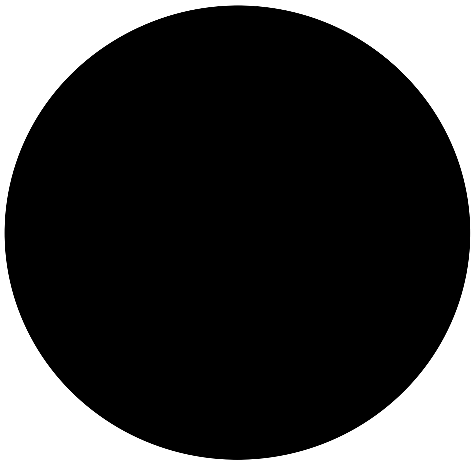
Exactly solvable irrelevant deformations and holography

Monica Guica

CERN/EPFL/IPhT

Motivation

- black hole entropy



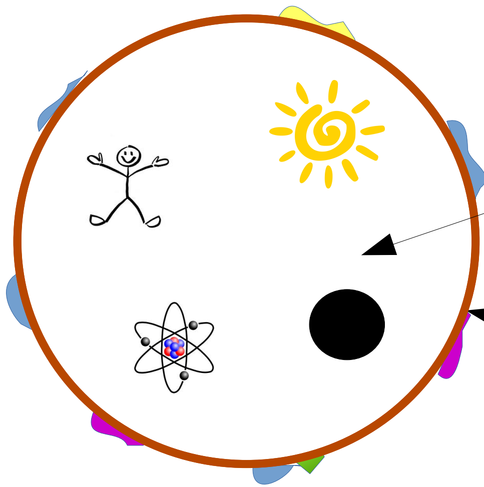
$$\boxed{S_{BH} = \frac{\mathcal{A}_H}{4G}}$$

universal

Introduction

- black hole entropy

Holography



Gravitational theory
d+1 dimensions

Field theory (no gravity)
d dimensions

✓ usual Quantum Mechanics

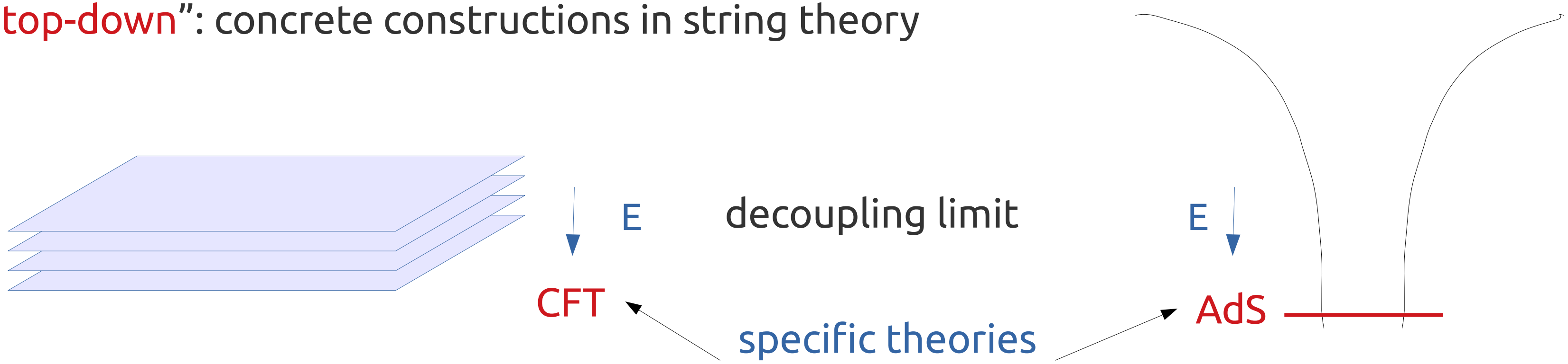
$$\boxed{S_{BH} = \frac{\mathcal{A}_H}{4G}}$$

universal

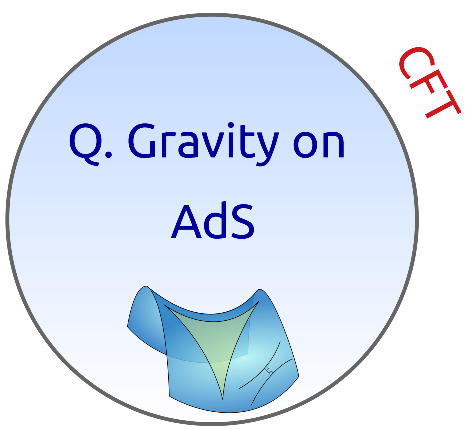
The AdS/CFT correspondence

- AdS/CFT correspondence: two approaches

I. “top-down”: concrete constructions in string theory



II. ”bottom-up”: universalist approach



- $\forall$ CFT_d with large N (large gap) → gravity in AdS_{d+1}
- symmetries → asymptotic symmetries (Virasoro in 3d)
- correlation functions → scattering
- axiomatic description of CFTs, even at strong coupling

Beyond AdS/CFT

~~universality~~

- non-AdS holography: **hard** → few concrete examples in string theory (funny decoupled asymptotics)

  → often non-local theories, strongly coupled (hard to study independently)

assume holographic dual exists &

- infer properties of dual QFT from spacetime: symmetries, thermodynamics, correlation functions

e.g. celestial holography programme

symmetries

(asymptotic symmetries)

entropy

(black holes)

correlation functions

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- this data is hard to assemble into a consistent whole w/o knowing the basic QFT structure/properties

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symmetries

Virasoro x Virasoro

local

2d CFT

entropy

Cardy

correlation functions

2&3 pf fixed by Δ

- this data is hard to assemble into a consistent whole w/o knowing the basic QFT structure/properties

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QFT properties

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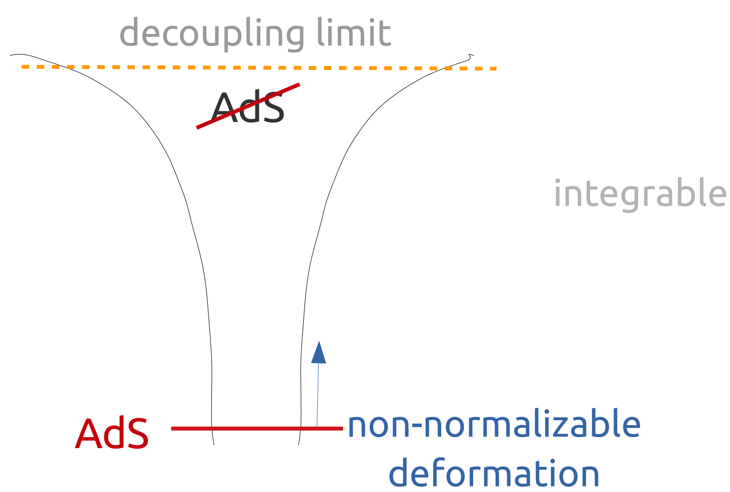
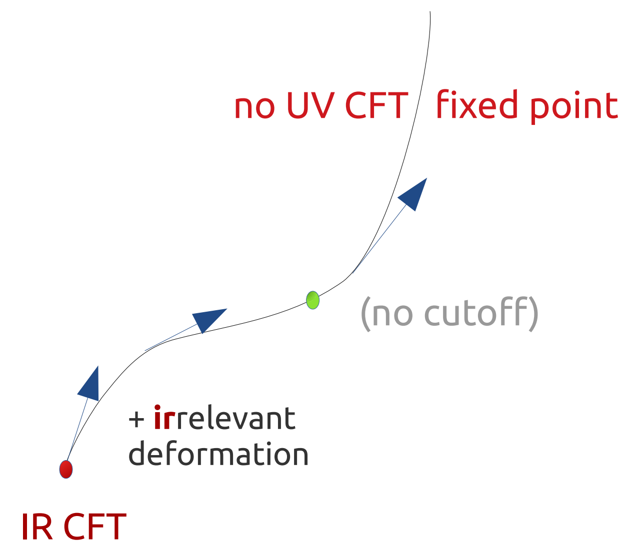
- this data is hard to assemble into a consistent whole w/o knowing the basic QFT structure/properties
- having an in principle independent QFT description is valuable

