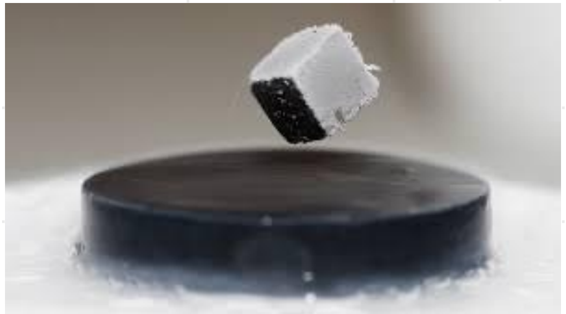
PTQFT

Breakdown of the Meissner effect at the exceptional point



Takano Taira[A]

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- Taira, T. (2024). J. Phys. A 57(5), 055001.

Question

How does non-Hermitian Hamiltonian appear ?

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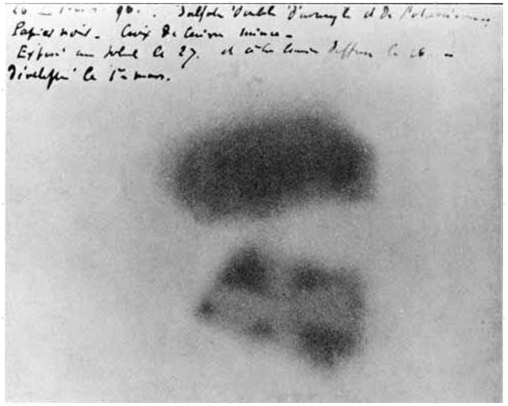

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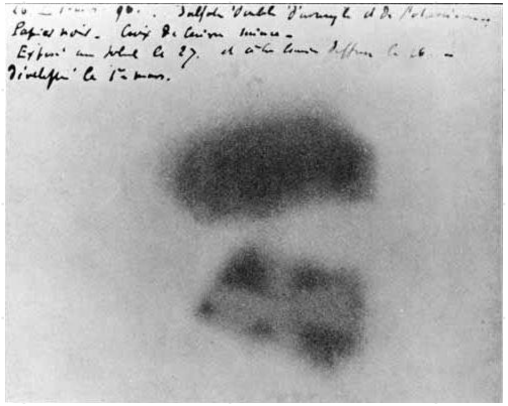

Für die Anregungswahrscheinlichkeit $|a|^2$ des oberen Zustandes ist von vornherein ein exponentieller Abfall zu erwarten. Wir versuchen daher den Ansatz

$$a = e^{-2\pi\Gamma t}. \tag{15a}$$

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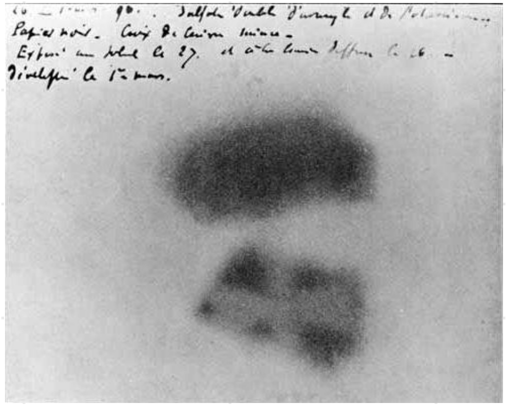

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$$H_{eff} = H - i\pi\,\Gamma$$

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Decay rate

Non-Hermitian Hamiltonian

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$$\Downarrow$$

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Reversible…

But decaying is irreversible…

Question

- 1970: Schulman proposed to use semi-group to implement irreversibility.
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- 1976 May: Gorini Kossakowski Sudarshan N-level
- 1976: Lindblad derived general form of equation solved by semi-group

Reminder

$$|\psi(t)\rangle$$

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$$\rho(t) = \sum_n p_{nm}(t)|n\rangle\langle m|$$

Non-Hermitian Hamiltonian

- 1930: Victor Weisskopf, Eugene Wigner

- 1976: Gorini, Kossakowski, Sudarshan and Lindblad

Non-Hermitian Hamiltonian

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$$H_{eff} = H - i\pi\,\Gamma$$

$$\frac{d\rho(t)}{dt} = -i\left(H_{eff}\rho(t) - \rho(t)H_{eff}^{\dagger}\right)$$

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$$\frac{d\rho(t)}{dt} = -i\left(H_{eff}\rho(t) - \rho(t)H_{eff}^{\dagger}\right) + 2\gamma\sum_{n} L_n\rho(t)L_n^{\dagger}$$

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- 1976: Gorini, Kossakowski, Sudarshan and Lindblad

$$\frac{d\rho(t)}{dt} = -i\,\mathcal{L}[\rho(t)]$$

← Non-Hermitian

Non-Hermitian Hamiltonian

- 1930: Victor Weisskopf, Eugene Wigner

$$H_{eff} = H - i\pi\,\Gamma \qquad N \times N$$

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$$\mathcal{L}[\rho] = -i\left(H_{eff}\rho(t) - \rho(t)H_{eff}^{\dagger}\right) + 2\gamma\sum_{n} L_n\rho(t)L_n^{\dagger} \qquad N^2 \times N^2$$

Non-Hermitian Hamiltonian

Assumption

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Non-Hermitian Hamiltonian

Markovian approximation

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Non-Hermitian Hamiltonian

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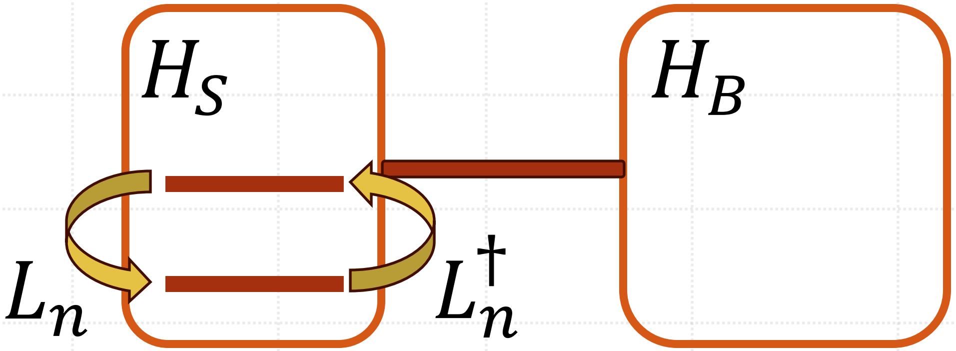
- 2024: Taira, Hatano, Nishino Non-Hermitian and non-linear Hamiltonian with Feshbach formalism. https://arxiv.org/abs/2406.17436

