VH Production

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Outline

State-of-the-art VH predictions in SM

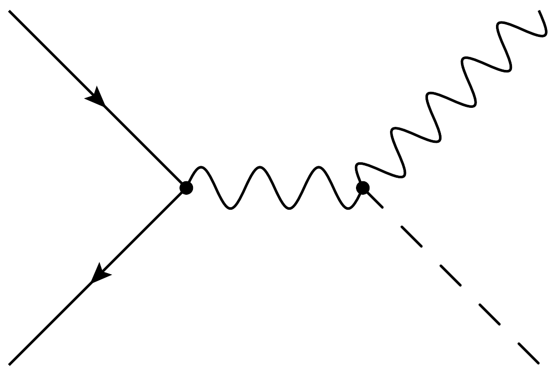
- Overview of $pp \to VH$
- Drell-Yan-like contributions
- VHj & Decays & Public Codes
- Gluon-induced contribution to $pp \to ZH$

Common technical aspects of VH & multiboson

- Numerics and Expansions
- Mass Scheme Uncertainties

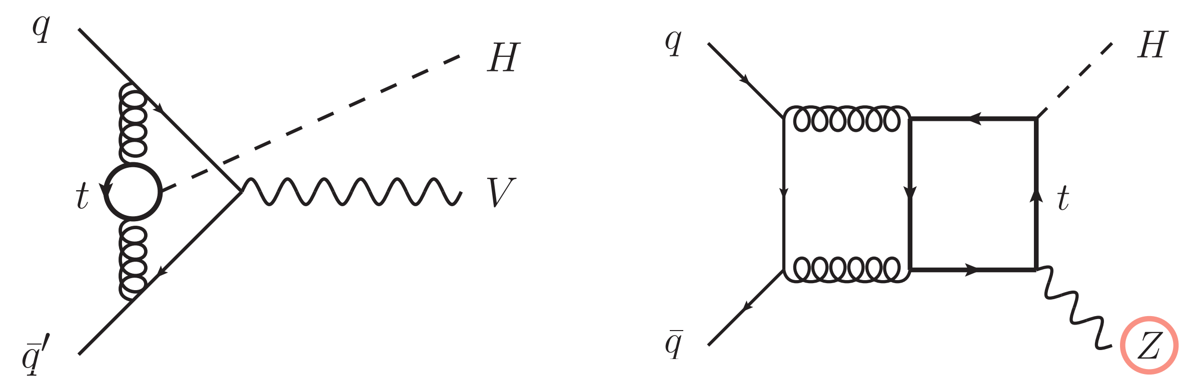
Summary

Overview of $pp \to VH$ @ Fixed-order



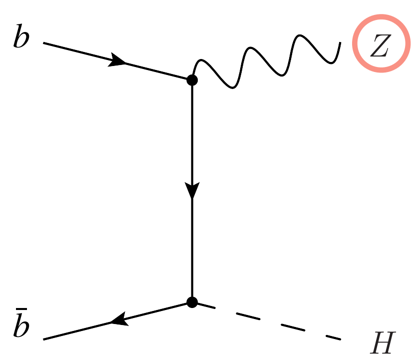
Drell-Yan piece ($\mathrm{N^3LO}$ known)

Brein, Djouadi, Harlander 03; Ferrera, Grazzini, Tramontano 14; Kumara, Mandal, Ravindran 14; Baglio, Duhr, Mistlberger, Szafron 22



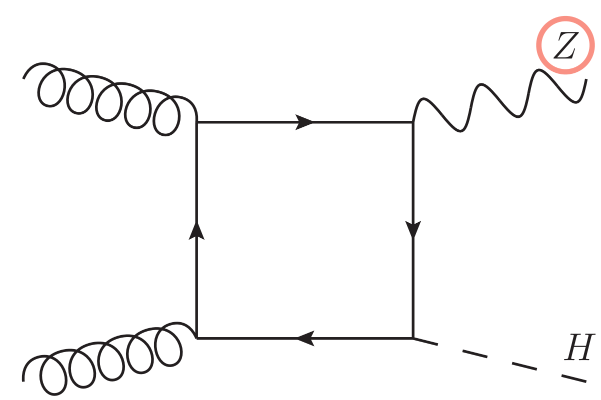
Non-Drell-Yan $q\bar{q}$ piece **~1-2%**

Brein, Harlander, Wiesemann, Zirke 11



$b\bar{b}$ piece (NNLO known)

Ahmed, Ajjath, Chen, Dhani, Mukherjee, Ravindran 19



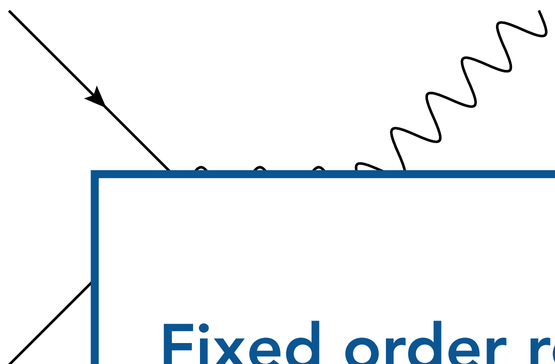
Gluon-induced (NLO known) **~10%**

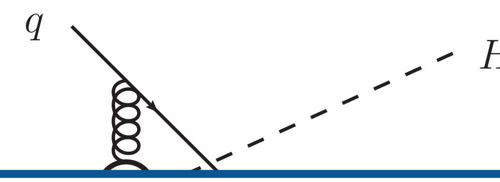
Dicus, Kao 88; Kniehl 90; Altenkamp, Dittmaier, Harlander, H. Rzehak, Zirke 12; Wang, Xu, Xu, Yang 21; Chen, Davies, Heinrich, Jones, Kerner, Mishima, Schlenk, Steinhauser 22; Degrassi, Gröber, Vitti, Zhao 22

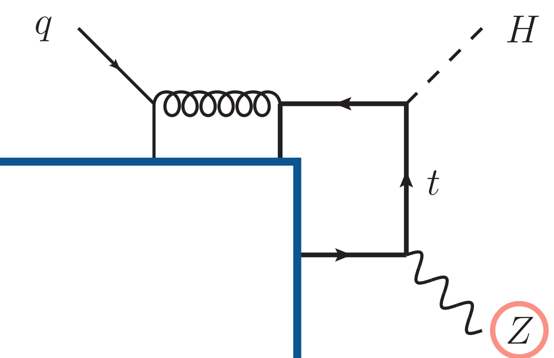
+ EW corrections (NLO known) ~ **-5-10%**

Ciccolini, Dittmaier, Krämer 02; Brein, Ciccolini, Dittmaier, Djouadi, Harlander, Krämer 04; Denner, Dittmaier, Kallweit, Mück 12

Overview of $pp \to VH$ @ Fixed-order







Drell-Ya

Brein, Djoua
Tramontano
Baglio, Duhr

%

$b\bar{b}$ piec

~10%

Ahmed, Ajjath, Chen, Dhani, Mukherjee, Ravindran 19

Harlander,
H. Rzehak, Zirke 12; Wang, Xu, Xu, Yang 21; Chen, Davies, Heinrich, Jones, Kerner, Mishima, Schlenk, Steinhauser 22; Degrassi, Gröber, Vitti, Zhao 22

Fixed order results available in various codes:

HAWK (NLO QCD + NLO EW)
Denner, Dittmaier, Kallweit, Mück 14

vh@nnlo (NNLO QCD + NLO EW)
Brein, Harlander, Zirke 12; Harlander, Klappert, Liebler, Simon 18

MCFM (NNLO QCD)
Campbell, Ellis, Williams 16

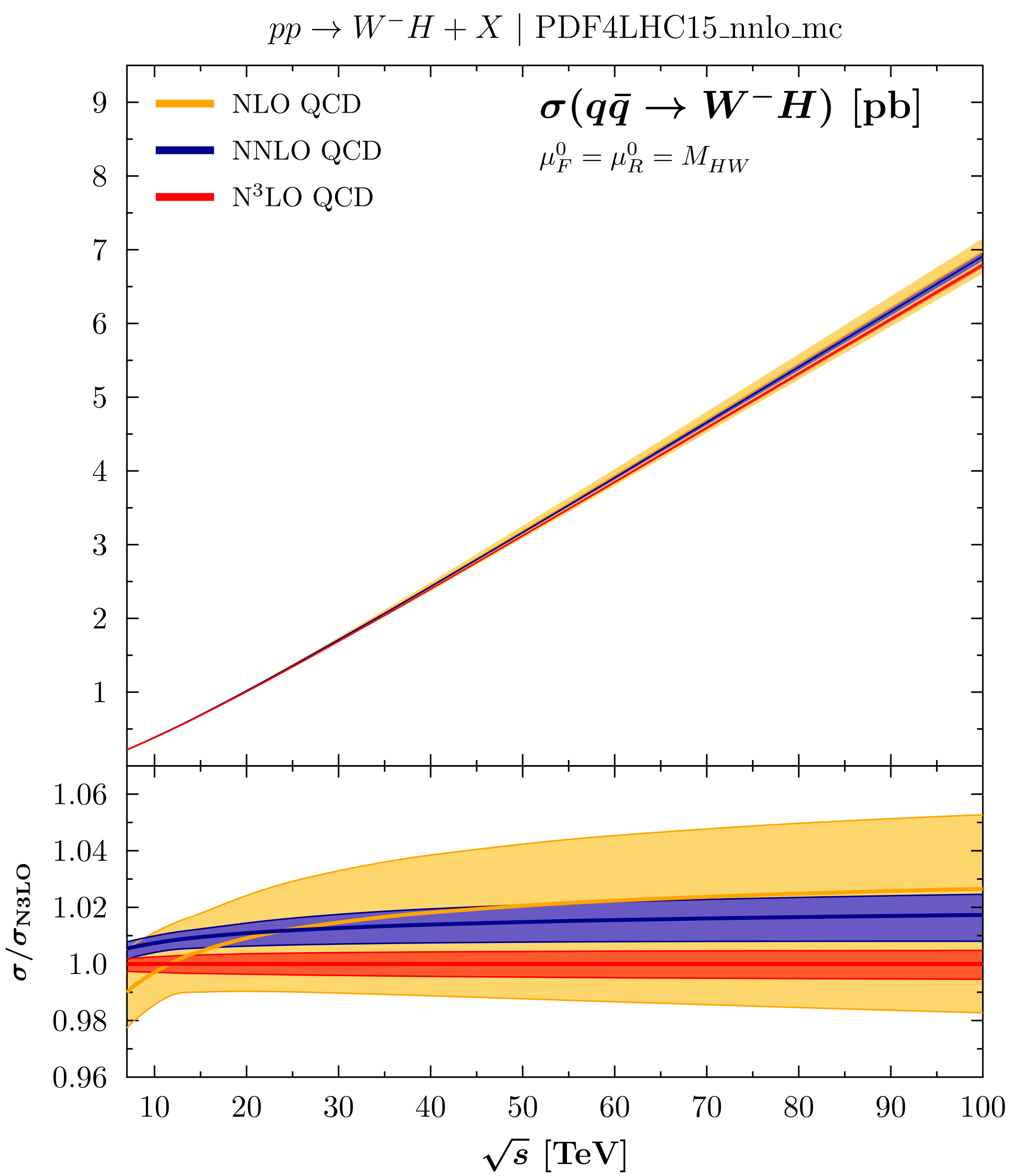
N3LO (N3LO DY QCD)
Baglio, Duhr, Mistlberger, Szafron 22

+ EW corrections (NLO known) ~ **-5-10%**

Ciccolini, Dittmaier, Krämer 02; Brein, Ciccolini, Dittmaier, Djouadi, Harlander, Krämer 04; Denner, Dittmaier, Kallweit, Mück 12

N³LO Drell-Yan $pp \to VH$

Process	$\sigma^{\rm LO}$ [pb]	$\sigma^{\rm NLO}$ [pb]	$K^{\rm NLO}$	$\sigma^{\rm NNLO}$ [pb]	$K^{\rm NNLO}$	$\sigma^{\rm N^3LO}$ [pb]	$K^{\rm N^3LO}$
W^+H	$0.758^{+2.43\%}_{-3.13\%}$	$0.883^{+1.38\%}_{-1.20\%}$	1.16	$0.891^{+0.28\%}_{-0.34\%}$	1.18	$0.884^{+0.27\%}_{-0.30\%}$	1.17
W^-H	$0.484^{+2.50\%}_{-3.26\%}$	$0.560^{+1.34\%}_{-1.23\%}$	1.16	$0.564^{+0.27\%}_{-0.34\%}$	1.17	$0.559^{+0.30\%}_{-0.33\%}$	1.16
ZH	$0.678^{+2.40\%}_{-3.11\%}$	$0.786^{+1.33\%}_{-1.16\%}$	1.16	$0.792^{+0.25\%}_{-0.32\%}$	1.17	$0.786^{+0.26\%}_{-0.29\%}$	1.16



