Finite volume effects in QFT and cosmological singularities

J. Alexandre, London 2024

- Null Energy Condition violation from QFT without modified Gravity or exotic matter
- Origin: finite volume

  Casimir effect

  Tunnelling between degenerate vacua
- Relevance to cosmic bounce?

The Leverhulme Trust & STFC

Null Energy Condition

- n^μ null vector $\longrightarrow$ $T_{\mu\nu} n^\mu n^\nu \geq 0$
- satisfied classically
- assumption for singularity theorems
- ideal fluid $\longrightarrow$ $\rho + p \geq 0$
- continuity equation $\dot{\rho} + 3H(\rho + p) = 0$

  ↳ intuitive behaviour

- can be violated in QFT

  ↳ possibility for cosmic bounce

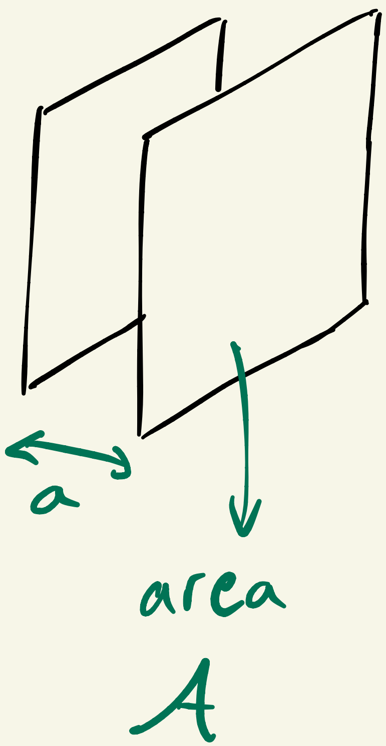
NEC violation from Casimir effect

$$E_{casimir} \equiv E_{discrete} - E_{continuous}$$

↳ cancellation of UV divergences

example

electromagnetic ground state energy



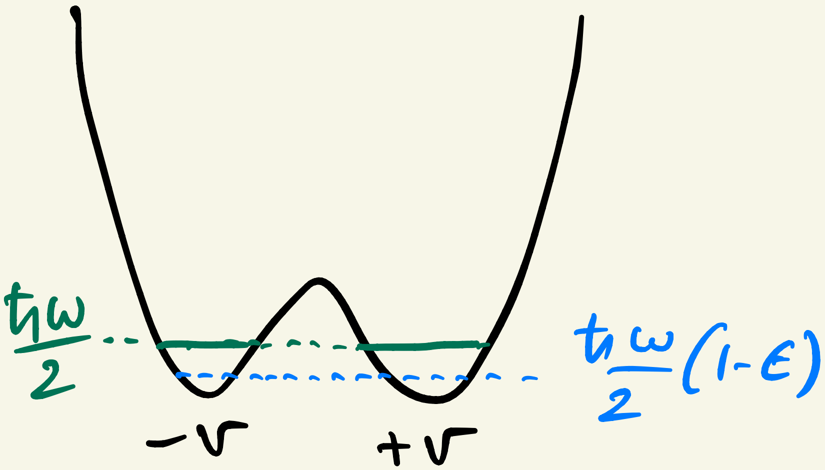
$$E_{cas} = -\frac{\pi^2 A}{720 a^3}$$

$$\rho = \frac{E_{cas}}{aA} = -\frac{\pi^2}{720 a^4}$$