Set up

- 1976: Gorini, Kossakowski, Sudarshan and Lindblad

$$\mathcal{L}[\rho] = -i\left(H_{eff}\rho(t) - \rho(t)H_{eff}^{\dagger}\right) + 2\gamma\sum_{n} L_n\rho(t)L_n^{\dagger}$$



$$H_{eff} = H_S - i\sum_{n} L_n^{\dagger}L_n$$

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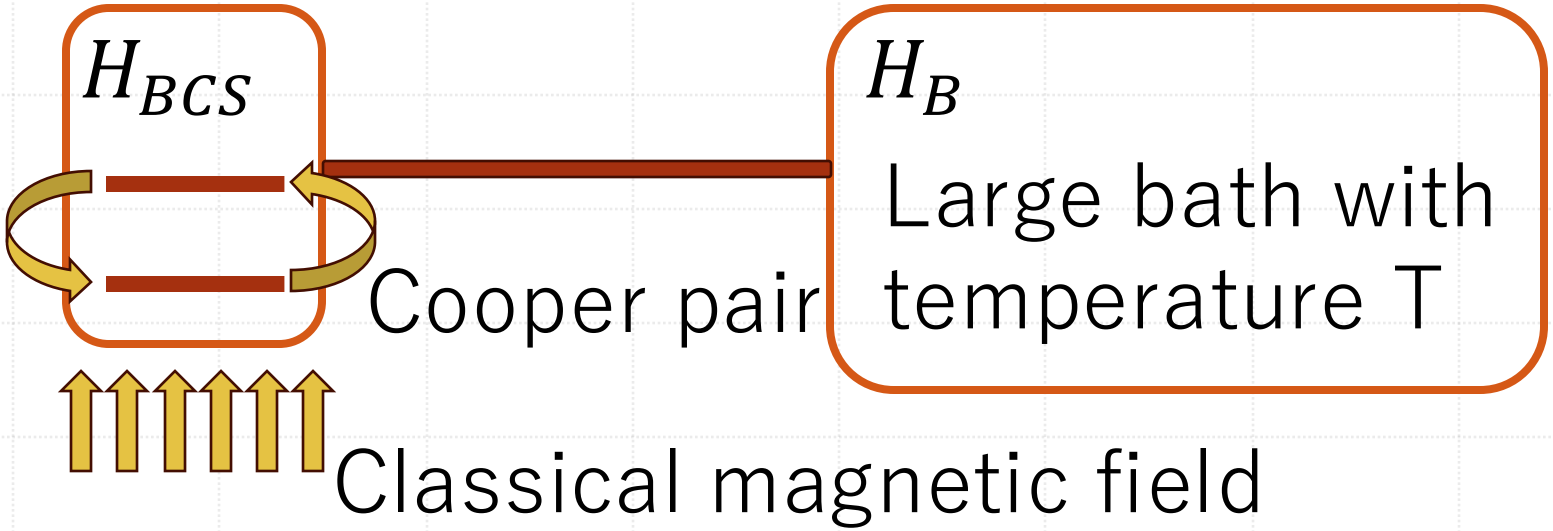

Model

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$$H_{\mathrm{Tot}} = H_{\mathrm{BCS}}[c_1^\dagger, c_1] + H_{\mathrm{Int}}[c_1^\dagger, c_2, c_1^\dagger, c_2] + H_{\mathrm{NH}}[c_2^\dagger, c_2] \tag{1.1}$$

$$H_{\mathrm{BCS}} = \int d^3r \sum_{\sigma=\uparrow,\downarrow} c_{1\sigma}^\dagger(\vec{r}) \left[-\frac{1}{2m_1} \left(\nabla - ie\vec{A} \right)^2 - \mu_1 \right] c_{1\sigma}(\vec{r}) - g c_{1\uparrow}^\dagger(\vec{r}) c_{1\downarrow}^\dagger(\vec{r}) c_{1\downarrow}(\vec{r}) c_{1\uparrow}(\vec{r}), \tag{1.2}$$

$$H_{\mathrm{NH}} = \int d^3r \sum_{\sigma=\uparrow,\downarrow} c_{2\sigma}^\dagger(\vec{r}) \left[-\frac{1}{2m_2} \left(\nabla - ie\vec{A} \right)^2 - \mu_2 \right] c_{2\sigma}(\vec{r}) \tag{1.3}$$

$$H_{\mathrm{int}} = -\mu \int d^3r \; c_{1\uparrow}^\dagger c_{1\downarrow}^\dagger c_{2\uparrow} c_{2\downarrow} + c_{2\uparrow}^\dagger c_{2\downarrow}^\dagger c_{1\uparrow} c_{1\downarrow}. \tag{1.4}$$

Method

- 2023: Taira, T. (2024). J. Phys. A 57(5), 055001.

$$H_{\mathrm{Tot}}[c_i, c_i^\dagger] = H_{\mathrm{Tot}}[c_i, c_i^\dagger]^\dagger$$

$$\mathcal{L}[\rho] = -i\left(H_{eff}\rho(t) - \rho(t)H_{eff}^\dagger\right) + 2\gamma\sum_n L_n\rho(t)L_n^\dagger$$

Jump term

$$H_{eff} = H_S - i\sum_n L_n^\dagger L_n$$

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$$H_{\mathrm{Tot}}[c_i, c_i^\dagger] = H_{\mathrm{Tot}}[c_i, c_i^\dagger]^\dagger$$

