Stability of axion-dilaton Euclidean wormholes

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Based on 2306.11129 and 2312.08971

with Jean-Luc Lehners and George Lavrelashvili

Motivation: Euclidean wormholes

Context: 4D Euclidean path integral approach to quantum gravity $Z \sim \int \mathcal{D}g\, e^{-S_{\mathrm{E}}[g]}$

$\Rightarrow$ are **topology changes** allowed?

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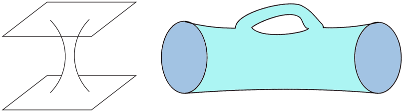

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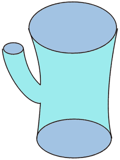

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Semi-wormholes interpreted as the production of baby universes

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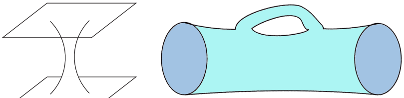

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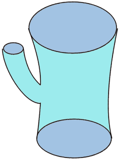

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First explicit solution: supported by an axion-flux [Giddings, Strominger 1988]

$\Rightarrow$ Non-locality & Non-unitary process made concrete

Paradoxes and puzzles

1. Initial interpretation: apparent non-unitary processes $\to$ loss of quantum coherence [Hawking 1987, 1988; Giddings, Strominger 1988; Rubakov et al. 1987]
2. Coleman's α parameter: [Coleman 1988] remove non-locality by introducing spacetime independent parameters in the path integral

$$Z = \int (\Pi_i \mathrm{d}\alpha_i)\, P(\alpha) \int \mathcal{D}g\mathcal{D}\phi\, e^{-S_\mathrm{E}[g,\phi,\lambda-\alpha]}$$

$\to$ multiverse picture, $\Lambda_\mathrm{eff} = 0$, overdensity of wormholes,...

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3. More puzzles from AdS/CFT and string theory solutions
   - AdS/CFT factorization puzzle Maldacena, Maoz [hep-th/0401024]

$$\langle O_1 O_2 \rangle = \langle O_1 \rangle \langle O_2 \rangle + \mathcal{O}(e^{-E\tau})$$

   - Coleman's α parameters do not exist in the dual CFT $\mathrm{AdS}_3 \times S^3 \times T^4$ Arkani-Hamed, Orgera, Polchinski [0705.2768]
   - ...

Negative modes of wormhole solutions: a way out?

Wormholes would not be **relevant saddle points** if they are unstable $\equiv$ if they possess negative modes: action can be lowered by adding perturbations

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- no dynamical dof for the axion in the homogeneous sector
- and no negative modes in higher angular harmonics

Upshot: **Axion-gravity wormholes are linearly stable**

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Giddings-Strominger wormholes [Giddings, Strominger 1988]

Axion-gravity with or without a massless dilaton

$$S_{\mathrm{E}} = \int \mathrm{d}^4x\sqrt{g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{12f^2}e^{-\beta\phi\kappa}H_{\mu\nu\rho}H^{\mu\nu\rho} + \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi\Big]$$

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Spherically symmetric & homogeneous ansatz:

$$\begin{cases} \mathrm{d}s^2 = h(t_{\mathrm{E}})^2\mathrm{d}t_{\mathrm{E}}^2 + a(t_{\mathrm{E}})^2\mathrm{d}\Omega_3^2\,, \\ \phi = \phi(t_{\mathrm{E}})\,, \\ H_{0ij} = 0\,,\ H_{ijk} = q\varepsilon^N_{ijk}\,. \end{cases}$$

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GS solution with a massless dilaton: regular solution $\forall\ \beta < \beta_c \simeq 1.632...$:

$$\begin{cases} \mathrm{d}s^2 = a_0^2\cosh(2t_{\mathrm{E}})\left(\mathrm{d}t_{\mathrm{E}}^2 + \mathrm{d}\Omega_3^2\right)\,, \\ \phi = \frac{1}{\beta}\ln\left[\frac{N^2}{3a_0^4}\cos^2\left(\frac{\beta}{\beta_c}\arccos\frac{1}{\cosh(2t_{\mathrm{E}})}\right)\right], \quad \text{with } N^2 = \frac{q^2}{2f^2}\,, \\ a_0^2 = \frac{N}{\sqrt{3}}\cos\left(\frac{\pi}{2}\frac{\beta}{\beta_c}\right). \end{cases}$$

