

# **Bosonic Dark Matter**

How to mix Particle Physics, Cosmology and Cold atoms

Alex Soto Science Gallery London – September 2024

Work with N. Proukakis and G. Rigopoulos:

*Phys.Rev.D* 108 (2023) 8, 083513 (ArXiv:2303.02049 [astro-ph.CO]) *Phys.Rev.D* 110 (2024) 2, 023504 (ArXiv:2311.05280 [astro-ph.CO]) ArXiv:2407.08860 [astro-ph.CO]

# **Motivation: Dark Matter**

Evidences suggest the existence of Dark Matter, but what is it?



80 orders of magnitude



### **Motivation: Particles**



Bosons



Fermions

or

# **Motivation: Particles**



Bosons

QCD Axion R. D. Peccei and H. R. Quinn. (1977)

- $\circ$  Scalar field (10<sup>-5</sup> to 10<sup>-3</sup> eV/c<sup>2</sup>, spin-0)
- It solves the CP problem
- Axion like particles A. Arvanitaki et al., arXiv:0905.4720 [hep-th]
  - Motivated by String models (Axiverse)
  - Wide range of masses

#### > Higher spin particles

✓ FDM is a model with a non-relativistic ultralight bosonic particle (around  $10^{-22}$  eV/c<sup>2</sup>)

$$\begin{split} &i\hbar\frac{\partial\Phi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Phi + m\,V\Phi\\ &\nabla^2V = 4\pi G\,m\,|\Phi|^2 \end{split}$$

- ✓ This simple model solves small scale problems of CDM:
  - Cusp-Core problem



ApJ 893, 21 (2020),

- This simple model solves small scale problems of CDM:
  - Cusp-Core problem
  - Missing satellites
  - Etc.

Images from: Mocz *et al.,* Phys. Rev. D **97**, 083519 (2018), arXiv:1801.03507 **[astro-ph.CO]** 





#### ✓ FDM behaves as CDM at large scales



Image from: Ferreira E.G.M. Astron Astrophys Rev **29**, 7 (2021), arXiv:2005.03254 [astro-ph.CO].

✓ FDM is a model with a non-relativistic ultralight bosonic particle (around  $10^{-22}$  eV/c<sup>2</sup>)

$$\begin{split} &i\hbar\frac{\partial\Phi}{\partial t}\!=\!-\frac{\hbar^2}{2m}\nabla^2\Phi+mV\Phi\\ &\nabla^2V\!=\!4\pi G\,m\,|\Phi|^2 \end{split}$$

Small masses give high occupation number  $\rightarrow$ 

Bose-Einstein Condensate

# **Motivation: Coherence**



### **Motivation: Coherence**

Similarities between Fuzzy Dark Matter and Ultracold Atom gases



Image from: N. Proukakis, G. Rigopoulos and A.S., *Phys. Rev. D* 108 (8 2023), p. 083513 arXiv:2303.02049 [astro-ph.CO]

# The Model: Starting point

Our basic object is very general:

 $S = \int dt [\int d^3x \, i\psi^*\psi - H]$ 

 $H = \int d^3x \left( -\frac{1}{2m} \psi^* \nabla^2 \psi + V_{\text{ext}} \psi^* \psi \right) + \frac{1}{2} \int d^3x \int d^3x' \, \psi^*(x) \, \psi^*(x') \, U(x, x') \, \psi(x') \, \psi(x)$ 

Interaction term including contact and long-range:  $U(x, x') = \alpha_1 \delta(x - x') + \alpha_2^2 \mathcal{O}^{-1}(x, x')$ 

> We can always write this action using a Hubbard-Stratonovich transformation as

$$S = \int d^4x \left( i\psi^* \dot{\psi} + \frac{1}{2m} \psi^* \nabla^2 \psi - \frac{\alpha_1}{2} (\psi^* \psi)^2 - V_{ext} \psi^* \psi + \frac{1}{2} V \mathcal{O} V - \alpha_2 V \psi^* \psi \right)$$

#### The Model: Starting point

► Dipolar gases: 
$$\alpha_1 = g - \frac{C_{dd}}{3}$$
,  $\alpha_2 = C_{dd}$ , and  $\mathcal{O} = C_{dd} \frac{\nabla^2}{(\mathbf{n} \cdot \nabla)^2}$ 

