Towards the Asymptotically Safe Landscape

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Applications of Field Theory to Hermitian and Non-Hermitian Systems

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UNIVERSITY OF SUSSEX

Path integral for metric quantum gravity

- Assumptions
  - Metric carries fundamental degrees of freedom
  - Diffeomorphism invariance
- Path integral

$$\int \mathcal{D}\hat{g}_{\mu\nu}\, e^{-S[\hat{g}]} \qquad \text{or} \qquad \int \mathcal{D}\hat{g}_{\mu\nu}\, e^{\mathrm{i}S[\hat{g}]}$$

Path integral for metric quantum gravity

- Assumptions
  - Metric carries fundamental degrees of freedom
  - Diffeomorphism invariance
- Path integral with gauge fixing, sources

$$Z[\bar{g}, J] \sim \int \mathcal{D}\hat{h}_{\mu\nu}\, e^{-S[\bar{g}+\hat{h}]-S_{\text{gf}}[\bar{g},\hat{h}]-S_{\text{gh}}[\bar{g},\hat{h},\hat{c},\hat{\bar{c}}]+\int_x \sqrt{\bar{g}}\, J^{\mu\nu}(x)\hat{h}_{\mu\nu}(x)}$$

- Metric split $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ required by gauge fixing and regulator
- Methods: Perturbation theory, lattice, functional methods, ...

Perturbative quantum gravity

Einstein-Hilbert action

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int_x \sqrt{\det\, g_{\mu\nu}}\,(2\Lambda - R(g_{\mu\nu}))$$

- Perturbatively non-renormalisable: $[G_{\mathrm{N}}] = -2$
- Need infinitely many counter terms: No predictivity ['t Hooft, Veltmann '74; Goroff, Sagnotti '85]

Higher-derivative action

$$S_{\mathrm{HD}} = S_{\mathrm{EH}} + \int_x \sqrt{\det\, g_{\mu\nu}} \left(\frac{1}{2\lambda}\, C^2_{\mu\nu\rho\sigma} - \frac{\omega}{3\lambda}\, R^2 \right)$$

- Perturbatively renormalisable: $[\omega] = [\lambda] = 0$
- Non-unitary propagator [Stelle '74]

$$G_{\mathrm{graviton}} \sim \frac{1}{p^2 + p^4/M_{\mathrm{Pl}}^2} = \frac{1}{p^2} - \frac{1}{M_{\mathrm{Pl}}^2 + p^2}$$

Non-perturbative tool: The Functional Renormalisation Group

Non-perturbative renormalisation group equation [Wetterich '93, Morris '94, Ellwanger '94, ...]

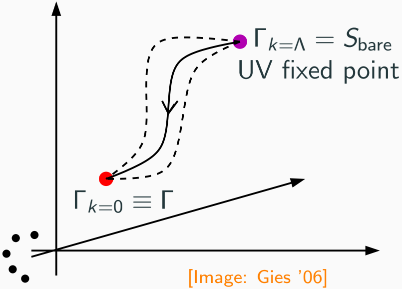
$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)} + R_k} k\partial_k R_k\right]$$

R_k = regulator function

Γ_k = scale-dependent effective action

Interpolation between

- bare action / UV FP
- quantum effective action Γ
- Wilsonian integrating out of momentum modes



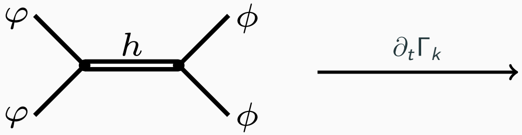
[Image: Gies '06]

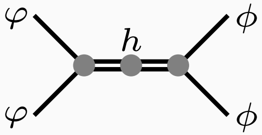
The Quantum Effective Action

- Legendre transform of the logarithm of the generating functional

$$\Gamma[\phi] = \sup_J \left\{ \int_x J(x)\,\phi(x) - \ln Z[J] \right\}$$

- Generates 1PI correlation functions and full quantum physics at tree level





Vertices from classical action S

No quantum effects

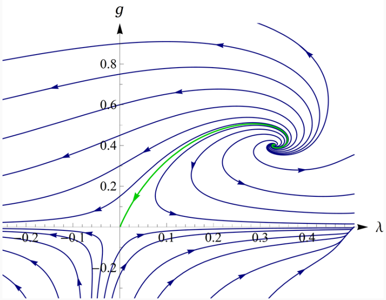
Vertices from quantum effective action Γ

Includes all quantum effects

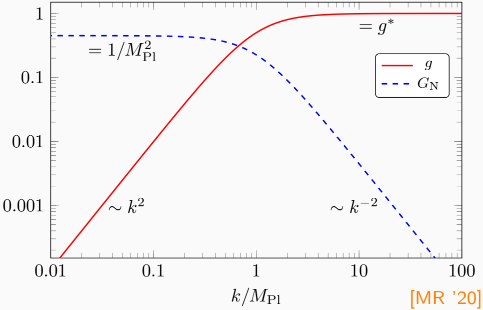
Asymptotically safe quantum gravity

QG could be non-perturbatively renormalisable via an interacting UV FP [Weinberg '76]

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int_x \sqrt{g}\,(2\Lambda - R)$$



[Reuter '96; Reuter, Saueressig '01]



[MR '20]

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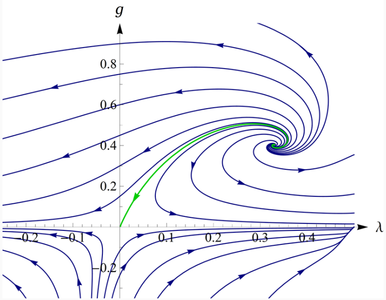

