

Bosonic Dark Matter

How to mix Particle Physics, Cosmology and Cold atoms

Alex Soto Science Gallery London – September 2024

Work with N. Proukakis and G. Rigopoulos:

Phys.Rev.D 108 (2023) 8, 083513 (ArXiv:2303.02049 [astro-ph.CO]) *Phys.Rev.D* 110 (2024) 2, 023504 (ArXiv:2311.05280 [astro-ph.CO]) ArXiv:2407.08860 [astro-ph.CO]

Motivation: Dark Matter

Evidences suggest the existence of Dark Matter, but what is it?

80 orders of magnitude

Motivation: Particles

Bosons Fermions

or

Motivation: Particles

Bosons

➢ QCD Axion R. D. Peccei and H. R. Quinn. (1977)

- o Scalar field (10−5 to 10−3 eV/c² , spin-0)
- o It solves the CP problem
- \triangleright Axion like particles A. Arvanitaki et al., arXiv:0905.4720 **[hep-th]**
	- o Motivated by String models (Axiverse)
	- o Wide range of masses

\triangleright Higher spin particles

 \checkmark FDM is a model with a non-relativistic ultralight bosonic particle (around 10⁻²² eV/c²)

$$
i\hbar \frac{\partial \Phi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Phi + m V \Phi
$$

$$
\nabla^2 V = 4\pi G m |\Phi|^2
$$

- \checkmark This simple model solves small scale problems of CDM:
	- ➢ Cusp-Core problem

ApJ **893**, 21 (2020),

- \checkmark This simple model solves small scale problems of CDM:
	- \triangleright Cusp-Core problem
	- \triangleright Missing satellites
	- \triangleright Etc.

Images from: Mocz *et al.*, Phys. Rev. D **97**, 083519 (2018), arXiv:1801.03507 **[astro-ph.CO]**

\checkmark FDM behaves as CDM at large scales

Image from: Ferreira E.G.M. *Astron Astrophys Rev* **29,** 7 (2021), arXiv:2005.03254 **[astro-ph.CO]**.

 \checkmark FDM is a model with a non-relativistic ultralight bosonic particle (around 10⁻²² eV/c²)

$$
i\hbar \frac{\partial \Phi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Phi + m V \Phi
$$

$$
\nabla^2 V = 4\pi G m |\Phi|^2
$$

Small masses give high occupation number \rightarrow Bose-Einstein Condensate

Motivation: Coherence

Motivation: Coherence

Similarities between Fuzzy Dark Matter and Ultracold Atom gases

Image from: N. Proukakis, G. Rigopoulos and A.S., *Phys. Rev. D* 108 (8 2023), p. 083513 arXiv:2303.02049 [astro-ph.CO]

The Model: Starting point

 \triangleright Our basic object is very general:

 $S = \int dt \left[\int d^3x \, i \psi^* \psi - H \right]$

 $H = \int d^3x \left(-\frac{1}{2m} \psi^* \nabla^2 \psi + V_{\text{ext}} \psi^* \psi \right) + \frac{1}{2} \int d^3x \int d^3x' \psi^*(x) \psi^*(x') U(x, x') \psi(x') \psi(x)$

Interaction term including contact and long-range: $U(x, x') = \alpha_1 \delta(x - x') + \alpha_2^2 \mathcal{O}^{-1}(x, x')$

 \triangleright We can always write this action using a Hubbard-Stratonovich transformation as

$$
S=\int d^4x \left(i\psi^*\dot{\psi}+\frac{1}{2m}\psi^*\nabla^2\psi-\frac{\alpha_1}{2}(\psi^*\psi)^2-V_{ext}\psi^*\psi+\frac{1}{2}V\mathcal{O}V-\alpha_2V\psi^*\psi\right)
$$

The Model: Starting point

$$
\triangleright \text{ Dipolar gases:} \quad \alpha_1 = g - \frac{C_{dd}}{3}, \qquad \alpha_2 = C_{dd}, \qquad \text{and} \qquad \mathcal{O} = C_{dd} \frac{\nabla^2}{(\mathbf{n} \cdot \nabla)^2}
$$