Non-AdS holography and irrelevant flows





- holographic dual ⟺ finely-tuned irrelevant flow → UV-complete theory decoupled from gravity

  (non-local QFT or string theory)

- string theory: concrete examples of such finely-tuned irrelevant flows dual to non-AdS backgrounds

  non-commutative N=4 SYM, dipole-deformed N=4 SYM, little string theory & generalisations

- largely intractable : rely on bottom-up methods (holography) to study them (! ∃ and basic properties)

Methodology

- combine the top-down set-ups ($\exists$ of decoupled theory & basic QFT properties)

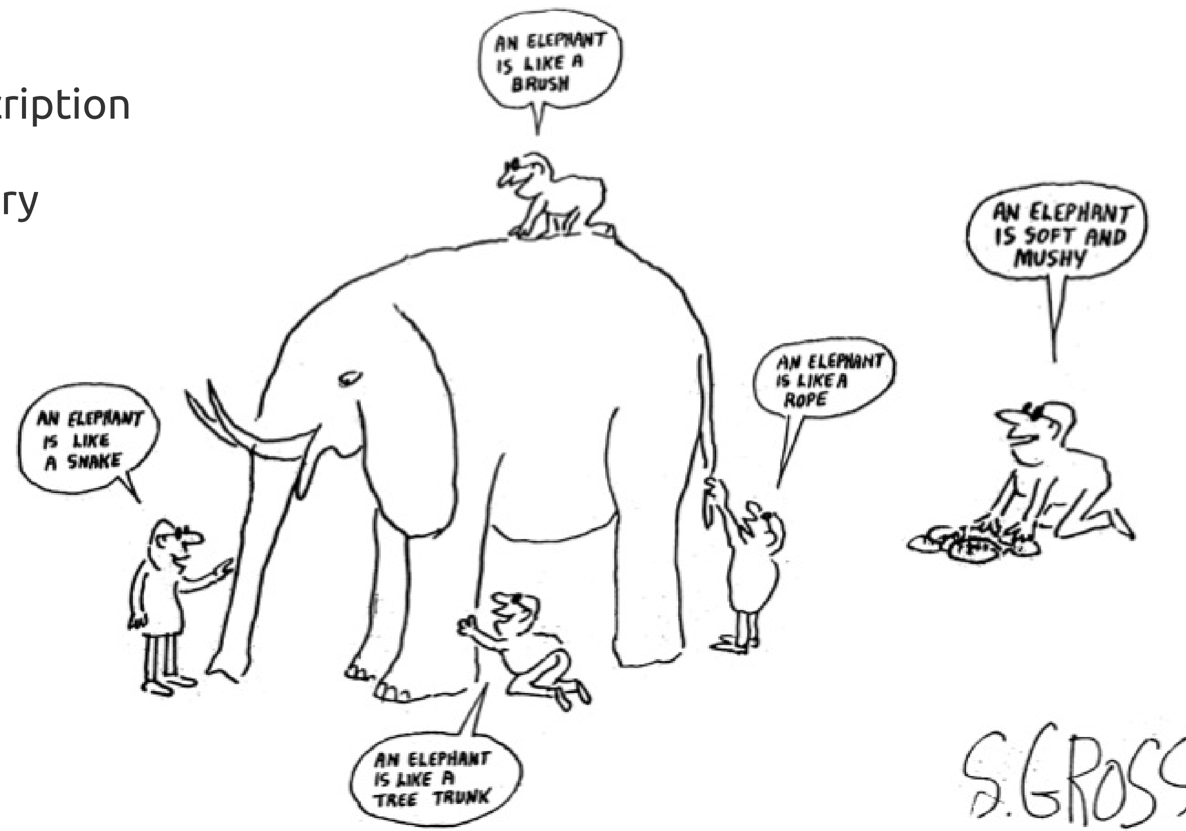
  with bottom-up approaches (symmetries, correlation functions, thermodynamics)

  and explicit computations in field-theory toy models (QFT structure)

  → more complete & consistent description

  of the non-AdS holographic dictionary

www.flickr.com



The field theory toy models

The field theory toy models:

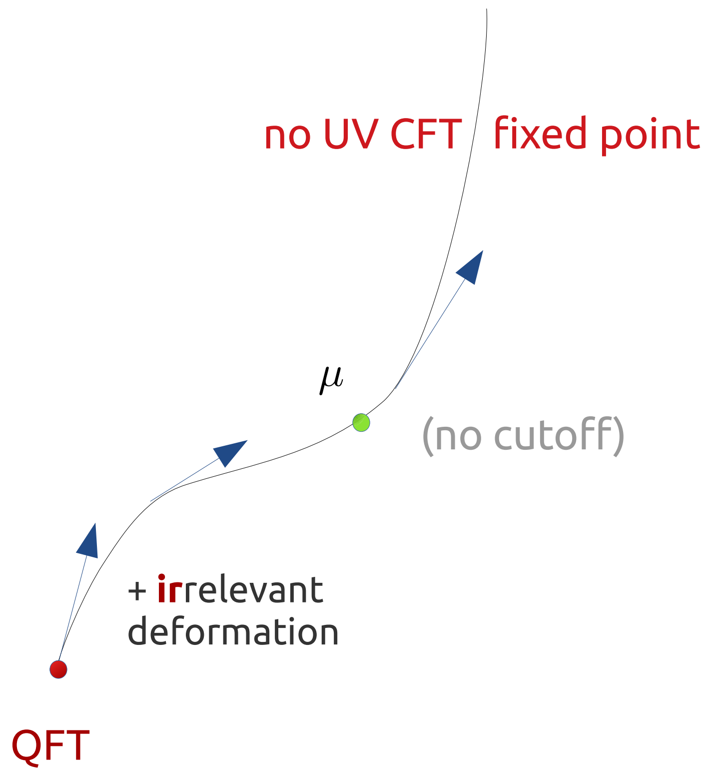
$T\bar{T}$ & $J\bar{T}$ – deformed CFTs

What is the $T\bar{T}$/ $J\bar{T}$ deformation?