$$p = -\frac{1}{A}\frac{\partial E_{cas}}{\partial a} = -\frac{\pi^2}{240 a^4}$$

$$\rho + p = -\frac{\pi^2}{180 a^4}$$

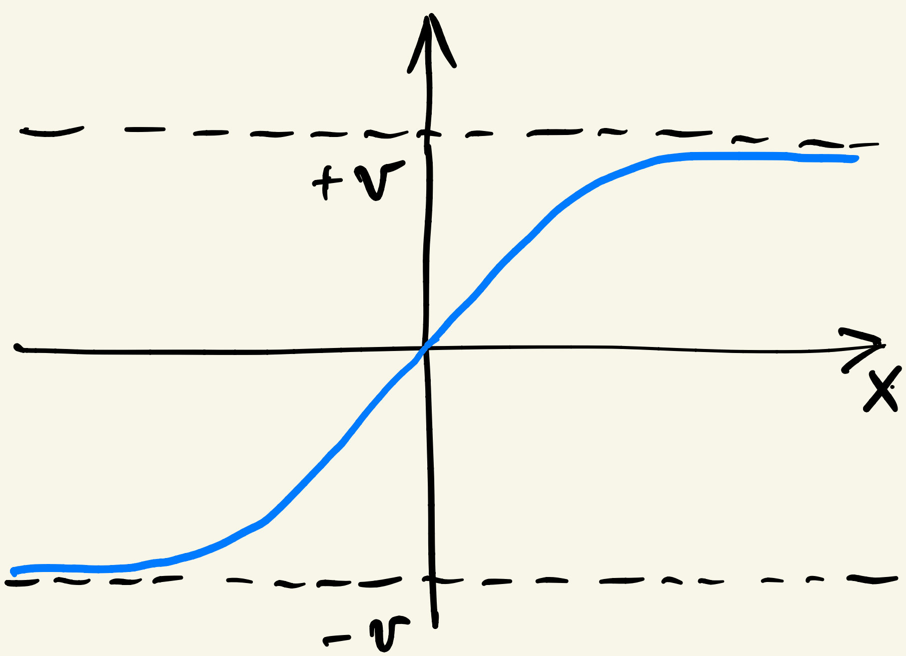
Tunnelling in Quantum Mechanics



ground state energy

$$\epsilon \propto \sqrt{S^0_{inst}}\, e^{-S^0_{inst}}$$

S^0_{inst} = instanton action

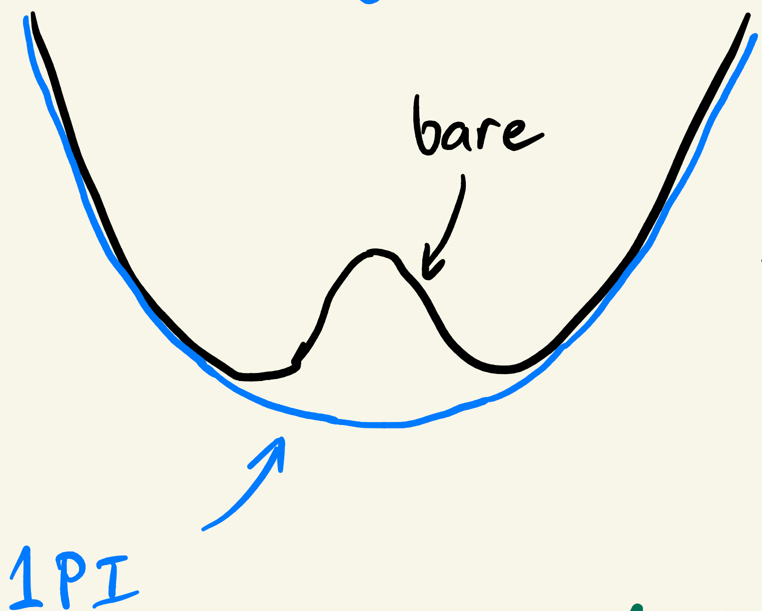


tunnelling

$\Downarrow$

decrease of ground state energy

Tunnelling in QFT (degenerate vacua)



symmetry restoration

tunnelling probability $\propto \sqrt{S_{inst}}\, e^{-S_{inst}}$

$$S_{inst} \propto (m\ell)^3/\lambda$$

infinite volume $\longrightarrow$ SSB

Convexity
- known since 70's
- explicit construction

J.A, J. Polonyi 2022 J.A, D. Backhouse 2023

$$U_{eff}(\phi) = U_0 + \frac{M^2}{2}\phi^2 + \cdots$$

$\hookrightarrow$ ground state energy density

Semi-classical approximation

$$Z[j] \simeq \sum_{\substack{\text{saddle}\\ \text{points } k}} \frac{\exp(-S[\phi_k])}{\sqrt{\det \delta^2 S[\phi_k]}}$$