N³LO Drell-Yan QCD corrections to $pp \to VH$ publicly available in code n3loxs

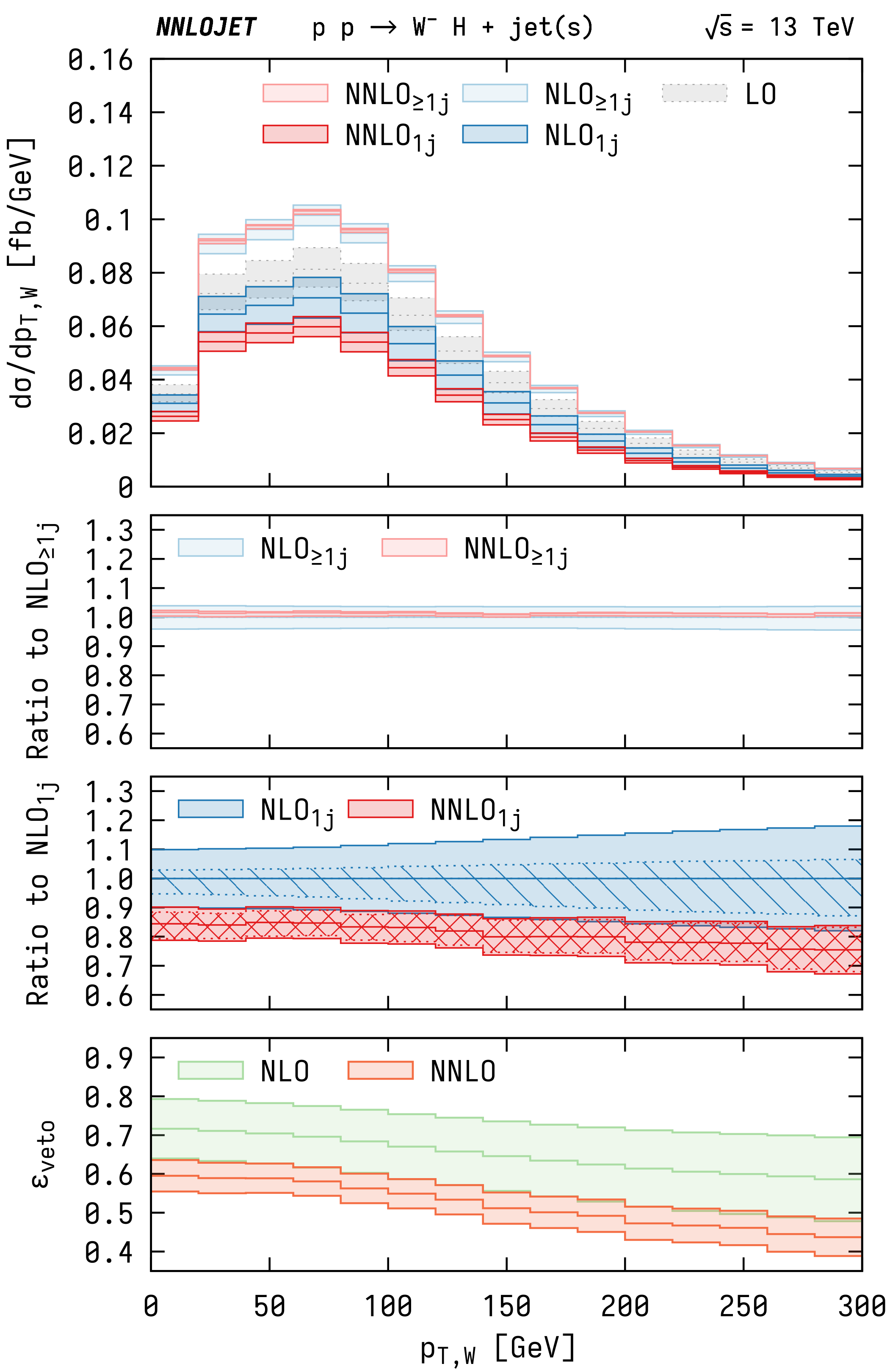
Baglio, Duhr, Mistlberger, Szafron 22

Result obtained using N³LO result for off-shell Drell-Yan production of W/Z then attaching the decay $V^* \to VH$

Behaviour of result broadly similar to Drell-Yan

- NNLO corrections unusually small
- N³LO corrections slightly larger than NNLO corrections and scale bands do not overlap

NNLO VH + jet(s)



σ [fb]	$\sigma^{\rm LO}$	$\sigma^{\rm NLO}_{\rm DY}$	$\sigma^{\rm NLO}_{\rm DY+R_I}$	$\sigma^{\rm NNLO}_{\rm DY}$	$\sigma^{\rm NNLO}$
$W^-H+\geq 1$ jet	$12.30^{+1.24}_{-1.09}$	$15.18^{+0.52}_{-0.56}$	$15.40^{+0.57}_{-0.59}$	$15.37^{+0.03}_{-0.12}$	$15.59^{+0.05}_{-0.15}$
W^-H+1 jet	$12.30^{+1.24}_{-1.09}$	$10.13^{+0.45}_{-0.83}$	$10.35^{+0.41}_{-0.78}$	$8.61^{+0.44}_{-0.47}$	$8.82^{+0.40}_{-0.43}$

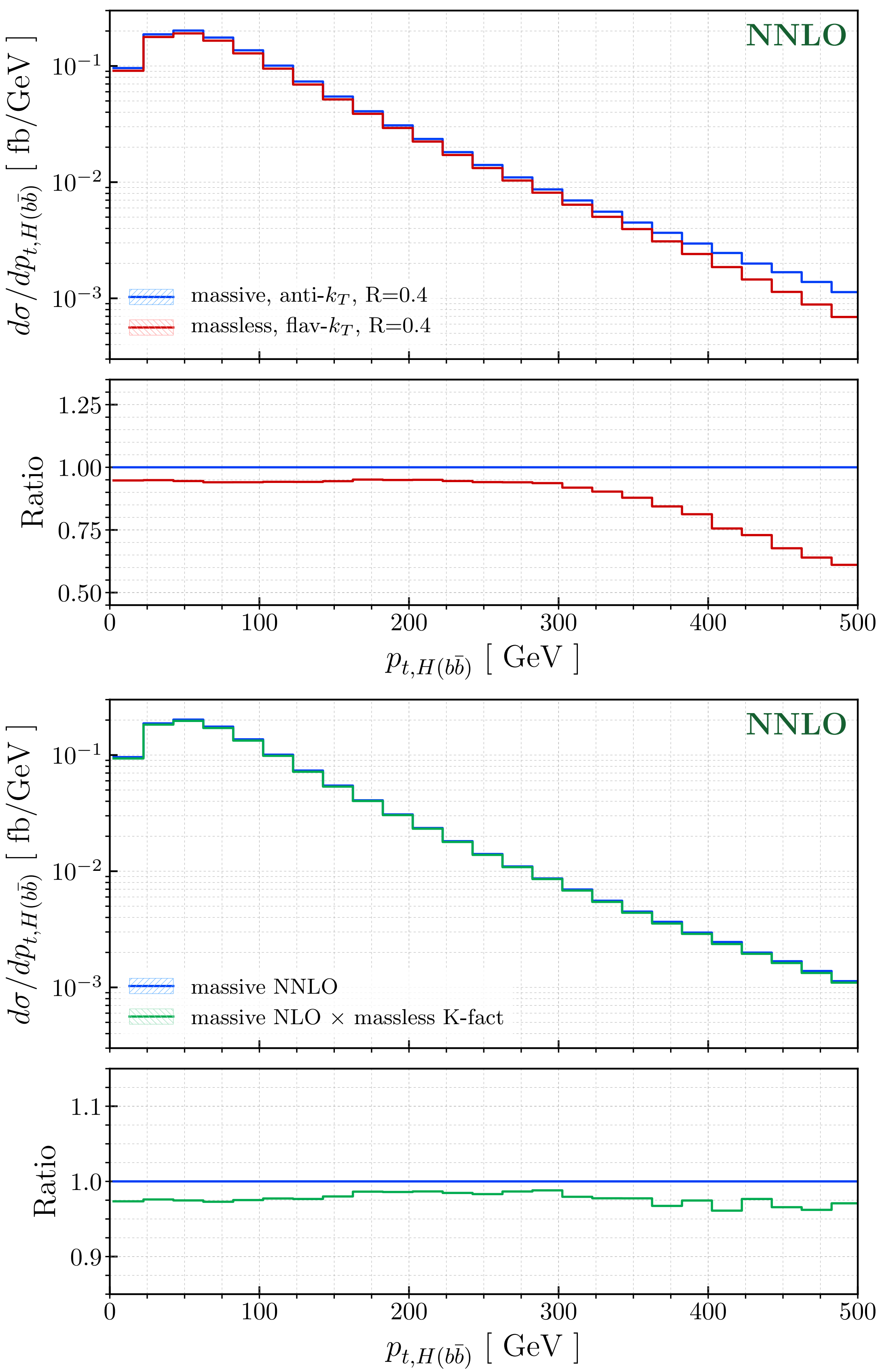
NNLO QCD including Drell-Yan type and top-loop induced contributions
Gauld, Gehrmann-De Ridder, Glover, Huss, Majer 19, 20, 21

Flat $\mathcal{O}(1\%)$ corrections for DY corrections to inclusive jet production within scale uncertainty

Corrections $\mathcal{O}(-10\%)$ for exclusive jet production (residual TH uncertainty $\mathcal{O}(5\%)$)

Top-loop induced contributions are small for WHj but larger for ZHj (gluon fusion contribution), have biggest impact for $p_{T,Z} > 150$ GeV

NNLO+PS $W(H \to b\bar{b})$ + b mass effects



WH production with $H \to b\bar{b}$ @ NNLO + PS

Astill, Bizon, Re, Zanderighi 18

+ NNLO with bottom mass effects

+ Anomalous couplings / SMEFT

Behring, Bizon, Caola, Melnikov, Röntsch 20;
Bizoń, Caola, Melnikov, Raoul Röntsch 21

Allows application of realistic jet algorithms for b jets (important for TH vs EXP)

