Quantum Field Theories in the Early Universe

1. Well-behaved QFTs in Minkowski space can develop pathologies when promoted to FRW.
2. This is especially acute for “higher-spin” QFTs (1, $3/2$, 2, …).
3. And some funny business for spin-0.
4. Is there a swampland of Minkowskian QFTs?
5. Or should we just accept restrictions on parameters of the QFTs (mass, couplings, etc.).
6. I will not have time to review EFT cutoffs, strong-coupling linits, nonlinearities, etc.

More complete treatment in
Cosmological gravitational particle production
EWK and Andrew Long
Reviews of Modern Physics (to appear) 2312.09042

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Inner Space/Outer Space Interface

Cosmological limits on particle properties:

1. neutrinos
2. axions
3. magnetic monopoles
4. all sort of BSM particles (e.g., SUSY)
5. cosmological defects
6. Kaluza-Klein modes
7. …

Mostly assume LTE

I will consider limits from "cosmological gravitational particle production (CGPP)

Image credit: Chris Stabb

CGPP

Chung, EWK, Riotto (1998); Kuzmin & Tkachev (1999)

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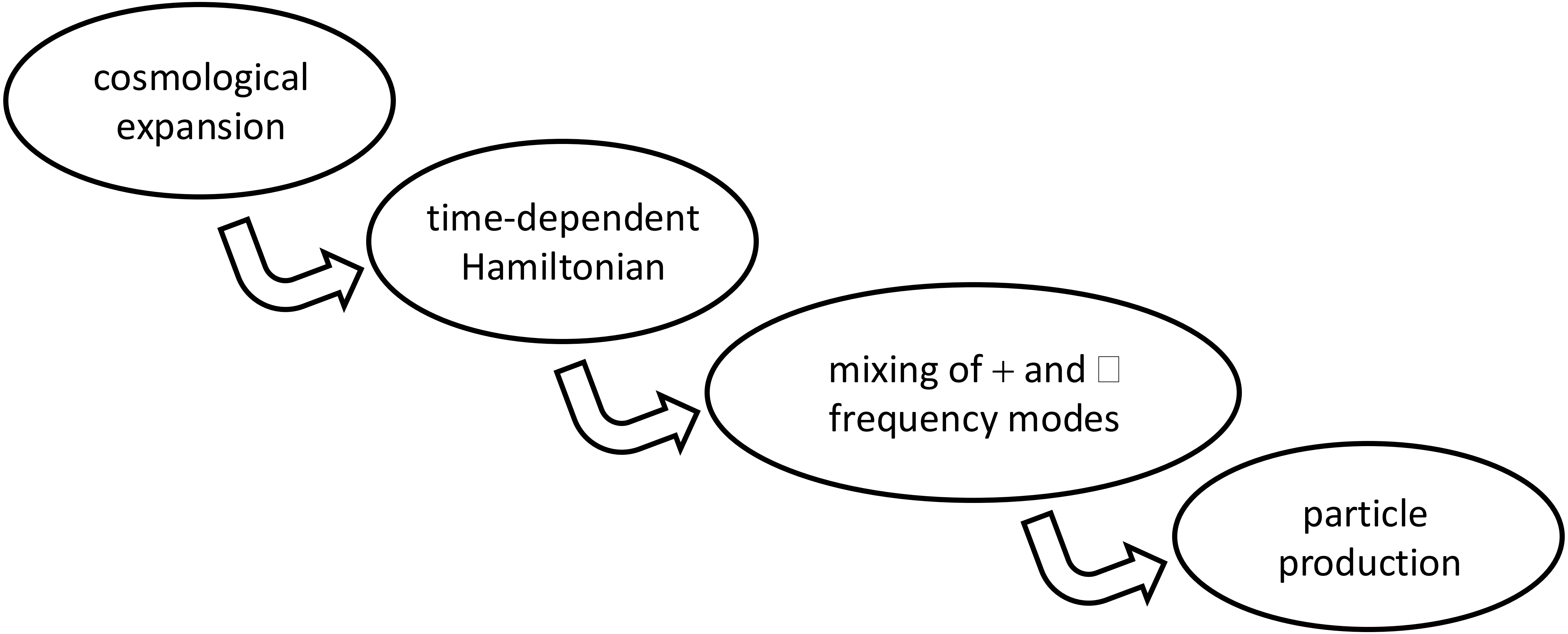
Cosmological Gravitational Particle Production (CGPP)

- In Minkowskian QFT, classify particle by IR of the Poincaré algebra.
- But, expanding universe not Poincaré invariant!
- Notion of a “particle” is approximate.

Schrodinger (1939); Parker (1965, 68); Fulling, Ford, & Hu; Zel’dovish; Starobinski; Grib, Frolov, Mamaev, & Mostepanenko; Mukhanov & Sasaki, Birrell & Davies…



Cosmological Gravitational Particle Production (CGPP)

- Expansion of the universe creates particles from the vacuum if $m \lesssim$ few $\times H$*.
- This is not optional—can't hide from gravity.
- "Semiclassical:" quantum spectator field in classical gravitational background.
- Calculation of GPP of massive fields is "straightforward."

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* And particle not conformally coupled (trace of stress tensor for matter field $g_{\mu\nu} T_M{}^{\mu\nu} \neq 0$).

Assume Standard Inflationary Picture

Quasi-de Sitter inflationary phase driven by vacuum energy of inflaton displaced from potential minimum, expansion rate changes (very) slowly during inflation (when $\ddot{a} > 0$); at the end of inflation, $H = H_e$.

Matter-dominated phase due to inflaton oscillations about minimum of potential.

Inflaton decays and leads to radiation-dominated phase characterized by a reheat temperature T_{RH} .

H_e and T_{RH} are unknown.

Scalar field in FRW background

covariant action

$$S[\varphi(x), g_{\mu\nu}(x)] = \int d^4x \sqrt{-g}\left[-\frac{1}{2}M_{\rm Pl}^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}m^2\varphi^2 + \frac{1}{2}\xi R\varphi^2\right]$$

Gravity enters the picture

in a spatially flat FRW background : $ds^2 = a^2(\eta)[d\eta^2 - d\mathbf{x}^2]$ (η is conformal time)

$$S[\varphi(\eta, \boldsymbol{x})] = \int_{-\infty}^{\infty} d\eta \int d^3\mathbf{x}\left[\frac{1}{2}a^2(\partial_\eta\varphi)^2 - \frac{1}{2}a^2(\nabla\varphi)^2 - \frac{1}{2}a^4m^2\varphi^2 + \frac{1}{2}a^4\xi R\varphi^2\right]$$

field rescaling

$$\phi(\eta, \boldsymbol{x}) = a(\eta)\varphi(\eta, \boldsymbol{x})$$

action for canonically-normalized field

$aH \to 0$ to zero at $\eta = \pm\infty$

$$S[\phi(\eta, \boldsymbol{x})] = \int_{-\infty}^{\infty} d\eta \int d^3\mathbf{x}\left[\frac{1}{2}(\partial_\eta\phi)^2 - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}m_{\rm eff}^2\phi^2 - \frac{1}{2}\partial_\eta(aH\phi^2)\right]$$

time-dependent effective mass

$$m_{\rm eff}^2(\eta) = a^2(\eta)\left[m^2 + \left(\frac{1}{6} - \xi\right)R(\eta)\right]$$

cosmological expansion ⇒
time-dependent background ⇒
time-dependent Hamiltonian for spectator fields

Scalar field in FRW background

covariant action

$$S[\varphi(x), g_{\mu\nu}(x)] = \int d^4x\sqrt{-g}\left[-\frac{1}{2}M_{\rm Pl}^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}m^2\varphi^2 + \frac{1}{2}\xi R\varphi^2\right]$$

nonminimal coupling term proportional to a "constant" ξ

$\xi = 0$: "minimal coupling"
$\xi = 1/6$: "conformal coupling"

in general, ξ should be a free parameter. $\xi = 1/6$ is an enhanced (classical) conformal symmetry point.

why not other nonminimal terms?

$\xi R\varphi^2$ is the only dimension–4 operator involving Ricci scalar, Ricci tensor, Riemann tensor

no symmetry forbids it, from EFT point of view should include it

furthermore, it should not be constant: there should be an RGE

Scalar field in FRW background

Fourier mode decomposition

$$\widehat{\phi}(\eta, \boldsymbol{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[\widehat{a}_{\mathbf{k}} \chi_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + \widehat{a}^{\dagger}_{\mathbf{k}} \chi^*_{\mathbf{k}}(\eta) e^{-i\mathbf{k}\cdot\mathbf{x}} \right]$$

mode functions satisfy wave equation

$$\partial_\eta^2 \chi_k(\eta) + \omega_k^2(\eta) \chi_k(\eta) = 0$$

but with a time-dependent dispersion relation

$$\omega_k^2(\eta) = |\mathbf{k}|^2 + m_{\text{eff}}^2(\eta)$$

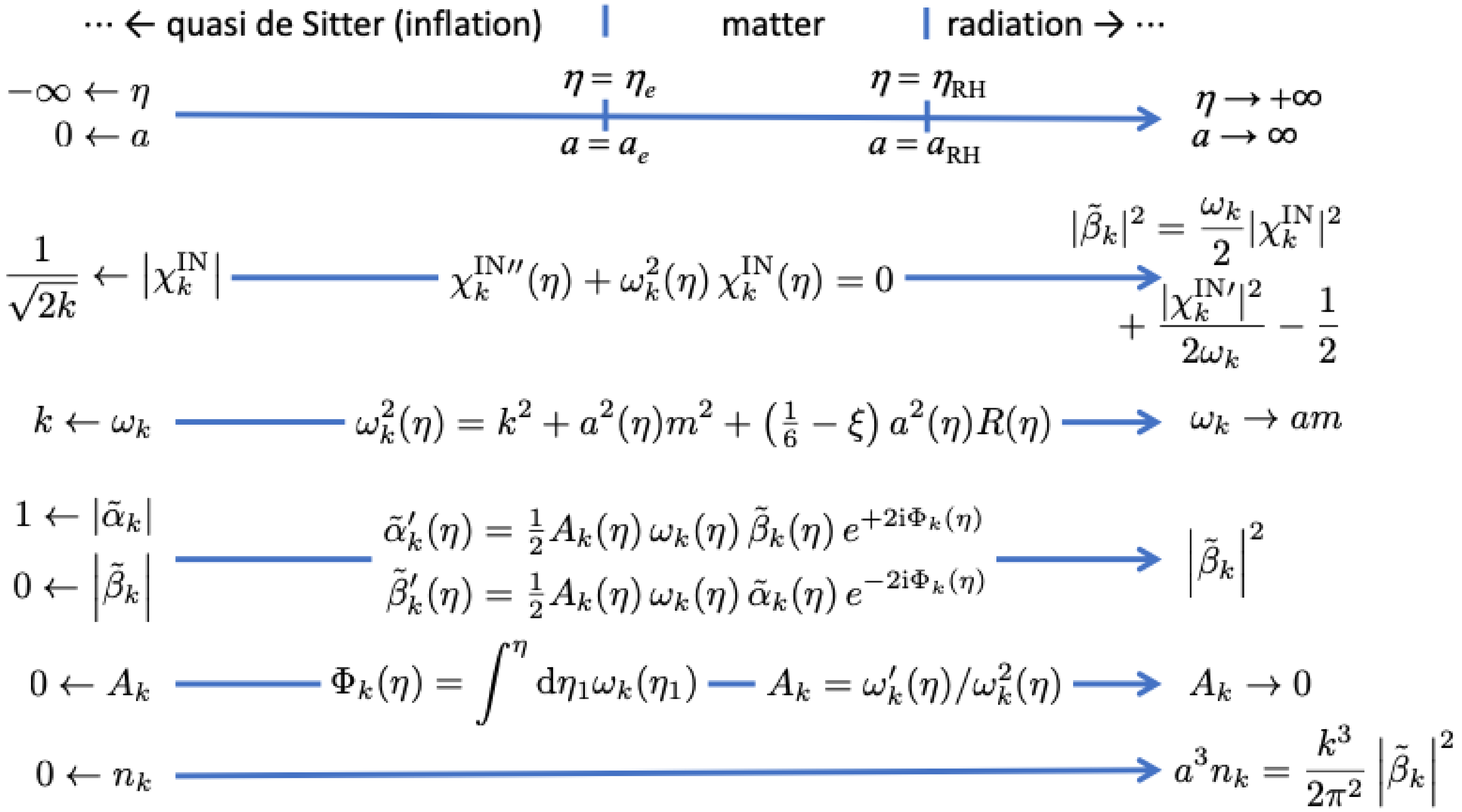
+ & – frequency modes

$$\chi_\pm^{(\text{early})}(\eta) \xrightarrow{\eta \to -\infty} \frac{1}{\sqrt{2k}} e^{\mp ik\eta} \qquad \chi_\pm^{(\text{late})}(\eta) \xrightarrow{\eta \to +\infty} \frac{1}{\sqrt{2am}} e^{\mp i \int^\eta d\eta' am}$$

leads to mode mixing

$$\begin{pmatrix} \chi_{k+}^{(\text{late})}(\eta) \\ \chi_{k-}^{(\text{late})}(\eta) \end{pmatrix} = \begin{pmatrix} \alpha_k & \beta_k \\ \beta_k^* & \alpha_k^* \end{pmatrix} \begin{pmatrix} \chi_{k+}^{(\text{early})}(\eta) \\ \chi_{k-}^{(\text{early})}(\eta) \end{pmatrix}$$

time-dependent Hamiltonian ⇒ mode mixing
⇒ – frequency modes from + frequency modes



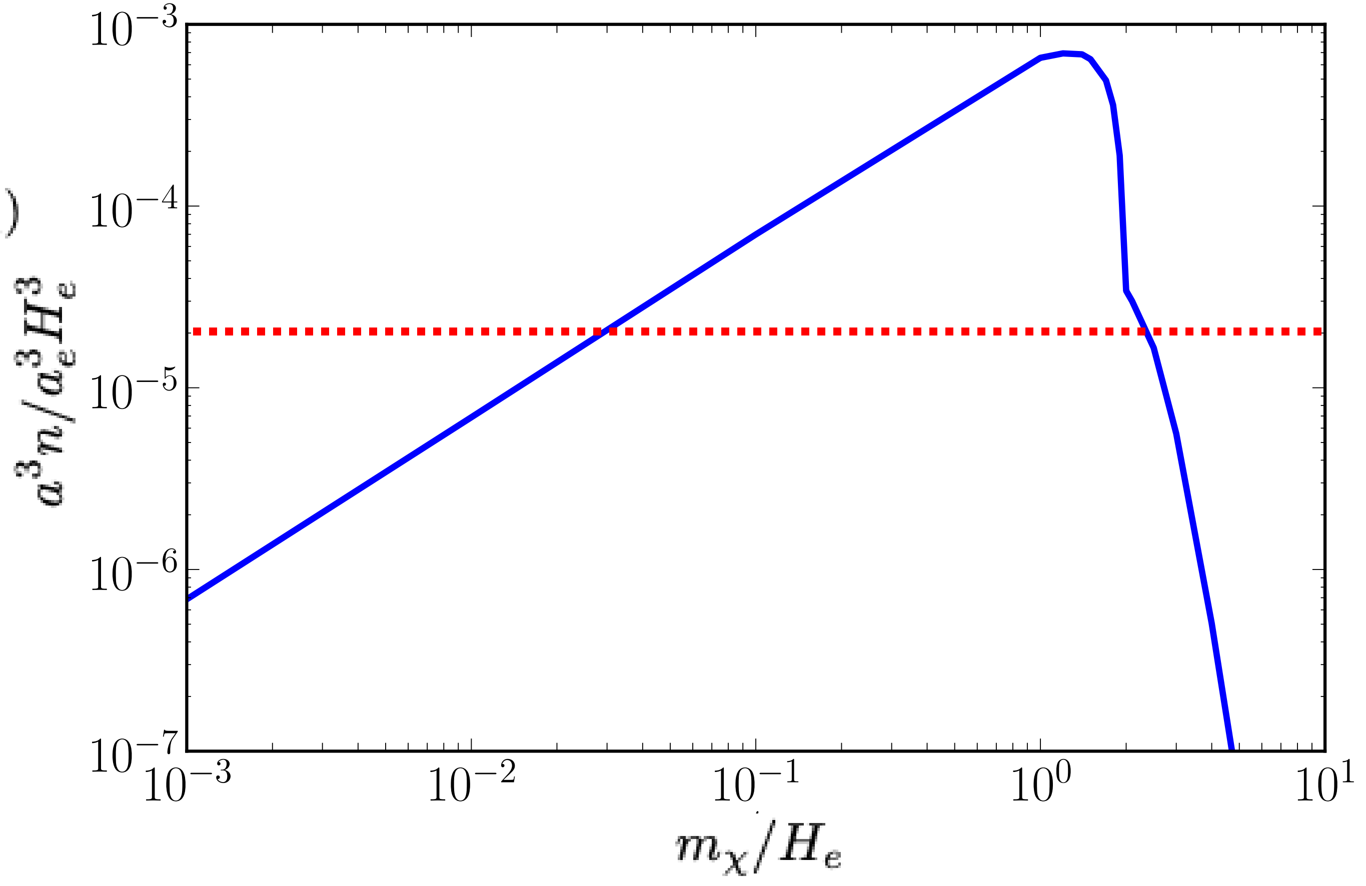
$\cdots \leftarrow$ quasi de Sitter (inflation)
matter
radiation $\rightarrow \cdots$
$-\infty \leftarrow \eta$
$0 \leftarrow a$
$\eta = \eta_e$
$a = a_e$
$\eta = \eta_{\rm RH}$
$a = a_{\rm RH}$
$\eta \rightarrow +\infty$
$a \rightarrow \infty$
$\frac{1}{\sqrt{2k}} \leftarrow |\chi_k^{\rm IN}|$
$\chi_k^{\rm IN\prime\prime}(\eta) + \omega_k^2(\eta)\,\chi_k^{\rm IN}(\eta) = 0$
$|\tilde{\beta}_k|^2 = \frac{\omega_k}{2}|\chi_k^{\rm IN}|^2 + \frac{|\chi_k^{\rm IN\prime}|^2}{2\omega_k} - \frac{1}{2}$
$k \leftarrow \omega_k$
$\omega_k^2(\eta) = k^2 + a^2(\eta)m^2 + \left(\frac{1}{6} - \xi\right) a^2(\eta) R(\eta)$
$\omega_k \rightarrow am$
$1 \leftarrow |\tilde{\alpha}_k|$
$0 \leftarrow \left|\tilde{\beta}_k\right|$
$\tilde{\alpha}_k'(\eta) = \frac{1}{2} A_k(\eta)\,\omega_k(\eta)\,\tilde{\beta}_k(\eta)\,e^{+2{\rm i}\Phi_k(\eta)}$
$\tilde{\beta}_k'(\eta) = \frac{1}{2} A_k(\eta)\,\omega_k(\eta)\,\tilde{\alpha}_k(\eta)\,e^{-2{\rm i}\Phi_k(\eta)}$
$\left|\tilde{\beta}_k\right|^2$
$0 \leftarrow A_k$
$\Phi_k(\eta) = \int^{\eta} {\rm d}\eta_1 \omega_k(\eta_1)$
$A_k = \omega_k'(\eta)/\omega_k^2(\eta)$
$A_k \rightarrow 0$
$0 \leftarrow n_k$
$a^3 n_k = \frac{k^3}{2\pi^2}\left|\tilde{\beta}_k\right|^2$

Quadratic Inflaton Potential for Conformally-Coupled Scalar: $\xi = 1/6$

$$\frac{\Omega h^2}{0.12} = \frac{m}{H_e}\left(\frac{H_e}{10^{12}\mathrm{GeV}}\right)^2\left(\frac{T_{\mathrm{RH}}}{10^9\mathrm{GeV}}\right)\ \frac{[a^3 n/\, a_e^3 H_e^3]}{10^{-5}}$$

