Axions, String Theory, and the Swampland

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Outline

I. The Landscape and the Swampland

II. No global symmetries and the ubiquity of axions

III. Weak Gravity Conjecture and axion inflation

IV. Approximate symmetries and axion bitowers

V. Summary and next steps

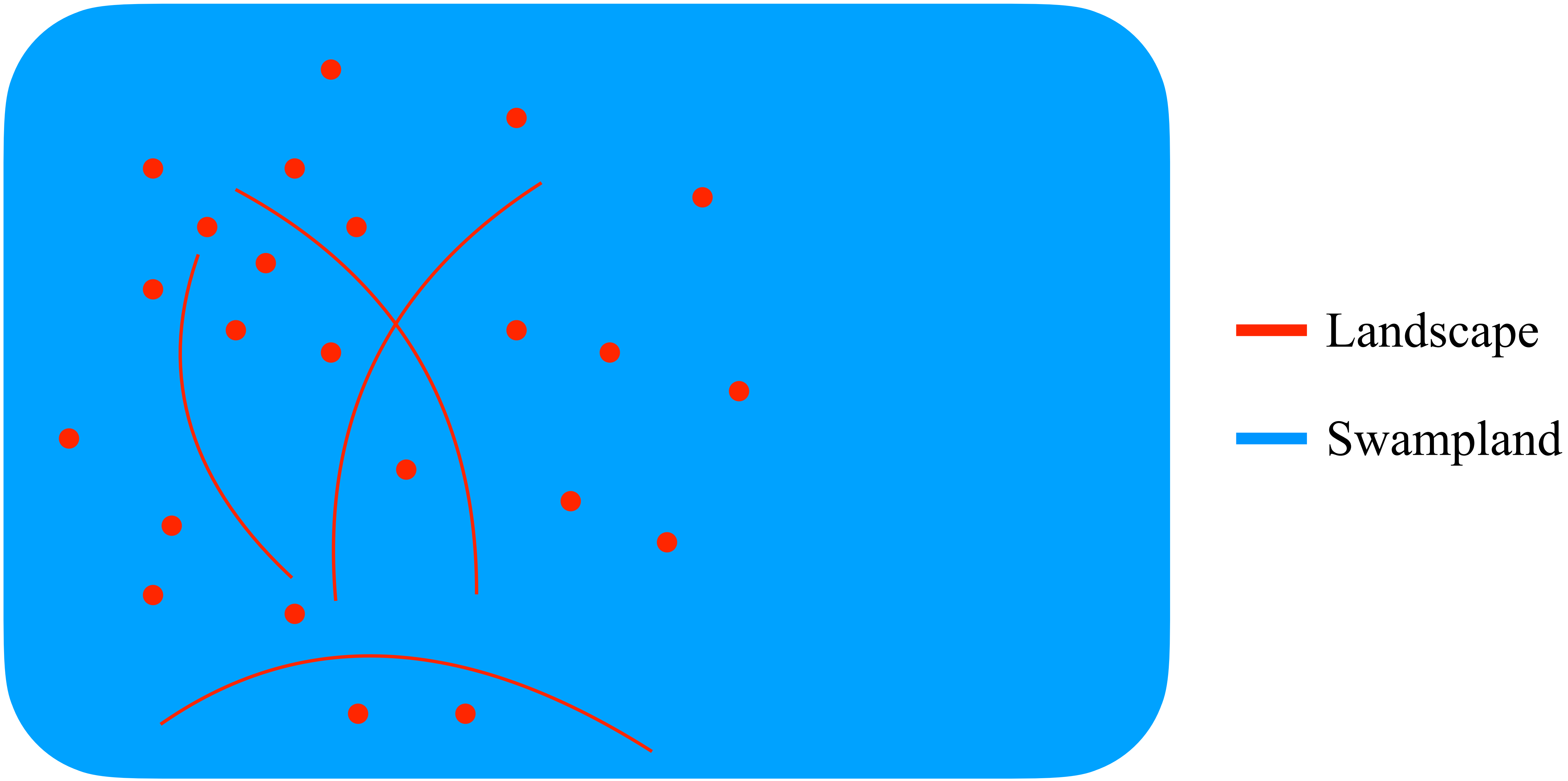
The Landscape and the Swampland

The Landscape

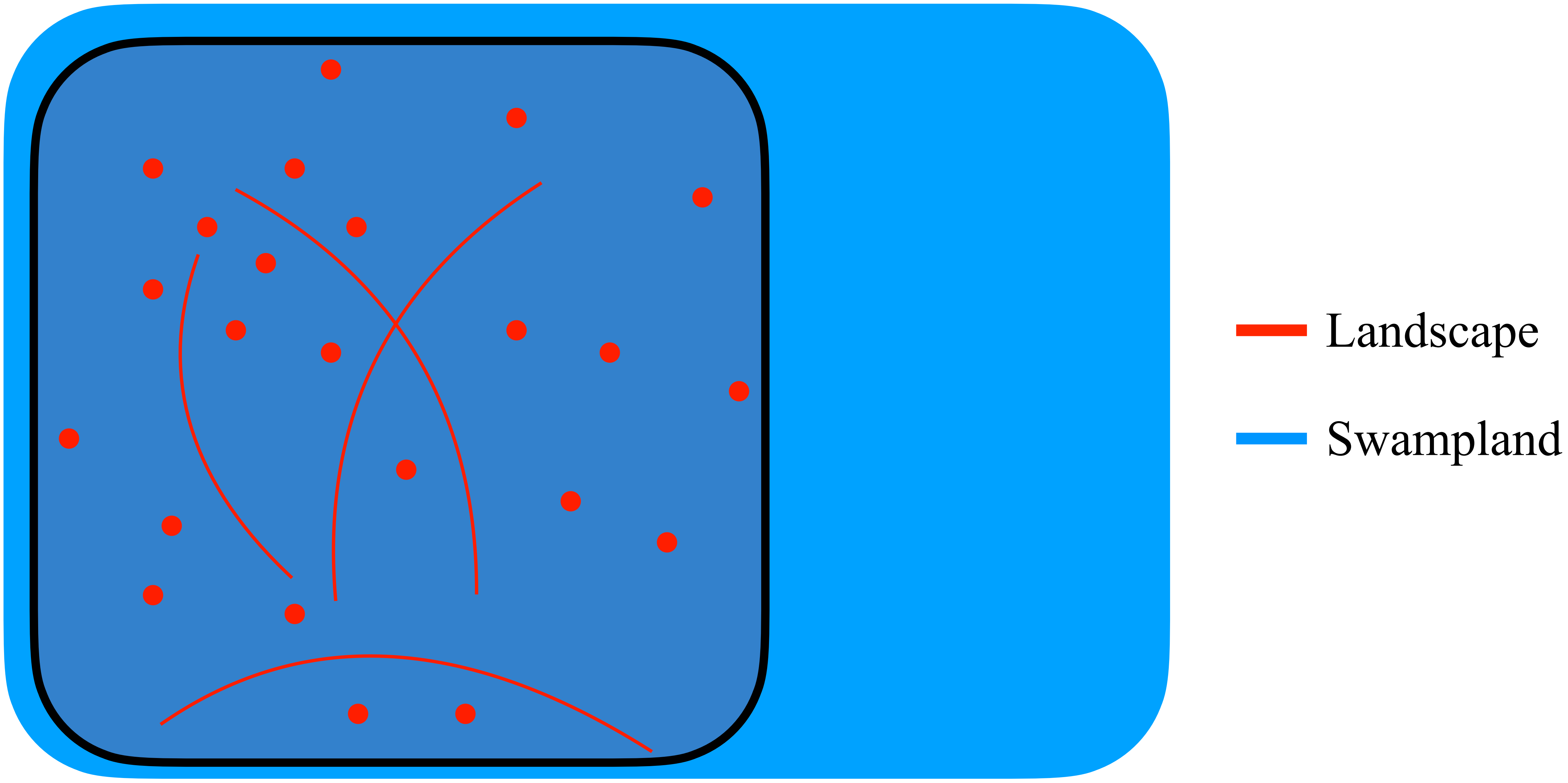
