QUANTUM ENTANGLEMENT IN STRING THEORY

w/ UPAMANYU MOITRA

2306.00990
2310.13735

2312.14253
2405.nnnnn.

GENDER IN HIGH ENERGY THEORY
GenHET CERN 2024

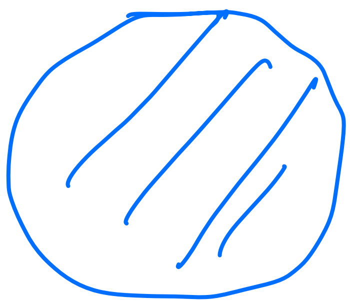
Entanglement entropy is of fundamental importance in QM & QFT and even more so in quantum gravity.

Can we define a notion of entropy?

Finiteness of entanglement entropy is at the heart of the information paradox in black hole physics.

Can we define an analog of von Neumann entropy in quantum gravity, in particular, string theory.

$$S = -\mathrm{Tr}\, \rho_0 \log \rho_0$$



Tracing over black hole interior naively gives a density matrix ρ_0 whose VN entropy <u>diverges</u>.

$\Rightarrow$ Infinite qbits for the black hole.

Ia) QE in QM

Classic example is the Bell pair with "Spooky" EPR long distance quantum correlations. A pair of photons in spin 0 state.

$$\longleftarrow \qquad\qquad \longrightarrow$$

$$\mathcal{H}_L \qquad\qquad \mathcal{H}_R$$

$$|\Omega\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Maximally entangled pure state in the Bipartite Hilbert space $\mathcal{H} = \mathcal{H}_L \times \mathcal{H}_R$

Very different from an unentangled pure state like $|U\rangle = \quad |\uparrow\uparrow\rangle \quad$ or $\quad |\uparrow\downarrow\rangle$

Entanglement Entropy

state $|\psi\rangle$ $\qquad \langle\psi|\psi\rangle = 1$

density matrix ρ $\qquad |\psi\rangle\langle\psi| \qquad \mathrm{Tr}\,\rho = 1$

Reduced density matrix $\rho_R = \mathrm{Tr}_{\mathcal{H}_L}\,\rho$ = partial Trace

$\mathrm{Tr}\,\rho_R = 1$. Sufficient to study correlators like

$$\langle\psi|\,O_R\,|\psi\rangle = \mathrm{Tr}_{\mathcal{H}_R}\,O_R\,\rho_R$$

Fine-grained (microscopic) von Neumann entropy

$$S_{EE} = -\mathrm{Tr}\,\rho_R \log \rho_R = \text{entanglement entropy.}$$

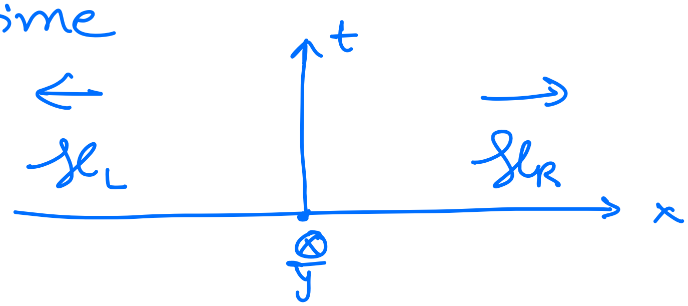
For $|\Omega\rangle$ $\qquad \rho_R = \mathrm{Tr}_{\mathcal{H}_L}\,|\Omega\rangle\langle\Omega| = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$$S_{EE} = \log 2 = \log(\dim \mathcal{H}_R)$$

$\Rightarrow$ maximum entanglement

QE in QFT

d+1 Spacetime



y = Transverse spatial slice

Divide space $x > 0$ $\mathcal{H}_R$

$x < 0$ $\mathcal{H}_L$

$$\mathcal{H} = \mathcal{H}_L \otimes \mathcal{H}_R$$ (Not really true)

$|\Omega\rangle$ = Minkowski vacuum

$$\rho_R = \mathrm{Tr}_{\mathcal{H}_L}\, \rho \qquad \rho = |\Omega\rangle\langle\Omega|$$

$$S_{EE} = -\,\mathrm{Tr}_{\mathcal{H}_R}\, \rho_R \log \rho_R$$

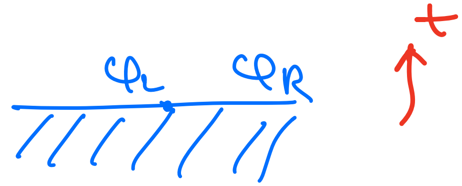
Area law with UV divergence $\boxed{\dfrac{A}{\varepsilon^{d-1}}}$

Path Integral

Wave-functional of $|\Omega\rangle$ in field basis $|\varphi(x)\rangle$

Divide $|\varphi(x)\rangle$ as $|\varphi_L(x)\rangle$ and $|\varphi_R(x)\rangle$

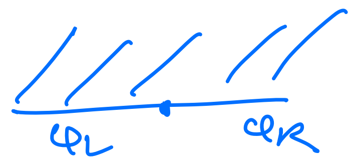
$$\langle \varphi_L \, \varphi_R | \Omega \rangle = \Psi(\varphi_L, \varphi_R) =$$



path integral in the lower time strip is a functional of the field values $\varphi_L(x, \vec{y})$ & $\varphi_R(x, \vec{y})$ and corresponds to the wavefunctional of the ket $|\Omega\rangle$.