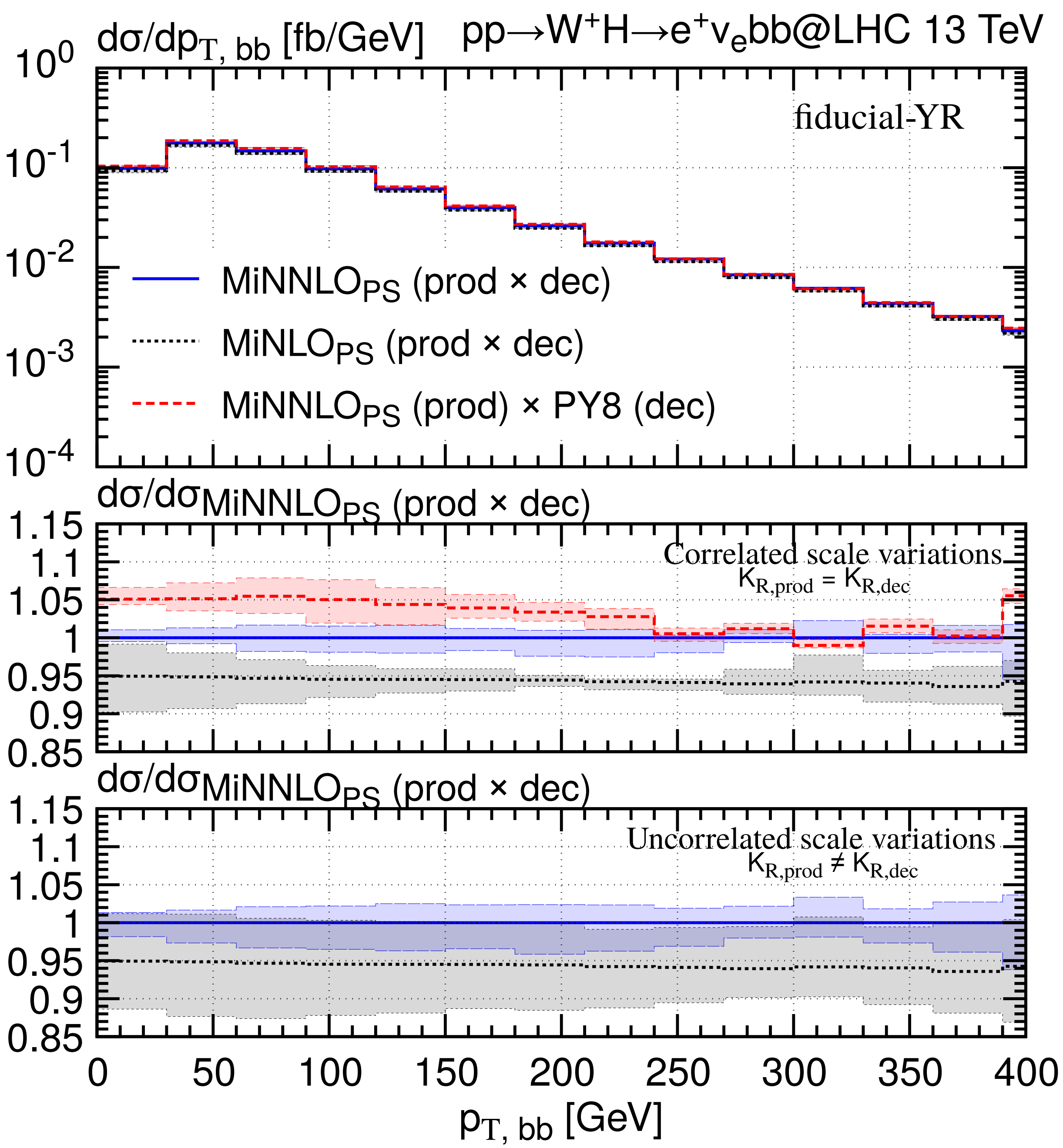
Including b mass increases σ_{tot} by $+6.3\,\%$ ($+7.7\,\%$ for boosted σ_{fid})

Rescaling massive NLO works for some distributions (e.g. $p_{t,H(b\bar{b})}$ but not $m_{H(b\bar{b})}$),

Results rely on use of flavour jet algorithms, saw a lot of theoretical development recently

Caletti, Larkoski, Marzani, Reichelt 22; Czakon, Mitov, Poncelet 22; Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler 23

POWHEG/MiNNLO/GENEVA for $pp \to VH$



QCD+EW (NLO with PS)

Granata, Lindert, Oleari, Pozzorini 17

GENEVA (NNLL′+NNLO with PS)

Alioli, Broggio, Kallweit, Lim, Rottoli 19

MiNNLO (NNLO with PS) also with decays

$pp \to l^+l^-b\bar{b}$, $pp \to \nu_l\bar{\nu}_l b\bar{b}$, $pp \to l^\pm\nu_l b\bar{b}$

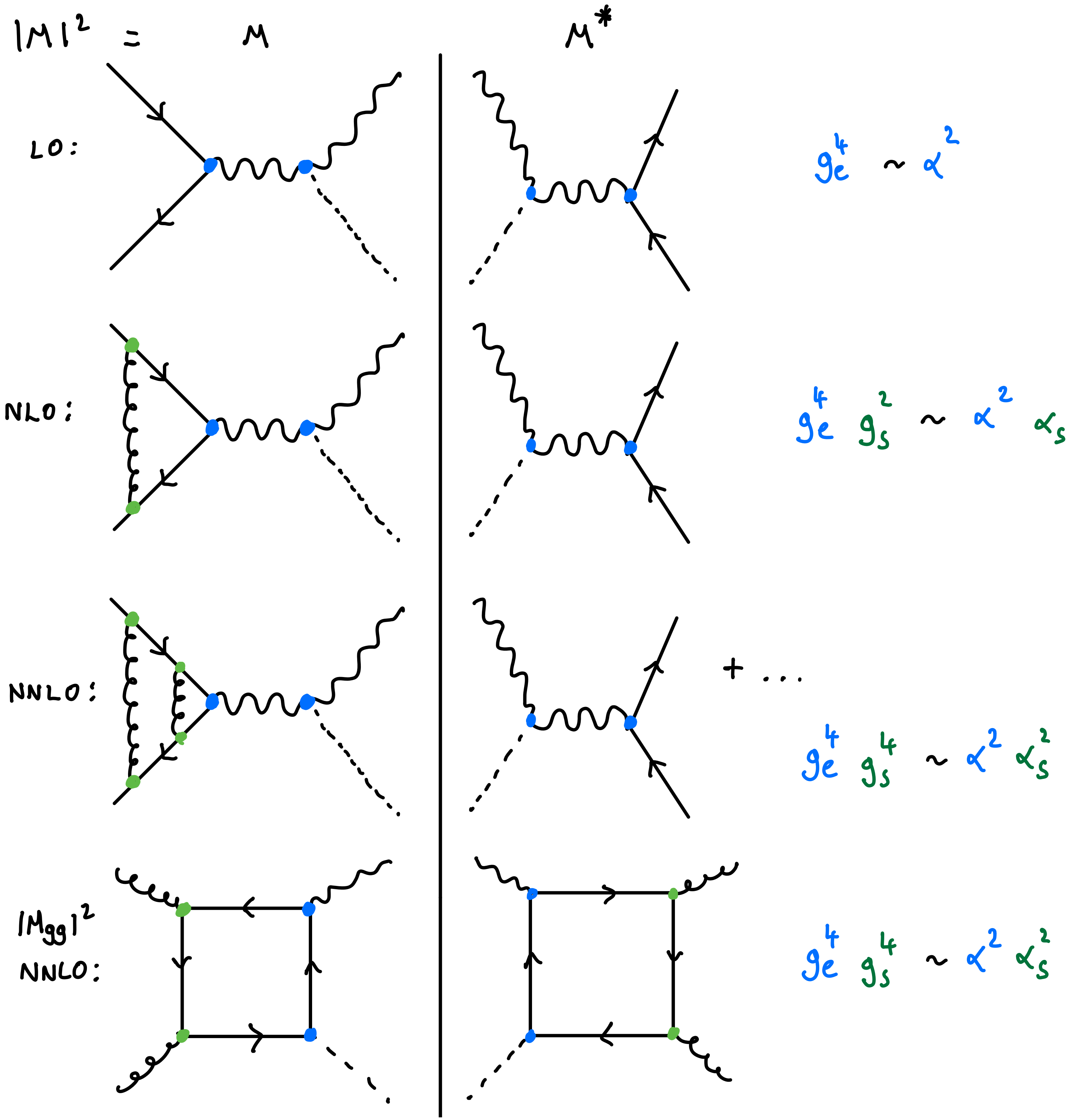
Zanoli, Chiesa, Re, Wiesemann, Zanderighi 21

MiNNLO SMEFT $pp \to ZH \to l^+l^-b\bar{b}$

Haisch, Scott, Wiesemann, Zanderighi, Zanoli 22

$pp \to W^+H \to e^+\nu_e b\bar{b}$		
σ [fb]	inclusive	fiducial-YR
MiNLO′	$54.04^{+6.6\%}_{-3.6\%}$	$20.13^{+2.3\%}_{-3.1\%}$
MiNNLO$_{PS}$	$57.44^{+1.7\%}_{-0.8\%}$	$21.27^{+1.3\%}_{-1.3\%}$
$pp \to W^-H \to e^-\bar{\nu}_e b\bar{b}$		
σ [fb]	inclusive	fiducial-YR
MiNLO′	$33.82^{+6.6\%}_{-3.6\%}$	$13.07^{+2.4\%}_{-3.3\%}$
MiNNLO$_{PS}$	$35.87^{+1.5\%}_{-0.7\%}$	$13.77^{+1.5\%}_{-1.6\%}$
$pp \to ZH \to e^+e^-b\bar{b}$		
σ [fb]	inclusive	fiducial-YR
MiNLO′	$14.88^{+6.7\%}_{-3.7\%}$	$5.21^{+2.2\%}_{-3.0\%}$
MiNNLO$_{PS}$ (no $gg \to ZH$)	$15.79^{+1.8\%}_{-0.9\%}$	$5.48^{+1.2\%}_{-1.2\%}$
MiNNLO$_{PS}$ (with $gg \to ZH$)	$16.99^{+3.6\%}_{-2.3\%}$	$6.07^{+3.4\%}_{-2.9\%}$

Overview of $pp \to ZH$ (II)



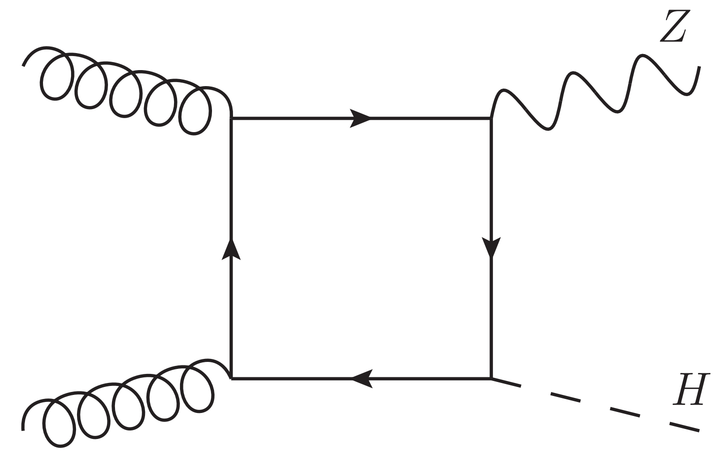
The $gg \to ZH$ channel contributes to $pp \to ZH$ starting at NNLO in QCD

However due the large gluon-gluon luminosity at the LHC it contributes significantly (~10%) to the total cross section

ZH in Gluon Fusion

Full leading order (loop induced)
Dicus, Kao 88; Kniehl 90

NLO in the limit of $m_t \to \infty$ ($K \approx 2$)
Altenkamp, Dittmaier, Harlander, H. Rzehak, Zirke 12



Virtual Corrections:

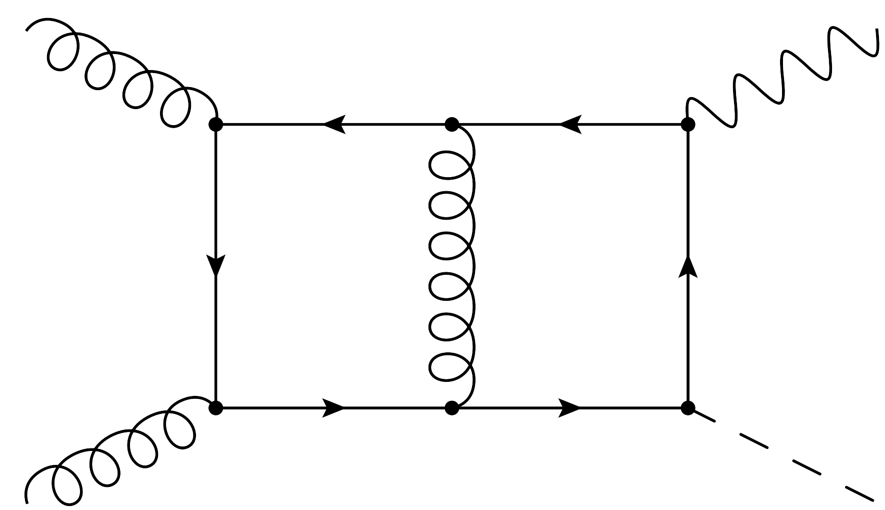
$1/m_t^8$ Expansion + Padé approx
Hasselhuhn, Luthe, Steinhauser 17

$1/m_t^{10}$ & m_t^{32} Expansion + Padé approx
Davies, Mishima, Steinhauser 20

Numerical
Chen, Heinrich, SPJ, Kerner, Klappert, Schlenk 20

p_T^4 Expansion
Alasfar, Degrassi, Giardino, Gröber, Vitti 21

$p_T^4 + m_t^{12}$ Expansion + Padé approx
Bellafronte, Degrassi, Giardino, Gröber, Vitti 22



NLO results:

small m_z, m_h Expansion
Wang, Xu, Xu, Yang 21

Numerical + m_t Expansion
Chen, Davies, Heinrich, SPJ, Kerner, Mishima, Schlenk, Steinhauser 22