The Swampland

Vafa ’05, Ooguri, Vafa ‘06

Landscape-Swampland Map



Goal: Delineate Boundary

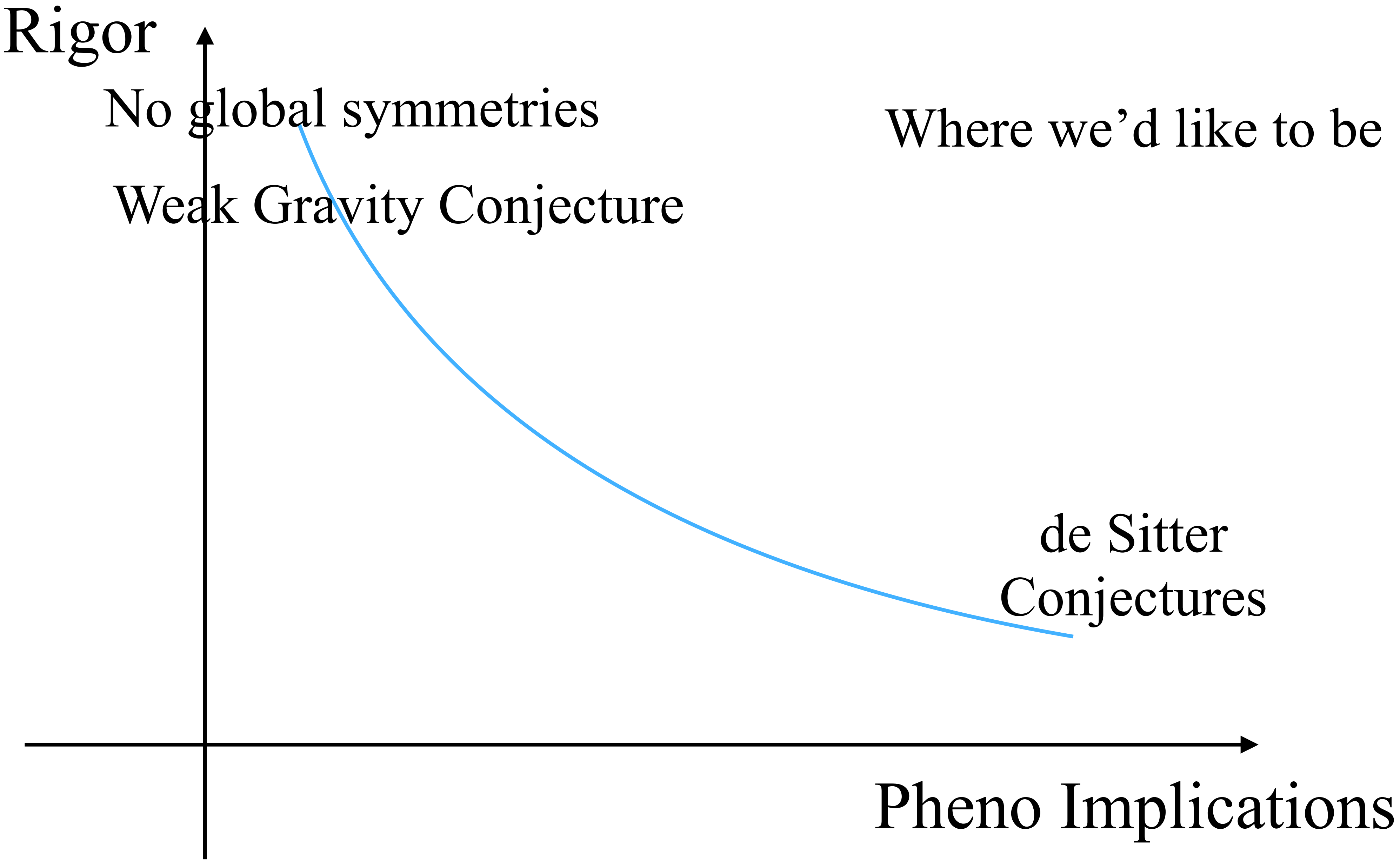


Some Swampland Conjectures

Vafa ’05, …

- **No Global Symmetries**
- **Weak Gravity Conjecture**
- Distance Conjecture
- Tower/Sublattice WGC
- Completeness of Spectrum
- No Free Parameters
- 0-form WGC
- Magnetic WGC
- Repulsive Force Conjecture
- Scalar WGC
- De Sitter Conjectures
- AdS Distance Conjecture
- Finiteness of Landscape
- No Eternal Inflation
- No Non-SUSY AdS
- No Non-SUSY Minkowski space
- Moduli spaces have finite volume
- Spin-2 Conjecture
- …

The Great Swampland Tradeoff



cf. talks from Hirosi Ooguri, Matthew Reece,…

No global symmetries and the ubiquity of axions

No Global Symmetries

Conjecture: There do not exist exact global symmetries in a consistent theory of quantum gravity in d>3 spacetime dimensions.