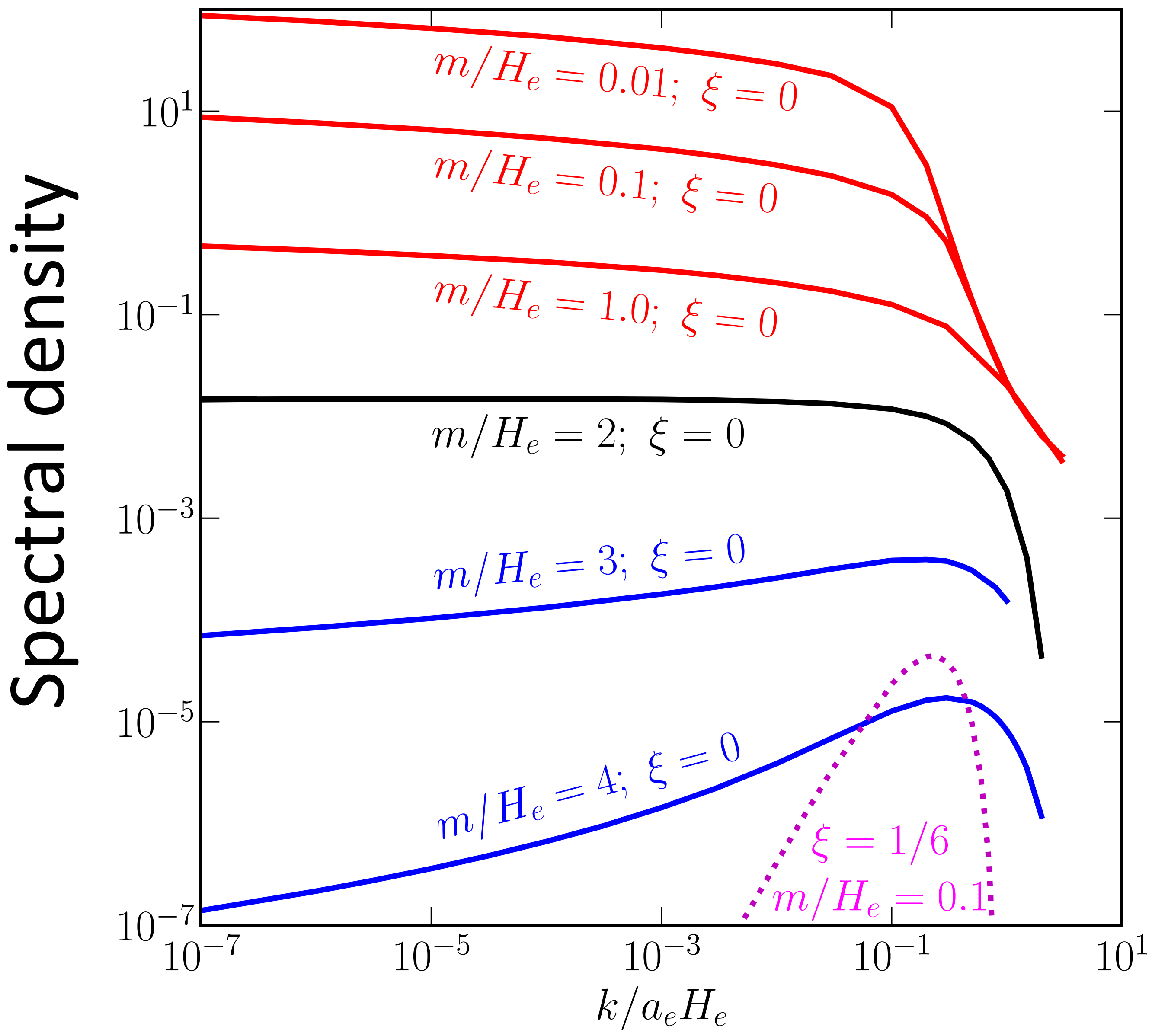
$$\sim \left(\frac{m}{10^{11}\mathrm{GeV}}\right)^2\left(\frac{T_{\mathrm{RH}}}{10^9\mathrm{GeV}}\right) \qquad (m \lesssim m_{\mathrm{inflaton}})$$

- This calculation assumes particular inflationary model (quadratic, which is ruled out).
- But general picture holds in other models since action occurs around end of inflation.
- We don’t know, but $H_e \approx 10^{12}$ GeV and $T_{\mathrm{RH}} \approx 10^9$ GeV are “common.”
- If stable and dark matter, $\Omega_\chi h^2 = 0.12 \ \rceil \ m \approx H_I$.
  Could have been anything!
- Perhaps inflation scale represents new physics scale, stable particle at that mass scale natural DM candidate.
- WIMPZILLA miracle!



Conformally-coupled scalar WIMPZILLA DM candidate
$m_\chi = \mathcal{O}(H_e) = \mathcal{O}(m_{\mathrm{inflaton}})$

Quadratic Inflaton Potential for Minimally-Coupled Scalar: $\xi = 0$



Red Spectrum leads to dangerous isocurvature fluctuations

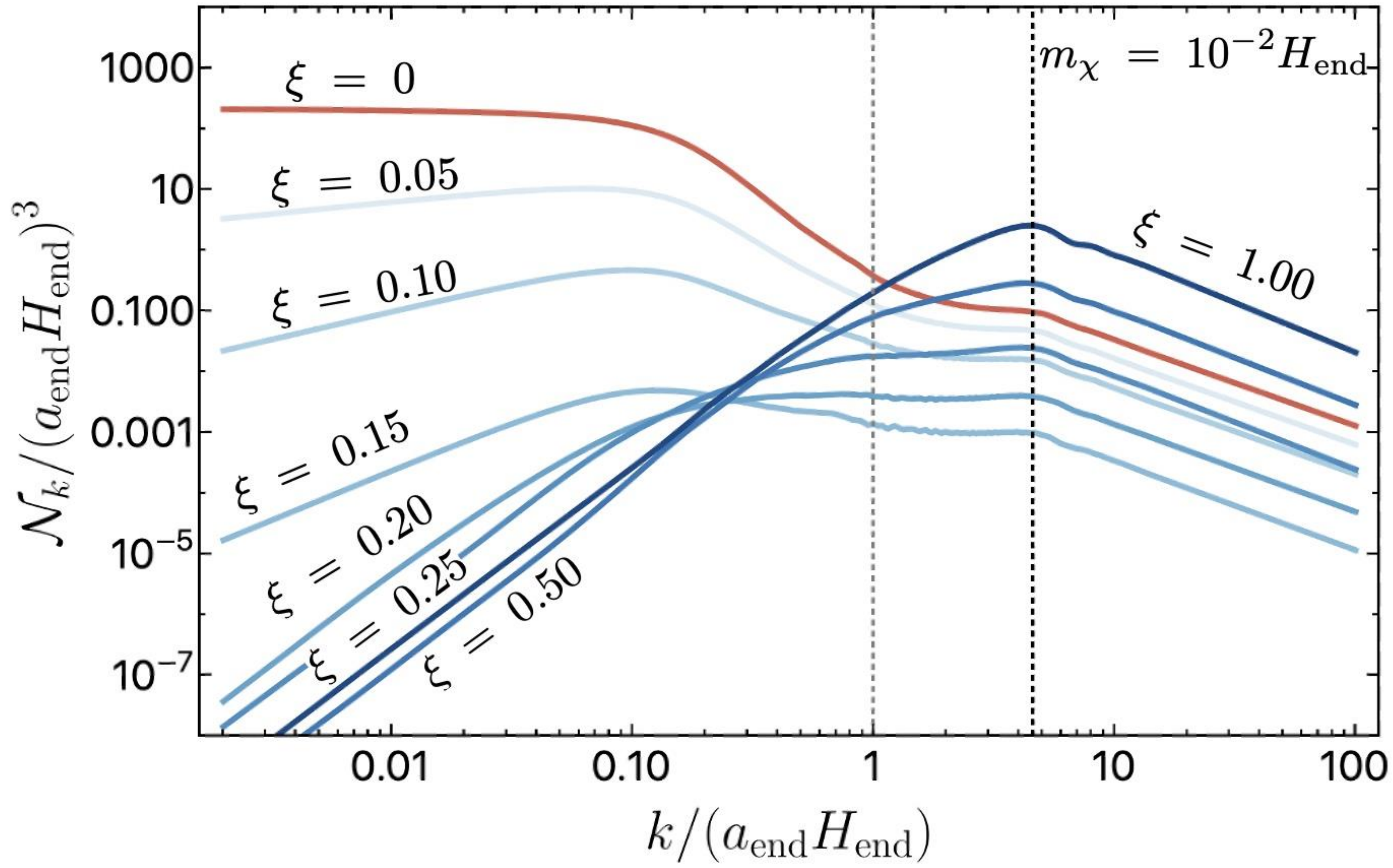
CMB limits

Chung, EWK, Riotto, Senatore (2005)

Stable, minimally-coupled scalars are disallowed if $m \lesssim$ few H_e

Model-T inflation model (Kallosh & Linde): $V(\varphi) = 10^{-10} M_{\rm Pl}^4 \tanh(\varphi/\sqrt{6}M_{\rm Pl})$

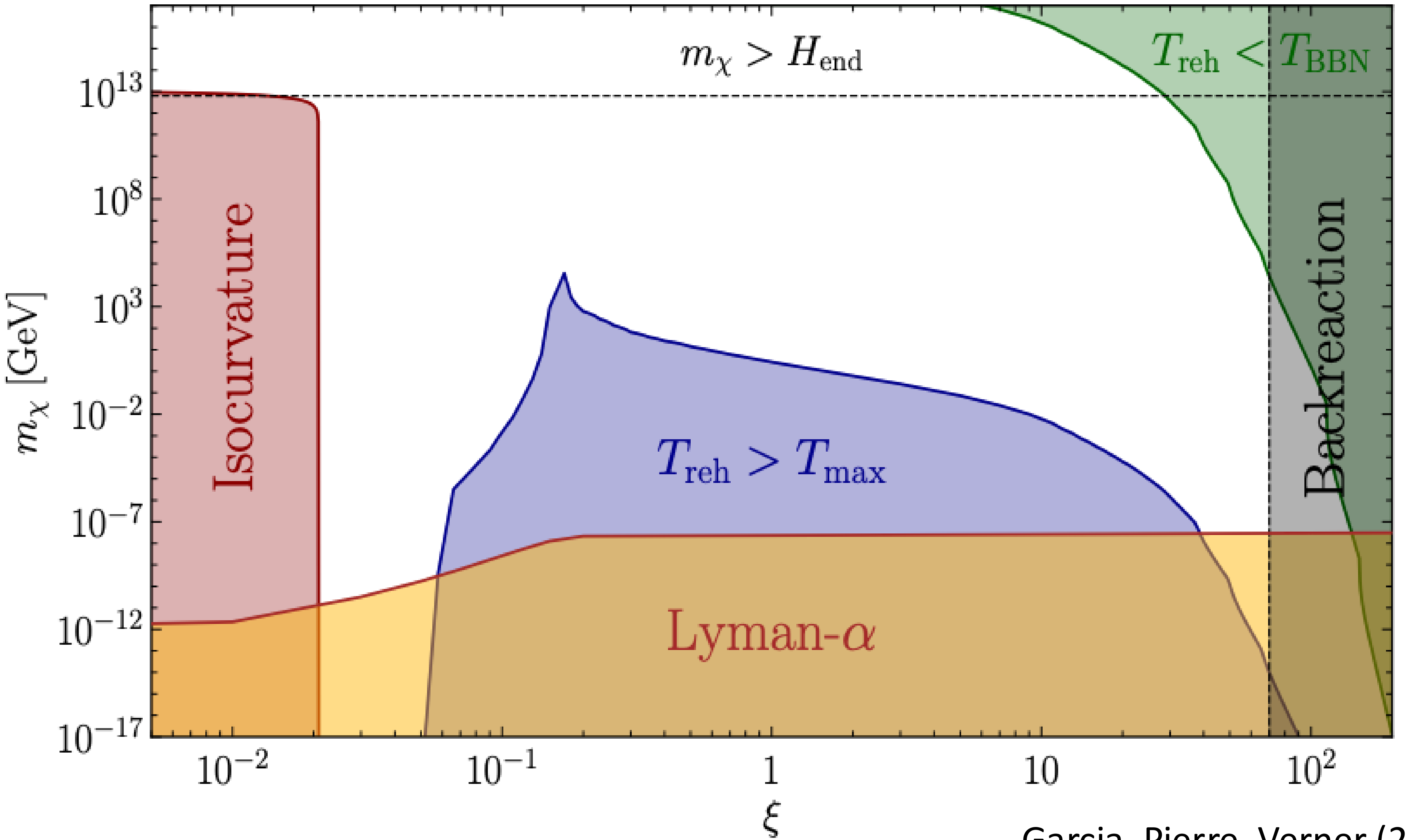
$$m_{\rm eff}^2 = m^2 + \tfrac{1}{6}(1-6\xi)R$$



Garcia, Pierre, Verner
2305.14446

also
EWK, Long, McDonough, Payeur
2211.14323

Fairbairn, Kainulainen, Markkanen, Nurmi
1808.08236



Garcia, Pierre, Verner (2023)

Scalar field in FRW background

- Stable, minimally-coupled scalar disallowed if $m \lesssim H_e$
- Other (ξ , m) combinations disallowed (too large Ω)
- Stable, conformally-coupled scalars (or minimally-coupled with $m \gtrsim H_e$) allowed, and could be dark matter.

WIMPZILLAS!

Dirac field in FRW background

$$\frac{\Omega h^2}{0.12} = \frac{m}{H_e}\left(\frac{H_e}{10^{12}\mathrm{GeV}}\right)^2\left(\frac{T_{\mathrm{RH}}}{10^9\mathrm{GeV}}\right)\frac{[a^3 n/a_e^3 H_e^3]}{10^{-5}}$$
$$\sim \left(\frac{m}{10^{11}\mathrm{GeV}}\right)^2\left(\frac{T_{\mathrm{RH}}}{10^9\mathrm{GeV}}\right) \qquad (m \lesssim m_{\mathrm{inflaton}})$$

No nonminimal dimension–4 operator

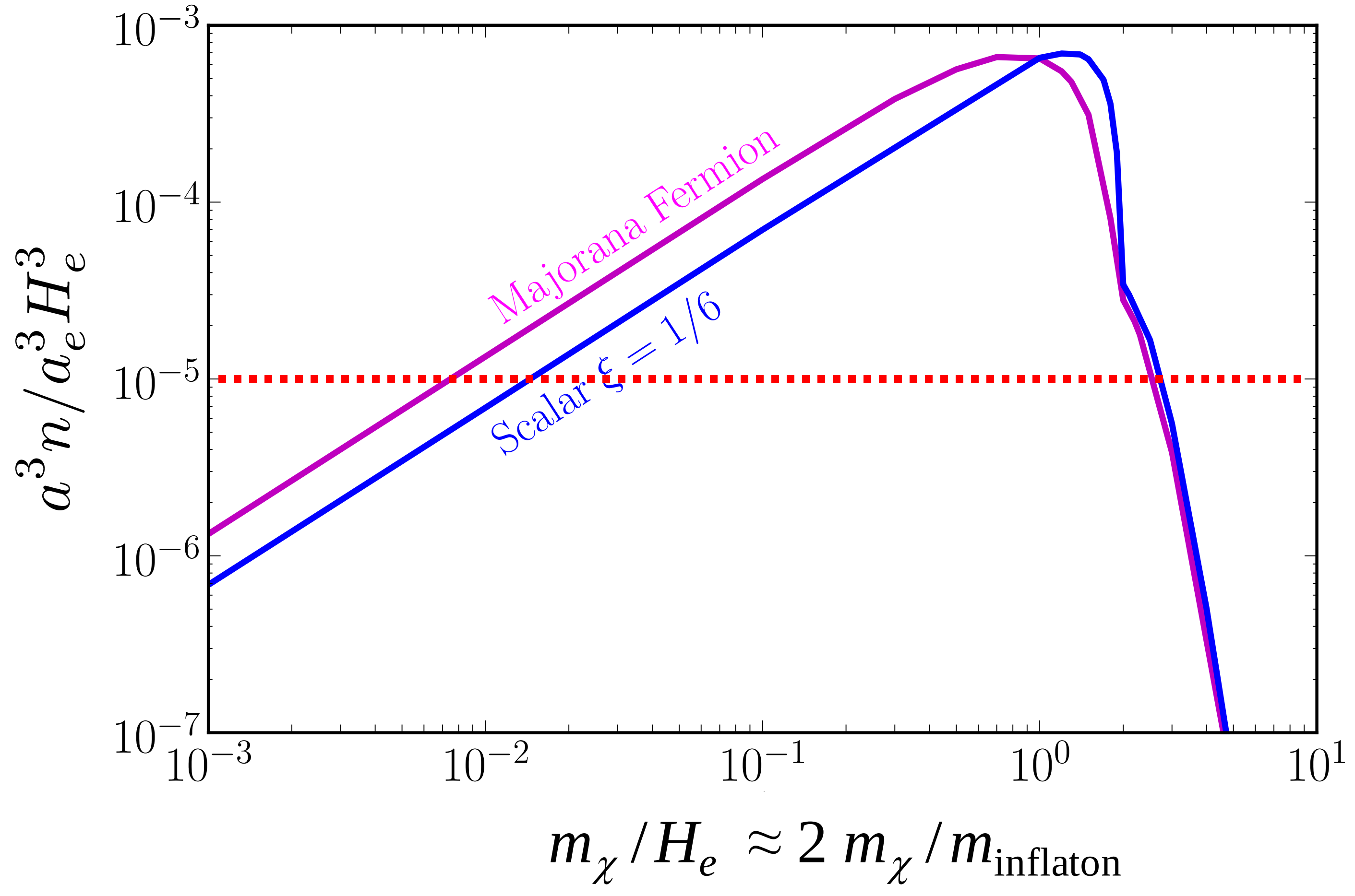
Dirac Equation in FRW:

$$i\partial_\eta \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix} = \begin{pmatrix} a(\eta)m & k \\ k & -a(\eta)m \end{pmatrix}\begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix}$$

Dispersion relation same as conformally-coupled scalar

Blue spectrum: no isocurvature issues

Dirac WIMPZILLA DM candidate for $m = \mathcal{O}(m_{\mathrm{inflaton}})$



7th Duc de Broglie—Proca field (dark photon) in FRW background

1742

Minimal coupling: Graham, Mardon, & Rajendran (2016); Ahmed, Grzadkowski, & Socha (2020); EWK & Long (2020)
Non-minimal coupling: Capanelli, Jenks, EWK, McDonough (2024)

Covariant action:

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m^2 g^{\mu\nu}A_\mu A_\nu - \frac{1}{2}\xi_1 R g^{\mu\nu}A_\mu A_\nu - \frac{1}{2}\xi_2 R^{\mu\nu}A_\mu A_\nu\right)$$

Two possible *nonminimal* dimension–4 terms: $Rg^{\mu\nu}A_\mu A_\nu$ and $R^{\mu\nu}A_\mu A_\nu$

Gauge invariance broken by mass term and by nonminimal terms, can fix via trick of
Baron Ernst Carl Gerlach Stueckelberg von Breidenbach zu Breidenstein und Melsbach (Abelian Higgs mechanism)

In FRW

$$S\left[A_\mu(t,\boldsymbol{x})\right] = \int d^4x\left[\frac{1}{2}a\left(\partial_0 A_i - \partial_i A_0\right)^2 - \frac{1}{4}a^{-1}\left(\partial_i A_j - \partial_j A_i\right)^2 + \frac{1}{2}a^3 m_t^2 A_0^2 - \frac{1}{2}a m_x^2 A_i^2\right]$$

A_0 is not dynamical and will be integrated out.

Two time-dependent mass terms $\left\{\begin{aligned} m_t^2 &= m^2 - \xi_1 R - \frac{1}{2}\xi_2 R - 3\xi_2 H^2 \\ m_x^2 &= m^2 - \xi_1 R - \frac{1}{6}\xi_2 R + \xi_2 H^2 \end{aligned}\right\}$ m_t^2 and m_x^2 can be positive or negative!

