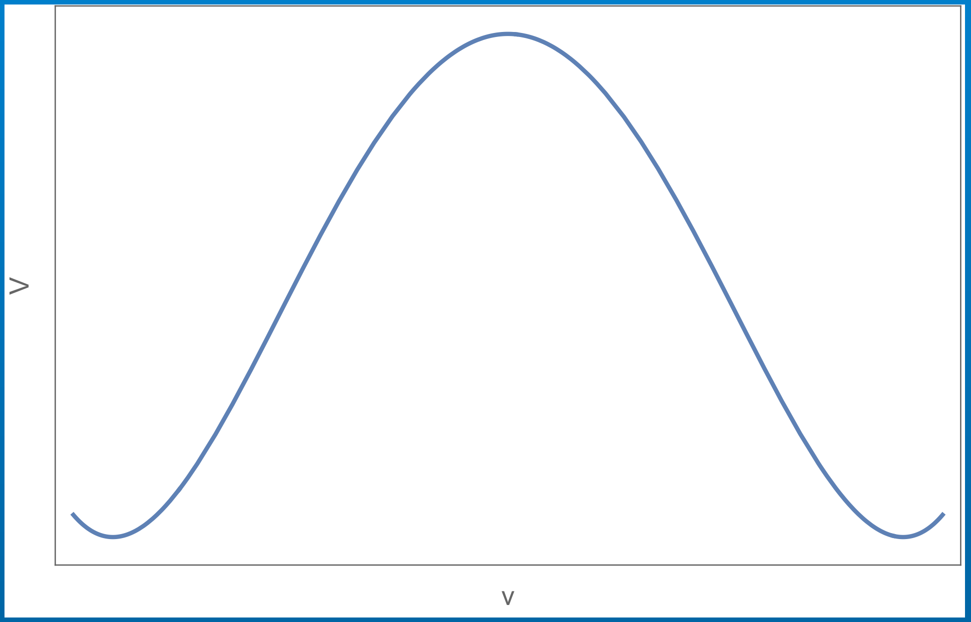
Why quantum gravity made me fall in love with domain walls

Graham White
Southampton

Basics of domain walls

Consider the potential

$$V = -\mu^2 v^2 + \lambda v^4$$



There are two minima with the same value

Basics of domain walls

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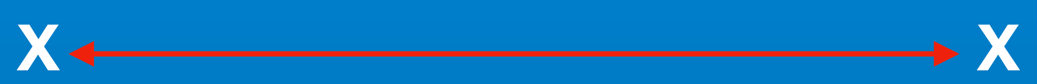


A solution to the equation of motion is obviously when we solve the Euler Lagrange equations

$$\partial^\mu \partial_\mu v = \frac{dV}{dv}$$

We have the usual boring solutions $v = \pm \sqrt{\frac{\mu^2}{2\lambda}}$

Basics of domain walls

We have the usual boring solutions $v = \pm\sqrt{\frac{\mu^2}{2\lambda}}$

We also have a solution which continuously goes from one vacuum to another

$$v = \frac{\mu \tanh[z\mu]}{\sqrt{2\lambda}}$$

Three of the derivative terms in the Euler Langrange equations vanish

$$\frac{\partial^2 v}{dt^2} = \frac{\partial^2 v}{dx^2} = \frac{\partial^2 v}{dy^2} = 0$$

This leaves just $\frac{\partial^2 v}{dz^2} = \frac{\partial V}{\partial v}$

Subbing in our Tanh solution we find both sides equals $-\frac{\sqrt{2}\mu^3 \mathrm{sech}[z\mu]^2 \tanh[z\mu]}{\sqrt{\lambda}}$

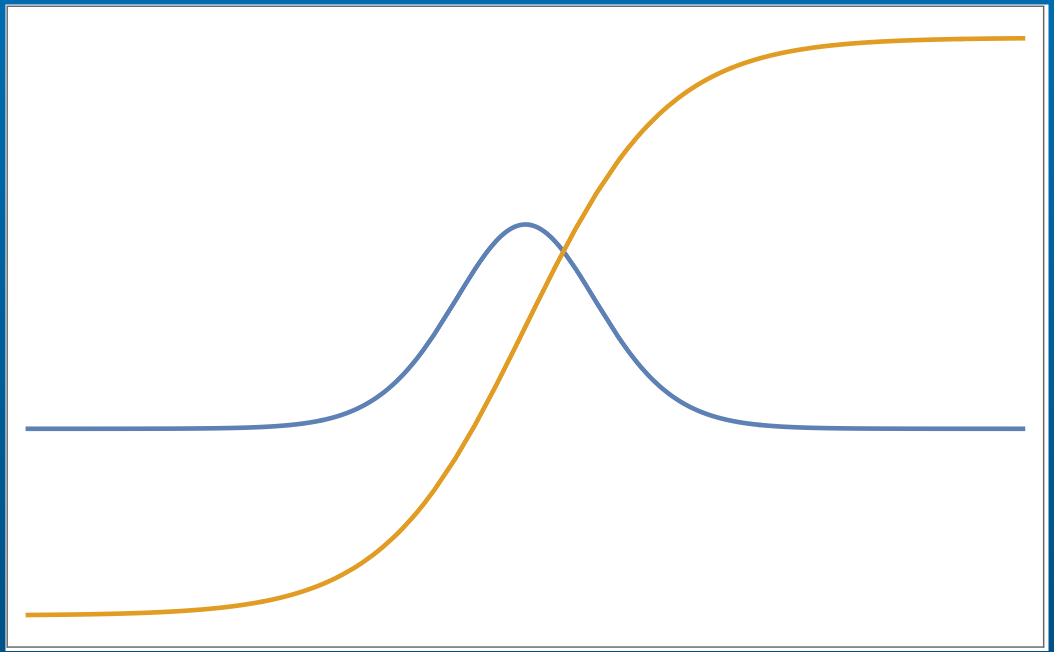
Basics of domain walls

So what is this weird Tanh solution?

When we put it into the Lagrangian it gives a localized energy distribution

$$\int dz L = \int dz \left[\frac{1}{2}\left(\frac{dv}{dz}\right) + V\right]$$

$$v = \frac{\mu \tanh[z\mu]}{\sqrt{2\lambda}} \rightarrow \int dz L = \int dz \frac{m^4(-1 + 2\mathrm{sech}[zu]^4)}{4\lambda}$$



The energy is clumpy! It looks like a wall of energy in space

This is why we call it a domain wall

Basics of domain walls

Contribution to energy density from strings and domain walls

$$E_{\rm DW} = \sigma R^2 \to \rho_{\rm DW} = \sigma R^2/R^3 = \sigma/R = \rho_{\rm DW\ initial}\frac{a_{\rm initial}}{a^1}$$

The energy density of radiation dilutes as a^{-4}, so the fraction of the total energy density of the Universe will *grow* as the Universe expands

If domain walls have no way of annihilating they will dominate the Universe!