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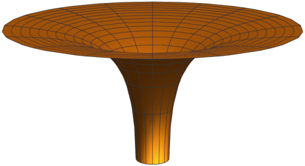
Boundary solutions:

- regularity at the throat:
  $\dot{a}(0) = 0\,,\ \dot{\phi}(0) = 0$
- asymptotic flat space:
  $\dot{a}(t\to\infty) = 1\,,\ \phi(t\to\infty) = 0$



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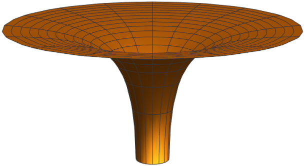

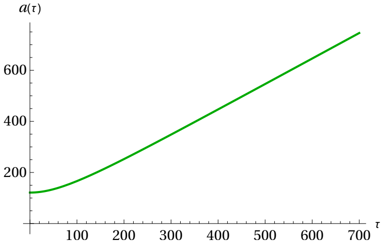

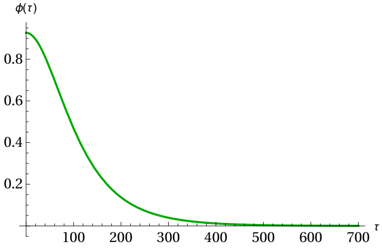

$\boldsymbol{m = 0.01}$, $\boldsymbol{N = 47089}$ **and** $\boldsymbol{\beta = 1.58}$

GS wormhole solution with a massive dilaton

- Andriolo, Shiu, Soler, Van Riet [2205.01119]: solutions exist above $\beta_c = 1.632...$
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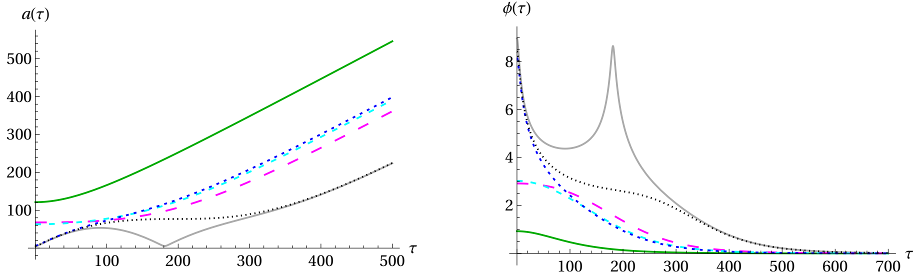

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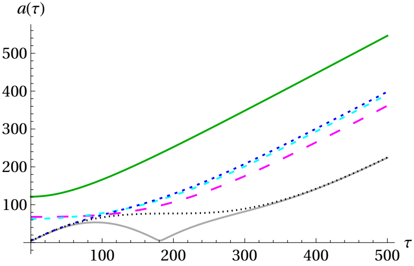
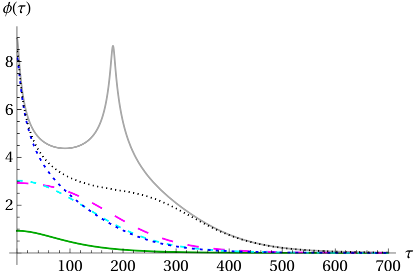
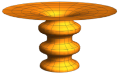
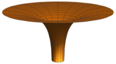




$S_{\mathrm{E}}[$  $] < S_{\mathrm{E}}[$  $]$

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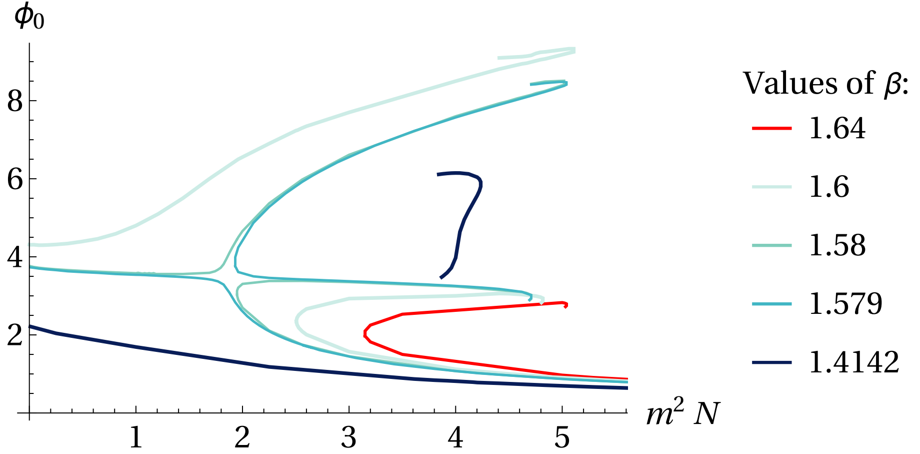