7th Duc de Broglie—Proca field (dark photon) in FRW background

Minimal coupling: Graham, Mardon, & Rajendran (2016); Ahmed, Grzadkowski, & Socha (2020); EWK & Long (2020)
Non-minimal coupling: Capanelli, Jenks, EWK, McDonough (2024)

Standard procedure:

1. Decompose action in terms of mode functions
2. Integrate out A_0
3. Introduce orthonormal set of transverse and longitudinal mode functions
4. Action separates into two pieces, transverse and longitudinal
5. Transverse mode action that of conformally-coupled scalar with mass m_x^2 (which can be positive or negative)
6. Longitudinal mode action more "interesting"

$$S^L = \int d\eta \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[\frac{1}{2}\frac{a^2 m_t^2}{k^2 + a^2 m_t^2}\left|\partial_\eta A_{\boldsymbol{k}}^L\right|^2 - \frac{1}{2}a^2 m_x^2 \left|A_{\boldsymbol{k}}^L\right|^2\right]$$

7. Since m_t^2 is time-dependent and not necessarily positive definite, a ghost can be propagated
8. If demand ghost-free for arbitrarily large k, must have $m_t^2 > 0$ during evolution
9. This will place limits on ξ_1 and ξ_2 as a function of m.
10. Longitudinal frequency "interesting"

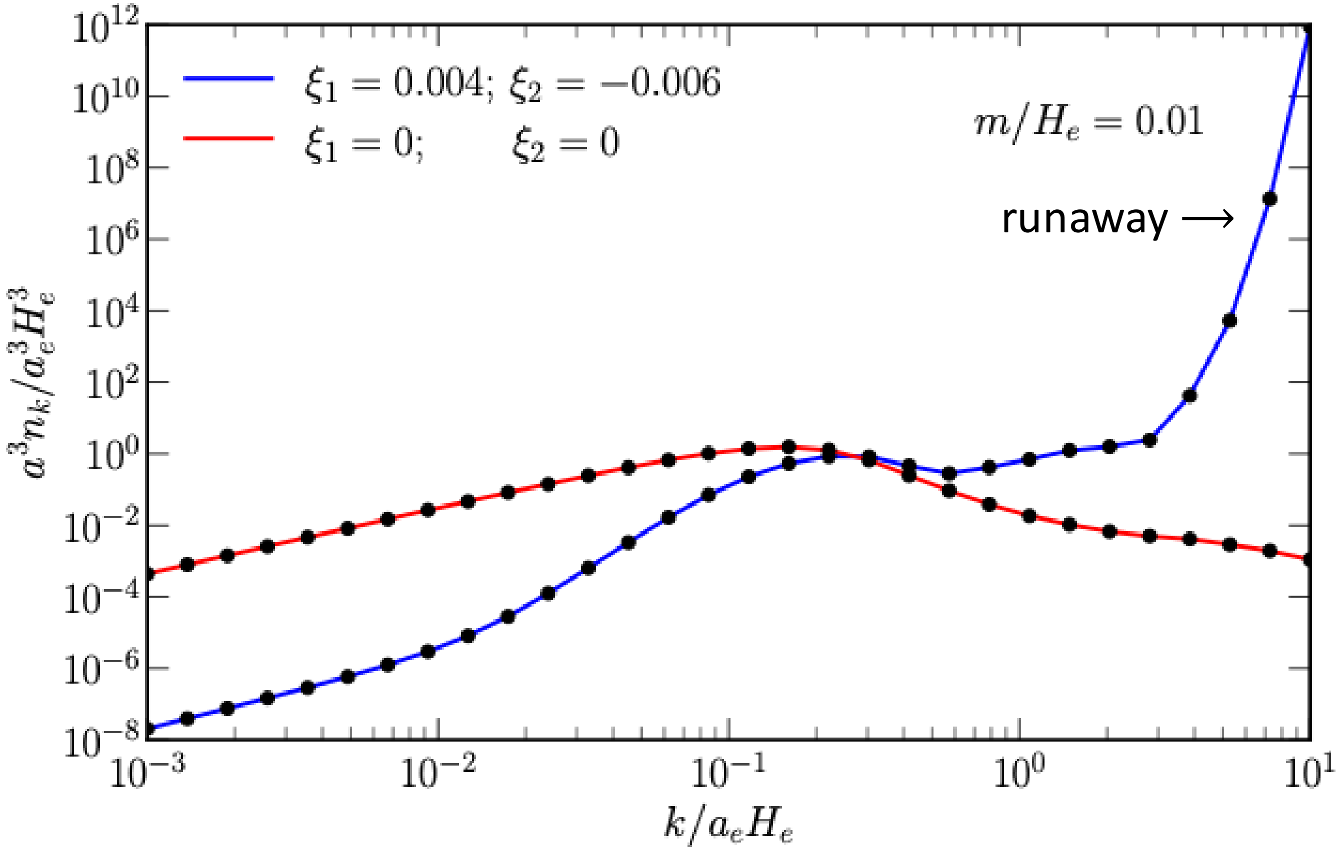
7th Duc de Broglie—Proca field (dark photon) in FRW background

Minimal coupling: Graham, Mardon, & Rajendran (2016); Ahmed, Grzadkowski, & Socha (2020); EWK & Long (2020)
Non-minimal coupling: Capanelli, Jenks, EWK, McDonough (2024)

$$\omega_L^2 = k^2 \frac{m_x^2}{m_t^2} + a^2 m_x^2 + \frac{3k^2 a^4 m_t^2 H^2}{(k^2 + a^2 m_t^2)^2} + \frac{k^2 a^2 R}{6(k^2 + a^2 m_t^2)} + \frac{Hak^2 m_t^{2\prime}}{m_t^2} \frac{(-k^2 + 2a^2 m_t^2)}{(k^2 + a^2 m_t^2)^2} + \frac{k^2 (m_t^{2\prime})^2}{4(m_t^2)^2} \frac{(k^2 + 4a^2 m_t^2)}{(k^2 + a^2 m_t^2)^2} - \frac{k^2 m_t^{2\prime\prime}}{2m_t^2 (k^2 + a^2 m_t^2)}$$

(prime denotes ∂_η)

1. In high-momentum limit (large k) the first term dominates $\omega_k^2 \to k^2 m_x^2 / m_t^2$
2. Have established that $m_t^2 > 0$ to be ghostless, if $m_x^2 < 0$ then ω_k^2 will be *negative*
3. Leading to an instability to particle production for arbitrarily large k modes.
4. Require $m_t^2 > 0$ *and* $m_x^2 > 0$
5. Depends (a little) on inflation model



7^{th} Duc de Broglie—Proca field (dark photon) in FRW background

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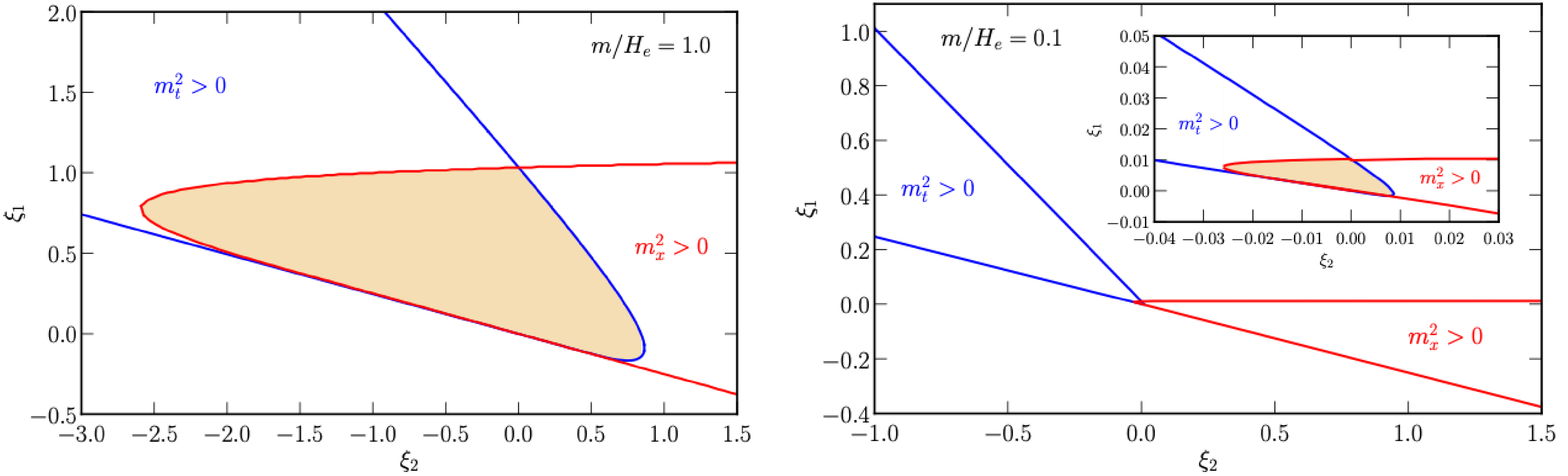


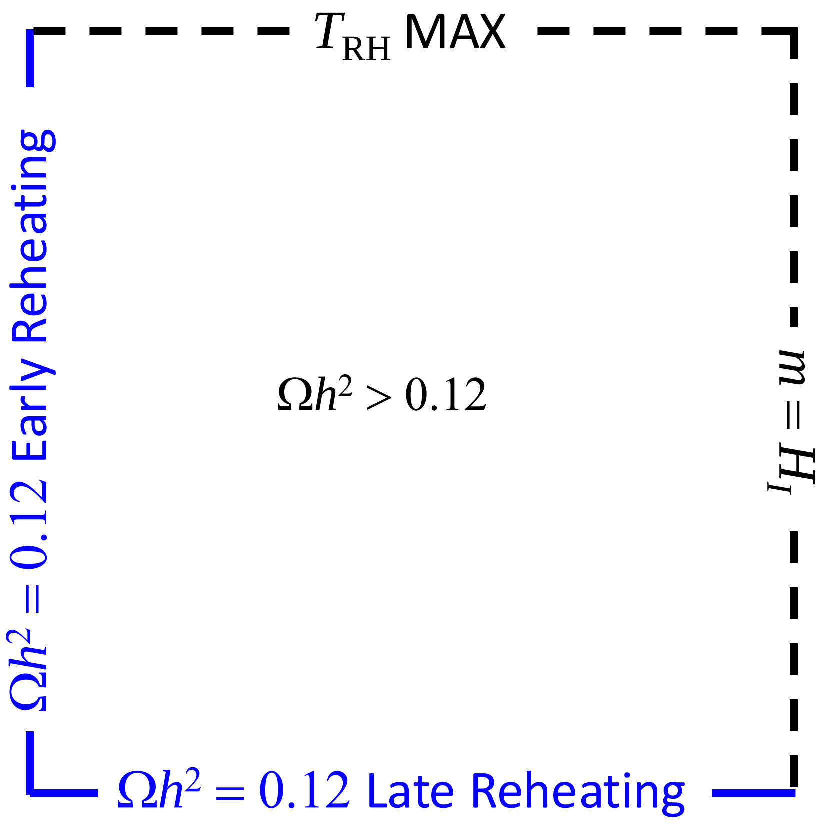
For “large” m/H_e, not very restrictive

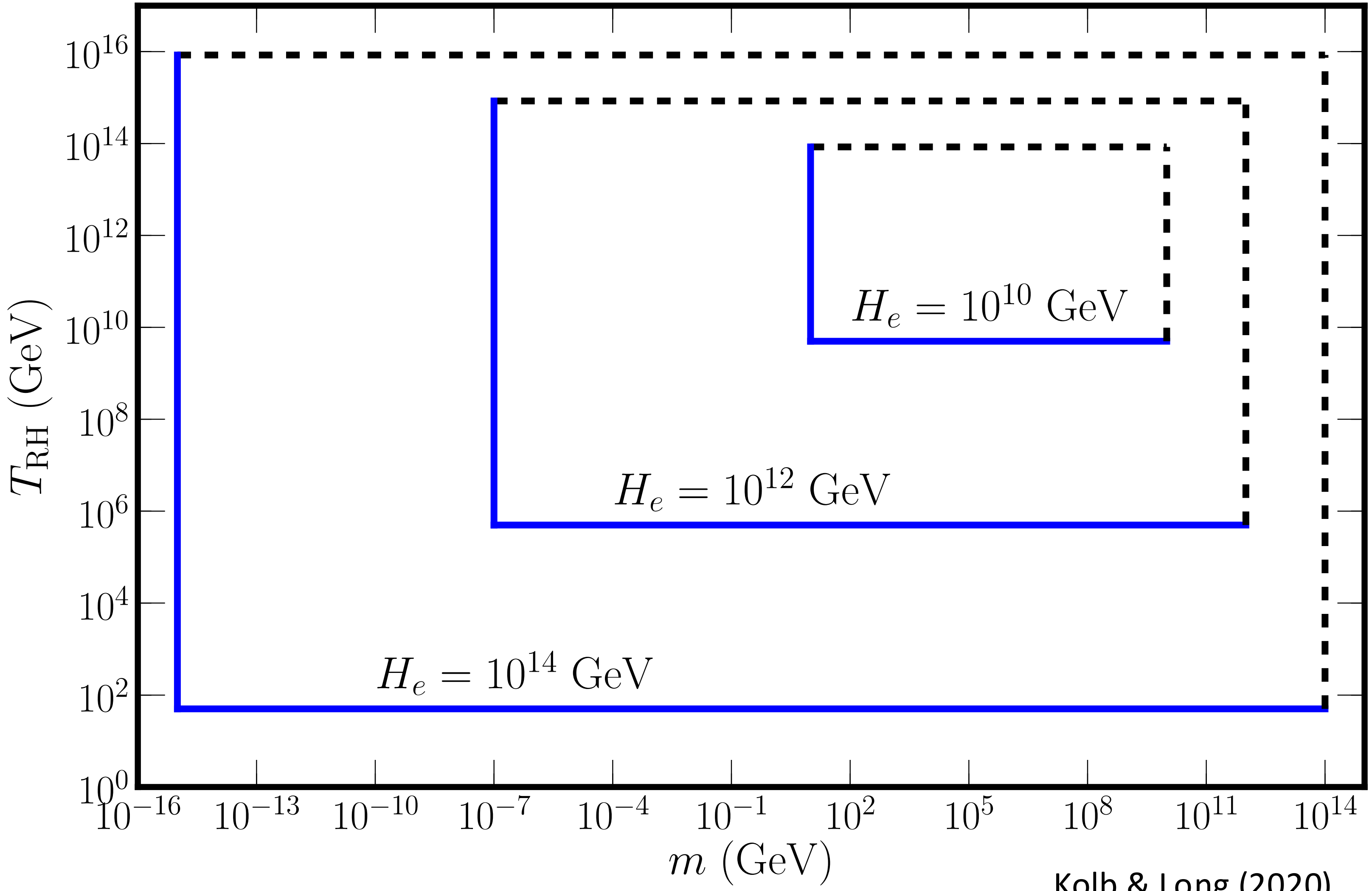
But for small m/H_e, as in dark photon models, very restrictive

Breakdown of EFT? Discussed in Capanelli et al.

Strong coupling? A. Hell

7th Duc de Broglie—Proca field (dark photon) in FRW background





Kolb & Long (2020)

Early/late reheating: $T_{\rm RH} \lesssim 8.4 \times 10^8 \left(\frac{m}{\rm GeV}\right)^{1/2} \rm GeV$

Very light (μeV) DM from GPP or very massive (10^{14} GeV) DM from GPP

Rarita-Schwinger field in FRW background

Kallosh, Kofman, Linde Van Proeyen (1999)
Giudice, Riotto, Takachev (1999)
EWK, Long, McDonough (2021)

Covariant action:

$$S = \int d^4x \sqrt{-g}\left[i\left(\tfrac{1}{4}\bar{\Psi}_\mu\,\gamma^{\mu\nu\rho}\nabla_\nu\Psi_\rho - \tfrac{1}{4}\bar{\Psi}_\mu\overleftarrow{\nabla}_\nu\gamma^{\mu\nu\rho}\Psi_\rho\right) - i\,\tfrac{1}{2}m\bar{\Psi}_\mu\gamma^{\mu\nu}\Psi_\nu\right]$$

Specialize to FRW and define new field: $\psi_\mu(\eta,\vec{x}) = a^{1/2}(\eta)\Psi_\mu(\eta,\vec{x})$

Impose constraints from field equations

Fourier decomposition $\psi_\mu(\eta,\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^3}\,\psi_{\mu,\vec{k}}(\eta)\,e^{i\vec{k}\cdot\vec{x}}$

Decompose $\psi_{\mu,\vec{k}} \longrightarrow \psi_{1/2,\vec{k}}$ and $\psi_{3/2,\vec{k}}$

Mode equations become
$$\begin{cases} \left[i\gamma^0\partial_\eta - \vec{k}\cdot\vec{\gamma} - am\right]\psi_{3/2,\vec{k}} = 0 \\ \left[i\gamma^0\partial_\eta - \left(C_A + iC_B\gamma^0\right)\vec{k}\cdot\vec{\gamma} - am\right]\psi_{1/2,\vec{k}} = 0 \end{cases}$$

C_A and C_B will be defined

Almost there!

Rarita-Schwinger field in FRW background

Parameterize spinor wavefunctions in terms of helicity eigenspinors with mode functions

$$\chi_{A,3/2,\vec{k}}(\eta),\ \chi_{B,3/2,\vec{k}}(\eta),\ \chi_{A,1/2,\vec{k}}(\eta),\ \chi_{B,1/2,\vec{k}}(\eta)$$

which satisfy

$$\begin{cases} i\partial_\eta \begin{pmatrix} \chi_{A,3/2,k}(\eta) \\ \chi_{B,3/2,k}(\eta) \end{pmatrix} = \begin{pmatrix} am & k \\ k & -am \end{pmatrix} \begin{pmatrix} \chi_{A,3/2,k}(\eta) \\ \chi_{B,3/2,k}(\eta) \end{pmatrix} \\ \\ i\partial_\eta \begin{pmatrix} \chi_{A,1/2,k}(\eta) \\ \chi_{B,1/2,k}(\eta) \end{pmatrix} = \begin{pmatrix} am & (C_A + iC_B)\,k \\ (C_A - iC_B)\,k & -am \end{pmatrix} \begin{pmatrix} \chi_{A,1/2,k}(\eta) \\ \chi_{B,1/2k}(\eta) \end{pmatrix} \end{cases}$$

Helicity $3/2$ mode equation is just like Dirac field

Helicity $1/2$ mode equation is more “interesting”

Nonzero for gravitini

Eigenvalues of $3/2$ mode equation are $\pm\sqrt{c_s^2 k^2 + a^2 m^2}$

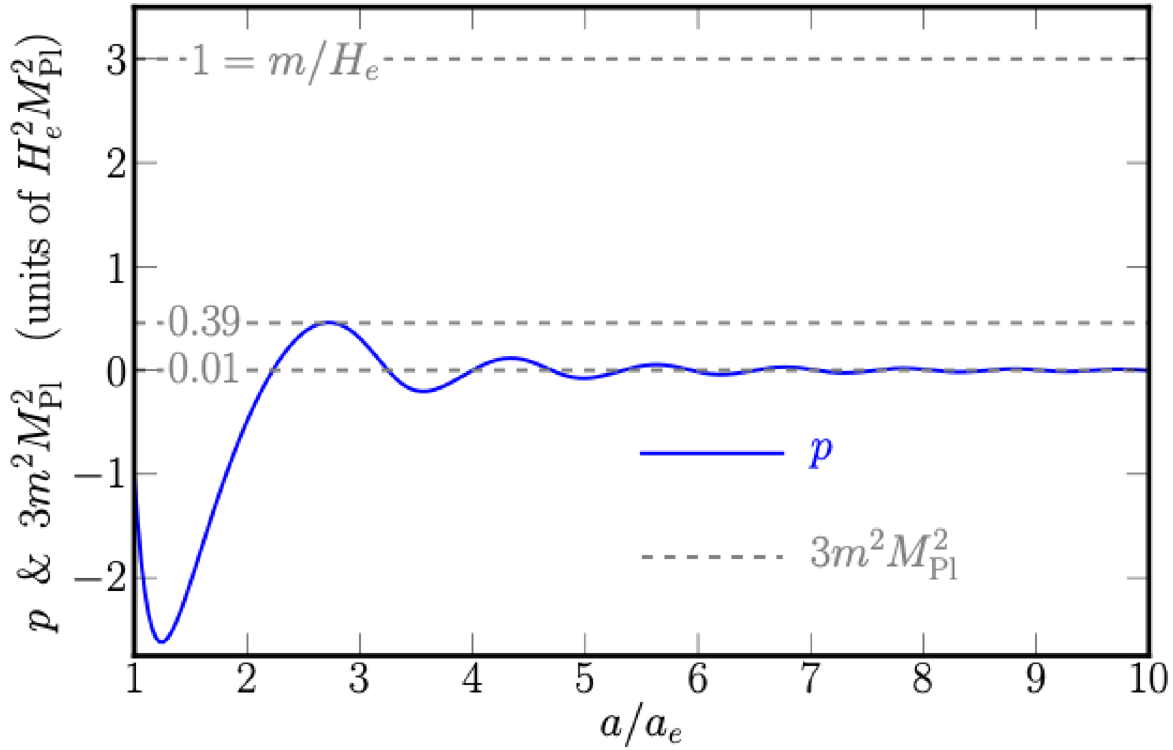
$$c_s^2 \equiv C_A^2 + C_B^2 = \frac{1}{9(H^2+m^2)^2}\left[\left(-\frac{1}{3}R - H^2 + 3m^2\right)^2 + 4\frac{(\partial_\eta m)^2}{a^2}\right]$$ is the sound speed