Cooper pair

$$L_n = c_{n\downarrow} c_{n\uparrow}$$

$$H_{eff} = H_S - i \sum_n L_n^\dagger L_n$$

Method

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$$H_{\text{eff}}[c_i, c_i^\dagger] \neq H_{\text{eff}}[c_i, c_i^\dagger]^\dagger$$

$$c_n|\psi_n\rangle = \psi_n|\psi_n\rangle$$

$$S_{\text{eff}}[\psi_i, \psi_i^\dagger]$$

Method

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$$S_{eff}[\psi_i, \psi_i^\dagger]$$

Non-Hermitian
Mean field theory

↕ ?

$$\tilde{S}[\psi_i, \psi_i^\dagger \Delta_i, \bar{\Delta}_i]$$

See appendix

Method

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$$\tilde{S}[\psi_i, \psi_i^{\dagger} \Delta_i, \bar{\Delta}_i]$$

Integrate out in partition function

$$S_{\mathrm{eff}}[\Delta_i, \bar{\Delta}_i]$$

See appendix

Complex field-theoretic action

- 2023: Taira, T. (2024). J. Phys. A 57(5), 055001.

$$S_{\text{eff}} = -\text{Tr}\log\begin{pmatrix} i\omega_n + \epsilon_{\vec{k}}^{(1)} & \Delta_1 + \frac{\mu}{i\gamma}\Delta_2 \\ \overline{\Delta}_1 + \frac{\mu}{i\gamma}\overline{\Delta}_2 & i\omega_n - \epsilon_{\vec{k}}^{(1)} \end{pmatrix} - \text{Tr}\log\begin{pmatrix} i\omega_n + \epsilon_{\vec{k}}^{(2)} & \Delta_2 + \frac{\mu}{g_1}\Delta_1 \\ \overline{\Delta}_2 + \frac{\mu}{g_1}\overline{\Delta}_1 & i\omega_n - \epsilon_{\vec{k}}^{(2)} \end{pmatrix} + \frac{\mu}{i\gamma g}(\overline{\Delta}_1\Delta_2 + \overline{\Delta}_2\Delta_1) + \frac{1}{g}\overline{\Delta}_1\Delta_1 + \frac{1}{i\gamma}\overline{\Delta}_2\Delta_2, \quad (5)$$

$\epsilon_k^{(i)} = k^2/2m_i$

$\omega_n = (2n+1)\pi T$ Matsubara frequency

Complex field-theoretic action

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$$
\begin{aligned}
S_{\text{eff}} = \int d^3r\ & \alpha_1 \nabla_i \overline{\Delta}_1 \nabla_i \Delta_1 \\
&+ \left(r_1 - \frac{1}{g} + r_2 \delta^2 \right) \overline{\Delta}_1 \Delta_1 + u_1 \left(\overline{\Delta}_1 \Delta_1 \right)^2 \\
&+ \alpha_2 \nabla_i \overline{\Delta}_2 \nabla_i \Delta_2 + \left(r_2 - \frac{1}{g} \frac{\epsilon}{i\delta} \right) \overline{\Delta}_2 \Delta_2 \\
&+ \left(-i\epsilon r_1 + \delta \left\{ r_2 - \frac{1}{g} \frac{\epsilon}{i\delta} \right\} \right) \left(\overline{\Delta}_1 \Delta_2 + \overline{\Delta}_2 \Delta_1 \right),
\end{aligned} \qquad (6)
$$

$\epsilon = \frac{\mu}{\gamma}$, $\delta = \frac{\mu}{g}$

See appendix

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$$\Delta_1, \Delta_2, \Delta_1^\dagger, \overline{\Delta_2}$$

$$\Delta := \begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} \qquad \bar{\Delta} := \begin{pmatrix} \Delta_1^\dagger \\ \overline{\Delta_2} \end{pmatrix}$$

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$$S_{eff} = \int d^3r \, (\vec{\nabla} - e\,\vec{A})\bar{\Delta}(\vec{\nabla} - e\,\vec{A})\Delta$$

$$+\bar{\Delta}M\Delta - u_1\left(\Delta_1^\dagger \Delta_1\right)^2$$

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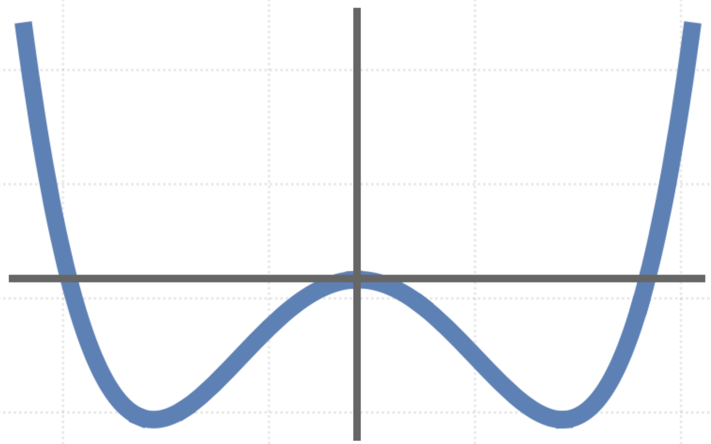

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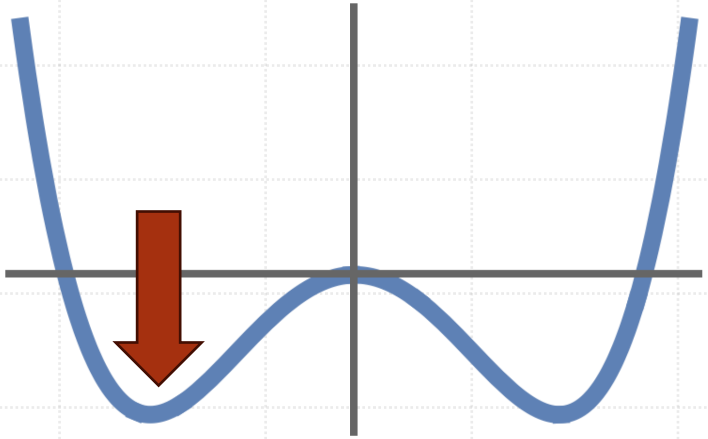