N. Proukakis, G. Rigopoulos and A.S., arXiv:2407.20178 **[cond-mat.quant-gas**]

$$
\triangleright \text{ Gravitational:} \quad \alpha_1 = g, \qquad \qquad \alpha_2 = m, \qquad \qquad \text{and} \qquad \qquad \mathcal{O} = \frac{1}{4\pi G} \, \nabla^2
$$

$$
S = \int d^4x \left(i\psi^* \dot{\psi} + \frac{1}{2m} \psi^* \nabla^2 \psi - \frac{g}{2} |\psi|^4 + \frac{1}{8\pi G} V \nabla^2 V - mV |\psi|^2 \right)
$$

N. Proukakis, G. Rigopoulos and A.S., *Phys. Rev. D* 108 (8 2023), p. 083513 arXiv:2303.02049 **[astro-ph.CO]** And arXiv:2407.08860 **[astro-ph.CO]**

Which is essentially the non-relativistic limit of

$$
S\!=\!\int d^4x\sqrt{-g}\!\left(\tfrac{1}{2\kappa^2}R-\tfrac{1}{2}g^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi-\tfrac{1}{2}m^2\phi^2-\tfrac{\lambda}{2}\phi^4\right)
$$

The Model: Starting point

To get our equations we split the field:

Image from: P. B. Blakie *Phys. Rev. E 7*8 (2 2008), p. 026704,

Similar in spirit to SPGPE models in cold atoms
arXiv:0803.3664 **[physics.comp-ph]**

Then we use Schwinger-Keldysh formalism, perturbation theory, approximations, Wigner transforms…

The Model: Main equations

We have three equations, one for the coherent part, one for the incoherent and one for the gravitational potential

$$
i\frac{\partial \Phi_0(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^2 + V_c(x)\right)\Phi_0(x) - iR\Phi_0(x) + \xi_1(x) - 2g\int d^4x' \Pi^R(x',x)V_{\text{nc}}(x')\Phi_0(x) + g\xi_2(x)\Phi_0(x)
$$

\n
$$
\frac{\partial f}{\partial t} + \frac{k}{m}\cdot\nabla f - \nabla V_{\text{nc}}\cdot\nabla_k f = \frac{1}{2}(I_a + I_b)
$$

\n
$$
\nabla^2 V^{\text{cl}}(x) = 4\pi G m\left(n_c(x) + \tilde{n}(x) + \frac{1}{2}\xi_2(x)\right) - 4\pi G m\int d^4x' \Pi^R(x',x)V_{\text{nc}}(x')
$$

\n
$$
V_c(x) = mV^{\text{cl}}(x) + g(n_c(x) + 2\tilde{n}(x))
$$

$$
V_{\rm nc}(x) = m V^{\rm cl}(x) + 2g(n_c(x) + \tilde{n}(x))
$$

$$
n_c = |\Phi_0|^2 \qquad \qquad \tilde{n} = \int \frac{d^3k}{(2\pi)^3} f
$$

Mean field potentials Coherent and incoherent number densities

The Model: FDM limit

For $g = 0$, order m, and all in the coherent part:

$$
i\frac{\partial \Phi_0(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^2 + V_c(x)\right)\Phi_0(x)
$$

 $\nabla^2 V^{\text{cl}}(x) = 4\pi G m n_c(x)$

 $V_c(x) = m V^{cl}(x)$

We recover FDM

$$
n_c = |\Phi_0|^2
$$

Mean field potentials Coherent and incoherent number densities

The Model: CDM limit

For $g = 0$, order m, and all in the non-coherent part:

$$
\frac{\partial f}{\partial t} + \frac{k}{m} \cdot \nabla f - \nabla V_{\text{nc}} \cdot \nabla_{\mathbf{k}} f = 0
$$