Branching structure of the GS-like wormhole solutions in the massive dilaton case: $\beta_c = 1.632...$

Perturbative stability via the Sturm-Liouville problem

Perturbative stability → negative modes of the quadratic part of the action:

$$S_{\mathrm{E}}[\bar{x} + X] \simeq S_{\mathrm{E}}[\bar{x}] + \frac{1}{2} \left. \frac{\delta^2 S_{\mathrm{E}}}{\delta x^2} \right|_{\bar{x}} X^2.$$

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For a certain gauge-invariant quantity $\mathcal{R}$, we will get:

$$S^{(2)} = \frac{1}{2} \int \mathrm{d}^4 x \sqrt{g} \left(\frac{\dot{\phi}^2}{Q(a,\phi)} \dot{\mathcal{R}}^2 + U(a,\phi) \mathcal{R}^2 \right)$$

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Associated to the eigenvalue equation (EoM for $\mathcal{R}$)

$$-\frac{1}{a^3}\frac{\mathrm{d}}{\mathrm{d}t_{\mathrm{E}}}\left(a^3\frac{\dot{\phi}^2}{Q}\dot{\mathcal{R}}\right) + U\mathcal{R} = \lambda w(t_{\mathrm{E}})\mathcal{R}$$

Perturbation ansatz and quadratic order action

Lorentzian conformal time η ansatz:

$$\begin{cases} \mathrm{d}s^2 = a^2(\eta)\left[-(1+2A(\eta,x^i))\,\mathrm{d}\eta^2 + (1-2\Psi(\eta,x^i))\,\mathrm{d}\Omega_3^2\right], \\ \phi = \phi(\eta) + \Phi(\eta,x^i)\,, \\ H_{0ij} = 0 + \varepsilon^N_{ijk}\delta^{kl}\partial_l W(\eta,x^i)\,, \\ H_{ijk} = q\varepsilon^N_{ijk}\left(1+Y(\eta,x^i)\right)\,. \end{cases}$$

- ⋆ axion perturbations W, Y non dynamical in the homogeneous sector $\nabla^2 = 0$
- ⋆ a good gauge-invariant variable is $\mathcal{R} = \Psi + \frac{\mathcal{H}}{\phi'}\Phi$.

Quadratic part of the Euclidean action in physical time t_E ($\mathrm{d}t_\mathrm{E} = -ia(\eta)\mathrm{d}\eta$)

$$S_\mathrm{E}^{(2)} = \frac{1}{2}\int_0^\infty dt_\mathrm{E}\int d^3x\sqrt{\gamma}a^3\left(\frac{\dot\phi^2}{Q}\dot{\mathcal{R}}^2 + U\mathcal{R}^2\right) - \int d^3x\sqrt{\gamma}a^3\,\frac{\dot\phi^2}{Q}\,\mathcal{R}\dot{\mathcal{R}}\Big|_{t_\mathrm{E}=0}^{t_\mathrm{E}=+\infty}$$

with $Q = H^2 - \kappa^2\dot\phi^2/6$ and $U = \frac{-2}{a^2Q^2}\left[\frac{V\dot\phi^2}{3}\left(1 - 3\frac{N^2}{a^4}e^{-\beta\phi}\right) + V_{,\phi}\dot\phi H\left(1 - \frac{N^2}{a^4}e^{-\beta\phi}\right) + \left(\frac{4}{3a^2}\dot\phi^2 - \beta H\dot\phi\left(V - \frac{2}{a^2}\right)\right)\frac{N^2}{a^4}e^{-\beta\phi}\right].$

Associated eigenvalue equation:

$$-\frac{1}{a^3}\frac{\mathrm{d}}{\mathrm{d}t_{\mathrm{E}}}\left(a^3\frac{\dot{\phi}^2}{Q}\dot{\mathcal{R}}\right)+U\mathcal{R}=\lambda w(t_{\mathrm{E}})\mathcal{R}$$

Analysing the asymptotic and throat limits:

- $\dot{\phi}^2/Q$ and U regular at the origin and asymptotically $\Rightarrow$ $w(t_{\mathrm{E}})=\dot{\phi}^2/Q$ is well-suited.
- $\dot{\mathcal{R}}(t_{\mathrm{E}}\to\infty)=0$ and $\dot{\mathcal{R}}(0)=0$ set the surface term to 0.