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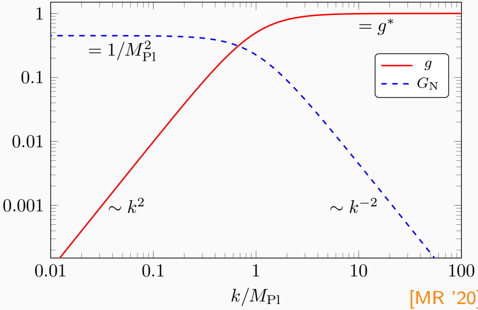


[MR '20]

Predictivity: number of free parameters = dimension of UV critical hypersurface
[Denz, Pawlowski, MR '16; Falls, Ohta, Percacci '20; Kluth, Litim '20; Knorr '21; . . .]

Unitarity: Positivity and finiteness of spectral functions and scattering amplitudes
[Bonanno, Denz, Pawlowski, MR '21; Fehre, Litim, Pawlowski, MR '21; . . .]

Expansion in curvature invariants

[Gies, Knorr, Lippoldt, Saueressig '16; Baldazzi, Falls, Kluth, Knorr '23]
[Falls, Litim, Schröder '18; Kluth, Litim '20; Morris, Stulga '22; . . .]
[Falls, Ohta, Percacci '20; Knorr '21, MR *(in prep)*]
[Denz, Pawlowski, MR '16; Knorr, Ripken, Saueressig '19; Bonanno, Denz, Pawlowski, MR '21; . . .]
[Knorr, Schiffer '21; Assant, MR *(in prep)*]

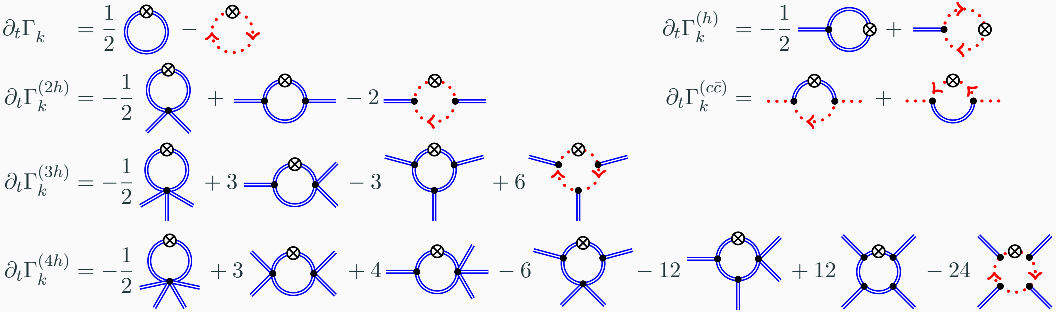
Λ				
R				
R^2	$R_{\mu\nu}R^{\mu\nu}$	$C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$		
R^3	$R\nabla^2 R$	$C_{\mu\nu\rho\sigma}\nabla^2 C^{\mu\nu\rho\sigma}$	${C_{\mu\nu}}^{\kappa\lambda}{C_{\kappa\lambda}}^{\rho\sigma}{C_{\rho\sigma}}^{\mu\nu}$	$\dots$
R^4	$R\nabla^4 R$	$C_{\mu\nu\rho\sigma}\nabla^4 C^{\mu\nu\rho\sigma}$	$\dots$	
$\vdots$		$\vdots$		

Fluctuation approach – expansion in scattering vertices

- Treat $\bar{g}$ and h independently, resolve fluctuation correlation functions

$$\Gamma_k[\bar{g}, h] = \sum_{n=0}^{\infty} \frac{1}{n!} \int \Gamma_k^{(0,h_{a_1}\dots h_{a_n})}[\bar{g}, 0] \cdot h_{a_1} \dots h_{a_n}$$

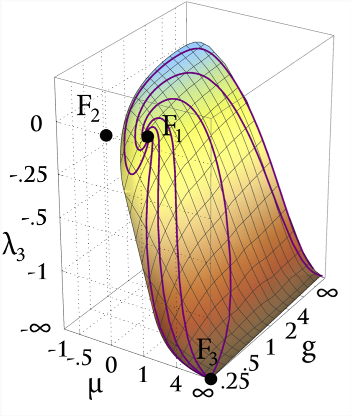
- Flat background $\bar{g} = \delta$ allows for momentum-space techniques



[Christiansen, Knorr, Meibohm, Pawlowski, MR '15; Denz, Pawlowski, MR '16; Pawlowski, MR '20; '23; . . .]

