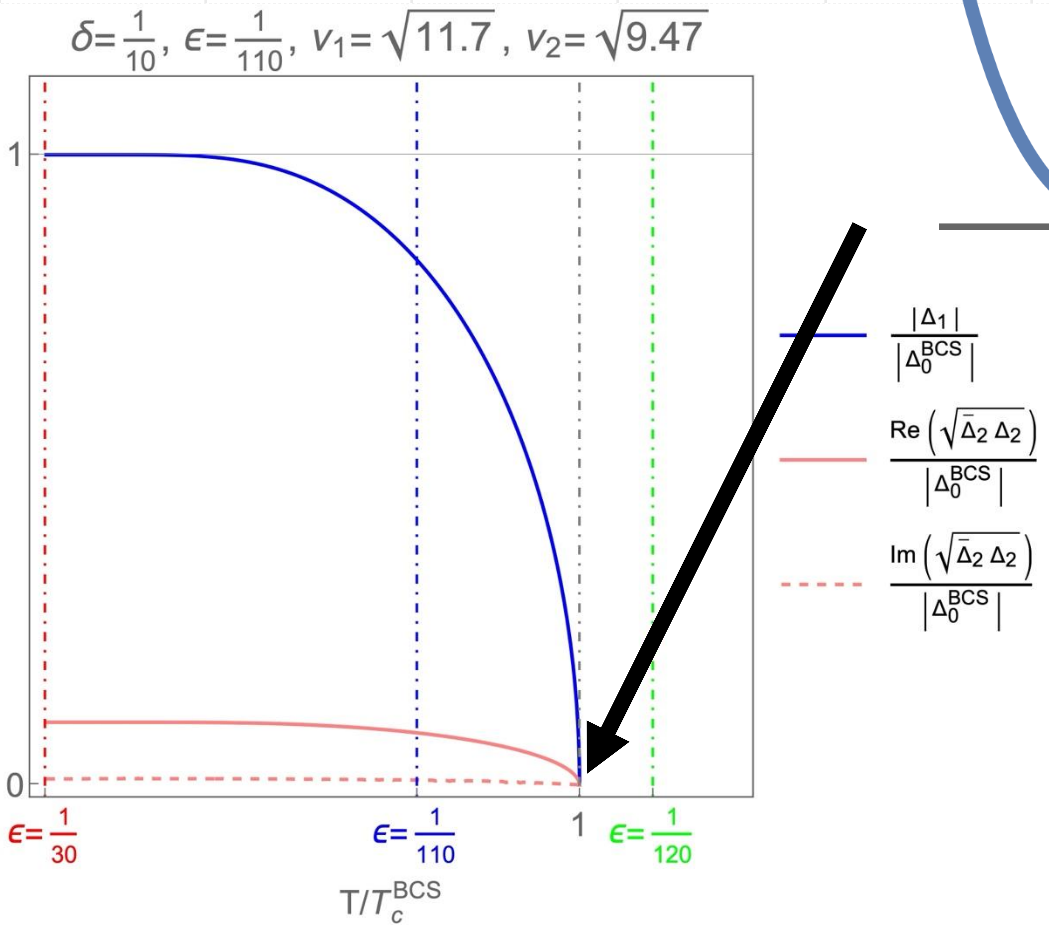
See appendix

Extra



$\delta=\frac{1}{10}$, $\epsilon=\frac{1}{110}$, $v_1=\sqrt{11.7}$, $v_2=\sqrt{9.47}$
1
0
$\epsilon=\frac{1}{30}$
$\epsilon=\frac{1}{110}$
1
$\epsilon=\frac{1}{120}$
T/T_c^{BCS}
$\frac{|\Delta_1|}{|\Delta_0^{BCS}|}$
$\frac{\mathrm{Re}\left(\sqrt{\bar{\Delta}_2 \Delta_2}\right)}{|\Delta_0^{BCS}|}$
$\frac{\mathrm{Im}\left(\sqrt{\bar{\Delta}_2 \Delta_2}\right)}{|\Delta_0^{BCS}|}$