Similarly for the bra

$$\langle \Omega | \varphi_L \, \varphi_R \rangle = \Psi^*(\varphi_L, \varphi_R) =$$



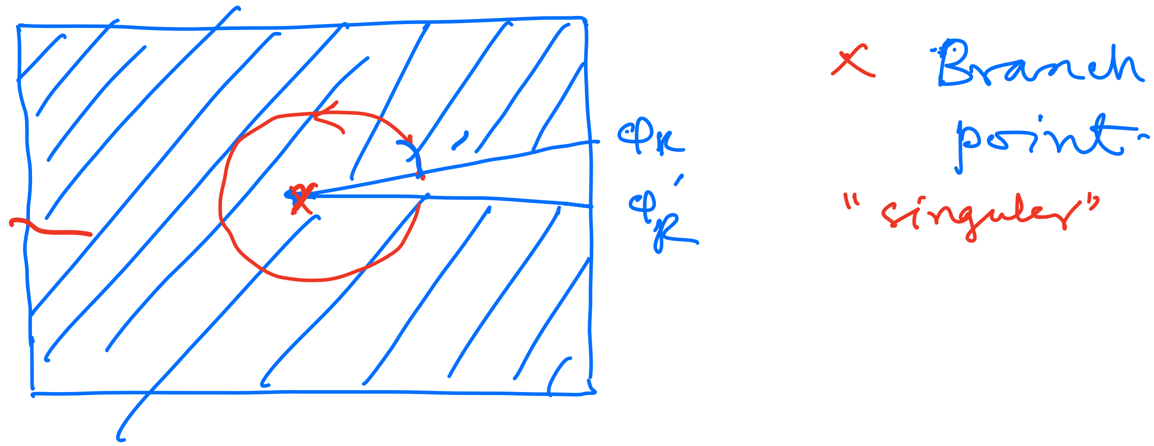
The density matrix in the field basis

$\langle \underline{\varphi_L'} \; \varphi_R' | \rho | \underline{\varphi_L} \; \varphi_R \rangle$ $\quad \rho = |\Omega\rangle\langle\Omega|$

Partial trace over the left states can be performed by identifying $\varphi_L' = \varphi_L(\vec{x}, \vec{y})$ and integrating

$$\boxed{\Rightarrow \langle \varphi_R' | \rho_R | \varphi_R \rangle = \int D\phi_L \underline{\langle \varphi_L \; \varphi_R' | \rho | \varphi_L \; \varphi_R \rangle}}$$

Rényi Entropy & Replica Method



path integral over cut plane gives a simple formula for the density matrix.

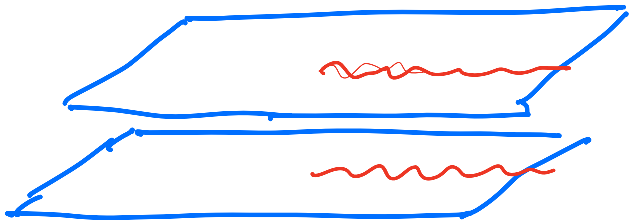
$$\langle \varphi_R' | \hat{\rho}_R | \varphi_R \rangle = \langle \varphi_R' | e^{-2\pi R} | \varphi_R \rangle$$

$$\langle \varphi_R' | \hat{\rho}_R^{\,n} | \varphi_R \rangle = \langle \varphi_R' | e^{-2\pi R n} | \varphi_R \rangle$$