- saddle points = dilute gas of instantons
- expand in j and infer background field

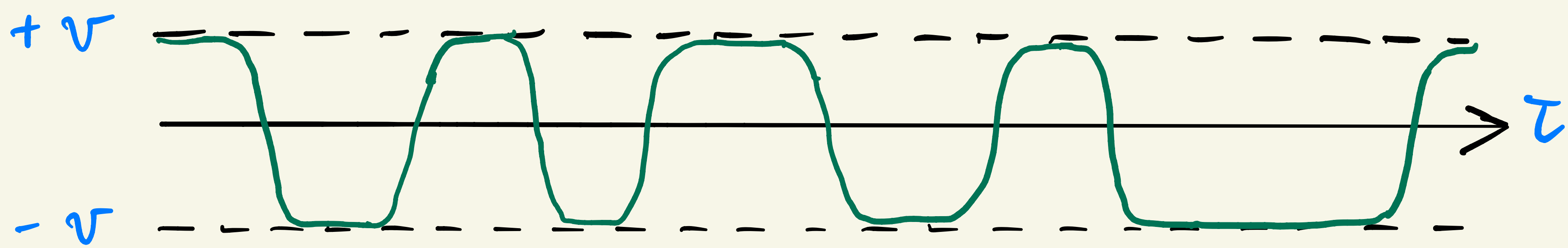
$$\phi_b = \langle \phi \rangle = \frac{-1}{Z} \frac{\delta Z}{\delta j}$$

- invert $\phi_b[j] \mapsto j[\phi_b]$
- Legendre transform $\rightarrow$ 1PI

$$S_{eff}[\phi_b] = -\ln Z - \int j \phi_b$$

$\hookrightarrow$ convex functional of ϕ_b

Dilute gas of instantons



$$Z \simeq \sum_{n=0}^{\infty} \frac{(m\beta)^n}{n!} \, F_n \, e^{-n S_{inst}}$$

j=0

zero-mode contribution

fluctuation factor (one-loop)

n-instanton action

QM approximation

$$Z \simeq \exp\left(-\beta E_{casimir} + m\beta \sqrt{\frac{6 S_{inst}}{\pi}} \, e^{-S_{inst}} \right)$$

Ground state energy

$$E_{true} = E_{casimir} + E_{tunnelling} = \ell^3 U_0$$

$$E_{tunnelling} \simeq -m\sqrt{\frac{(m\ell)^3}{\lambda}}\, e^{-(m\ell)^3/\lambda}$$