$p_T + m_t$ Expansion
Degrassi, Gröber, Vitti, Zhao 22

Inclusive Cross Section $gg \to ZH$

Wang et al. 21: $\sqrt{s} = 13$ TeV

$\mu_r = \mu_f$	$\sigma_{\rm LO}^{gg}$	$\sigma_{\rm NLO}^{gg}$	$\sigma_{pp\to ZH}^{\rm w/o\ gg}$	$\sigma_{pp\to ZH}$
$M_{ZH}/3$	73.56(7)	129.4(3)	784.0(7)	913.4(7)
M_{ZH}	51.03(5)	101.7(2)	781.1(7)	882.9(7)
$3M_{ZH}$	36.62(4)	80.4(2)	780.7(8)	861.1(8)

Chen et al. 22: $\mu_R = \mu_F = m_{ZH}$

$\sqrt{s}$	LO [fb]	NLO [fb]
13 TeV	$52.42^{+25.5\%}_{-19.3\%}$	$103.8(3)^{+16.4\%}_{-13.9\%}$
13.6 TeV	$58.06^{+25.1\%}_{-19.0\%}$	$114.7(3)^{+16.2\%}_{-13.7\%}$
14 TeV	$61.96^{+24.9\%}_{-18.9\%}$	$122.2(3)^{+16.1\%}_{-13.6\%}$

Degrassi et al. 22: $\sqrt{s} = 13$ TeV, $\mu_R = \mu_F = m_{ZH}/2$

Top-mass scheme	LO [fb]	$\sigma_{LO}/\sigma_{LO}^{OS}$	NLO [fb]	$\sigma_{NLO}/\sigma_{NLO}^{OS}$	$K = \sigma_{NLO}/\sigma_{LO}$
On-Shell	$64.01^{+27.2\%}_{-20.3\%}$	-	$118.6^{+16.7\%}_{-14.1\%}$	-	1.85
$\overline{\rm MS}$, $\mu_t = M_{ZH}/4$	$59.40^{+27.1\%}_{-20.2\%}$	0.928	$113.3^{+17.4\%}_{-14.5\%}$	0.955	1.91
$\overline{\rm MS}$, $\mu_t = m_t^{\overline{\rm MS}}(m_t^{\overline{\rm MS}})$	$57.95^{+26.9\%}_{-20.1\%}$	0.905	$111.7^{+17.7\%}_{-14.6\%}$	0.942	1.93
$\overline{\rm MS}$, $\mu_t = M_{ZH}/2$	$54.22^{+26.8\%}_{-20.0\%}$	0.847	$107.9^{+18.4\%}_{-15.0\%}$	0.910	1.99
$\overline{\rm MS}$, $\mu_t = M_{ZH}$	$49.23^{+26.6\%}_{-19.9\%}$	0.769	$103.3^{+19.6\%}_{-15.6\%}$	0.871	2.10

Find good agreement between groups when same inputs used

NLO corrections are very large ($K \approx 2$)

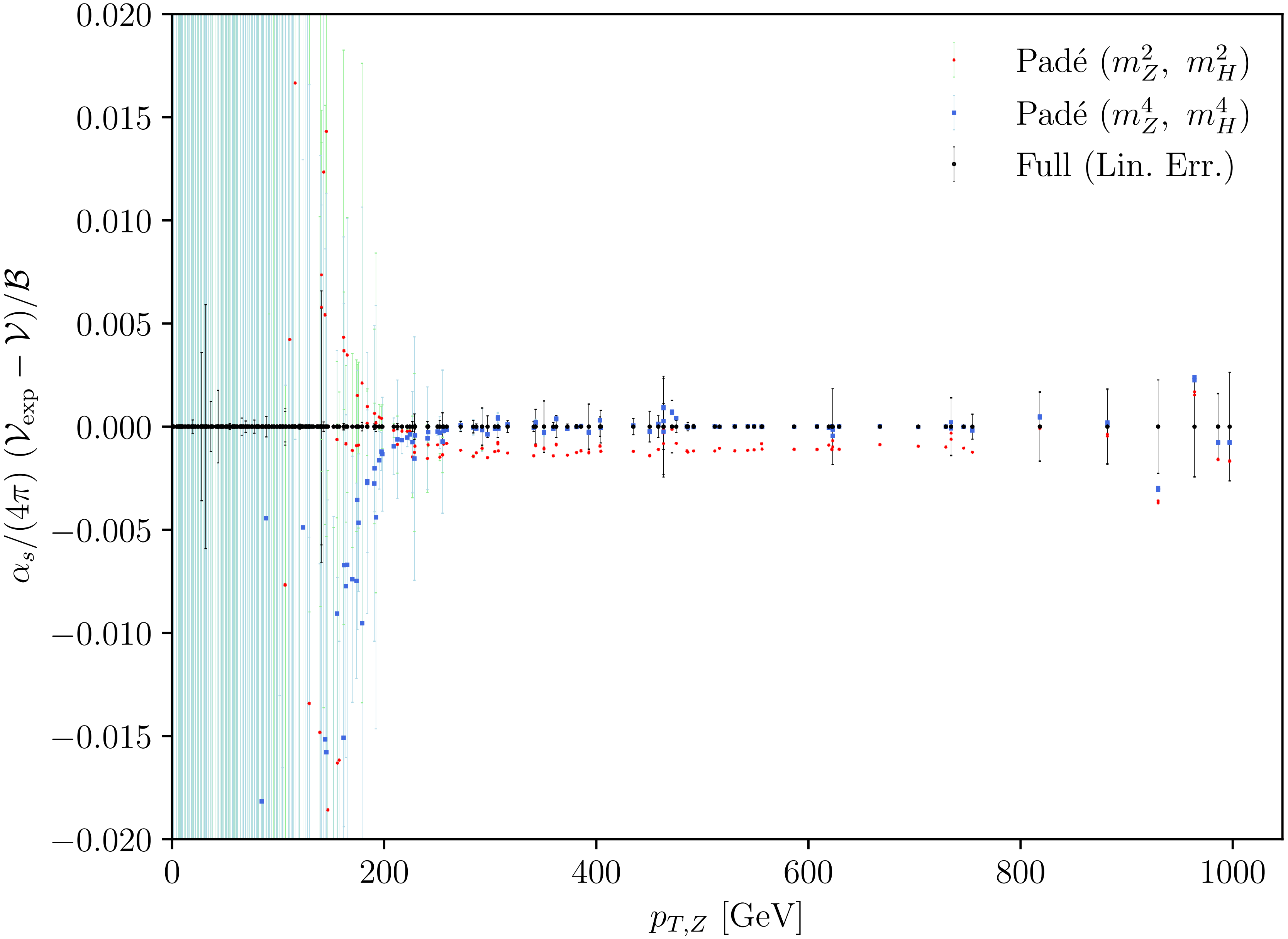
Currently in process of updating Higgs WG recommendation using these results

Comparison Numeric/Small m_t Expansion

The amplitude can be expanded around small m_t, m_h, m_z

Known to $(m_z^4,\ m_h^4,\ m_t^{32})$

Davies, Mishima, Steinhauser 20; Mishima 18; Chen, Davies, Heinrich, SPJ, Kerner, Mishima, Schlenk, Steinhauser 22



Find **acceptable** agreement for $p_{T,Z} \geq 150$ GeV (<2.8% difference)

Comparison Small p_T/ Small m_t Expansion

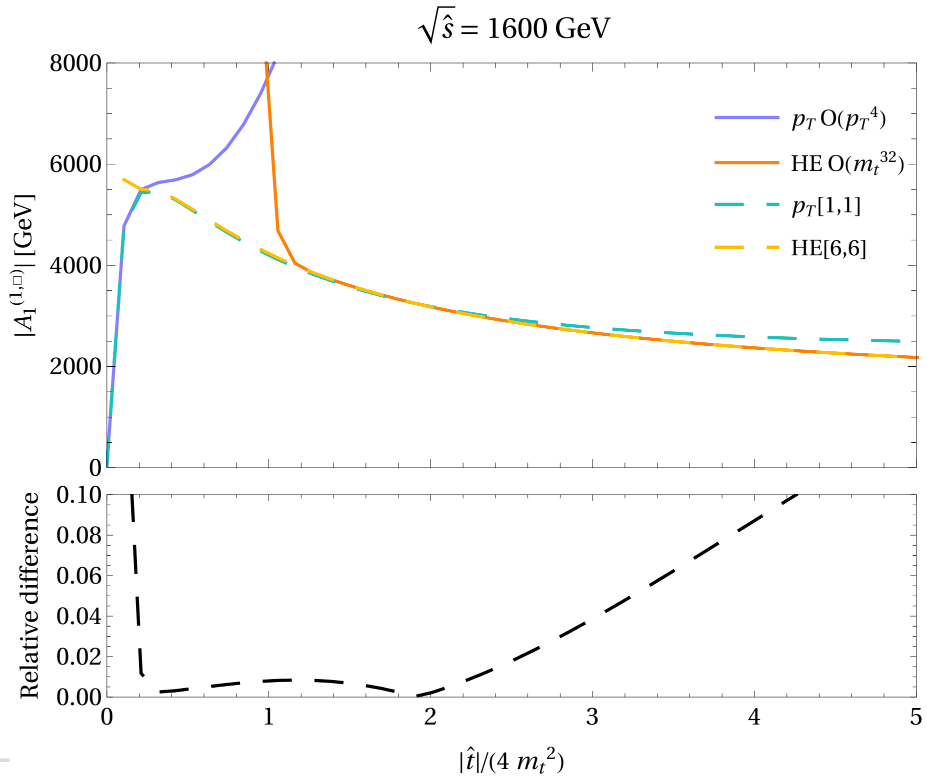
Merging pT and HE Expansions at NLO

Improve the convergence of a series expansion by matching the coefficients of the **Pade approximant** [m/n] [e.g. Fleisher, Tarasov ('94)]

$$f(x) \overset{x\to 0}{\simeq} c_0 + c_1 x + \cdots + c_q x^q \qquad f(x) \simeq [m/n](x) = \frac{a_0 + a_1 x + \cdots + a_m x^m}{1 + b_1 x + \cdots + b_n x^n} \qquad (q = m+n)$$

[Bellafronte, Degrassi, Giardino, Gröber, MV -2103.06225]

- For each FF we merged the following results
  - pT exp improved by [1/1] Padé
  - HE exp improved by [6/6] Padé
- Padé results are stable and comparable in the region $|\hat{t}| \sim 4 m_t^2 \rightarrow$ can switch without loss of accuracy (% level or below)
- Evaluation time for a phase-space point below 0.1 *s* $\Rightarrow$ suitable for Monte Carlo



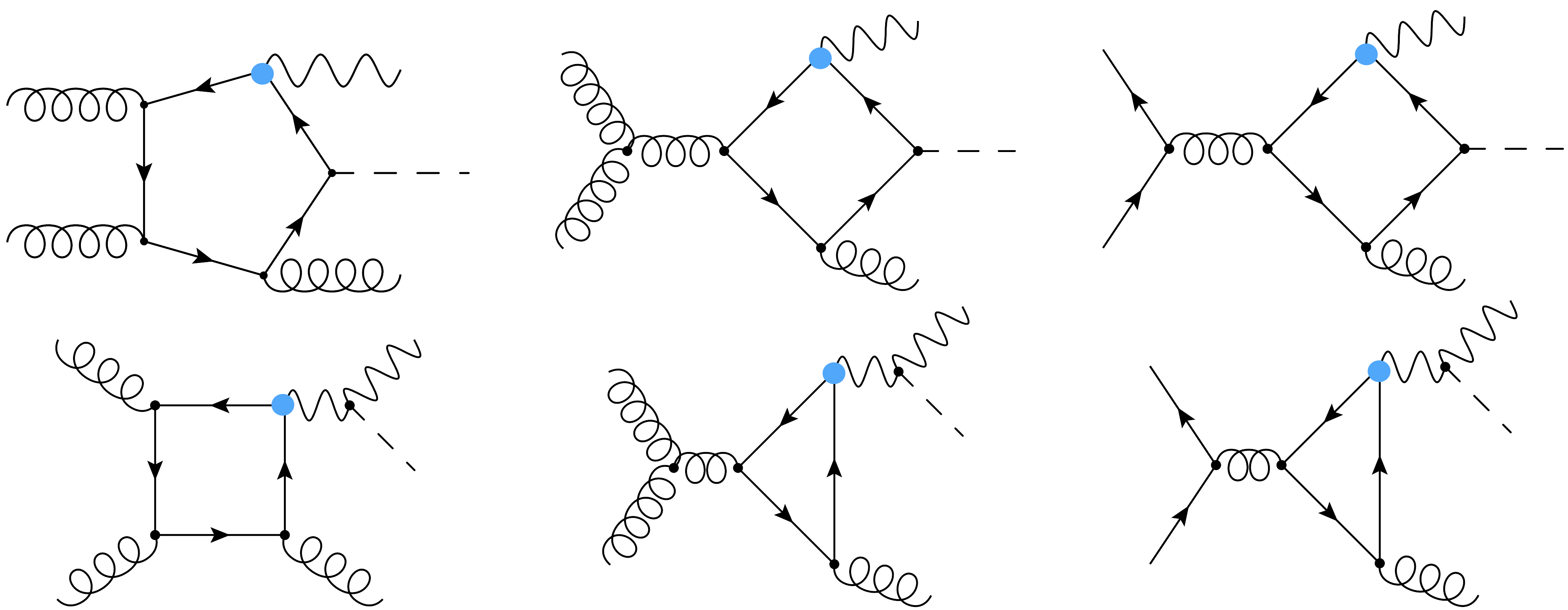
Slide: Marco Vitti (20th Workshop of the LHC Higgs Working Group)