N. Proukakis, G. Rigopoulos and A.S., arXiv:2407.20178 [cond-mat.quant-gas]

➢ Gravitational: 
$$\alpha_1 = g$$
,  $\alpha_2 = m$ , and  $\mathcal{O} = \frac{1}{4\pi G} \nabla^2$ 

$$S = \int d^4x \left( i\psi^* \dot{\psi} + \frac{1}{2m} \psi^* \nabla^2 \psi - \frac{g}{2} |\psi|^4 + \frac{1}{8\pi G} V \nabla^2 V - m V |\psi|^2 \right)$$

N. Proukakis, G. Rigopoulos and A.S., *Phys. Rev. D* 108 (8 2023), p. 083513 arXiv:2303.02049 **[astro-ph.CO]** And arXiv:2407.08860 **[astro-ph.CO]** 

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{2} \phi^4 \right)$$

# The Model: Starting point

To get our equations we split the field:





Image from: P. B. Blakie *Phys. Rev. E* 78 (2 2008), p. 026704, arXiv:0803.3664 **[physics.comp-ph]** 

Similar in spirit to SPGPE models in cold atoms

Then we use Schwinger-Keldysh formalism, perturbation theory, approximations, Wigner transforms...

### The Model: Main equations

We have three equations, one for the coherent part, one for the incoherent and one for the gravitational potential

$$\begin{split} \dot{e} \frac{\partial \Phi_0(x)}{\partial t} &= \left( -\frac{1}{2m} \nabla^2 + V_c(x) \right) \Phi_0(x) - iR \Phi_0(x) + \xi_1(x) - 2g \int d^4 x' \Pi^R(x', x) V_{\rm nc}(x') \Phi_0(x) + g \xi_2(x) \Phi_0(x) \\ \frac{\partial f}{\partial t} &+ \frac{k}{m} \cdot \nabla f - \nabla V_{\rm nc} \cdot \nabla_k f = \frac{1}{2} (I_a + I_b) \\ \nabla^2 V^{\rm cl}(x) &= 4\pi G \, m \left( n_c(x) + \tilde{n}(x) + \frac{1}{2} \xi_2(x) \right) - 4\pi G \, m \int d^4 x' \, \Pi^R(x', x) V_{\rm nc}(x') \\ V_c(x) &= m \, V^{\rm cl}(x) + g(n_c(x) + 2\tilde{n}(x)) \end{split}$$

$$V_{\rm nc}(x) = m V^{\rm cl}(x) + 2g(n_c(x) + \tilde{n}(x))$$

Mean field potentials

$$n_c = |\Phi_0|^2$$
  $\tilde{n} = \int \frac{d^3k}{(2\pi)^3} f$ 

Coherent and incoherent number densities

#### The Model: FDM limit

For g = 0, order m, and all in the coherent part:

$$i\frac{\partial\Phi_0(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^2 + V_c(x)\right)\Phi_0(x)$$

 $\nabla^2 V^{\rm cl}(x) = 4\pi G m \ n_c(x)$ 

 $V_c(x) = m V^{\rm cl}(x)$ 

Mean field potentials

We recover FDM

$$n_c = |\Phi_0|^2$$

Coherent and incoherent number densities

#### The Model: CDM limit

For g = 0, order m, and all in the non-coherent part:

$$\begin{aligned} &\frac{\partial f}{\partial t} + \frac{\mathbf{k}}{m} \cdot \nabla f - \nabla V_{\rm nc} \cdot \nabla_{\mathbf{k}} f = 0 \\ &\nabla^2 V^{\rm cl}(x) = 4\pi G m \qquad \tilde{n}(x) \end{aligned}$$

We recover Vlasov-Poisson used in CDM

$$V_{\rm nc}(x) = m V^{\rm cl}(x)$$

Mean field potentials

$$\tilde{n} = \int \frac{d^3k}{(2\pi)^3} f$$

Coherent and incoherent number densities

#### The Model: Main equations

$$\begin{split} &i\frac{\partial\Phi_{0}(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^{2} + V_{c}(x)\right)\Phi_{0}(x) - iR\Phi_{0}(x) + \xi_{1}(x) - 2g\int d^{4}x'\Pi^{R}(x',x)V_{\rm nc}(x')\Phi_{0}(x) + g\xi_{2}(x)\Phi_{0}(x) \\ &\frac{\partial f}{\partial t} + \frac{k}{m}\cdot\nabla f - \nabla V_{\rm nc}\cdot\nabla_{k}f = \frac{1}{2}(I_{a} + I_{b}) \\ &\nabla^{2}V^{\rm cl}(x) = 4\pi G m \left(n_{c}(x) + \tilde{n}(x) + \frac{1}{2}\xi_{2}(x)\right) - 4\pi G m \int d^{4}x' \Pi^{R}(x',x)V_{\rm nc}(x') \end{split}$$