Renyi Entropy

$$Z_n = \mathrm{Tr}\,\hat{\rho}_R^{\,n}$$

= path-integral over n-sheeted cover



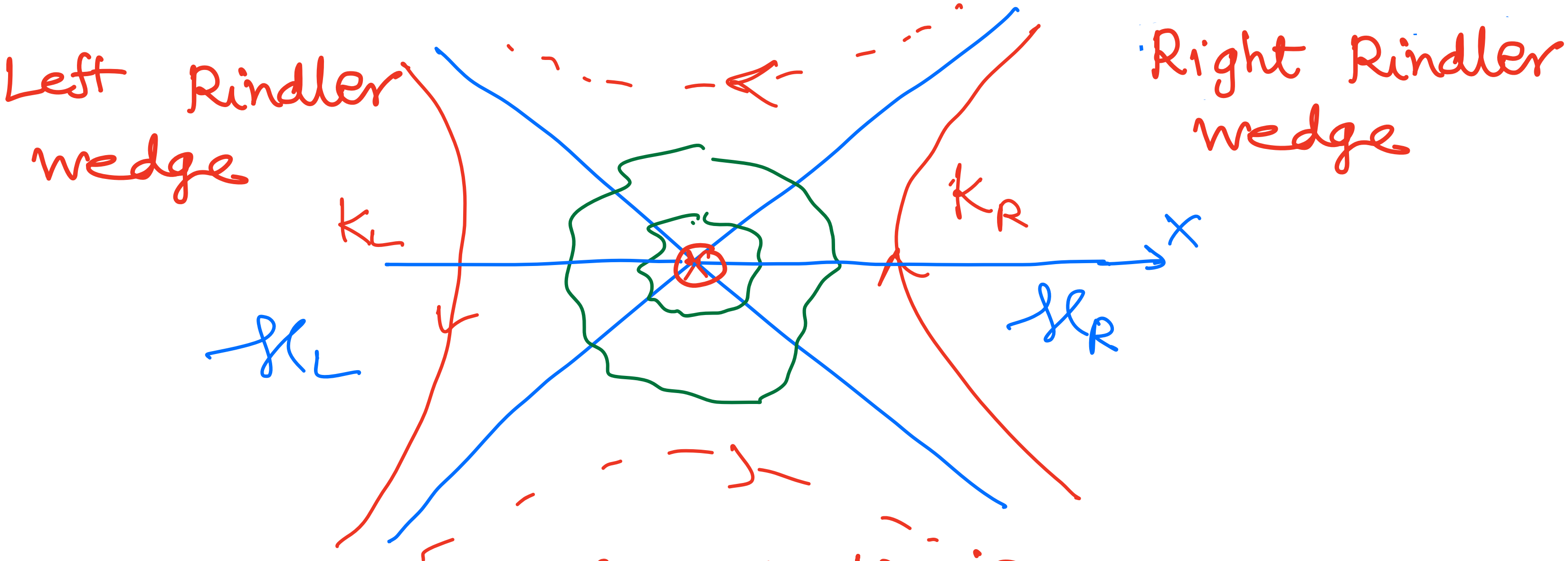
Replica Method

$\theta = 2\pi n$ = surplus opening angle.

$$S_{EE} = -\frac{d}{dn} Z_n \Big|_{n=1} = -\frac{d}{dn} \mathrm{Tr}\left(e^{n \log \hat{\rho}}\right)\Big|_{n=1}$$

Needs analytic continuation in n.

Rindler Spacetime



$\otimes$ = Bifurcate Horizon

$K = K_R - K_L$ = Lorentz Boost

A quantum fluctuation in Minkowski vacuum lives for ever and appears like thermal bath of particles

$$T(x) = \frac{1}{2\pi x}$$ Unruh

$$ds^2 = -dt^2 + dx^2 + (d\vec{y})^2$$

$$= -\rho^2 d\tau^2 + d\rho^2 + (d\vec{y})^2$$

$t = \rho \cosh\tau \qquad x = \rho \sinh\tau \qquad t^2 - x^2 = \rho^2$

Wick rotation : Periodic Euclidean time

$\tau = -i\theta \qquad ds^2 = \rho^2 d\theta^2 + \rho^2$

$$\boxed{T(\rho) = \frac{1}{2\pi\rho}}$$ KMS condn.

$$\rho_R = \exp\left[-2\pi K_R\right]$$

K_R = Partial Lorentz Boost in right wedge

$$K = K_R - K_L$$

Entropy density $s(T) = T^{d-1}(x)$

$$\boxed{S = \int_{\varepsilon}^{L} dx\, d\vec{y}\; T^{d-1}(x)}$$

$$= \frac{A_y}{\varepsilon^{d-1}} \qquad d > 1$$

$$= \frac{c}{6} \log L/\varepsilon \qquad d = 1$$

UV divergence because of strong short distance correlations & entanglement or very high Unruh temperature.

$$\varphi(t,x,\vec{y}) = \sum a_{\vec{k}\omega}\, e^{-i\omega t + i\vec{k}\cdot\vec{y}} f_{\vec{k}\omega}(x) + h.c.$$

$$\varphi_R = \sum b_{\vec{k},\omega}\, e^{-i\omega\tau + i\vec{k}\cdot\vec{y}} f_{\vec{k},\omega}(\rho) + h.c.$$

$$\varphi_L = \sum \tilde{b}_{\vec{k},\omega}\, e^{-i\omega\tau + i\vec{k}\cdot\vec{y}} f_{\vec{k},\omega}(\rho) + h.c.$$

$$a_\omega |\Omega\rangle = 0 \qquad b_\omega |R\rangle = \tilde{b}_\omega |R\rangle = 0$$

Exponential map $t = \rho \cosh\tau \simeq e^{\tau}$

$\Rightarrow$ Mixing of positive & negative frequencies

$\Rightarrow$ Bogoliubov transformations.

$$\boxed{a_\omega = b_\omega - e^{-\pi\omega}\,\tilde{b}^\dagger_\omega}$$

$$b_\omega |\Omega\rangle = e^{-\pi\omega}\,\tilde{b}^\dagger_\omega |\Omega\rangle$$

Coherent state:

$b_\omega |R\rangle = 0$

$\tilde{b}_\omega |R\rangle = 0$

$$b_\omega |\beta\rangle = \beta |\beta\rangle$$

$$\Rightarrow |\beta\rangle \sim e^{-\beta b^\dagger} |R\rangle$$

$|R\rangle$ = Rindler vacuum

Unruh = Bell.