Towards a solution of the cosmological domain walls problem

Zygmunt Lalak (Warsaw U.)
Jul, 1996

5 pages
Part of High energy physics : Proceedings, 28th International Conference, ICHEP'96, Warsaw, Poland, July 25-31, 1996. Vol. 1, 2, 1545-1549
Contribution to: ICHEP 96, 1545-1549
e-Print: hep-ph/9702405 [hep-ph]
View in: ADS Abstract Service

Spontaneous discrete symmetry breaking during inflation and the NMSSM domain wall problem

John McDonald (Helsinki U.)
Sep, 1997

27 pages
Published in: *Nucl.Phys.B* 530 (1998) 325-345
e-Print: hep-ph/9709512 [hep-ph]
DOI: 10.1016/S0550-3213(98)00414-3
View in: ADS Abstract Service

Domain wall problem of axion and isocurvature fluctuations in chaotic inflation models

S. Kasuya (Tokyo U., ICRR), M. Kawasaki (Tokyo U., ICRR), T. Yanagida (Tokyo U.)
Sep, 1997

8 pages
Published in: *Phys.Lett.B* 415 (1997) 117-121
e-Print: hep-ph/9709202 [hep-ph]
DOI: 10.1016/S0370-2693(97)01270-7
Report number: ICRR-397-97-20
View in: ADS Abstract Service

On the cosmological domain wall problem in supersymmetric models

Tomohiro Matsuda (Tokyo Inst. Tech.)
Apr, 1998

10 pages
Published in: *Phys.Lett.B* 436 (1998) 264-268
e-Print: hep-ph/9804409 [hep-ph]
DOI: 10.1016/S0370-2693(98)00861-2
Report number: TIT-HEP-390
View in: ADS Abstract Service, AMS MathSciNet

A solution of the Randall-Sundrum model and the mass hierarchy problem

S. Ichinose (Shizuoka U., Ohya)
2001

12 pages
Published in: *Class.Quant.Grav.* 18 (2001) 421-432
DOI: 10.1088/0264-9381/18/3/305
View in: AMS MathSciNet

Evading the cosmological domain wall problem

Sebastian E. Larsson (Oxford U.), Subir Sarkar (Oxford U.), Peter L. White (Oxford U.)
Aug, 1996

14 pages
Published in: *Phys.Rev.D* 55 (1997) 5129-5135
e-Print: hep-ph/9608319 [hep-ph]
DOI: 10.1103/PhysRevD.55.5129
Report number: OUTP-96-11-P

Basics of domain walls

Making domain walls metastable

1) Local discrete symmetry - need to be eaten by strings
2) Global discrete symmetry - need to be annihilated

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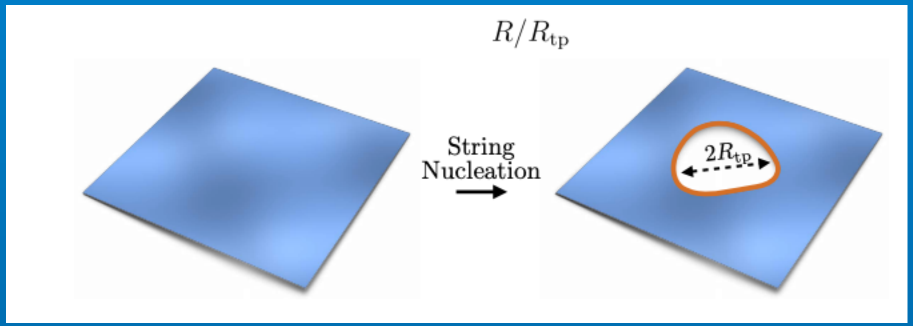

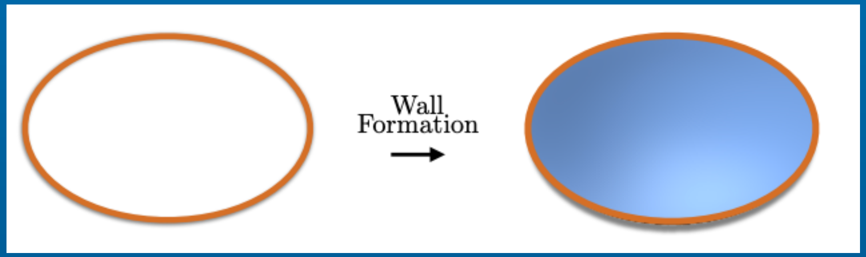

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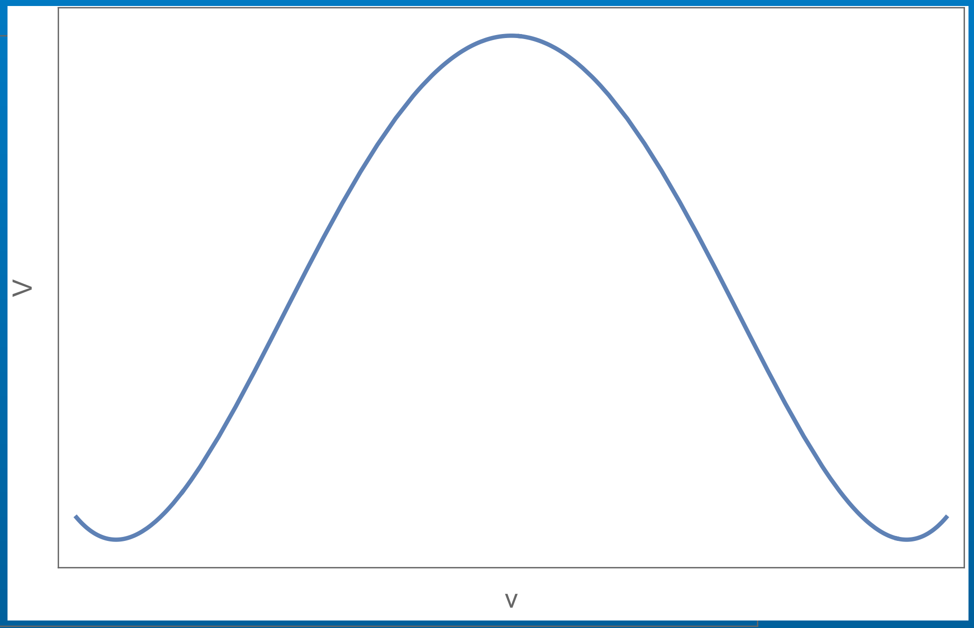