# The Model: Particle collisions

 $i\frac{\partial\Phi_{0}(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^{2} + V_{c}(x)\right)\Phi_{0}(x) - iR\Phi_{0}(x) + \xi_{1}(x) - 2g\int d^{4}x'\Pi^{R}(x',x)V_{nc}(x')\Phi_{0}(x) + g\xi_{2}(x)\Phi_{0}(x)$ 

 $\frac{\partial f}{\partial t} + \frac{\mathbf{k}}{m} \cdot \nabla f - \nabla V_{\rm nc} \cdot \nabla_{\mathbf{k}} f = \frac{1}{2} \left( I_a + I_b \right) \quad \longleftarrow \quad Colling \quad Colli$ 

Collisional terms

 $\nabla^2 V^{\rm cl}(x) = 4\pi G m \left( n_c(x) + \tilde{n}(x) + \frac{1}{2} \xi_2(x) \right) - 4\pi G m \int d^4 x' \, \Pi^R(x', x) V_{\rm nc}(x')$ 



Image from: N. Proukakis and B. Jackson J. Phys. B : At. Mol. Opt. Phys 41 (2008), arXiv:0810.0210 [cond-mat.other]

# The Model: Particle collisions

 $i\frac{\partial\Phi_0(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^2 + V_c(x)\right)\Phi_0(x) - iR\Phi_0(x) + \xi_1(x) - 2g\int d^4x'\Pi^R(x',x)V_{\rm nc}(x')\Phi_0(x) + g\xi_2(x)\Phi_0(x)$ 

 $\frac{\partial f}{\partial t} + \frac{\mathbf{k}}{m} \cdot \nabla f - \nabla V_{\rm nc} \cdot \nabla_{\mathbf{k}} f = \frac{1}{2} \left( I_a + I_b \right) \quad \longleftarrow \quad \text{Collisional terms}$ 

 $\nabla^2 V^{\rm cl}(x) = 4\pi G m \left( n_c(x) + \tilde{n}(x) + \frac{1}{2} \xi_2(x) \right) - 4\pi G m \int d^4x' \, \Pi^R(x', x) V_{\rm nc}(x')$ 



mal State

Condensate Atom

# The Model: Particle collisions

 $\nabla^2 V^{\rm cl}(x) = 4\pi G m \left( n_c(x) + \tilde{n}(x) + \frac{1}{2} \xi_2(x) \right) - 4\pi G m \int d^4 x' \, \Pi^R(x', x) V_{\rm nc}(x')$ 



$$I_{a} = 4g^{2}n_{c}\int \frac{d^{3}p_{1} d^{3}p_{2} d^{3}p_{3}}{(2\pi)^{2}\hbar^{4}} \delta(\varepsilon_{q} + \varepsilon_{p_{1}} - \varepsilon_{p_{2}} - \varepsilon_{p_{3}})\delta(p_{2} - p_{1} - q + p_{3})$$
  
× $(\delta(p_{1} - p) - \delta(p_{2} - p) - \delta(p_{3} - p))[(1 + f_{1})f_{2}f_{3} - f_{1}(1 + f_{2})(1 + f_{3})]$ 



 $i\frac{\partial\Phi_{0}(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^{2} + V_{c}(x)\right)\Phi_{0}(x) - iR\Phi_{0}(x) + \xi_{1}(x) - 2g\int d^{4}x'\Pi^{R}(x',x)V_{\mathrm{nc}}(x')\Phi_{0}(x) + g\xi_{2}(x)\Phi_{0}(x)$ 

 $\frac{\partial f}{\partial t} + \frac{\mathbf{k}}{m} \cdot \nabla f - \nabla V_{\rm nc} \cdot \nabla_{\mathbf{k}} f = \frac{1}{2} (I_a + I_b)$ 

 $\nabla^2 V^{\rm cl}(x) = 4\pi G m \left( n_c(x) + \tilde{n}(x) + \frac{1}{2} \xi_2(x) \right) - 4\pi G m \int d^4x' \,\Pi^R(x', x) V_{\rm nc}(x')$ 