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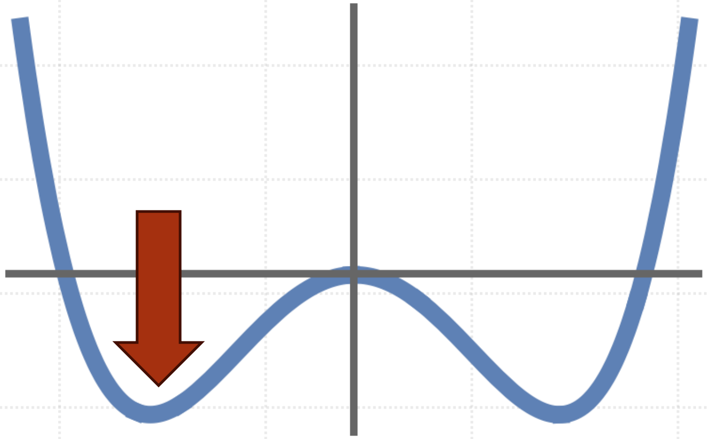
$$+\bar{\Delta}M_{eff}\Delta + \cdots$$

Non-Hermitian mass matrix



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Meissner effect

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Meissner effect

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$$\nabla \times \left.\frac{\delta S_{eff}}{\delta A_i}\right|_{A=A_0} = 0 \Rightarrow \nabla \times \vec{B} = \overline{\Delta_{vac}}\Delta_{vac}\vec{B}$$

$$\vec{B} = \begin{pmatrix} B_x \\ 0 \\ 0 \end{pmatrix}$$

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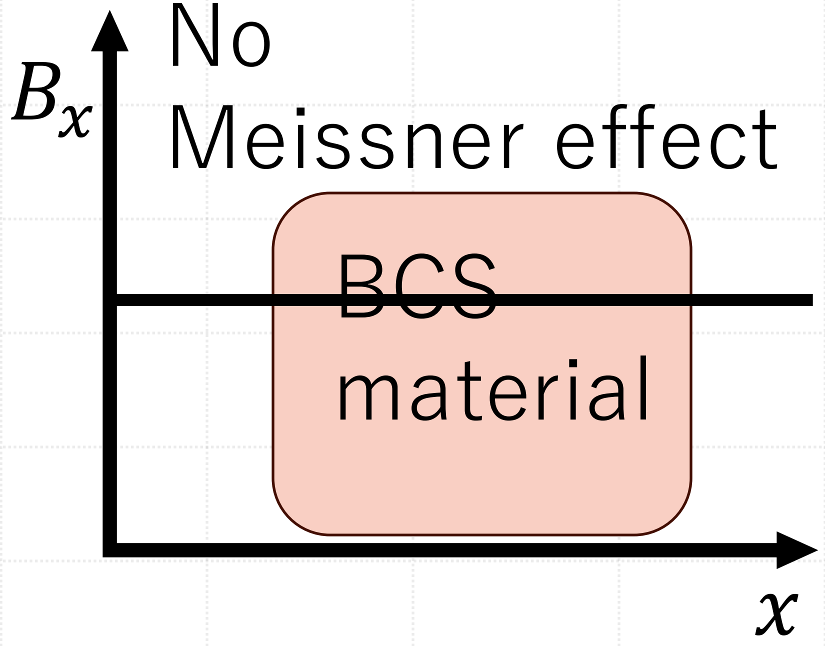

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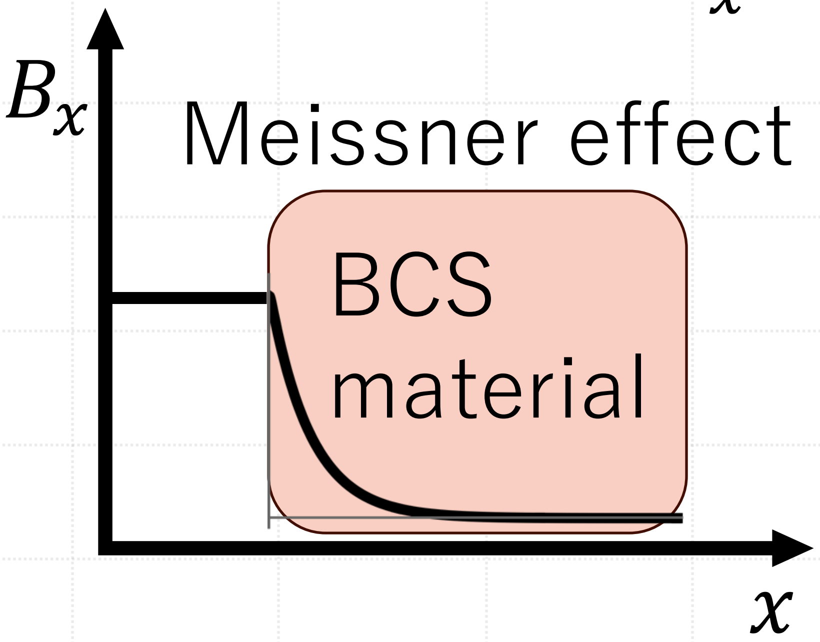