Real Emission Diagrams

There is some **freedom** regarding which real diagrams we include in gg vs $q\bar{q}$

Must be careful not to double count when combining all channels for $pp \to ZH$

Diagrams included in all works ($n_f = 5$ + massive top in loop) + crossings

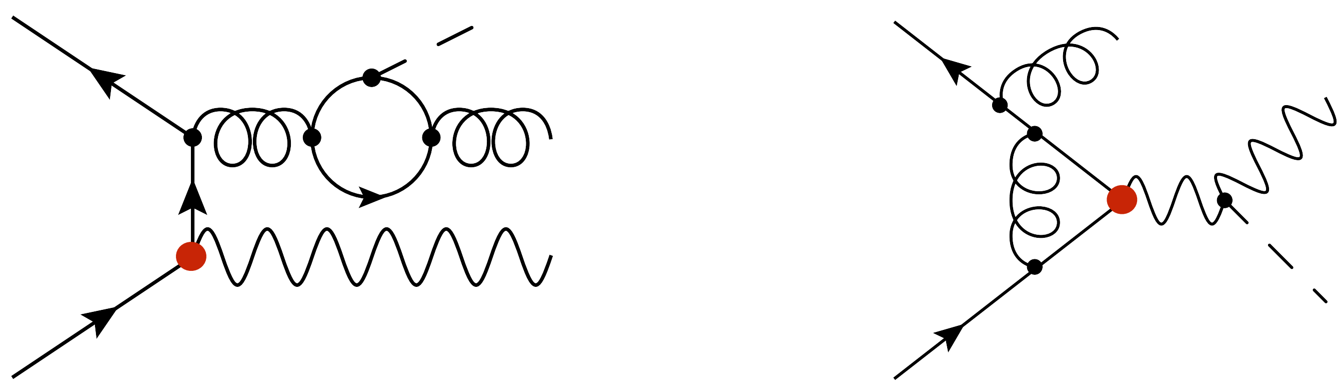


We require:

1) A closed fermion loop
2) A Z-boson or Goldstone boson coupled to that loop

Real Emission Diagrams (II)

Diagrams excluded in Wang et al./Chen et al., included in Degrassi et al

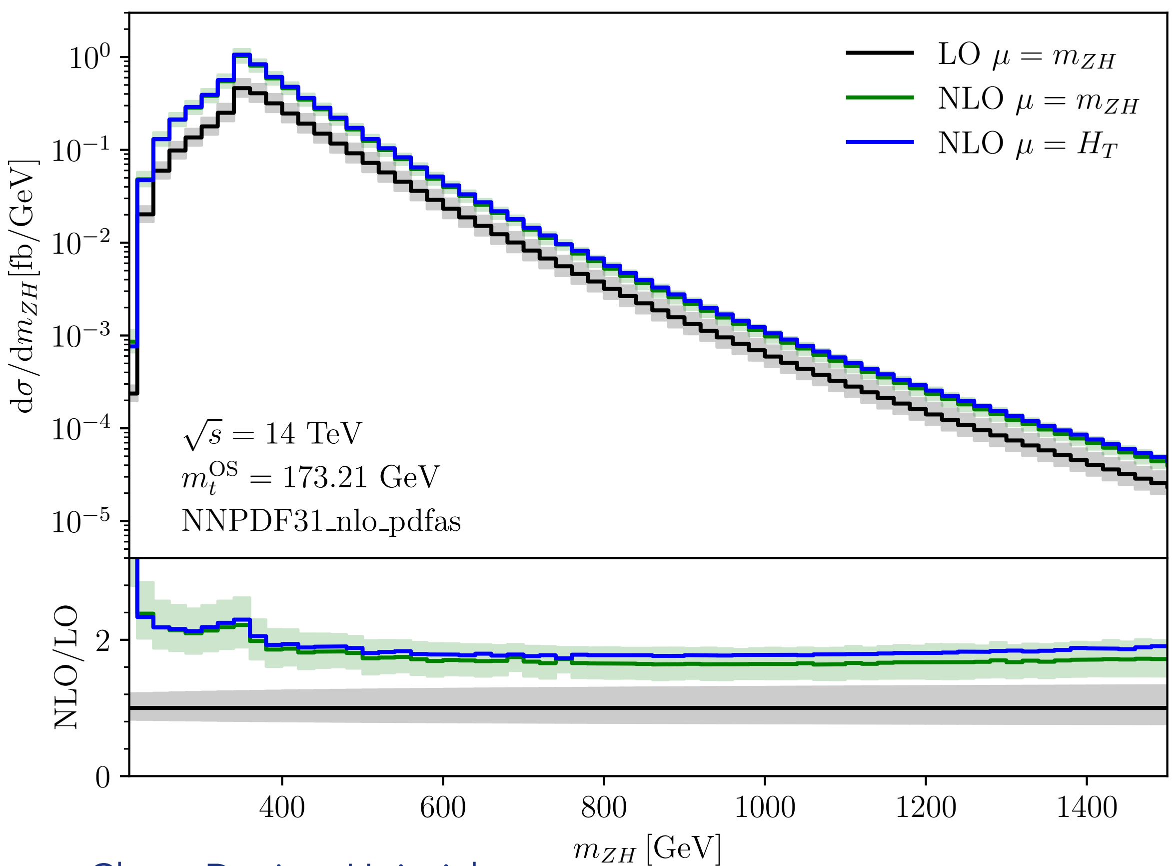


Left class of diagrams: separately UV/IR finite & gauge invariant
Previously studied in detail See e.g. Brein, Harlander, Wiesemann, Zirke 12

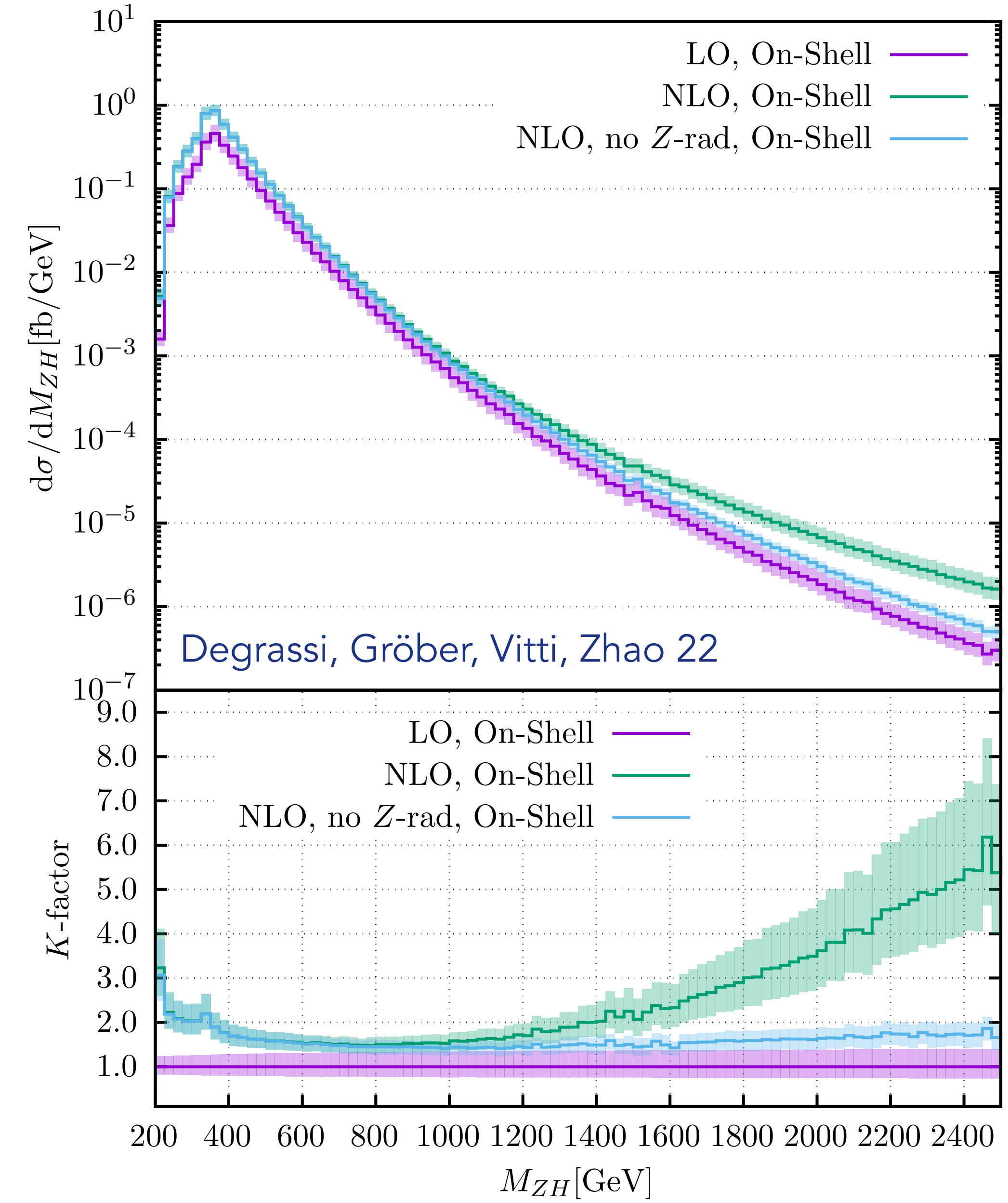
Right class of diagrams: belongs to real corrections to Drell-Yan (i.e. $q\bar{q}$)
Included in DY calculations

Brein, Djouadi, Harlander 03;
Ferrera, Grazzini, Tramontano 14;
See also: Kumara, Mandal, Ravindran 14