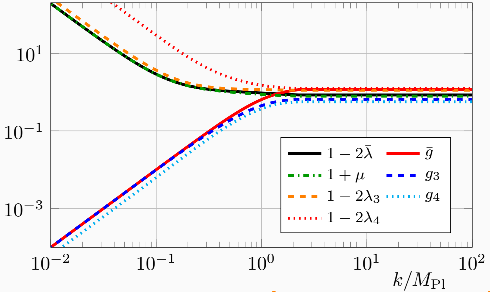
Number of relevant directions

Phase diagram



[Christiansen, Knorr, Meibohm, Pawlowski, MR '15]

UV-IR trajectories

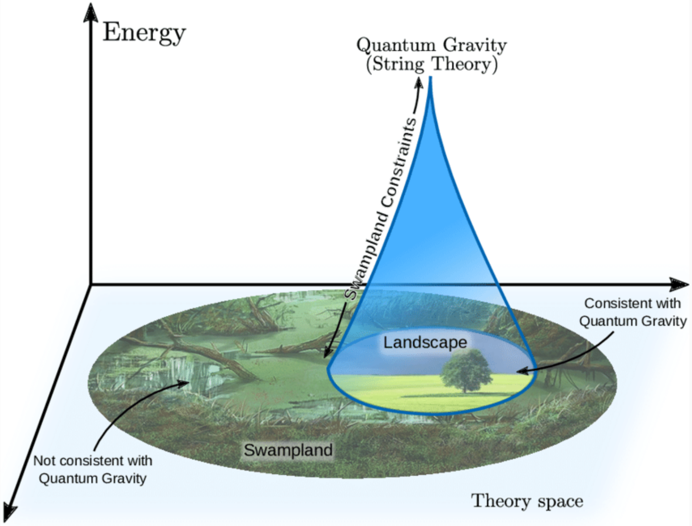


[Denz, Pawlowski, MR '16]

- Three relevant directions: Λ, R, R^2
- Irrelevant direction, e.g., $C^2_{\mu\nu\rho\sigma}$

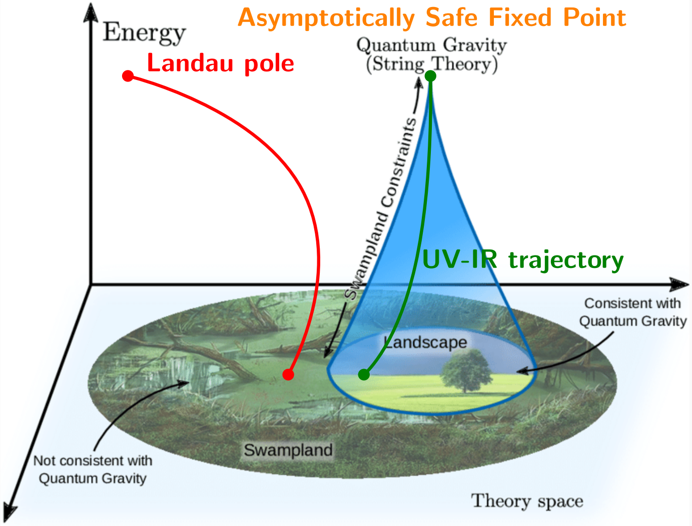
Gravity-matter systems

The Asymptotically Safe Landscape



[van Beest, José Calderón-Infante, Mirfendereski, Valenzuela '20]

The Asymptotically Safe Landscape



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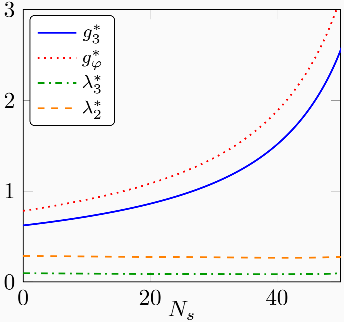
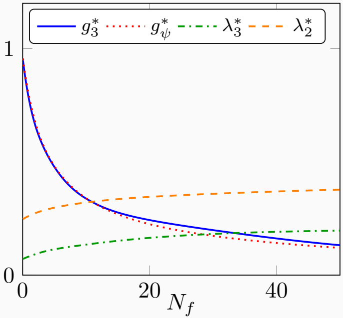
UV fixed point with matter

Scalars



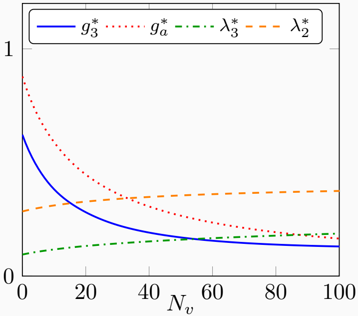
[Eichhorn, Labus, Pawlowski, MR '18]

Fermions



[Eichhorn, Lippoldt, Schiffer '18]

Gauge bosons

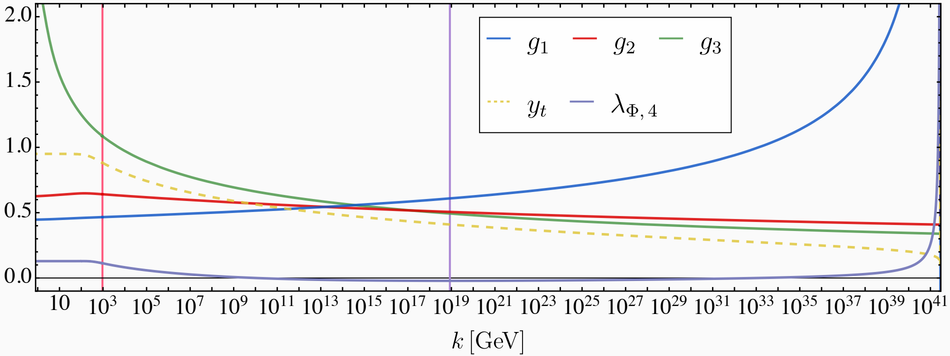


[Christiansen, Litim, Pawlowski, MR '17]

- Fermions and gauge bosons stabilise, scalars destabilise
- SM matter safely included

Standard Model without Gravity

Does gravity help with Landau poles in the matter sector?