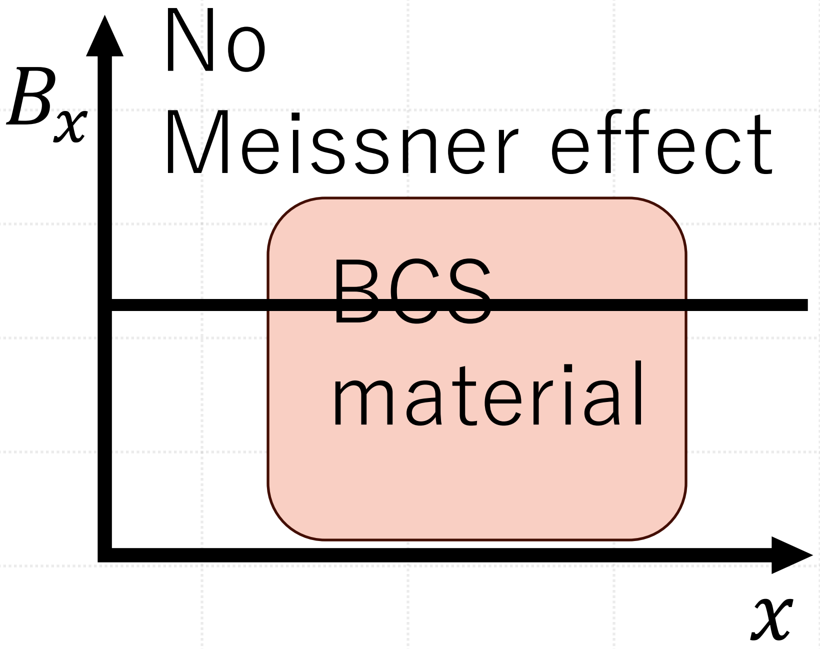

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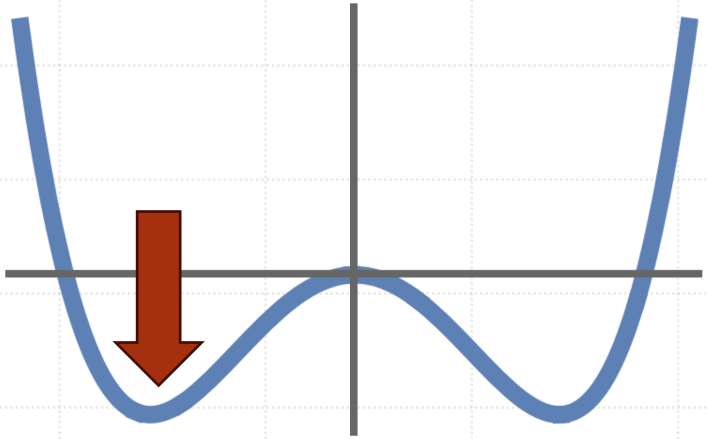
Non-Hermitian mass matrix



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$$\frac{\delta S_{eff}}{\delta \Delta} = 0 \Rightarrow M_{eff}\Delta_{vac} = 0 \text{ Right eigenvector}$$

$$\frac{\delta S_{eff}}{\delta \overline{\Delta}} = 0 \Rightarrow \overline{\Delta_{vac}} M_{eff} = 0 \text{ Left eigenvector}$$

$\overline{\Delta_{vac}}\Delta_{vac} = 0$ at the exceptional point!!

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Meissner effect

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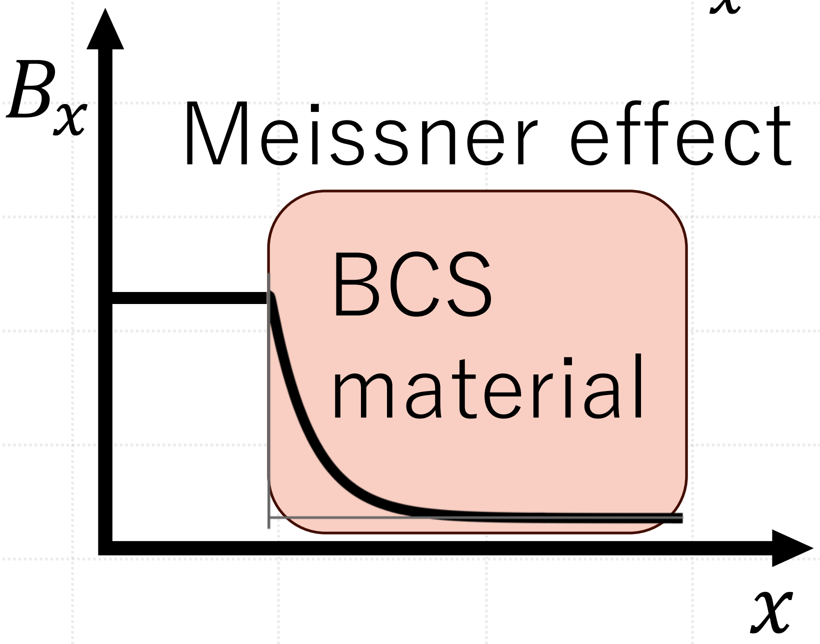

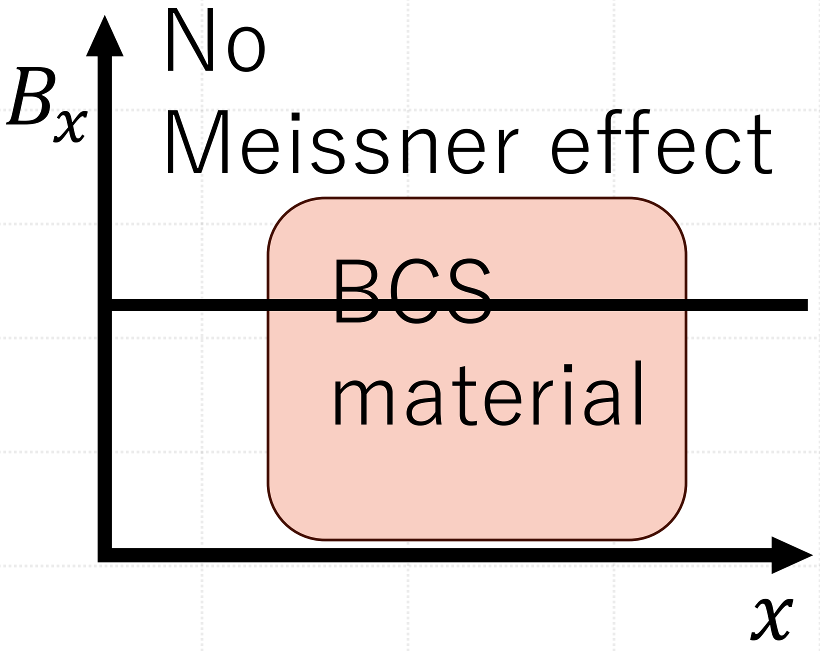

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Meissner effect

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$$B_x = e^{-\sqrt{\overline{\Delta_{vac}}\Delta_{vac}}\ x}$$

$$\nabla^2 \vec{B} = \frac{2e^2\alpha_2}{\kappa}\overline{\Delta}_1^{\mathrm{sol}}\Delta_1^{\mathrm{sol}}\left[1+\nu\left(-\delta+\frac{\epsilon}{\gamma}\frac{r_1}{r_2^2+\frac{1}{\gamma^2}}+i\epsilon\frac{r_1 r_2}{r_2^2+\frac{1}{\gamma^2}}\right)^2\right]\vec{B}.$$