Squeezed state

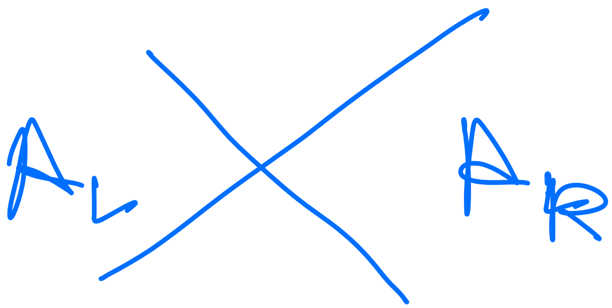
$$\Rightarrow \quad |\Omega\rangle = \prod_{\omega, \vec{k}} \sqrt{1 - e^{-2\pi\omega}}\, \exp\left[e^{-\pi\omega}\, b^\dagger_\omega\, \tilde{b}^\dagger_\omega \right] |R\rangle$$

Algebraic QFT

It's not quite correct to assume $\mathcal{H} = \mathcal{H}_R \otimes \mathcal{H}_L$ because of strong correlations at the boundary.

Hilbert space not factorized

Algebra of local observables is nevertheless factorized.



$$[A_R, A_L] = 0$$

Von Neumann Algebra

Algebra $\mathcal{A}$ of local observables :

$$\mathcal{A} = \{ \mathcal{O}(x) \}$$

① $\mathcal{O}(x)$ Bounded

② Closed under conjugation

$\mathcal{O}(x)^{\dagger}$ also belongs to $\mathcal{A}$

③ Closed under weak limits

$$\langle \psi | \mathcal{O}_n(x) | \phi \rangle \longrightarrow \langle \psi | \mathcal{O}(x) | \phi \rangle$$

$\{ \mathcal{O}_n(x) \in \mathcal{A} \}$ Then $\mathcal{O}(x) \in \mathcal{A}$

Tomita-Takesaki Theory

Given a von Neumann algebra and a cyclic separating state, one can define a modular Hamiltonian K. For right Rindler wedge it's the Rindler Hamiltonian or Lorentz Boost

$$\boxed{K_R = \int_{x \geq 0} dx\, dy \; x\, T_{00}}$$

Type-I vs Type-III.

Type-I admits an irrep.

Ex: $\mathcal{H} = \mathcal{H}_L \otimes \mathcal{H}_R$ in QM

A_R admits irrep on $\mathcal{H}_R$.

Type-III $\mathcal{H} \neq \mathcal{H}_L \otimes \mathcal{H}_R$ in QFT

$$[A_R, A_L] = 0$$

If A_R admitted an irrep, this would be inconsistent w/ Shur's lemma.

Divergence of the entanglement entropy is the property of the algebra and not the state.

(1) Bekenstein Bound

(Bekenstein, Casini)

$$E \cdot L \gtrsim \frac{\hbar S}{2\pi}$$

Follows from positivity of Relative Entropy

$$\langle H \rangle_\rho - \langle H \rangle_\sigma \geq \frac{\hbar}{2\pi} \left(S_\rho - S_\sigma \right)$$

σ = ground state

ρ = any state

(2) Strong subadditivity (Info) paradox

(Mathur, AMPS..)

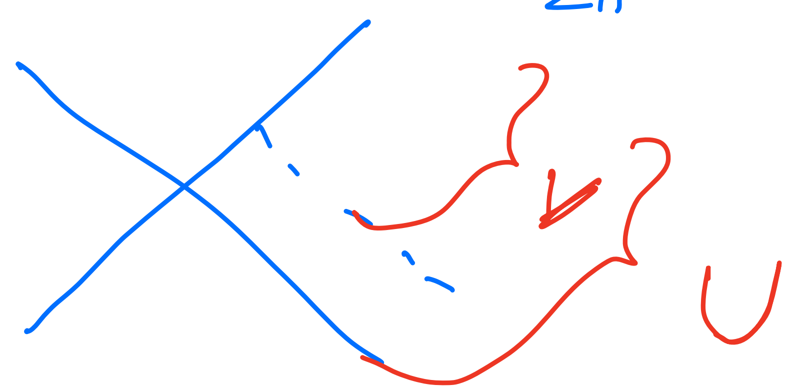
(3) Generalized Second law:

$$\Delta S_{gen} = \Delta\left(\frac{A}{4G} + S_{out}\right) \geq 0$$

Follows from monotonicity (under inclusion) of Relative Entropy.