$E_{casimir}$ depends on geometry + boundary conditions

<u>example: 3-torus</u>

$$E_{casimir} \simeq -\frac{(m\ell)^{3/2}}{\ell} e^{-m\ell} \qquad m\ell \gg 1$$

$\longrightarrow$ dominant for large box size

Full calculation

W. Ai, J.A, M. Carosi, B. Garbrecht, S. Pla 2024

resolvent method $\quad (\hat{A} + s)\, G_A(s; x, y) = \delta^{(4)}(x-y)$

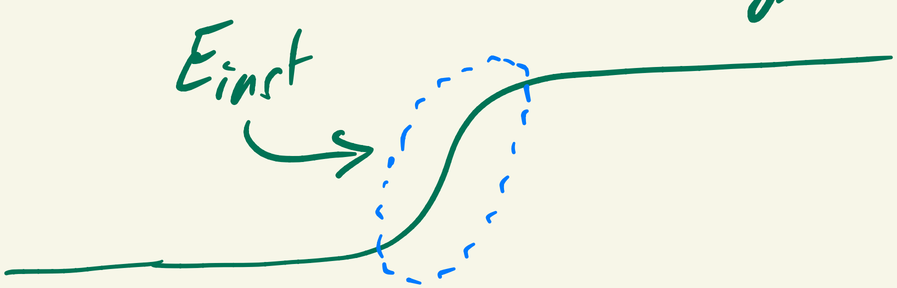
$$\ln \frac{\det \hat{A}}{\det \hat{B}} = -\int_0^\infty ds \int d^4x \left[G_A(s;x,y) - G_B(s;x,y) \right]$$

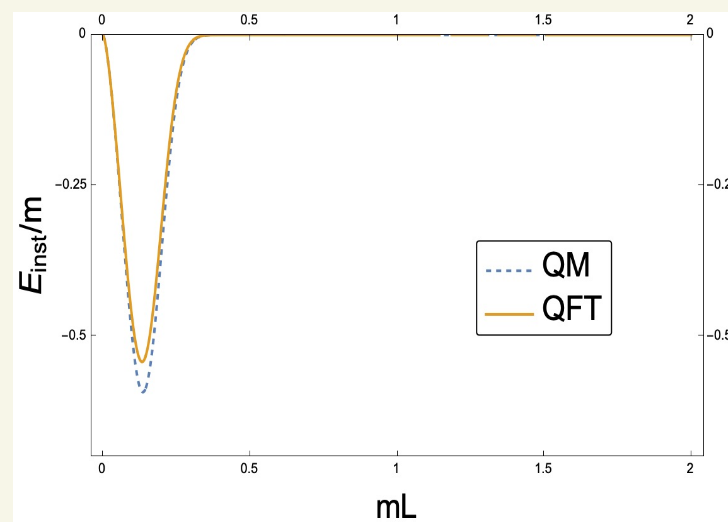
\+ spectral decomposition of Green's functions

\+ ansatz $\quad G(s;x,y) = \frac{1}{\ell^3} \sum_{\vec{n} \in \mathbb{Z}^3} F(s;\tau,\tau')\, e^{-i\vec{k}_n \cdot (\vec{x}-\vec{y})}$

\+ renormalisation

= instanton energy





Massless field on a (rigid) 3-sphere

J.A, D. Backhouse

$$S = \int d\tau \int d^3x \sqrt{g} \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{\lambda}{24} \left(\phi^2 - \frac{6|\xi| R}{\lambda} \right)^2 \right)$$

- effective $(\text{mass})^2 = 2|\xi| R$
- instanton action $= \frac{48\pi^2}{\lambda} \sqrt{3|\xi|^3}$

↳ independent of radius

→ no exponential suppression with radius

But R^2 term → modified gravity for dynamical 3-sphere

Relevance to Cosmology?

Assumptions

- space = 3-torus
- FLRW flat metric
- adiabatic expansion
- fluid = true ground state

