Complex Metastable Condensates in Chern-Simons Quantum Gravity

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Applications of Field Theory to Hermitian and Non-Hermitian Systems
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1. Outline

- **The (string-inspired) Model : Chern-Simons gravity with gravitational axions**

- **Origin of Gravitational Anomalies**

- **The role of primordial gravitational waves in inducing complex gravitational anomaly condensates – weak quantum gravity estimates**

- **Induced Linear-axion monodromy inflation of Running-Vacuum-Model (RVM) type (metastable de Sitter vacuum, compatible with swampland)**

- **Gravitational anomaly condensates and matter-antimatter asymmetry (i.e. reason for our existence)**

- **Conclusions: a flash of the entire cosmological history of this Universe**

- The (string-inspired) Model : Chern-Simons gravity with gravitational axions

- Origin of Gravitational Anomalies

- The role of primordial gravitational waves in inducing complex gravitational anomaly condensates – weak quantum gravity estimates

- Induced Linear-axion monodromy inflation of Running-Vacuum-Model (RVM) type (metastable de Sitter vacuum, compatible with swampland)

- Gravitational anomaly condensates and matter-antimatter asymmetry (i.e. reason for our existence)

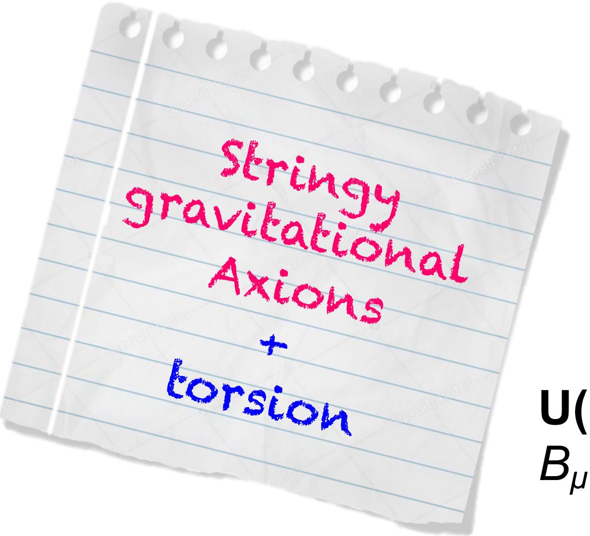
- Conclusions: a flash of the entire cosmological history of this Universe

- Conjecture: current-era accelerated expansion of the Universe as a PT symmetric phase of Chern-Simons gravity ? → Sarben Sarkar's talk

2. The Model:

String-inspired Chern-Simons gravity

Gross and Sloan, Metsaev and Tseytlin



Stringy gravitational Axions + torsion

Massless Gravitational multiplet of (closed) strings:

- **spin 0 scalar (dilaton Φ)**
- **spin 2 traceless symmetric rank 2 tensor (graviton $g_{\mu\nu}$)**
- **spin 1 antisymmetric rank 2 tensor**

U(1) gauge symmetry
$B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_{[\mu}\theta_{\nu]}$

KALB-RAMOND FIELD

$$B_{\mu\nu} = -B_{\nu\mu}$$

4-DIM action

$$S_B = \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}[-R+2\,\partial_\mu\Phi\,\partial^\mu\Phi] - \frac{1}{6\kappa^2}\,e^{-4\Phi}\,H_{\lambda\mu\nu}H^{\lambda\mu\nu}+\ldots\Big)$$

$\kappa^2 = 8\pi\, G$

Green, Schwarz

String Anomaly Cancellation requires modification in definition of $H_{\mu\nu\rho}$

$$H_{\mu\nu\rho} = \partial_{[\mu}\, B_{\nu\rho]} \Rightarrow \boxed{\boldsymbol{H} = \mathrm{dB} + \frac{\alpha'}{8\,\kappa}\Big(\Omega_{3\mathrm{L}} - \Omega_{3\mathrm{Y}}\Big)}$$

$$\Omega_{3\mathrm{L}} = \omega^a{}_c \wedge \mathbf{d}\omega^c{}_a + \frac{2}{3}\omega^a{}_c \wedge \omega^c{}_d \wedge \omega^d{}_a, \quad \Omega_{3\mathrm{Y}} = \mathbf{A} \wedge \mathbf{dA} + \mathbf{A} \wedge \mathbf{A} \wedge \mathbf{A},$$

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$$\overline{R}(\overline{\Gamma})$$

generalised curvature

Φ = constant throughout

$$\boxed{\overline{\Gamma}^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + \frac{\kappa}{\sqrt{3}}H^{\mu}_{\nu\rho} \neq \overline{\Gamma}^{\mu}_{\rho\nu}}$$

Contorsion

Gross and Sloan, Metsaev and Tseytlin

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$$\overline{R}(\overline{\Gamma})$$

generalised curvature

quantum torsion →

gravitational axion b "dual" to H torsion

Campbell, Duncan, Kaloper, Olive
Svrcek, Witten

$$\boxed{\overline{\Gamma}^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + \underbrace{\frac{\kappa}{\sqrt{3}}H^{\mu}_{\nu\rho}}_{\text{Contorsion}} \neq \overline{\Gamma}^{\mu}_{\rho\nu}}$$

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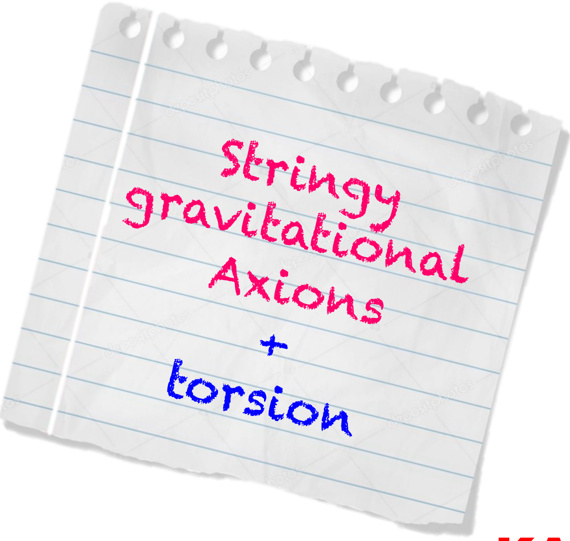

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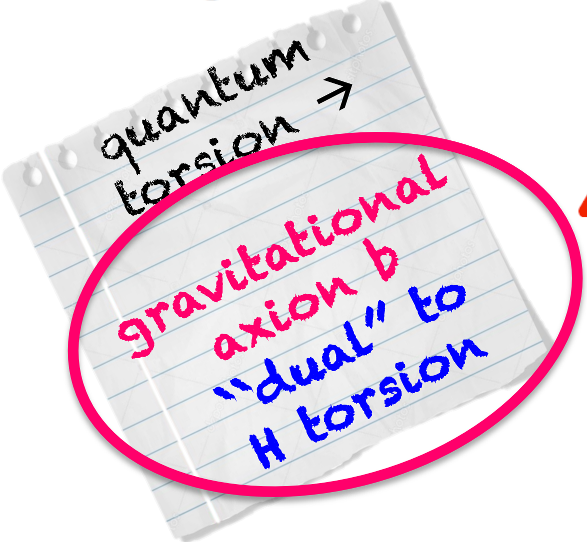

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$$\overline{\Gamma}^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + \frac{\kappa}{\sqrt{3}}H^{\mu}_{\nu\rho} \neq \overline{\Gamma}^{\mu}_{\rho\nu}$$

Contorsion

Effective Actions & Anomaly Cancellation – Addition of Counterterms

Φ = constant throughout, e.g. → 0

Green, Schwarz

$$S_B = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa^2}[-R+2\,\cancel{\partial_\mu\Phi\partial^\mu\Phi}] - \frac{1}{6\kappa^2}e^{-4\cancel{\Phi}}H_{\lambda\mu\nu}H^{\lambda\mu\nu}+\dots\right)$$

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$\alpha' = M_s^{-2}$

Modified Bianchi Constraint

$$\boxed{\varepsilon_{abc}{}^{\mu}\,\nabla_\mu\,H^{abc} = \frac{\alpha'}{32}\sqrt{-g}\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) \equiv \sqrt{-g}\,\mathscr{G}(\omega,\mathbf{A})}$$

Effective Actions & Anomaly Cancellation – Addition of Counterterms

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$$\Downarrow$$

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Implement in path-integral as a field theory δ(…) via Lagrange multiplier b(x) pseudoscalar (axion-like) field (Kalb-Ramond (KR) Axion) becomes dynamical after H-torsion integration

CP -invariant

pseudoscalar

$$\Pi_x\, \delta\Big(\varepsilon^{\mu\nu\rho\sigma}\, \overline{\boldsymbol{H}_{\nu\rho\sigma}(x)_{;\mu} - } \mathcal{G}(\omega, \mathbf{A})\Big) = \int Db \exp\Big[i \int d^4x \sqrt{-g}\, \frac{1}{\sqrt{3}} b(x) \Big(\varepsilon^{\mu\nu\rho\sigma} H(x)_{\nu\rho\sigma;\mu} - \mathcal{G}(\omega, \mathbf{A})\Big)\Big]$$

$$= \int Db\ \exp\Big[-i \int d^4x \sqrt{-g}\, \Big(\partial^\mu b(x)\, \frac{1}{\sqrt{3}}\, \epsilon_{\mu\nu\rho\sigma}\ \boldsymbol{H}^{\nu\rho\sigma} + \frac{b(x)}{\sqrt{3}} \mathcal{G}(\omega, \mathbf{A})\Big)\Big]$$

$$\mathcal{Z} = \int DH\, Db\, \exp(-H \wedge *H + c_1 b(dH - \mathcal{G}) + \dots)$$

Effective action after H-torsion (exact) path-integration

$$\boxed{S_B^{\rm eff} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2\kappa^2} R + \frac{1}{2}\partial_\mu b\, \partial^\mu b + \frac{\sqrt{2}\, \alpha'}{96\, \kappa \sqrt{3}} b(x) \Big(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\, \widetilde{F}^{\mu\nu}\Big) + \dots\Big]}$$

$$\Pi_x\,\delta\Big(\varepsilon^{\mu\nu\rho\sigma}\ \overset{\text{CP -invariant}}{\overline{\boldsymbol{H}_{\nu\rho\sigma}(x)_{;\mu} - \mathcal{G}(\omega,\mathbf{A})}}\Big) = \int Db\exp\Big[i\int d^4x\sqrt{-g}\frac{1}{\sqrt{3}}b(x)\Big(\varepsilon^{\mu\nu\rho\sigma}H(x)_{\nu\rho\sigma;\mu} - \mathcal{G}(\omega,\mathbf{A})\Big)\Big]$$

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KR-axion anomalous CP-conserving interaction with gravity

cf. classically in 4 dim: b-field "dual" to H-torsion

$$\boxed{-3\sqrt{2}\partial_\sigma b = \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}}$$

Inclusion of Fermions

$$S^{\rm eff}_B = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+\, S^{Free}_{Dirac} + \int d^4x\sqrt{-g}\,\Big(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\,J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

$$\mathcal{F}^d = \varepsilon^{abcd}\,e_{b\lambda}\partial_a e^\lambda_c,$$

vielbeins

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

Axial Current

KR-axion anomalous
CP-conserving interaction

torsion

cf. classically in 4 dim:
b-field "dual" to H-torsion

$$\boxed{-3\,\sqrt{2}\partial_\sigma b \;=\; \sqrt{-g}\;\epsilon_{\mu\nu\rho\sigma}\,H^{\mu\nu\rho}}$$

The Model

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\, b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+\, S_{\rm Dirac}^{\rm Free} + \int d^4x\sqrt{-g}\,\Big(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\, J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\, J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$ All fermion species $\mathcal{F}^d = \varepsilon^{abcd}\, e_{b\lambda}\partial_a e^\lambda_c,$

The Model

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa_{\sqrt{3}}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

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$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

All fermion species

$$\mathcal{F}^d = \varepsilon^{abcd}\,e_{b\lambda}\partial_a e^\lambda_c,$$

Vanishes for Friedmann-Lemaitre-Roberston-Walker backgrounds

The Model

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[- \frac{1}{2\kappa^2}\, R + \frac{1}{2}\, \partial_\mu b\, \partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\, b(x)\, \Big(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\, \widetilde{F}^{\mu\nu}\Big) + \dots \Big]$$

$$+ S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\,\Big(\quad \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\, J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\, J^5_\mu J^{5\mu} + \dots \Big] + \dots$$

$$J^{5\,\mu} = \bar{\psi}_j\, \gamma^\mu\, \gamma^5 \psi_j$$

All fermion species

The Model

Anomaly terms

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{2\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

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$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

All fermion species

3. Gravitational (Chern-Simons) Anomalies

NB:

Anomalies in Quantum Field Theory:

Classical Symmetry → Conserved Current

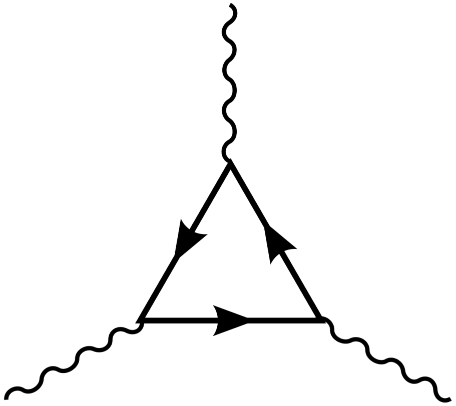
Quantum Theory: Failure of current conservation in ANY REGULARIZATION of the quantum theory

or equivalently:

Path-Integral measure NOT INVARIANT under symmetry transformation

Fujikawa

OF INTEREST HERE: GAUGE & GRAVITATIONAL CHIRAL ANOMALIES



CHIRAL FERMIONIC LOOP in graphs with **1+D/2 external legs** (gauge fields or gravitons) in D- space-time dimensions **D=4 → triangular graphs**

Alvarez-Gaume, Witten

NB:

Mixed Anomalies (Gravitational + Gauge)

$$\nabla_\mu J^{5\mu} = c_1 \left(R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \widetilde{F}^{\mu\nu} \right)$$

gravitational covariant derivative

$$\boxed{J^{5\mu} = \sum_j \overline{\psi}_j \gamma^\mu \gamma^5 \psi_j}$$

Axial Current

$$\widetilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\pi} R^{\lambda\pi}{}_{\rho\sigma}, \qquad \widetilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$$

$$\varepsilon_{\mu\nu\rho\sigma} = \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}, \qquad \varepsilon^{\mu\nu\rho\sigma} = \frac{\mathrm{sgn}(g)}{\sqrt{-g}}\,\epsilon^{\mu\nu\rho\sigma}$$

Anomaly terms are total derivatives:

$$\sqrt{-g}\left(R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \widetilde{F}^{\mu\nu} \right) = \sqrt{-g}\,\mathcal{K}^\mu_{\mathrm{mixed}}(\omega)_{;\mu} = \partial_\mu\left(\sqrt{-g}\,\mathcal{K}^\mu_{\mathrm{mixed}}(\omega)\right)$$

$$= 2\,\partial_\mu\Big[\epsilon^{\mu\nu\alpha\beta}\,\omega_\nu^{ab}\left(\partial_\alpha\,\omega_{\beta ab} + \frac{2}{3}\,\omega_{\alpha a}{}^{c}\,\omega_{\beta cb}\right) - 2\epsilon^{\mu\nu\alpha\beta}\left(A_\nu^i\,\partial_\alpha A_\beta^i + \frac{2}{3}\,f^{ijk}\,A_\nu^i\,A_\alpha^j\,A_\beta^k\right)\Big]$$

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Anomaly terms are total derivatives – can couple to axion-like fields *b*

$$S_B^{\rm eff} \ni \frac{1}{f_b} \int d^4x \sqrt{-g} \Big[\, b(x) \left(R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \widetilde{F}^{\mu\nu} \right) + \dots \Big];$$

Axion coupling

e.g. gravitational axion $f_b = 96 \sqrt{\frac{3}{2} \frac{\kappa}{\alpha'}} = 96 \sqrt{\frac{3}{2} \frac{M_s^2}{M_{\rm Pl}}}$

NB: Mixed Anomalies (Gravitational + Gauge)

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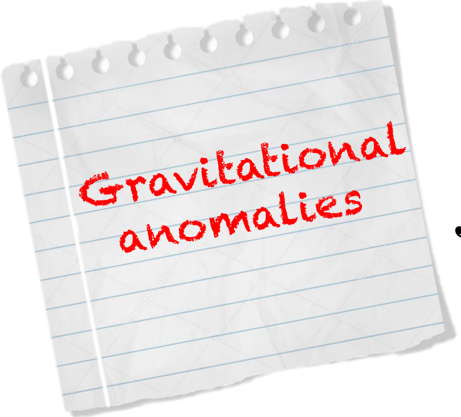
Contributions to Stress tensor **YES** **NO**

Gravitational Anomalies & Diffeomorphism Invariance



$$\int d^4x\,\sqrt{-g}\,b(x)\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big)$$

Spoils conservation of stress tensor (diffeomporphism invariance affected in quantum theory)

Topological, does NOT contribute to stress tensor

$$\delta\Big[\int d^4x\sqrt{-g}\,b\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\Big] = 4\int d^4x\sqrt{-g}\,\mathcal{C}^{\mu\nu}\,\delta g_{\mu\nu} = -4\int d^4x\sqrt{-g}\,\mathcal{C}_{\mu\nu}\,\delta g^{\mu\nu}$$

Cotton tensor

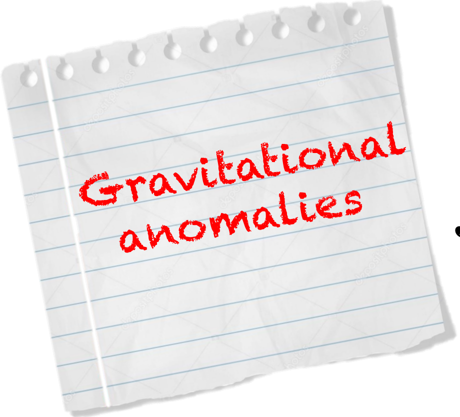
$$\mathcal{C}^{\mu\nu} = -\frac{1}{2}\Big[v_\sigma\Big(\varepsilon^{\sigma\mu\alpha\beta}R^{\nu}{}_{\beta;\alpha} + \varepsilon^{\sigma\nu\alpha\beta}R^{\mu}{}_{\beta;\alpha}\Big) + v_{\sigma\tau}\Big(\widetilde{R}^{\tau\mu\sigma\nu} + \widetilde{R}^{\tau\nu\sigma\mu}\Big)\Big] = -\frac{1}{2}\Big[\Big(v_\sigma\,\widetilde{R}^{\lambda\mu\sigma\nu}\Big)_{;\lambda} + (\mu\leftrightarrow\nu)\Big]$$

$$v_\sigma \equiv \partial_\sigma b = b_{;\sigma},\;\; v_{\sigma\tau} \equiv v_{\tau;\sigma} = b_{;\tau;\sigma}$$

Traceless $g_{\mu\nu}\,\mathcal{C}^{\mu\nu} = 0$

Jackiw, Pi (2003)

Gravitational Anomalies & Diffeomorphism Invariance


Gravitational anomalies

$$\int d^4x \sqrt{-g}\, b(x) \Big(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\, \widetilde{F}^{\mu\nu} \Big)$$

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Cotton tensor

$$\mathcal{C}^{\mu\nu} = -\frac{1}{2}\Big[v_\sigma \Big(\varepsilon^{\sigma\mu\alpha\beta} R^{\nu}{}_{\beta;\alpha} + \varepsilon^{\sigma\nu\alpha\beta} R^{\mu}{}_{\beta;\alpha} \Big) + v_{\sigma\tau} \Big(\widetilde{R}^{\tau\mu\sigma\nu} + \widetilde{R}^{\tau\nu\sigma\mu} \Big) \Big] = -\frac{1}{2}\Big[\Big(v_\sigma\, \widetilde{R}^{\lambda\mu\sigma\nu} \Big)_{;\lambda} + (\mu \leftrightarrow \nu) \Big]$$

$$v_\sigma \equiv \partial_\sigma b = b_{;\sigma}, \;\; v_{\sigma\tau} \equiv v_{\tau;\sigma} = b_{;\tau;\sigma}$$

not necessarily positive contributions to vacuum energy ⚠

Traceless

$$g_{\mu\nu}\, \mathcal{C}^{\mu\nu} = 0$$

Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \mathcal{C}^{\mu\nu} = \kappa^2 T^{\mu\nu}_{\text{matter}}$$

$$\mathcal{C}^{\mu\nu}{}_{;\mu} = - \frac{1}{8} v^{\nu} R^{\alpha\beta\gamma\delta} \widetilde{R}_{\alpha\beta\gamma\delta}$$

$$v_{\sigma} \equiv \partial_{\sigma} b$$

$$\kappa^2 T^{\mu\nu}_{\text{matter}\,;\mu} = - \mathcal{C}^{\mu\nu}{}_{;\mu} \neq 0$$

Diffeomoprphism invariance breaking by gravitational anomalies ?

Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \mathcal{C}^{\mu\nu} = \kappa^2 T^{\mu\nu}_{\text{matter}}$$

$$\mathcal{C}^{\mu\nu}{}_{;\mu} = -\frac{1}{8} v^{\nu} R^{\alpha\beta\gamma\delta} \widetilde{R}_{\alpha\beta\gamma\delta}$$

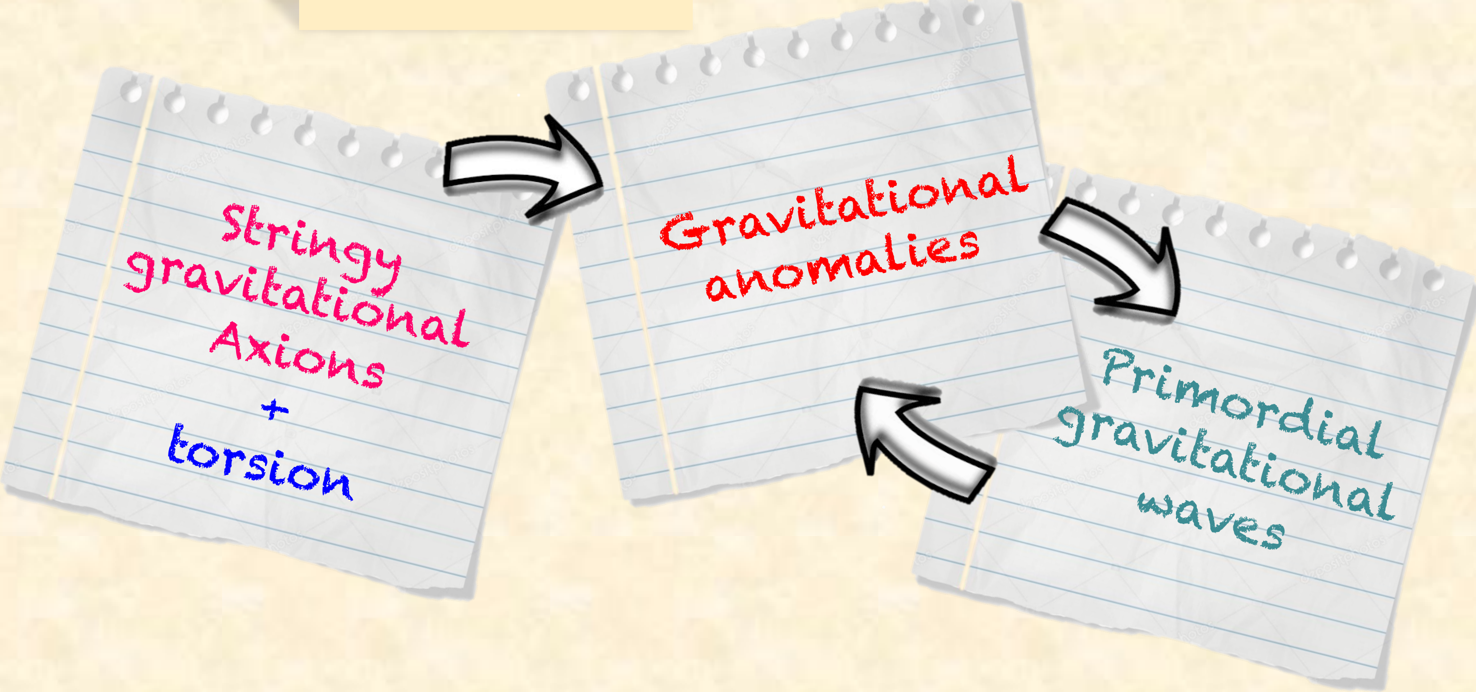
$$v_{\sigma} \equiv \partial_{\sigma} b$$

$$\kappa^2 T^{\mu\nu}_{\text{matter}\,;\mu} + \mathcal{C}^{\mu\nu}{}_{;\mu} = 0$$

No problem with diffeo

Conserved Modified stress-energy tensor

The Parts

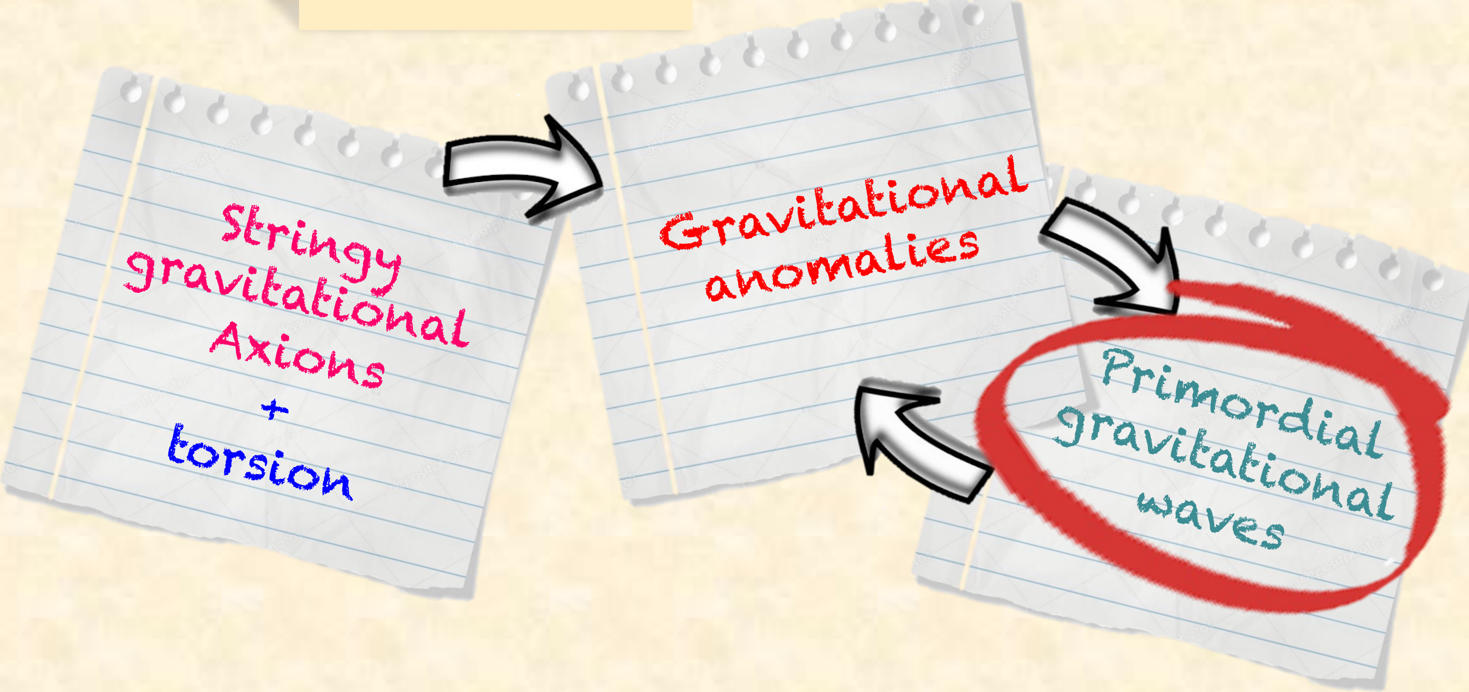


Stringy gravitational Axions
+
torsion

Gravitational anomalies

Primordial gravitational waves

The Parts



Stringy gravitational Axions
+
torsion

Gravitational anomalies

Primordial gravitational waves

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\, b(x)\, R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

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$$\begin{aligned} S_B^{\rm eff} &= \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R+\frac{1}{2}\,\partial_\mu b\,\partial^\mu b+\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}+\dots\Big] \\ &= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R+\frac{1}{2}\,\partial_\mu b\,\partial^\mu b-\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu+\dots\Big]\,, \end{aligned}$$

Chiral Fermionic matter & radiation fields are supposed to be generated by the decay of the false (running) vacuum (cf below) at the end of inflation

⚠

The Model in Early Universe:
only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

NB:

absent before formation of GW

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[\boxed{-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b} + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,\xcancel{R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}} + \dots\Big]$$

$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\xcancel{\partial_\mu b(x)\,\mathcal{K}^\mu} + \dots\Big]\,,$$

No potential for KR axion before generation of GW

→ stiff-matter, equation of state **w=+1**

→stiff-axion-matter dominance during very early (pre-inflationary) Universe

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only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

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$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,\cancel{R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}} + \dots\Big]$$

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No potential for KR axion before generation of GW

→ stiff-matter, equation of state **w=+1**

→stiff-axion-matter dominance during very early (pre-inflationary) Universe

c.f. Zeldovich but for baryons in his model

The Model in Early Universe: only gravitational d.o.f. ($b, g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\,\partial_\mu b\, \partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\, b(x)\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} + \dots \Big]$$

$$= \int d^4x\, \sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\,\partial_\mu b\, \partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\, \partial_\mu b(x)\, \mathcal{K}^\mu + \dots \Big]\,,$$

Primordial Gravitational Waves

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

$$S^{\rm eff}_B = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

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Primordial Gravitational Waves

Potential Origins in pre-inflationary era?

NEM,Solà
EPJ-ST
(2020)

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

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$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

Primordial Gravitational Waves

Potential Origins in pre-inflationary era?

(i) merging of primordial Black Holes formed from collapse of massive brane/stringy defects

NEM,Solà
EPJ-ST
(2020)

The Model in Early Universe:
only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

$$S_B^{\rm eff} = \int d^4x \sqrt{\ }\Big[\quad \frac{1}{\ } \; R + \frac{1}{\ } \; \partial \; b \, \partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\, b(x)\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} + \dots \Big]$$

$$= \int \quad b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\, \partial_\mu b(x)\, \mathcal{K}^\mu + \dots \Big]\,,$$

Carr, Garcia-Bellido, Khlopov, Malomed, Zeldovich, Hawking, Page, Ricotti, Ostriker, Mack, Clese, Fleury, Kuhnel, Peloso, Sandstand, Unal, Sendouda, Yokoyama, ...

Primordial Gravitational Waves
Potential Origins in pre-inflationary era?

(i) merging of primordial Black Holes formed from collapse of massive brane/stringy defects

NEM,Solà EPJ-ST (2020)

Ellis, NEM, Nanopoulos, Sakellariadou, Elghozi, Yusaf....

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

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Primordial Gravitational Waves

Potential Origins in pre-inflationary era?

(ii) Collapse/collisions of Domain walls formed in theories with (approxmate) discrete symmetry breaking, e.g. via bias in double-well potentials of some condensate (gravitino or gaugino)

NEM,Solà EPJ-ST (2020)

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

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Zeldovich,Kobzarev, Okun,
Kibble,Vilenkin, Sikivie,
Gelimini, Gleiser, Kolb, ...

Lalak, Ovrut,
Lola, G. Ross,
Thomas

Takahashi,
Yanagida,
Yonekura

Primordial Gravitational Waves

Potential Origins in pre-inflationary era?

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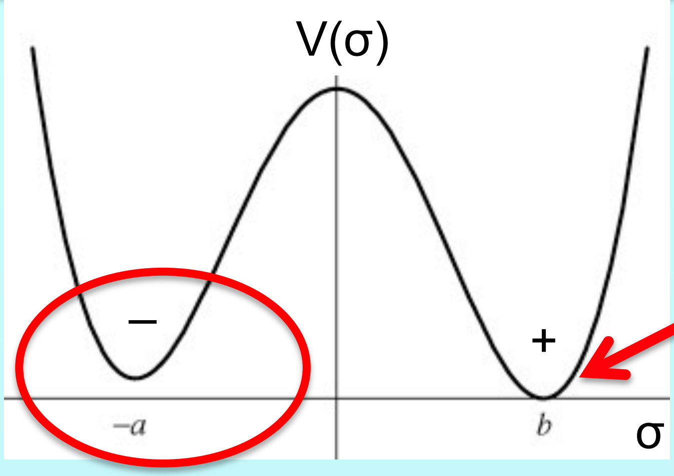
NEM,Solà
EPJ-ST
(2020)

Ellis, NEM,
Alexandre,
Houston

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)



SUGRA broken gravitino Condensate stabilised →
RVM GW-induced Inflation

Zeldovich,Kobzarev, Okun, Kibble,Vilenkin, Sikivie, Gelimini, Gleiser, Kolb, ...

Lalak, Ovrut, Lola, G. Ross, Thomas

Takahashi, Yanagida, Yonekura

Primordial Gravitational Waves

Potential Origins in pre-inflationary era?

(ii) Collapse/collisions of Domain walls formed in theories with (approxmate) discrete symmetry breaking, e.g. via bias in double-well potentials of some condensate (gravitino or gaugino)

NEM,Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

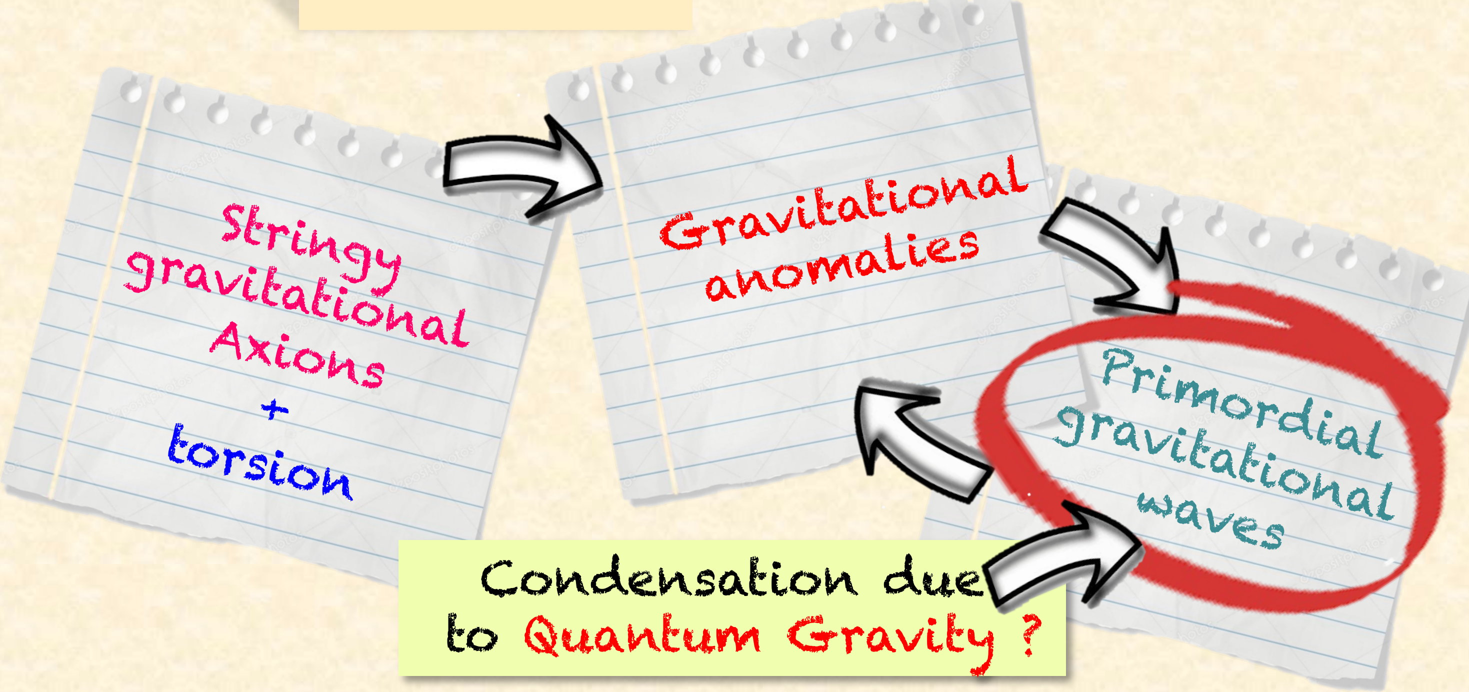
Non-trivial if GW present

$$
\begin{aligned}
S_B^{\rm eff} &= \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R+\frac{1}{2}\,\partial_\mu b\,\partial^\mu b+\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}+\dots\Big]\\
&= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R+\frac{1}{2}\,\partial_\mu b\,\partial^\mu b-\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu+\dots\Big]\,,
\end{aligned}
$$

Primordial Gravitational Waves

3. Gravitational waves & Grav. Anomaly condensates

The Parts



Stringy gravitational Axions + torsion

Gravitational anomalies

Primordial gravitational waves

Condensation due to Quantum Gravity ?