Extra



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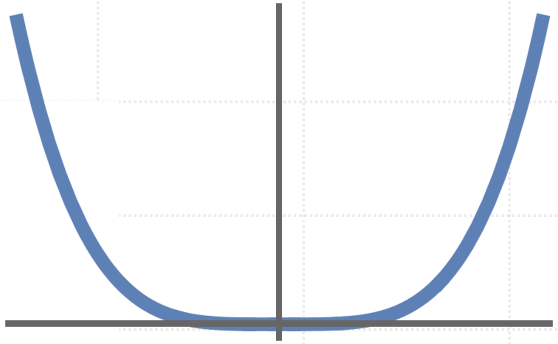
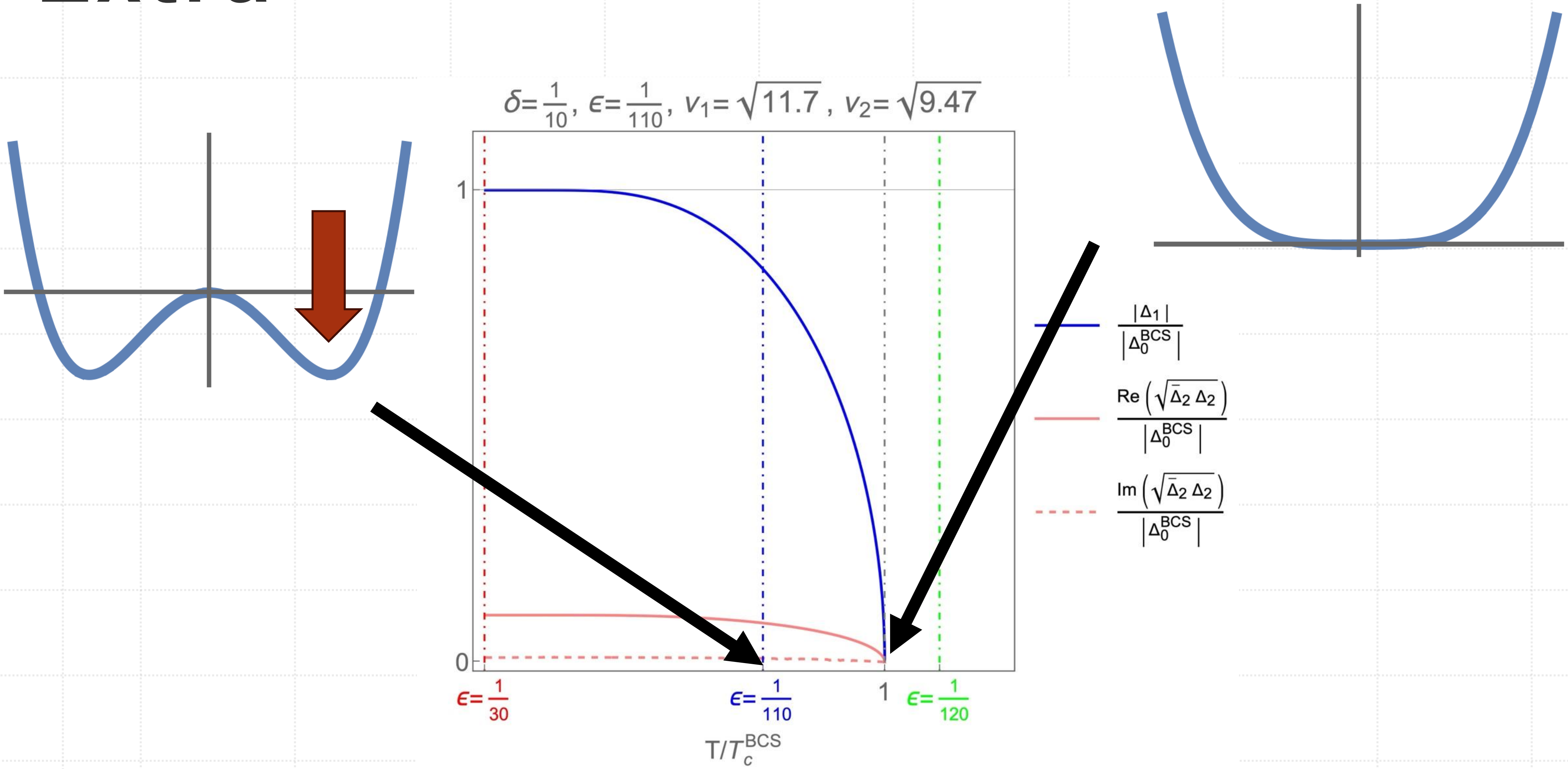


PTQFT

Extra



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$\frac{\mathrm{Im}\left(\sqrt{\bar{\Delta}_2\,\Delta_2}\right)}{|\Delta_0^{BCS}|}$

Future prospect

- Can same argument hold with jump term?
- What about non-Markovian case?
- Application to increase the critical temperature of superconductor?

Thank you

Takano Taira[A]

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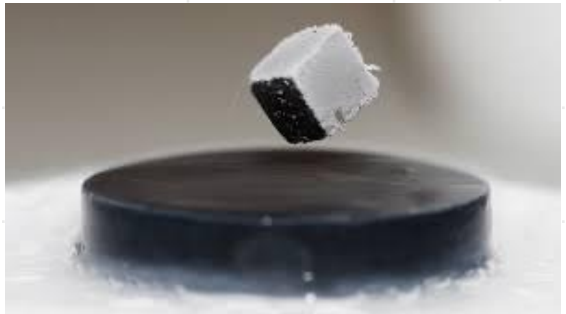

- 2023: Taira, T. (2024). J. Phys. A 57(5), 055001.