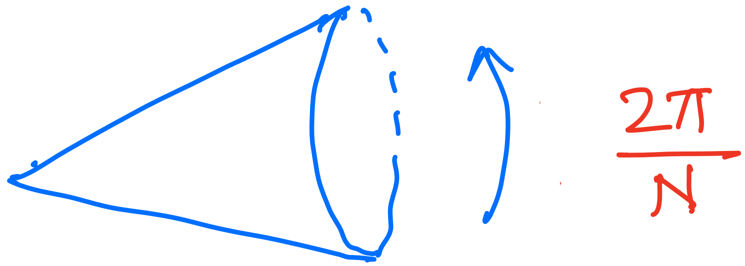
Uses Null Raichaudhari equation to relate the change in "energy" to change in area. But this term is UV divergent.

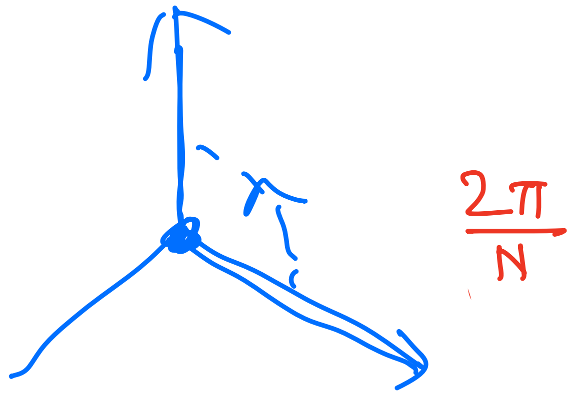
$$\Delta S_{rel}(\rho|\sigma) = \Delta H_\sigma - \frac{\Delta S_\rho}{2\pi} \leq 0$$



Orbifold Method

Instead of considering n-fold cover with surplus opening angle one can consider an order N orbifold with deficit opening angle.





Compute $\mathrm{Tr}\,\rho^N$ as the partition function of $\mathbb{C}/\mathbb{Z}_N$ orbifold with $N = \frac{1}{n}$.

Strings on a Cone

$$\hat{Z}(N) = \mathrm{Tr}\left[\exp\left(-\frac{2\pi}{N} H_R\right)\right] \qquad \beta = \frac{2\pi}{N}$$

$$\underline{K_R} \equiv \underline{H_R}$$

$$S = -\beta \frac{\partial \log \hat{Z}}{\partial \beta} + \log \hat{Z}$$

$$S = \frac{d}{dN}\left(N \log \hat{Z}(N)\right)\Big|_{N=1}$$

deficit angle $\delta_N = 2\pi\left(1 - \frac{1}{N}\right)$

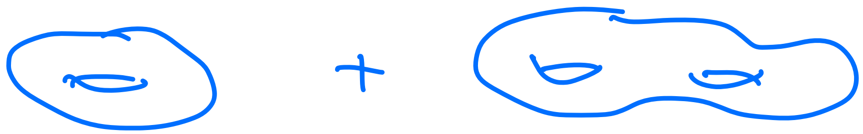
conical curvature singularity at the tip

$$R(x) = 4\pi\left(1 - \frac{1}{N}\right)\delta^{(2)}(x)$$

$$S = S^{(0)} + S_q$$

$$\log \hat{Z}_q(N) = Z_q(N)$$

$$\hat{Z}_q(N) = \sum_{g=1}^{\infty} Z^{(g)}(N)$$



$S^{(0)} =$ "classical entanglement"

At classical level $\log \hat{Z}^{(0)} \neq Z^{(0)}(N)$

LHS is nonzero even though RHS $= 0$

Analytic continuation gives

$$S^{(0)} = \frac{A}{4G}$$

$\mathbb{C}/\mathbb{Z}_N$ Orbifold

Type-IIB in lightcone gauge on $\mathbb{R}^6 \times \mathbb{C}$

$$SO(8) \supset SO(6) \times SO(2)$$

$$X^i \qquad 8_v = 6(0) + \underline{1(1) + 1(-1)}$$

$$S^a \qquad 8_s = \underline{4(\tfrac{1}{2})} + \underline{\bar{4}(-\tfrac{1}{2})}$$

$$\tilde{S}^{\dot{a}} \qquad 8_c = \underline{4(-\tfrac{1}{2})} + \underline{\bar{4}(\tfrac{1}{2})}$$

$N = \text{odd}$

$$SO(2) \simeq U(1) \simeq \text{spin}(2)$$

Fields charged under it get twisted

$$\mathbb{Z}_N = \{1, g, g \ldots g^{N-1}\} \subset U(1)$$

$$g = \exp\left[\frac{4\pi i R}{N}\right]$$

Partition Function

$$\mathcal{Z}(\tau,\bar\tau,N) = \sum_{k,l=0}^{N-1} \left| \frac{\vartheta^4\left(\frac{k\tau}{N}+\frac{l}{N}\middle|\tau\right)}{\eta^9(\tau)\,\vartheta\left(\frac{2k\tau}{N}+\frac{2l}{N}\middle|\tau\right)} \right|^2$$