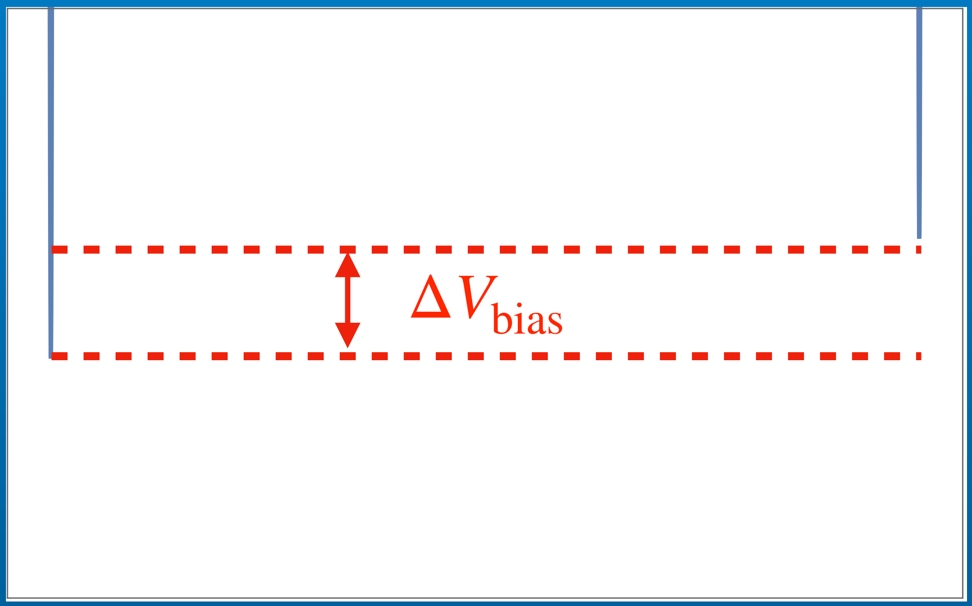

Basics of domain walls

Gravitational waves, let's do some scaling relations

$$\frac{d\rho_{\mathrm{GW}}}{dt} = -\, n_{\mathrm{dw}} P_{\mathrm{GW}}, \; \Omega_{\mathrm{GW}} = f \, \frac{d\rho_{\mathrm{GW}}/df}{\rho_c}$$

Start with the power

$$P_{\mathrm{GW,dw}} \sim G\sigma M_{\mathrm{DW}}$$

$$M = \sigma R^2$$

$$P_{\mathrm{GW,dw}} \sim G\sigma^2 R^2,$$

Basics of domain walls

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$$\frac{d\rho_{\rm GW}}{dt} = -n_{\rm dw}P_{\rm GW},\ \Omega_{\rm GW} = f\,\frac{d\rho_{\rm GW}/df}{\rho_c}$$

Next the number density

$$P_{\rm GW,dw} \sim G\sigma^2R^2,$$

$$n_{\rm dw} = R^{-3} \rightarrow \frac{d\rho_{\rm GW}}{dt} \sim R^{-3}G\sigma^2R^2 \sim H$$

Basics of domain walls

Gravitational waves, let's do some scaling relations

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Now put it together

$$\frac{d\rho_{\rm GW}}{dt} \sim H$$

$$\frac{1}{\rho_{\rm rad}}\frac{d\rho}{dt} \sim H^{-2}H \sim H^{-1}$$

Basics of domain walls

Gravitational waves

$$\frac{d\rho_{\rm GW}}{dt} = -\, n_{\rm defect} P_{\rm GW},\ \Omega_{\rm GW} = f\,\frac{d\rho_{\rm GW}/df}{\rho_c}$$

Finally convert from time to frequency $\dfrac{d\rho/dt}{\rho_{\rm rad}} \sim \dfrac{1}{H}$

For radiation domination $a \sim t^{1/2}, f \sim a^{-1}, H \sim a^2$

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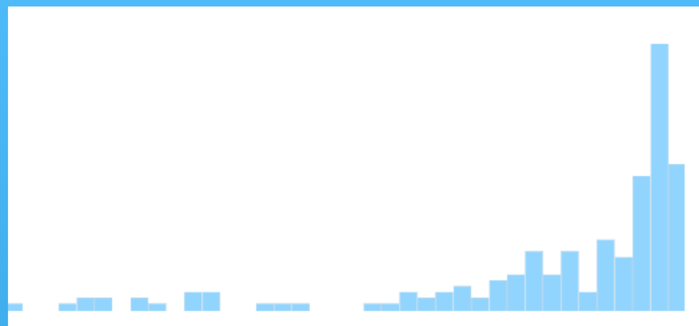
For radiation domination $a \sim t^{1/2}, f \sim a^{-1}, H \sim a^2$

This implies $\frac{df}{dt} \sim t^{-3/2} \sim a^{-3}$

Using chain rule $\frac{1}{\rho_{\rm rad}}\frac{d\rho_{\rm GW}}{df} = \frac{1}{\rho_{\rm rad}}\frac{d\rho_{\rm GW}}{dt}\left(\frac{df}{dt}\right)^{-1} \sim a^{-2}a^3 \sim a \sim f^{-1}$

And this is exactly what we find in simulations

$$\Omega_{\rm GW}(f) = \Omega_{\rm max}\left(\Theta(f - f_{\rm peak})\left[\frac{f}{f_{\rm peak}}\right]^{-1} + \Theta(f_{\rm peak} - f)\left[\frac{f}{f_{\rm peak}}\right]^{3}\right)$$



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Yunjia Bao (Chicago U., KICP), Keisuke Harigaya (Chicago U., KICP and Tokyo U., IPMU), Lian-Tao Wang (Chicago U., KICP) (Jul 22, 2024)

e-Print: 2407.17525 [hep-ph]

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D. Grüber (Porto U. and Porto U., Astron. Dept.), L. Sousa (Porto U. and Porto U., Astron. Dept.), P.P. Avelino (Porto U. and Porto U., Astron. Dept.) (Mar 14, 2024)

Published in: *Phys.Rev.D* 110 (2024) 2, 023505 • e-Print: 2403.09816 [gr-qc]

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Junseok Lee (Tohoku U.), Kai Murai (Tohoku U.), Fuminobu Takahashi (Tohoku U.), Wen Yin (To
Tokyo Metropolitan U.) (Jul 12, 2024)

e-Print: 2407.09478 [hep-ph]

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Hieu The Pham (Ho Chi Minh City U. and Vietnam Natl. U., Ho Chi Minh), Eibun Senaha (Vietnam Natl. U., Ho Chi Minh and ICST, Ho Chi Minh City) (Mar 25, 2024)

Published in: *Phys.Rev.D* 109 (2024) 9, 095048 • e-Print: 2403.16568 [hep-ph]

pdf DOI cite claim reference search 0 citations

Why I hated global domain walls

$$V = -mu^2\phi^2 + \lambda\phi^4 + \frac{1}{\Lambda}\phi^5$$

How small is the bias?

$$T_{\rm ann} \sim 3.41 \times 10^{-2}\mathrm{GeV}\left(\frac{\sigma}{\mathrm{TeV}^3}\right)^{-1/2}\left(\frac{V_{\rm bias}}{\mathrm{MeV}^4}\right)^{1/2}$$

$$\sigma \sim v^3,\ V_{\rm bias} \sim \frac{v^5}{\Lambda} \rightarrow T_{\rm ann} \sim 10^9\frac{v}{\sqrt{\Lambda}}$$

If you just wanted to get rid of them just set $T_{\rm ann} = v$ you already have an unnatural scale separation

$$\Lambda \sim 10^{18}\mathrm{GeV}$$

If we to do something fun with domain walls, we need them to last long enough to contribute nontrivially to the energy density which means *an effective beyond the Planck scale*

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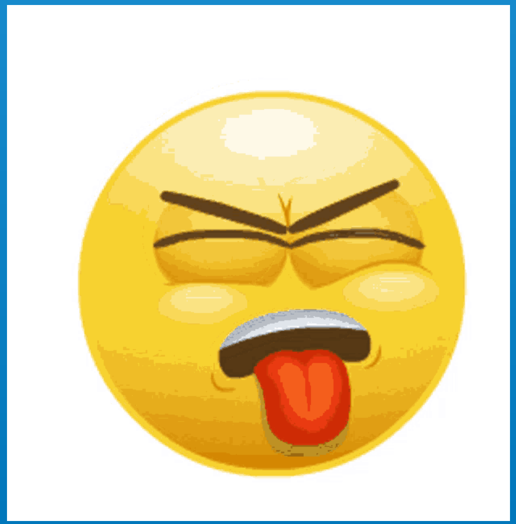


Are other operators better?