$$
\nabla^2 V^{\text{cl}}(x) = 4\pi G m \qquad \tilde{n}(x)
$$

We recover Vlasov-Poisson used in CDM

$$
V_{\rm nc}(x) = m V^{\rm cl}(x)
$$

$$
\tilde{n}=\int \! \frac{d^3k}{(2\pi)^3} f
$$

Mean field potentials Coherent and incoherent number densities

The Model: Main equations

 $i\frac{\partial \Phi_0(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^2 + V_c(x)\right)\Phi_0(x) - iR\Phi_0(x) + \xi_1(x) - 2g\int d^4x' \Pi^R(x',x)V_{\text{nc}}(x')\Phi_0(x) + g\xi_2(x)\Phi_0(x)$ $\frac{\partial f}{\partial t} + \frac{k}{m} \nabla f - \nabla V_{\text{nc}} \nabla_{\mathbf{k}} f = \frac{1}{2} (I_a + I_b)$ $\nabla^2 V^{cl}(x) = 4\pi G m (n_c(x) + \tilde{n}(x) + \frac{1}{2}\xi_2(x)) - 4\pi G m \int d^4x' \Pi^R(x',x) V_{nc}(x')$

The Model: Particle collisions

 $i\frac{\partial \Phi_0(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^2 + V_c(x)\right)\Phi_0(x) - iR\Phi_0(x) + \xi_1(x) - 2g\int d^4x' \Pi^R(x',x)V_{\text{nc}}(x')\Phi_0(x) + g\xi_2(x)\Phi_0(x)$

 $\frac{\partial f}{\partial t} + \frac{\mathbf{k}}{m} \cdot \nabla f - \nabla V_{\text{nc}} \cdot \nabla_{\mathbf{k}} f = \frac{1}{2} (I_a + I_b)$ Collisional terms

 $\nabla^2 V^{cl}(x) = 4\pi G m (n_c(x) + \tilde{n}(x) + \frac{1}{2}\xi_2(x)) - 4\pi G m \int d^4x' \Pi^R(x',x) V_{\text{nc}}(x')$

Image from: N. Proukakis and B. Jackson *J. Phys. B : At. Mol. Opt. Phys 41* (2008), arXiv:0810.0210 **[cond-mat.other]**

The Model: Particle collisions

 $i\frac{\partial \Phi_0(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^2 + V_c(x)\right)\Phi_0(x) - iR\Phi_0(x) + \xi_1(x) - 2g\int d^4x' \Pi^R(x',x)V_{\text{nc}}(x')\Phi_0(x) + g\xi_2(x)\Phi_0(x)$

 $\frac{\partial f}{\partial t} + \frac{\mathbf{k}}{m} \cdot \nabla f - \nabla V_{\text{nc}} \cdot \nabla_{\mathbf{k}} f = \frac{1}{2} (I_a + I_b)$ Collisional terms

 $\nabla^2 V^{\text{cl}}(x) = 4\pi G m (n_c(x) + \tilde{n}(x) + \frac{1}{2}\xi_2(x)) - 4\pi G m \int d^4x' \Pi^R(x',x) V_{\text{nc}}(x')$

$$
I_b = 4 g^2 \int \frac{d^3 p_2 d^3 p_3 d^3 p_4}{(2\pi)^5 \hbar^7} \delta(\varepsilon_{\mathbf{p}_3} + \varepsilon_{\mathbf{p}_4} - \varepsilon_{\mathbf{p}_2} - \varepsilon_{\mathbf{p}}) \delta(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4)
$$

$$
\times [f_3 f_4 (f+1)(f_2+1) - f f_2 (f_3+1)(f_4+1)]
$$

The Model: Particle collisions

 $i\frac{\partial \Phi_0(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^2 + V_c(x)\right)\Phi_0(x) - iR\Phi_0(x) + \xi_1(x) - 2g\int d^4x' \Pi^R(x',x)V_{\text{nc}}(x')\Phi_0(x) + g\xi_2(x)\Phi_0(x)$ $\frac{\partial f}{\partial t} + \frac{\mathbf{k}}{m} \cdot \nabla f - \nabla V_{\text{nc}} \cdot \nabla_{\mathbf{k}} f = \frac{1}{2} (I_a + I_b)$ Collisional terms

 $\nabla^2 V^{\text{cl}}(x) = 4\pi G\, m \big(n_c(x) + \tilde n(x) + \tfrac{1}{2}\xi_2(x)\big) - 4\pi G\, m \! \int d^4x' \, \Pi^R(x',x) V_{\text{nc}}(x')$