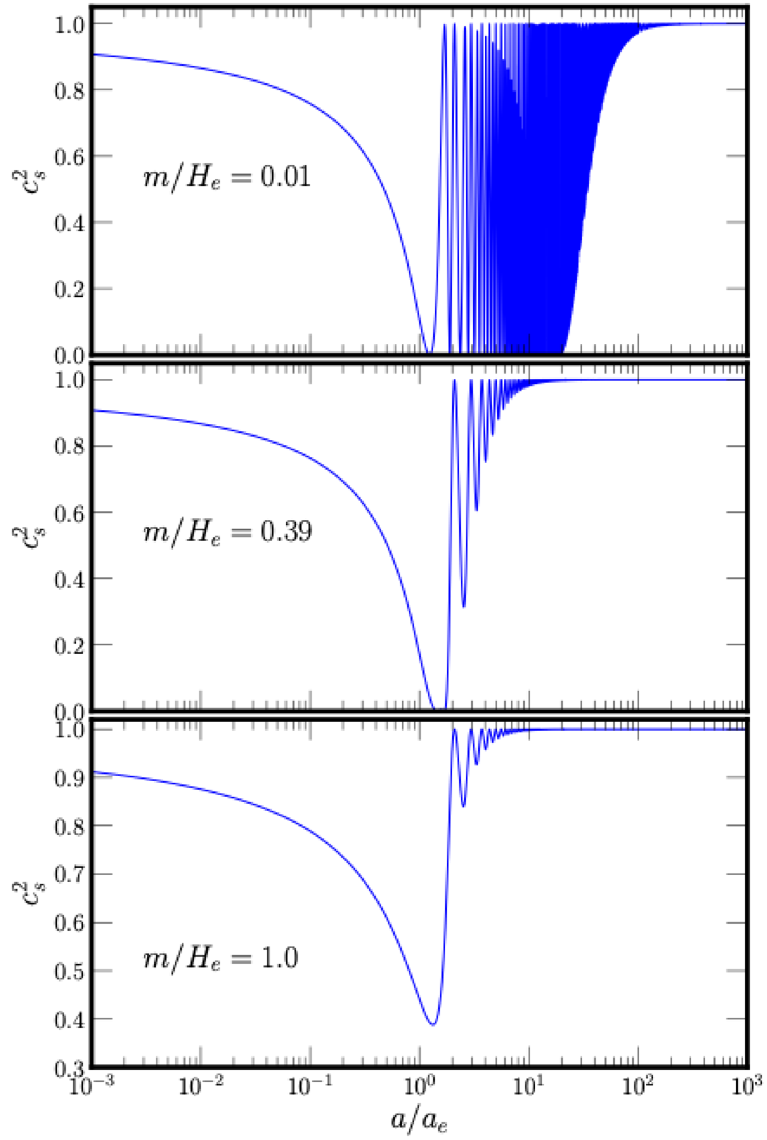
Rarita-Schwinger field in FRW background

New feature (or is it a bug?): sound speed can vanish!

$$c_s = \frac{\left|-\frac{1}{3}R(\eta) - H^2(\eta) + 3m^2\right|}{3(H^2(\eta)+m^2)} = \frac{\left|p(\eta) - 3m^2 M_{\rm Pl}^2\right|}{\rho(\eta) + 3m^2 M_{\rm Pl}^2}$$

$c_s = 0$ when $p(\eta) = 3m^2 M_{\rm Pl}^2$





$m/H_e \gtrsim 1 \leftrightarrow c_s = 0$ avoided

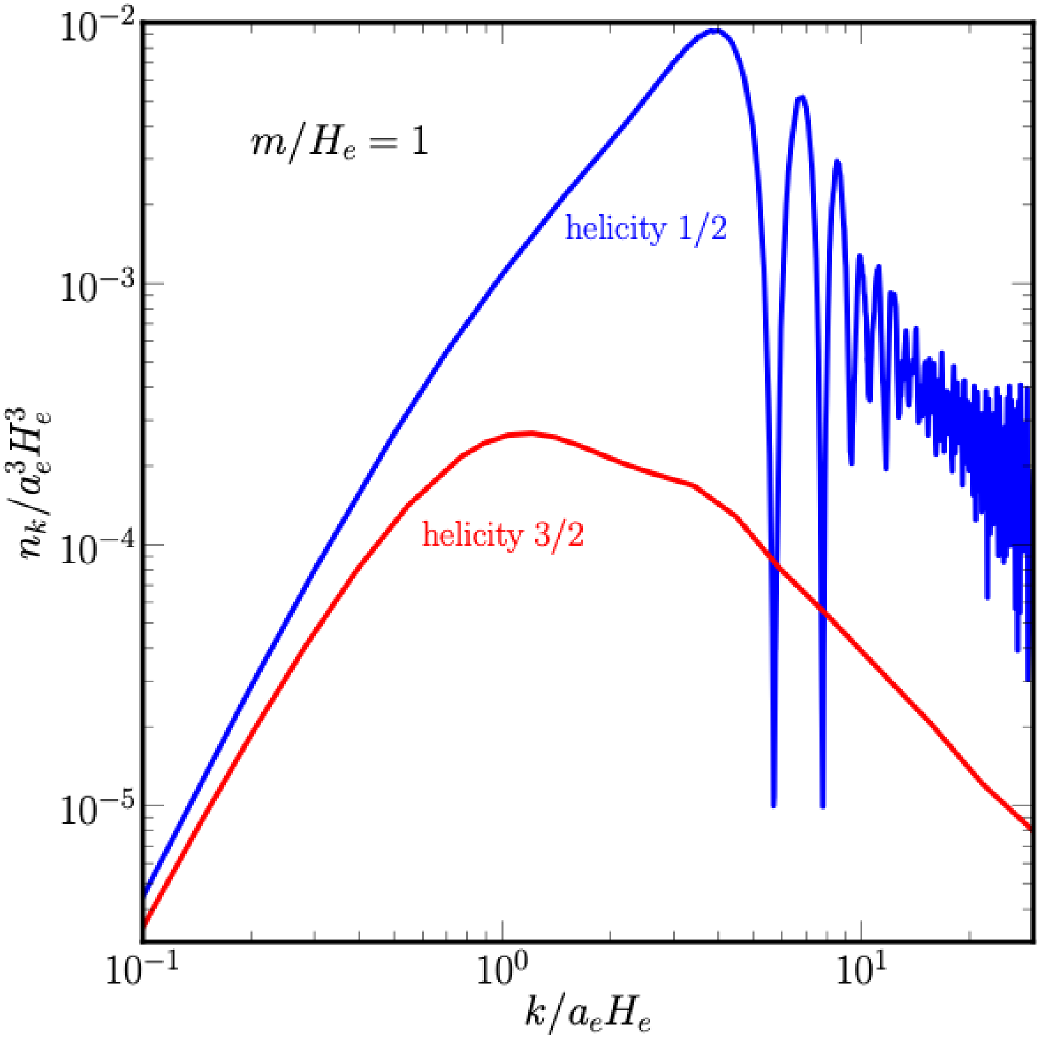
$m/H_e \lesssim 1 \leftrightarrow c_s = 0$ allowed

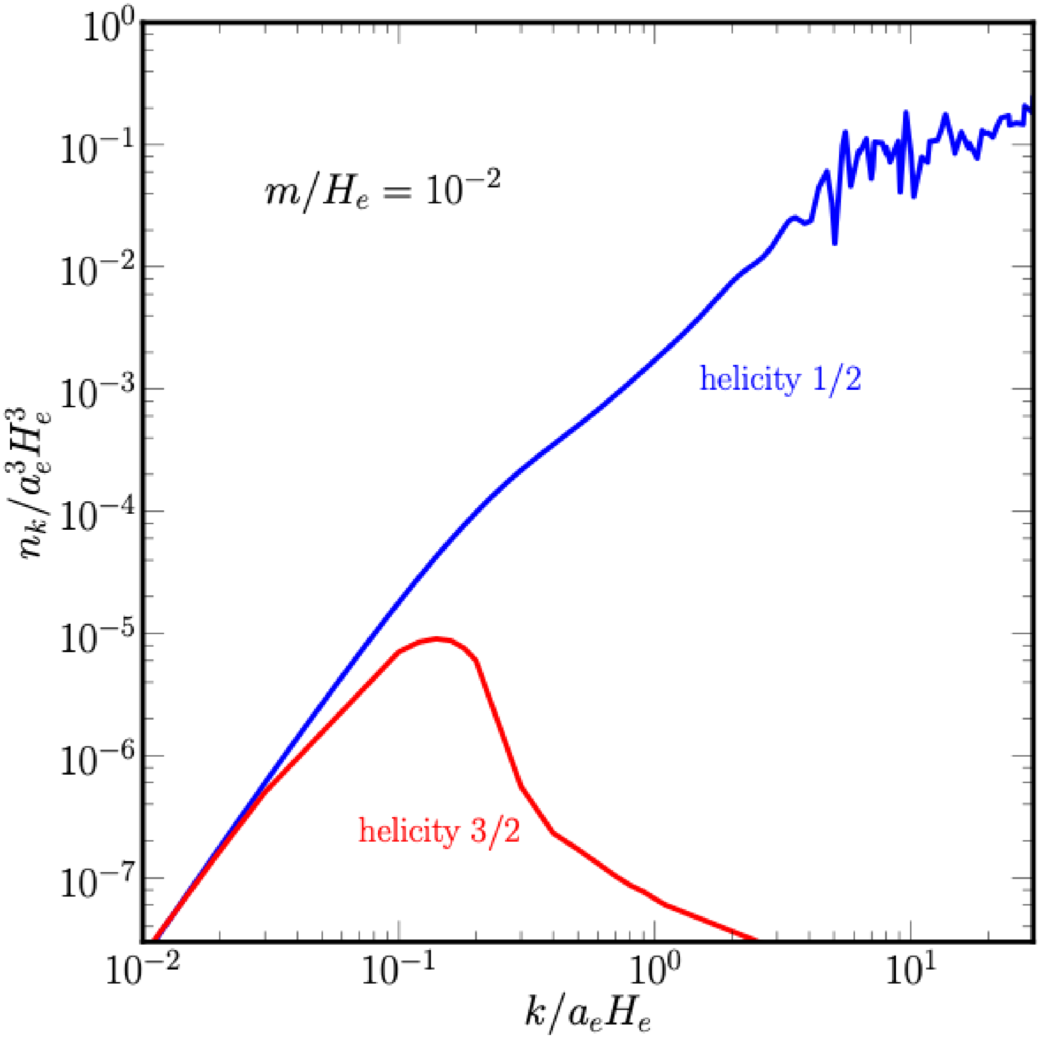
Rarita-Schwinger field in FRW background

$$\omega_k^2(\eta) = c_s^2(\eta)k^2 + a^2(\eta)m^2$$

If $c_s \neq 0$, then ω^2 approx. constant for high-k modes, GPP suppressed

If $c_s = 0$, then ω^2 independent of k, GPP unsuppressed for high-k modes





Rarita-Schwinger field in FRW background

Spin-$3/2$ particles arise in theories of supergravity; $s = 3/2$ gravitino is superpartner of $s = 2$ graviton

Does supergravity have a catastropic production of gravitinos? It depends on the model!

For models with a single chiral superfield

$$K(\Phi, \bar{\Phi}) = \Phi\bar{\Phi}; \quad W(\Phi) = \frac{1}{2} m_\phi \Phi^2 \qquad m_{3/2} = e^{K(\Phi,\bar{\Phi})/2M_{\rm Pl}^2} \frac{W(\Phi)}{M_{\rm Pl}^2} = e^{\phi^2/4M_{\rm Pl}^2} \frac{m_\phi}{4M_{\rm Pl}^2} \phi^2$$

gravitino mass is time-dependent (depends on rolling inflaton)
implying $c_s = 1$ at all times & no catastrophic production

For models with multiple chiral superfields

c_s depends on relative orientation of inflaton direction & SUSY breaking
mixing between the goldstino & inflatino may avoid the catastrophe (explicit calculation needed)
$c_s = 0$ occurs in models with a nilpotent superfield $S^2 = 0$ and orthogonal constraint $S \cdot (\Phi - \bar{\Phi}) = 0$

EWK, Long, McDonough (2021)
Dudas, Garcia, Mambrini,Olive, Peloso, Verner (2021)
Antoniadis, Benaki, Ke (2021)

Fierz-Pauli field in FRW background*

EWK, Ling, Long, Rosen (2022)

We wish to study cosmological gravitational particle production of a massive spin-2 field in FRW background.

Field content will be a massless graviton, a massive spin–2 field, and fields necessary to source an FRW background to provide a cosmology that leads to inflation → matter domination → radiation domination → our universe.

Story of massive spin–2 fields begins in 1939 with work of Fierz and Pauli, but first …

Start with EH action: $S[g_{\mu\nu}] = \int d^4x\,\sqrt{-g}\;\frac{M_P^2}{2}\,R[g]$

Linearize about Minkowski spacetime: $g_{\mu\nu} \to \eta_{\mu\nu} + \frac{2}{M_P}h_{\mu\nu}$

Quadratic action: $S[h_{\mu\nu}] = \int d^4x\left[-\frac{1}{2}\nabla_\lambda h_{\mu\nu}\nabla^\lambda h^{\mu\nu} + \nabla_\mu h^{\nu\lambda}\nabla_\nu h^\mu{}_\lambda - \nabla_\mu h^{\mu\nu}\nabla_\nu h + \frac{1}{2}\nabla_\mu h\nabla^\mu h\right]$

* I should have listened to Ruth!

Fierz-Pauli field in FRW background*

Now add a mass term: Fierz-Pauli (1939) (see reviews by Hinterbichler 1105.3735; de Rahm 1401.4173)

$$\delta S[h_{\mu\nu}] = \int d^4x \left[-\tfrac{1}{2}m_1^2 h_{\mu\nu}h^{\mu\nu} - \tfrac{1}{2}m_2^2 h^2\right]$$

Introduces unwanted 6th degree of freedom (a ghost) of mass $m^2_{\rm ghost} = \dfrac{m_1^2}{4}\dfrac{m_1^2+4m_2^2}{m_1^2+m_2^2}$

So, choose $m_2^2 = -m_1^2$ to banish ghost to ∞ (but no symmetry enforces this!)

$$\delta S[h_{\mu\nu}] = \int d^4x \left[-\tfrac{1}{2}m^2\left(h_{\mu\nu}h^{\mu\nu} - h^2\right)\right]$$ Fierz-Pauli mass term

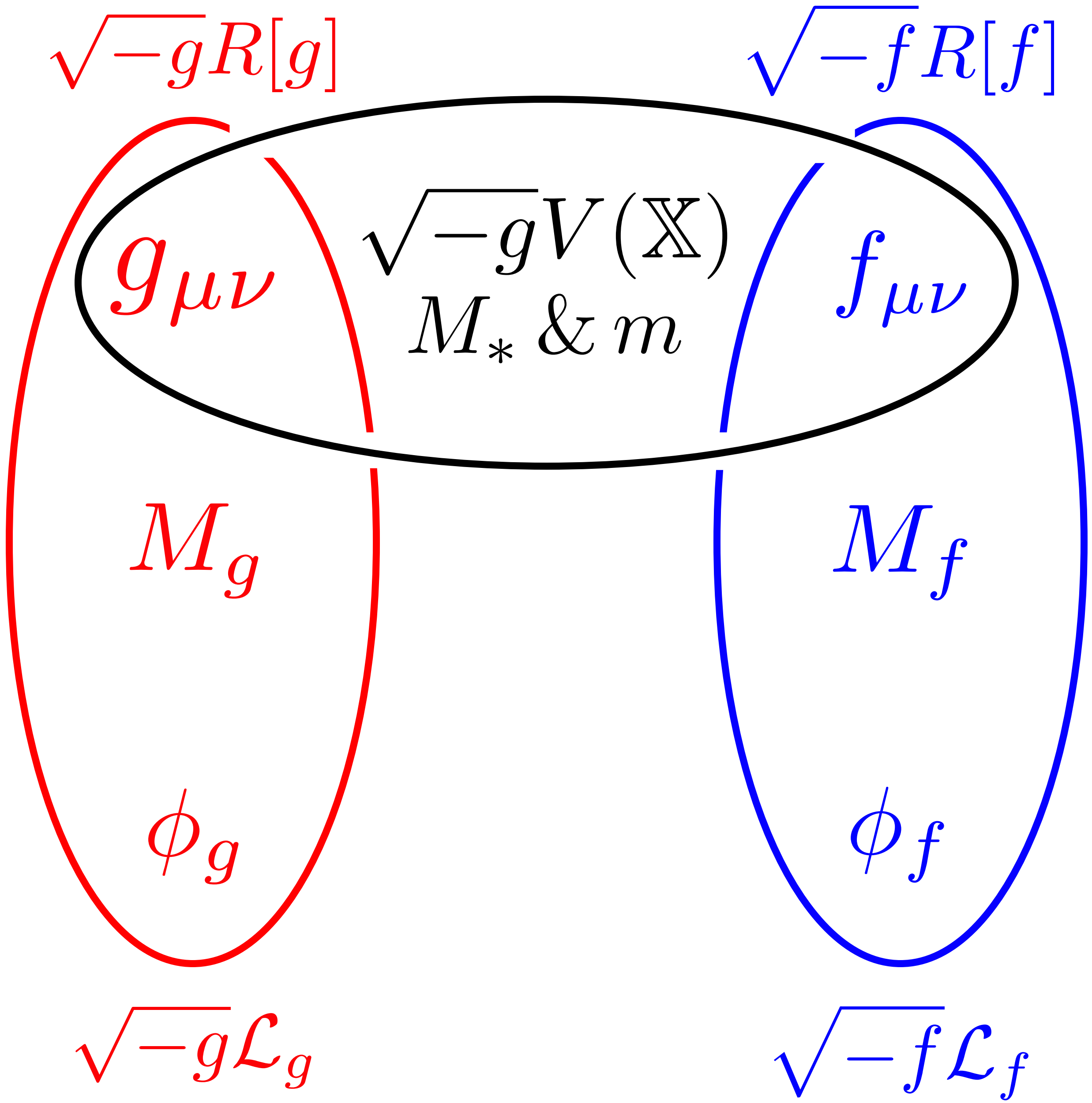
Boulware and Deser (1972) discovered that Fierz-Pauli tuning breaks down with generic nonlinear extensions.

Once thought that all Lorentz-invariant massive spin-2 theories were ghostly, until de Rahm-Gabadadze-Tolley (dRGT) introduced second “reference” metric, taken to be Minkowski. Have 2 metrics interacting via potential.