For $T_{\rm ann} = \epsilon\phi$

$$V_{\rm bias} = \frac{1}{\Lambda}\phi^5 \rightarrow \Lambda \sim \frac{1}{\epsilon^2}10^{17}\ ({\rm GeV})$$

$$V_{\rm bias} = \frac{1}{\Lambda}v_h^2\phi^3 \rightarrow \Lambda \sim \frac{1}{\phi^2\epsilon^2}10^{22}\ ({\rm GeV})$$

$$V_{\rm bias} = \frac{1}{\Lambda}v_h^4\phi \rightarrow \Lambda \sim \frac{10^{27}}{\phi^4\epsilon^2}\ ({\rm GeV})$$

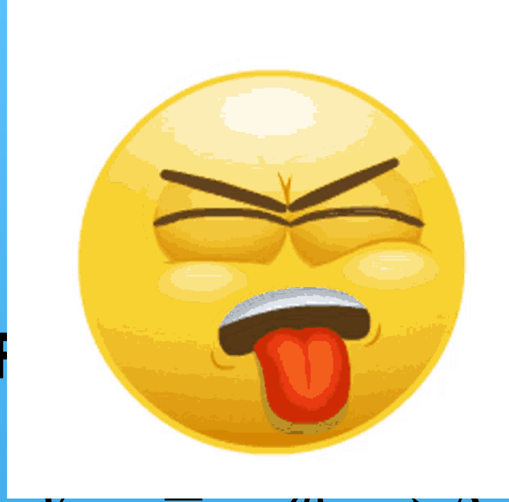
If you want interesting pheno you want to be close to domain wall domination, which occurs at

$$T_{\rm dom} \sim \sqrt{\frac{\phi^3}{M_{\rm pl}}}$$

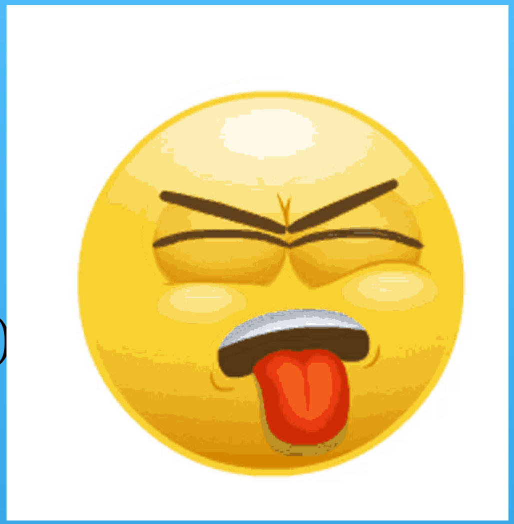
$$V_{\rm bias} = \frac{1}{\Lambda}\phi^5 \rightarrow \Lambda \sim \frac{M_{\rm pl}}{\phi}10^{18}\ ({\rm GeV})$$

$$V_{\rm bias} = \frac{1}{\Lambda}\phi^3 v_h^2 \rightarrow \Lambda \sim \frac{M_{\rm pl}}{\phi^3}10^{22}\ ({\rm GeV})$$

$$V_{\rm bias} = \frac{1}{\Lambda}\phi^1 v_h^4 \rightarrow \Lambda \sim \frac{M_{\rm pl}}{\phi^5}10^{27}\ ({\rm GeV})$$



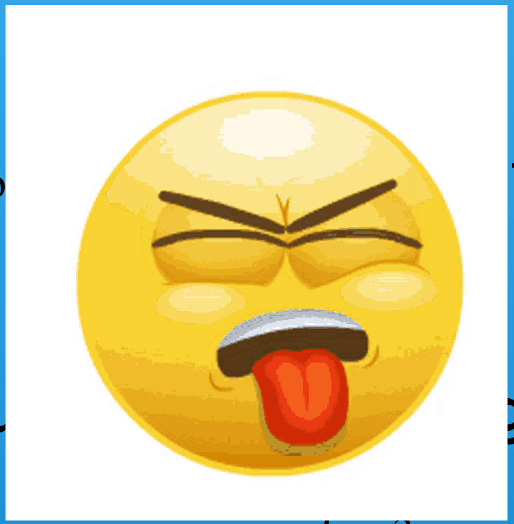
Are ... **better?**



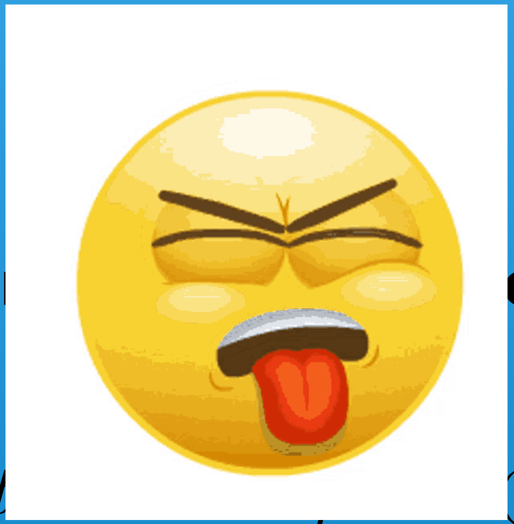
$V_{\rm bias} = \frac{\;}{\Lambda}\phi \to \Lambda \sim \frac{1}{\epsilon^2}10^{17}$ (GeV)

$V_{\rm bias} = \frac{1}{\Lambda} v_h^2 \phi^3 \to \Lambda \sim \frac{1}{\phi^2\epsilon^2}10^{22}$ (GeV)

$V_{\rm b}$ ... $\frac{10^{27}}{\phi^4\epsilon^2}$ (GeV)