- irrelevant deformation of 2d QFTs $\rightarrow$ UV complete QFTs that are non-local

- finely tuned irrelevant flow

integrability preserved



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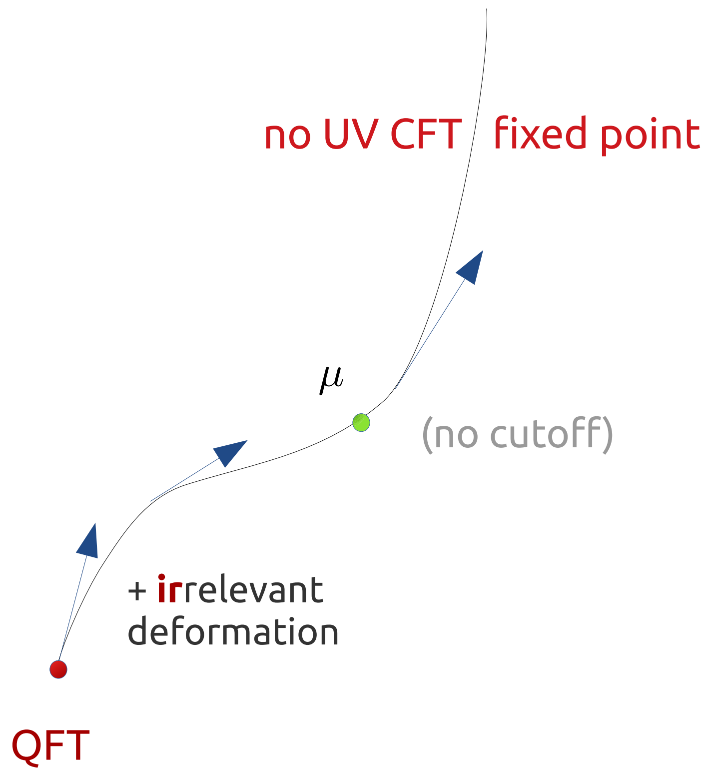


integrability preserved

- finely tuned irrelevant flow

  - bilinear of two conserved currents J^A, J^B

$$\lim_{y\to x} \epsilon^{\alpha\beta} J^A_\alpha(x) J^B_\beta(y) = \mathcal{O}_{J^A J^B} + \text{ derivative terms}$$

nice factorization properties

$$\boxed{\frac{\partial S(\mu)}{\partial \mu} = \int d^2x \, \mathcal{O}_{J^A J^B}(\mu)}$$

Smirnov & Zamolodchikov '16

  - examples of universal deformations

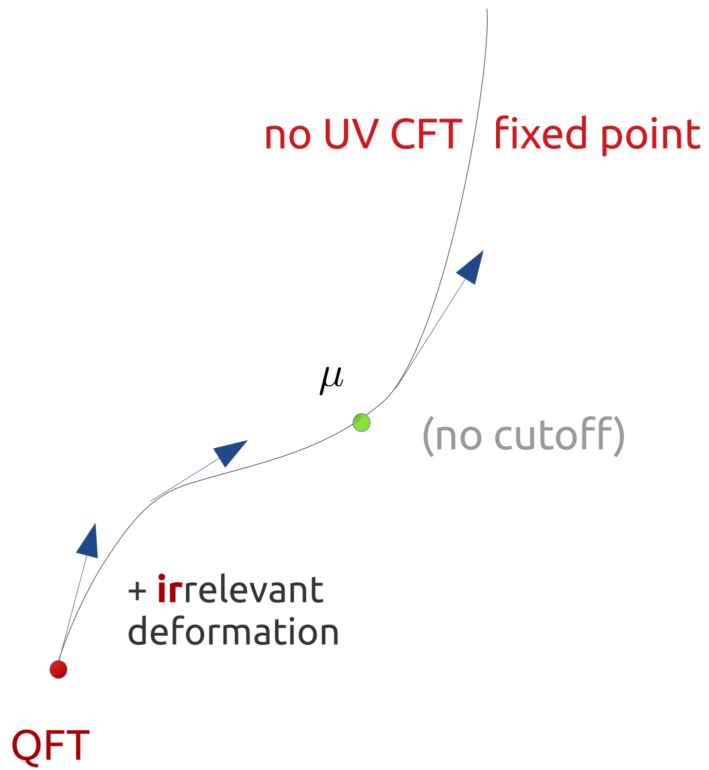
$$T\bar{T}: \quad J^A_\alpha = T_\alpha{}^A \,, \quad J^B_\beta = T_\beta{}^B \ (\times \epsilon_{AB}) \qquad (2,2)$$

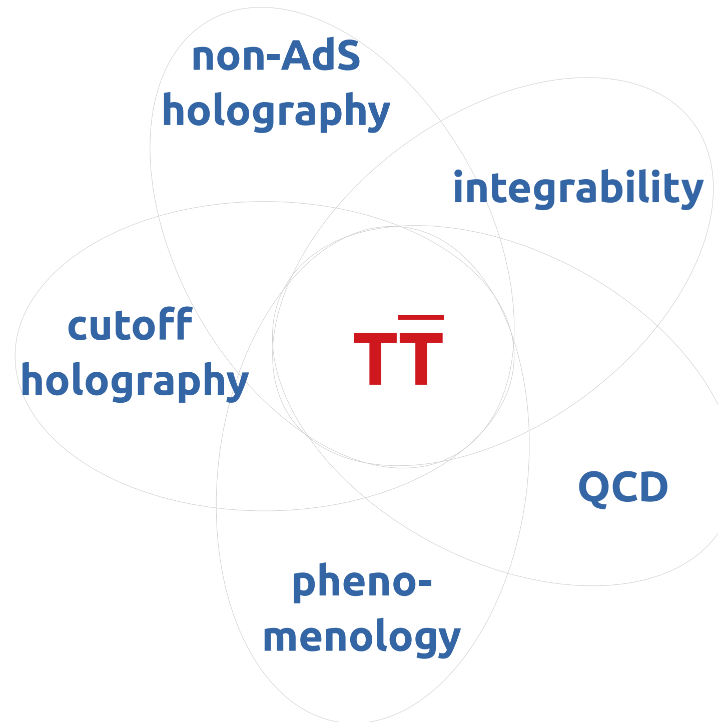
$$J\bar{T}: \quad J^A_\alpha = J_\alpha \,, \quad J^B_\beta = T_{\beta\bar{z}} \qquad \cancel{\text{Lorentz}} \qquad (1,2)$$

$$\underbrace{SL(2,\mathbb{R})_L}_{\text{local \& conformal}} \times \underbrace{U(1)_R}_{\text{non-local}}$$

Why are they interesting?

- irrelevant deformations of 2d QFTs → UV complete QFTs that are non-local





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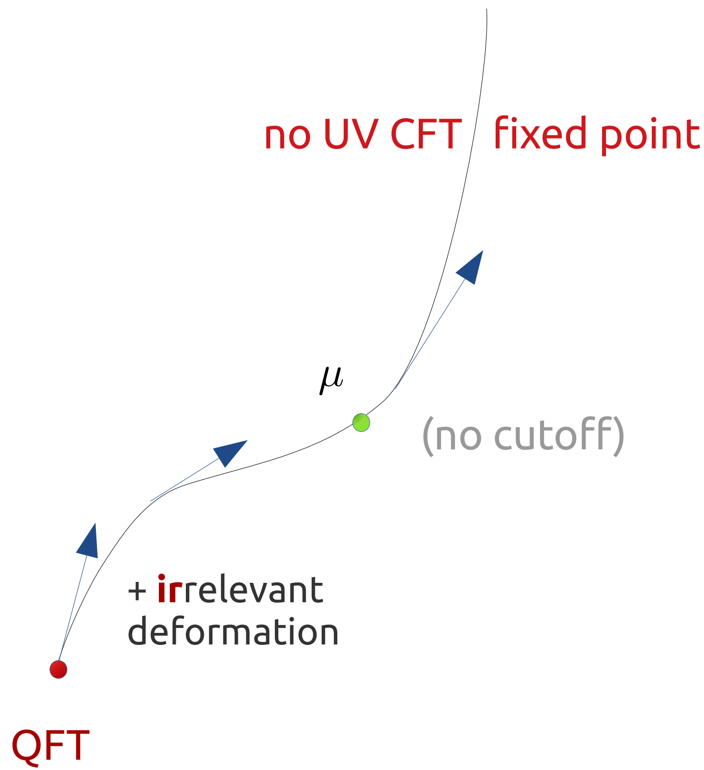