 $i\frac{\partial\Phi_{0}(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^{2} + V_{c}(x)\right)\Phi_{0}(x) - \left(iR\Phi_{0}(x) + \xi_{1}(x) - 2g\int d^{4}x'\Pi^{R}(x',x)V_{\mathrm{nc}}(x')\Phi_{0}(x) + g\xi_{2}(x)\Phi_{0}(x)\right) + \frac{1}{2}\left(-\frac{1}{2m}\nabla^{2} + V_{c}(x)\right)\Phi_{0}(x) - \frac{1}{2}\left($ 

 $\frac{\partial f}{\partial t} + \frac{\mathbf{k}}{m} \cdot \nabla f - \nabla V_{\rm nc} \cdot \nabla_{\mathbf{k}} f = \frac{1}{2} (I_a + I_b)$ 

 $\nabla^2 V^{\rm cl}(x) = 4\pi G m \left( n_c(x) + \tilde{n}(x) + \frac{1}{2} \xi_2(x) \right) - 4\pi G m \int d^4 x' \,\Pi^R(x', x) V_{\rm nc}(x')$ 



The condensate can grow or decrease





The term changes the energy of the coherent part



For each dissipative term we will have a noise term (Fluctuation-Dissipation theorem)

$$\langle \xi_1^*(x')\xi_1(x)\rangle = \frac{i}{2}\Sigma_{(c)}^K(x)\delta(x-x')$$

Noise can induce condensate production



For each dissipative term we will have a noise term (Fluctuation-Dissipation theorem)

$$\langle \xi_2(x)\xi_2(x')\rangle = -2i\Pi^K(x,x')$$



#### **The Model: Poisson Equation**



# **Fuzzy Dark matter constraints**

Lyman- $\alpha$  Forest sets important bounds on this model



#### Hydrodynamic equations

We work up to order g with universe expansion:  $abla^2 V = 4\pi G a^2 (
ho_c + ilde{
ho})$ 

Condensate  $\rightarrow$  Madelung transformation

$$\frac{\partial \rho_c}{\partial t} + 3H\rho_c + \frac{1}{a}\nabla \cdot (\rho_c \boldsymbol{v}) = 0$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{1}{a} \boldsymbol{v} \cdot \nabla \boldsymbol{u} = -\nabla \left( -\frac{\hbar^2}{2m^2 a^3} \frac{\nabla^2 \sqrt{\rho_c}}{\sqrt{\rho_c}} + \frac{1}{a} V + \frac{g}{m^2 a} (\rho_c + 2\tilde{\rho}) \right)$$

Non-condensed particles

$$\begin{aligned} \frac{\partial \tilde{\rho}}{\partial t} + 3H\tilde{\rho} + \frac{1}{a}\nabla \cdot (\tilde{\rho}\tilde{\boldsymbol{v}}) &= 0\\ \frac{\partial \tilde{\boldsymbol{u}}}{\partial t} + \frac{1}{a}\tilde{\boldsymbol{v}}\cdot\nabla \tilde{\boldsymbol{u}} &= -\nabla \left(\frac{1}{a}V + \frac{2g}{m^2a}(\rho_c + \tilde{\rho})\right) - \frac{1}{a\tilde{\rho}}\nabla P\\ \frac{\partial P}{\partial t} + 5HP + \frac{1}{a}\nabla \cdot (\tilde{\boldsymbol{v}}P) &= -\frac{2}{3a}P\nabla \cdot \tilde{\boldsymbol{v}} \end{aligned}$$

#### **Linear Regime**

Our system admits consistently a particle pressure  $P = \kappa \tilde{\rho}^{\,5/3}$ 

# **Linear Regime**



Hybrid model looks like FDM of higher mass.

We must be careful with statements about constraints on the mass.

N. Proukakis, G. Rigopoulos and A.S., *Phys. Rev. D* 110 (2 2024), p. 023504 ArXiv:2311.05280 **[astro-ph.CO]** 

# **Summary and Comments**

We have a general model for bosonic Dark Matter (condensate + particles). Known models are recovered under the appropriate limits.

> This full picture can give a rich phenomenology and physical effects.

The model with both components with self-interaction can mimic FDM, so we need to be careful with placing constraints.



Thanks!