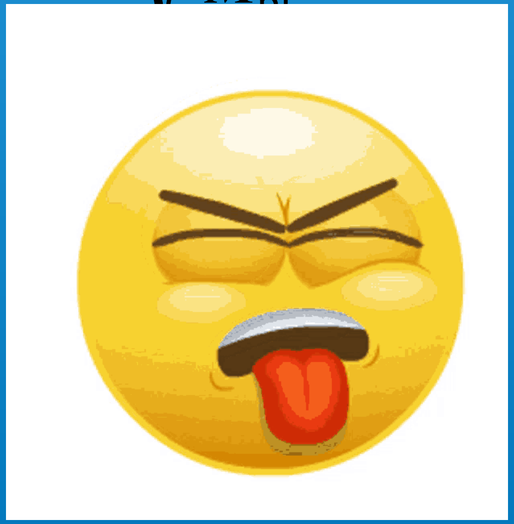


If you ... **g pheno you wa** ... **omination, which occurs at**



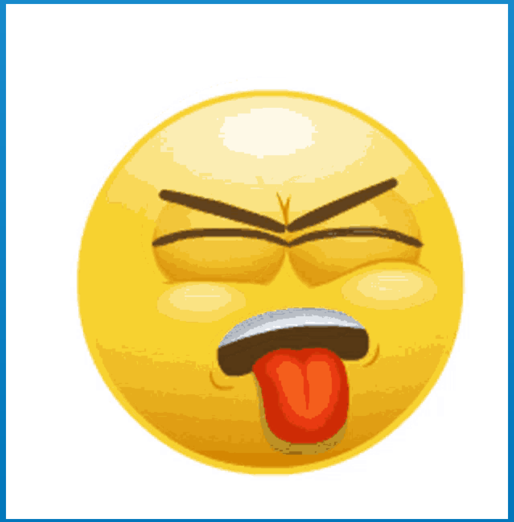
$T_{\rm dom} \sim \sqrt{\frac{\phi^3}{M_{\rm pl}}}$ ...

$V_{\rm bias} = \frac{1}{\Lambda}\phi$ ... $\frac{\;}{\phi}$ (GeV)



$V_{\rm bias} = \frac{1}{\Lambda}\phi^3 v_h^2 \to \Lambda \sim \frac{M_{\rm pl}}{\phi^3}10^{22}$

$V_{\rm bias} = \frac{1}{\Lambda}\phi^1 v_h^4 \to \Lambda \sim \frac{M_{\rm pl}}{\phi^5}10^{27}$



Part 2: Quantum Gravity to the rescue….

Evidence for quantum gravity spoiling global charge

1) True if blackhole thermodynamics is correct

$$S_{\text{BH}} = \frac{\text{Area}}{4G}$$

(the above is violated if you are allowed to have a continuous global charge)

2) Empirically true of every discrete global symmetry in specific string theory compactifications

3) Can be proven in the case of AdS/CFT for both discrete and continuous symmetries

Note that the violation of a global symmetry is non-perturbative

Since it is a QG effect one might naively

think $\frac{1}{\Lambda_{\rm QG}}\mathcal{O}_{\rm sym\ br} \leftrightarrow \Lambda_{\rm QG} = M_{\rm pl}$

However, since in specific cases in string theory, the global symmetry is violated by a non-perturbative process such as a gravitational instanton (wormhole!)

$$\Lambda_{\rm QG} = e^{S_{\rm wh}} M_{\rm Pl}$$

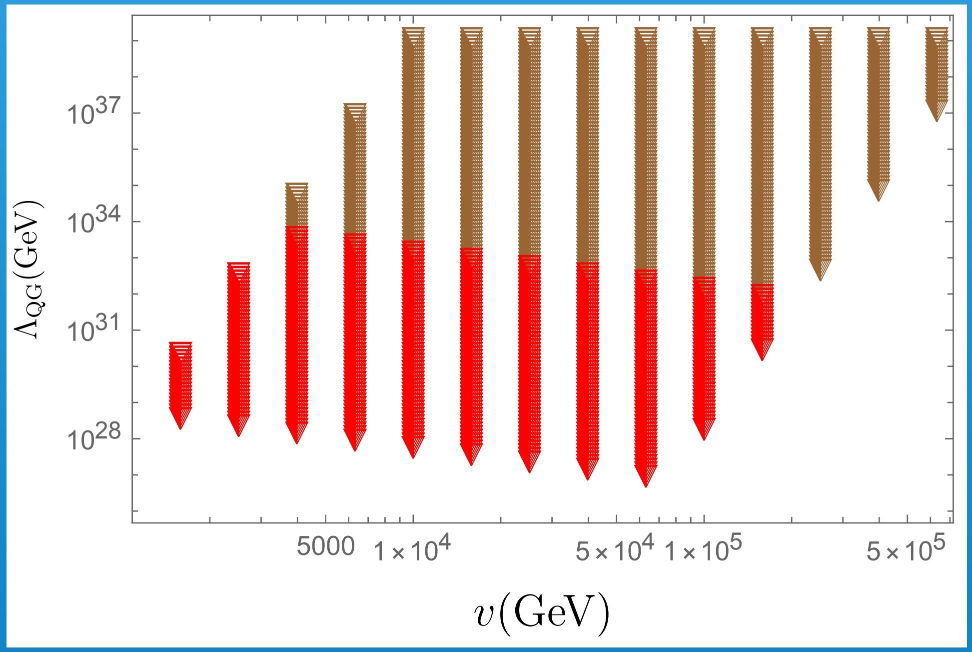
Where $S_{\rm wh}$ is the action evaluated for a wormhole solution

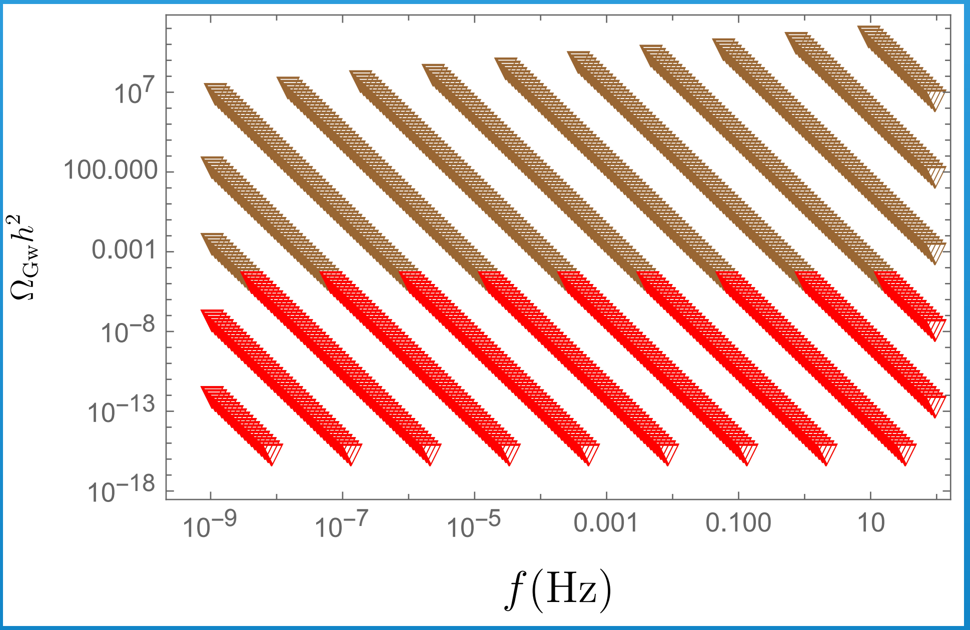
What scale do visible domain walls like the scale of QG to be?