Basilakos, NEM, Solà

Dorlis, NEM, Vlachos

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

Gravitational Chern-Simons (gCS)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\, \partial_\mu b\, \partial^\mu b + \sqrt{\frac{2}{3}}\, \frac{\alpha^\prime}{96\, \kappa}\, b(x)\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} + \dots \Big]$$

$$= \int d^4x\, \sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\, \partial_\mu b\, \partial^\mu b - \sqrt{\frac{2}{3}}\, \frac{\alpha^\prime}{96\, \kappa}\, \partial_\mu b(x)\, \mathcal{K}^\mu + \dots \Big]\,,$$

Primordial Gravitational Waves →
Condensate < …> of Gravitational Anomalies

$$g\mathcal{CS} = \sqrt{\frac{2}{3}}\, \frac{\alpha^\prime}{96\, \kappa}\, \int d^4x\, \sqrt{-g}\, \Big(\langle\, b(x)\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\, \rangle + \, : b(x)\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} : \Big)$$

quantum ordered

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

Gravitational Chern-Simons (gCS)

$$S^{\rm eff}_B = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

$$+\sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\,\int d^4x\sqrt{-g}\,\langle b(x)\,R_{\mu\mu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle$$

Cosmological-Constant-like

Condensate < …> of Gravitational Anomalies

$$g\mathcal{CS} = \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\,\int d^4x\,\sqrt{-g}\,\Big(\langle\, b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\,\rangle + \,:b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}:\Big)$$

quantum ordered

$$\sqrt{-g}\,\mathcal{K}^{\mu}(\omega)_{;\mu}$$

Effective action contains **CP violating axion-like coupling**

$$S_B^{\rm eff} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} \qquad\qquad '\Big) + \dots\Big]$$

Effective action contains **CP violating axion-like coupling**

$$\partial_\mu\Big(\sqrt{-g}\,\mathcal{K}^\mu(\omega)\Big)$$

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}b(x)\Big(R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma} \qquad '\Big) + \dots\Big]$$

$$\begin{aligned} ds^2 = dt^2 - a^2(t)\Big[&(1-h_+(t,z))\,dx^2 \\ &+ (1+h_+(t,z))\,dy^2 \\ &+ 2h_\times(t,z)\,dx\,dy + dz^2\Big] \end{aligned}$$

Average over inflationary space time in the presence of **primordial Gravitational waves**

$b(x)=b(t)$

Alexander, Peskin, Sheikh –Jabbari
Lyth, Rodriguez, Quimbay

$$\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\kappa^2\int^{\mu}\frac{d^3k}{(2\pi)^3}\,\frac{H^2}{2k^3}\,k^4\,\Theta + O(\Theta^3)$$

$$\Theta = \sqrt{\frac{2}{3}}\,\frac{\kappa^3}{12}H\dot{b} \ll 1$$

$$\kappa = M_{\rm Pl}^{-1},$$

$$\dot{b} \equiv db/dt$$

$$a(t) \sim e^{Ht}$$

μ = low-energy UV cutoff ~ M_s

H ≈ const. (inflation)

Effective action contains **CP violating axion-like coupling**

$$\partial_\mu\left(\sqrt{-g}\,\mathcal{K}^\mu(\omega)\right)$$

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} \qquad '\Big) + \ldots\Big]$$

$$ds^2 = dt^2 - a^2(t)\Big[(1-h_+(t,z))\,dx^2 + (1+h_+(t,z))\,dy^2 + 2h_\times(t,z)\,dx\,dy + dz^2\Big]$$

Average over inflationary space time in the presence of **primordial Gravitational waves**

$b(x)=b(t)$

Alexander, Peskin, Sheikh –Jabbari
Lyth, Rodriguez, Quimbay

EFT

$$\langle R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\,\kappa^2\int^{\mu}\frac{d^3k}{(2\pi)^3}\,\frac{H^2}{2k^3}\,k^4\,\Theta + O(\Theta^3)$$

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$$\dot{b} \equiv db/dt$$

$$a(t) \sim e^{Ht}$$

H* ≈ const. *(inflation)

Effective action contains **CP violating axion-like coupling**

$\partial_\mu\left(\sqrt{-g}\,\mathcal{K}^\mu(\omega)\right)$

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\right) + \dots\Big]$$

$$ds^2 = dt^2 - a^2(t)\Big[(1-h_+(t,z))\,dx^2 + (1+h_+(t,z))\,dy^2 + 2h_\times(t,z)\,dx\,dy + dz^2\Big]$$

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$$\Theta = \sqrt{\frac{2}{3}}\,\frac{\kappa^3}{12}\,H\dot{b} \ll 1$$

$\kappa = M_{\rm Pl}^{-1}$,

$\dot{b} \equiv db/dt$

$a(t) \sim e^{Ht}$

H ≈ const. (inflation)

μ = low-energy UV cutoff ~ M_s

Revisiting-Reestimating the Calculation of Condensate

In weak quantum-gravity framework

Dorlis, NEM, Vlachos
Phys.Rev.D 110 (2024), 063512,
arXive:2403.09005 [gr-qc]

Dorlis, NEM, Vlachos
2404.18741 (PoS Corfu 2023)

Improvement on approximations made in previous works

Alexander, Peskin, Sheikh –Jabbari
Lyth, Rodriguez, Quimbay

Revisiting-Reestimating the Calculation of Condensate

In weak quantum-gravity framework

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$$S^{\rm eff} = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \frac{1}{2}(\partial_\mu b)(\partial^\mu b) - A\, b\, R_{CS} \right] , \quad A = \sqrt{\frac{2}{3}} \frac{\alpha'}{48\kappa}$$

$$2 < R_{CS} > = -\langle R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} \rangle = -\int DbDg_{\mu\nu} e^{-S^{\rm eff}}\, R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} \quad = \text{constant}$$

WE DO NOT KNOW THE FULL QG THEORY!

Dorlis, NEM, Vlachos, arXive:2403.09005 [gr-qc], PRD 110 (2024) , 063512

$$S^{\rm eff} = \int d^4x \sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}(\partial_\mu b)(\partial^\mu b) - A\, b\, R_{CS}\right], \quad A = \sqrt{\frac{2}{3}}\frac{\alpha'}{48\kappa}$$

$$\boxed{2 < R_{CS} > = -\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle = -\int DbDg_{\mu\nu}e^{-S^{\rm eff}}\, R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma} \quad = \text{constant}}$$

WE DO NOT KNOW THE FULL QG THEORY!

Only Estimate at Stationary points of $S^{\rm eff}$

$$\frac{\delta S^{\rm eff}}{\delta b} = \frac{\delta S^{\rm eff}}{\delta g_{\mu\nu}} = 0$$

Dorlis, NEM, Vlachos, arXive:2403.09005 [gr-qc], PRD 110 (2024) , 063512

$$S^{\rm eff} = \int d^4x \sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}(\partial_\mu b)(\partial^\mu b) - A\, b\, R_{CS}\right], \quad A = \sqrt{\frac{2}{3}}\frac{\alpha'}{48\kappa}$$

$$2<R_{CS}>= -\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle = -\int DbDg_{\mu\nu}\, e^{-S^{\rm eff}}\, R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma} \quad = \text{constant}$$

WE DO NOT KNOW THE FULL QG THEORY!

EFT approach in expanding universe

Only Estimate at Stationary points of $S^{\rm eff}$

$$ds^2 = -dt^2 + \alpha^2(t)(\delta_{ij} + h_{ij})dx^i dx^j \qquad h_{ij} = h_+\epsilon^{(+)}_{ij} + h_\times\epsilon^{(\times)}_{ij} \qquad [h_{ij}] = \begin{bmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

GW tensor perturbations

$$\lambda = R, L$$

$$h_{ij}(t,\vec{x}) = h_L\, \epsilon^{(L)}_{ij} + h_R\, \epsilon^{(R)}_{ij} = \sum_{\lambda=L,R} h_\lambda(t,\vec{x})\epsilon^{(\lambda)}_{ij},$$

Chiral GW waves needed for $R_{CS} \neq 0$ ⚠

Dorlis, NEM, Vlachos, arXive:2403.09005 [gr-qc], PRD 110 (2024) , 063512

$$S^{\text{eff}} = \int d^4x \sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}(\partial_\mu b)(\partial^\mu b) - A\, b\, R_{CS}\right], \quad A = \sqrt{\frac{2}{3}}\frac{\alpha'}{48\kappa}$$

$$2 < R_{CS} > = -\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle = -\int DbDg_{\mu\nu}e^{-S^{\text{eff}}}\, R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma} \quad = \text{constant}$$

WE DO NOT KNOW THE FULL QG THEORY!

EFT approach in expanding universe

Only Estimate at Stationary points of S^{eff}

$$\boxed{ds^2 = -dt^2 + \alpha^2(t)(\delta_{ij} + h_{ij})dx^i dx^j}$$

$$h_{ij} = h_+\epsilon^{(+)}_{ij} + h_\times\epsilon^{(\times)}_{ij} \qquad [h_{ij}] = \begin{bmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

GW tensor perturbations

$$\lambda = R, L$$

$$h_{ij}(t,\vec{x}) = h_L\ \epsilon^{(L)}_{ij} + h_R\ \epsilon^{(R)}_{ij} = \sum_{\lambda=L,R} h_\lambda(t,\vec{x})\epsilon^{(\lambda)}_{ij},$$

Chiral GW waves needed for $R_{CS} \neq 0$ ⚠

$$h_{ij} = \kappa\sum_\lambda\int\frac{d^3k}{(2\pi)^{3/2}}e^{i\vec{k}\cdot\vec{x}}\frac{\psi_{\lambda,\vec{k}}(\eta)}{\alpha\sqrt{1 - l_\lambda l_{\vec{k}} L_{CS}(\eta)}}\epsilon^\lambda_{ij}, \qquad L \to R \qquad \text{and} \qquad \vec{k} \to -\vec{k}.$$

$$L_{CS}(\eta) = k\xi\ , \qquad \xi = \frac{4Ab'\kappa^2}{\alpha^2}, \quad b' = db/d\eta \qquad \left[\epsilon^{(R)}_{ij}\right] = \frac{1}{\sqrt{2}}\left(\left[\epsilon^{(+)}_{ij}\right] + i\left[\epsilon^{(\times)}_{ij}\right]\right) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i & 0 \\ i & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \left[\epsilon^{(L)}_{ij}\right]^\dagger$$

conformal time

Canonical quantization of weak gravity

Introduce complex scalar fields

$$\phi(\eta,\vec{x}) = \psi_L(\eta,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \widetilde{\psi}_{L,\vec{k}}(\eta), \qquad \widetilde{\phi}_{\vec{k}} = \widetilde{\psi}_{L,\vec{k}}\,,$$

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Quantum operators (creation,annihilation acting on appropriate vacuum state |0> (Bunch-Davis)

$$\hat{\widetilde{\phi}}_{\vec{k}}(\eta) = \widetilde{v}_{\vec{k}}\hat{\alpha}^-_{\vec{k}} + v^*_{-\vec{k}}\hat{b}^+_{-\vec{k}}\,,$$

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Lyth, Rodriguez, Quimbay

$$u_{L,\vec{k}} = \kappa\frac{\widetilde{v}_{\vec{k}}}{z_{L,\vec{k}}}\,, \quad u_{R,\vec{k}} = \kappa\frac{v_{\vec{k}}}{z_{R,\vec{k}}}\,.$$

$$z_{\lambda,\vec{k}}(\eta) = \alpha\sqrt{1 - l_\lambda l_{\vec{k}} L_{CS}(\eta)}$$

$$\left[\hat{\alpha}^-_{\vec{k}}\,,\,\hat{\alpha}^+_{\vec{k}'}\right] = \left[\hat{b}^-_{\vec{k}}\,,\,\hat{b}^+_{\vec{k}'}\right] = \delta^{(3)}(\vec{k}-\vec{k}')$$

& the rest zero

$$\boxed{\langle R_{CS}\rangle = \frac{2i}{\alpha^4}\left[\langle\partial_z^2 h_L \partial_z h'_R\rangle + \langle h''_L\partial_z h'_R\rangle - \langle\partial_z^2 h_R\partial_z h'_L\rangle - \langle h''_R\partial_z h'_L\rangle\right]}$$

$$\langle\ldots\rangle = \langle 0|\ldots|0\rangle$$

Trivial for non-chiral GW

Keep **ALL terms** (further to approximations made in **Lyth, Rodriguez, Quimbay**)

Dorlis, NEM, Vlachos, arXive:2403.09005 [gr-qc], PRD 110 (2024) , 063512

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& the rest zero

$$\boxed{\langle R_{CS}\rangle = \frac{2}{\alpha^4} \int^{\alpha\mu} \frac{d^3\vec{k}}{(2\pi)^3} l_{\vec{k}} \left[k^3 \left(u_{L,\vec{k}} u^{*\prime}_{L,\vec{k}} - u_{R,\vec{k}} u^{*\prime}_{R,\vec{k}}\right) + k \left(u''_{R,\vec{k}} u^{*\prime}_{R,\vec{k}} - u''_{L,\vec{k}} u^{*\prime}_{L,\vec{k}}\right)\right]}$$

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μ = low-energy
UV cutoff ~ M_s

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& the rest zero

$$A = \sqrt{\frac{2}{3}}\frac{\alpha'}{48\kappa}$$

$$\boxed{\langle R_{CS}\rangle^I = -\frac{A}{\pi^2}\frac{\dot{b}_I}{M_{\rm Pl}}\left(\frac{H_I}{M_{\rm Pl}}\right)^3\mu^4 < 0\,,}$$

$H_I \approx$ *constant during inflation*

$$\langle \ldots \rangle = \langle 0|\ldots|0\rangle$$

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UV cutoff ~ M_s

Canonical quantization of weak gravity

Introduce complex scalar fields

$$\phi(\eta,\vec{x}) = \psi_L(\eta,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}}\widetilde{\psi}_{L,\vec{k}}(\eta), \qquad \widetilde{\phi}_{\vec{k}} = \widetilde{\psi}_{L,\vec{k}},$$

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& the rest zero

If you have N_I sources of GW Linear superposition of GW perturbations

$$\boxed{\langle R_{CS}\rangle_I^{total} = -\mathcal{N}_I\,\frac{A\kappa^4\mu^4}{\pi^2}\,\dot{b}_I H_I^3}$$

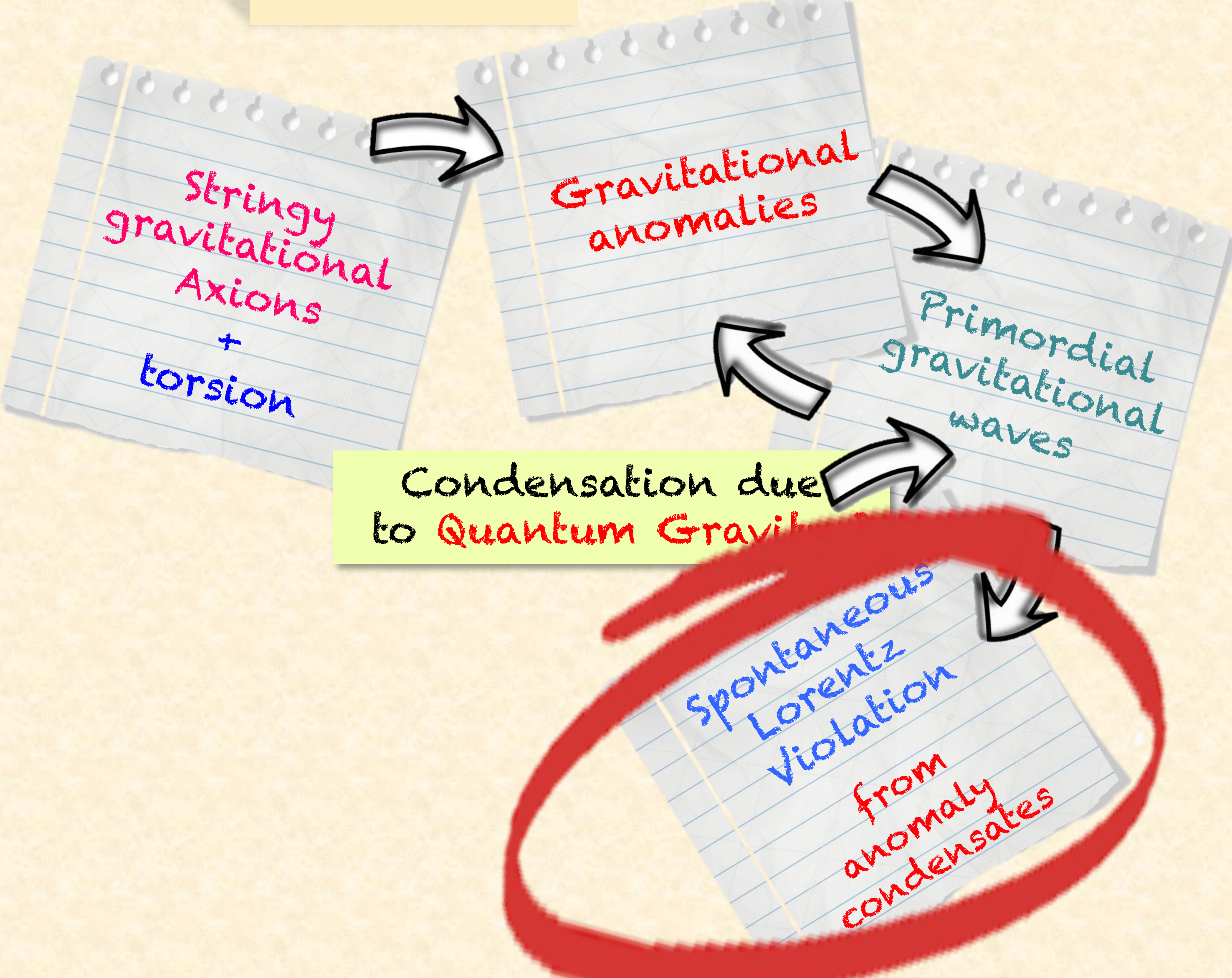
$H_I \approx$ *constant during inflation*

$$\langle\ldots\rangle = \langle 0|\ldots|0\rangle$$

$\kappa^{-1} = M_{Pl}$

μ = low-energy UV cutoff ~ M_s

The Parts



Stringy gravitational Axions + torsion

Gravitational anomalies

Primordial gravitational waves

Condensation due to Quantum Gravi

Spontaneous Lorentz Violation from anomaly condensates

Basilakos, NEM, Solà

Solutions (backgrounds) to the Eqs of Motion

Homogeneity
& Isotropy

$$\boxed{\alpha' = M_s^{-2}}$$

$$S_B^{\text{eff}} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2\kappa^2} R + \frac{1}{2}\partial_\mu b\, \partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\, \mathcal{K}^\mu + \dots \Big] ,$$

$$\boxed{\partial_\alpha\Big[\sqrt{-g}\Big(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^\alpha(t)\Big)\Big] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^0 \sim \text{constant}}$$

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$$\Downarrow$$

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μ = low-energy UV cutoff ~ M_s

FLRW spacetime

$\Big(A = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{48\kappa}\Big)$

Planck Data $H/M_{\rm Pl} \lesssim 10^{-5}$

time evolution of Anomaly

$$\boxed{\mathcal{K}^0(t) \simeq \mathcal{K}^0_{\rm begin}(0)\,\exp\Big[-3H\,t\,\Big(1 - 0.5\,N_I\;10^{-4}\Big(\frac{H}{M_{\rm Pl}}\Big)^2\Big(\frac{\mu}{M_s}\Big)^4\Big)\Big]}$$

Homogeneity & Isotropy

Solutions (backgrounds) to the Eqs of Motion

$$S_B^{eff} = \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

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FLRW spacetime

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to ensure approximately constant anomaly current during inflation

$$\mathcal{K}^0 = \text{const.}$$

Spontaneous LV solution

$$\boxed{N_I \gtrsim O(10^{14})}$$

@ the beginning of RVM inflation

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \boxed{\dot{\bar{b}} = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^0 \sim \text{constant}}$$

$$\dot{\bar{b}} \sim \varepsilon_{ijk} H^{ijk} \approx \textit{constant}$$

Spontaneous
LV solution
(constant spatial
components of H-torsion)