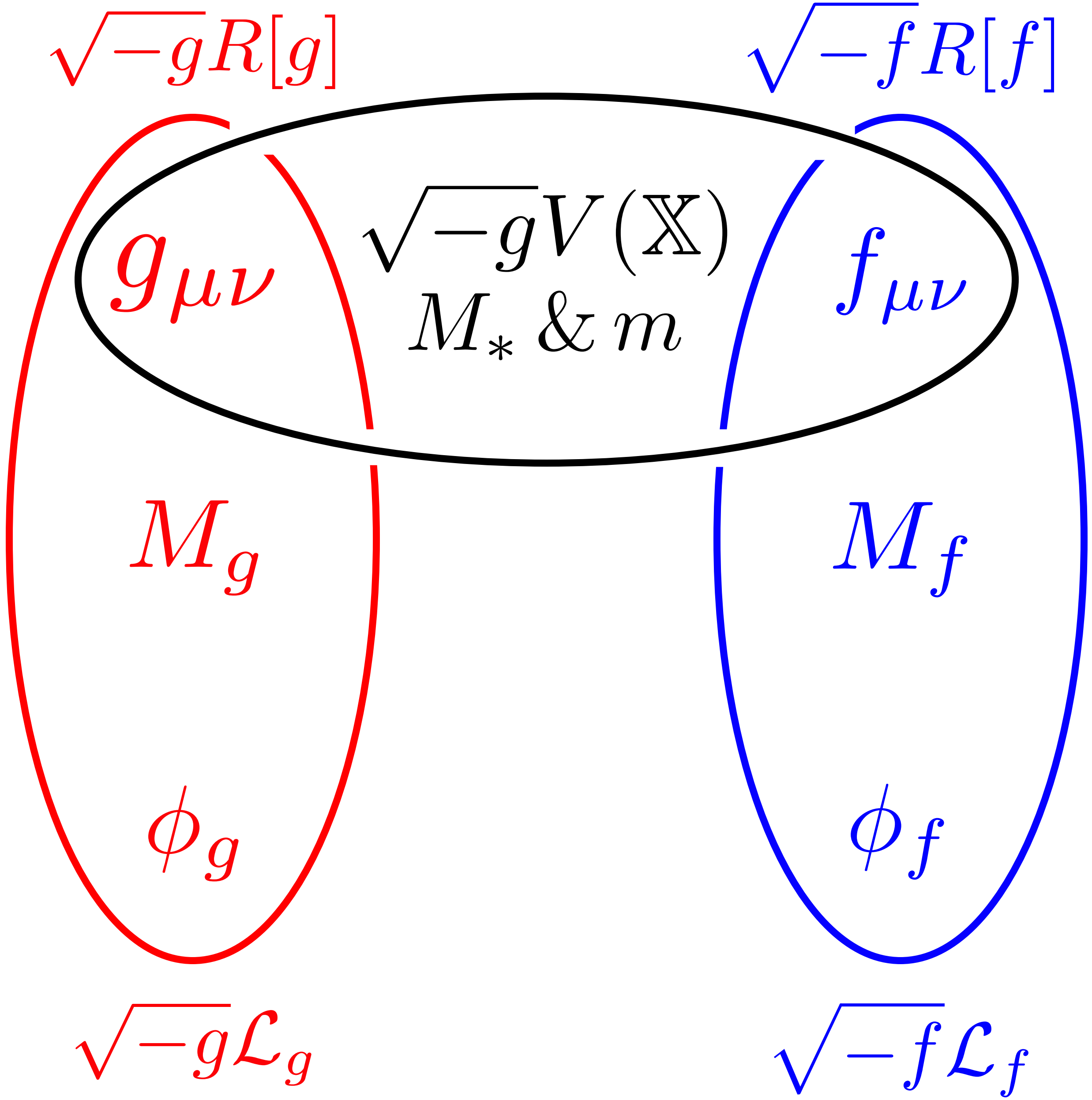
Extended/completed to general metric by Hassan & Rosen → ghost-free bigravity (2011).

This is our starting point. Field content: two metric fields, $g_{\mu\nu}$ and $f_{\mu\nu}$, coupling to two scalar fields, ϕ_g and ϕ_f.

Inflationary Bigravity



Inflationary Bigravity



Mirroring and Background Fields (bar denotes background)

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{2}{M_g} h_{\mu\nu}$$

$$f_{\mu\nu} = \bar{f}_{\mu\nu} + \frac{2}{M_f} k_{\mu\nu}$$

$$\bar{g}_{\mu\nu} = \bar{f}_{\mu\nu} = \mathrm{FRW}$$

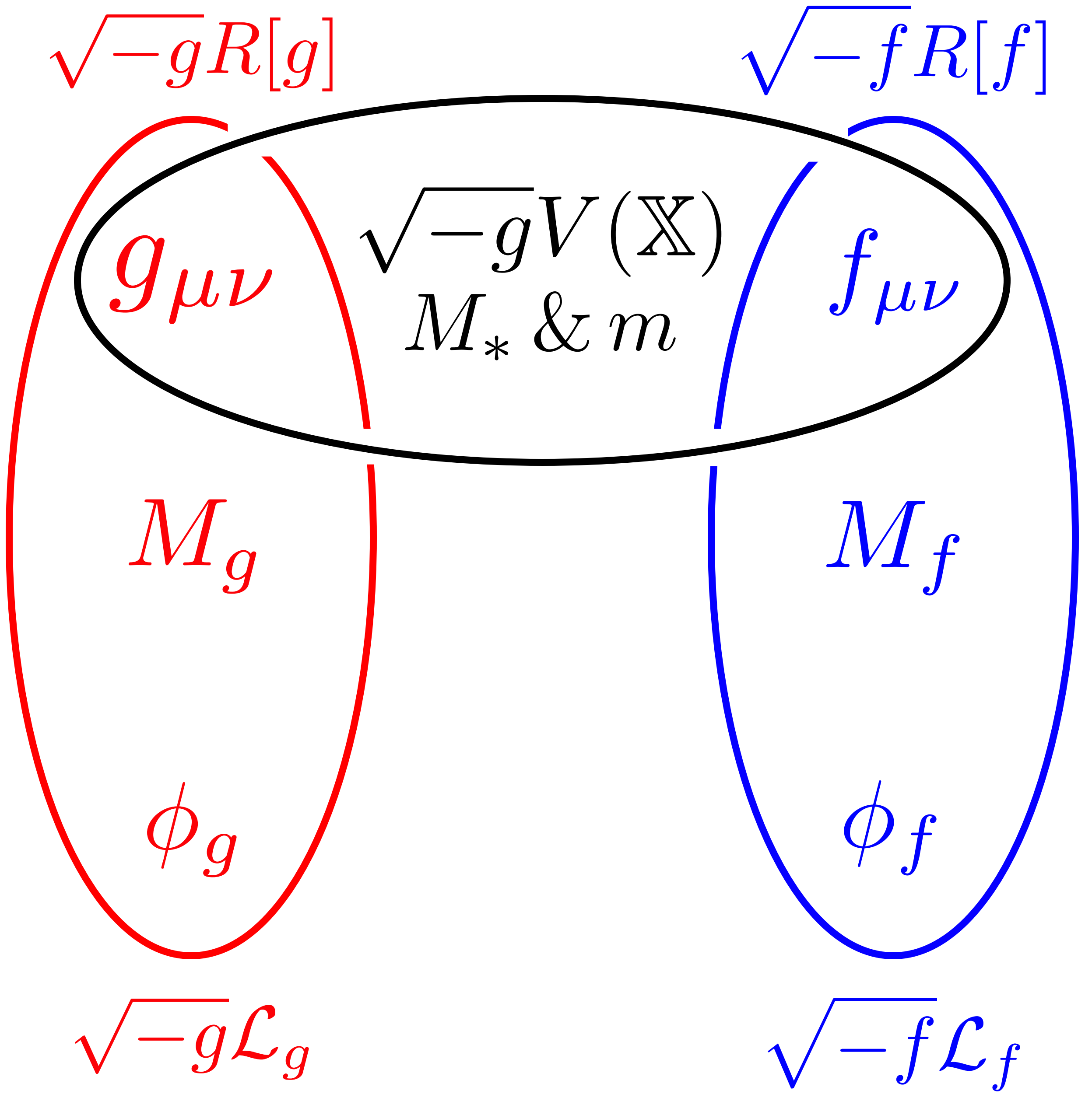
$$\phi_g = \bar{\phi}_g + \varphi_g$$

$$\phi_f = \bar{\phi}_f + \varphi_f$$

$$\frac{1}{M_g}\bar{\phi}_g = \frac{1}{M_f}\bar{\phi}_f \equiv \frac{1}{M_P}\bar{\phi}$$

$$\frac{1}{M_g^2} V_g\left(\frac{M_g}{M_P}\phi\right) = \frac{1}{M_f^2} V_f\left(\frac{M_f}{M_P}\phi\right) \equiv \frac{1}{M_P} V(\phi)$$

Inflationary Bigravity



Transform to mass eigenstates

$$\{h_{\mu\nu},\ k_{\mu\nu}\} \rightarrow \{u_{\mu\nu},\ v_{\mu\nu}\}$$

$$\frac{u_{\mu\nu}}{M_*} = \frac{h_{\mu\nu}}{M_f} + \frac{k_{\mu\nu}}{M_g}$$

$$\frac{v_{\mu\nu}}{M_*} = \frac{h_{\mu\nu}}{M_g} - \frac{k_{\mu\nu}}{M_f}$$

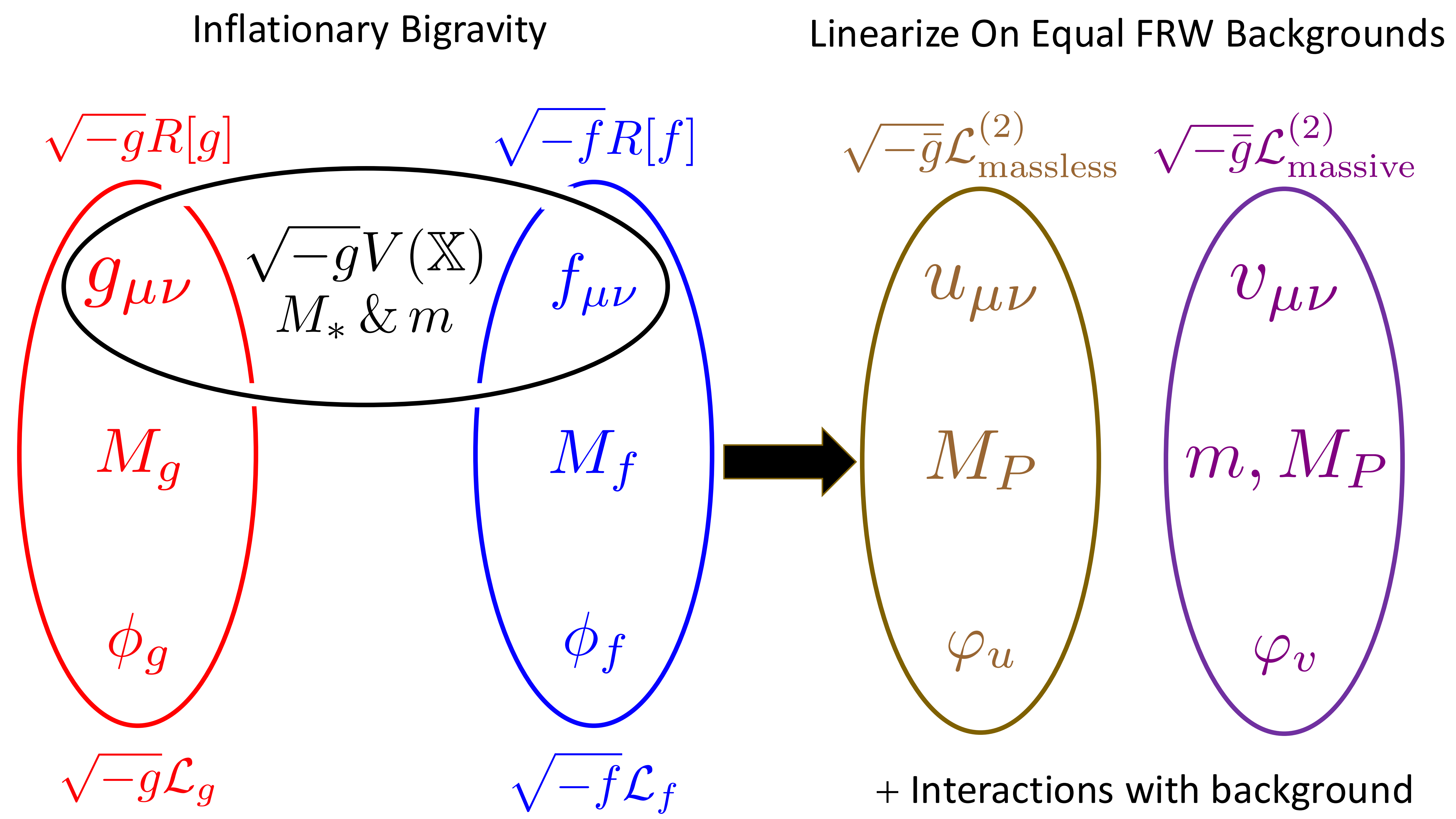
$$\{\varphi_g,\ \varphi_f\} \rightarrow \{\varphi_u,\ \varphi_v\}$$

$$\frac{\varphi_u}{M_*} = \frac{\varphi_g}{M_f} + \frac{\varphi_f}{M_g}$$

$$\frac{\varphi_v}{M_*} = \frac{\varphi_g}{M_g} - \frac{\varphi_f}{M_f}$$

$$M_P^2 = M_g^2 + M_f^2$$

$$M_*^2 = (M_g^{-2} + M_f^{-2})^{-1}$$


Inflationary Bigravity
Linearize On Equal FRW Backgrounds
$\sqrt{-g}R[g]$
$\sqrt{-f}R[f]$
$\sqrt{-g}V(\mathbb{X})$
$M_* \,\&\, m$
$g_{\mu\nu}$
M_g
ϕ_g
$f_{\mu\nu}$
M_f
ϕ_f
$\sqrt{-g}\mathcal{L}_g$
$\sqrt{-f}\mathcal{L}_f$
$\sqrt{-\bar{g}}\mathcal{L}^{(2)}_{\text{massless}}$
$\sqrt{-\bar{g}}\mathcal{L}^{(2)}_{\text{massive}}$
$u_{\mu\nu}$
M_P
φ_u
$v_{\mu\nu}$
m, M_P
φ_v
+ Interactions with background

Scalar/Vector/Tensor (SVT) Decomposition Of Massive Spin-2 Field

Represent 4-tensor by variables that transform under spatial rotations as 3–scalars/3–vectors/3–tensors

$$v_{00} = a^2 E \qquad v_{0i} = a^2 (\partial_i F + G_i) \qquad v_{ij} = a^2 (\delta_{ij} A + \partial_i \partial_j B + \partial_i C_j + \partial_j C_i + D_{ij})$$

Subject to transverse/traceless constraints on G, C, D:

At quadratic order S/V/T decouple

For S/V/T

1. Remove nondynamical DoFs.
2. Express in terms of Fourier modes.
3. Canonically normalize kinetic term.
4. Check for ghosts, gradient instabilities.
5. Find mode equations and ω_k.
6. Solve with appropriate boundary conditions.
7. Integrate over k.
8. Write paper.

$L_S = L_S(A, B, E, F, \varphi_v)$ and φ_u. After removing non-propagating DoFs, and defining $\hat{\varphi}_v = \varphi_v - A a^{-1} \bar{\phi}' / M_P H$

$$L_{S,k} = K_\varphi |\tilde{\hat{\varphi}}'_v|^2 - M_\varphi |\tilde{\hat{\varphi}}_v|^2 + K_B |\tilde{B}'|^2 - M_B |\tilde{B}|^2 + L_2 \tilde{\hat{\varphi}}^{*\prime}_v \tilde{B}' + L_1 \tilde{\hat{\varphi}}^*_v \tilde{B}' - L_0 \tilde{\hat{\varphi}}^*_v \tilde{B}$$

Field redefinition to diagonalize kinetic terms: $\{\tilde{\hat{\varphi}}_v, \tilde{B}\} \Rightarrow \{\tilde{\Pi}, \tilde{\mathcal{B}}\}$

$$L_{S,k} = K_\Pi |\tilde{\Pi}'|^2 - M_\Pi |\tilde{\Pi}|^2 + K_{\mathcal{B}} |\tilde{\mathcal{B}}'|^2 - M_{\mathcal{B}} |\tilde{\mathcal{B}}|^2 + \lambda_1 \tilde{\Pi}^* \tilde{\mathcal{B}}' - \lambda_0 \tilde{\Pi}^* \tilde{\mathcal{B}}$$

$$L_{S,k} = K_\varphi \, |\tilde{\hat{\varphi}}_v'|^2 - M_\varphi \, |\tilde{\hat{\varphi}}_v|^2 + K_B \, |\tilde{B}'|^2 - M_B \, |\tilde{B}|^2 + L_2 \, \tilde{\hat{\varphi}}_v^{*\prime} \tilde{B}' + L_1 \, \tilde{\hat{\varphi}}_v^{*} \tilde{B}' - L_0 \, \tilde{\hat{\varphi}}_v^{*} \tilde{B}$$

$$K_B = \frac{a^6 m^2}{8} \frac{\big(8m^2H^2 - 6H^2m_H^2 - m^2m_H^2\big)k^4}{H^2k^4 + 3a^2\big(m^2 - m_H^2\big)H^2k^2 + \frac{3}{8}a^4m^2\big(6m^2H^2 - 4H^2m_H^2 - m_H^4\big)} \tag{3.17c}$$

$$M_B = \frac{a^6 m^2}{8} \frac{c_{10}k^{10} + c_8k^8 + c_6k^6 + c_4k^4}{\big[H^2k^4 + 3a^2\big(m^2 - m_H^2\big)H^2k^2 + \frac{3}{8}a^4m^2\big(6m^2H^2 - 4H^2m_H^2 - m_H^4\big)\big]^2} \tag{3.17d}$$

$$\begin{aligned}
c_{10} &= H^2\big(8m^2H^2 - 8H^4 - 2H^2m_H^2 - m^2m_H^2\big) \\
c_8 &= a^2H^2\Big[\big(30m^4H^2 + 32m^2H^4 - 96H^6 - 3m^4m_H^2 - 56m^2H^2m_H^2 \\
&\qquad + 48H^4m_H^2 + 5m^2m_H^4 + 6H^2m_H^4\big) \\
&\qquad + \big(4m^2 - 24H^2\big)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2}\Big] \\
c_6 &= \tfrac{3}{8}a^4m^2\Big[\big(96m^4H^4 + 144m^2H^6 - 6m^4H^2m_H^2 - 252m^2H^4m_H^2 - 192H^6m_H^2 \\
&\qquad + 8m^2H^2m_H^4 + 200H^4m_H^4 - 10H^2m_H^6 - m^2m_H^6\big) \\
&\qquad + \big(8m^2m_H^2 - 16H^2m_H^2\big)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2}\Big] \\
c_4 &= \tfrac{3}{8}a^6m^4\Big[\big(36m^4H^4 - 48m^2H^6 + 64H^8 - 12m^2H^4m_H^2 - 32H^6m_H^2 \\
&\qquad - 12m^2H^2m_H^4 + 4H^4m_H^4 + 12H^2m_H^6 - 3m^2m_H^6 + 2m_H^8\big) \\
&\qquad - \big(24m^2H^2 - 16H^4 - 12m^2m_H^2 - 8H^2m_H^2 + 8m_H^4\big)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2}\Big]
\end{aligned}$$

$$L_2 = \frac{a^3m^2\bar\phi'}{2M_PH} \frac{H^2k^4 + \frac{3}{2}a^2\big(m^2 - m_H^2\big)H^2k^2}{H^2k^4 + 3a^2\big(m^2 - m_H^2\big)H^2k^2 + \frac{3}{8}a^4m^2\big(6m^2H^2 - 4H^2m_H^2 - m_H^4\big)} \tag{3.17e}$$

$$L_1 = -\frac{a^4m^2\bar\phi'}{M_P} \frac{\big(H^2 - \frac{1}{4}m_H^2 - \frac{1}{2}\frac{aHV'(\bar\phi)}{\bar\phi'}\big)k^4 - \frac{3}{2}a^2\big(m^2 - m_H^2\big)\big(H^2 + \frac{1}{4}m_H^2 + \frac{1}{2}\frac{aHV'(\bar\phi)}{\bar\phi'}\big)k^2}{H^2k^4 + 3a^2\big(m^2 - m_H^2\big)H^2k^2 + \frac{3}{8}a^4m^2\big(6m^2H^2 - 4H^2m_H^2 - m_H^4\big)} \tag{3.17f}$$

$$L_0 = \frac{a^3m^2\bar\phi'}{2M_PH} \frac{c_{10}k^{10} + c_8k^8 + c_6k^6 + c_4k^4 + c_2k^2}{\big[H^2k^4 + 3a^2\big(m^2 - m_H^2\big)H^2k^2 + \frac{3}{8}a^4m^2\big(6m^2H^2 - 4H^2m_H^2 - m_H^4\big)\big]^2} \tag{3.17g}$$