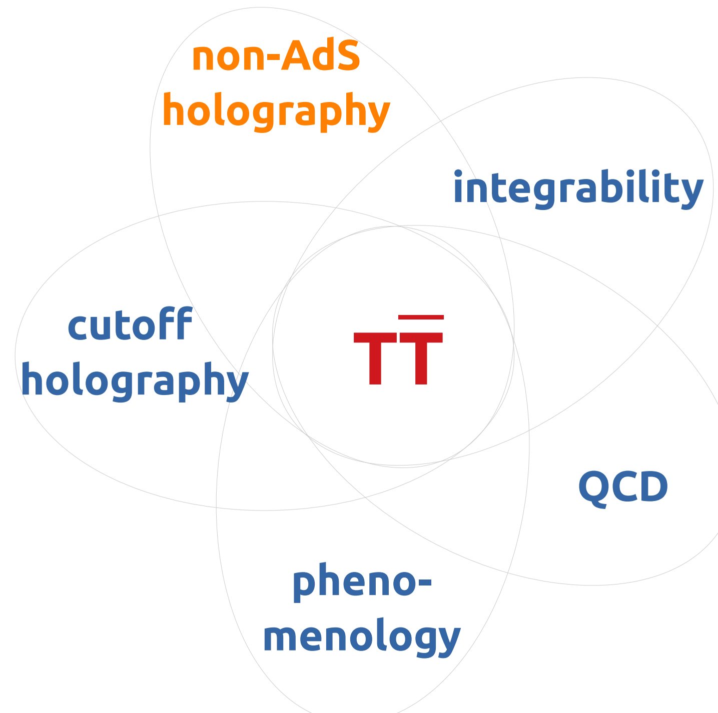

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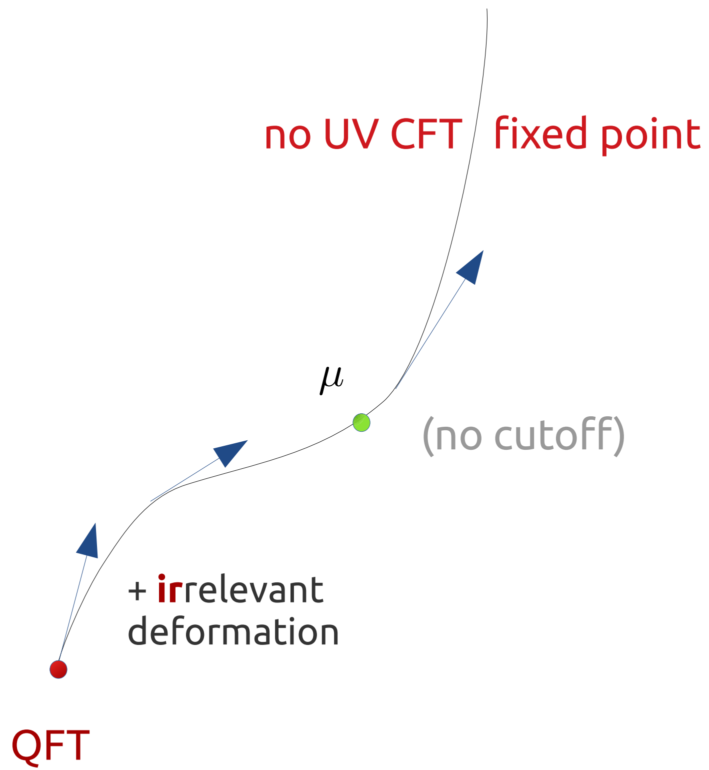
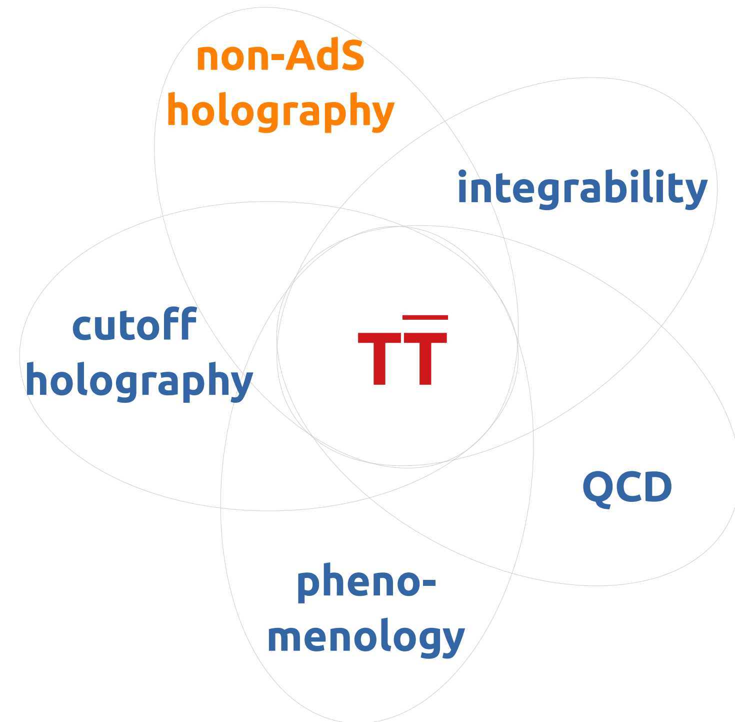
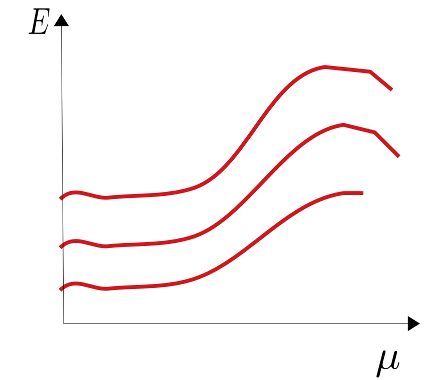




exactly solvable

Exact solvability of $T\bar{T}$/ $J\bar{T}$ deformations

- place SZ-deformed theory on a cylinder (R) & study flow of energies and eigenstates



$$\partial_\mu E_n = \langle n_\mu | \underbrace{\partial_\mu H}_{\mathcal{O}_{J^A J^B}} | n_\mu \rangle \qquad \partial_\mu | n_\mu \rangle = \sum_{m \neq n} \frac{\langle m_\mu | \partial_\mu H | n_\mu \rangle}{E_n^\mu - E_m^\mu} | m_\mu \rangle \equiv \mathcal{X}_{J^A J^B} | n_\mu \rangle$$

Observables:

- deformed finite-size spectrum & thermodyn. $\qquad E_\mu(R) = E_0(R + \mu E_\mu)$
- S-matrix $\qquad S_\mu(p_i) = e^{i\mu \sum_{i<j} \epsilon^{\alpha\beta} p_\alpha^i p_\beta^j} S_0(p_i)$
- correlation functions $\qquad$ "momentum-dependent spectral flow"
- extended symmetries $\qquad$ "Virasoro x Virasoro"