Parametrisation

@ end of
Inflationary
era

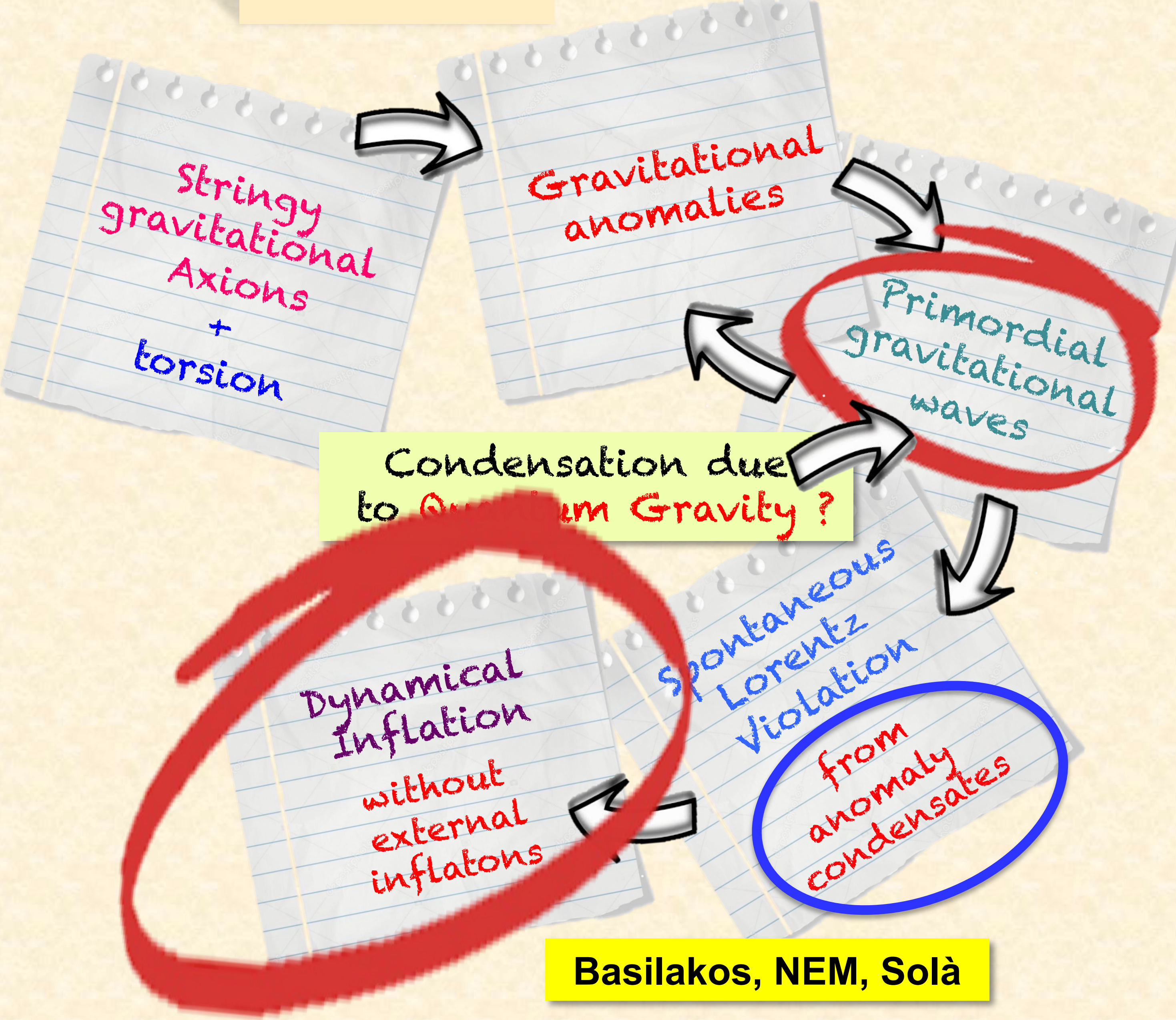
$$\boxed{\dot{\bar{b}} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H}$$

$$H = H_{\rm infl} \simeq \text{const.}$$

How can we estimate ε ?

4. Grav. Anomaly Condensates & inflation

The Parts



Stringy gravitational Axions
+
torsion

Gravitational anomalies

Primordial gravitational waves

Spontaneous Lorentz Violation

from anomaly condensates

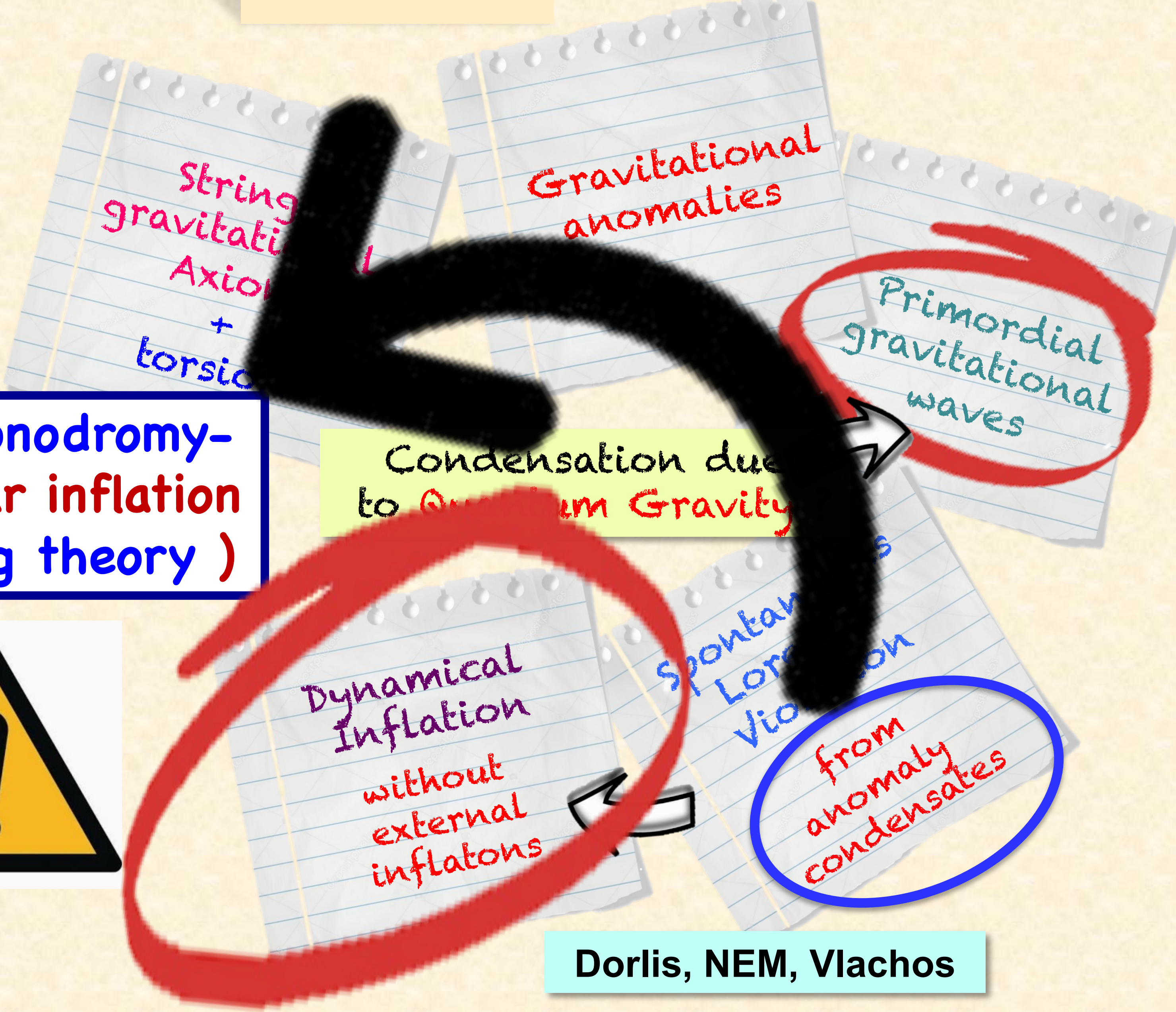
Dynamical Inflation

without external inflatons

Condensation due to Quantum Gravity ?

Basilakos, NEM, Solà

The Parts



Axion-monodromy-like Linear inflation (cf. string theory)

Dorlis, NEM, Vlachos

Dorlis, NEM, Vlachos, arXive:2403.09005 [gr-qc], PRD 110 (2024) , 063512

Torsion axions, Condensates & Inflation

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

Primordial string Universe Gravitational Waves (GW): e.g. from collapse of (rotating) primordial black holes (PBH) sourced by Torsion-induced axion field *b(x)*

$$\Box b \propto R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}$$

constant

can induce **condensates** of gravitational Chern-Simons terms $\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle$

Compute using weak (perturbative)
Quantum gravity techniques
With GW perturbation modes

Condensates lead to linear axion potentials

$$V(b) \ni b(x)\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle$$

Lyth, Rodriguez, Qiumbay
Alexander, Peskin, Sheikh-Jabbari
Dorlis, Vlachos, NEM 2403.09005 [gr-qc]

(cf. string/brane theory linear axion monodromy potentials:
Silverstein, Mc Allister, Westphal, …, but here different origin)

Dorlis, Vlachos, NEM 2403.09005 [gr-qc]

Linear axion potential and Running Vacuum Model (RVM) Inflation

Dynamical System approach to inflation

$$\left.\begin{aligned} 3H^2 &= \kappa^2\left(\frac{\dot{b}^2}{2}+V(b)\right) \\ 2\dot{H}+3H^2 &= -\kappa^2\left(\frac{\dot{b}^2}{2}-V(b)\right) \\ \ddot{b}+3H\dot{b}&+V_{,b}=0 \end{aligned}\right\} \quad \begin{aligned} x' &= -\frac{3}{2}\left[2x-x^3+x\left(y^2-1\right)-\frac{\sqrt{2}}{\sqrt{3}}\lambda y^2\right] \\ y' &= -\frac{3}{2}y\left[-x^2+y^2-1+\frac{\sqrt{2}}{\sqrt{3}}\lambda x\right] \\ \lambda' &= -\sqrt{6}\left(\Gamma-1\right)\lambda^2 x \\ \lambda &= -\frac{V_{,b}}{\kappa V} \quad \text{and} \quad \Gamma=\frac{V V_{,bb}}{V_{,b}^2} \end{aligned}$$

$$V_{,b} \equiv \frac{\delta V}{\delta b}$$

$$x = cos\varphi \;,\; y = sin\varphi \quad \Longrightarrow \quad \boxed{\begin{aligned} \varphi' &= \left(3\cos\varphi-\frac{\sqrt{6}}{2}\lambda\right)\sin\varphi \\ \lambda' &= -\sqrt{6}(\Gamma-1)\lambda^2\cos\varphi \end{aligned}}$$

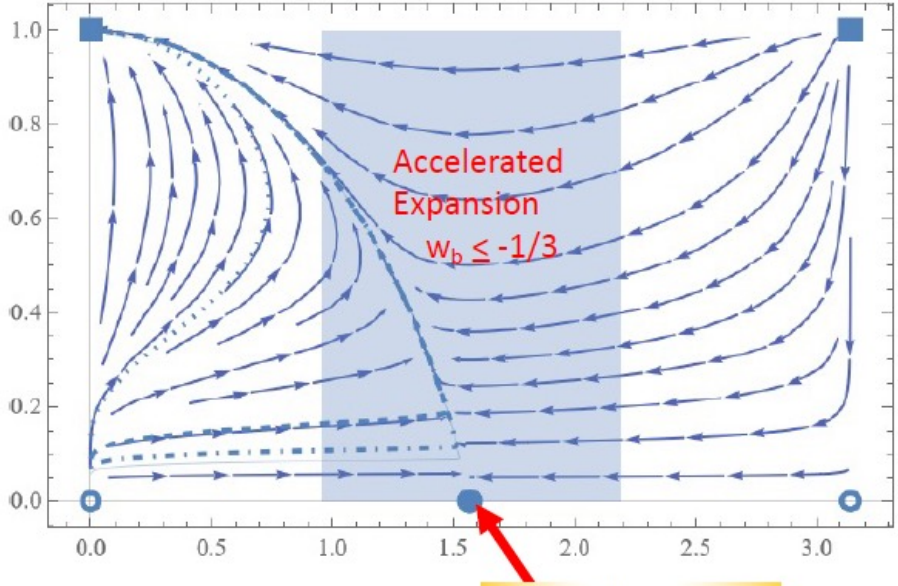
$$\zeta = \frac{\lambda}{\lambda + 1}$$

$$\varphi' = \left(3\cos\varphi - \frac{\sqrt{6}}{2}\frac{\zeta}{1-\zeta}\right)\sin\varphi$$

$$\zeta' = -\sqrt{6}(\Gamma - 1)\zeta^2\cos\varphi$$

Dorlis, Vlachos, NEM 2403.09005 [gr-qc]

Phase Portait for the Linear Potential



Inflation (exponential Expansion w_b = -1)

$\zeta - \varphi$ plane.

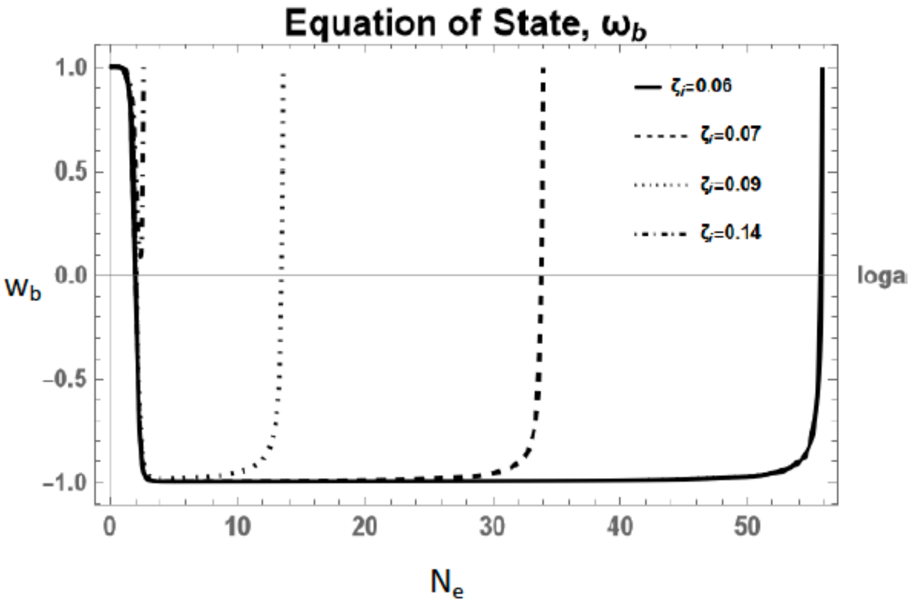
$$\zeta = \frac{\lambda}{\lambda + 1} \qquad \varphi' = \left(3\cos\varphi - \frac{\sqrt{6}}{2}\frac{\zeta}{1-\zeta}\right)\sin\varphi$$

$$\zeta' = -\sqrt{6}(\Gamma - 1)\zeta^2 \cos\varphi$$

Dorlis, Vlachos, NEM 2403.09005 [gr-qc]

The condensate $\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle$ induced

by quantum graviton fluctuations of chiral (left-right asymmetric) GW type



$$\text{-Re}\ \langle R_{CS}\rangle^{total} = \mathcal{N}_I \frac{A\kappa^4\mu^4}{\pi^2}\dot{b}_I H_I^3$$

$$A = \sqrt{\frac{2}{3}\frac{\alpha'}{48\kappa}}$$

String EFT:
$\mu = M_s = (\alpha')^{-1/2}$

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Dorlis, Vlachos, NEM 2403.09005 [gr-qc]

The condensate $\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle$ induced

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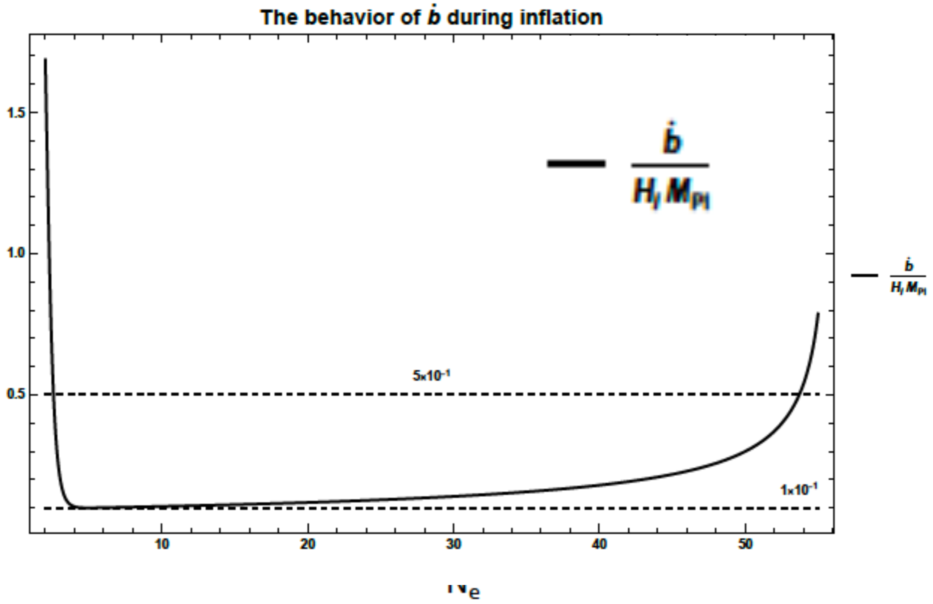

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Hubble

GW sources

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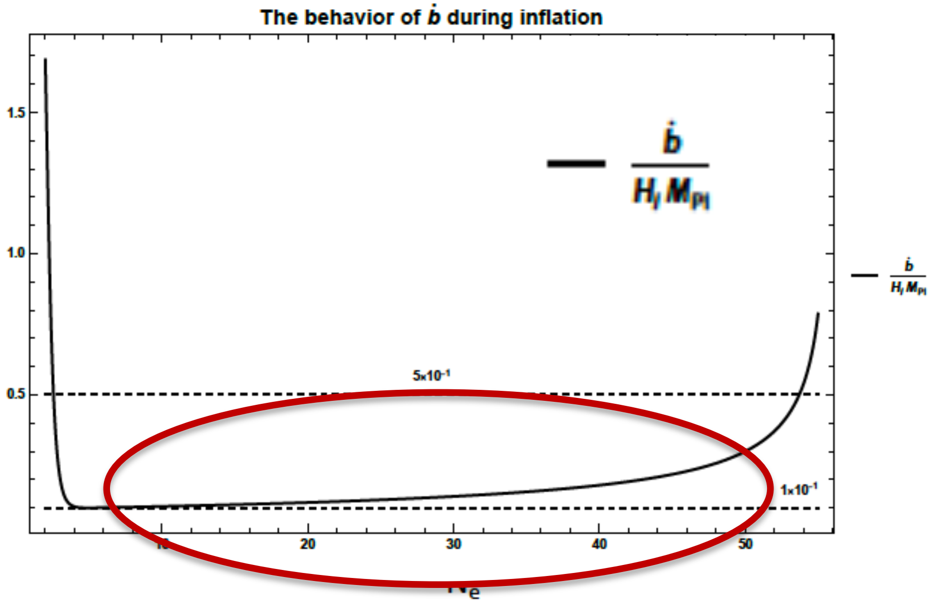

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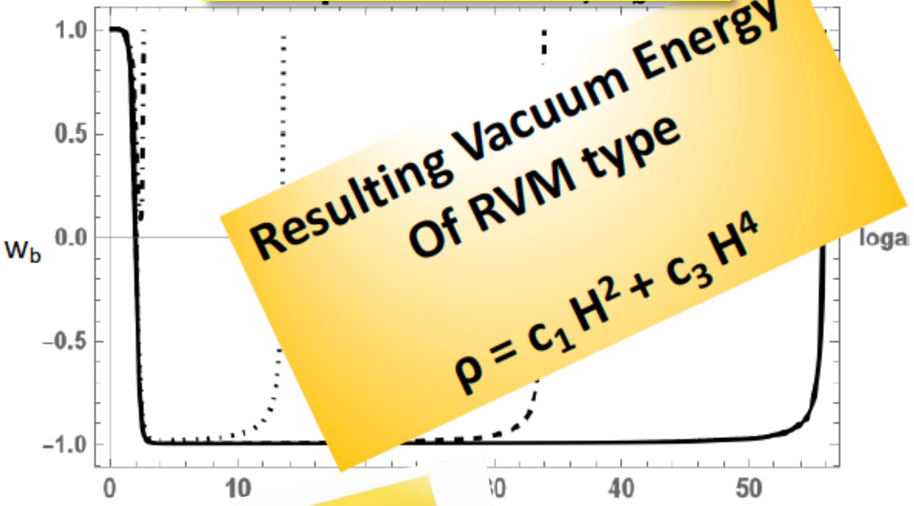