$\theta = 2\pi n \longrightarrow \frac{2\pi}{N} \Rightarrow N = \frac{1}{n}$

$$Z^{(1)}(N) = \frac{A_H}{N} \int \frac{d^2\tau}{\tau_2{}^5}\, \mathcal{Z}(\tau,\bar\tau,N)$$

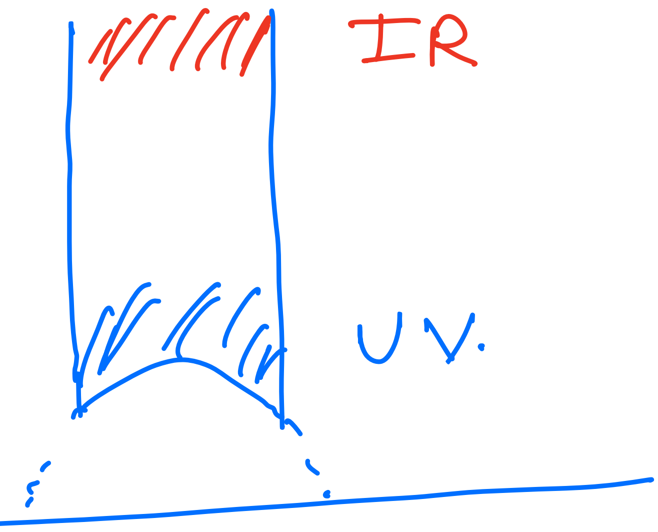
$$A_H = V_8 / (2\pi l_s)^8 \qquad l_s{}^2 = \alpha'$$

= Horizon Area.

Tachyonic Divergence

The $\mathbb{Z}_N$ orbifold breaks supersymmetry.
The spectrum is replete with tachyons

$\Rightarrow$ The modular integral is badly divergent in the infrared



L2

String theory provides a natural UV cutoff

Presence of tachyons need not be a cause for despair. In QM, $\mathrm{Tr}\,\rho^n$ converges with $\mathrm{Tr}\,\rho = 1$

$\mathrm{Tr}\,\rho^{\frac{1}{n}}$ can diverge.

We are computing $\mathrm{Tr}\,\rho^N = \mathrm{Tr}\,\rho^{\frac{1}{n}}$

$$N = \frac{1}{n}$$

Tachyonic divergence could be a signal of this fact that $\mathrm{Tr}\,\rho^{\frac{1}{n}}$ can be divergent.

How can we find the analytic continuation to the physical region $0 < N \leq 1$ given the data for $N > 1$ for all odd integers?

One can achieve this for a <u>simpler problem</u> of open strings on the Rindler horizon computing the entanglement entropy on the D-brane worldvolume.

Open strings on Rindler Horizon

Witten (2018) was able to find an analytic continuation of the annulus partition function

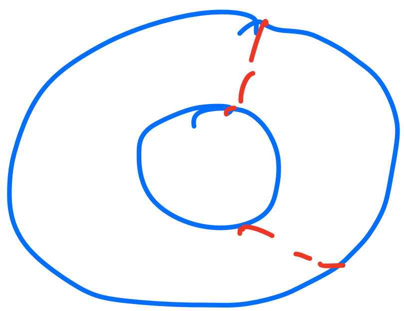
Open strings offer a double simplification. = Entanglement on D-brane

(1) No twisted states ⇒ only N terms in the orbifold sum

(2) Only one set of oscillator modes.

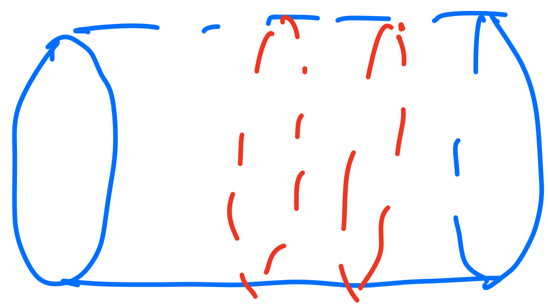
Witten found an analytic continuation. Unique by Carlson's theorem.

One can access the closed string sector by viewing the annulus as the conformally equivalent cylinder diagram.



Open string channel

$\simeq$



closed string channel

Disappearing Tachyons

Even though for $N > 1$ there are tachyons in the closed string channel, happily there are not tachyons in the physically interesting regime $0 < N \leq 1$ corresponding to $1 \leq n < \infty$

recall $$n = \frac{1}{N}$$

Closed Strings on Rindler Horizon

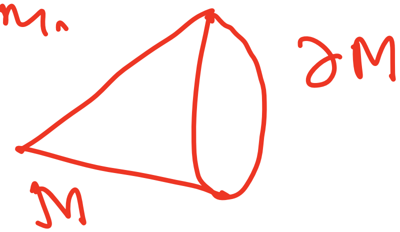
Analytic continuation of the closed string partition function is a considerably harder problem and is not yet available.