(correlation functions and extended symmetries: seed = CFT)

are simple, universal , non-local deformations

of the original CFT/QFT observables

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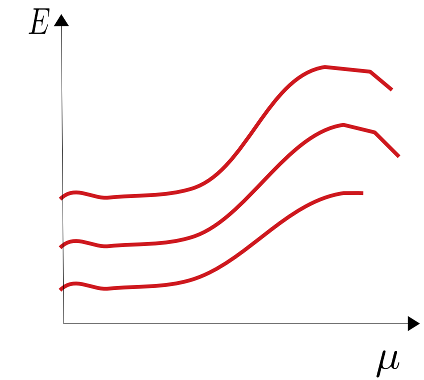

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seed = CFT

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$$\partial_\mu \tilde{L}_m^\mu = [\mathcal{X}_{J^A J^B}, \tilde{L}_m^\mu]$$

$$T\bar{T}: \quad \tilde{L}_m = (R + 2\mu H_R) Q_m$$

$$J\bar{T}: \quad \tilde{L}_m = L_m - \mu J_m H_R + \frac{\mu^2}{4} H_R^2 \delta_{m,0}$$

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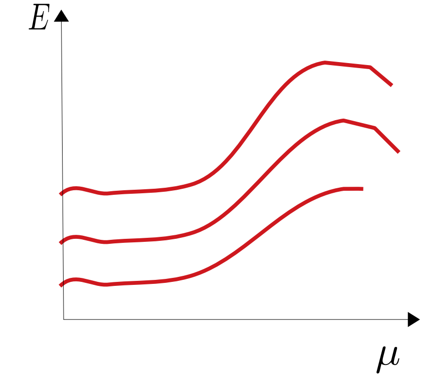

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“momentum-dependent spectral flow”

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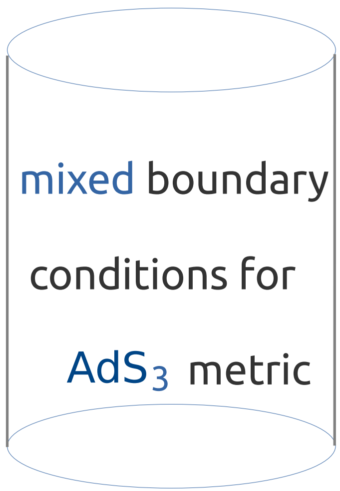
Holography for $T\bar{T}$, $J\bar{T}$ - deformed CFTs

$T\bar{T}$, $J\bar{T}$ deformations : **double trace**

- universal , $\forall$ large c CFT



precision holography : perfect match of bulk/ boundary spectrum & (non-linear) infinite symmetry group ($T\bar{T}$)

Single-trace $T\bar{T}$ /$J\bar{T}$ deformation

- seed symmetric product orbifold CFT $\mathcal{M}^p/S_p$

"single-trace $T\bar{T}$" deformation

$$\sum_{i=1}^{p} T_i\bar{T}_i \Rightarrow (T\bar{T}_{def.}\,\mathcal{M})^p/S_p$$

- similarly solvable   Chakraborty, Georgescu, MG '23
- linked to non-AdS holography
  - s.tr $T\bar{T}$ ~ asympt. flat with linear dilaton
  - s.tr $J\bar{T}$ ~ stringy warped AdS_3

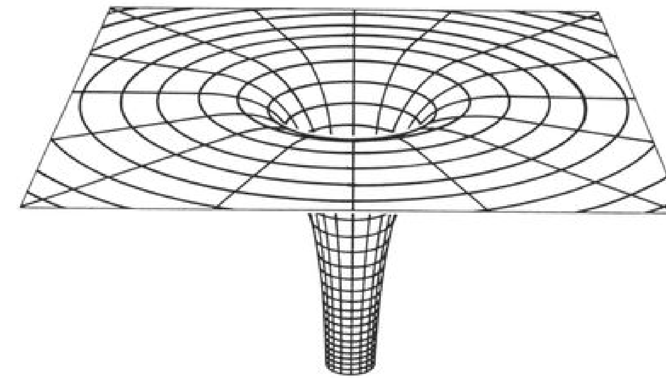
    ~ extremal black holes

Example I : the Kerr/CFT correspondence

The Kerr / CFT correspondence

- extreme Kerr black hole $GM^2 \simeq J$ e.g. GRS 1915+105

$$ds^2 = 2J\,\Omega^2(\theta)\Big[\underbrace{-r^2dt^2 + \frac{dr^2}{r^2}}_{SL(2,\mathbb{R})_L} + \frac{\sin^2\theta}{\Omega^4(\theta)}\underbrace{(d\phi + rdt)^2}_{U(1)_R} + d\theta^2\Big]$$



Near Horizon Extreme Kerr

$\boxed{warped\ AdS_3}$

- Kerr/CFT : $\begin{cases} \text{infinite \# of symmetries} \rightarrow \text{Virasoro} \quad c = 12J \\ \text{Kerr entropy reproduced by Cardy} \quad \approx \frac{1}{2}\ \text{CFT}_2 \end{cases}$

- universality (all extremal black holes have Virasoro + entropy match)

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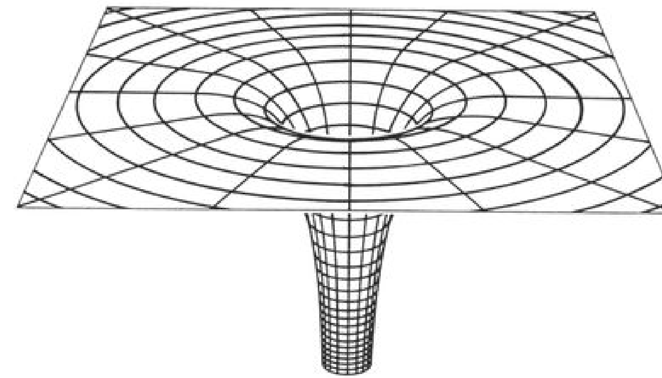

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- universality (all extremal black holes have Virasoro + entropy match)

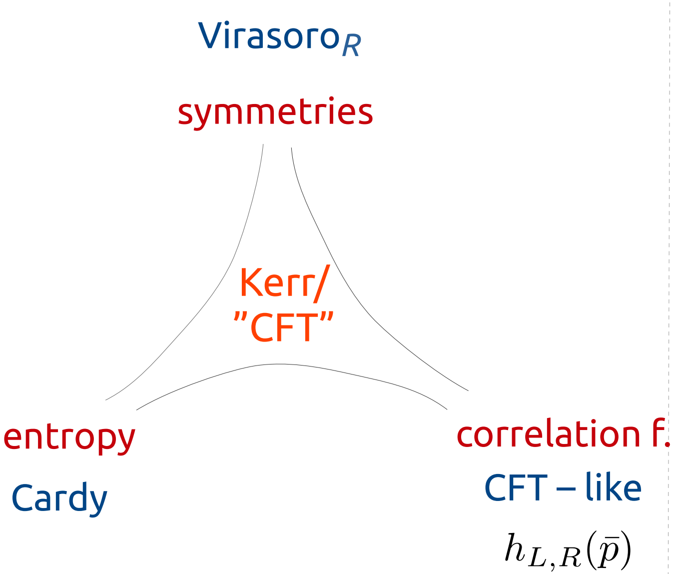
- scattering amplitudes look like momentum-space CFT_2 correlation functions (on both sides), but