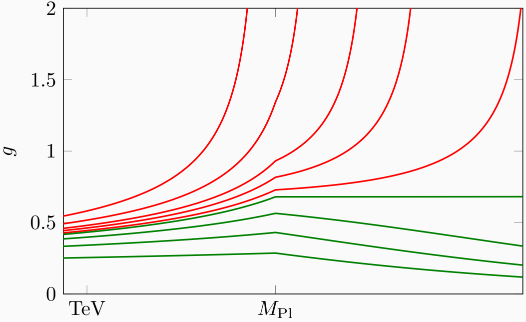
Avoiding Landau poles in the U(1) sector

Simple parameterisation of $U(1)$ beta function

$$\beta_{g_1} = \beta_{g_1,\text{matter}} - f_g g_1$$

[Eichhorn, Versteegen '17]

$$\text{with } f_g \propto g_N \approx \begin{cases} \text{const} & k > M_{\text{pl}} \\ 0 & k < M_{\text{pl}} \end{cases}$$



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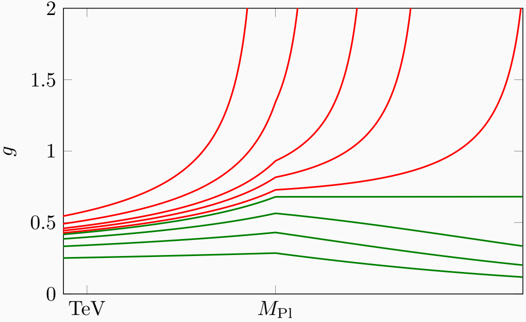
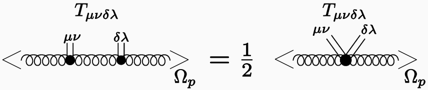


- f_g is gauge and regulator dependent
- Kinematic identity

[Folkerts, Litim, Pawlowski '11]



guarantees $f_g \geq 0$

[Christiansen, Litim, Pawlowski, MR '17, Pastor-Gutiérrez, Pawlowski, MR '22, . . .]

- Gauge coupling is relevant

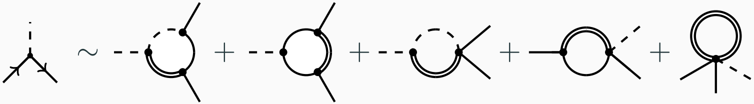
$$\theta_g = -\partial_g \beta_g \big|_{g=0} = f_g \geq 0$$

Relevance of the Yukawa coupling

[de Brito, MR, Schiffer (in prep)]

- Leading order: $y\sqrt{|g|}\phi\bar{\psi}\psi$ + wave function renormalisations

  Contains strong regulator dependence [Eichhorn, Held '17; Pastor-Gutiérrez, Pawlowski, MR '22]

$k\partial_k$ 

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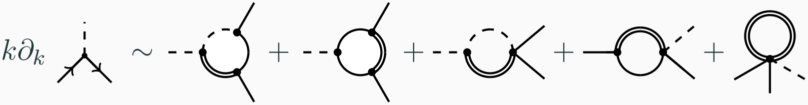

- NLO (dim-6 operators): $y_R\sqrt{|g|}R\phi\bar{\psi}\psi + \ldots$

- NNLO (dim-8 operators): $y_{R^2}\sqrt{|g|}R^2\phi\bar{\psi}\psi + y_{C^2}\sqrt{|g|}C^2_{\mu\nu\rho\sigma}\phi\bar{\psi}\psi + \ldots$

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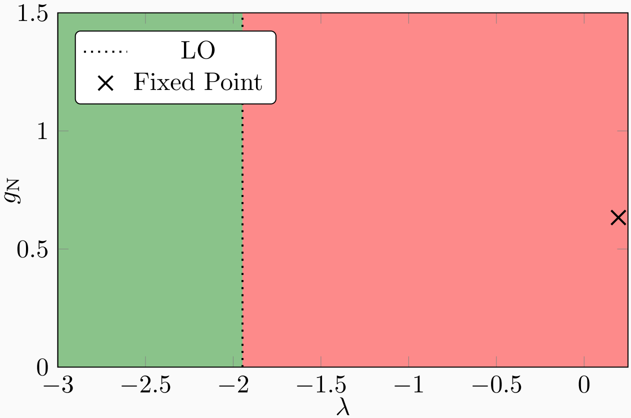

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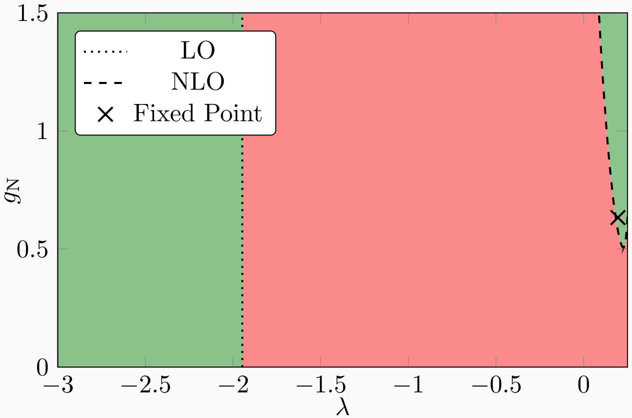
Green region: Yukawa relevant → finite Yukawa couplings in IR