$\Rightarrow$ We look for eigenmodes with BC: $\dot{\mathcal{R}}(0)=0\,,\ \mathcal{R}(0)=1\,,\ \dot{\mathcal{R}}(\infty)\to 0\,,\ \mathcal{R}(\infty)\to 0$

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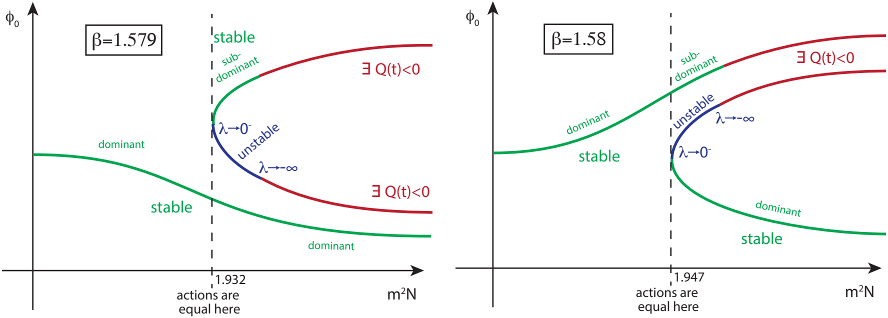


Conclusions and outlook

Current status:

- Better understanding of **axion-dilaton GS-like wormhole solutions**
  phase space of solutions, connection to massless case, perturbative stability of homogeneous sector (also inhomogeneous sector, see Hertog, Maenaut, Missoni, Tielemans, Van Riet, to appear)
- new puzzles related to **oscillating solutions** → lower Euclidean action
- **bifurcating structure** appearance of additional negative mode → new symmetry?

Future prospects:

- better suited variables to avoid the **negative Q problem**
  → stability of oscillating solutions?
- different types of asymptotic boundaries / boundary conditions for the axion
  - ⋆ axion-gravity in de Sitter Aguilar-Gutierrez et al. [2306.13951]
  - ⋆ AdS wormholes in axion-scalar-gravity Petzios, Papadoulaki [2403.17046]
- uplift to 10D asymptotically AdS as in Loges, Shiu, Van Riet [2302.03688]

Coleman's α parameters: removing bilocality Coleman 1988

For a detailed explanation: Preskill 1989, Arkani-Hamed, Orgera, Polchinski [0705.2768] or Hebecker, Mikhail, Soler [1807.00824]

Partition function for one wormhole:

$$Z_{1,w} = \int \mathcal{D}g\mathcal{D}\phi\, e^{-S[g,\phi]} \left(\int \mathrm{d}^4x \sqrt{g(x)} \int \mathrm{d}^4y \sqrt{g(y)} e^{-S_w[x,y,g,\phi]} \right).$$

Approximating S_w with bilocal operators: $S_w[x,y,g,\phi] = S_w + \sum_{i,j} \tilde{\Delta}_{ij} \mathcal{O}_i(x)\mathcal{O}_j(y)$.

In the dilute gas approx (N wormholes independent of one another, with length of the throat $\gg$ its diameter), the sum exponentiate as in the instanton limit:

$Z_w = \int \mathcal{D}g\mathcal{D}\phi\, e^{-S[g,\phi]+I}$ with

$$I = \frac{1}{2} \int \mathrm{d}^4x \sqrt{g} \int \mathrm{d}^4y \sqrt{g} \sum_{i,j} \Delta_{ij} \mathcal{O}_i(x)\mathcal{O}_j(y), \quad \Delta_{ij} \propto e^{-S_w}.$$