Hawking ‘80s, Banks, Seiberg ’10

Generalized Global Symmetries

Gaiotto, Kapustin, Seiberg, Willett ‘14

- A “*q*-form global symmetry” is a global symmetry for which the charged operators are *q*-dimensional
  - $q = 0$ corresponds to an ordinary global symmetry
- Global symmetry transformations form a group, *G*.
- *G* may be discrete or continuous. For $q = 0$, it may be non-Abelian or Abelian. For $q > 0$, it must Abelian.
- If *G* is continuous, it has (under reasonable assumptions) a conserved *d-q-1*-form “Noether current” *J*:

$$dJ^{(d-q-1)} = 0$$

Chern-Weil Global Symmetries

- G gauge theory has conserved currents of the form

$$J = \mathrm{Tr}(F^k) := \mathrm{Tr}(\underbrace{F \wedge F ... \wedge F}_{k})$$

- Their conservation follows from $dF = 0$ (G abelian)/the Bianchi identity $dF + [A, F] = 0$ (G non-abelian)
  - As a result, they are not easy to break
- In 4d, $\mathrm{Tr}(F^2)$ is a 4-form, so trivially conserved. Nonetheless, there is a sense in which it generates a (-1)-form symmetry, as it has quantized (integral) periods. The associated charge is instanton number.

Heidenreich, McNamara, Montero, Reece, TR, Valenzuela, ‘21

Eliminating CW Symmetries

- Since QG does not have exact global symmetries, these CW symmetries must either be *broken* or *gauged*

Broken	Gauged
Add monopoles	Add d-4 form C_{d-4}
$dF \neq 0$	$\mathcal{L} \supset C_{d-4} \wedge F \wedge F$
$\Rightarrow d(F \wedge F) \neq 0$	$\Rightarrow F \wedge F = d(\cdots)$
$\Rightarrow$ symmetry broken	$\Rightarrow$ symmetry gauged

Heidenreich, McNamara, Montero, Reece, TR, Valenzuela, ‘21

Breaking CW Symmetries by Unification

- Consider GUT symmetry breaking in d dimensions:

$$SU(5) \to SU(3) \times SU(2) \times U(1)$$

- UV: expect one CW current, $\mathrm{Tr}\, F^2_{SU(5)}$, gauged by C_{d-4}
- IR: expect three CW currents, $\mathrm{Tr}\, F^2_{SU(3)}$, $\mathrm{Tr}\, F^2_{SU(2)}$, $\mathrm{Tr}\, F^2_{U(1)}$
- An IR theorist might over-count CW symmetries and expect more d-4 forms than actually exist. One CW symmetry will be gauged, other CW currents in IR are broken in UV by unification

Heidenreich, McNamara, Montero, Reece, TR, Valenzuela, ‘21

Axions and Quantum Gravity

- Axions are ubiquitous in string compactifications
- CW currents gauged by $C_{d-4} \wedge \mathrm{Tr}(F \wedge F)$ Chern-Simons terms in d > 4 dimensions, which reduce in 4d to $\theta\, \mathrm{Tr}(F \wedge F)$
- Chern-Weil perspective helps explain prevalence of axions in quantum gravity: they are needed to remove would-be (-1)-form global symmetries by gauging them (not just "looking under the lamppost")

Heidenreich, McNamara, Montero, Reece, TR, Valenzuela, '21

Implications for Axion Physics

- Common concern about axions for solving CP problem is the axion quality problem. Misaligned contributions to potential could spoil the solution:

$$\Lambda_{\mathrm{UV}}^4 \left[e^{-S_{\mathrm{QCD}}+i\theta} + e^{-S_{\mathrm{other}}+i\theta+i\delta} + \mathrm{h.c.}\right]$$

- If $\delta \neq 0$, need $S_{\mathrm{other}} \gg S_{\mathrm{QCD}}$
- The Chern-Weil perspective ameliorates this worry: given two kinds of instantons, expect either two different axions (both symmetries gauged), or else expect some way to transform instantons into one another (one symmetry broken)
- Suggests we only need worry about gauge sectors that can be unified with QCD