Red region: Yukawa irrelevant → vanishing Yukawa couplings in IR

Relevance of the Yukawa coupling



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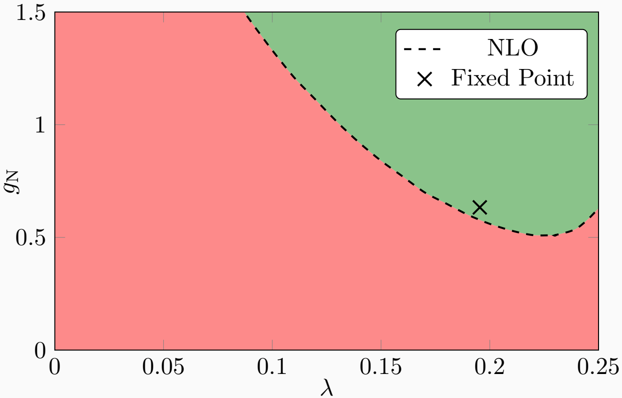

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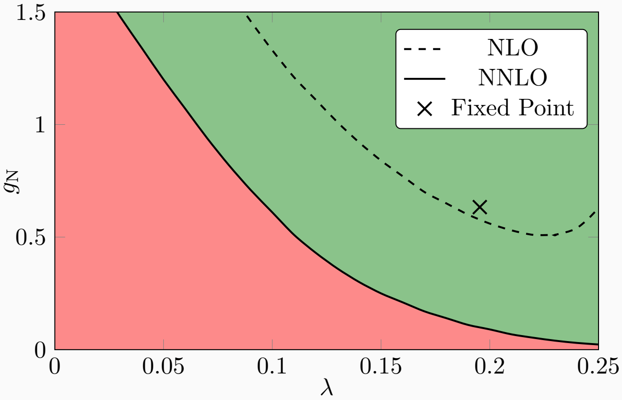

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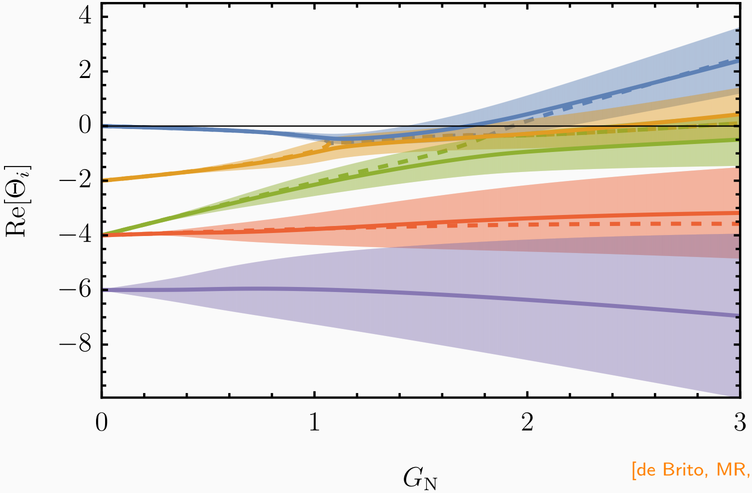

[de Brito, MR, Schiffer *(in prep)*]

For $\Lambda = 0$, the Yukawa coupling becomes relevant at $G_\mathrm{N} \sim 1.4 - 2.1$

Relevance of the quartic scalar coupling

- The quartic scalar coupling is irrelevant…
  [Eichhorn, Hamada, Lumma, Yamada '17; Pawlowski, MR, Wetterich, Yamada '18, …]
- …but generated below the Planck scale

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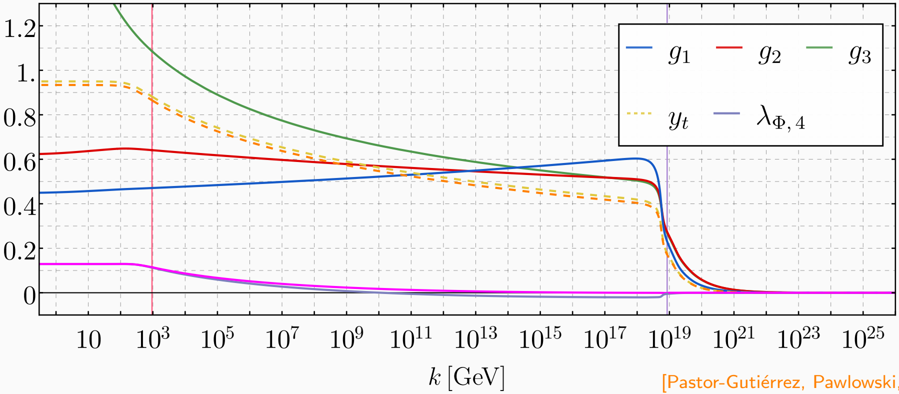
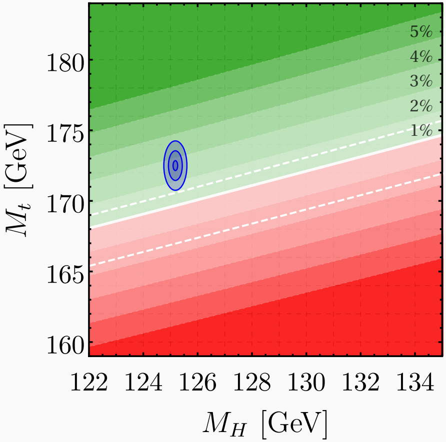


[Pastor-Gutiérrez, Pawlowski, MR '22]