Resulting Vacuum Energy Of RVM type

$\rho = c_1 H^2 + c_3 H^4$

J. Solà talk

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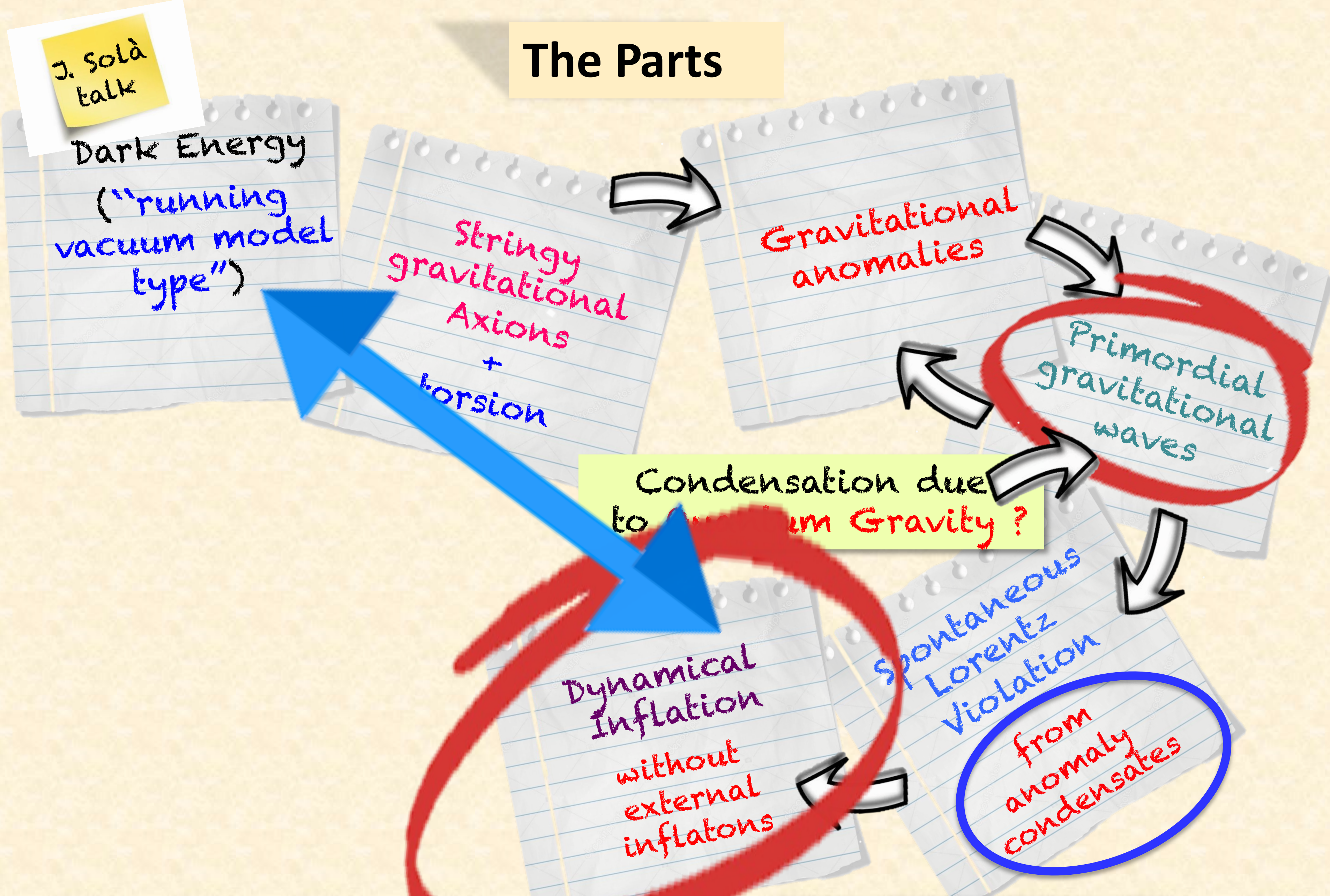

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NB: Metastable Inflationary Vacua

Dorlis, NEM, Vlachos
2404.18741 (PoS Corfu 2023)

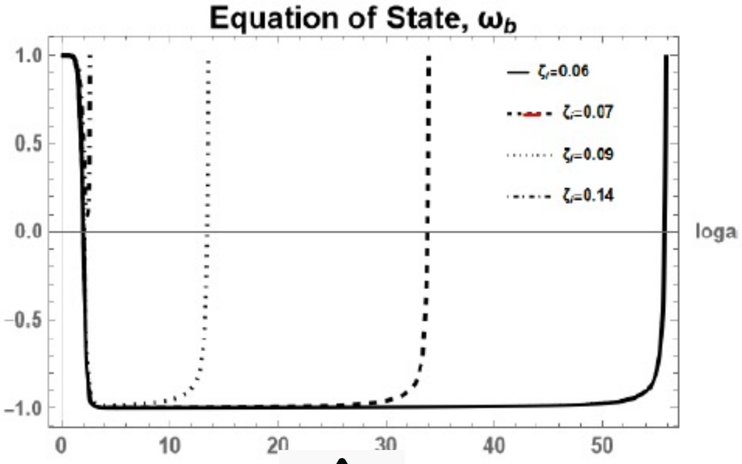
Compatibility with swampland (Ooguri, Vafa, …)

CONDENSATE WITH CUTOFF GRAVITON MODES HAS IMAGINARY PARTS

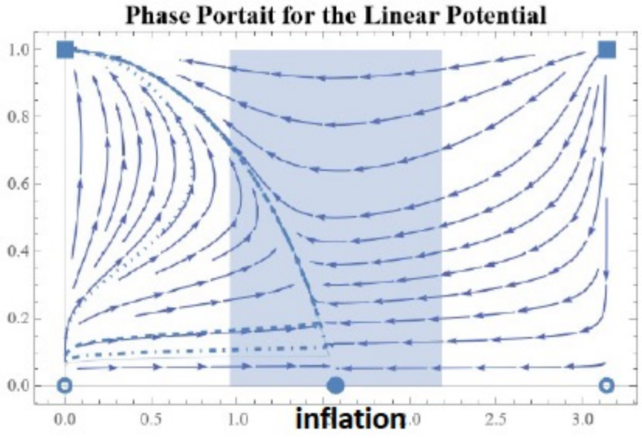
(ENVIRONMENT OF MODES WITH MOMENTA ABOVE THE CUTOFF μ)

Consistent with swampland ↔ INSTABILITY OF CONDENSATE PHASE → FINITE LIFE TIME OF THIS PHASE

→ CONSISTENT WITH *50 e-FOLDINGS* AS STEMS FROM DYNAMICAL SYSTEM ANALYSIS !



$\zeta - \varphi$ plane.



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Quantum chiral GW computation yields a non-Hermitian Chern-Simons-anomaly operator

$$\widehat{R}_{CS} - \widehat{R}^{\dagger}_{CS} = \frac{2}{a^4}\int \frac{d^3k\, d^3k'}{(2\pi)^3}\, e^{i(\vec{k}+\vec{k}')\cdot\vec{x}} \Big\{k^2 k' l_{\vec{k}'} \Big(\Big[\widehat{h}_{L,\vec{k}}, \widehat{h}'_{R,\vec{k}'}\Big] + \Big[\widehat{h}'_{L,\vec{k}'}, \widehat{h}_{R,\vec{k}}\Big]\Big) - k' l_{\vec{k}'} \Big(\Big[\widehat{h}''_{L,\vec{k}}, \widehat{h}'_{R,\vec{k}'}\Big] + \Big[\widehat{h}'_{L,\vec{k}'}, \widehat{h}''_{R,\vec{k}}\Big]\Big)\Big\}$$

Quantum commutators

Quantum commutators

$$h'_{L(R),\vec{k}} \equiv \frac{d}{d\eta} h_{L(R),\vec{k}},$$
$$\eta = \text{conformal time}$$

Quantum graviton fluctutations destabilise the CS condensate vacuum !

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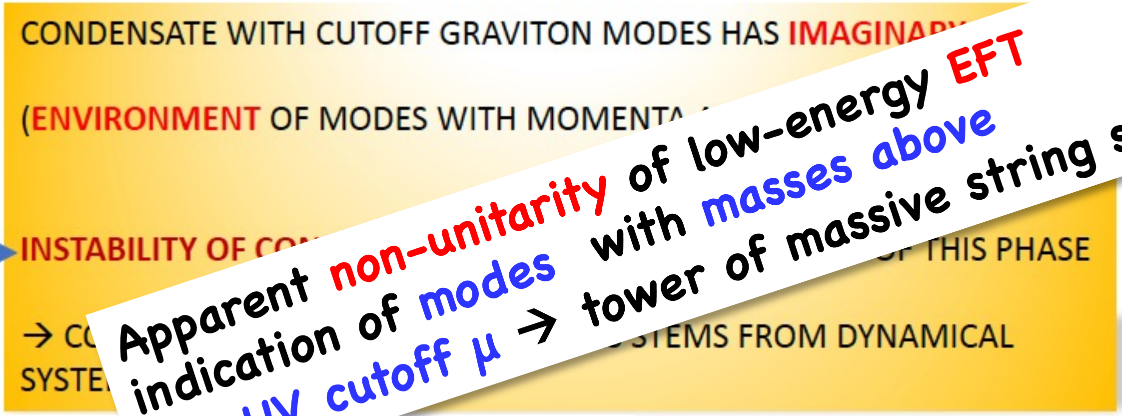
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SYSTE

Apparent non-unitarity of low-energy EFT
indication of modes with masses above
the UV cutoff μ → tower of massive string states ?

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$$\mathrm{Im}\left(\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\nu\mu\rho\sigma}\rangle\right) = \frac{16A\dot{b}\mu^7}{7M_{\rm Pl}^4\,\pi^2}\left[1+\left(\frac{H_I}{\mu}\right)^2\left(\frac{21}{10}-6\left(\frac{A\mu\dot{b}}{M_{\rm Pl}^2}\right)^2\right)\right]$$

They induce imaginary parts in the Hamiltonian of GW perturbations

$$\mathrm{Im}\,(\mathcal{H}) = \int d^3x\,\frac{1}{2}A\,b\,\mathrm{Im}\left(\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\nu\mu\rho\sigma}\rangle\right) \approx V_{dS}^{(3)}\,\frac{8bA^2\dot{b}\mu^7}{7M_{\rm Pl}^4\,\pi^2}\,.$$

$V_{dS}^{(3)}$ denotes the de Sitter 3-volume $\quad V_{dS}^{(3)}T^E = \dfrac{24\pi^2}{M_{\rm Pl}^2\Lambda}\,, \quad \Lambda \approx 3H_I^2$ *T^E = Euclidean time*

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Dorlis, NEM, Vlachos
2404.18741 (PoS Corfu 2023)

Life time of Inflationary vacua

$$\boxed{\tau \sim (\mathrm{Im}\mathcal{H})^{-1}}$$

To ensure $T^E \sim (50-60)H_I^{-1}$ (**phenomenologically consistent inflation duration**)

$$H_I\tau \sim \frac{7H_I^2 M_{\rm Pl}^6}{64 b A^2 \dot{b} M_s^7}\left(H_I T^E\right) \sim 10^{-2}\left(\frac{M_{\rm Pl}}{M_s}\right)^3 \cdot (H_I T^E)$$

$$\boxed{\frac{M_s}{M_{\rm Pl}} \lesssim 0.215}$$

Restriction on the string scale ⚠

We have seen

The rate of KR axion background during Inflation

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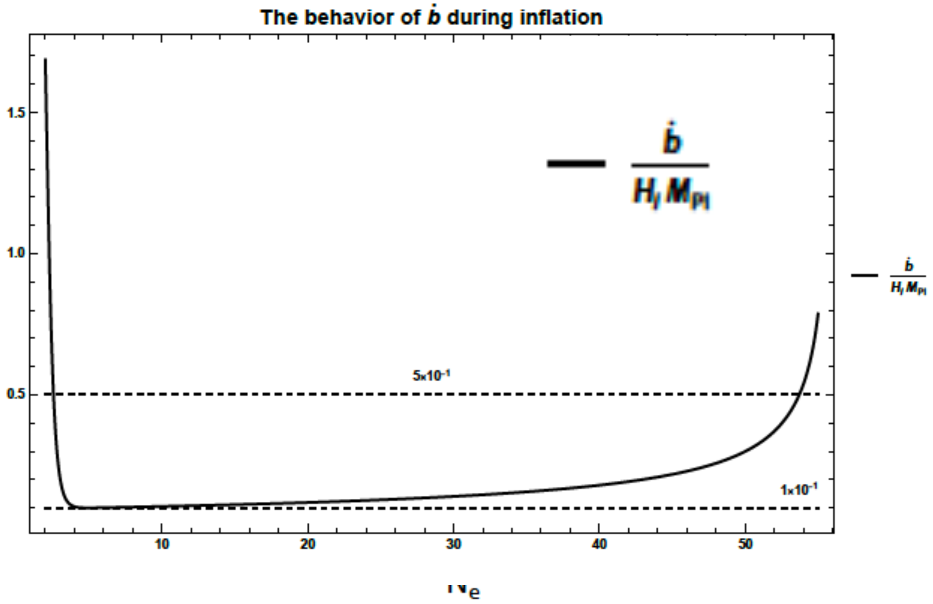

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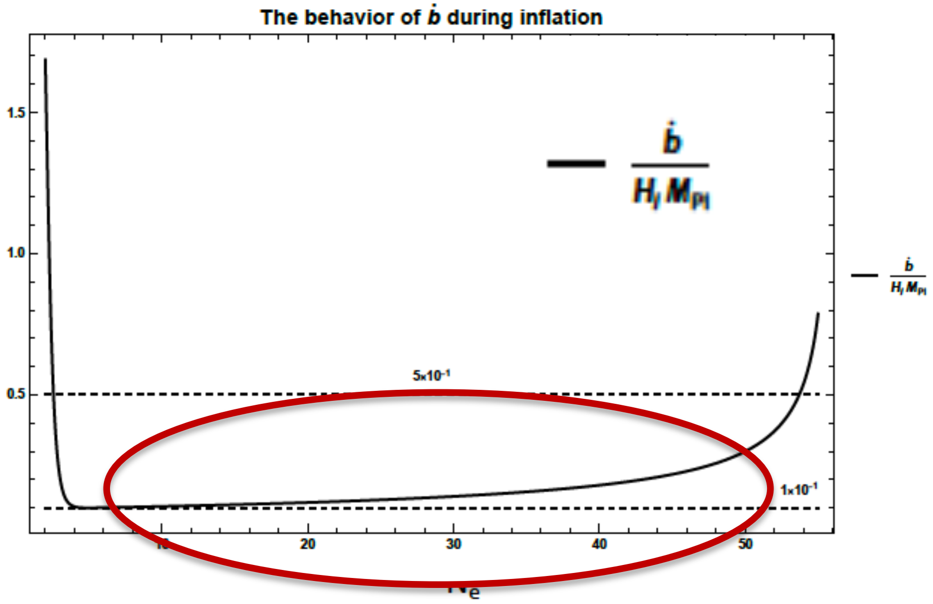

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Hence

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \boxed{\dot{\bar{b}} = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\mathcal{K}^0 \sim \text{constant}}$$

$$\dot{\bar{b}} \sim \varepsilon_{ijk}H^{ijk} \approx \textit{constant}$$

$$\dot{b}_I \sim 10^{-1} H_I M_{\rm Pl}$$

Spontaneous LV solution (constant spatial components of H-torsion)

Parametrisation

ε = O(10^{-2})

@ end of Inflationary era

$$\boxed{\dot{\bar{b}} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H}$$

$$H = H_{\rm infl} \simeq \text{const.}$$

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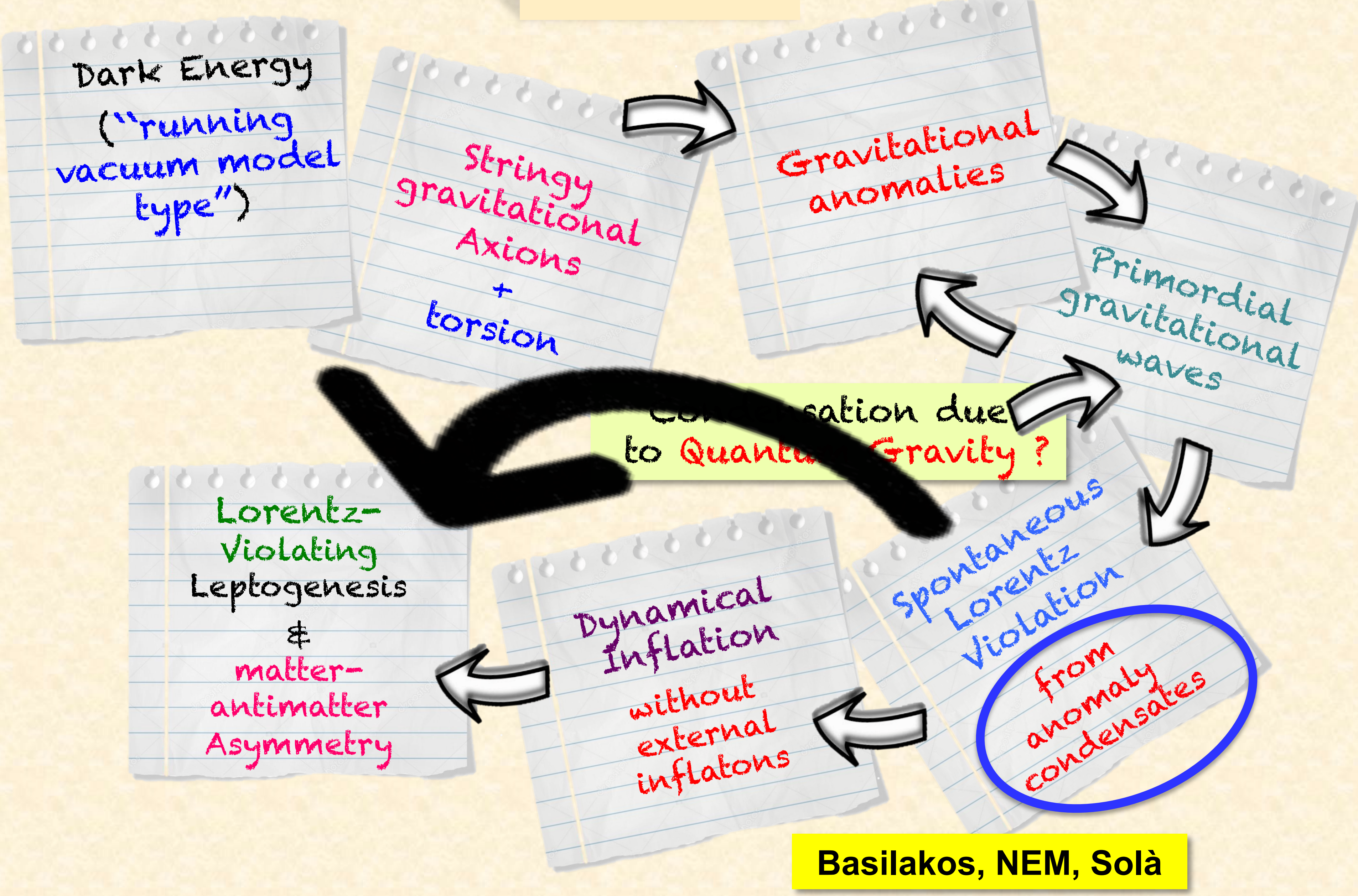
Important for Leptogenesis @ radiation era

5. Post-inflationary RVM era & Matter-antimatter asymmetry

or: do we exist because of gravitational anomalies?