Bredberg, Hartman, Song, Strominger '09

with momentum-dependent conformal dimensions $h_L(\bar{p}), h_R(\bar{p})$ **non-local ?!**

Kerr/“CFT” from different viewpoints

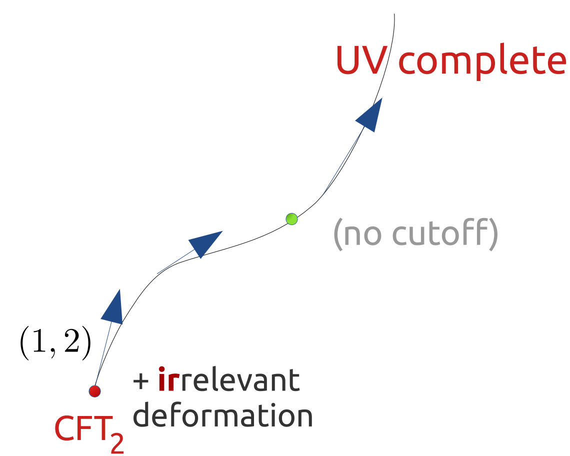
bottom-up



- universal $warped\ AdS_3$
- isometry: $SL(2,\mathbb{R})_L \times U(1)_R$
- ! arbitrariness in ASG

top-down

- $warped\ AdS_3$ can be easily modeled in string theory
- e.g. dipole-deformed D1-D5 + flow to IR → f.th. not tractable



$$\underbrace{SL(2,\mathbb{R})_L}_{\text{local \& conformal}} \times \underbrace{U(1)_R}_{\text{non-local!}}$$

QFT toy model

- $J\bar{T}$ – deformed CFT
- Virasoro x Virasoro symmetry

$$\tilde{L}_n^{\mu} = R\, L_n - \lambda H_R J_n + \frac{\lambda^2 H_R^2}{4} \delta_{n,0}$$

$$\tilde{J}_n^{\mu} = J_n - \frac{\lambda H_R}{2} \delta_{n,0} \quad \text{\& RM}$$

- choose operators $\mathcal{O}(p)$ ~ standard Ward identities w.r.t. L_n
- all their correlation functions are entirely determined by original CFT correlators; 2 & 3pf : CFT with

$$h_{L,R} \to h_{L,R}(\bar{p})$$

Conclusion & next steps

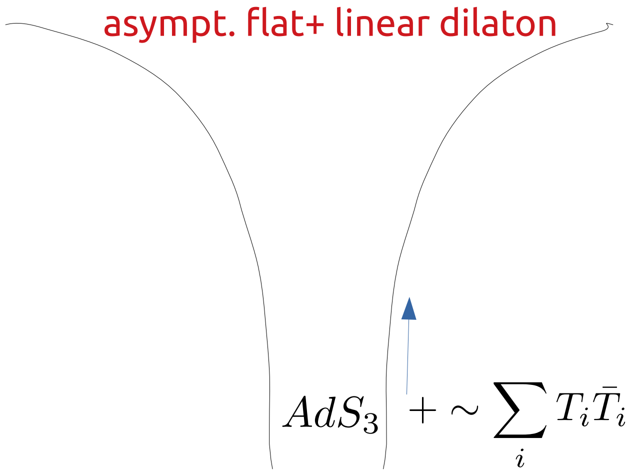
- explicit QFT toy model shows that it **is** possible to have both Virasoro symmetry and non-locality
- action of Virasoro generators is subtly modified & non-locality of correlators highly structured

Next steps :

- revisit symmetries of $J\bar{T}$ – deformed CFTs from spacetime point of view (ASG) & reproduce both sets of symmetry generators
- use this intuition to revisit ASG calculations for warped AdS_3 (in string theory) → Virasoro or subtly $\neq$?
- revisit black hole entropy calculations: Cardy or single-trace $J\bar{T}$ (modified Cardy)?
- do we need to revisit the Kerr/CFT ASG calculations?
- universal properties of field theories dual to $warped\ AdS_3$ ("dipole CFT axioms")?

Example II : $T\bar{T}$ and asymptotically linear dilaton holography

The asymptotic linear dilaton background and $T\bar{T}$



k NS5 and p F1 strings in the NS5 decoupling limit

$g_s \to 0\,, \quad \alpha'$ fixed

p large

UV: Little String Theory $/ K3$

non-gravitational, non-local theory with Hagedorn growth

IR: AdS_3 long strings: described by $(\mathcal{M}_{6k})^p/S_p$ orbifold

short strings: not ~

Interpolating geometry

- dual to CFT source for an irrelevant $(2,2)$ single-trace operator ~ $\sum_{i=1}^{p} T_i \bar{T}_i$

Giveon, Itzhaki, Kutasov '17

- worldsheet σ - model tractable

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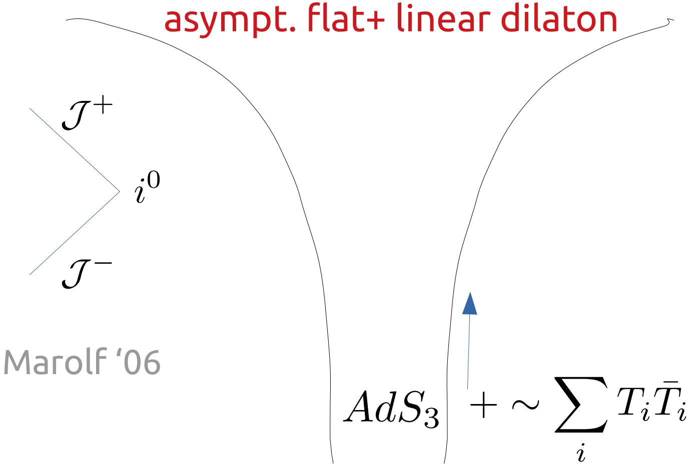

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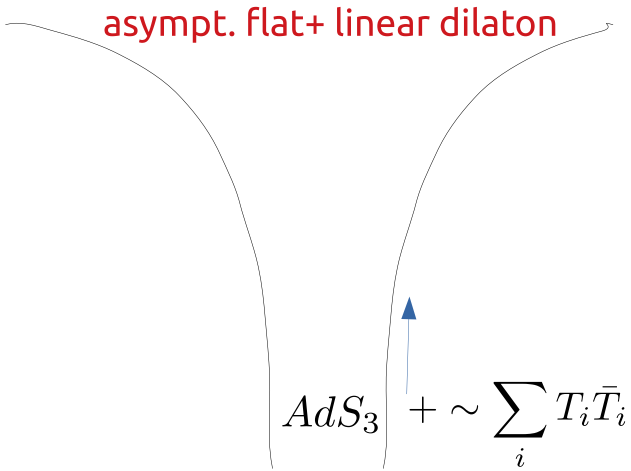

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- long string subsector (only!) well-described by single-trace $T\bar{T}$
- black hole entropy and asymptotic symmetries perfectly match single-trace $T\bar{T}$