CONDENSATE - THERMAL COLLISION

$$
I_a = 4g^2 n_c \int \frac{d^3 p_1 d^3 p_2 d^3 p_3}{(2\pi)^2 \hbar^4} \delta(\varepsilon_{\mathbf{q}} + \varepsilon_{\mathbf{p}_1} - \varepsilon_{\mathbf{p}_2} - \varepsilon_{\mathbf{p}_3}) \delta(\mathbf{p}_2 - \mathbf{p}_1 - \mathbf{q} + \mathbf{p}_3)
$$

$$
\times (\delta(\mathbf{p}_1 - \mathbf{p}) - \delta(\mathbf{p}_2 - \mathbf{p}) - \delta(\mathbf{p}_3 - \mathbf{p}))[(1 + f_1) f_2 f_3 - f_1(1 + f_2)(1 + f_3)]
$$

 $i\frac{\partial \Phi_0(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^2 + V_c(x)\right)\Phi_0(x) - iR\Phi_0(x) + \xi_1(x) - 2g\int d^4x' \Pi^R(x',x)V_{\text{nc}}(x')\Phi_0(x) + g\xi_2(x)\Phi_0(x)$

 $\frac{\partial f}{\partial t} + \frac{k}{m} \cdot \nabla f - \nabla V_{\text{nc}} \cdot \nabla_k f = \frac{1}{2} (I_a + I_b)$

 $\nabla^2 V^{\text{cl}}(x) = 4\pi G m (n_c(x) + \tilde{n}(x) + \frac{1}{2}\xi_2(x)) - 4\pi G m \int d^4x' \Pi^R(x',x) V_{\text{nc}}(x')$

 $i\frac{\partial \Phi_0(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^2 + V_c(x)\right)\Phi_0(x) - \left(iR\Phi_0(x) + \xi_1(x) - 2g\int d^4x' \Pi^R(x',x)V_{\text{nc}}(x')\Phi_0(x) + g\xi_2(x)\Phi_0(x)$

 $\frac{\partial f}{\partial t} + \frac{k}{m} \nabla f - \nabla V_{\text{nc}} \nabla_k f = \frac{1}{2} (I_a + I_b)$

 $\nabla^2 V^{cl}(x) = 4\pi G m (n_c(x) + \tilde{n}(x) + \frac{1}{2}\xi_2(x)) - 4\pi G m \int d^4x' \Pi^R(x',x) V_{nc}(x')$

The condensate can grow or decrease

The term changes the energy of the coherent part

 $i\frac{\partial \Phi_0(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^2 + V_c(x)\right)\Phi_0(x) - iR\Phi_0(x) + \xi_1(x) - 2g\int d^4x' \Pi^R(x',x)V_{\text{nc}}(x')\Phi_0(x) + g\xi_2(x)\Phi_0(x)$ $\frac{\partial f}{\partial t} + \frac{\mathbf{k}}{m} \cdot \nabla f - \nabla V_{\text{nc}} \cdot \nabla_{\mathbf{k}} f = \frac{1}{2} (I_a + I_b)$ Complex Noise $\nabla^2 V^{cl}(x) = 4\pi G m (n_c(x) + \tilde{n}(x) + \frac{1}{2}\xi_2(x)) - 4\pi G m \int d^4x' \Pi^R(x',x) V_{nc}(x')$

For each dissipative term we will have a noise term (Fluctuation-Dissipation theorem)

$$
\langle \xi_1^*(x')\xi_1(x)\rangle = \frac{i}{2}\Sigma_{(c)}^K(x)\delta(x - x')
$$

Noise can induce condensate production

 $i\frac{\partial \Phi_0(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^2 + V_c(x)\right)\Phi_0(x) - iR\Phi_0(x) + \xi_1(x) - 2g\int d^4x' \Pi^R(x',x)V_{\text{nc}}(x')\Phi_0(x) + g\xi_2(x)\Phi_0(x)$ Real noise $\frac{\partial f}{\partial t} + \frac{k}{m} \nabla f - \nabla V_{\text{nc}} \nabla_k f = \frac{1}{2} (I_a + I_b)$ $\nabla^2 V^{cl}(x) = 4\pi G m (n_c(x) + \tilde{n}(x) + \frac{1}{2}\xi_2(x)) - 4\pi G m \int d^4x' \Pi^R(x',x) V_{\text{nc}}(x')$