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$$= \overline{\Delta_{vac}}\Delta_{vac}$$

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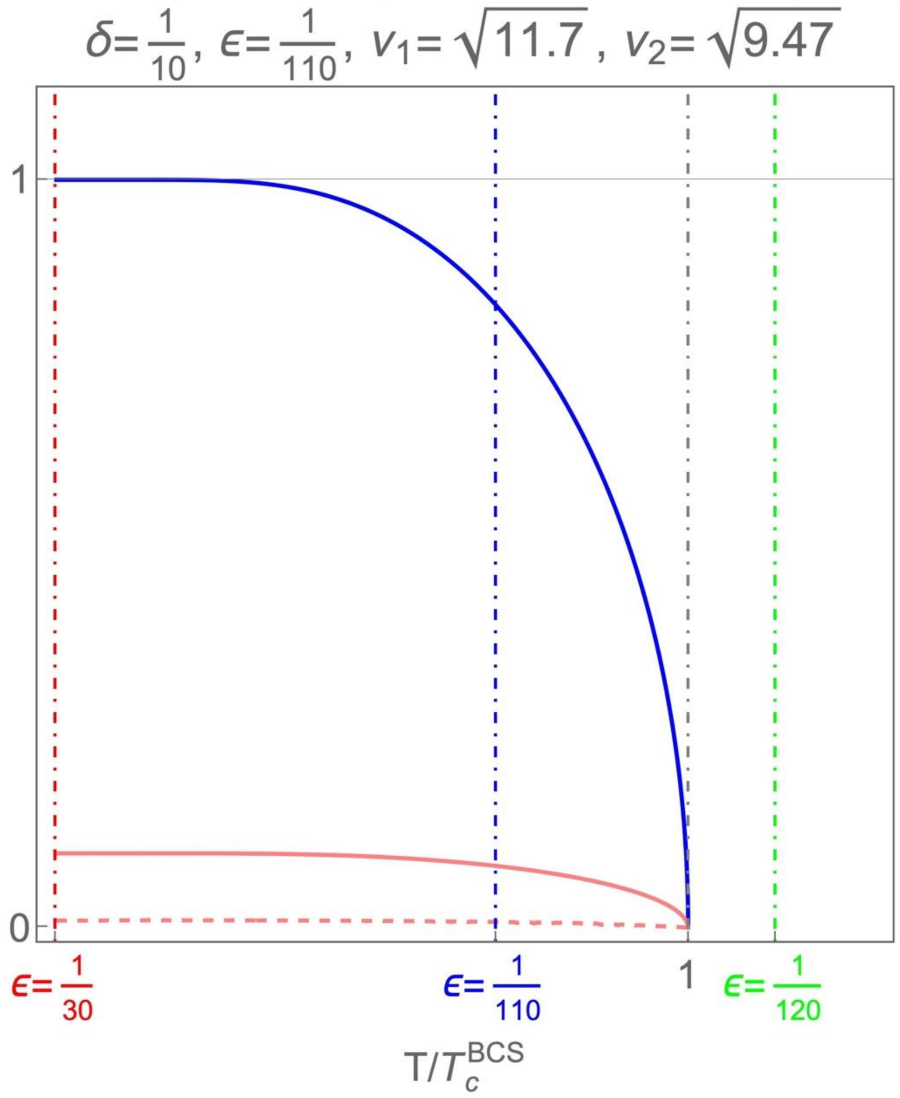
1935 London equation

$$= \overline{\Delta_{vac}}\Delta_{vac}$$

Meissner effect

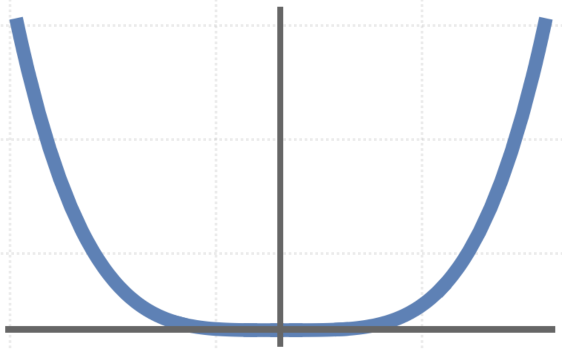
$$\nabla^2 \vec{B} = \frac{2e^2\alpha^2}{\kappa} \overline{\Delta_1^{\rm sol}} \Delta_1^{\rm sol} \left[1 + \nu \left(-\delta + \frac{\epsilon}{\gamma} \frac{r_1}{r_2^2 + \frac{1}{\gamma^2}} + i\epsilon \frac{r_1 r_2}{r_2^2 + \frac{1}{\gamma^2}} \right)^2 \right] \vec{B}.$$