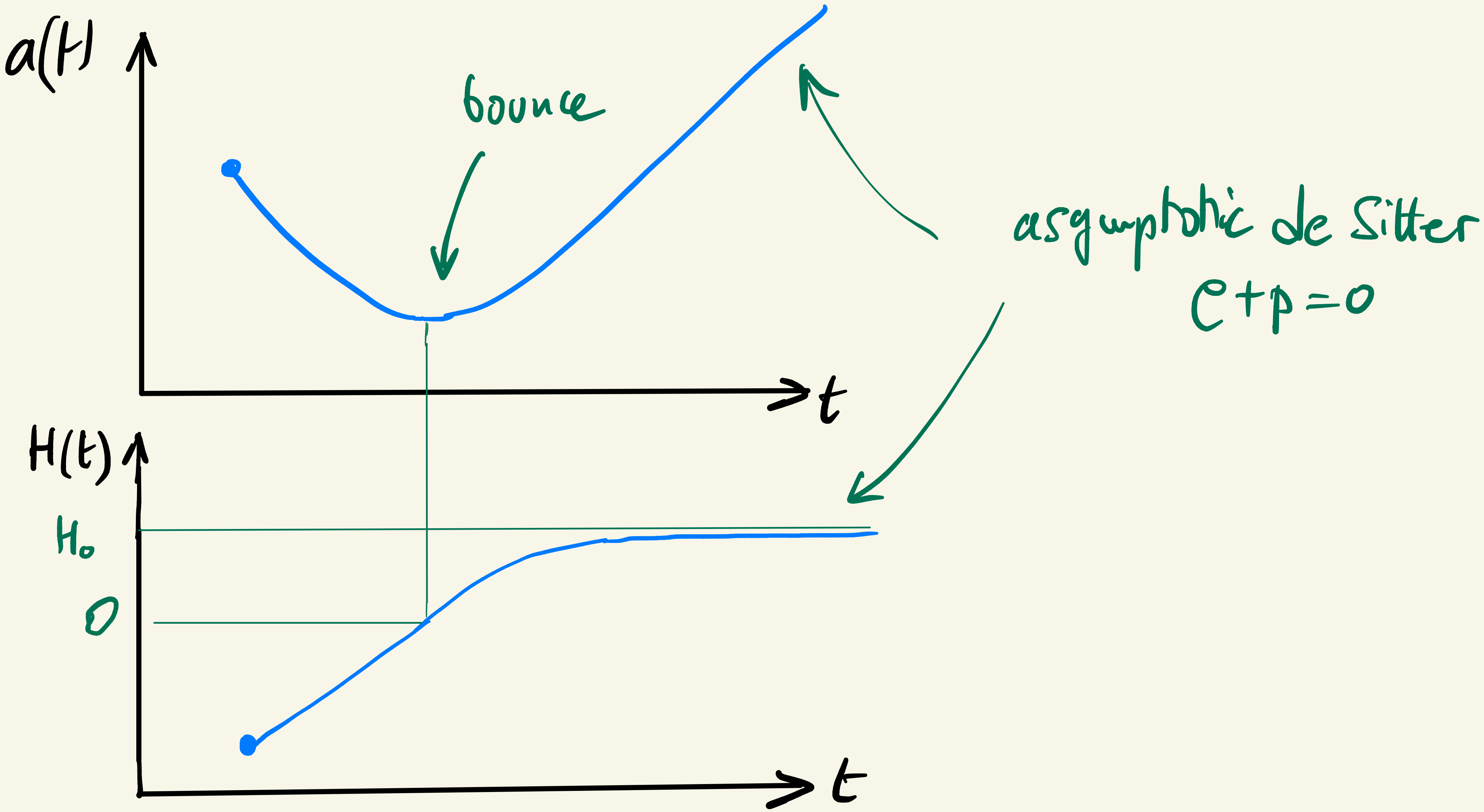
→ solve Friedmann equations with

$$\rho = \rho_{casimir} + \Lambda - \rho_0 \frac{e^{-\alpha^3}}{\alpha^{3/2}} \qquad \left(\alpha^3 \equiv a^3(t) S_{inst}\right)$$

$$p = p_{casimir} - \Lambda - \rho_0 \left(\frac{1}{2\alpha^{3/2}} - \alpha^{3/2}\right) e^{-\alpha^3}$$

J.A, S. Pla 2023

Cosmic bounce



$\longrightarrow$ dynamically induced bounce

Anisotropic Universe

- Friedmann equations in terms of average H and anisotropy $\sigma^2 = \frac{1}{18}\left[(H_1-H_2)^2+(H_2-H_3)^2+(H_1-H_3)^2\right]$

$$H^2 = \frac{8\pi G}{3}\rho + \sigma^2$$

$$H^2 + \dot{H} = -\frac{4\pi G}{3}(\rho+3p) - 2\sigma^2$$

$\rightarrow$ σ^2 tends to restore NEC $\left(\sigma^2 \propto a^{-6}(t)\right)$

- a_c critical scale factor assume $H(t_0)=0$

bounce $(\dot{H}(t_0)>0)$ if $a(t_0)>a_c$

collapse $(\dot{H}(t_0)<0)$ if $a(t_0)<a_c$

J.A, K. Clough, S. Pla 2023

Future studies

- Cosmology with R^2 term?
- Relevance to black hole singularity?