This can be rewritten in a local expression by integrating over spacetime independent parameters α_i:

$$e^I = \prod_i \left(\int \mathrm{d}\alpha_i \right) e^{-\frac{1}{2}\sum_{i,j} \alpha_i \Delta^{-1}_{ij} \alpha_j}\, e^{\sum_i \alpha_i \int \mathrm{d}^4x \sqrt{g}\, \mathcal{O}_i(x)}$$

Coleman's α parameters: multiverse interpretation

Since $S[g,\lambda] = \sum_i \lambda_i \int d^4x \sqrt{g}\, \mathcal{O}_i(x)$ where λ_i are the coupling constants, the effect of the wormholes is to shift them $\lambda_i \to \lambda_i - \alpha_i$. Then the partition function reads

$$Z(\alpha) = \int \mathcal{D}\alpha\, G(\alpha) \left[\int \mathcal{D}g\, e^{-S[g,\lambda-\alpha]} \right], \quad G(\alpha) = e^{-\frac{1}{2}\sum_{i,j} \alpha_i \Delta^{-1}_{ij} \alpha_j}.$$

Multiverse interpretation: each baby universe state correspond to a specific set of fixed α's:

- Each species of baby universe labeled by $i \Rightarrow a_i^\dagger,\ a_i:\ [a_i, a_i^\dagger] = 1$
- $S_{\mathrm{EFT}} = S_0 + \sum_i A_i \int d^4x\, \mathcal{L}_i$, with $A_i = a^\dagger_{i*} + a_i$: $A_i\,|\alpha\rangle = \alpha_i\,|\alpha\rangle$
- $[H, A_i] = 0 \Rightarrow$ different $|\alpha\rangle$ = different superselection sectors of the EFT
- On a $|\alpha\rangle$– eigenstate: $S_{\mathrm{EFT}} = S_0 + \sum_i \alpha_i \int d^4x\, \mathcal{L}_i$: α_i are the coupling constants

Loss of unitarity elegantly explained by the $|\alpha\rangle$– eigenstate:

$$|\Psi\rangle = |\Psi_{\text{EFT}}\rangle \times |\Psi_{\text{BU}}\rangle \longrightarrow |\Psi'\rangle = \sum_i c_i \left|\Psi^i_{\text{EFT}}\right\rangle \times \left|\Psi^i_{\text{BU}}\right\rangle$$

→ tracing out the baby universe state:

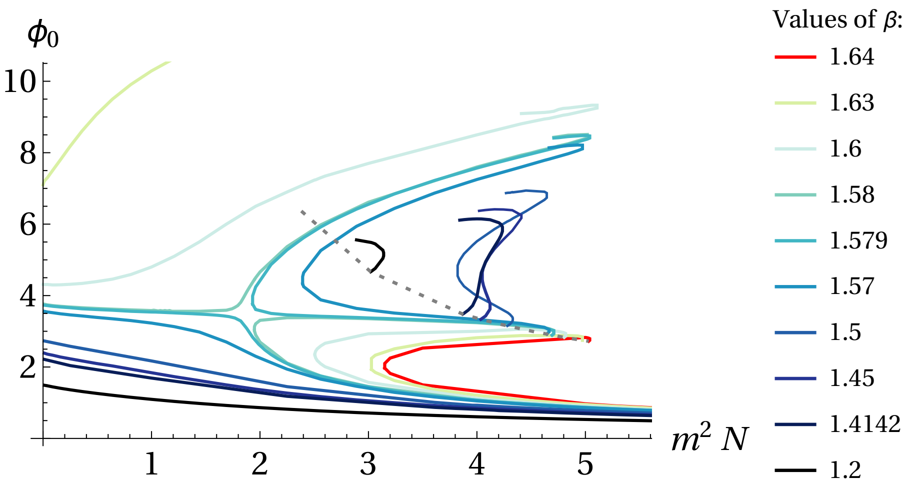
$$|\Psi_{\text{EFT}}\rangle \langle\Psi_{\text{EFT}}| \longrightarrow \sum_i |c_i|^2 \left|\Psi^i_{\text{EFT}}\right\rangle \left\langle\Psi^i_{\text{EFT}}\right| .$$

Using Coleman's α parameters: $|\Psi_{\text{BU}}\rangle = |\alpha\rangle$ and $[H, A_i] = 0 \Rightarrow |\alpha\rangle$ is invariant under time evolution and each $\{\alpha_i\}$– set defines a well-defined and unitary EFT.