The Parts



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("running vacuum model type")

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+
torsion

Gravitational anomalies

Primordial gravitational waves

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Spontaneous Lorentz Violation

from anomaly condensates

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Lorentz-Violating Leptogenesis
&
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Basilakos, NEM, Solà

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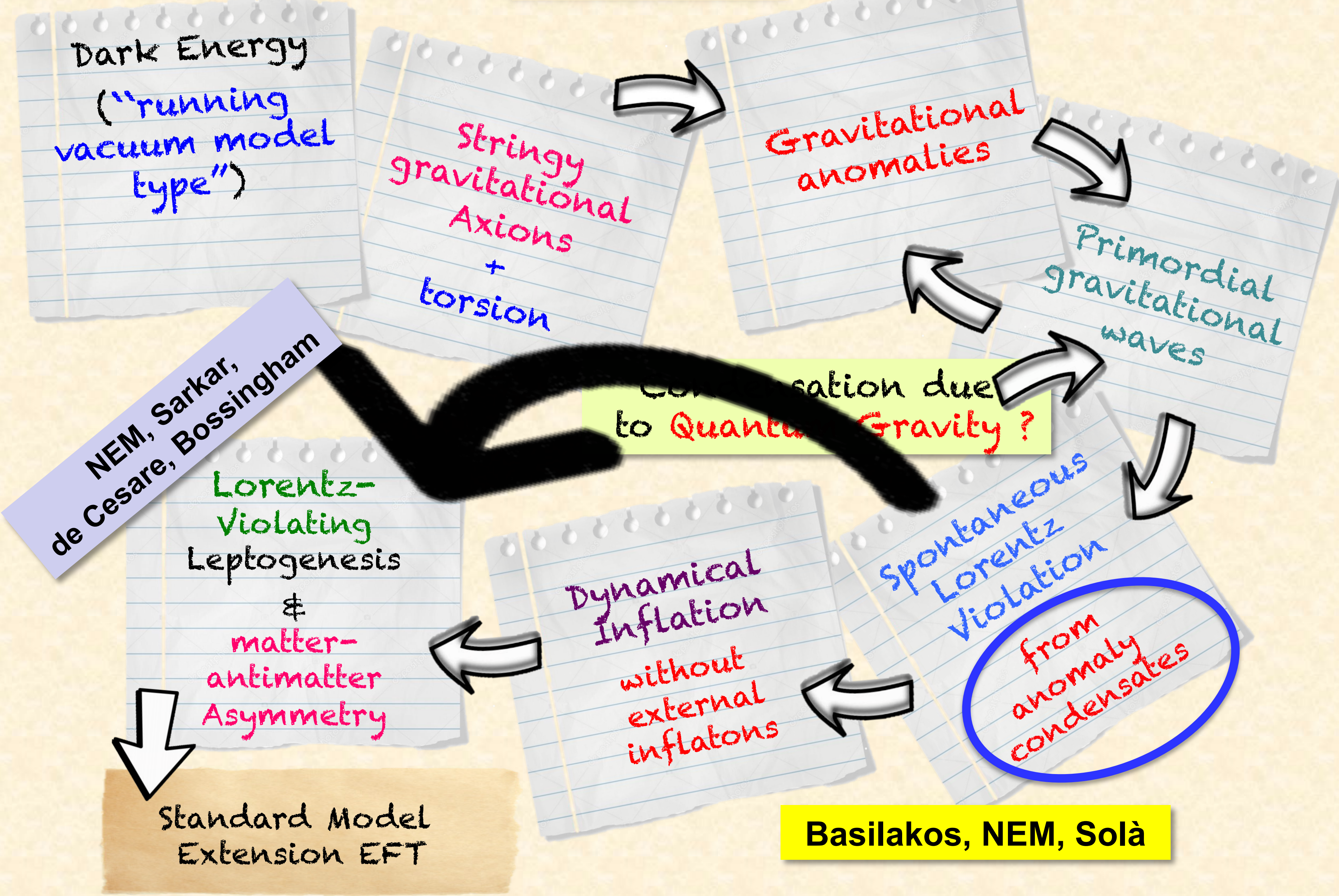

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Standard Model Extension EFT

NEM, Sarkar, de Cesare, Bossingham

Basilakos, NEM, Solà

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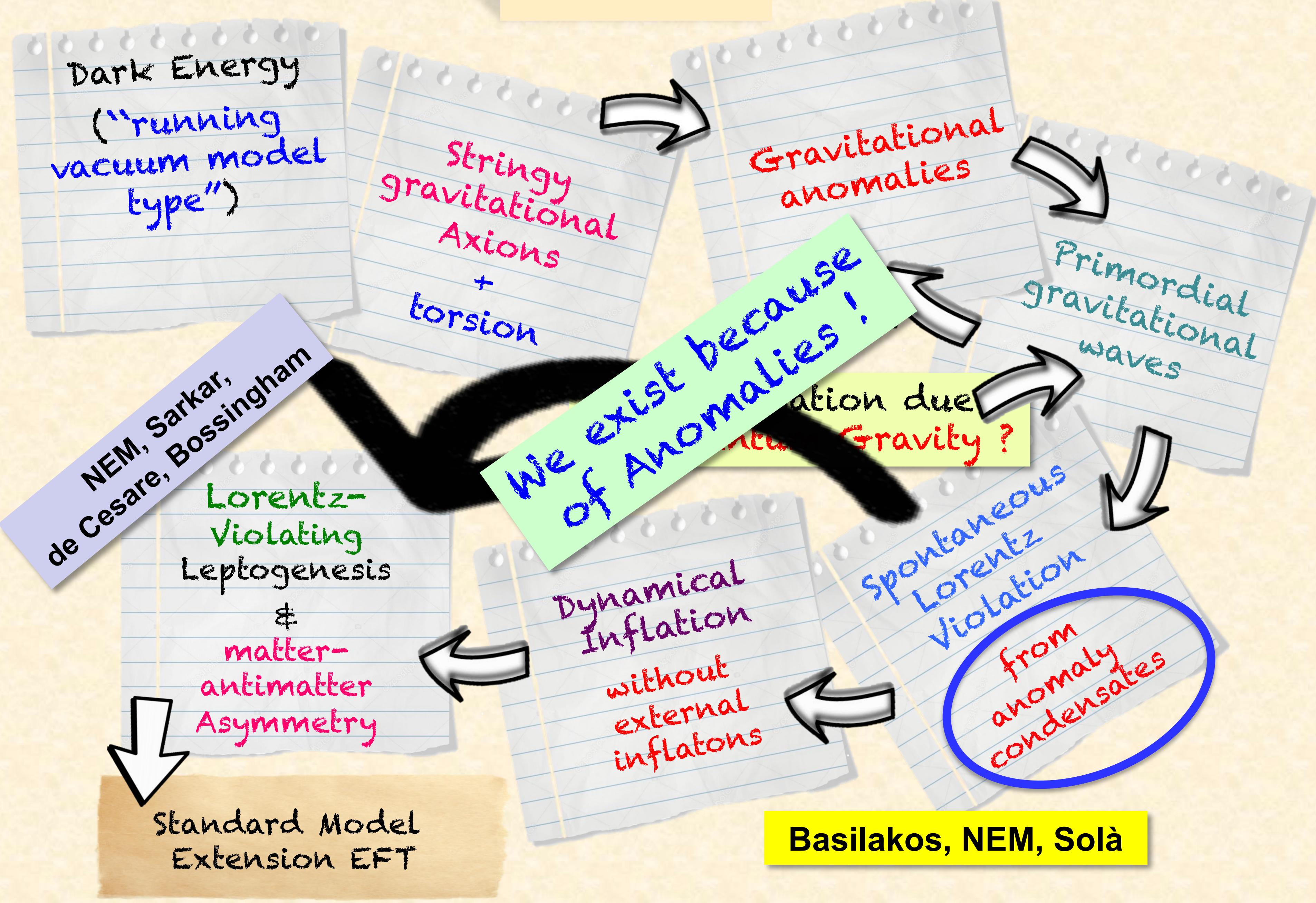

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We exist because of Anomalies !

NEM, Sarkar, de Cesare, Bossingham

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Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

$$S^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \kappa\, b(x)\nabla_\mu\Big(\sqrt{\frac{2}{3}}\frac{1}{96}\mathscr{K}^\mu - \sqrt{\frac{3}{8}}J^{5\mu}\Big)\Big] + \dots,$$

$J^{5\mu} = \sum_j \bar{\psi}_j\gamma^\mu\gamma^5\psi_j;$ Chiral current, including RHN

$$\boxed{\partial_\mu\Big(\sqrt{-g}\Big[\sqrt{\frac{3}{8}}J^{5\mu} - \sqrt{\frac{2}{3}}\frac{1}{96}\mathscr{K}^\mu\Big]\Big) = \text{"chiral U(1) anomalies"}.}$$

Possibly also QCD type

Eqs of Motion for b-field → $\partial_\mu\Big(\sqrt{-g}\,\partial^\mu b(x)\Big) = \text{"chiral U(1) anomalies"},$

Scale factor *a(t)* ~ T^{-1}

Possibly also QCD

$$\boxed{\dot{b} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U(1) anomaly terms}}$$

Viewed as sufficiently slow moving to induce Leptogenesis

Bossingham, NEM, Sarkar (2018)

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Basilakos, NEM,Solà (2019-20)

Required by consistency of quantum theory of matter and radiation (**diffeomorphism invariance**)

Scale factor ***a(t)* ~ *T*$^{-1}$**

Possibly also QCD

$$\boxed{\dot{\bar{b}} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U}(1) \text{ anomaly terms}}$$

sufficiently slowly varying during leptogenesis (brief) epoch → qualitatively similar to approximately const. background

Bossingham, NEM, Sarkar

Lorentz- & CPT-Violating

Leptogenesis →

→ Baryogenesis

in models with Massive Right-handed Neutrinos

**NEM, Sarkar,
+ de Cesare, Bossingham**

**Early Universe
T >> T_{EW}**

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

NEM, Sarkar,
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Heavy RHN interact with axial (approximately) constant background

with only temporal component $B_0 \neq 0$ $\quad B_\mu = M_{\rm Pl}^{-1}\,\dot{\bar{b}}\,\delta_{\mu 0}$

STANDARD MODEL EXTENSION EFT

Kostelecky, Bluhm, Colladay, Lehnert, Potting, Russell *et al.*

Lorentz & CPT Violation

$$\mathcal{L} = \tfrac{1}{2}i\bar{\psi}\Gamma^\nu \bar{\partial}_\nu \psi - \bar{\psi}M\psi,$$

$$M \equiv m + a_\mu\gamma^\mu + b_\mu\gamma_5\gamma^\mu + \tfrac{1}{2}H^{\mu\nu}\sigma_{\mu\nu}$$

Spontaneous Violation of Lorentz Symmetry
(LV coefficients are v.e.v. of tensor-valued field quantities)
$B_0 \approx$ constant is H-torsion background in our model

CPT Violation

de Cesare, NEM, Sarkar
Eur.Phys.J. C75, 514 (2015)

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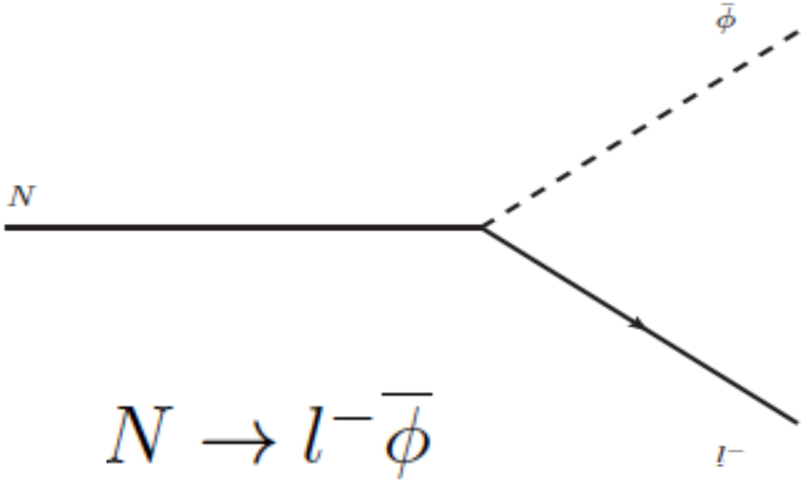
with only temporal component $B_0 \neq 0$

Produce Lepton asymmetry

Lepton number & CP Violations
@ tree-level due to
Lorentz/CPTV Background



$N \rightarrow l^+\phi$

$N \rightarrow l^-\overline{\phi}$

$$\Gamma_1 = \sum_k \frac{|Y_k|^2}{32\pi^2}\frac{m^2}{\Omega}\frac{\Omega + B_0}{\Omega - B_0} \neq \Gamma_2 = \sum_k \frac{|Y_k|^2}{32\pi^2}\frac{m^2}{\Omega}\frac{\Omega - B_0}{\Omega + B_0}$$

$B_0 \neq 0$

CPV & LV

$$\Omega = \sqrt{B_0^2 + m^2}$$

$$\mathcal{L} = i\overline{N}\not{\partial} N - \frac{m}{2}(\overline{N^c}N \;{}^{m}\overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Constant B_0 Background

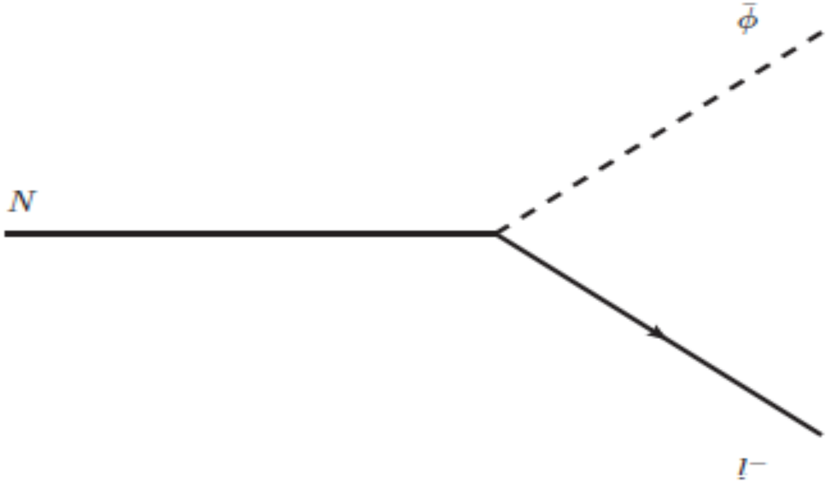
CPT Violation

Early Universe
T >> T_{EW}

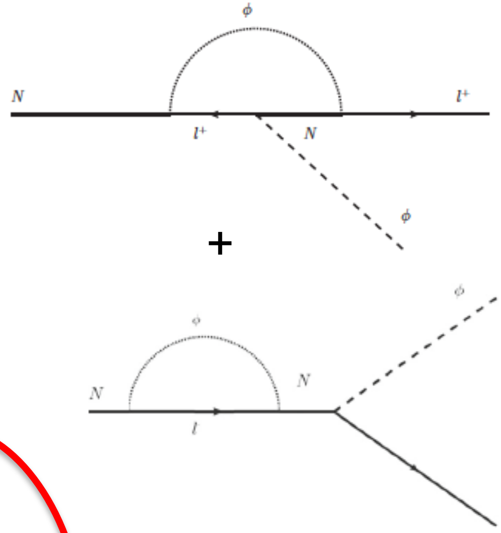
Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \to \overline{\phi}\,\ell\,,\;\phi\,\overline{\ell}$$

Produce Lepton asymmetry



Contrast with one-loop conventional CPV Leptogenesis (in absence of H-torsion)



Fukugita, Yanagida,

CPTV Thermal

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Early Universe

$T \gg T_{EW}$

CPT Violation

Constant $B^0 \neq 0$ background

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \rightarrow \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10},$$

$$\frac{B_0}{m} \simeq 10^{-8}$$

Produce Lepton asymmetry

Solving system of Boltzmann eqs

$$\frac{\Delta L^{TOT}}{s} \simeq \frac{g_N}{7\,e\,(2\pi)^{3/2}}\frac{B_0}{m} \simeq 0.007\,\frac{B_0}{m}$$

CPTV Thermal

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Early Universe
$T > 10^5$ GeV

CPT Violation

Constant $B^0 \neq 0$ background

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \rightarrow \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

$$\boxed{\frac{\Delta L}{n_\gamma} \simeq 10^{-10},} \rightarrow \boxed{\frac{B_0}{m} \simeq 10^{-8}}$$

Produce Lepton asymmetry

$$Y_k \sim 10^{-5}$$

$$m \geq 100\text{TeV} \rightarrow$$

$$B^0 \sim 1\text{MeV}$$

Solving system of Boltzmann eqs

$$\boxed{\frac{\Delta L^{TOT}}{s} \simeq \frac{g_N}{7\,e\,(2\pi)^{3/2}}\frac{B_0}{m} \simeq 0.007\,\frac{B_0}{m}}$$

$$T_D \simeq m \sim 100\ \text{TeV}$$

consistent with :
light neutrino masses in SM +
stability of Higgs vacuum

CPTV Thermal

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

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$$Y_k \sim 10^{-5}$$
$$m \geq 100\text{TeV} \to$$
$$B^0 \sim 1\text{MeV}$$

$$T_D \simeq m \sim 100 \text{ TeV}$$

Similar order of magnitude estimates if $B^0 \sim T^3$ during Leptogenesis era

Bossingham, NEM, Sarkar

CPTV Thermal

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Early Universe
$T > 10^5$ GeV

CPT Violation

nt $B^0 \neq 0$
round

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

$N_I \rightarrow \overline{\phi}\ell\,,\ \phi$

$\simeq 10^{-8}$

Solving
system
of Boltzm
eqs

$\rightarrow$

$\sim 1\mathrm{MeV}$

$I^{'}_D \simeq m \sim 100$ TeV

This Leptogenesis scenario can be embedded/extend existing scenarios of Leptogenesis
(Pilaftsis, Deppisch, Underwood, …)

Also can be accommodated within the νMSM (Shaposhnikov, Asaka, Blanchet, Canetti, Drewes, Gorbunov, Laine, Boyarski, Ruchaiskiy, Tkachev …)

Simila … of magnitude estimates
if $B^0 \sim T^3$ during Leptogenesis era

Bossingham, NEM,
Sarkar

CPTV Thermal

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Early Universe
T > 10^5 GeV

CPT Violation

Constant $B^0 \neq 0$
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Baryogenesis

?