Basic bibliography

First works involving Casimir effect in Cosmology
Mamayev, Mostepanenko, Starobinsky, Zeldovich (1980's)

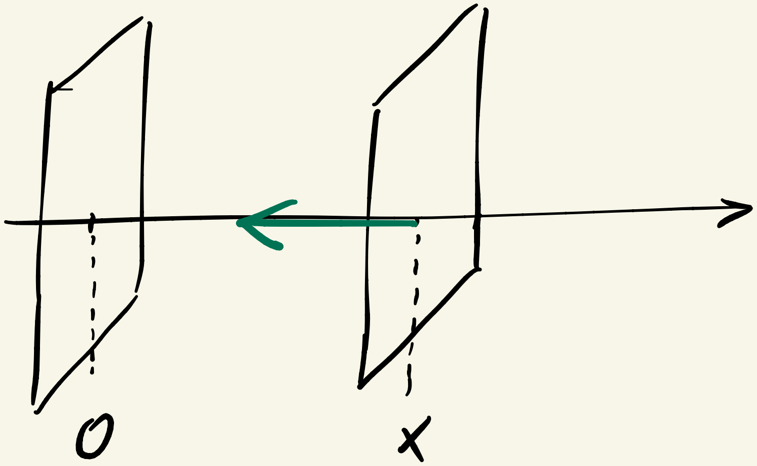
Reviews on NEC Rubakov 2014
Kontou, Sanders 2020

Resolvent method Baacke, Junker 1993

Review on tunnelling Kleinert (book)

Casimir effect on 3-sphere + Cosmological bounce
Herdeiro, Sampaio (2005)

Moving mirror

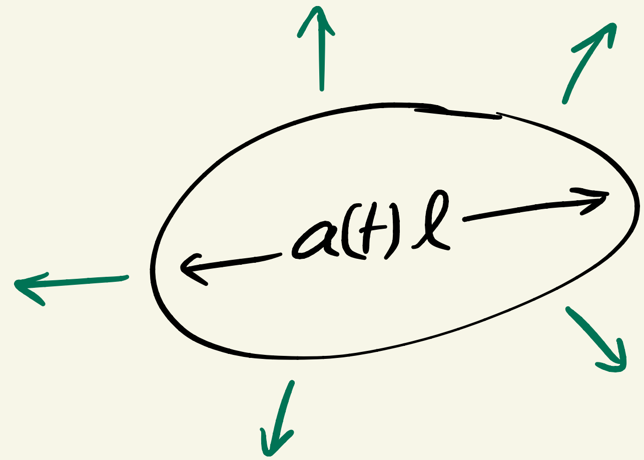


$$m\ddot{x} = F_{casimir}$$

$$F_{casimir} = -\frac{d}{dx}\left(-\frac{A}{x^3}\right)$$

$$A > 0 \longrightarrow \ddot{x} < 0$$

Cosmic bounce



$$\frac{\ddot{a}}{a} = \underset{\substack{\uparrow \\ 0}}{H^2} - \frac{\kappa}{2}(\rho + p)$$