$$V_{\rm bias} \simeq \frac{1}{\Lambda_{\rm QG}}\left(v_1^5 + \frac{v_1^3 v_h^2}{2} + \frac{v_1 v_h^4}{4}\right) ,$$

$$f_p \simeq 3.75\times 10^{-9}\ {\rm Hz}\ C_{\rm ann}^{-1/2}\mathcal{A}^{-1/2}\widehat{\sigma}^{-1/2}\widehat{V}_{\rm bias}^{1/2} ,$$
$$\Omega_p h^2 \simeq 5.3\times 10^{-20}\ \widetilde{\epsilon}\mathcal{A}^4 C_{\rm ann}^2\widehat{\sigma}^4\widehat{V}_{\rm bias}^{-2} ,$$

let $10^2\ ({\rm GeV}) < v < M_{\rm pl}$





This corresponds to $\Lambda_{\rm QG} = M_{\rm pl}e^{S_{\rm wh}}$ with $23 \lesssim S_{\rm wh} \lesssim 35$

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This is pretty darn plausible

1) **Unlike Peccei Quinn, which requires $S \gtrsim 100$, my stringy collaborator tells me this is a pretty plausible range**
2) **If string theory is true -> zillions of "moduli". Approximate discrete symmetries common enough**

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This is at least on par with other GW sources

Phase transitions -> require a very strong transition to be visible
Cosmic strings -> debate over field theoretic treatments perhaps not settled
SIGWs -> requires a period of matter domination to last long enough and end abruptly enough

So what can we do with Domain walls

1) test quantum gravity (qualitatively)
2) Producing dark matter
3) Explain NANOGrav

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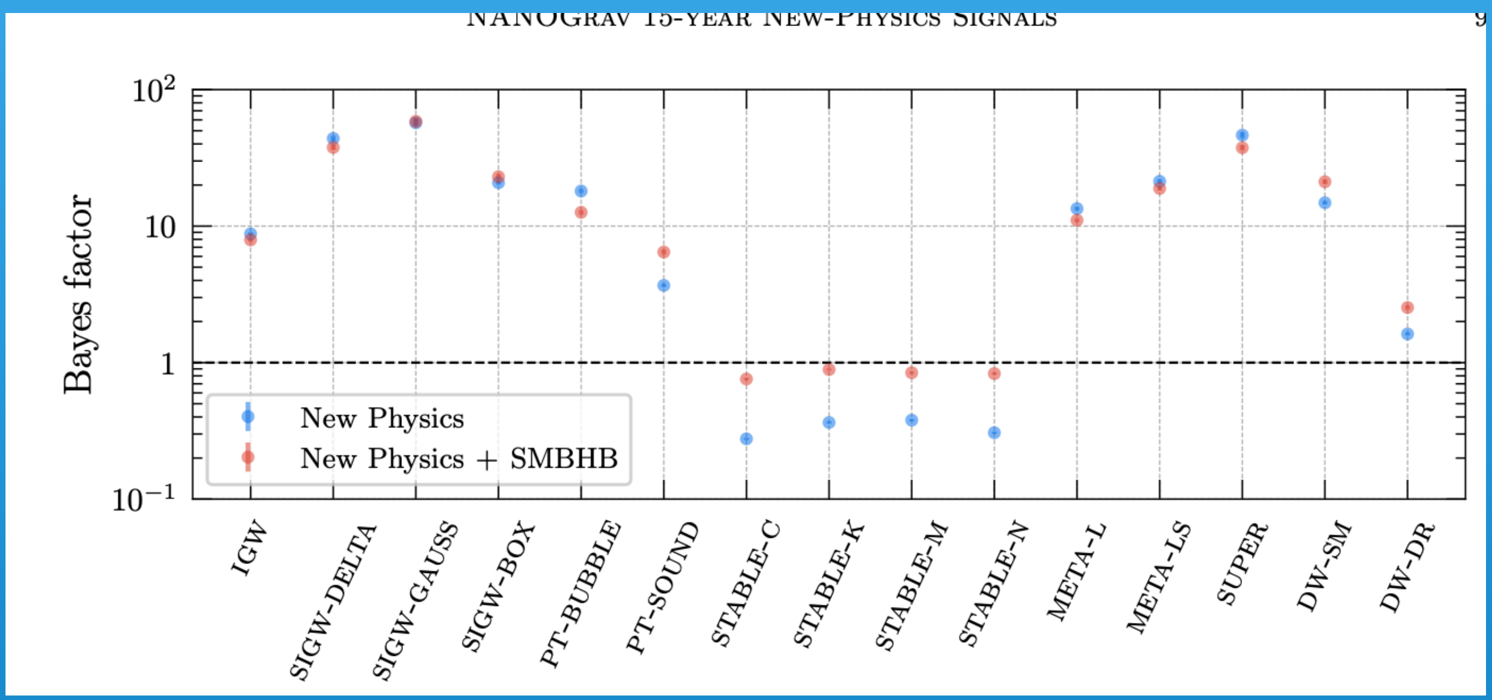