Heidenreich, McNamara, Montero, Reece, TR, Valenzuela, ‘21

Weak Gravity Conjecture and axion inflation

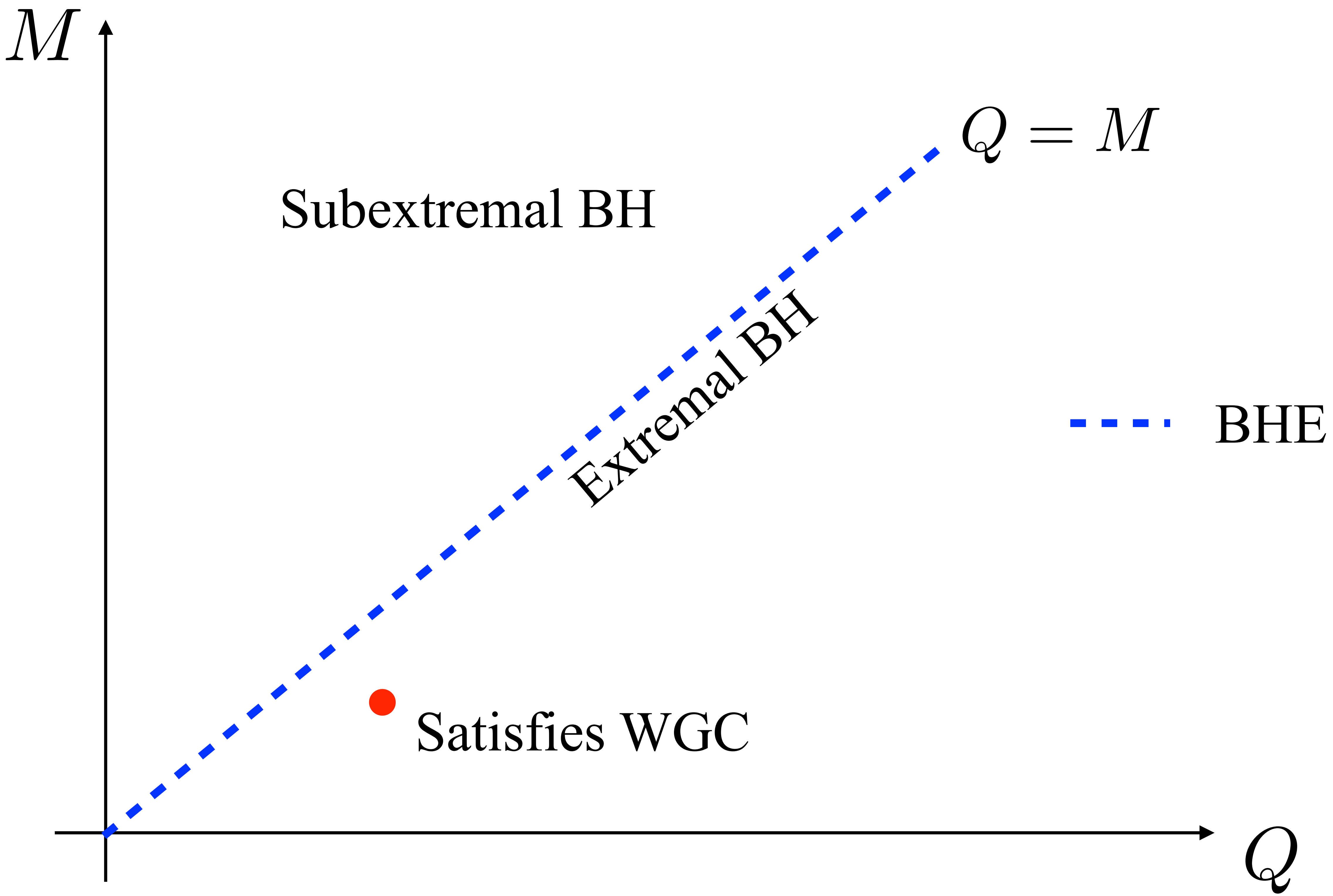
Weak Gravity Conjecture (WGC)

In any d-dimensional U(1) gauge theory coupled to quantum gravity, there must exist a "superextremal" particle of charge *q*, mass *m*, with

$$\frac{q}{m} \geq \frac{Q}{M}\bigg|_{\text{ext}} \gtrsim \frac{1}{M_{\text{Pl};d}^{\frac{d-2}{2}}} \qquad (q = \underset{\text{coupling constant}}{\underset{\uparrow}{g}}\overset{\text{charge quantum}}{\overset{\downarrow}{n}})$$

Arkani-Hamed, Motl, Nicolis, Vafa, '06

Weak Gravity Conjecture (WGC)



The Generalized WGC

p-form WGC: Given a p-form gauge field in d dimensions, there exists an electrically charged object of dimension p-1 and a magnetically charged object of dimension d-p-3 with

$$T_{el} \lesssim \left(\frac{g^2}{G_N}\right)^{1/2} \quad , \quad T_{mag} \lesssim \left(\frac{1}{g^2 G_N}\right)^{1/2}$$