$$c_{10} = H^4 \tag{3.17h}$$

$$\begin{aligned}
c_8 &= \tfrac{1}{2}a^2H^4\Big[\big(9m^2 + 12H^2 - 13m_H^2\big) - 4\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big] \\
c_6 &= \tfrac{3}{8}a^4H^2\Big[\big(18m^4H^2 + 32m^2H^4 + 64H^6 - 48m^2H^2m_H^2 - 64H^4m_H^2 \\
&\qquad + m^2m_H^4 + 28H^2m_H^4\big) \\
&\qquad + 8\big(-4m^2H^2 + 4H^4 + m^2m_H^2\big)\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big] \\
c_4 &= \tfrac{3}{16}a^6m^2H^2\Big[\big(18m^4H^2 - 24m^2H^4 + 256H^6 - 54m^2H^2m_H^2 - 160H^4m_H^2 \\
&\qquad + 9m^2m_H^4 + 60H^2m_H^4 - 7m_H^6\big) \\
&\qquad + 4\big(-30m^2H^2 + 32H^4 + 12m^2m_H^2 + 4H^2m_H^2 - 7m_H^4\big)\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big] \\
c_2 &= \tfrac{9}{16}a^8m^4H^2\big(2H^2 - m_H^2\big)\Big[-\big(4H^2 + m_H^2\big)\big(3m^2 - 4H^2 - m_H^2\big) \\
&\qquad + 4\big(-3m^2 + 2H^2 + 2m_H^2\big)\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big]
\end{aligned}$$

$$K_\varphi = \frac{a^2}{2} \frac{H^2k^4 + 3a^2\big(m^2 - m_H^2\big)H^2k^2 + \frac{9}{4}a^4m^2\big(m^2 - m_H^2\big)H^2}{H^2k^4 + 3a^2\big(m^2 - m_H^2\big)H^2k^2 + \frac{3}{8}a^4m^2\big(6m^2H^2 - 4H^2m_H^2 - m_H^4\big)} \tag{3.17a}$$

$$M_\varphi = \frac{a^2}{2} \frac{c_{10}k^{10} + c_8k^8 + c_6k^6 + c_4k^4 + c_2k^2 + c_0}{\big[H^2k^4 + 3a^2\big(m^2 - m_H^2\big)H^2k^2 + \frac{3}{8}a^4m^2\big(6m^2H^2 - 4H^2m_H^2 - m_H^4\big)\big]^2} \tag{3.17b}$$

$$\begin{aligned}
c_{10} &= H^4 \\
c_8 &= \tfrac{1}{2}a^2H^2\Big[\big(12m^2H^2 + 8H^4 - 14H^2m_H^2 - m_H^4\big) + 4\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} + 2H^2V''(\bar\phi)\Big] \\
c_6 &= \tfrac{3}{8}a^4H^2\Big[\big(36m^4H^2 + 72m^2H^4 - 82m^2H^2m_H^2 - 64H^4m_H^2 \\
&\qquad - 7m^2m_H^4 + 40H^2m_H^4 + 8m_H^6\big) \\
&\qquad + 8\big(3m^2 - 4m_H^2\big)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\qquad + 16\big(m^2 - m_H^2\big)H^2V''(\bar\phi)\Big] \\
c_4 &= \tfrac{3}{8}a^6\Big[4H^2\big(9m^6H^2 + 36m^4H^4 + 16m^2H^6 - 30m^4H^2m_H^2 - 76m^2H^4m_H^2 \\
&\qquad - 3m^4m_H^4 + 31m^2H^2m_H^4 + 24H^4m_H^4 + 6m^2m_H^6 - 6H^2m_H^6 - 3m_H^8\big) \\
&\qquad - 4m^2H^2\big(H^2 - m_H^2\big)\tfrac{V'(\bar\phi)^2}{M_P^2} \\
&\qquad + \big(36m^4H^2 + 8m^2H^4 - 94m^2H^2m_H^2 + m^2m_H^4 + 48H^2m_H^4\big)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\qquad + \big(36m^4H^2 - 58m^2H^2m_H^2 - m^2m_H^4 + 24H^2m_H^4\big)H^2V''(\bar\phi)\Big] \\
c_2 &= \tfrac{9}{32}a^8m^2\Big[H^2\big(18m^6H^2 + 120m^4H^4 + 128m^2H^6 - 78m^4H^2m_H^2 - 384m^2H^4m_H^2 \\
&\qquad - 9m^4m_H^4 + 132m^2H^2m_H^4 + 128H^4m_H^4 + 23m^2m_H^6 - 32H^2m_H^6 - 16m_H^8\big) \\
&\qquad - 8H^2\big(2m^2H^2 - 2m^2m_H^2 + m_H^4\big)\tfrac{V'(\bar\phi)^2}{M_P^2} \\
&\qquad + 4\big(6m^4H^2 - 22m^2H^2m_H^2 + m^2m_H^4 + 14H^2m_H^4\big)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\qquad + 4\big(m^2 - m_H^2\big)\big(12m^2H^2 - 10H^2m_H^2 - m_H^4\big)H^2V''(\bar\phi)\Big] \\
c_0 &= \tfrac{27}{32}a^{10}m^4\Big[-2H^2\big(2m^2H^2 - 2m^2m_H^2 + m_H^4\big)\tfrac{V'(\bar\phi)^2}{M_P^2} \\
&\qquad - m^2\big(2H^2 - m_H^2\big)\big(4H^2 + m_H^2\big)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\qquad + \big(m^2 - m_H^2\big)\big(6m^2H^2 - 4H^2m_H^2 - m_H^4\big)H^2V''(\bar\phi)\Big]
\end{aligned}$$

Should have listened to Ruth!

Fierz-Pauli field in FRW background

Degrees of freedom: $u_{\mu\nu}$ (tensor) , φ_u , $v_{\mu\nu}$ (tensor, vector) , Π , $\mathcal{B}$

No DoF left behind: $2 \quad + \; 1 \; + \quad 2 \quad + \; 2 \quad +1+1 \; = \; 9$

$$K_\Pi = \frac{a^2}{2}\frac{H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{9}{4}a^4m^2\left(m^2 - m_H^2\right)H^2}{H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)}$$

$$K_{\mathcal{B}} = \frac{3a^6m^2(m^2 - m_H^2)}{4k^4 + 12a^2(m^2 - m_H^2)k^2 + 9a^4m^2(m^2 - m_H^2)} \qquad m_H^2(\eta) = 2H^2(\eta)\left[1 - \epsilon(\eta)\right] \qquad \epsilon(\eta) = -H'/(aH^2)$$

If $m < m_H(\eta)$, theory propagates a ghost in $\tilde{\mathcal{B}}$ (spin–2 sector)!

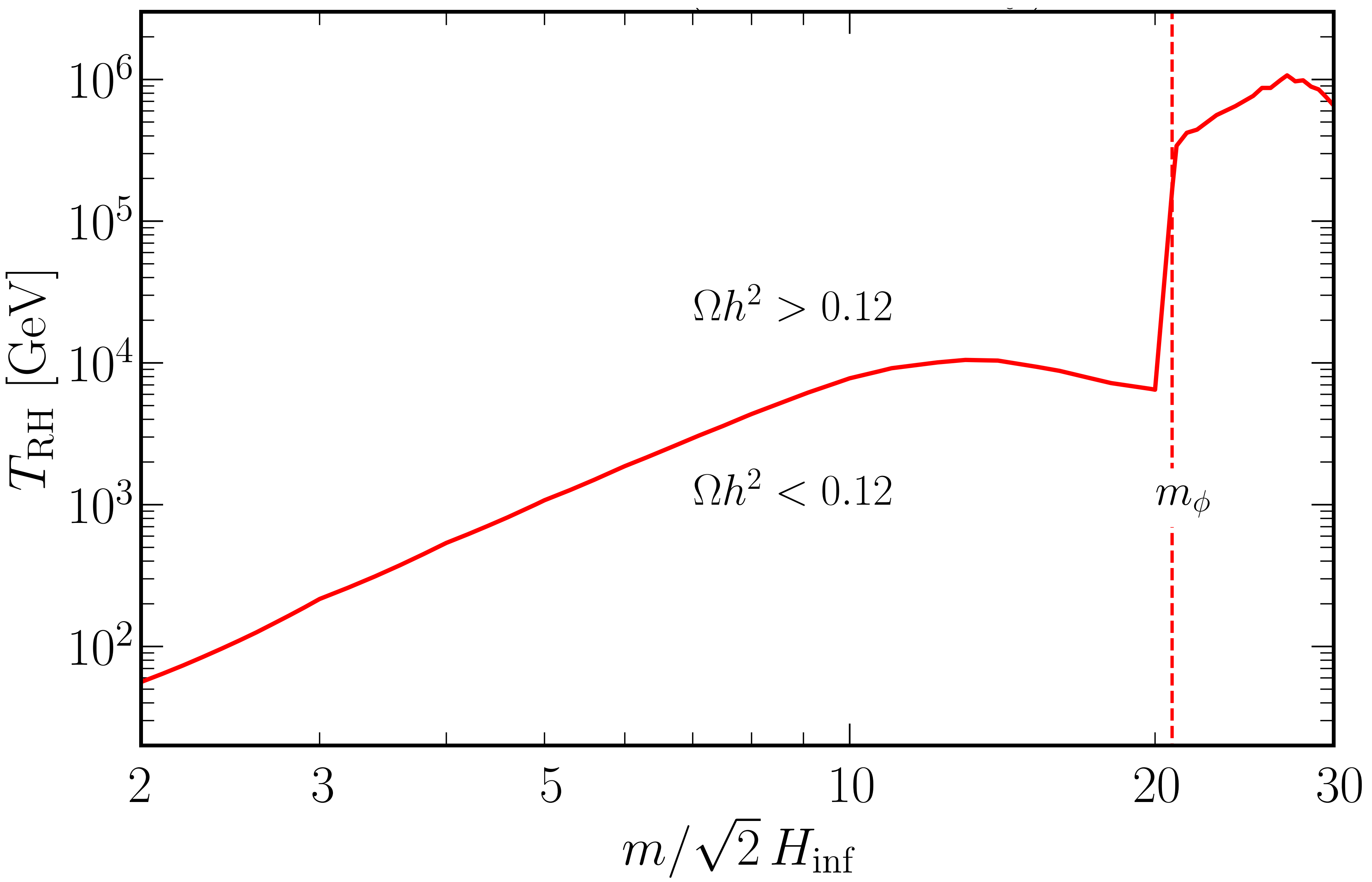
In 1986 Higuchi studied perturbations of massive gravity on a de Sitter background and found a ghost if $m^2 < 2H^2$.

We find a ghost in a general FRW background if $m^2 < 2H^2(\eta)\,[\,1 - \epsilon(\eta)\,]$. (In dS $\epsilon = 0$.)

In dS $m^2 = 2H^2$ is a “partially massless” point: mass term also vanishes.

FRW ghost is not generally a “partially massless” point.

Fierz-Pauli field in FRW background



Fierz-Pauli field in FRW background

- First comprehensive study of CGPP of massive spin-2 fields.
- We employ ghost-free bigravity: two spin-2 fields $+ 2$ inflatons. CMB implications?
- Calculated CGPP.
- Can massive spin-2 field be stable; if so, DM candidate (WIMPzilla).
- If massive spin-2 field unstable but long lived, decay could have interesting effects (baryogenesis, entropy generation, decay produces DM, …).
- Also studied another bigravity model (nonminimal) that is stable—promising DM candidate.
- Derived generalized FRW Higuchi bound.
- Studied ghosts (and gradient instabilities for nonminimal model). What do they signify?

Quantum Field Theories in the Early Universe

How should one regard QFTs, perfectly healthy in Minkowski spacetime, but have issues in a non-pathological, classical gravitational background?

1. (H_e–dependent, T_{RH} –dependent, and spin –dependent) limits on stable particles masses from Ω.
   Is that an issue with the QFT, or just a result like $m_\nu \lesssim \mathrm{eV}$?

2. Stable, *minimally-coupled* scalars have infrared issues unless $m \sigma H_e$.
   Is that an issue with the QFT, or just "not in our universe"?

3. Dark photons have issues with runaway production if non-minimally coupled.
   Shared with massive Kalb-Ramond fields.

4. Massive Rarita-Schwinger fields can have catastrophic production unless $m \sigma H_e$.
   SUGRA people should pay attention.

5. Massive Fierz-Pauli fields can develop ghosts and gradient instabilities unless $m \sigma H_e$.
   Is there a better formulation of massive gravity?

6. Do we have to look at different gravity theories at high-energy.
   Torsion, contorted geometry (Mavromatos & Sarkar); disformal gravity (Hell).

7. Is there a Flatland Swampland?

https://louisianaswamp.com/

A swamp can be beautiful and teeming with life (that will sting, bite, or eat you)

Quantum Field Theories in the Early Universe

1. Well-behaved QFTs in Minkowski space can develop pathologies when promoted to FRW.
2. This is especially acute for "higher-spin" QFTs (1, $3/2$, 2, …).
3. And some funny business for spin-0.
4. Is there a swampland of Minkowskian QFTs?
5. Or should we just accept restrictions on parameters of the QFTs (mass, couplings, etc.).
6. I will not have time to review EFT cutoffs, strong-coupling linits, nonlinearities, etc.

More complete treatment in
Cosmological gravitational particle production
EWK and Andrew Long
Reviews of Modern Physics (to appear) 2312.09042

Kavli Institute
for Cosmological Physics
at The University of Chicago

Rocky Kolb
University of Chicago

King's College London
September 2024

Backup Slides

Inner Space/Outer Space Interface

Particle physics (**Inner Space**)
is required to understand the universe

- dark matter
- dark energy
- baryon asymmetry
- CMB fluctuations
- origin of structure

The universe (**Outer Space**)
is a particle accelerator

- big bang as particle accelerator
- limits on Beyond Standard Model physics
- long lifetime/path length
- stellar energy loss
- large B fields

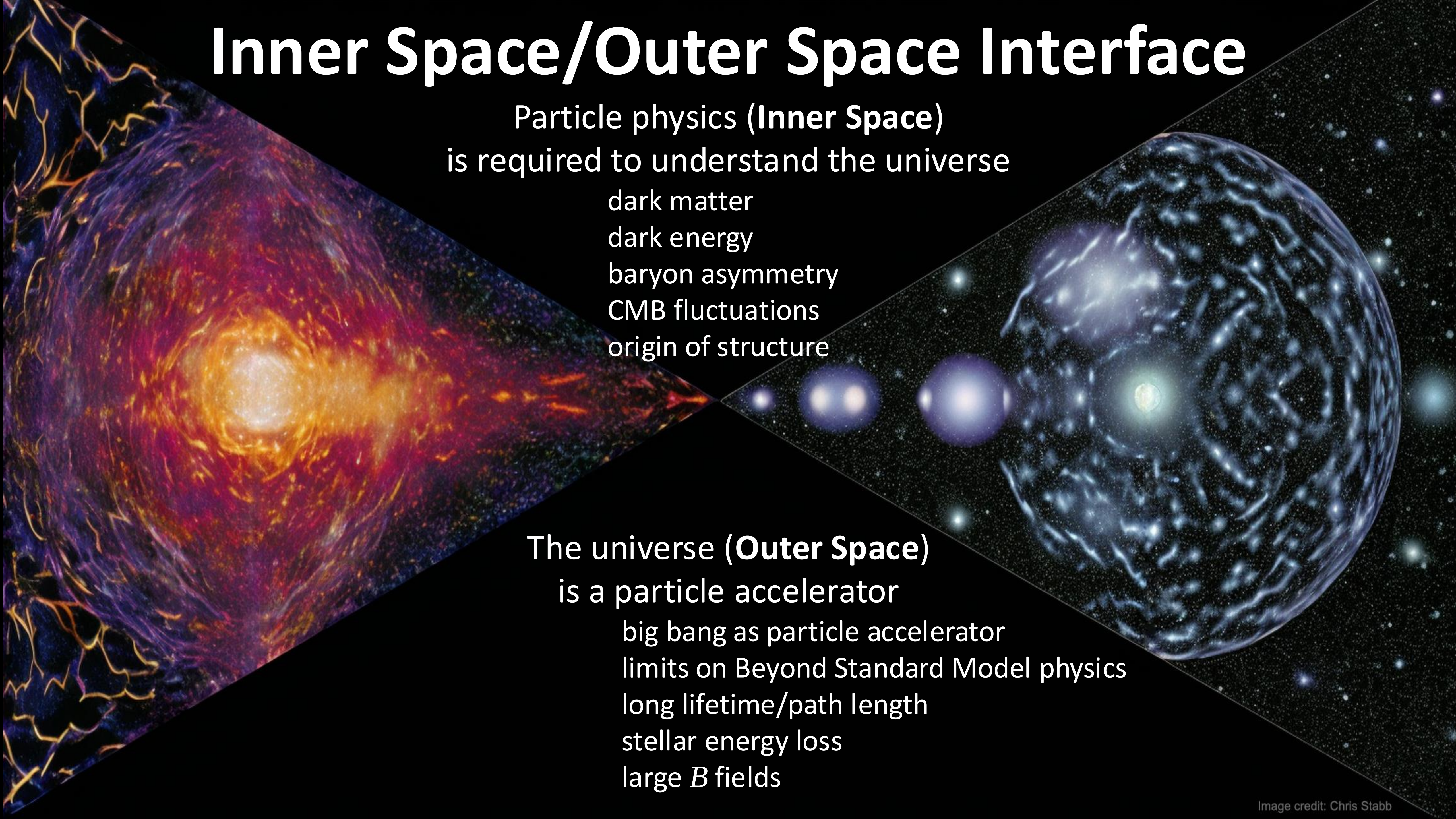


Image credit: Chris Stabb

Inner Space/Outer Space Interface

Big Bang as accelerator assumes

1. at some point temperature larger than some mass scale m
2. particle interacts with SM plasma

BUT

1. Maximum temperature of the radiation-dominated universe is the "reheat" temperature after inflation, $T_{\rm RH}$

   $T_{\rm RH}$ may be as low as $8\ \mathrm{MeV}$ (to set stage for BBN)!