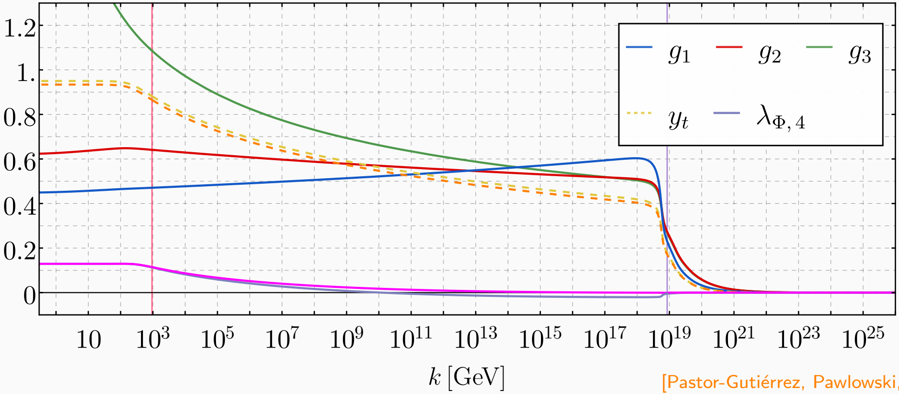
Higgs vs top mass in the asymptotically safe Standard Model



[Pastor-Gutiérrez, Pawlowski, MR '22]

- Predicted Higgs mass of 125 GeV
  [Shaposhnikov, Wetterich '12]
- Small mismatch between predicted and measured Higgs-top mass ratio in pure SM
- Can be fixed with BSM physics, e.g., dark matter [Reichert, Smirnov '19]

Standard Model with Gravity – Summary



[Pastor-Gutiérrez, Pawlowski, MR '22]

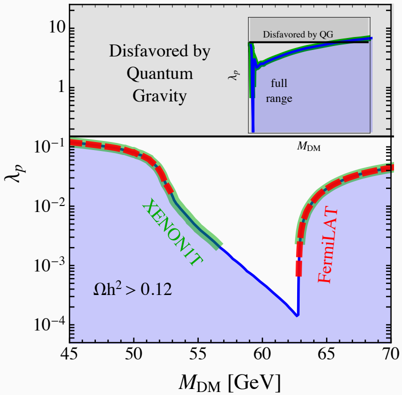
Standard Model *just* outside of AS Landscape?

Dark Matter meets Quantum Gravity

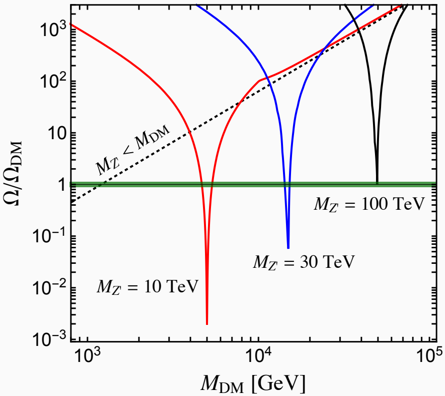
[MR, Smirnov '19]

$$\mathcal{L}_\text{D} \sim \text{Higgs-portal-scalar} + \text{dark-fermion} + \text{dark-}U(1)_X\text{-with-dark-photon}$$

Scalar dark matter

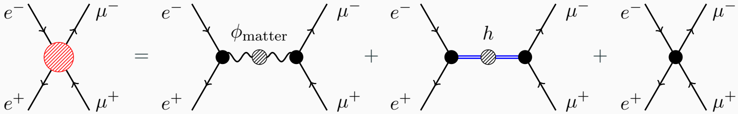


Fermionic dark matter



Scattering amplitudes

Towards scattering amplitudes

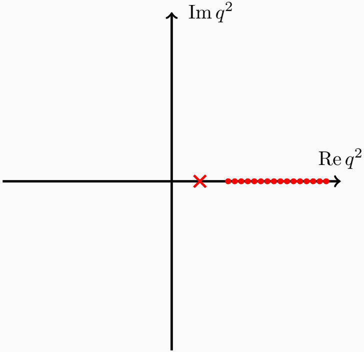


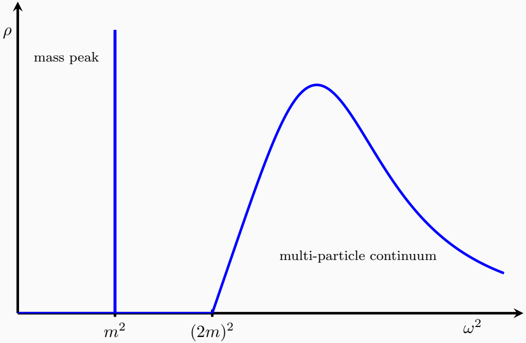
- Need well-behaved propagators without ghost or tachyonic instabilities
- Test bounds on scattering amplitudes, e.g., violated by GR
- Need access to correlation functions on Lorentzian signature at time-like momenta

Källén-Lehmann spectral representation

[Källén '52; Lehmann '54]

$$G(q^2) = \int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi} \frac{\rho(\lambda^2)}{q^2 - \lambda^2} \qquad \text{with} \qquad \rho(\omega^2) = -\lim_{\varepsilon \to 0} \operatorname{Im} G(\omega^2 + i\varepsilon)$$





Spectral Renormalisation Group

[Fehre, Litim, Pawlowski, MR '21; fQCD, MR '22]