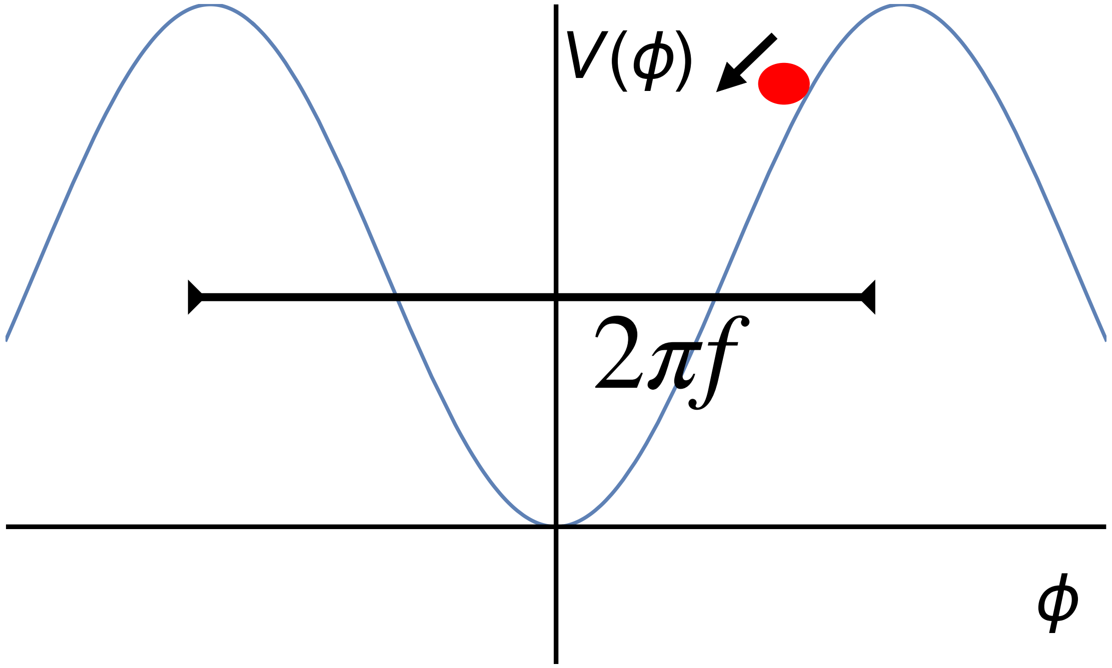
Axion (0-form) WGC: Given an axion with decay constant *f*, there exists an instanton of action *S* such that

$$fS \lesssim M_{\rm Pl}$$

Arkani-Hamed, Motl, Nicolis, Vafa, ’06

Natural Inflation

- One popular model of inflation is called "natural inflation," which involves a sinusoidal potential:



$$V(\phi) = \Lambda_{\rm UV}^4 e^{-S} \left(1 - \cos(\frac{\phi}{f})\right) + O(e^{-2S})$$

- Need $f \gg M_{\rm Pl}$ for inflation, $S \gg 1$ for parametric control

Freese, Frieman, Olinto '90

Natural Inflation

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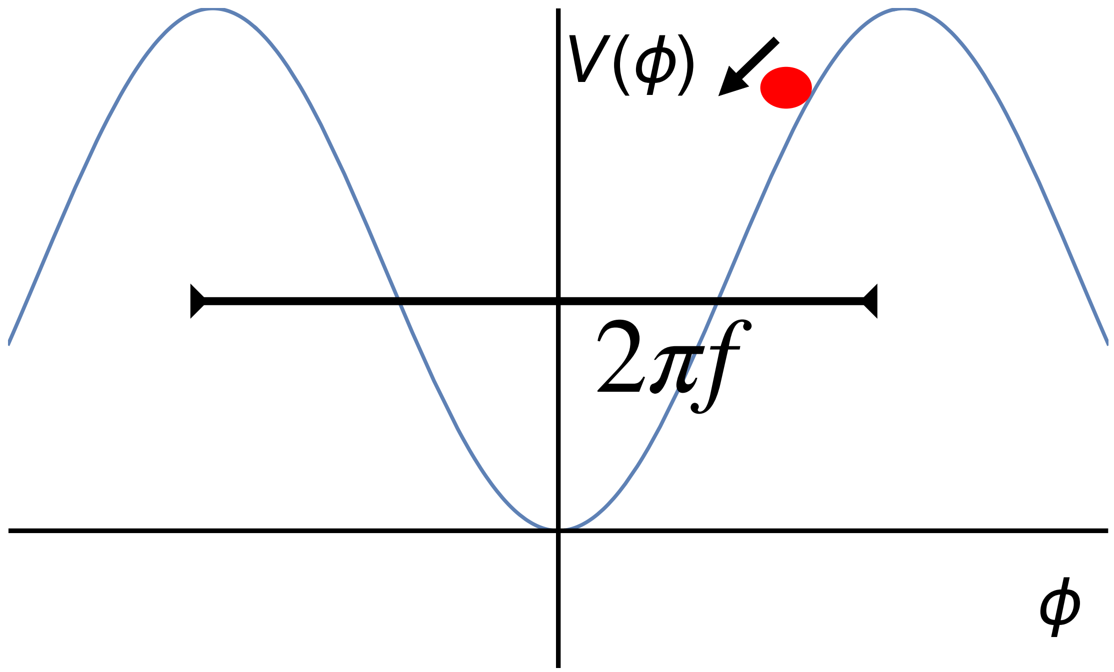