$$\rho + p < 0 \longrightarrow \ddot{a} > 0$$

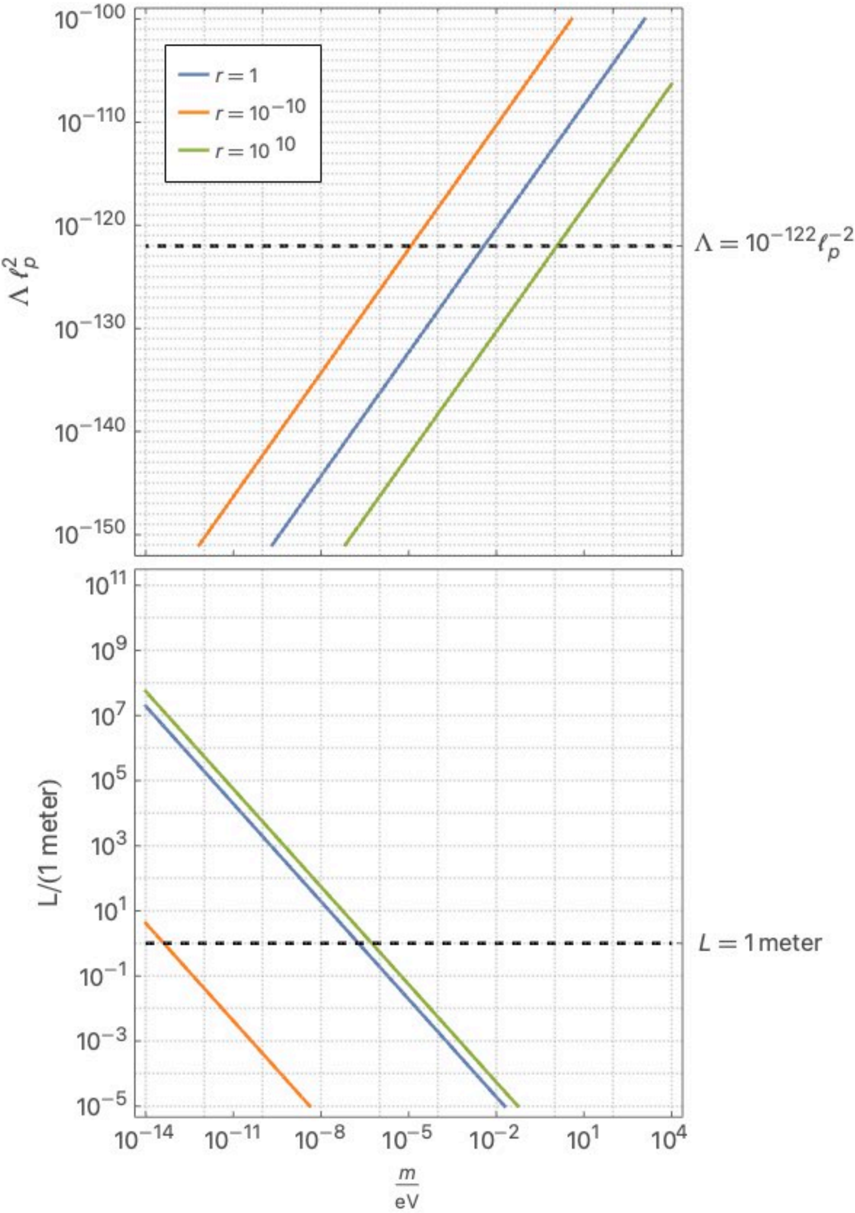
Phenomenology

typical size of the Universe

- now $L \simeq 10^{26}$ meters

  ↳ no sign of periodicity

- Grand Unification $L \simeq 1$ meter



$r=1$
$r=10^{-10}$
$r=10^{10}$
$\Lambda\,\ell_p^2$
$\Lambda = 10^{-122}\ell_p^{-2}$
10^{-100}
10^{-110}
10^{-120}
10^{-130}
10^{-140}
10^{-150}
L/(1 meter)
$L = 1\,\mathrm{meter}$
10^{11}
10^{9}
10^{7}
10^{5}
10^{3}
10^{1}
10^{-1}
10^{-3}
10^{-5}
10^{-14}
10^{-11}
10^{-8}
10^{-5}
10^{-2}
10^{1}
10^{4}
$\frac{m}{\mathrm{eV}}$