Extra

- 2023: Taira, T. (2024). J. Phys. A 57(5), 055001.

$\nabla \times A \equiv B$

$$\nabla^2 \vec{B} = \frac{2e^2\alpha_2}{\kappa} \overline{\Delta}_1^{\mathrm{sol}} \Delta_1^{\mathrm{sol}} \left[1 + \nu \left(-\delta + \frac{\epsilon}{\gamma} \frac{r_1}{r_2^2 + \frac{1}{\gamma^2}} + i\epsilon \frac{r_1 r_2}{r_2^2 + \frac{1}{\gamma^2}} \right)^2 \right] \vec{B}.$$

$\lambda_L(T)$

$$\vec{B}_x = e^{-\sqrt{\lambda_L} x}$$

See appendix

Method

$\Delta_1^\dagger \neq \bar{\Delta}_1, \Delta_2^\dagger \neq \bar{\Delta}_2$: Auxiliary fields in MFT

$$\frac{\delta}{\delta\bar{\Delta}_1}\tilde{S}[\psi_i,\psi_i^\dagger\Delta_i,\bar{\Delta}_i] = 0$$

$$\begin{aligned}
\frac{1}{g}\left(\Delta_1+\frac{\mu}{i\gamma}\Delta_2\right) &= \langle\psi_{1\downarrow}\psi_{1\uparrow}\rangle+\frac{\mu}{g}\langle\psi_{2\downarrow}\psi_{2\uparrow}\rangle,\\
\frac{1}{i\gamma}\left(\Delta_2+\frac{\mu}{g}\Delta_1\right) &= \langle\psi_{2\downarrow}\psi_{2\uparrow}\rangle+\frac{\mu}{i\gamma}\langle\psi_{1\downarrow}\psi_{1\uparrow}\rangle,\\
\frac{1}{g}\left(\bar{\Delta}_1+\frac{\mu}{i\gamma}\bar{\Delta}_2\right) &= \langle\psi_{1\uparrow}^\dagger\psi_{1\downarrow}^\dagger\rangle+\frac{\mu}{g}\langle\psi_{2\uparrow}^\dagger\psi_{2\downarrow}^\dagger\rangle,\\
\frac{1}{i\gamma}\left(\bar{\Delta}_2+\frac{\mu}{g}\bar{\Delta}_1\right) &= \langle\psi_{2\uparrow}^\dagger\psi_{2\downarrow}^\dagger\rangle+\frac{\mu}{i\gamma}\langle\psi_{1\uparrow}^\dagger\psi_{1\downarrow}^\dagger\rangle,
\end{aligned}$$

Method

Want to expand trace log in $\Delta_i, \bar{\Delta}_i$

$$S_{\text{eff}} = -\text{Tr}\log\begin{pmatrix} i\omega_n + \epsilon^{(1)}_{\vec{k}} & \Delta_1 + \frac{\mu}{i\gamma}\Delta_2 \\ \overline{\Delta}_1 + \frac{\mu}{i\gamma}\overline{\Delta}_2 & i\omega_n - \epsilon^{(1)}_{\vec{k}} \end{pmatrix} - \text{Tr}\log\begin{pmatrix} i\omega_n + \epsilon^{(2)}_{\vec{k}} & \Delta_2 + \frac{\mu}{g_1}\Delta_1 \\ \overline{\Delta}_2 + \frac{\mu}{g_1}\overline{\Delta}_1 & i\omega_n - \epsilon^{(2)}_{\vec{k}} \end{pmatrix} + \frac{\mu}{i\gamma g}(\overline{\Delta}_1\Delta_2 + \overline{\Delta}_2\Delta_1) + \frac{1}{g}\overline{\Delta}_1\Delta_1 + \frac{1}{i\gamma}\overline{\Delta}_2\Delta_2, \quad (5)$$

$$\frac{\delta S_{\text{eff}}}{\delta \bar{\Delta}_i} = 0$$

Gap equations

Method

Gap equations

Dropping $\mathcal{O}(\epsilon^3)$ from gap eq

$$\Delta_\epsilon \equiv \Delta_1 - i\epsilon\Delta_2$$

$$\Delta_\delta \equiv \Delta_2 + \delta\Delta_1$$

$$\epsilon = \frac{\mu}{\gamma}, \delta = \frac{\mu}{g}$$

g — Hermitian BCS theory ⟷ (μ) ⟷ Non-Hermitian theory — γ

$|\Delta_\epsilon|$

$(\bar{\Delta}_\delta \Delta_\delta)^{1/2}$