CPTV Thermal

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

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$$\frac{B_0}{m} \simeq 10^{-8}$$

Produce Lepton asymmetry

Equilibrated electroweak B+L violating sphaleron interactions

B-L conserved

Environmental Conditions Dependent

$$L = \frac{2}{M} l_L l_L \phi\phi + \mathrm{H.c.}$$

where

$$l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

Fukugita, Yanagida,

Observed Baryon Asymmetry In the Universe (BAU)

$$\frac{n_B - \overline{n}_B}{n_B + \overline{n}_B} \sim \frac{n_B - \overline{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

T > 1 GeV

CPTV Thermal

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Early Universe
T > 10^5 GeV

CPT Violation

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T > 1 GeV

NB:

NEM, Sarkar,
+ de Cesare, Bossingham

Early Universe
$T >> T_{EW}$

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k \overline{L}_k \tilde{\phi} N + h.c.$$

NB: in our stringy models this mass **could be generated dynamically**, **e.g. through non-perturbative instanton effects that break shift-symmetry by coupling KR axions to right-handed neutrinos**

NEM, Pilaftsis

$$\begin{aligned}\mathcal{S} = \int d^4x\sqrt{-g}\,\Big[&\frac{1}{2}(\partial_\mu b)^2 + \frac{b(x)}{192\pi^2 f_b} R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma} \\ &+ \frac{1}{2f_b^2}J^5_\mu J^{5\mu} + \gamma(\partial_\mu b)\,(\partial^\mu a) + \frac{1}{2}(\partial_\mu a)^2 \\ &- y_a i a\left(\overline{\psi}_R^{\,C}\psi_R - \overline{\psi}_R\psi_R^{\,C}\right)\Big]\,,\end{aligned}$$

NB:

NEM, Sarkar,
+ de Cesare, Bossingham

Early Universe
$T >> T_{EW}$

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

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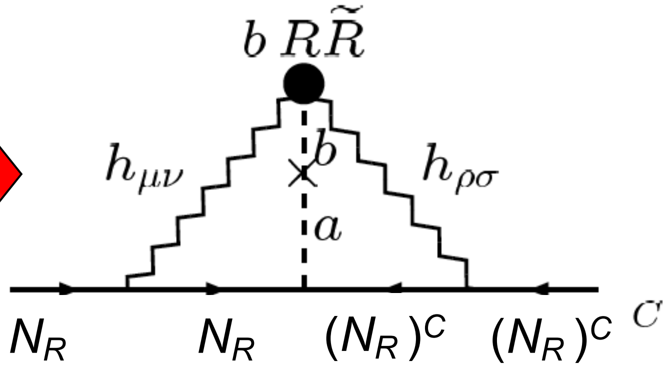


Radiatively-induced RHN mass

6. The Whole:

Stringy-RVM Cosmological Evolution

Summary of (stringy-RVM) Cosmological Evolution

Cosmic
Time

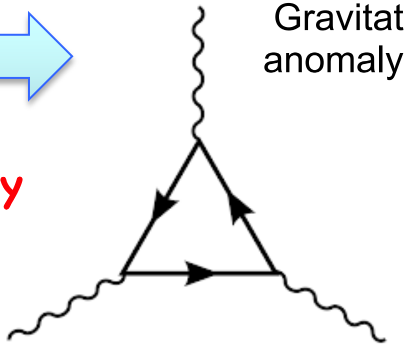
Big-Bang, pre-inflationary phase

Basilakos, NEM, Solà

RVM Inflationary (de Sitter) Phase

Primordial
Gravitational
Waves

From a pre-inflationary
era after Big-Bang



Gravitational
anomaly (GA)

Undiluted constant
KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$$

$$\dot{\overline{b}} \sim \sqrt{2\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

chiral matter
generation
@ inflation exit

forward direction

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

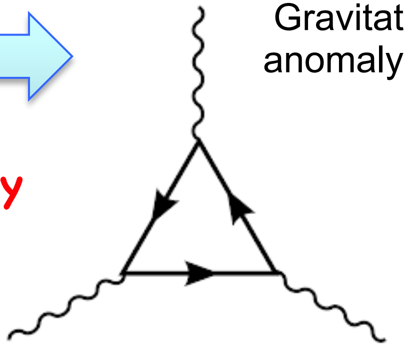
Big-Bang, pre-inflationary phase

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



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From a pre-inflationary era after Big-Bang

chiral matter generation @ inflation exit

Cancellation of GA

Summary of (stringy-RVM) Cosmological Evolution

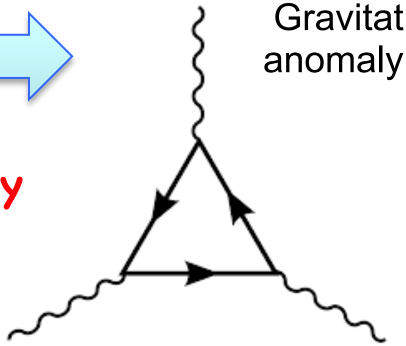
Cosmic Time

Big-Bang, pre-inflationary phase

Basilakos, NEM, Solà

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves →



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From a pre-inflationary era after Big-Bang

chiral matter generation @ inflation exit

Radiation Era

Cancellation of GA

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

B-L conserving sphelaron processes → Baryongenesis

forward direction

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

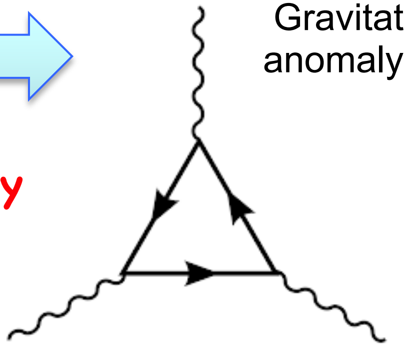
Big-Bang, pre-inflationary phase

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



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chiral matter generation @ inflation exit

From a pre-inflationary era after Big-Bang

Radiation Era

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Cancellation of GA

B-L conserving sphelaron processes → **Baryongenesis**

Matter Era

Possible potential generation for b → axion Dark matter

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

forward direction

Basilakos, NEM, Solà

KR mass: $m_b = \frac{\Lambda^2_{\rm QCD}}{\tilde{f}_b}$

$$\boxed{\frac{M_s}{M_{\rm Pl}} \lesssim 0.215}$$

Inflation pheno

$$V_b^{\rm QCD} \simeq \Lambda^4_{\rm QCD}\Big(1 - \cos(\frac{b}{\tilde{f}_b})\Big) \, ,$$

$$\tilde{f}_b = \frac{3}{\pi^2}\sqrt{\frac{3}{2}\frac{\kappa}{\alpha'}} = \frac{3}{\pi^2}\sqrt{\frac{3}{2}}\Big(\frac{M_s}{M_{\rm Pl}}\Big)^2 M_{\rm Pl}$$

$= 1.7 \times 10^{-2}\ M_{\rm Pl}$

$\Lambda_{\rm QCD} \sim 218$ MeV

@ QCD era

$$\boxed{S_b^{\rm eff} = \int d^4x\sqrt{-g}\Big[\frac{1}{2}\partial_\mu b\, \partial^\mu b - \tilde{f}_b^{-1}\ \frac{\alpha_s}{8\pi}\, b(x)\, G^a_{\mu\nu}\, \tilde{G}^{a\mu\nu}\Big]}$$

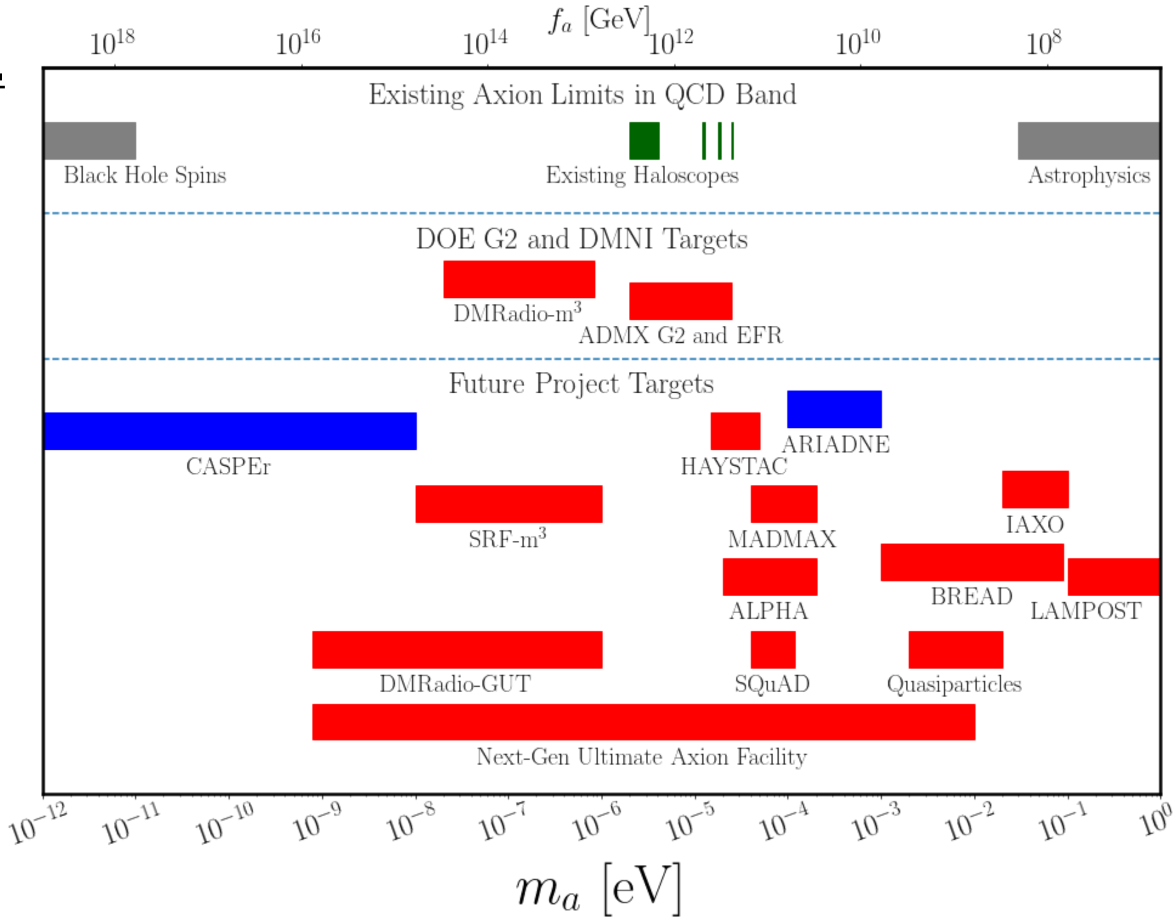
$$\boxed{m_b \simeq 1.15 \times 10^{-6}\,{\rm eV}}$$

Instanton-effects-induced KR-axion potential and mass due to QCD chiral anomaly

Matter Era

Possible potential generation for b → axion Dark matter

Axion Dark Matter - **Adams, C.B.** ***et al*** **- arXiv:2203.14923**



$$V_b^{\rm QCD} \simeq \Lambda^4_{\rm QCD}\Big(1 - \cos$$

$$\Lambda_{\rm QCD} \sim 218 \ {\rm MeV}$$

forward direction

@ QCD era

Compatible with QCD-Axion Phenomenology

$$m_b \simeq 1.15 \times 10^{-6}\,{\rm eV}$$

Instanton-effects-induced KR-axion potential and mass due to QCD chiral anomaly

Matter Era

Possible potential generation for b → axion Dark matter

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

forward direction

Basilakos, NEM, Solà

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$= 1.7 \times 10^{-2}$ $M_{\rm Pl}$

$$\Lambda_{\rm QCD} \sim 218 \ {\rm MeV}$$

@ QCD era

Compatible with
QCD-Axion
Phenomenology

But in string theory the scale Λ in the instanton potential V_b might be different

$$m_b \simeq 1.15 \times 10^{-6}\,{\rm eV}$$

Instanton-effects-induced KR-axion potential and mass due to QCD chiral anomaly

Matter Era

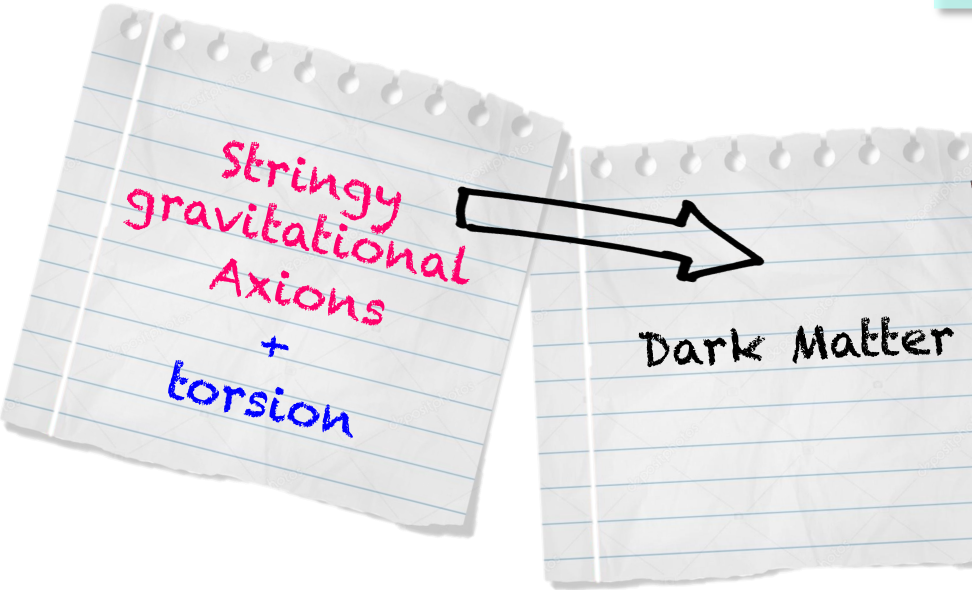
Possible potential generation for b → axion Dark matter

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

Basilakos, NEM, Solà

forward direction



KR (gravitational or model-independent) axions connected to "torsion" in string theory
→ Geometrical origin of Dark Matter

Matter Era

Possible potential generation for b → axion Dark matter

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

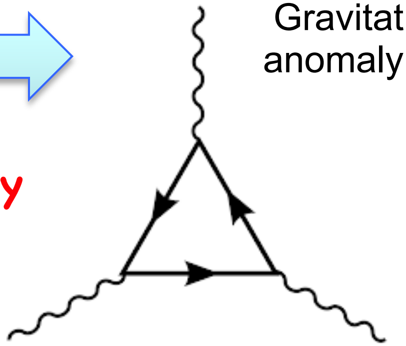
Big-Bang, pre-inflationary phase

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



Gravitational anomaly (GA)

Undiluted constant KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$$

$$\dot{\overline{b}} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

chiral matter generation

From a pre-inflationary era after Big-Bang

Radiation Era

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

Cancellation

OUTLOOK: Incorporate other model-dependent stringy axions → Axiverse Interesting Cosmology (*eg* Marsh 2015) could be ultralight → AION etc

B-L conserving sphelaron processes → Baryongenesis

Matter Era

Possible potential generation for b → axion Dark matter

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

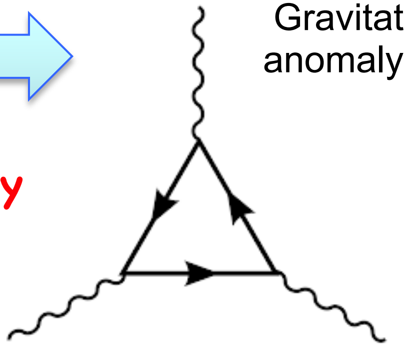
Big-Bang, pre-inflationary phase

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves →



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chiral matter generation @ inflation exit

From a pre-inflationary era after Big-Bang

Cancellation of GA

Radiation Era

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Matter Era

Possible potential generation for b → axion Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$ **Phenomenology**

Summary of (stringy-RVM) Cosmological Evolution

Basilakos, NEM, Solà

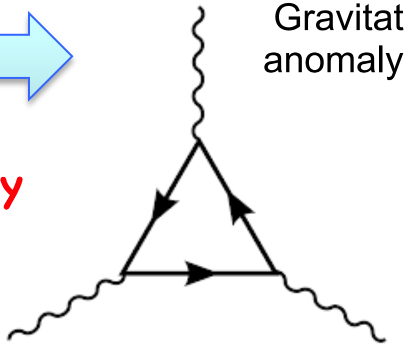
Cosmic Time

forward direction

Big-Bang, pre-inflationary phase

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



Gravitational anomaly (GA)

From a pre-inflationary era after Big-Bang

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$$\dot{\overline{b}} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

chiral matter generation @ inflation exit

Radiation Era

$$B_0 \propto \boxed{\dot{\overline{b}} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U(1) anomaly terms}}$$

$$B_0\Big|_{\rm today} \sim 2.435 \times 10^{-34}\,{\rm eV}$$

Consistent with current bounds on LV & CPTV
$B_0 < 10^{-2}$ eV,
$B_i < 10^{-22}$ eV

Matter Era

Possible potential generation for b – → axion Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$

Phenomenology

$H_0 \sim 10^{-42}$ GeV
$\approx 10^{-60}\,M_{\rm Pl} \approx 10^{-33}$ eV

Summary of (stringy-RVM) Cosmological Evolution

Basilakos, NEM, Solà

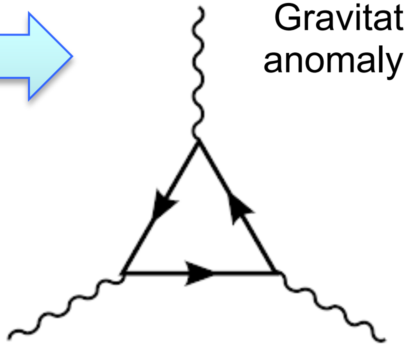
Cosmic Time

forward direction

Big-Bang, pre-inflationary phase

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



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chiral matter generation @ inflation exit

Cancellation of GA

Radiation Era

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B-L conserving sphel

Consistent with current bounds on LV & CPTV
$B_0 < 10^{-2}$ eV,
$B_i < 10^{-22}$ eV

Need to understand Modern Era better

Matter Era

F

Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$

$H_0 \sim 10^{-42}$ GeV
$\approx 10^{-60}\,M_{\rm Pl} \approx 10^{-33}$ eV

Phenomenology

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

Big-Bang, pre-inflationary phase

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Undiluted constant KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\bar{b}}\,\delta_{\mu 0}$$

$$\dot{\bar{b}} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

Primordial
Gr
Wa

Gravitational

Distinguishing feature from ΛCDM
Alleviate cosmological data tensions

$$\boxed{\rho^{\rm today}_{\rm RVM}(H) = 3M_{\rm Pl}^4\left(c_0 + \nu\left(\frac{H_0}{M_{\rm Pl}}\right)^2\right)}$$

$$\nu = \mathcal{O}(10^{-3})$$

$$\frac{3}{\kappa^2}\,c_0 \simeq 10^{-122}\,M_{\rm Pl}^4$$

Gómez-Valent
Solà

chiral matter
generation
lation exit

Ra

B_0

Le
RH

N_I

B-L

Ma

ion matter

$\varepsilon' M_{\rm Pl} H_0$

$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$

Modern de-Sitter Era

GA resurfacing

J. Solà
talk

RVM-type
Running Dark Energy

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

Big-Bang, pre-inflationary phase

Basilakos, NEM, Solà

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Gómez-Valent
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NB: Stringy CS RVM can lead to
H² Log H corrections to gravity
that can alsoalleviate the
data tensions