2. What about particles with no SM interactions (or) interactions too weak to be populated in the primordial soup?

   (No evidence that dark matter interacts with SM particles)

Image credit: Chris Stabb

Bigravity With Minimal Coupling To Matter (Minimal Model)

$$S=\int \mathrm{d}^4x\left[\frac{M_g^2}{2}\sqrt{-g}\,R[g]+\frac{M_f^2}{2}\sqrt{-f}\,R[f]-m^2M_*^2\sqrt{-g}\,V(\mathbb{X};\beta_n)+\sqrt{-g}\,\mathcal{L}_g(g,\phi_g)+\sqrt{-f}\,\mathcal{L}_f(f,\phi_f)\right]$$

Kinetic terms for *f* and *g* + dRGT potential + Matter Lagrangians

dRGT Potential: $\mathbb{X}^\mu{}_\nu=(\sqrt{g^{-1}f})^\mu{}_\nu$

$$V(\mathbb{X};\beta_n)\equiv\sum_{n=0}^{4}\beta_nS_n(\mathbb{X})\,,\qquad S_n(\mathbb{X})\equiv\mathbb{X}^{\mu_1}_{[\mu_1}\ldots\mathbb{X}^{\mu_n}_{\mu_n]}$$

Matter Lagrangians:

$$\left.\begin{aligned}\mathcal{L}_g(g,\phi_g)&=-\tfrac{1}{2}g^{\mu\nu}\nabla_\mu\phi_g\nabla_\nu\phi_g-V_g(\phi_g)\\ \mathcal{L}_f(f,\phi_f)&=-\tfrac{1}{2}f^{\mu\nu}\nabla_\mu\phi_f\nabla_\nu\phi_f-V_f(\phi_f)\end{aligned}\right\}\ \text{Source FRW background}$$

After sausage making, want to end with: massless spin–2, massive spin–2, two scalar fields

DOFs: $2 + 5 + 2 = 9$

dRGT Potential

$$V(\mathbb{X};\beta_n) \equiv \sum_{n=0}^{4} \beta_n S_n(\mathbb{X})\,, \quad S_n(\mathbb{X}) \equiv \mathbb{X}^{\mu_1}_{[\mu_1} \ldots \mathbb{X}^{\mu_n}_{\mu_n]}\,, \quad \mathbb{X}^{\mu}{}_{\nu} = (\sqrt{g^{-1}f})^{\mu}{}_{\nu}$$

Five parameters: β_0 … $\beta_{4.}$ Only three combinations enter at quadratic order

$\beta_1 + 2\beta_2 + \beta_3 = 1$ (Normalizes Fierz-Pauli mass to be *m*)

$$\Lambda_g = m^2\big(\beta_0 + 3\beta_1 + 3\beta_2 + \beta_3\big)$$

$$\Lambda_f = m^2\big(\beta_1 + 3\beta_2 + 3\beta_3 + \beta_4\big)$$

$\Lambda = 0$; inflation driven by ϕ_g and ϕ_f

Three masses: $M_g^2,\ M_f^2,\ m^2$

$$M_*^2 = (M_g^{-2} + M_f^{-2})^{-1}$$

$$M_P^2 = M_g^2 + M_f^2$$

$$V(\mathbb{X};\beta_n) \equiv \sum_{n=0}^{4} \beta_n S_n(\mathbb{X})\,, \quad S_n(\mathbb{X}) \equiv \mathbb{X}^{\mu_1}_{[\mu_1} \ldots \mathbb{X}^{\mu_n}_{\mu_n]}\,, \quad \mathbb{X}^{\mu}{}_{\nu} = (\sqrt{g^{-1}f})^{\mu}{}_{\nu}$$

$$S_0(\mathbb{X}) = \tfrac{1}{0!}\left([\mathbb{X}]^0\right) \qquad [\mathbb{X}] \equiv \mathrm{Tr}(\mathbb{X})$$

$$S_1(\mathbb{X}) = \tfrac{1}{1!}\left([\mathbb{X}]^1\right)$$

$$S_2(\mathbb{X}) = \tfrac{1}{2!}\left([\mathbb{X}]^2 - [\mathbb{X}^2]\right)$$

$$S_3(\mathbb{X}) = \tfrac{1}{3!}\left([\mathbb{X}]^3 - 3[\mathbb{X}^2][\mathbb{X}] + 2[\mathbb{X}^3]\right)$$

$$S_4(\mathbb{X}) = \tfrac{1}{4!}\left([\mathbb{X}]^4 - 6[\mathbb{X}^2][\mathbb{X}]^2 + 8[\mathbb{X}^3][\mathbb{X}] + 3[\mathbb{X}^2]^2 - 6[\mathbb{X}^4]\right)$$

$$g_{\mu\nu} = f_{\mu\nu} \Rightarrow \mathbb{X}^{\mu}{}_{\nu} = \delta^{\mu}_{\nu}$$

$$S_0(\mathbb{X}) = 1$$

$$S_1(\mathbb{X}) = 4$$

$$S_2(\mathbb{X}) = 6$$

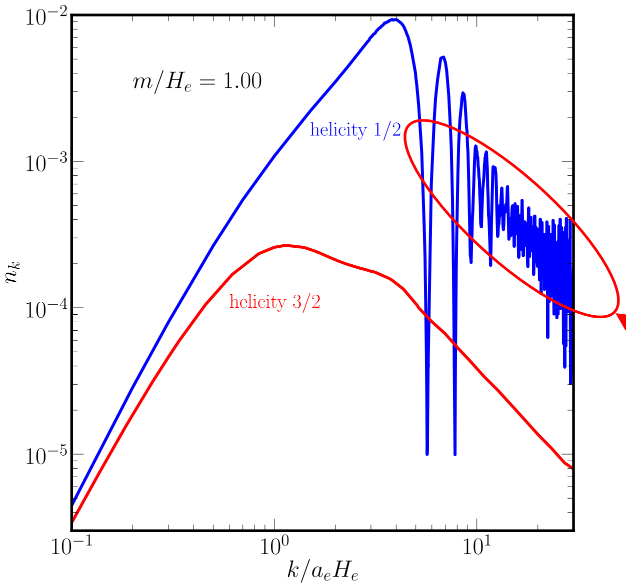
$$S_3(\mathbb{X}) = 4$$

$$S_4(\mathbb{X}) = 1$$

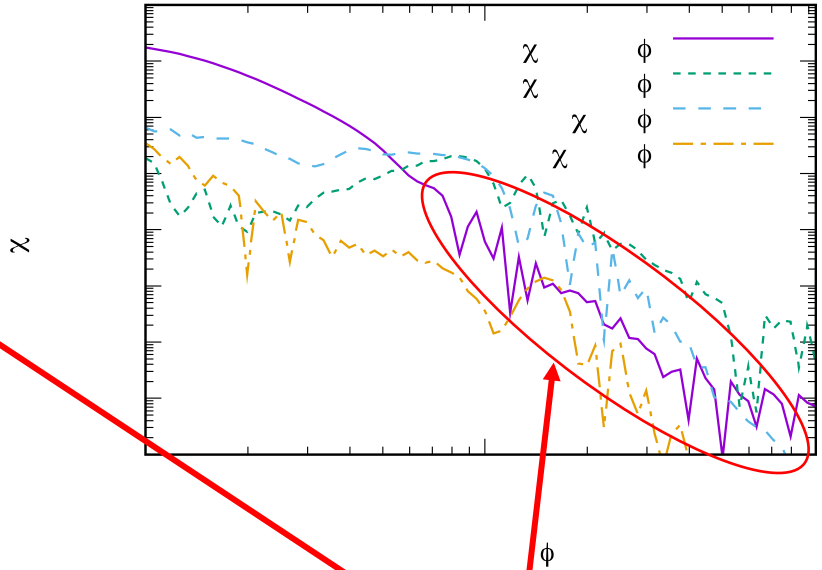
Quantum interference in gravitational particle production

(Basso, Chung, EWK, Long)

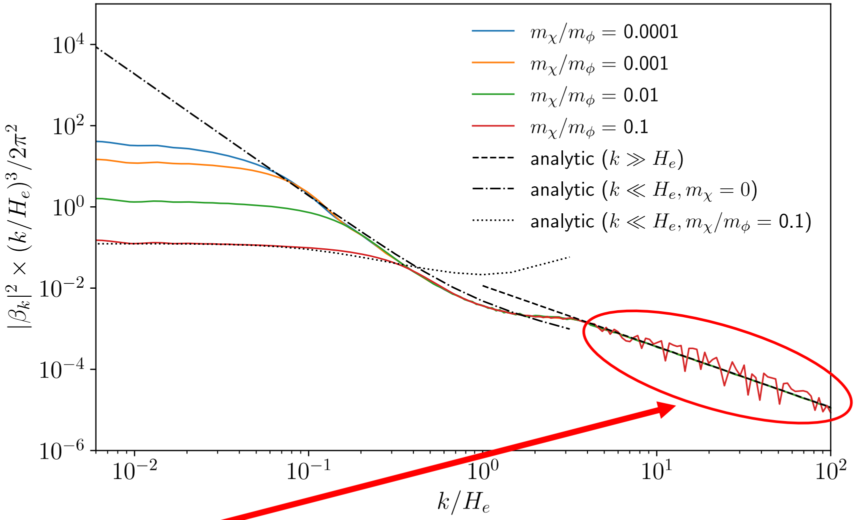
Catastrophic Production of Slow Gravitinos
EWK, Long, Mcdonough



Production of Purely Gravitational Dark Matter
Ema, Nakayama, Tang

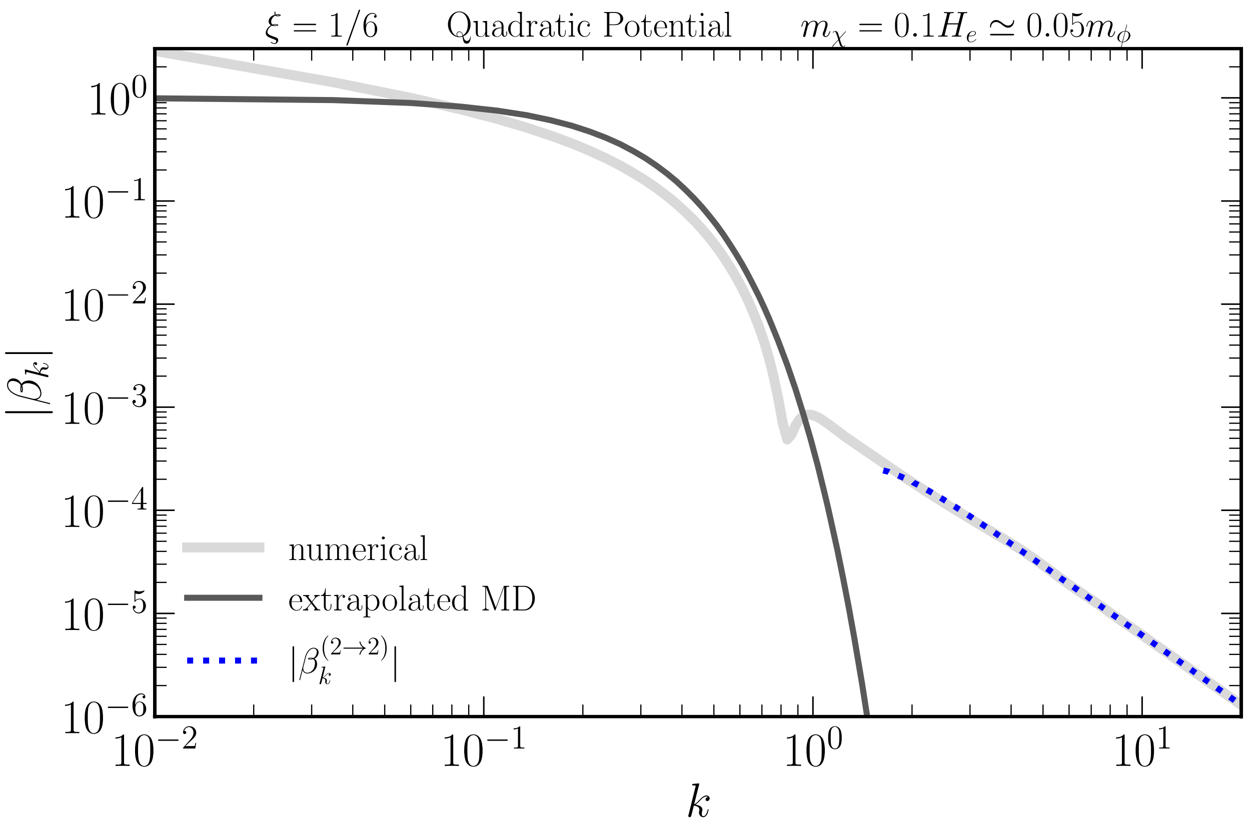


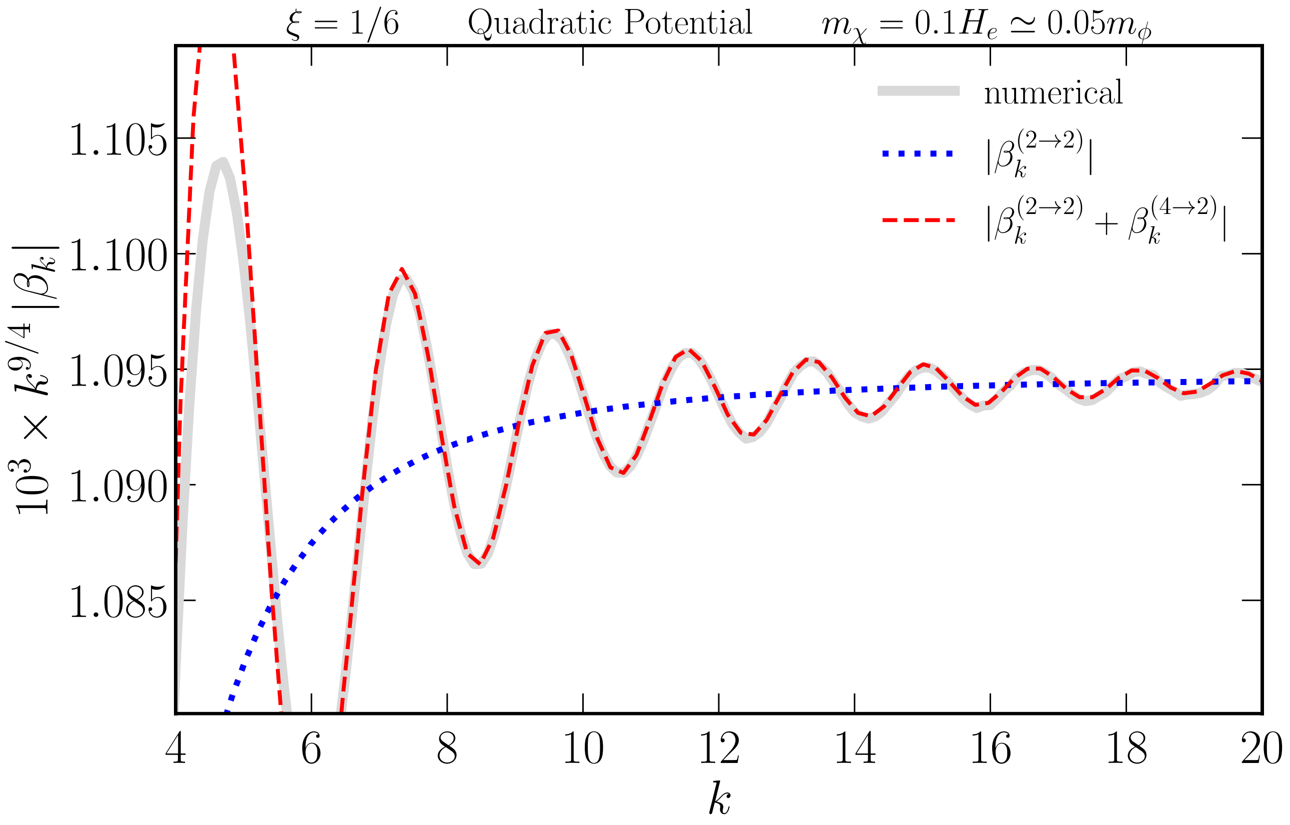
Boltzmann or Bogoliubov?
Kaneta, Mook, Oda



WTF? (Why These Features?) also, power-law decrease instead of exponential

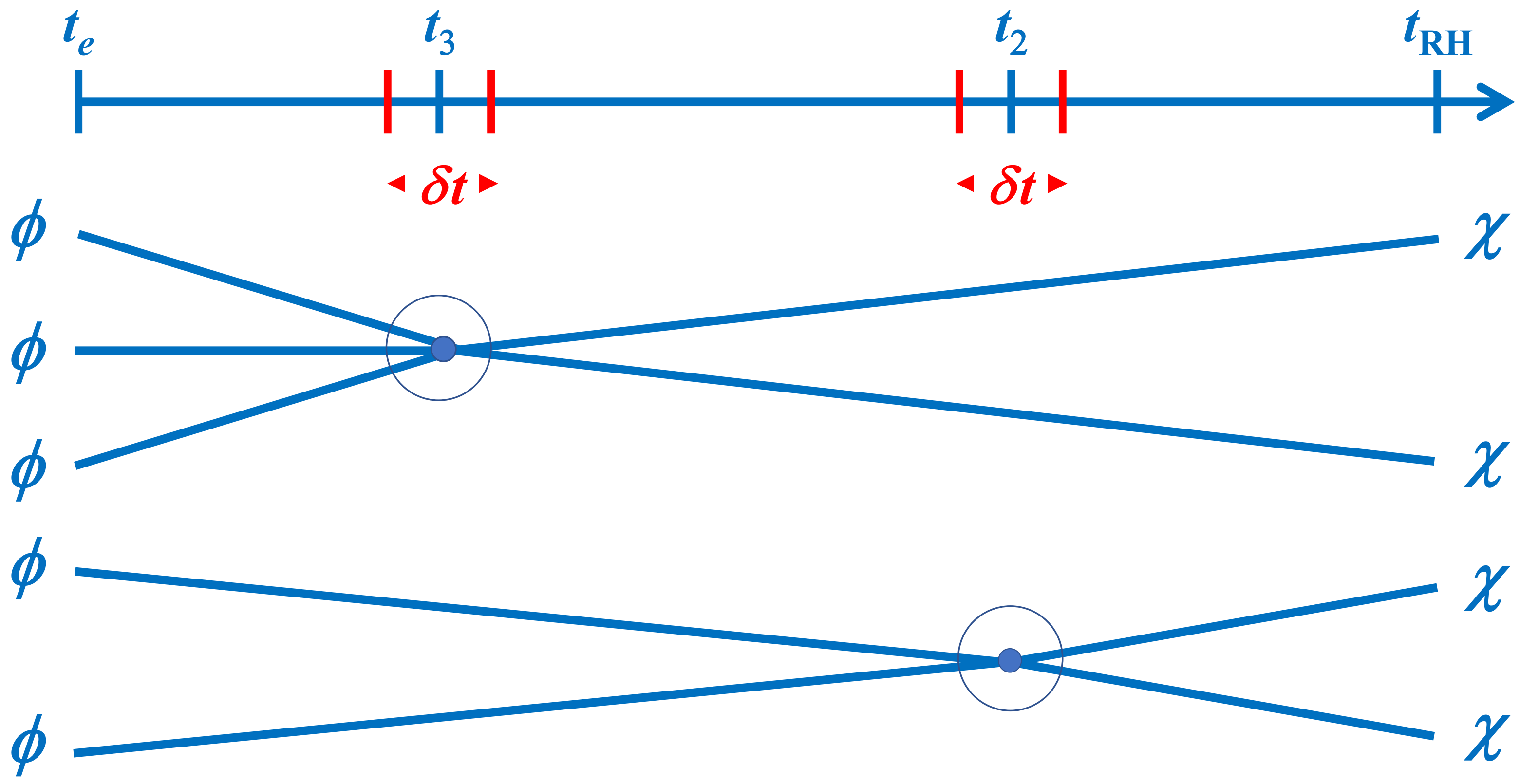
We argue that these features are due to the quantum interference of coherent scattering reactions. We find analytic formulae for the particle production amplitude for a conformally-coupled scalar field, including an interference effect in the kinematic region where the production can be interpreted as inflaton scattering into scalar final states via graviton exchange.