- Callan-Symanzik cutoff uniquely preserves causality and Lorentz invariance

$$R_k = Z_\phi \, k^2$$

- Finite flow equation with counterterms

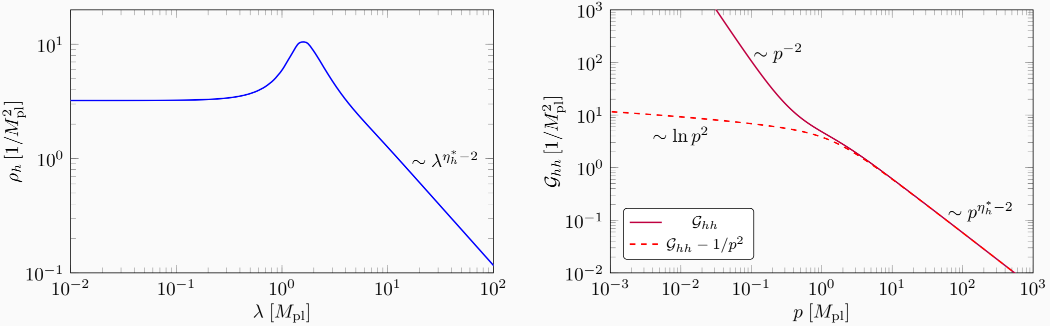
$$\boxed{\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \, \mathcal{G}_k \, \partial_t R_k - \partial_t S_{\mathrm{ct},k}}$$

- Dimensional regularisation of UV divergences in $d = 4 - \varepsilon$ possible

- Use ρ_h in flow diagrams

$\partial_t \rho_h \propto$  $+ \dots$ with $\mathcal{G}_h(q^2) = \int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi} \frac{\rho_h(\lambda^2)}{q^2 - \lambda^2}$

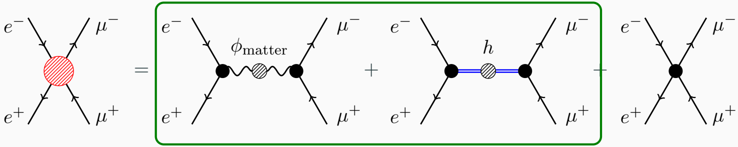
Graviton spectral function

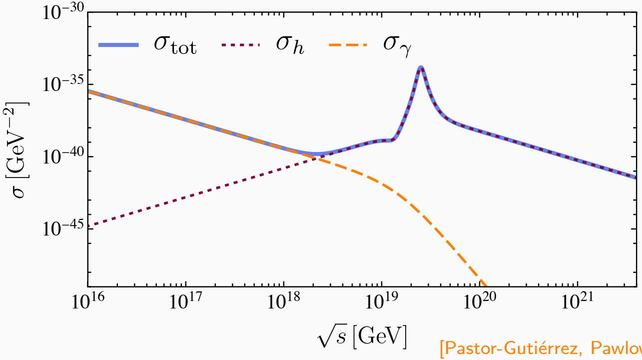


[Fehre, Litim, Pawlowski, MR '21]

- Massless graviton delta-peak with positive multi-graviton continuum
- No ghosts and no tachyons $\longrightarrow$ no indications for unitarity violation
- Good agreement with reconstruction results and EFT [Bonanno, Denz, Pawlowski, MR '21]

Towards graviton-mediated scattering cross-sections





[Pastor-Gutiérrez, Pawlowski, MR, Ruisi *(in prep)*]

Summary

- Asymptotically safety is a strong contender for the fundamental theory of quantum gravity with few fundamental parameters
- Straightforward inclusion of matter degrees of freedom
- Standard Model *just* outside of AS Landscape?
- Predictive theory that can constrain BSM physics
- Direct Lorentzian computation of graviton spectral function with spectral fRG
- Key step towards scattering processes and unitarity

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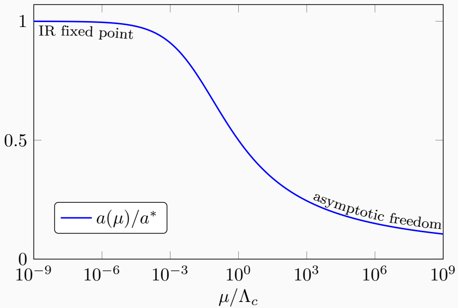
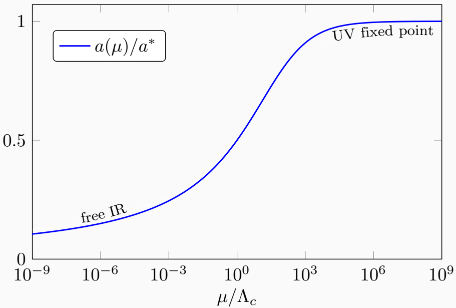
Thank you for your attention!

Back-up slides

Asymptotic freedom vs safety & fixed points





- Banks-Zaks
- Wilson-Fisher in $d = 4 - \varepsilon$

- Litim-Sannino (gauge-Yukawa)
- Quantum Gravity in $d = 2 + \varepsilon$

Asymptotic safety is a natural generalisation of asymptotic freedom

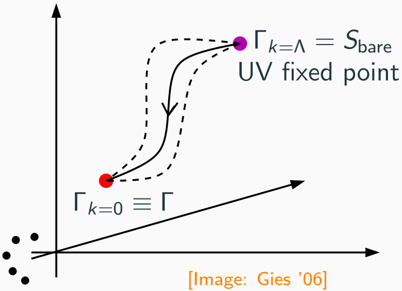
The scale-dependent Quantum Effective Action