Fortunately, the tachyonic terms do admit a resummation and analytic continuation to a function that is finite for $0 < N \leq 1$

Classical Entanglement in String Theory

$$I = \frac{1}{16\pi G}\int_M [dx]\, \underline{e^{-2\phi}}\left[R + 4(\nabla\phi)^2 - \ell^{2(x)}\, V(T)\right] \quad \text{“} I_M \text{”}$$

$$+ \frac{1}{8\pi G}\int_{\partial M} [dx]\, e^{-2\phi}\, K. \quad \text{“} I_{\partial M} \text{”}$$



Gibbons-Hawking term : $K \doteq$ Extrinsic Curvature

$$\Rightarrow \log\left(Z^{(0)}(N)\right) = \frac{A}{4G\hbar}\left(\frac{1}{N} - 1\right)$$

Dilaton equations are satisfied exactly for the exact CFT background

$\Rightarrow$ No dilaton tadpoles. $\Rightarrow I_M = 0.$

Note that a priori this has nothing to do with a black hole. or holography

- Could be viewed as a generalization of von Neumann entropy in quantum gravity even though it is defined quite differently.

- Are quantum corrections calculable?

$$S = \frac{A}{4G} + S_q .$$

Analytic continuation of Tachyons

$$Z(N) = \int \frac{d^2\tau}{\tau_2^2} \mathcal{F}(\tau) \qquad \tau = \tau_1 + i\tau_2$$

$$\mathcal{F}(\tau) = \sum e^{2\pi i n \tau_1} \mathcal{F}_n(\tau_2)$$

$$\therefore \; Z(N) = \int \frac{d\tau_2}{\tau_2^2} \mathcal{F}_0(\tau_2)$$

Exponentially growing terms in $\mathcal{F}_0(\tau_2)$ correspond to tachyons.

Tachyonic Terms $T(\tau_2)$

$$T(\tau_2) = \frac{2}{\tau_2^3} \sum_{k=1}^{\frac{N-1}{2}} \exp\left(\frac{4\pi\tau_2 k}{N}\right) f_k(\tau_2)$$

$$f_k(\tau_2) = \sum_{r=0}^{\infty} e^{-r\frac{(2N-4k)}{N}2\pi\tau_2}$$

$$T(\tau_2, N) = \frac{2}{\tau_2^3} \sum_{r=0}^{\infty} e^{-4\pi r \tau_2} \frac{1 - e^{\alpha_r (N-1)\tau_2}}{1 - e^{-2\alpha_r \tau_2}}$$

$$\alpha_r = \frac{\pi(4r+2)}{N} > 0$$

since $r \geq 0$ $\quad N > 0$

Remarkably $T(z_2, N)$ has <u>no</u> exponentially growing "tachyonic" terms for $0 < N \leq 1$ even though it has tachyonic divergences for $N > 1$

We can write

$$\mathcal{F}_0(z_2, N) = T(z_2, N) + \mathcal{R}(z_2, N)$$

$\mathcal{R}(z_2, N)$ has no divergences and can be easily integrated.

The analytic continuation of Tachyons is not accidental and depends on three 'just so' properties of string orbifolds

(1) There are exactly (N-1) twisted sectors each containing tachyon.

(2) In the k-twisted sector there is precisely one leading tachyon whose mass-squared is linear in k

$$M^2 = -\frac{4\pi k}{N}$$

(3) There are many subleading tachyons but they all have unit multiplicity.

Calabi-Yau Compactifications

Our ability to obtain a finite analytic continuation in the physical region $0 < N \leq 1$ depended on a very specific structure of the tachyonic spectrum.

Is this a feature of 10d superstring or is it more generally valid, in Calabi-Yau compactifications.

There could be additional tachyons.

$T^6/\mathbb{Z}_3$ orbifold

$$\underbrace{(01)}_{\text{Rindler plane}}\ \underbrace{(23)\ (45)\ (67)}_{T^6}\ \underbrace{(89)}_{\text{Light cone.}}$$

$$g = \exp\left[\frac{4\pi i J_{01}}{N}\right]$$

$$T = \exp\left[\frac{2\pi i}{3}\left(J_{23} + J_{45} - 2J_{67}\right)\right]$$

SU(3) holonomy.

- There is a danger that there are new tachyons in the doubly-twisted sectors $(T^r g^k)$.
- One finds that all states in these sectors are either massless or massive, and the analytic continuation in N of the tachyons as in 10d
- For a $K3 = T^4/\mathbb{Z}_3$ these are new tachyons.

$T^4/\mathbb{Z}_3$

Now there are additional tachyons in the doubly-twisted sectors, with ground state energies as below.

$r = 1, 2$

$$\epsilon = \begin{cases} 0 & 0 \le \frac{k}{N} \le \frac{1}{3} \\ 2\left(\frac{1}{3} - \frac{k}{N}\right) & \frac{1}{3} < \frac{k}{N} < \frac{1}{2} \\ 2\left(\frac{1}{3} - \frac{N-k}{N}\right) & \frac{1}{2} < \frac{k}{N} < \frac{2}{3} \\ 0 & 2/3 \le k/N < 1 \end{cases}$$

$$\sum_{n=0}^{n_K} \sum_{\lfloor \frac{N+1}{3} \rfloor}^{\frac{N-1}{2}} \exp^{-2\pi\tau_2 \left[2\left(\frac{1}{3} - \frac{k}{N}\right) + 2n\left(1 - \frac{2k}{N}\right)\right.}$$

$$\frac{N}{3} < k \leq \frac{N}{2}$$

Now we take $n_K \to \infty$ and do the k-sum first.