Invariant Mass $gg \to ZH$

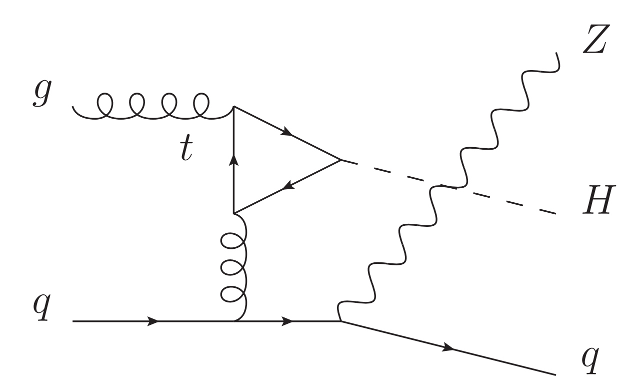


Chen, Davies, Heinrich,
SPJ, Kerner, Mishima, Schlenk, Steinhauser 22

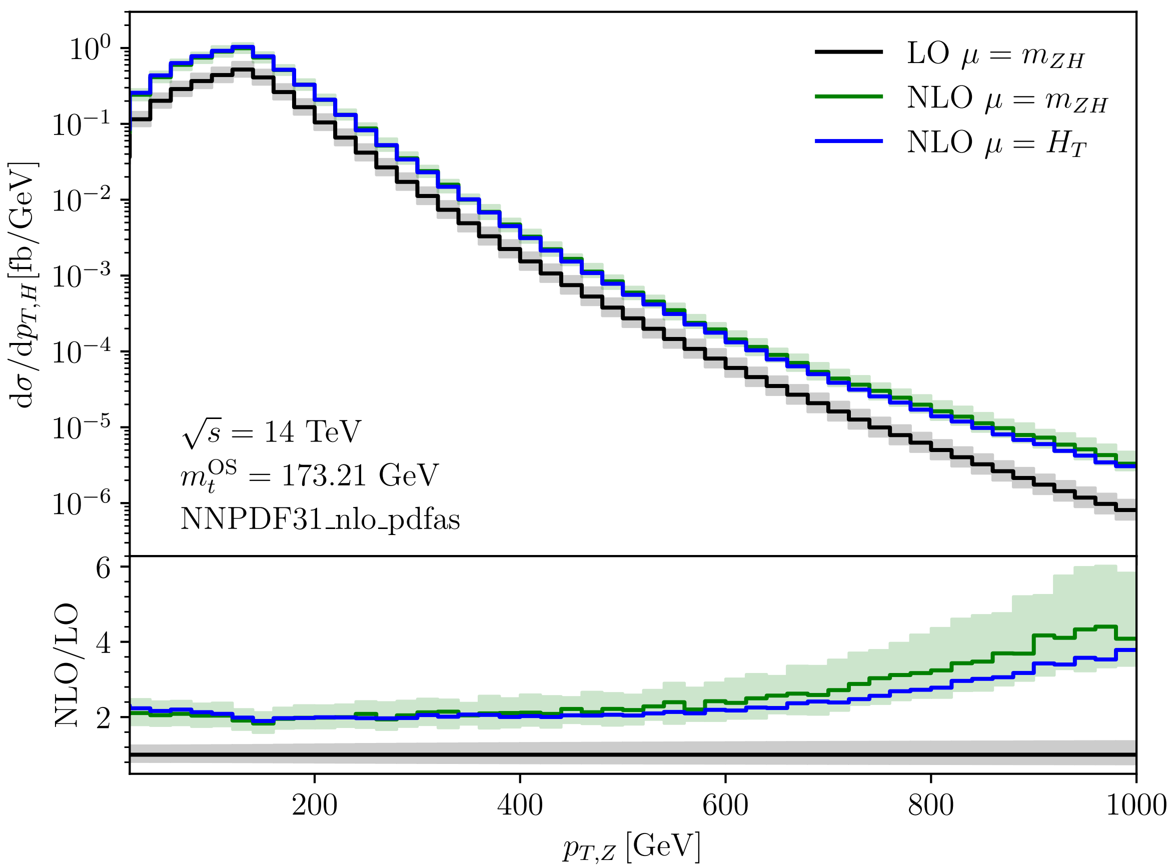


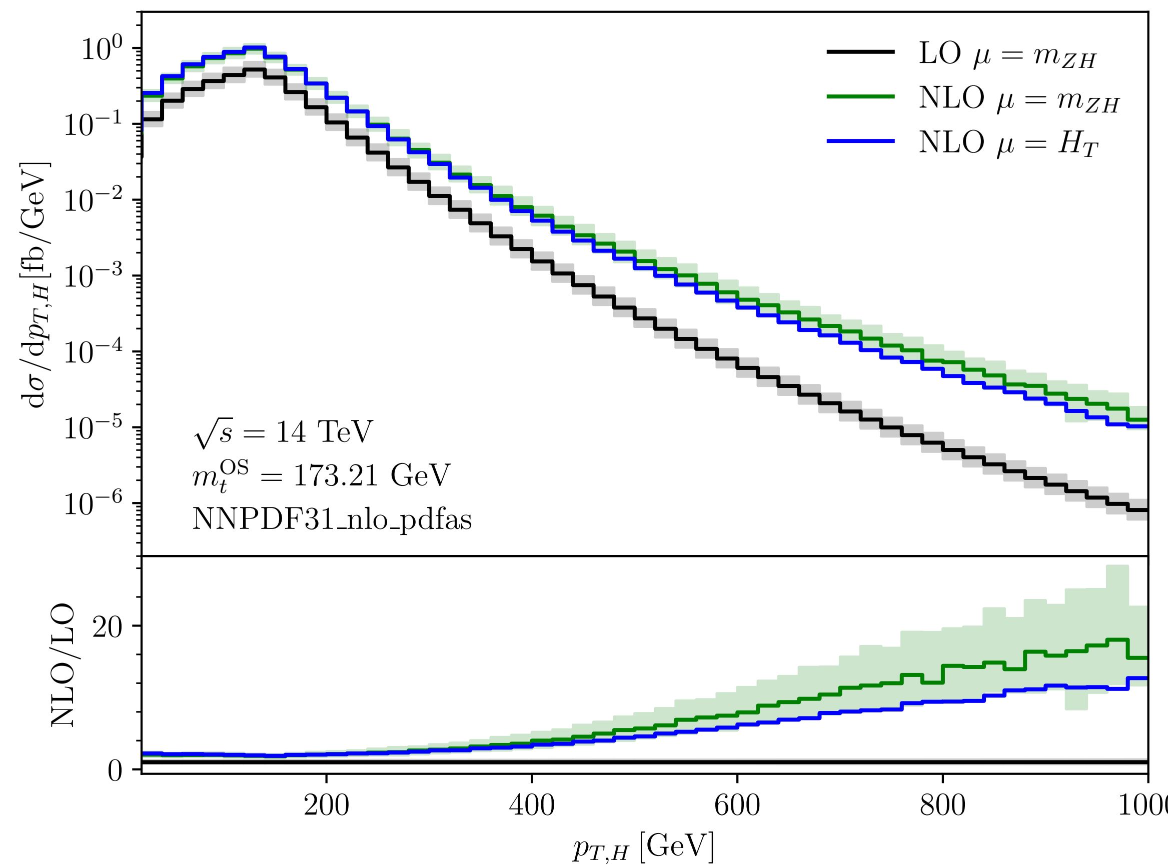
NLO/LO somewhat flat except at ZH and $t\bar{t}$ production thresholds

Large rise at high-m_{ZH} due to real diagrams with Z radiated from external quark line (Z-rad)



Z Transverse Momentum





$p_{T,Z}$: **Large** NLO corrections, rising sharply at large $p_{T,Z}$
Placing cuts on soft H emission only slightly tames growth

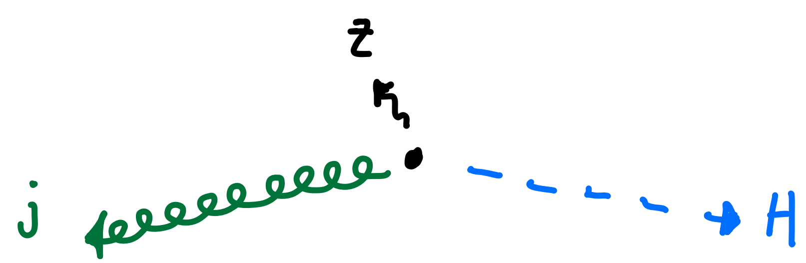
$p_{T,H}$: **Extremely** large pathological NLO corrections, rising very sharply at large $p_{T,H}$
Placing cuts on soft Z emission tames growth somewhat

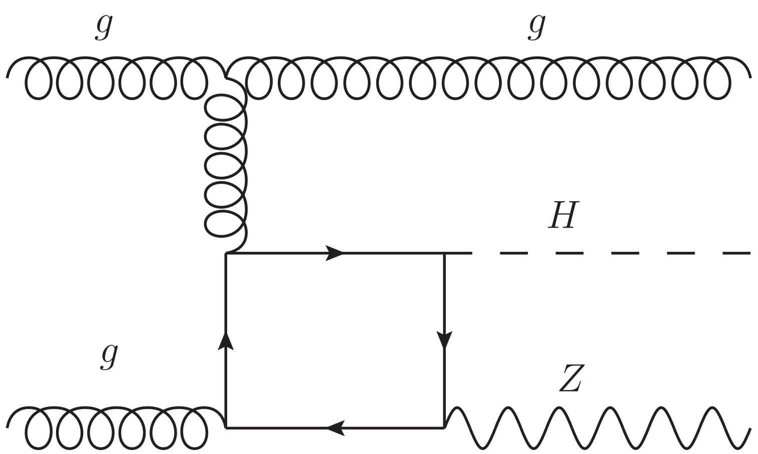
Very important to include higher order corrections in this region

Transverse Momentum: Z vs H

The different behaviour of $p_{T,Z}$ and $p_{T,H}$ was observed previously in $gg \to ZH + j$

Hespel, Maltoni, Vryonidou 15; Les Houches 19





Traced to configurations where Higgs recoils against a hard jet, with a soft Z

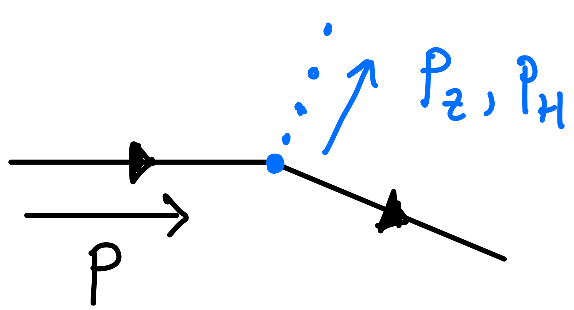
Maltoni et al. attributed this t-channel gluon exchange

One observation

If we apply an eikonal approximation to such diagrams, the enhancement of soft Z bosons can be understood

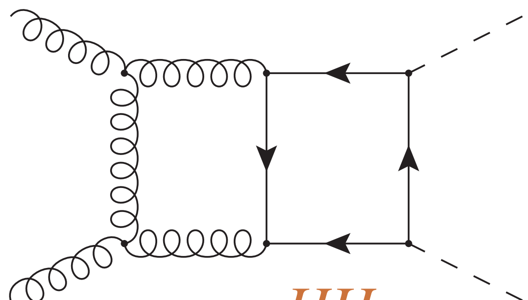
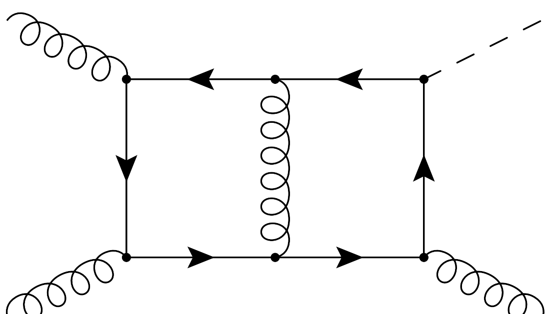
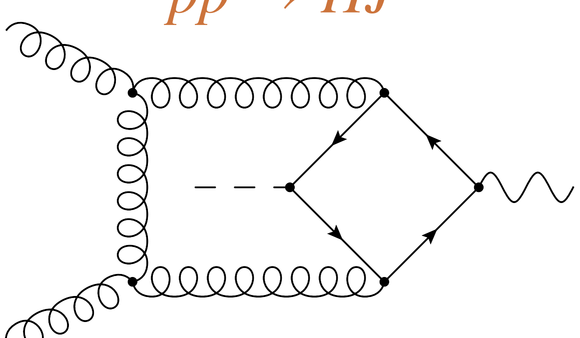
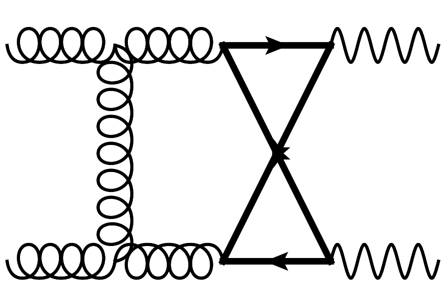
$$(\text{Soft } Z \text{ emission}) : \frac{p^\mu}{p \cdot p_Z}$$