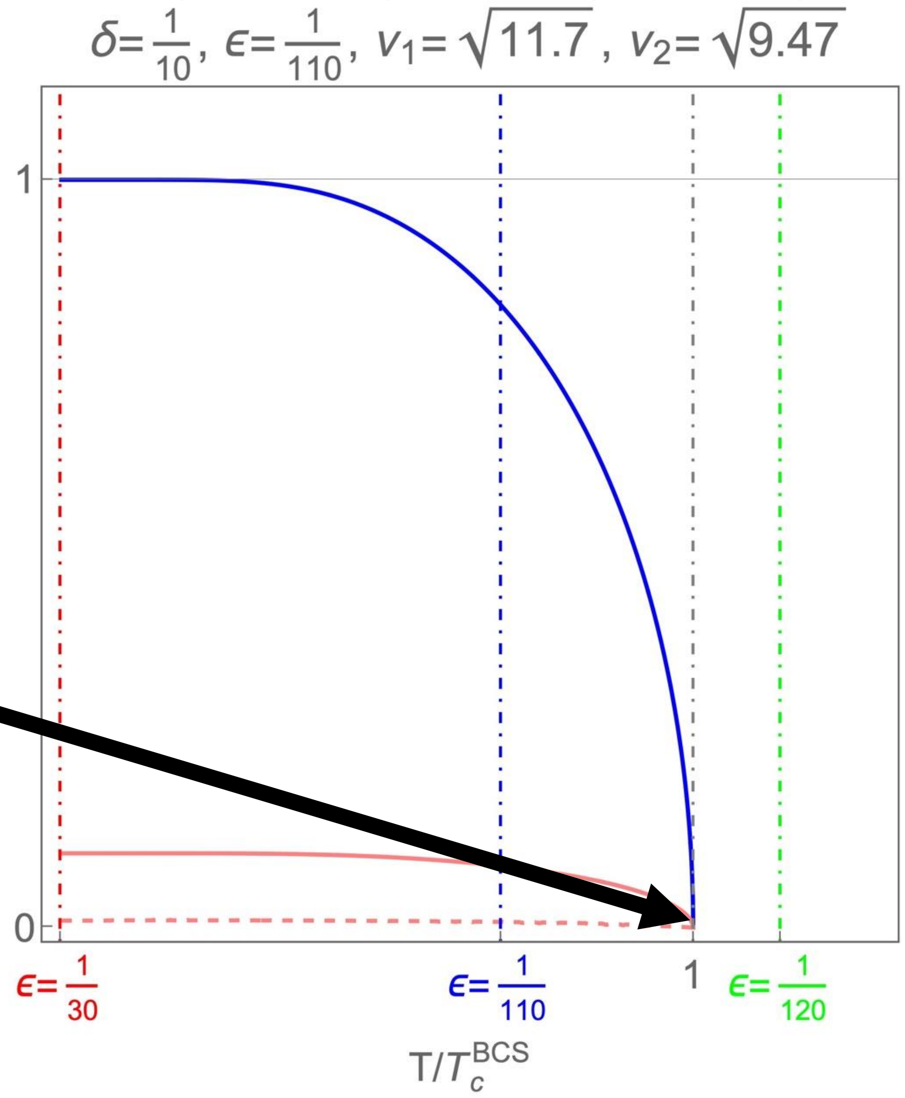


PTQFT

Meissner effect

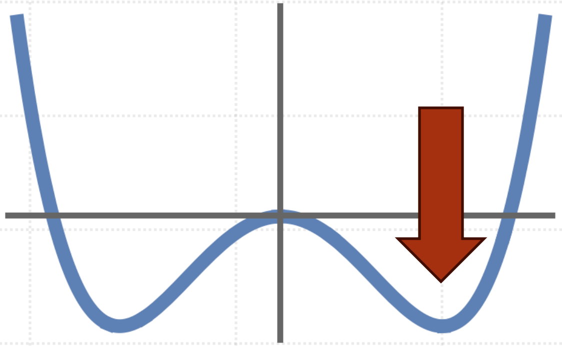
$$\nabla^2 \vec{B} = \frac{2e^2\alpha_2}{\kappa} \bar{\Delta}_1^{\rm sol} \Delta_1^{\rm sol} \left[1 + \nu \left(-\delta + \frac{\epsilon}{\gamma} \frac{r_1}{r_2^2 + \frac{1}{\gamma^2}} + i\epsilon \frac{r_1 r_2}{r_2^2 + \frac{1}{\gamma^2}} \right)^2 \right] \vec{B}.$$