Gómez-Valent, NEM, Solà
CQG 41 (2024) 1, 015026

H

forward direction

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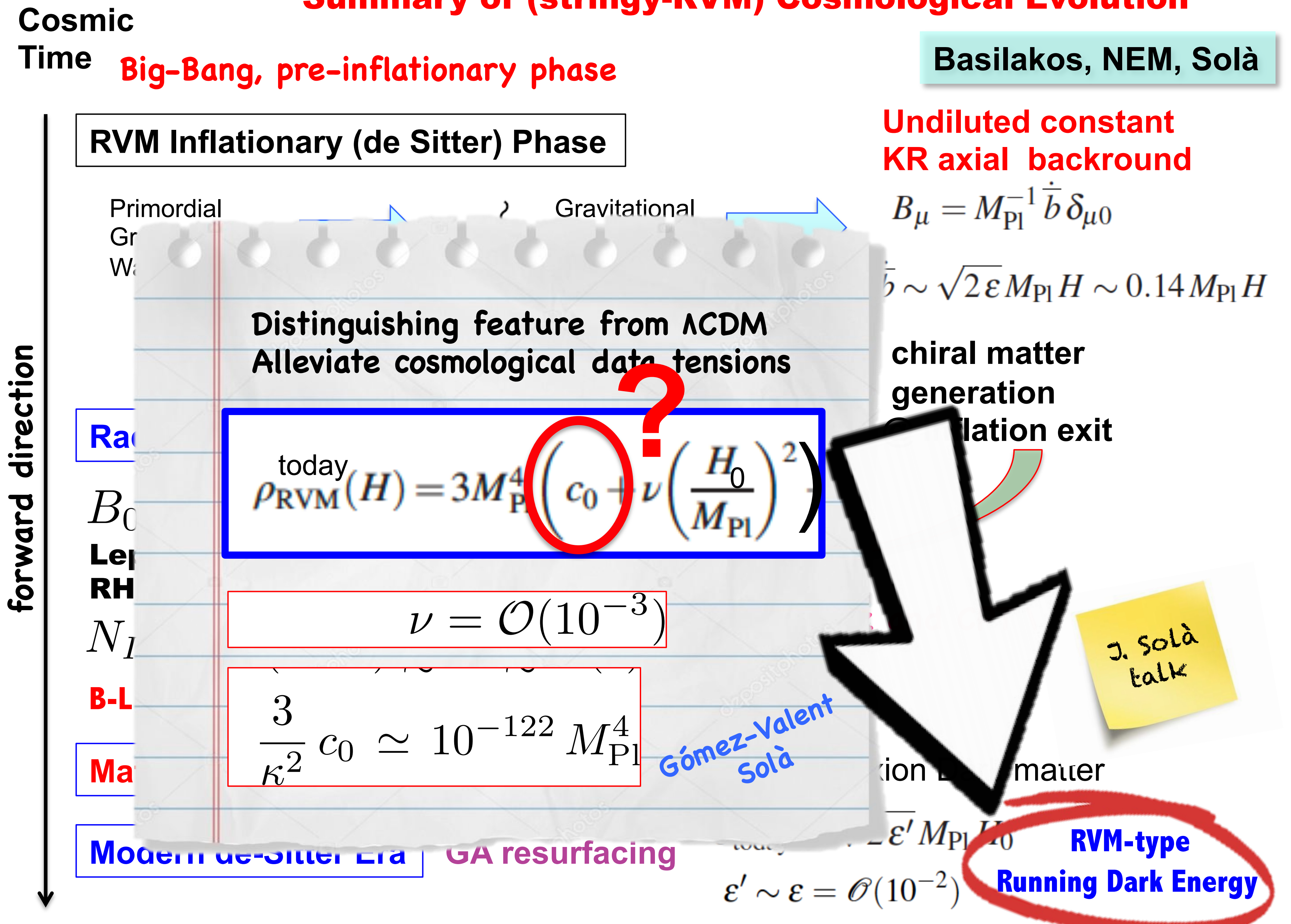

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Gómez-Valent
Solà

Currently observed
Cosmic acceleration?

J. Solà
talk

A conjecture ?

NEM & Sarben Sarkar,
Phys.Rev.D 110 (2024), 045004
e-Print: 2402.14513 [hep-th]

Instead of Cosmological Constant or Dark Energy dominance → a repulsive PT symmetric (non Hermitian) Infrared (IR) Phase of Chern Simons gravity

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Hermitian theory:
Axion gauge sector → flow from Ultraviolet (UV) to IR,
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Eichhorn, Gies, Rocher (2012)

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$$\mathcal{L}_E = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{2} \partial_\mu a \partial_\mu a + \frac{1}{2} m^2 a^2 + \frac{1}{4} i g a F_{\mu\nu} \tilde{F}_{\mu\nu}$$

axion

U(1) field

Axion QED in flat spacetime

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Axion QED in flat spacetime

$$\mathfrak{g} = \sqrt{\frac{2}{3}\frac{\alpha'}{24\kappa}}$$

Embed in stringy Chern-Simons gravity

A conjecture ?

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Romatschke, Kamata, Grable, Weiner

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$$\mathfrak{g} = \sqrt{\frac{2}{3}\frac{\alpha'}{24\kappa}} \longrightarrow i\,\mathfrak{g} \longrightarrow \kappa \to i\tilde{\kappa}, \qquad \tilde{\kappa} \in \mathbb{R}$$

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⇓ Phase transition?

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See: S.Sarkar's talk

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Phase transition?

→ Repulsive gravity (``change of sign of $G_N = \kappa^2$ ")

→ Accelerated expansion , no need for
Cosmological constant c_0
(but …. RVM behaviour might co-exist)

Eichhorn, Gies, Rocher (2012)

Inspired by Ai, Bender, Sarkar (2022) ?

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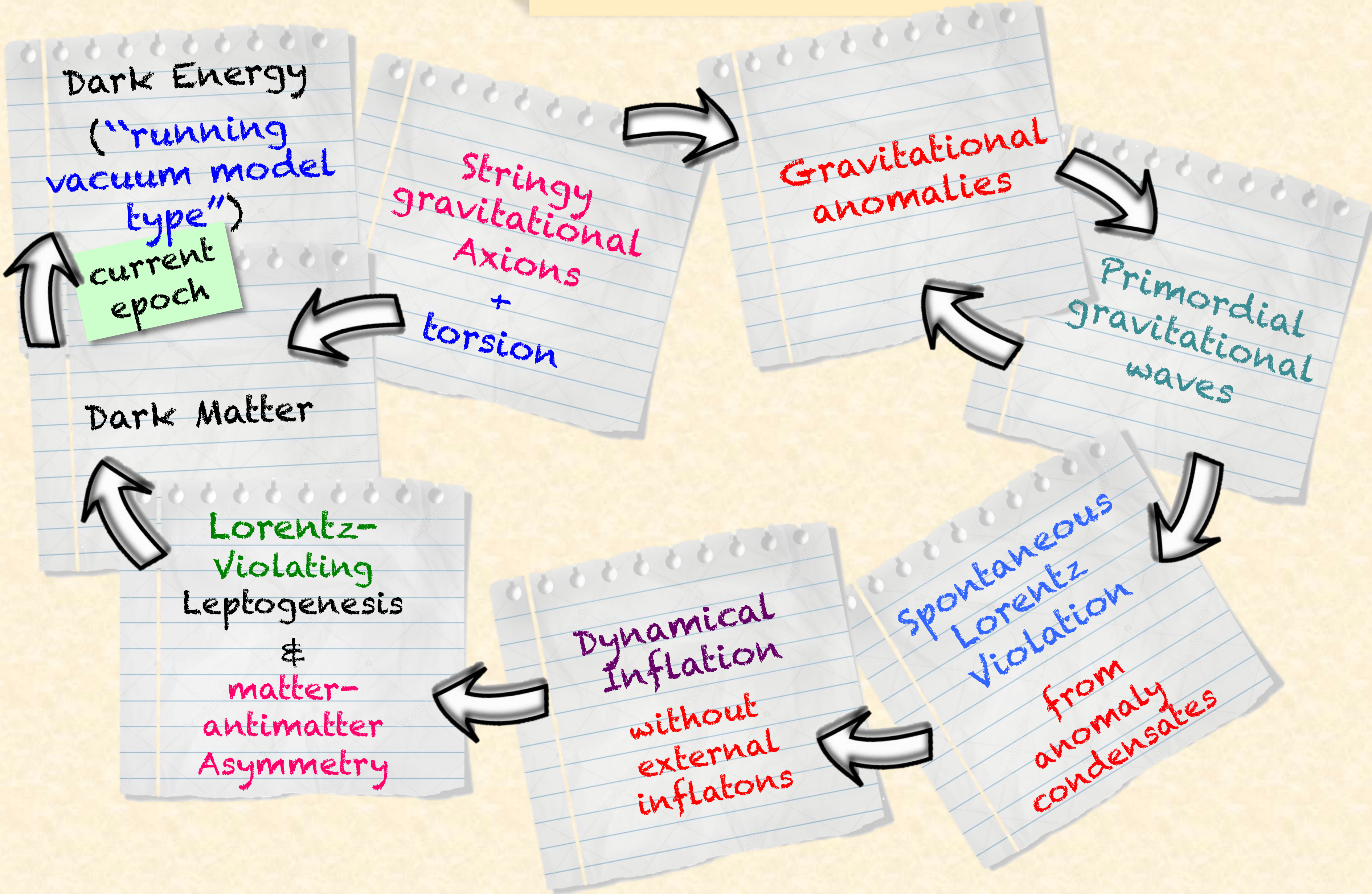
The Cosmic Evolution



Stringy gravitational Axions + torsion

Gravitational anomalies

Primordial gravitational waves

Spontaneous Lorentz Violation from anomaly condensates

Dynamical Inflation without external inflatons

Lorentz-Violating Leptogenesis & matter-antimatter Asymmetry

Dark Matter

Dark Energy ("running vacuum model type")

current epoch

The Cosmic Evolution

Or, **instead,** a PT symmetric IR Phase of (repulsive) gravity?

Stringy gravitational Axions + torsion

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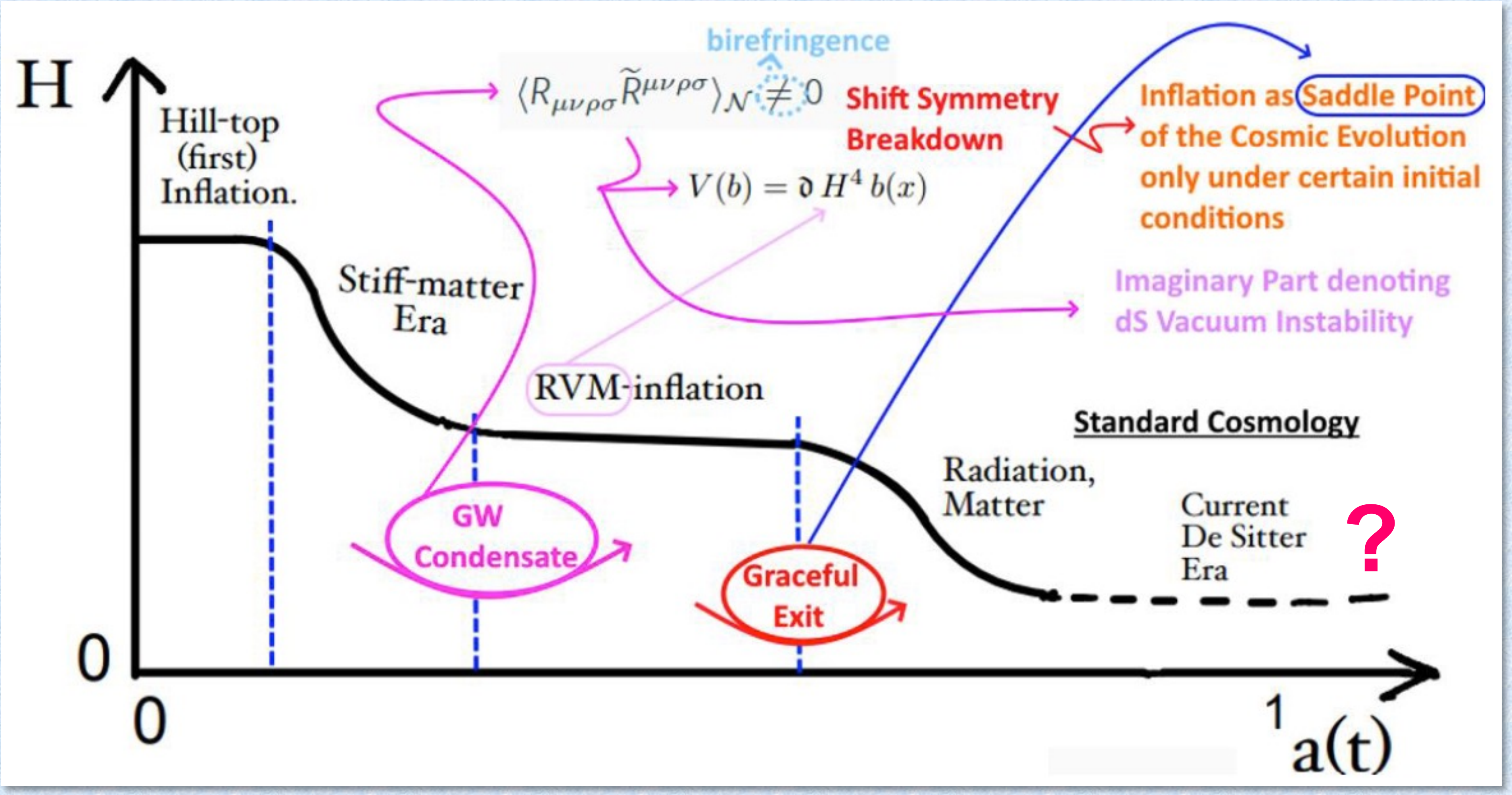
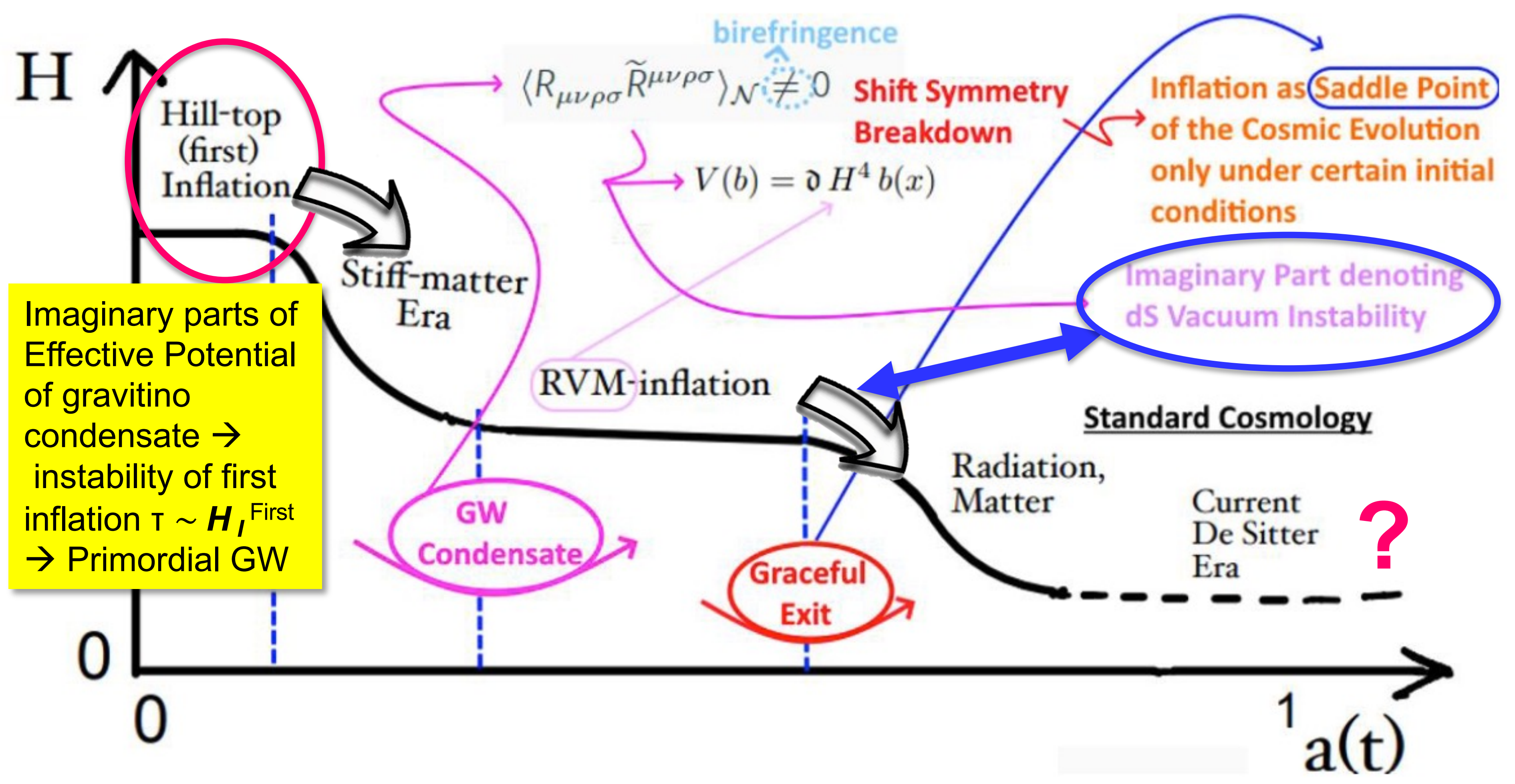
Synopsis: The Evolution of the Stringy RVM



NEM, Solà, Dorlis, Vlachos

Synopsis: The Evolution of the Stringy RVM



Imaginary parts of Effective Potential of gravitino condensate → instability of first inflation τ ~ H_I^{First} → Primordial GW

NEM, Solà, Dorlis, Vlachos

Cosmological (stringy RVM) Evolution: the Whole & its Parts

Cosmic Time

forward direction

RVM Inflationary (de Sitter) Phase

Undiluted constant KR axial backround

Primordial Gravitational Waves

Gravitational anomaly (GA)

Radiation Era

Leptogenesis induced by RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

Matter Era

axion Dark matter

Modern de-Sitter Era

RVM-type Running Dark Energy

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We exist because of Anomalies !

Paraphrasing C. Sagan: We are "anomalously" made of star stuff

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Or: a PT symmetric IR
Phase of repulsive gravity ?

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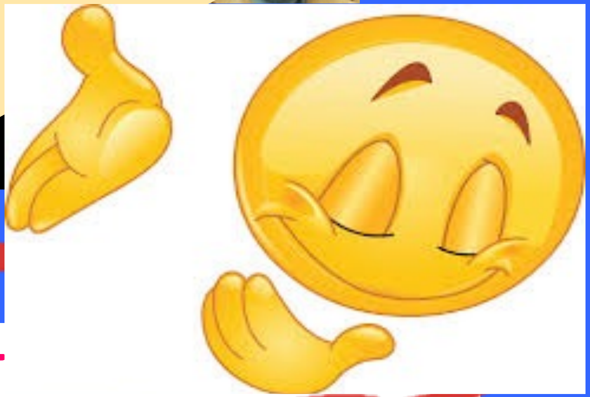
Thank you !



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(vi) Eur. Phys.J.ST 230 (2021),2077

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[c] Gómez-Valent, NEM, Solà

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Links with : spontaneous Lorentz violation (via (gravitational axion) backgrounds) and Matter-Antimatter Asymmetry in theories with Right-Handed Neutrinos, PT symmetric gravity phase

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