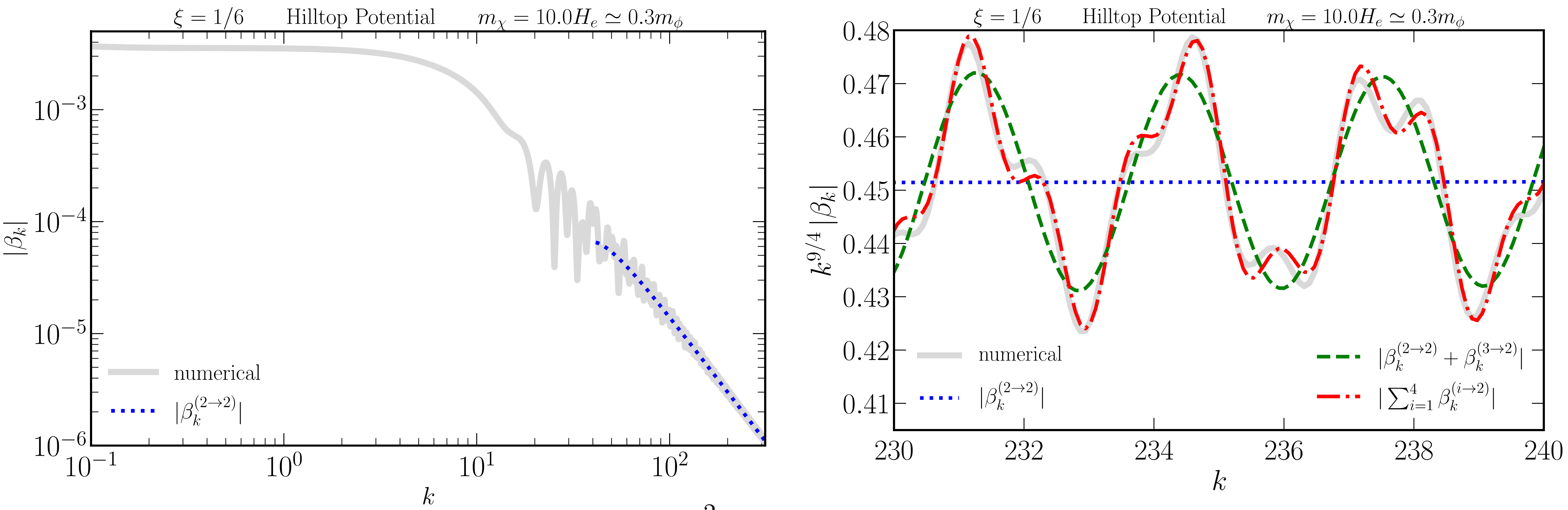


- Extrapolated MD (matter dominated-no inflaton oscillations) $|\beta_k| \propto \exp(-k^{3/2})$ for $k > 1$
- Numerical (quadratic inflaton potential with inflaton oscillations) $|\beta_k| \propto k^{-9/4}$ for $k > 1$
- Power-law behavior can be understood as $\phi + \phi \;\square\; \chi + \chi$ via a classical Boltzmann approach
- But, $\phi + \phi \;\square\; \chi + \chi$ via a classical Boltzmann approach cannot explain oscillations
- Oscillations due to quantum interference $|\, c_1\langle\chi\chi|U|\phi\phi\rangle + c_2\langle\chi\chi|U|\phi\phi\phi\phi\rangle \,|^2$

Quantum interference in gravitational particle production



- Initial macroscopic inflaton scattering state can be viewed as cold coherent superposition of $n\phi$ states
- Bogoliubov treatment allows processes that can be interpreted as $|\, c_1\langle\chi\chi|U|\phi\phi\rangle + c_2\langle\chi\chi|U|\phi\phi\phi\phi\rangle\,|^2$



- Hilltop inflaton potential $V \sim \left(1 - \frac{\phi^6}{v^6}\right)^2$ effective cubic term
- Quantum interference much more pronounced

Change Perturbation Variables: Massive and Massless Modes Decouple

$$\mathcal{L}^{(2)}_{\text{massless}} = \mathcal{L}^{(2)}_{uu} + \mathcal{L}^{(2)}_{u\varphi_u} + \mathcal{L}^{(2)}_{\varphi_u\varphi_u}$$

$$\begin{aligned}\mathcal{L}^{(2)}_{uu} = &-\tfrac{1}{2}\nabla_\lambda u_{\mu\nu}\nabla^\lambda u^{\mu\nu} + \nabla_\mu u^{\nu\lambda}\nabla_\nu u^\mu{}_\lambda \\ &-\nabla_\mu u^{\mu\nu}\nabla_\nu u + \tfrac{1}{2}\nabla_\mu u\nabla^\mu u \\ &+\left(\bar{R}_{\mu\nu} - M_P^{-2}\,\nabla_\mu\bar{\phi}\nabla_\nu\bar{\phi}\right) \\ &\quad\times\left(u^{\mu\lambda}u_\lambda{}^\nu - \tfrac{1}{2}u^{\mu\nu}u\right)\end{aligned}$$

$$\mathcal{L}^{(2)}_{\text{massive}} = \mathcal{L}^{(2)}_{vv} + \mathcal{L}^{(2)}_{v\varphi_v} + \mathcal{L}^{(2)}_{\varphi_v\varphi_v}$$

$$\begin{aligned}\mathcal{L}^{(2)}_{vv} = &-\tfrac{1}{2}\nabla_\lambda v_{\mu\nu}\nabla^\lambda v^{\mu\nu} + \nabla_\mu v^{\nu\lambda}\nabla_\nu v^\mu{}_\lambda \\ &-\nabla_\mu v^{\mu\nu}\nabla_\nu v + \tfrac{1}{2}\nabla_\mu v\nabla^\mu v \\ &+\left(\bar{R}_{\mu\nu} - M_P^{-2}\,\nabla_\mu\bar{\phi}\nabla_\nu\bar{\phi}\right) \\ &\quad\times\left(v^{\mu\lambda}v_\lambda{}^\nu - \tfrac{1}{2}v^{\mu\nu}v\right) \\ &\boxed{-\tfrac{1}{2}m^2\left(v^{\mu\nu}v_{\mu\nu} - v^2\right)}\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{(2)}_{u\,\varphi_u} = M_P^{-1}\Big[&\left(\nabla_\mu\bar{\phi}\nabla_\nu\varphi_u + \nabla_\nu\bar{\phi}\nabla_\mu\varphi_u\right) \\ &\times\left(u^{\mu\nu} - \tfrac{1}{2}\bar{g}^{\mu\nu}u\right) - V'(\bar{\phi})\varphi_u u\Big]\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{(2)}_{v\,\varphi_v} = M_P^{-1}\Big[&\left(\nabla_\mu\bar{\phi}\nabla_\nu\varphi_v + \nabla_\nu\bar{\phi}\nabla_\mu\varphi_v\right) \\ &\times\left(v^{\mu\nu} - \tfrac{1}{2}\bar{g}^{\mu\nu}v\right) - V'(\bar{\phi})\varphi_v v\Big]\end{aligned}$$

$$\mathcal{L}^{(2)}_{\varphi_u\varphi_u} = -\tfrac{1}{2}\nabla_\mu\varphi_u\nabla^\mu\varphi_u - \tfrac{1}{2}V''(\bar{\phi})\varphi_u^2$$

$$\mathcal{L}^{(2)}_{\varphi_v\varphi_v} = -\tfrac{1}{2}\nabla_\mu\varphi_v\nabla^\mu\varphi_v - \tfrac{1}{2}V''(\bar{\phi})\varphi_v^2$$

Tensor Sector (Prime Denotes ∂_η)

$$L_T = \frac{1}{2}a^2\Big[D'_{ij}D'_{ij} - \partial_k D_{ij}\partial_k D_{ij} - a^2m^2 D_{ij}D_{ij}\Big]$$

Canonically normalized kinetic term: $\chi_{ij} = aD_{ij}$

Fourier modes of $\chi_{ij}(\eta, \mathbf{x})_{\boldsymbol{\alpha}}\, \tilde{\chi}_{ij}(\eta, \mathbf{k})$; can take $\mathbf{k} = (0, 0, k)$

$$[\tilde{\chi}_{ij}] = \begin{bmatrix} \tilde{\chi}_+ & \tilde{\chi}_\times & 0 \\ \tilde{\chi}_\times & -\tilde{\chi}_+ & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\chi}''_{\substack{+\\ \times}}(\eta, k) + \omega_k^2(\eta)\, \tilde{\chi}_{\substack{+\\ \times}}(\eta, k) = 0$$

$$\omega_k^2(\eta) = k^2 + a^2m^2 - a''/a$$

If $m = 0$, mode equation for gravitational wave propagating on an FRW background, familiar from studies of tensor perturbations in inflation

Vector Sector (Prime Denotes ∂_η)

$$L_V = a^2\Big[\partial_j\big(G_i - C_i'\big)\partial_j\big(G_i - C_i'\big) + a^2m^2\big(G_iG_j - \partial_jC_i\partial_jC_i\big)\Big]$$

G_i not dynamical, can be Integrated out

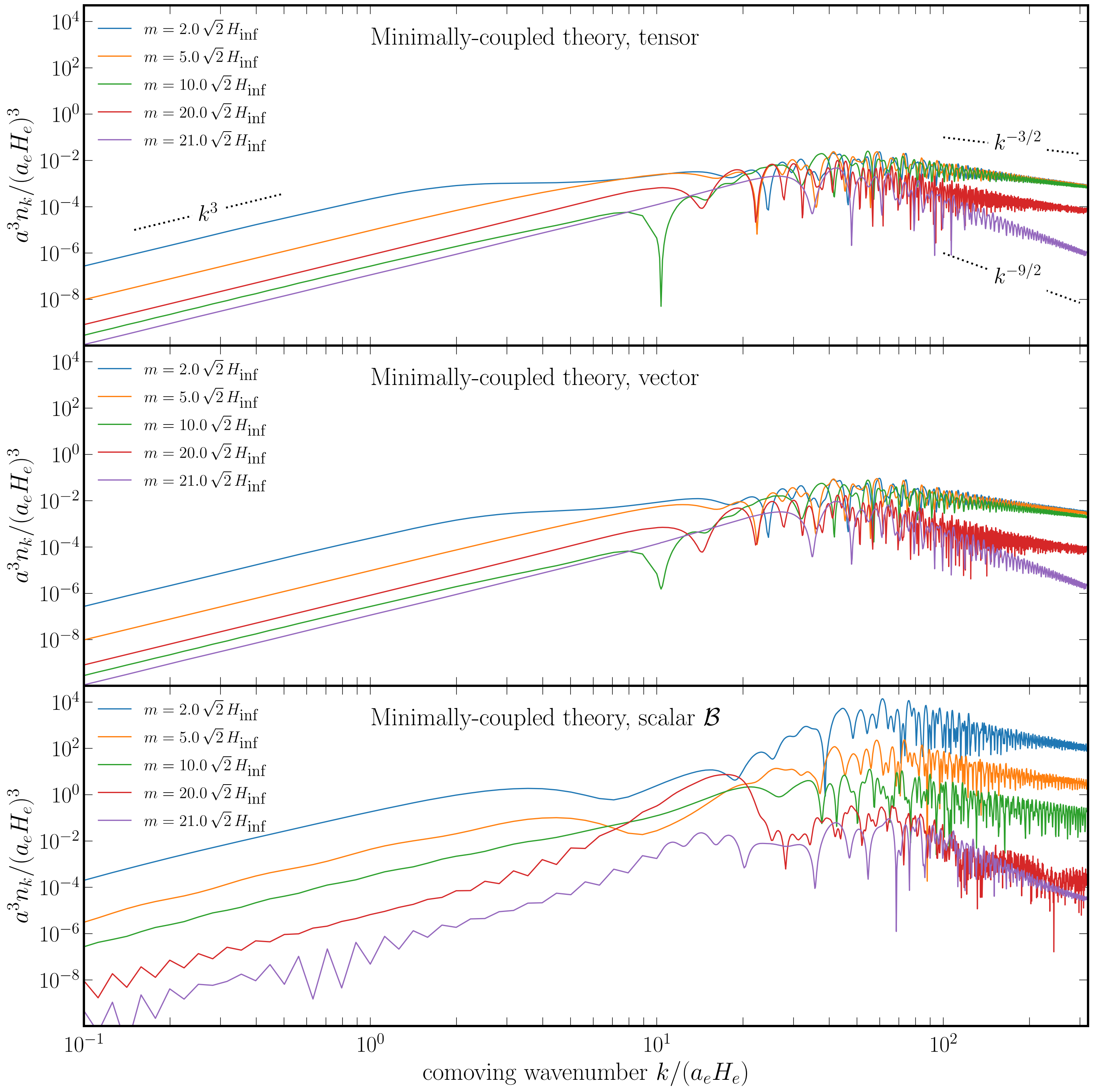
In Fourier space: $$L_{V,k} = \frac{a^4k^2m^2}{k^2+a^2m^2}|\tilde{C}_i'|^2 - a^4k^2m^2|\tilde{C}_i|^2$$

If $m=0$, Lagrangian vanishes trivially since massless theory does not propagate vector modes.

Canonically normalize, again taking $\mathbf{k}=(0,0,k)$, and defining $\tilde{\chi}_\pm(\eta,k) = (\tilde{\chi}_1 \mp i\tilde{\chi}_2)/\sqrt{2}$:

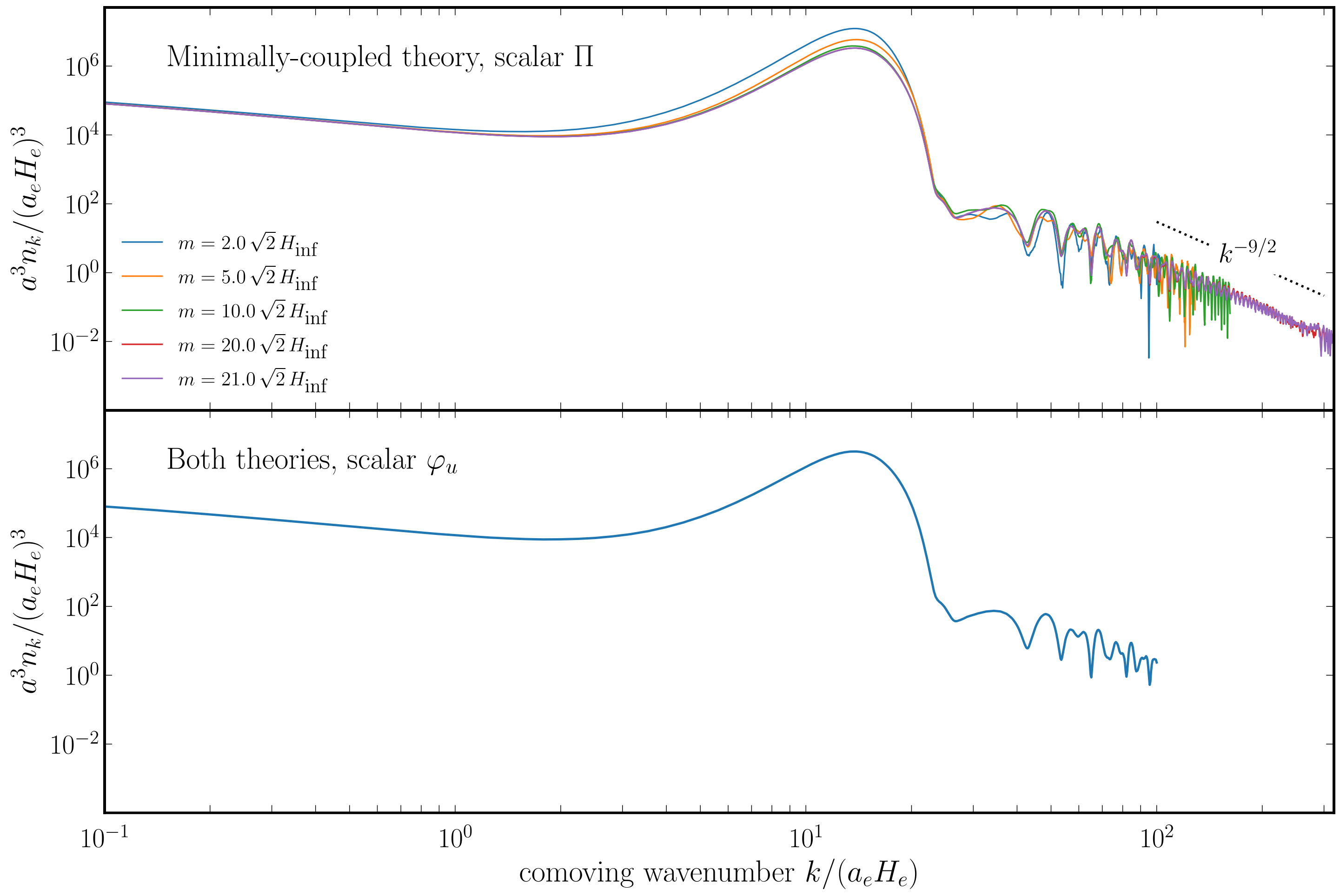
$$\tilde{\chi}_\pm''(\eta,k) + \omega_k^2(\eta)\,\tilde{\chi}_\pm(\eta,k) = 0$$
$$\omega_k^2(\eta) = k^2 + a^2m^2 - f''/f$$
$$f = a^2/\sqrt{k^2+a^2m^2}$$

Massive Spin–2 Spectra



- Modes with $k/a_eH_e < 1$ left horizon before end of inflation
- Modes with $k/a_eH_e > 1$ always sub-horizon
- Only consider non-ghostly masses
- Low-k oscillations explained
- High-k oscillations explained
  Basso, Chung, EWK, Long 2209.01713
- Low-k scaling (k^3) explained
- High-k scaling ($k^{-3/2}$ or $k^{-9/2}$) explained
  Basso, Chung, EWK, Long 2209.01713
- Scalar (helicity-0 mode) dominates

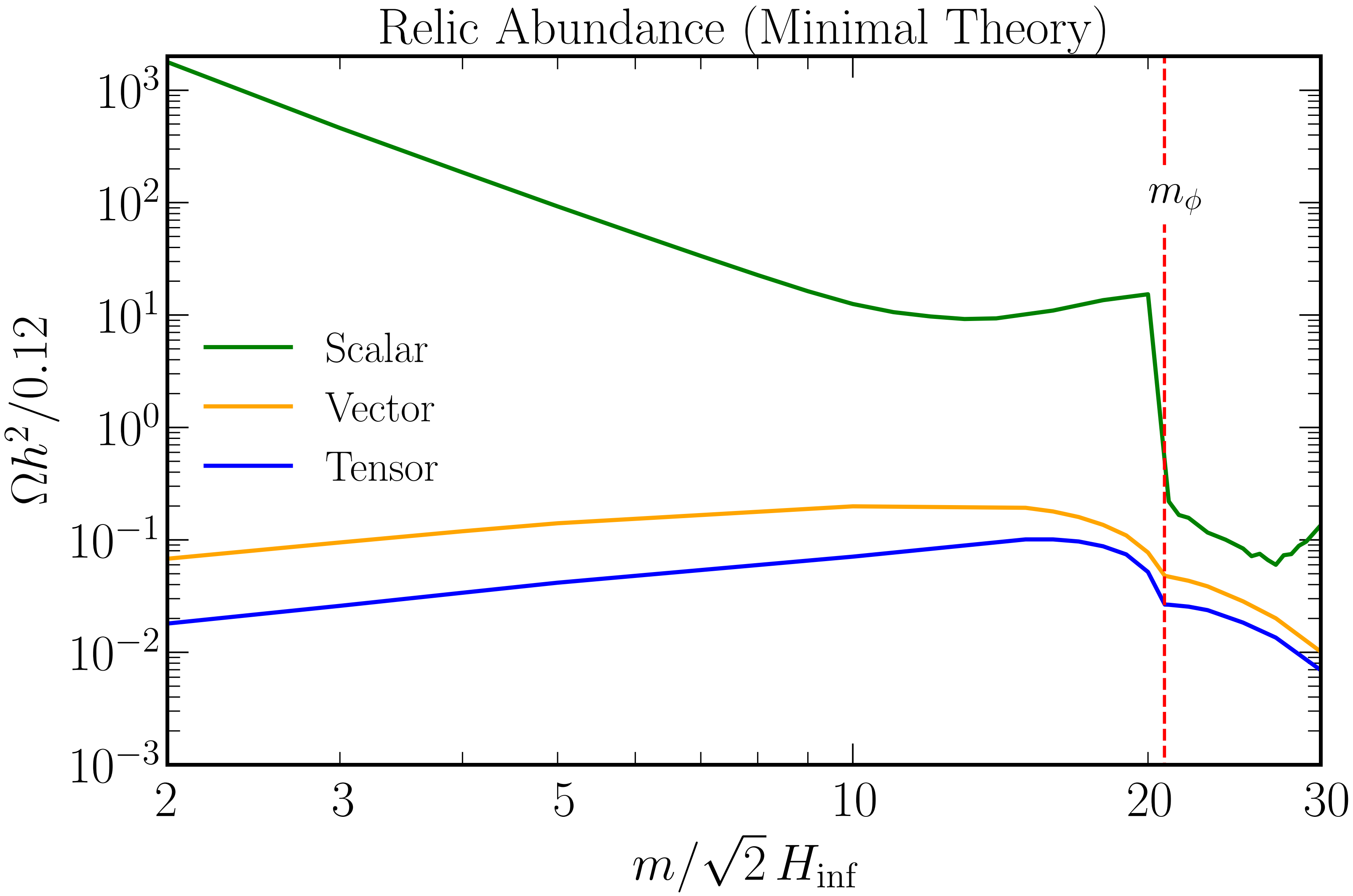
Inflaton Spectra



- Modes with $k/a_e H_e < 1$ left horizon before end of inflation
- Modes with $k/a_e H_e > 1$ always sub-horizon
- Only consider non-ghostly Π masses
- High-k oscillations explained
  Basso, Chung, EWK, Long 2209.01713
- Low-k scaling explained
- High-k scaling ($k^{-9/2}$) explained
  Basso, Chung, EWK, Long 2209.01713

Relic Abundance (Assuming Stable)

$$\frac{\Omega h^2}{0.12} \approx \frac{m}{10^{10}\mathrm{GeV}} \; \frac{H_e}{10^{10}\mathrm{GeV}} \; \frac{T_{\mathrm{RH}}}{10^8\mathrm{GeV}} \; \frac{a^3 n}{a_e^3 H_e^3}$$



For $T_{\mathrm{RH}} = 10^5$ GeV