- Need $f \gg M_{\rm Pl}$ for inflation, $S \gg 1$ for control
- Axion WGC $\Rightarrow fS \lesssim M_{\rm Pl}$
- Natural inflation incompatible with WGC!

Arkani-Hamed, Motl, Nicolis, Vafa ’06, TR ’14, ’15, Brown, Cottrell, Shiu, Soler ‘15

Natural Inflation in String Theory

- Surveys of string landscape have repeatedly observed $\mathscr{R} \equiv \pi f_{\text{eff}} \lesssim M_{\text{Pl}}$ Banks, Dine, Fox, Gorbatov '04, TR '14, '15, Bachlechner, Long, McAllister '14, Montero, Uranga, Valenzuela '15, Conlon, Krippendorf '16, Long, McAllister, Stout '16

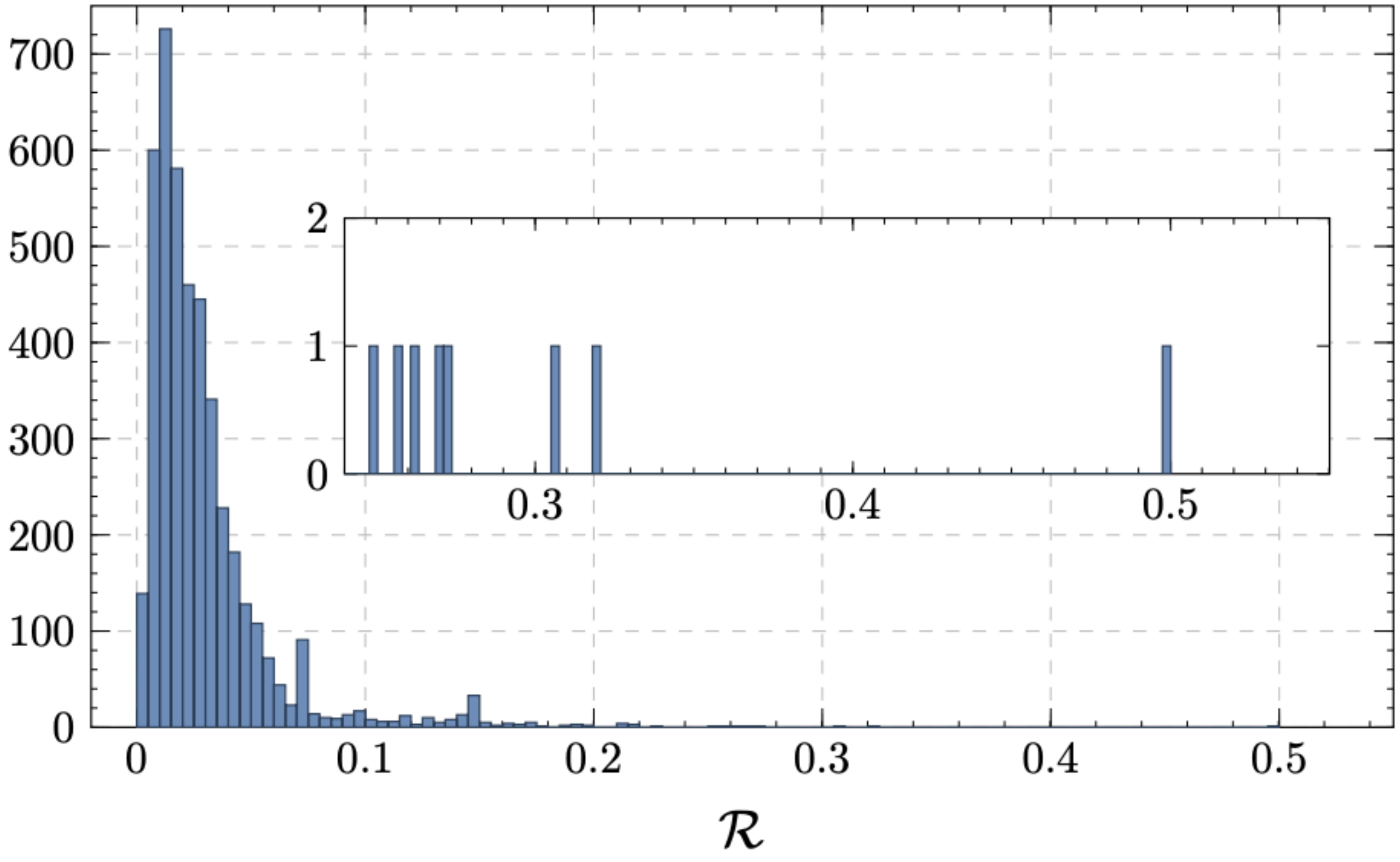

Figure adapted from Long, McAllister, Stout '16

Approximate Symmetries and Axion Bitowers

Approximate Symmetries in QG

- An approximate symmetry is defined in terms of some small parameter g, which vanishes as the symmetry becomes exact
- Evidence suggests that global symmetries in must be not only broken, but *badly broken*, in the context of quantum gravity, meaning that g becomes O(1) at the Planck scale:

$$g(E = M_{\mathrm{Pl}}) \gtrsim 1$$

Cordova, Ohmori, Rudelius ‘22

Fichet, Saraswat ’19

Axion Bitowers

- Axion has approximate shift symmetry, $\theta \to \theta + c$ (exact gauge symmetry when $c = 2\pi$)
- One common way to break is an axion bitower:

$$\text{Particles:} \quad m_{p,q}^2 = m_0^2 \left((p - q\frac{\theta}{2\pi})^2 + \frac{q^2 S^2}{(2\pi)^2} \right)$$

$$\text{Strings:} \quad T_{p,q}^2 = T_0^2 \left((p - q\frac{\theta}{2\pi})^2 + \frac{q^2 S^2}{(2\pi)^2} \right)$$

- Symmetry badly broken at Planck scale $\Rightarrow fS \lesssim M_{\text{Pl}}$
  - Matches axion WGC!

Etheredge, Heidenreich, Reece, Rudelius, to appear