We can ignore the floor function since the extra terms are nontachyonic.

$$\rho = e^{-2\pi\tau_2} \to 0 \quad \text{as} \quad \tau_2 \to \infty$$

$$\sum_{K=1}^{\frac{N-1}{2}} \rho^{\,2\left(\frac{1}{3} - \frac{k}{N}\right) + 2n\left(1 - \frac{2k}{N}\right)}$$

We again find the tachyons sum to

$$\epsilon^{\frac{2}{3}+2n} \frac{\left(1-\epsilon^{-\frac{2}{N}(1+2n)\left(\frac{N-1}{2}\right)}\right)}{\left(1-\epsilon^{\frac{2}{N}(1+2n)}\right)}$$

well behaved as $\epsilon \to 0$ and $0 < N \leq 1$

All follows from the geometric sum

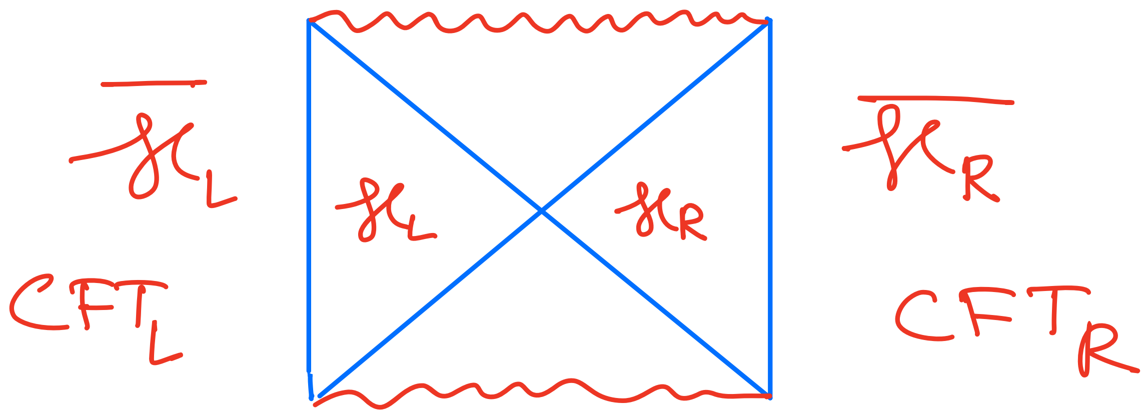
$$\boxed{\sum_{k=1}^{(N-1)/2} x^k = \frac{-x\left(1-x^{\frac{N-1}{2}}\right)}{(1-x)}}$$

Thus, there is good evidence that entanglement entropy computed using the orbifold method (a stringy analog of replica method) is <u>finite</u>. and <u>calculable</u>.

This feature appears to continue for generic Calabi-Yau compactifications to models with less supersymmetry.

Holography & Entanglement

Rindler spacetime = Near horizon geometry of the two-sided eternal Schwarzschild-AdS Black Hole



$|\Omega\rangle$

$\simeq$ Minkowski vacuum

$$|\overline{\Omega}\rangle = \sum e^{-\pi\omega} |n_\omega \bar{n}_\omega\rangle\langle n_\omega \bar{n}_\omega|$$

Thermofield double entangled state

Quantum Entanglement on BH Horizon & Holography

$AdS_3 \times S^3 \times T^4$

NS5-F1 system $(Q5, Q1)$

Exact worldsheet CFT

WZW $SL(2, \mathbb{C})$ model at $k = (L/l_s)^2$

BTZ Black hole mass M spin J

$Z(N, M, J, k)$ can be exactly computed for $\mathbb{Z}_N$ orbifolds of BTZ.

Once again tachyonic divergences get tamed in the physical region

Holographic Entanglement

$$\bar{\rho}_R = \operatorname{Tr}_{\mathcal{H}} |\bar{\Omega}\rangle\langle\bar{\Omega}|$$

= Thermal density matrix of CFT at finite temperature.

$$\bar{S} = -\operatorname{Tr} \bar{\rho}_R \log \bar{\rho}_R = f(\lambda, N^2)$$

$$S = \bar{S} = \frac{A}{4G} + S_q$$

$$\lambda = g_s N \simeq \frac{l_s^2}{A} \qquad g_s^2 = \frac{1}{N^2}$$

Expect finite and nonzero corrections.

Renormalization of Newton's constant

Divergences in the entanglement entropy in the bulk theory of gravity can be absorbed in the renormalization of Newton's constant.

Susskind & Uglum

$$\boxed{S = \frac{A}{4G} + S_q}$$ is finite

$$(a+b)^2 = a^2 + b^2 + 2ab$$