(Cardy → Hagedorn) (non-linear modif. Virasoro) Georgescu, MG '22

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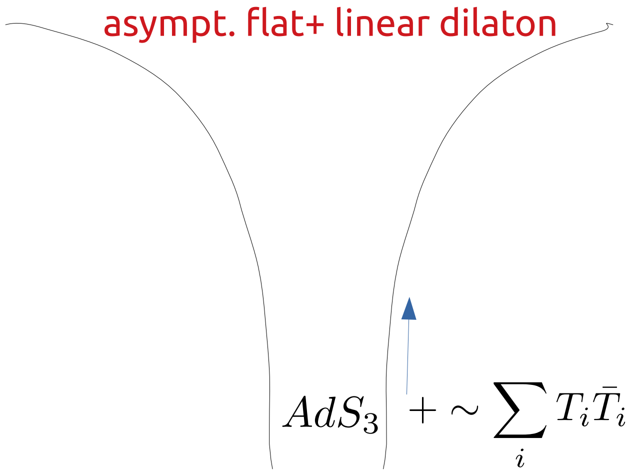

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- long string subsector (only!) well-described by single-trace $T\bar{T}$ ⟵ **not exact $T\bar{T}$**
- black hole entropy and asymptotic symmetries perfectly match single-trace $T\bar{T}$

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Details of the symmetry algebra calculation

- explicitly solve flow equation $\partial_\mu \tilde{L}^\mu_m = [\mathcal{X}_{T\bar{T}}, \tilde{L}^\mu_m]$ in the classical limit MG, Monten, Tsiares '22

- translate result to Lagrangian formalism → field-dependent diffoemorphism $\hat{u} \sim \sigma + t + 2\mu \int^\sigma \mathcal{H}_R$

$$\xi^U = -\left(f(\hat{u}) - \frac{2\mu(\hat{u}-U)}{R_H} Q_{\hat{f}'} \right) \qquad \xi^V = \frac{2\mu}{R_u} \left(\int d\tilde{\sigma} f'(\hat{\tilde{u}}) \tilde{\mathcal{H}}_L [G(\tilde{\sigma}-\sigma) - \Delta \hat{\tilde{u}}] + \frac{Q_{\hat{f}'}}{R_H}(v + 2\mu H_R V) \right)$$

$$U, V = \sigma \pm t$$

- compare with asymptotic allowed diffeos in AdS_3 with mixed bnd. cond. dual to double-trace $T\bar{T}$

$$U \to U + f(u) + \mu \int_{c_{\mathcal{L}_f}}^v \bar{\mathcal{L}} \bar{f}' \qquad V \to V + \bar{f}(v) + \mu \int_{c_{\bar{\mathcal{L}}_{\bar{f}}}}^u \mathcal{L} f'$$

winding! $\propto \oint \mathcal{L} f'_p \to Q_{f'_p}$

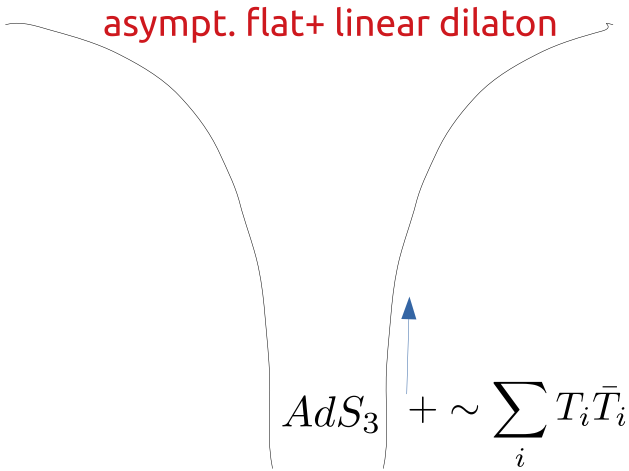
- charge algebra : $Virasoro \times Virasoro$ or non-linear modification, depending on chosen basis

- the asymptotic symmetries of the ALD background, fixed by $\omega(\mathcal{L}_{\xi^{ASG}} \bar{g}, \delta M) = \omega(\mathcal{L}_{\xi^{ASG}} \bar{g}, \delta J) = 0$

  are the same functions of field-dependent coordinates as in the double-trace case

- charge algebra the same up to the order checked (single-trace version) Georgescu, MG '22

The asymptotic linear dilaton background and $T\bar{T}$



k NS5 and p F1 strings in the NS5 decoupling limit

$g_s \to 0\,, \quad \alpha'$ fixed

p large

UV: Little String Theory $/ K3$

non-gravitational, non-local theory with Hagedorn growth

IR: AdS_3 long strings: described by $(\mathcal{M}_{6k})^p / S_p$ orbifold

short strings: not ~

Interpolating geometry

- dual to CFT source for an irrelevant $(2,2)$ single-trace operator $\sim \sum_{i=1}^{p} T_i \bar{T}_i$ Giveon, Itzhaki, Kutasov '17
- long string subsector (only!) well-described by single-trace $T\bar{T}$
- black hole entropy and asymptotic symmetries perfectly match single-trace $T\bar{T}$ ← **$T\bar{T}$ – like !**

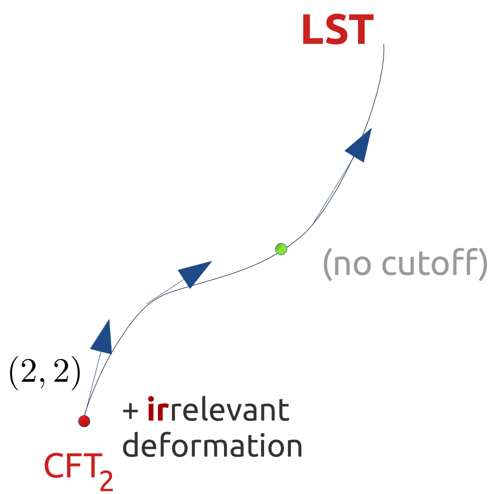
(Cardy → Hagedorn) (non-linear modif. Virasoro) Georgescu, MG '22

Different viewpoints on ALD holographic dual

top-down

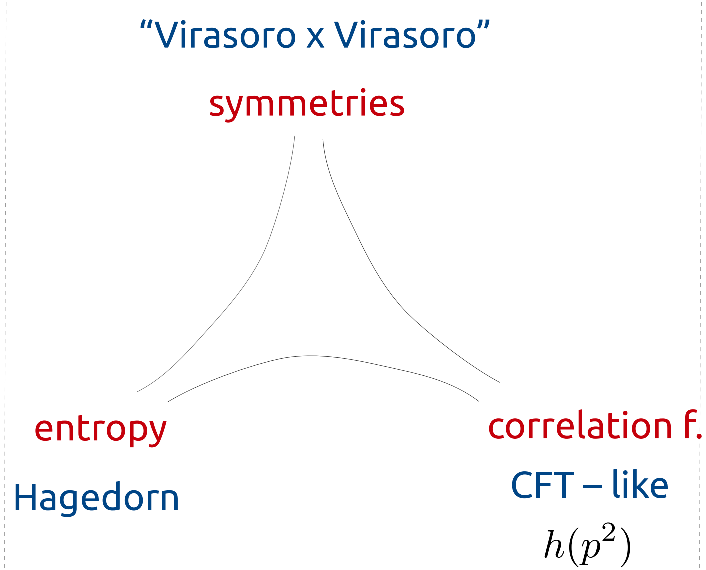
- decoupled bckgnd. interpolates b/w

AdS_3 (IR) & flat w/ linear dilaton (UV)