Axion Bitowers in String Theory

- Type IIB string theory in 10d

(p, q)-strings: $$T_{p,q}^2 = T_0^2 \left((p - q\frac{\theta}{2\pi})^2 + \frac{q^2 S^2}{(2\pi)^2} \right)$$

$$T_0 = \frac{1}{2\pi\alpha'} \qquad S = T_{\mathrm{D}(-1)} = \frac{2\pi}{g_s}$$

- Circle compactification of tower of charged particles in 5d

$$m_{p,q}^2 = m_0^2 \left((p - q\frac{\theta}{2\pi})^2 + \frac{q^2 S^2}{(2\pi)^2} \right)$$

$$m_0 = \frac{1}{R} \qquad S = 2\pi m_5 R$$

Etheredge, Heidenreich, Reece, Rudelius, to appear

Axion Bitowers and Potentials

- One important consequence of a particle bitower is that it generates a sinusoidal potential:

$$V(\theta) = -\sum_{q=1}^{\infty}\sum_{l=1}^{\infty}\frac{m_0^4 S^2 q^2}{32\pi^4 l^3}e^{-2\pi q l S}\cos(q l \theta)\left(1+\frac{3}{2\pi q l S}+\frac{3}{(2\pi q l S)^2}\right)$$

- Such potentials could play a role in dark matter, inflation, etc.

Fan, Fraser, Reece, Stout ‘21

Etheredge, Heidenreich, Reece, Rudelius to appear

Summary and Next Steps

Summary

- Surveys of the quantum gravity landscape and swampland suggest that
  - Axions are ubiquitous in the landscape
  - Natural inflation is difficult (perhaps impossible) to achieve
  - Axions often couple to matter through bitowers
- These can be understood as consequences of the absence of global symmetries and/or the Weak Gravity Conjecture

Next Steps

- More precise constraints on axions in QG—less $\sim$, $\gtrsim$ and more $=$, $>$, $\geq$
- Stronger arguments for WGC, absence of approximate symmetries at the Planck scale
- Further investigation of cosmological and phenomenological applications

Thank you!