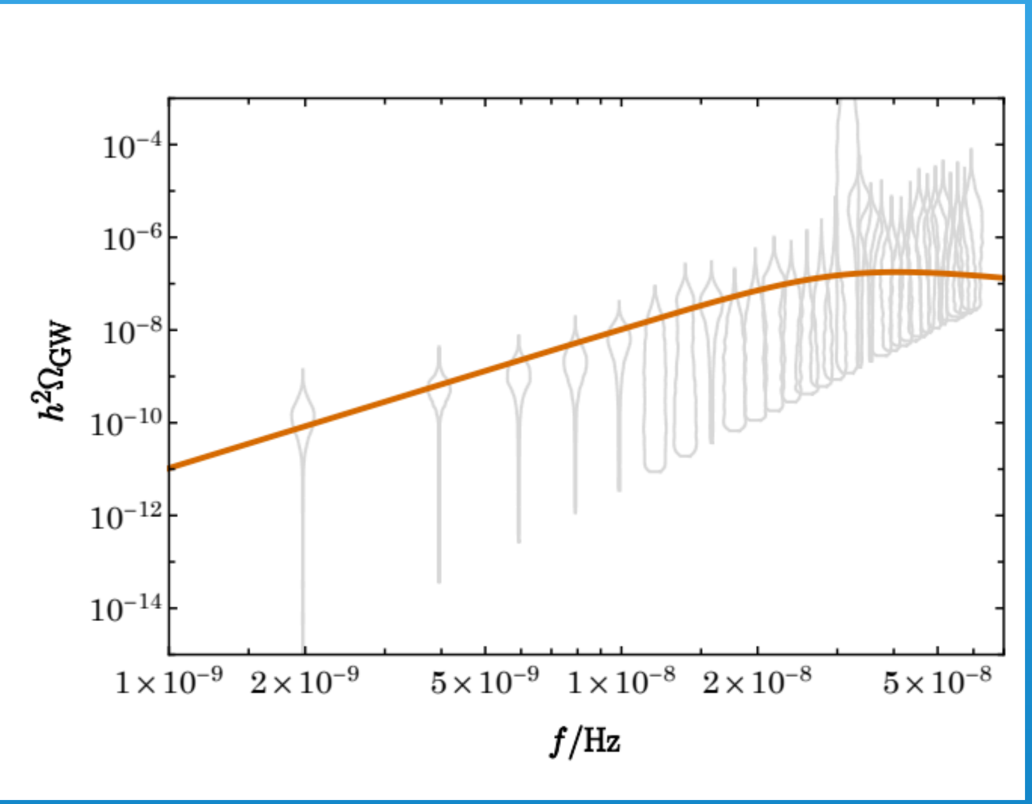

arXiv: 2306.16219

arXiv: 2308.03724

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   a. Finding DWs with a bias scale well above the Planck scale proves a qualitative feature of quantum gravity
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   c. Might be able to get an independent measure of Λ_{QG}

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      - Need an observable sensitive to physics above the Planck scale

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1) CMB polarization power spectrum can be sensitive to incredible decay times $\tau \sim 10^{26}\mathrm{s}$
2) $0\nu\beta\beta$ decay
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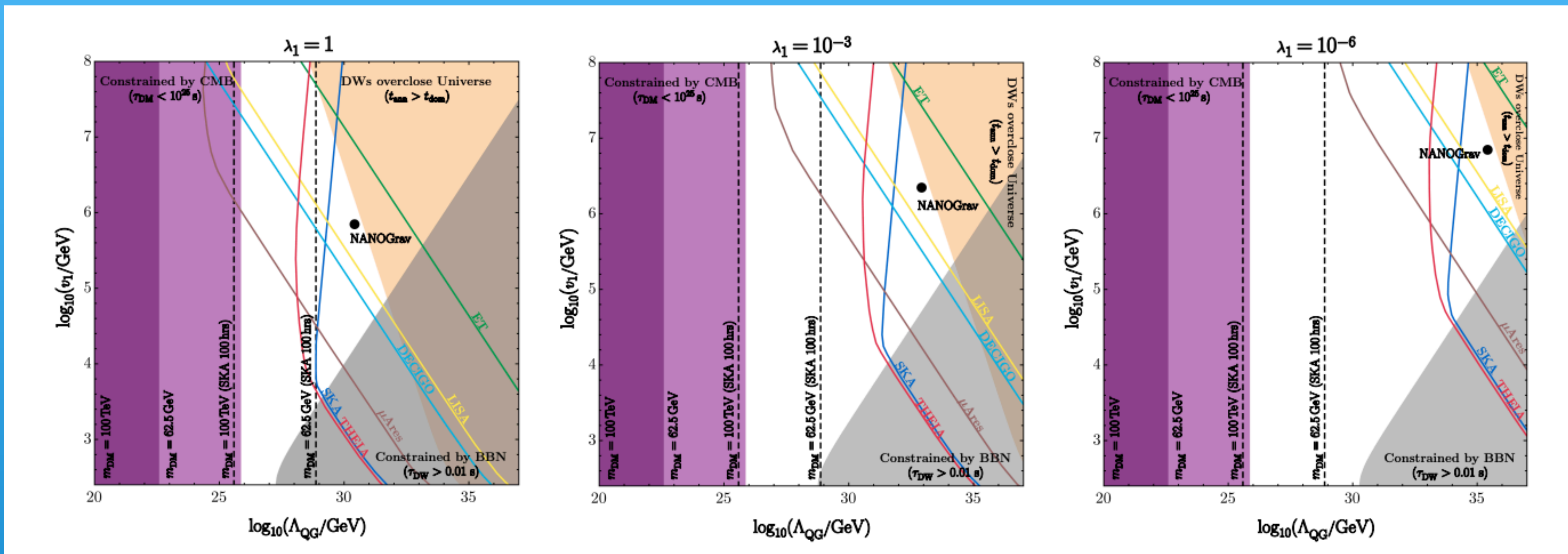
For scalar dark matter

$$\frac{1}{\Lambda_{\rm QG}} S_{\rm DM} H^4 \to \quad \sin\theta = \frac{v_h^3}{(m_h^2 - m_{\rm DM}^2)\Lambda_{\rm QG}} . \quad \to \Gamma_{\rm DM\to SM} = \sin^2\theta\,\Gamma_h(m_{\rm DM}) \propto \frac{1}{\Lambda_{\rm QG}^2}$$

SKA can also be sensitive to radio waves produced in DM rich clusters/ galaxies and can probe the range $\Gamma_{\rm DM\to SM} \lesssim 10^{30}\mathrm{s}$



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3) **Diffuse background (x/γ ray)**

$$\mathcal{L}_{\rm break} = \sum_{i=1,2,3} \frac{\beta_i}{\Lambda_{\rm QG}} S\overline{\ell_{L_i}}\widetilde{H}\chi\,, \longrightarrow \mathcal{L}_{\rm break} = \sum_{i=1,2,3} \frac{\beta_i v_s v_h}{\sqrt{2}\Lambda_{\rm QG}} \overline{\nu_{L_i}}\chi\,. \longrightarrow \theta \simeq \sum_{i=1,2,3}\left(\frac{m_{D_i}}{m_{\rm DM}}\right) = \sum_{i=1,2,3}\left(\frac{\beta_i v_s v_h}{\sqrt{2}\Lambda_{\rm QG} m_{\rm DM}}\right)$$

$$\tau_{\chi\to\nu\gamma} \simeq \left(\frac{9\alpha_{\rm EM}\sin^2\theta}{1024\pi^4} G_F^2 m_{\rm DM}^5\right)^{-1} \simeq 1.8\times10^{17}\ {\rm s}\left(\frac{10{\rm MeV}}{m_{\rm DM}}\right)^5\left(\frac{\sin\theta}{10^{-8}}\right)^{-2}$$

$$\tau_{\chi\to e^+e^-\nu} \simeq \left(\frac{c_\alpha\sin^2\theta}{96\pi^3} G_F^2 m_{\rm DM}^5\right)^{-1} \simeq 2.4\times10^{15}\ {\rm s}\left(\frac{10{\rm MeV}}{m_{\rm DM}}\right)^5\left(\frac{\sin\theta}{10^{-8}}\right)^{-2}$$