- Expand effective action in all operators compatible with symmetry

$$\Gamma_k = \sum_i g_i(k)\, \mathcal{O}_i$$

- Flow equation defines a vector field in space of operators

$$k\partial_k \Gamma_k = \sum_i k\partial_k g_i(k)\, \mathcal{O}_i = \sum_i \beta_{g_i}\, \mathcal{O}_i$$



[Image: Gies '06]

Non-perturbative functional approach to path integral

- Path integral with suppressed IR modes

$$Z_k[J] \sim \int \mathcal{D}\varphi_{p^2 \geq k^2} \; e^{-S[\varphi] + \int_x J(x)\,\varphi(x)}$$

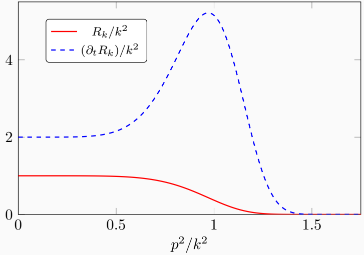
- Regulator function R_k to suppress of IR modes

$$\int \mathcal{D}\varphi_{p^2 \geq k^2} = \int \mathcal{D}\varphi \; e^{-\frac{1}{2}\int_p \phi R_k \phi}$$

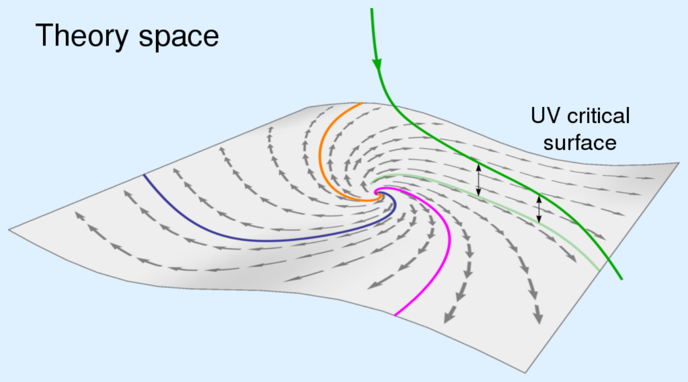
- Wilsonian integrating out of momentum shells

$$k\partial_k Z_k[J] = -\frac{1}{2}\int_p \frac{\delta^2 Z_k[J]}{\delta J(p)\delta J(-p)} k\partial_k R_k(p^2)$$

- Turns path integral into functional differential equation
  - For $k \to \infty$: no quantum effects / classical action
  - For $k \to 0$: full quantum theory



UV critical hypersurface



[Picture: Wikipedia]

- UV repulsive (irrelevant) direction *determines* parameter
- UV attractive (relevant) direction leaves parameter *free*

Predictivity

Linearized beta functions around fixed point

$$\beta_{g_i}(\vec{g}) = \underbrace{\beta_{g_i}(\vec{g}^{\,*})}_{=0} - \sum_j B_{ij}(\vec{g}^{\,*})(g_j - g_j^*) + \mathcal{O}\big((g_j - g_j^*)^2\big)$$

Stability matrix

$$B_{ij}(\vec{g}) = -\frac{\partial \beta_{g_i}(\vec{g})}{\partial g_j}$$

Critical exponents (eigenvalues of stability matrix): $\theta_j = d_{\mathcal{O}_j} +$ quantum

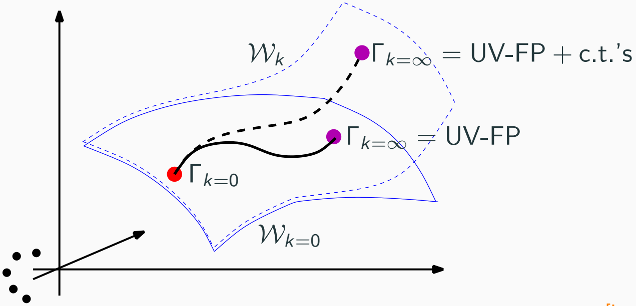
$$\begin{aligned} \Re(\theta_j) > 0 \quad &\longleftrightarrow \quad \text{UV attractive/IR repulsive} \\ \Re(\theta_j) < 0 \quad &\longleftrightarrow \quad \text{UV repulsive/IR attractive} \\ \Re(\theta_j) = 0 \quad &\longleftrightarrow \quad \text{marginal} \end{aligned}$$

At the Gaußian fixed point $\theta_j = d_{\mathcal{O}_j}$

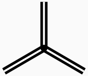
Controlling the diffeomorphism symmetry

- Background metric $\bar{g}_{\mu\nu}$ and fluctuation field $h_{\mu\nu}$ are treated independently
- Diffeomorphism symmetry is governed by non-trivial Ward identity

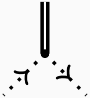
$$\mathcal{W}_k = \mathcal{G}\Gamma_k + \mathcal{G}S_{\text{regulator}} - \langle \mathcal{G}(S_{\text{gf}} + S_{\text{gh}} + \Delta S_{\text{regulator}}) \rangle = 0$$



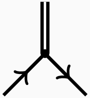
[Image: Gies '06]



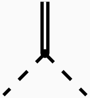
$\longrightarrow G_3(p_1, p_2, p_3)$



$\longrightarrow G_c(p_1, p_2, p_3)$



$\longrightarrow G_\psi(p_1, p_2, p_3)$



$\longrightarrow G_\varphi(p_1, p_2, p_3)$

$\ldots$

- Momentum dependent couplings
- Related by symmetry identities
- Reduce to G_N + higher-order terms for $k \to 0$