$$(\text{Soft } H \text{ emission}) : \frac{m_t}{p \cdot p_H}$$



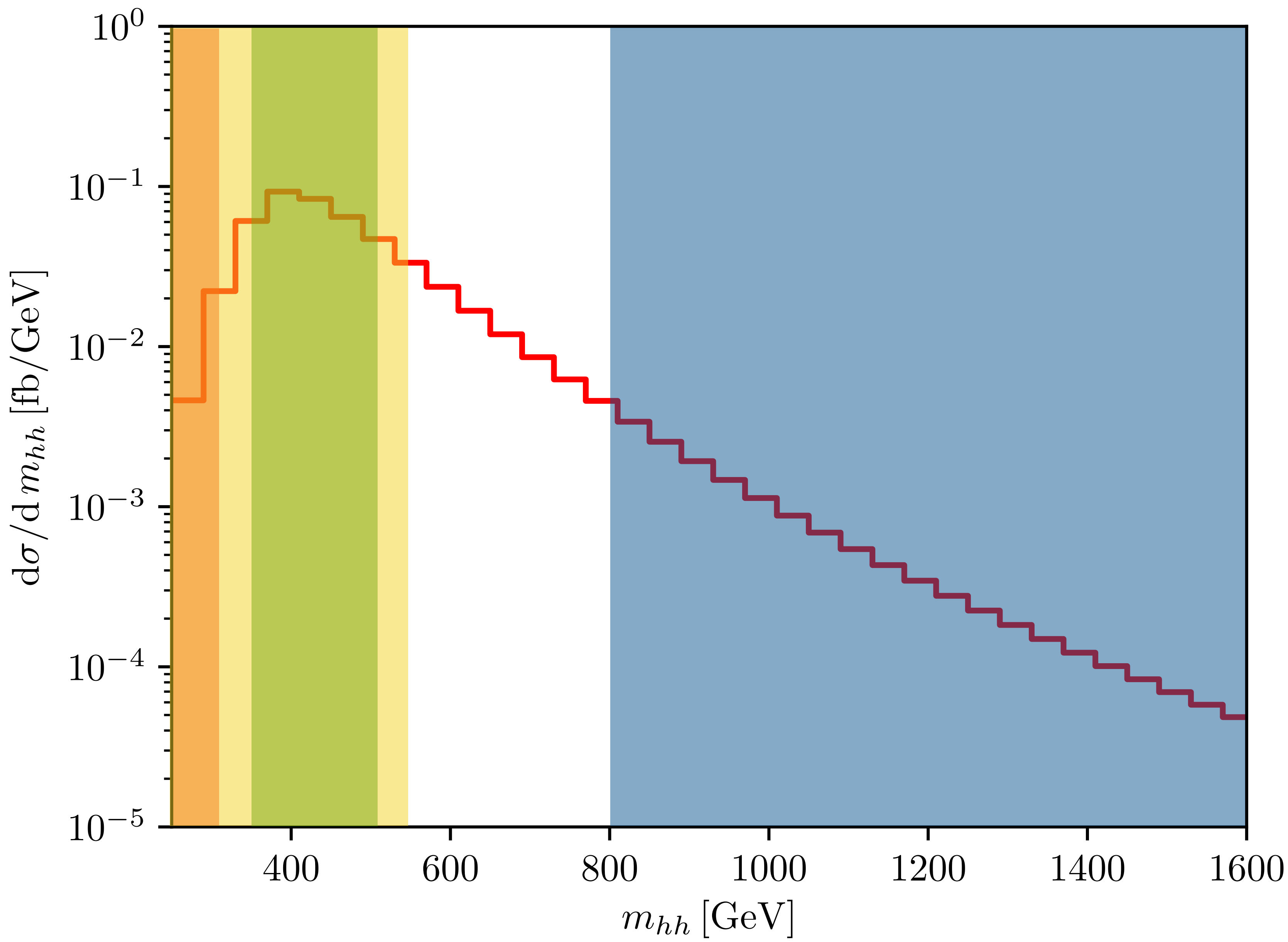
Ratio for large radiator (transverse) momentum $\sim p_T/m_t \gg 1$

Numerically Computing $2 \to 2$ Amplitudes

	#Tensor Structures	#Master Integrals	
 $pp \to HH$	2	327*	*Not fully reduced
 $pp \to HJ$	4+2	458	
 $gg \to HZ$	6	452	Can be attacked with same tool chain (e.g. pySecDec) Heinrich, SPJ, Kerner, Magerya, Olsson, Schlenk 23
 $gg \to ZZ$	20	264	For reduction we mostly fixed m_z^2/m_t^2 etc

Expansions for 2 → 2 Amplitudes

VV, VH, HH virtual 2 → 2 amplitudes often share the same master integrals after expansion, can combine up these expansions to cover the phase-space



Low invariant mass:

expand in $1/m_t^2$

known to NNLO

Grigo, Hoff, Steinhauser 15, …

Around Peak:

threshold expansion

Gröber, Maier, Rauh 17, …

High energy:

known at NLO

Davies, Mishima, Steinhauser, Wellmann 18, 19, …

Small p_T or t:

known at NLO

Alasfar, Degrassi, Giardino Groeber, Vitti 21; Davies, Mishima, Schönwald, Steinhauser 23, …

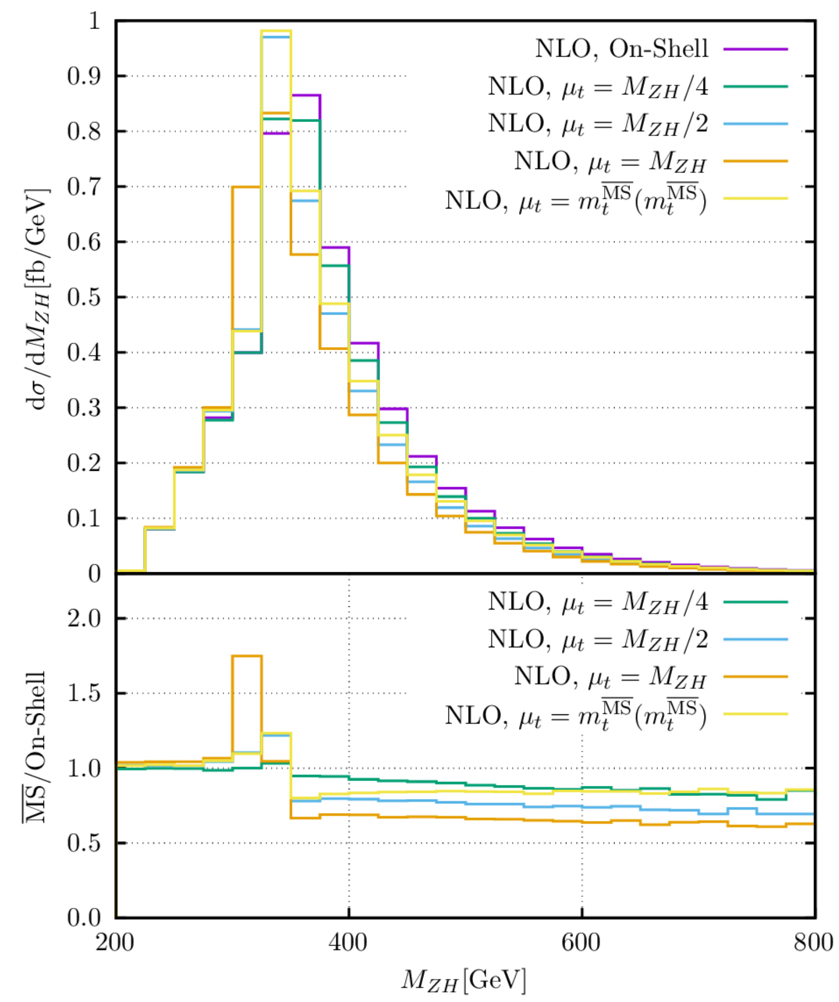
Mass Scheme Uncertainty

Top Mass Scheme Uncertainty

- Envelope of deviations of $\overline{\text{MS}}$ schemes wrt OS result
  Same method already used for HH production
  [Baglio et al. - 1811.05692, 2003.03227]
- Uncertainty sensitive to the binning of top-pair threshold peak

Avoid overestimate of uncertainty

Bin Width [GeV]	LO	NLO
1	$64.01^{+15.6\%}_{-35.9\%}$	$118.6^{+17.2\%}_{-27.0\%}$
5	$64.01^{+15.3\%}_{-35.6\%}$	$118.6^{+14.7\%}_{-24.9\%}$
25	$64.01^{+14.0\%}_{-33.1\%}$	$118.6^{+10.9\%}_{-20.8\%}$
100	$64.01^{+2.0\%}_{-25.3\%}$	$118.6^{+0.6\%}_{-13.7\%}$
∞	$64.01^{+0\%}_{-23.1\%}$	$118.6^{+0\%}_{-12.9\%}$



- Top-mass uncertainty ~ scale uncertainty
- Agreement with [Chen et al. - 2204.05225] for $M_{ZH} > 400\,\text{GeV}$

[Degrassi, Gröber, MV, Zhao - 2205.02769]

Slide: Marco Vitti (20th Workshop of the LHC Higgs Working Group)

Mass Scheme Uncertainty

Comparing to $gg \to HH$, we see a different high-energy behaviour

$$A_i^{\text{fin}} = a_s A_i^{(0),\text{fin}} + a_s^2 A_i^{(1),\text{fin}} + \mathcal{O}(a_s^3) \quad \text{with} \;\; a_s = \alpha_s/4\pi$$

HH Davies, Mishima, Steinhauser, Wellmann 18; Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira, Streicher 20

$$A_i^{(0)} \sim m_t^2 f_i(s,t)$$

$$A_i^{(1)} \sim 6 C_F A_i^{(0)} \log\left[\frac{m_t^2}{s}\right]$$

LO: m_t^2 from y_t^2

NLO: leading $\log(m_t^2)$ from mass c.t.

converting to $\overline{\text{MS}}$ gives $\log\left[\mu_t^2/s\right]$

motivating scale choice of $\mu_t^2 \sim s$

ZH Davies, Mishima, Steinhauser 20

$$A_i^{(0)} \sim m_t^2 f_i(s,t) \;\; \log^2\left[\frac{m_t^2}{s}\right]$$

$$A_i^{(1)} \sim \frac{(C_A - C_F)}{6} A_i^{(0)} \;\; \log^2\left[\frac{m_t^2}{s}\right]$$

LO: one m_t from y_t

NLO: leading $\log(m_t^2)$ not coming from mass c.t. (C_A)

Similar power-suppressed mass logarithms have been studied in single H

Liu, Penin 18; Anastasiou, Penin 20; Liu, Modi, Penin 22; Liu, Neubert, Schnubel, Wang 22; Schnubel 23

Conclusion

Fixed Order

- $pp \to WH$: dominant pieces known to N3LO/NNLO accuracy
- $pp \to ZH$: largest theoretical uncertainty still from gluon-induced contribution
- NLO EW corrections available

Public MC/ Parton Shower Codes

- Available at NNLO + PS accuracy for all processes and with relevant decays
- Also available in SMEFT, useful for global analyses/combined constraints

Future…

- Higgs WG recommendation for $gg \to ZH$
- Incorporate NLO $gg \to ZH$ corrections into public codes for $pp \to ZH$
- Handling mass scheme uncertainties in Higgs processes (?)
- Further EW corrections necessary (?)

Thank you for listening!