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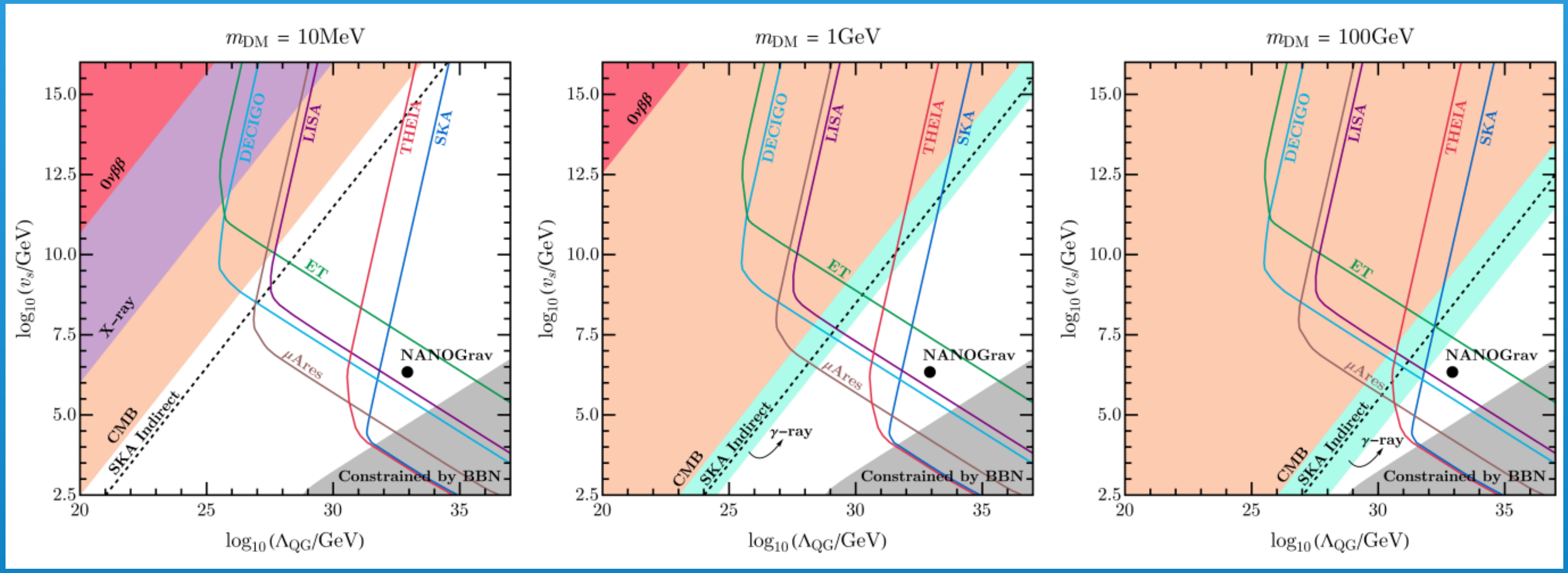

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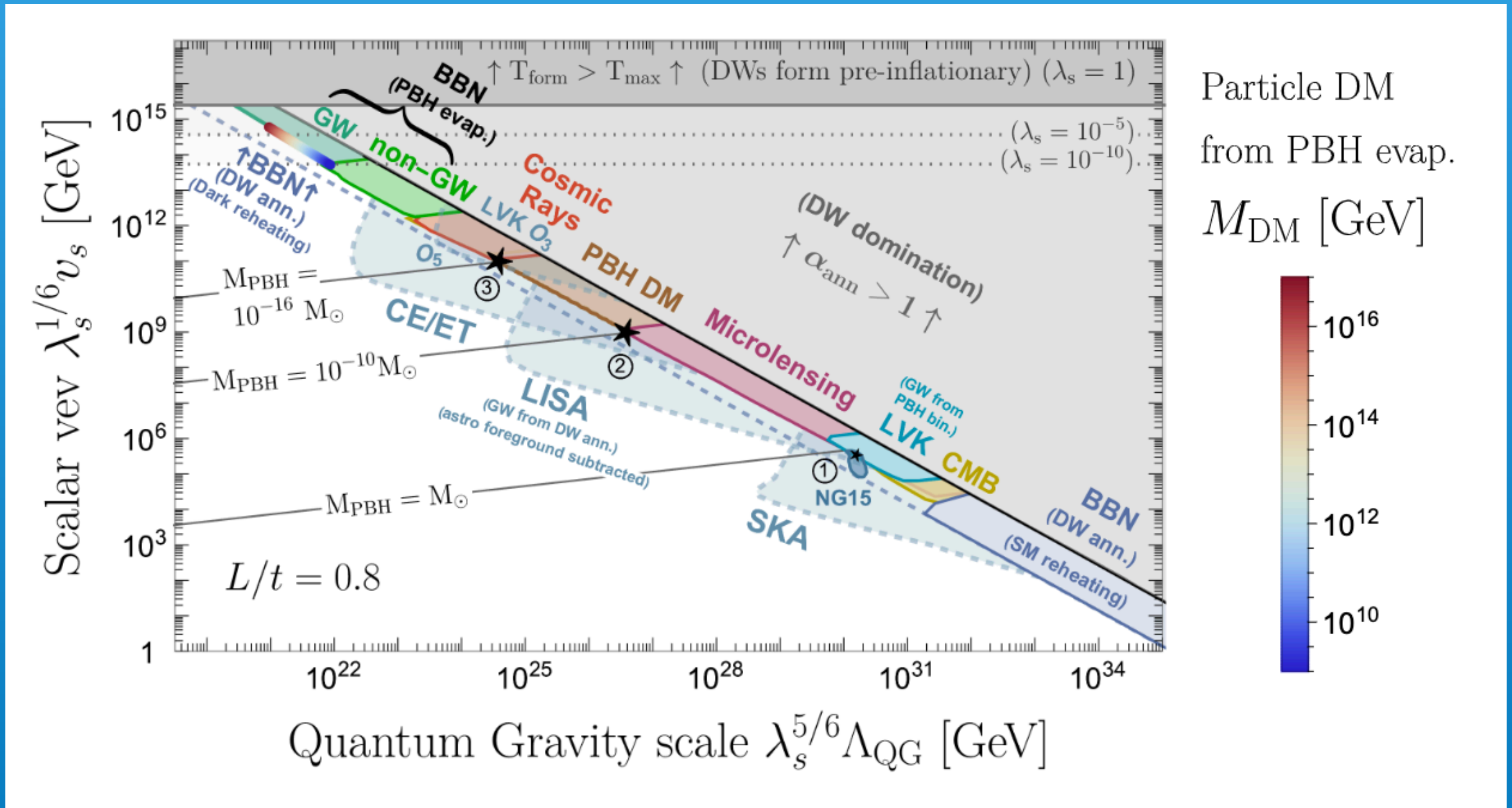
1) test quantum gravity (qualitatively)
2) Producing dark matter

- Domain walls can induce production of primordial black holes
- Superhorizon size domain walls must grow as R~a due to causality
- Their schwarzchild radius can exceed their radius at annihilation forming a pbh



5. Application of the PBH, GW, and BBN constraints of Fig. 4 on the parameter space of the Z_2 symmetric singlet bro

Summary and conclusion

Quantum gravity makes domain walls a *compelling* source of gravitational waves in the early Universe

This is because the relevant QG process is non-perturbative and effective opperators are suppressed by a scale above the Planck scale

Can use GWs to qualitatively test QG

Can cross check the scale (assuming 1 QG scale!)

Domain walls can explain NANOGrav and DM