Statistical prediction of the effective coupling constant:

- Coleman's calculation that $\Lambda_{\text{eff}} = 0$ Coleman 1988
- FKS catastrophe (overdensity of wormholes) calculated similarly for $\mathcal{O}(x) = R^2$. Fischler, Kaplunovsky, Susskind 1989

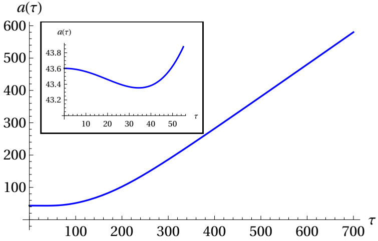
Massive dilaton potential $V(\phi) = m^2\phi^2/2$

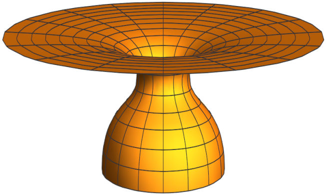


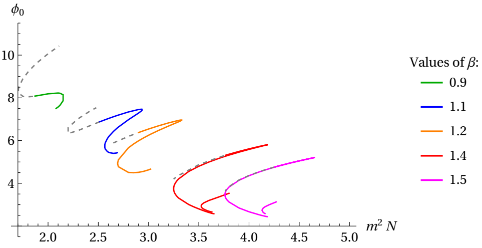
All GS-like solutions for the massive dilaton case: $\beta_c = 1.632...$

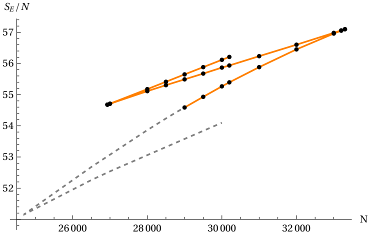
Expanding baby-universe solutions

One example - $\beta = 1.2$, $N = 30000$, $m = 0.01$



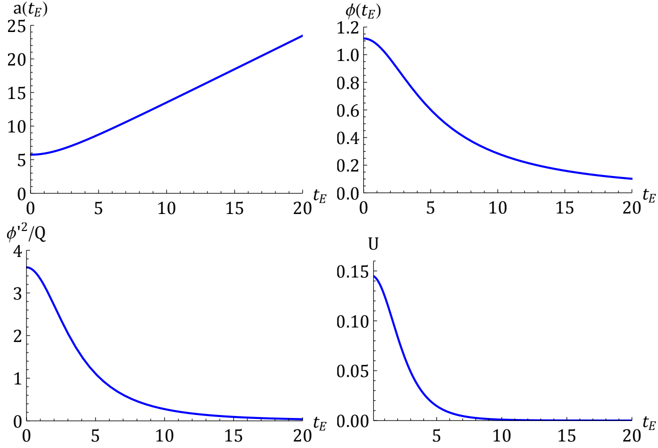






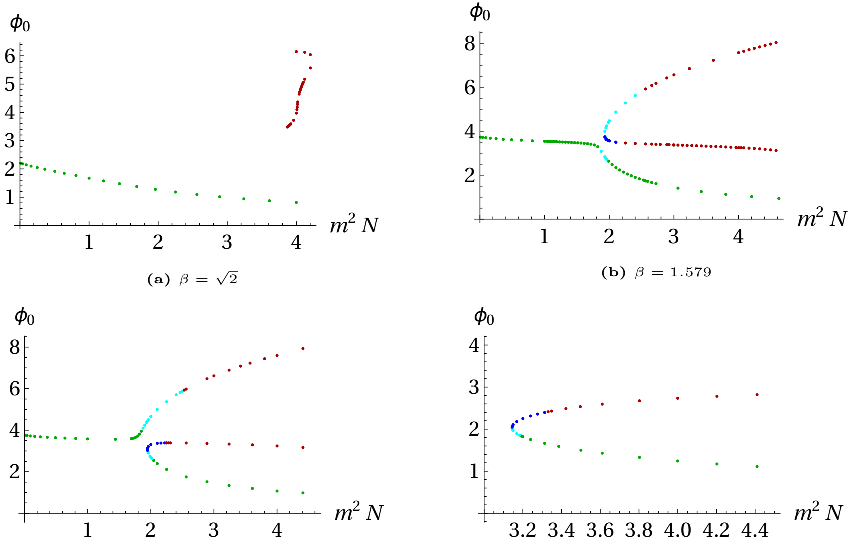
Stability of massless dilaton solutions

$\beta = 1$ massless dilaton solutions:



Potential and kinetic term positive throughout $\rightarrow$ Perturbatively stable

Stability of massive dilaton solutions



(a) $\beta = \sqrt{2}$

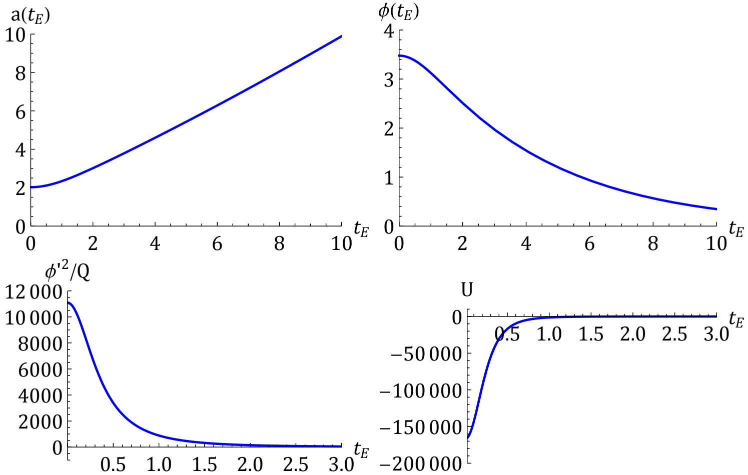
(b) $\beta = 1.579$

(c) $\beta = 1.58$

(d) $\beta = 1.64$

Example with one negative mode

$\beta = 1.579$, $m^2 N = 2.2$, $\phi_0 \approx 3.477 \rightarrow$ $U < 0$ initially



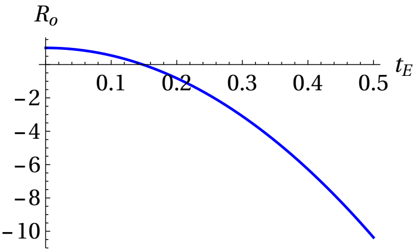
Example with one negative mode

Nodal Theorem:

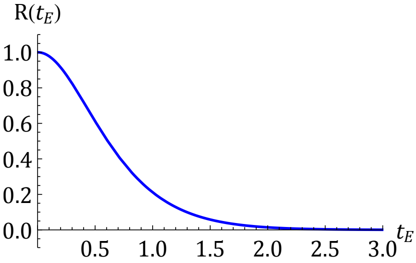
nodes of solution to the zero eigenvalue equation = # negative modes

$$-\frac{1}{a^3}\frac{\mathrm{d}}{\mathrm{d}t_{\mathrm{E}}}\left(a^3\frac{\dot{\phi}^2}{Q}\dot{\mathcal{R}}_\circ\right)+U\cdot\mathcal{R}_\circ=0\,,\ \text{with } \mathcal{R}_\circ(0)=1,\ \dot{\mathcal{R}}_\circ(0)=0\,.$$

$\beta = 1.579$, $m^2N = 2.2$, $\phi_0 \approx 3.477$:



(a) Even zero eigenmode with one node



(b) Corresponding negative mode with $\lambda \approx -10.36$

Negative Q problem

Negative kinetic term $\rightarrow$ presence of a ghost (infinitely many negative modes)

- Similar problems for certain Euclidean bounce solutions [Rubakov et al. 1985]/ CDL bounces [Coleman, De Luccia 1980]: tunneling negative modes with on top an infinite tower with much higher frequency.
- Negative mode problem: much studied for the last 30 years but no solution so far [Gratton, Turok, Lavrelashvili, Rubakov, Tinyakov, Lee, Weinberg, ...]
- Physical or mathematical problem? $\rightarrow$ location of the singularity can be shifted by a canonical transformation / in Hamiltonian treatment Q is singular for all wormhole solutions

$\Rightarrow$ solved by finding suitable variables?

NB: "expanding" wormhole solutions all have $Q = H^2 - \kappa^2\dot{\phi}^2/6$ negative when $\dot{a} = 0 \rightarrow$ stability cannot be analysed here