Backup

Mass Scheme Uncertainty

Can assess impact of changing top quark mass renormalisation scheme for $p_{T,H} \geq 140$ GeV & $p_{T,Z} \geq 150$ GeV using full B + full R + expanded virtuals

HH: Baglio, Campanario, Glaus, Mühllleitner, Spira, Streicher 18 + Ronca 20, 21;
$t\bar{t}$: Catani, Devoto, Grazzini, Kallweit, Mazzitelli 20; $t\bar{t}H$: Martin, Moch, Saibel 21
$t\bar{t}j$: Alioli, Fuster, Garzelli, Gavardi, Irles, Melini, Moch, Uwer, Voß 22; Various: Les Houches 19

Convert $m_t \to \overline{m}_t(\overline{m}_t)$ using 4-loops, then use RGE at 5-loops with $n_f = 6$
Gives $m_t = 173.21$ GeV $\to$ $\overline{m}_t(\overline{m}_t) = 163.39$ GeV
Chetyrkin, Kuhn, Steinhauser 00; Herren, Steinhauser 18

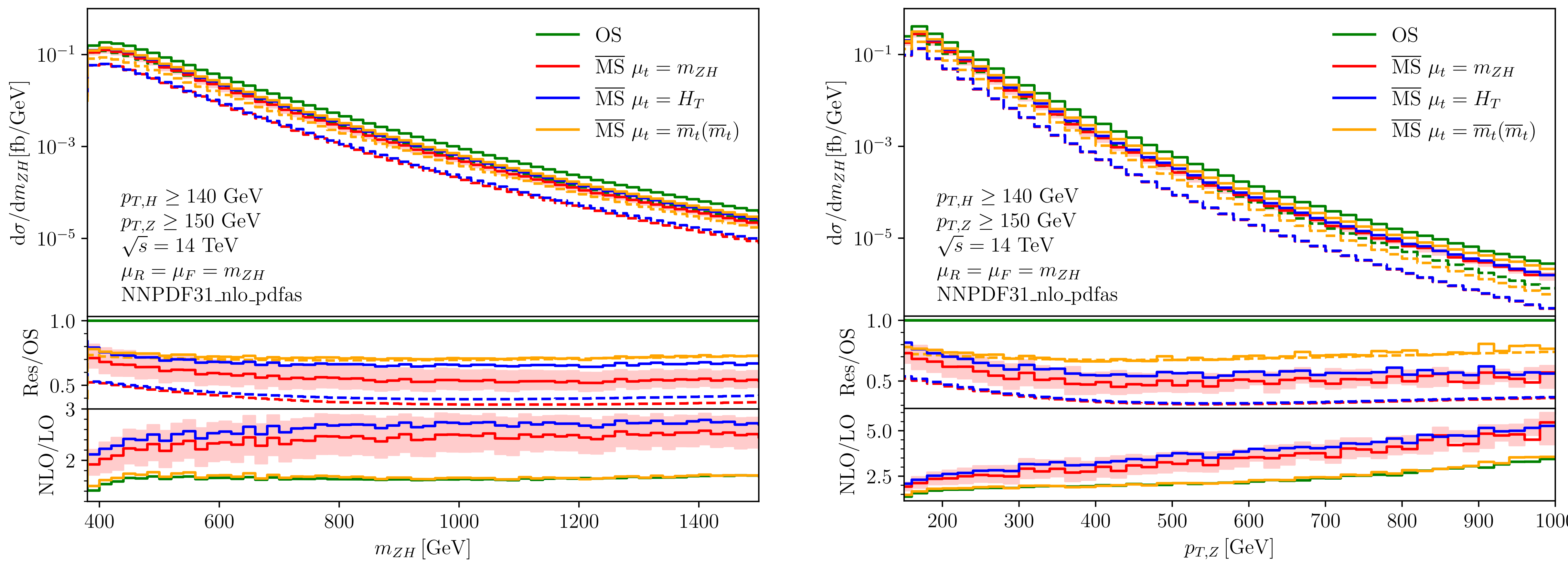
Go from OS to $\overline{\mathrm{MS}}$ mass counter term using:

$$m_t \to \overline{m}_t(\mu_t) \left(1 + \frac{\alpha_s(\mu_R)}{4\pi} C_F \left\{ 4 + 3 \log \left[\frac{\mu_t^2}{\overline{m}_t(\mu_t)^2} \right] \right\} \right)$$

Study 3 different renormalisation scales:

$$\mu_t = m_{ZH},$$
$$\mu_t = H_T = \sum_{i=H,Z} \sqrt{m_i^2 + p_{T,i}^2} + \sum_k |p_{T,k}|$$
$$\mu_t = \overline{m}_t(\overline{m}_t)$$

Mass Scheme Uncertainty



Observations @ $m_{ZH} = 1$ TeV

Large difference between different schemes

LO: OS result ~2.9x $\overline{\text{MS}}$ result

NLO: Difference reduced ~1.9x

Scale $\mu_t = \overline{m}_t(\overline{m}_t)$ most similar to OS (OS is 1.4x)

Scale $\mu_t = m_{ZH}$ differs most from OS (OS is 1.9x)

If taken as a theoretical uncertainty, is much larger than scale uncertainty

Comparison to Expansion (Small m_h, m_z)

Can expand in only m_h, m_z and retain full m_t dependence Wang, Xu, Xu, Yang 21

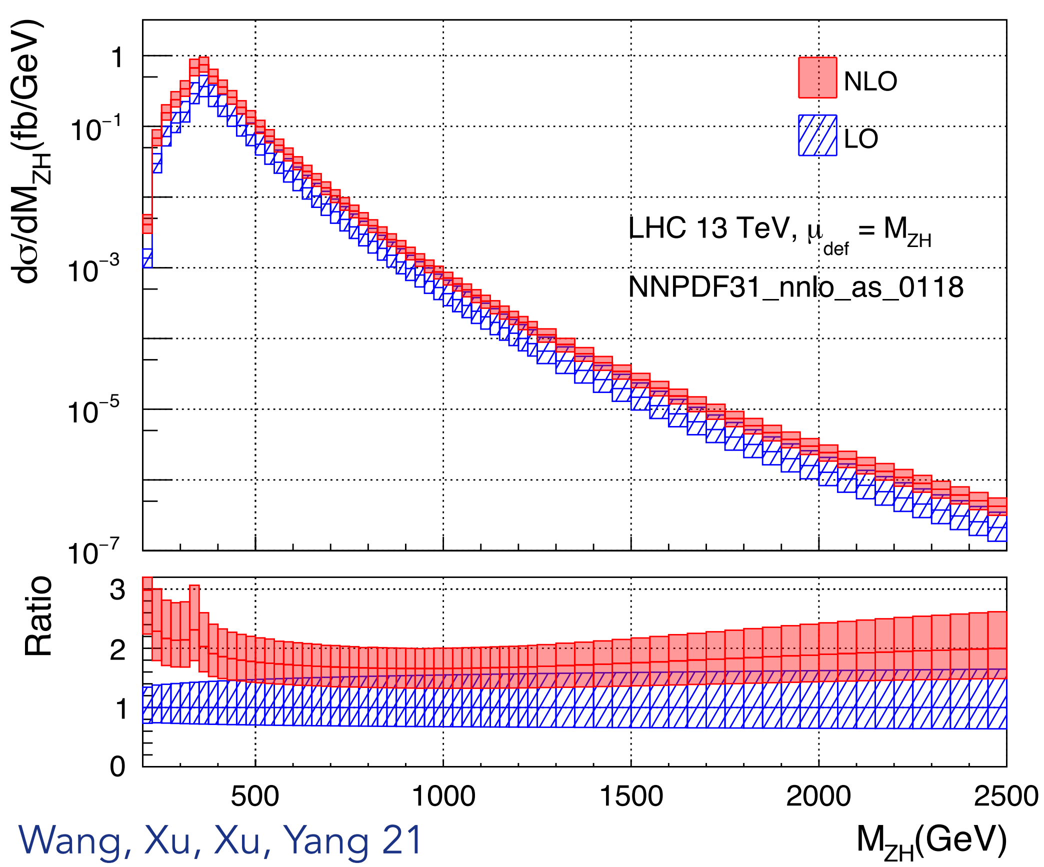
Integrals appearing in the expansion (scales s, t, m_t^2) are known

Caron-Huot, Henn 14; Becchetti, Bonciani 18; Xu, Yang 18; Wang, Wang, Xu, Xu, Yang 20;

Expansion shows good agreement with numerical result

No breakdown near top threshold

$\hat{s}/m_t^2$	$\hat{u}/m_t^2$	$\mathcal{V}'_{\text{fin}}$			
		pySecDec	$\mathcal{O}(m^0)$	$\mathcal{O}(m^2)$	$\mathcal{O}(m^4)$
1.707133657190554	−0.441203767016323	35.429092(6)	35.9823	35.5530	35.4478
3.876056604162662	−1.616287256345735	4339.045(1)	4319.37	4336.63	4338.73
4.130574250302561	−1.750372271104745	6912.361(3)	6870.47	6906.92	6911.64
4.130574250302561	−2.595461551488002	6981.09(2)	6979.28	6980.14	6980.85
134.5142052093564	−70.34125943305149	−153.9(4)	−154.543	−154.458	−154.460
134.5142052093564	−105.1770655376327	527(4)	524.585	525.958	525.965



Wang, Xu, Xu, Yang 21

Authors obtained NLO results for $gg \to ZH$

Virtuals: small m_h, m_z expansion

Reals: GoSam Cullen et al. 11, 14

We find agreement with their total cross section result and uncertainty

(2% difference ascribed to different choice of PDFs and masses)

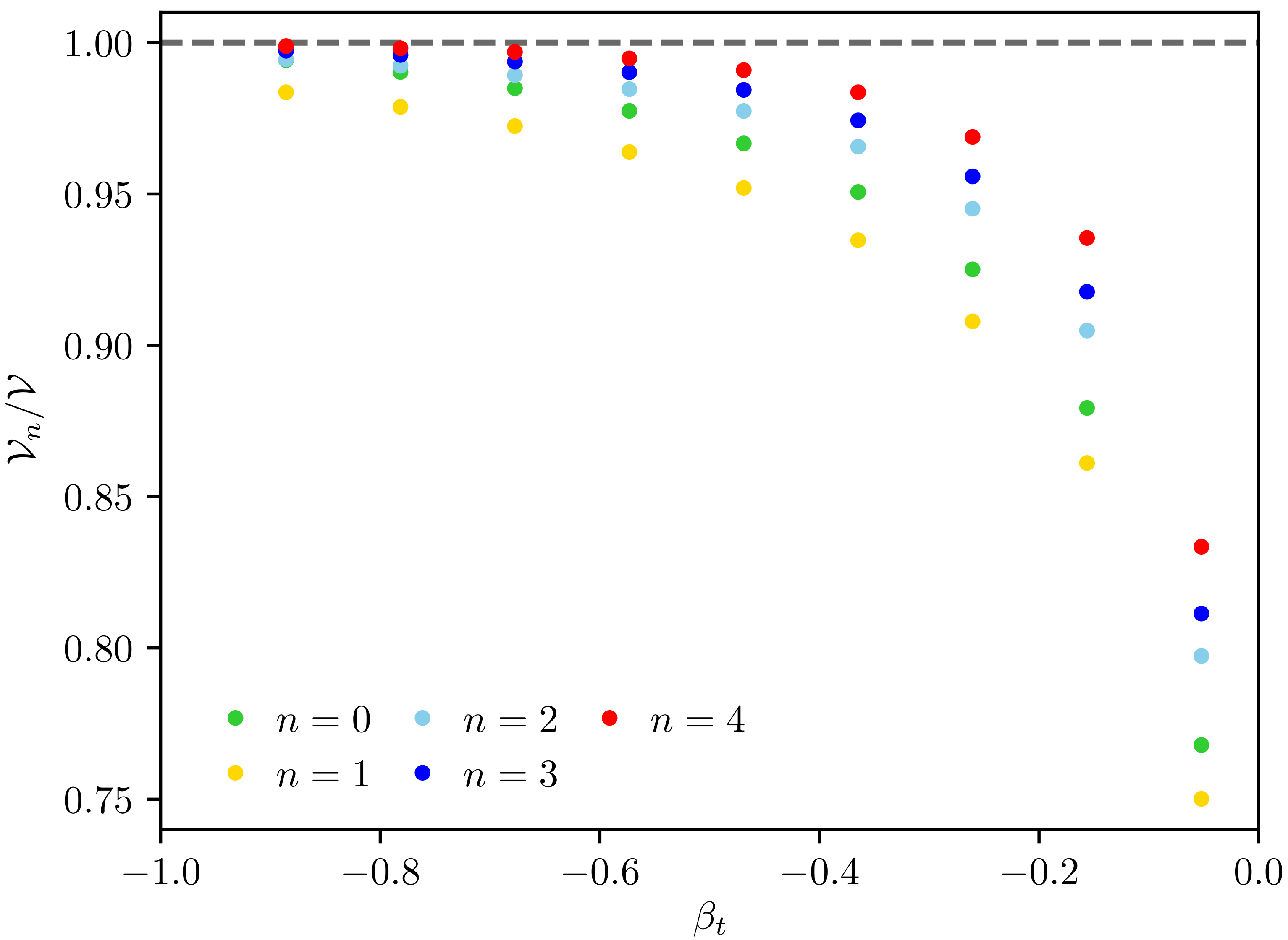
Comparison to Large m_t Expansion

The amplitude has been expanded around large-m_t and computed analytically

Hasselhuhn, Luthe, Steinhauser 17; Davies, Mishima, Steinhauser 20

Let us compare our result to the Born reweighted $1/m_t^{2n}$ expansion:

$$\mathcal{V}_n = \frac{\mathcal{B}}{\mathcal{B}_n}\widetilde{\mathcal{V}}_n + \mathcal{V}^{\mathrm{1PR}}$$



Per mille level agreement far below top quark threshold:

$$\mathscr{V}_4/\mathscr{V} = 0.9989$$

Expansion breaks down at threshold, observe that it differs from our result

Observation: $n = 1$ apparently worse than $n = 0$