For each dissipative term we will have a noise term (Fluctuation-Dissipation theorem)

$$
\langle \xi_2(x) \xi_2(x') \rangle \; = \; -2i\Pi^K(x,x')
$$

The Model: Poisson Equation

Fuzzy Dark matter constraints

Lyman- α Forest sets important bounds on this model

Hydrodynamic equations

We work up to order g with universe expansion: $\nabla^2 V = 4\pi G a^2 (\rho_c + \tilde{\rho})$

Condensate Madelung transformation

$$
\frac{\partial \rho_c}{\partial t} + 3H\rho_c + \frac{1}{a}\nabla \cdot (\rho_c \boldsymbol{v}) = 0
$$

$$
\frac{\partial \boldsymbol{u}}{\partial t} + \frac{1}{a} \boldsymbol{v} \cdot \nabla \boldsymbol{u} = -\nabla \left(-\frac{\hbar^2}{2m^2 a^3} \frac{\nabla^2 \sqrt{\rho_c}}{\sqrt{\rho_c}} + \frac{1}{a} V + \frac{g}{m^2 a} (\rho_c + 2\tilde{\rho}) \right)
$$

Non-condensed particles \longrightarrow Moments and truncation (assuming isotropy and distribution function even respect bulk velocity)

$$
\frac{\partial \tilde{\rho}}{\partial t} + 3H\tilde{\rho} + \frac{1}{a}\nabla \cdot (\tilde{\rho}\tilde{\boldsymbol{v}}) = 0
$$

$$
\frac{\partial \tilde{\boldsymbol{u}}}{\partial t} + \frac{1}{a}\tilde{\boldsymbol{v}} \cdot \nabla \tilde{\boldsymbol{u}} = -\nabla \left(\frac{1}{a}V + \frac{2g}{m^2 a}(\rho_c + \tilde{\rho})\right) - \frac{1}{a\tilde{\rho}}\nabla P
$$

$$
\frac{\partial P}{\partial t} + 5HP + \frac{1}{a}\nabla \cdot (\tilde{\boldsymbol{v}}P) = -\frac{2}{3a}P\nabla \cdot \tilde{\boldsymbol{v}}
$$

Linear Regime

Our system admits consistently a particle pressure $P = \kappa \tilde{\rho}^{\,5/3}$

$$
\ddot{\delta}_c + 2H\dot{\delta}_c + \left(\frac{\hbar^2 k^4}{4m^2 a^4} - 4\pi G \bar{\rho} f + \frac{g \bar{\rho} f k^2}{m^2 a^2}\right) \delta_c - (1 - f) \left(4\pi G \bar{\rho} - \frac{2g \bar{\rho} k^2}{m^2 a^2}\right) \delta_{\text{nc}} = 0
$$

$$
\ddot{\delta}_{\text{nc}} + 2H\dot{\delta}_{\text{nc}} - \left(4\pi G \bar{\rho} (1 - f) - \frac{1}{a^2} \left(\frac{2g \bar{\rho} (1 - f)}{m^2} + \frac{5\kappa \bar{\rho}^{2/3} (1 - f)^{2/3}}{3}\right) k^2\right) \delta_{\text{nc}} - f \left(4\pi G \bar{\rho} - \frac{2g \bar{\rho} k^2}{m^2 a^2}\right) \delta_c = 0
$$

$$
f : \text{coson mass}
$$

$$
f : \text{condensate fraction}
$$

$$
g : \text{self-interaction}
$$

$$
\kappa : \text{particle pressure } (P = \kappa \tilde{\rho}^{5/3})
$$

Linear Regime

Hybrid model looks like FDM of higher mass.

We must be careful with statements about constraints on the mass.

N. Proukakis, G. Rigopoulos and A.S., *Phys. Rev. D* 110 (2 2024), p. 023504 ArXiv:2311.05280 **[astro-ph.CO]**

Summary and Comments

 \triangleright We have a general model for bosonic Dark Matter (condensate + particles). Known models are recovered under the appropriate limits.

 \triangleright This full picture can give a rich phenomenology and physical effects.

 \triangleright The model with both components with self-interaction can mimic FDM, so we need to be careful with placing constraints.

Thanks!