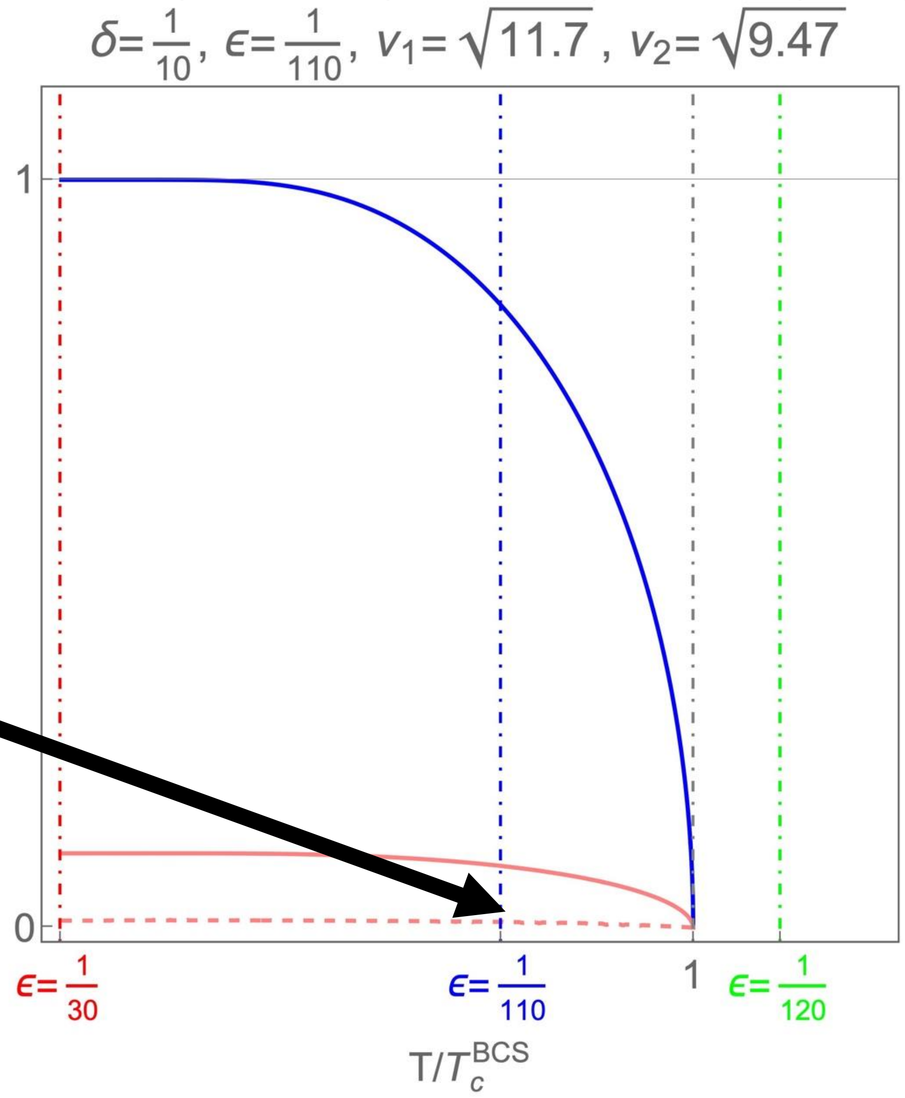


Meissner effect

$$\nabla^2 \vec{B} = \frac{2e^2\alpha^2}{\kappa}\overline{\Delta}_1^{\mathrm{sol}}\Delta_1^{\mathrm{sol}}\left[1+\nu\left(-\delta+\frac{\epsilon}{\gamma}\frac{r_1}{r_2^2+\frac{1}{\gamma^2}}+i\epsilon\frac{r_1 r_2}{r_2^2+\frac{1}{\gamma^2}}\right)^2\right]\vec{B}.$$



SSB happens but still Meissner effect



Future prospect

- Can same argument hold with jump term?
- What about non-Markovian case?
- Application to increase the critical temperature of superconductor?

Thank you

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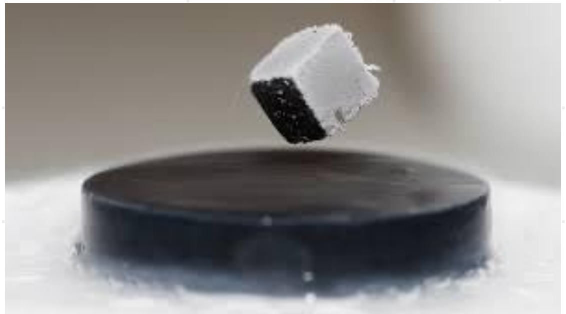

- 2023: Taira, T. (2024). J. Phys. A 57(5), 055001.

Consistency equation

$\Delta_1^\dagger \neq \bar{\Delta}_1, \Delta_2^\dagger \neq \bar{\Delta}_2$: Auxiliary fields in MFT

$$\frac{\delta}{\delta\bar{\Delta}_1}\tilde{S}[\psi_i, \psi_i^\dagger \Delta_i, \bar{\Delta}_i] = 0$$

$$\frac{1}{g}\left(\Delta_1 + \frac{\mu}{i\gamma}\Delta_2\right) = \langle\psi_{1\downarrow}\psi_{1\uparrow}\rangle + \frac{\mu}{g}\langle\psi_{2\downarrow}\psi_{2\uparrow}\rangle,$$

$$\frac{1}{i\gamma}\left(\Delta_2 + \frac{\mu}{g}\Delta_1\right) = \langle\psi_{2\downarrow}\psi_{2\uparrow}\rangle + \frac{\mu}{i\gamma}\langle\psi_{1\downarrow}\psi_{1\uparrow}\rangle,$$

$$\frac{1}{g}\left(\bar{\Delta}_1 + \frac{\mu}{i\gamma}\bar{\Delta}_2\right) = \langle\psi_{1\uparrow}^\dagger\psi_{1\downarrow}^\dagger\rangle + \frac{\mu}{g}\langle\psi_{2\uparrow}^\dagger\psi_{2\downarrow}^\dagger\rangle,$$

$$\frac{1}{i\gamma}\left(\bar{\Delta}_2 + \frac{\mu}{g}\bar{\Delta}_1\right) = \langle\psi_{2\uparrow}^\dagger\psi_{2\downarrow}^\dagger\rangle + \frac{\mu}{i\gamma}\langle\psi_{1\uparrow}^\dagger\psi_{1\downarrow}^\dagger\rangle,$$

Extra

- 2023: Taira, T. (2024). J. Phys. A 57(5), 055001.

$\nabla \times A \equiv B$

$$\nabla^2 \vec{B} = \frac{2e^2\alpha_2}{\kappa}\overline{\Delta}_1^{\mathrm{sol}}\Delta_1^{\mathrm{sol}}\left[1+\nu\left(-\delta+\frac{\epsilon}{\gamma}\frac{r_1}{r_2^2+\frac{1}{\gamma^2}}+i\epsilon\frac{r_1 r_2}{r_2^2+\frac{1}{\gamma^2}}\right)^2\right]\vec{B}.$$

$\lambda_L(T)$

$$\vec{B}_x = e^{-\sqrt{\lambda_L}x}$$

See appendix

Method

Want to expand trace log in $\Delta_i, \bar{\Delta}_i$

$$S_{\text{eff}} = -\mathrm{Tr}\log\begin{pmatrix} i\omega_n + \epsilon^{(1)}_{\vec{k}} & \Delta_1 + \frac{\mu}{i\gamma}\Delta_2 \\ \bar{\Delta}_1 + \frac{\mu}{i\gamma}\bar{\Delta}_2 & i\omega_n - \epsilon^{(1)}_{\vec{k}} \end{pmatrix} \quad (5)$$

$$-\mathrm{Tr}\log\begin{pmatrix} i\omega_n + \epsilon^{(2)}_{\vec{k}} & \Delta_2 + \frac{\mu}{g_1}\Delta_1 \\ \bar{\Delta}_2 + \frac{\mu}{g_1}\bar{\Delta}_1 & i\omega_n - \epsilon^{(2)}_{\vec{k}} \end{pmatrix}$$

$$+\frac{\mu}{i\gamma g}(\bar{\Delta}_1\Delta_2 + \bar{\Delta}_2\Delta_1) + \frac{1}{g}\bar{\Delta}_1\Delta_1 + \frac{1}{i\gamma}\bar{\Delta}_2\Delta_2,$$

$$\frac{\delta S_{\text{eff}}}{\delta \bar{\Delta}_i} = 0$$

Gap equations

Method

Gap equations

Dropping $\mathcal{O}(\epsilon^3)$ from gap eq

$$\Delta_\epsilon \equiv \Delta_1 - i\epsilon\Delta_2$$

$$\Delta_\delta \equiv \Delta_2 + \delta\Delta_1$$

$$\epsilon = \frac{\mu}{\gamma},\ \delta = \frac{\mu}{g}$$