- existence of decoupled theory (string theory)
- hard to track down (! susy)

bottom-up



QFT toy model

- $T\overline{T}$ – deformed CFT
- Hagedorn entropy
- "Virasoro x Virasoro" symmetry

$$\tilde{L}_n^{\mu} = (R + 2\mu H_R) Q_n$$ & RM

- correlation functions

$$\langle \mathcal{O}(p)\mathcal{O}(-p)\rangle \sim \int d^2\sigma\, e^{ip\sigma} \frac{\#}{\sigma^{2h+\mu p^2}}$$

CFT with $h \to h(p^2) = h + \mu p^2/2$

- similar structure, but not the same

function as the spacetime

universal quantities match (only)

Additional examples

Georgescu, MG, Kovensky '24

- classify all maximally supersymmetric irrelevant deformations of the D1-D5 CFT (22 for K3 comp. sp.)
- in supergravity, they infinitesimally correspond to deformations of $AdS_3\,(\,H_3^+\,)$ char. by 3-form fluxes

$$\mathcal{O}_{H_3^+}\,,\qquad \mathcal{O}_{H_3^-}\,,\mathcal{O}_{F_3^-}\,,\underbrace{\mathcal{O}_{F_3^{-,I}}}_{19}$$

LST

$$F_5 = F_3^{-,I}\wedge\omega_I$$

- full supergravity solutions

$$ds_6^2 = \frac{1}{\sqrt{H_\Lambda H^\Lambda}}(-dt^2+d\sigma^2)+\sqrt{H_\Lambda H^\Lambda}(dr^2+r^2 d\Omega_3^2) \qquad H_\Lambda = \frac{q_\Lambda}{r^2}+c_\Lambda$$

- asymptotics degenerate if $c_\Lambda \eta^{\Lambda\Sigma} c_\Sigma = 0$ (minimum amount of $\mathcal{O}_{H_3^+}$)
- all such backgrounds correspond to known decoupling limits of string theory

open brane LST → NS5 branes in critical RR 2- & 4- form fields

Gopakumar, Minwalla, Seiberg, Strominger '00
Harmark '00

- black hole entropy : Cardy → Hagedorn

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Conclusions

- argued it is useful to approach non-AdS holography from a multifaceted viewpoint

- information about the "QFT structure" of the dual theory, in complement to symmetries, correlators, entropy (← bottom-up) can be quite useful, if not essential

- once structure is understood, QFT toy models can also give insight into observables & their special prop

- generalisations of single-trace TT/ JT – deformed CFTs with the same universal properties?

Thank you !

The primary condition

- main idea: use interplay of the two sets of symmetry generators

$$\begin{cases} \tilde{L}^{\mu}_n = R\, L_n - \lambda H_R J_n + \dfrac{\lambda^2 H_R^2}{4}\delta_{n,0}\,, \qquad \tilde{J}^{\mu}_n = J_n - \dfrac{\lambda H_R}{2}\delta_{n,0} \\ \tilde{\bar{L}}^{\mu}_n = R_v \bar{L}_n - \lambda : H_R \bar{J}_n : + \dfrac{\lambda^2 H_R^2}{4}\delta_{n,0}\,, \qquad \tilde{\bar{J}}^{\mu}_n = \bar{J}_n - \dfrac{\lambda H_R}{2}\delta_{n,0} \end{cases}$$

assumed

full quantum

- algebra LM (L_n, J_n) : Virasoro-Kac-Moody ; algebra RM $(\bar{L}_n, \bar{J}_n)$: non-linear modification of Vir.-KM

- LM: operators should be primary w.r.t. L_n, J_n $\leftarrow$ implement conformal & affine U(1) transf.

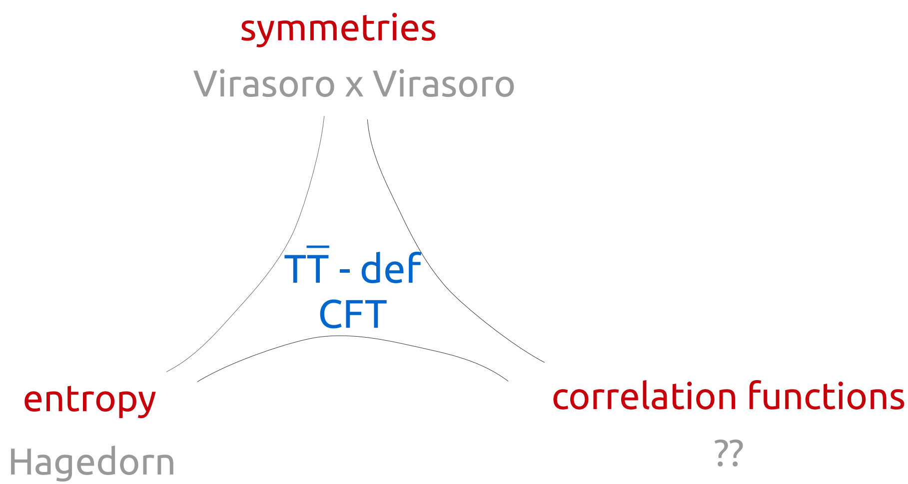
Ward id: $$[L_n, \mathcal{O}(w)] = e^{nw}(nh\mathcal{O} + \partial_w \mathcal{O})_{n \geq -1} \quad \text{w/} \quad h = \tilde{h} + \lambda \bar{p}\tilde{q} + \frac{\lambda^2 \bar{p}^2}{4}$$

- introduce auxiliary ops. $\tilde{\mathcal{O}}(w,\bar{w})$ defined via $\partial_\lambda \tilde{\mathcal{O}}(w,\bar{w}) = [\mathcal{X}_{J\bar{T}}, \tilde{\mathcal{O}}(w,\bar{w})]$ $\leftarrow$ identical correlation functions and Ward identities w.r.t. $\tilde{L}_n$ etc., as the operators in the undeformed CFT

$$\boxed{\mathcal{O}(w,-) = e^{Aw} e^{\lambda \bar{p} \sum_{n=1}^{\infty} e^{nw} \tilde{J}_{-n}} \tilde{\mathcal{O}}(w,-) e^{-\lambda \bar{p} \sum_{n=1}^{\infty} e^{-nw} \tilde{J}_n} \times RM}$$

Conclusions

- we have shown that, despite their non-locality, TT – deformed CFTs posess infinite symmetries
- various perspectives: abstract QM, classical Hamiltonian, Lagrangian, holographic + single-trace
- we have shown that the asymptotic symmetries of the asymptotically linear dilaton background in string theory are precisely those of single-trace $T\bar{T}$ – deformed CFTs
- this further suggests the relevant "QFT structure" for these bckgnds is closely related to that of



- a better understanding of both field theory and gravity (both doable!) may pave the way for precision holography in this background

Setup

- as we anticipate field-dependent symmetries → turn on non-trivial background to see this

- we thus consider the asymptotically linear dilaton black hole backgrounds

- $ALD \times S^3 \times T^4$ → use consistent truncation to 3d $\quad ds^2 = ds_3^2 + \ell^2 ds_{S_3}^2 \,, \qquad H = 2\ell^2\, \omega_{S^3} + b\, e^{2\phi}\, \omega_3$

$$d\bar{s}^2 = \frac{r^2}{\alpha' r^4 + \beta r^2 + \alpha' L_u L_v}\left(r^2 dU dV + L_u dU^2 + L_v dV^2 + \frac{L_u L_v}{r^2} dU dV\right) + k\frac{dr^2}{r^2}$$

$$e^{2\bar{\phi}} = \frac{k r^2}{\alpha' r^4 + \beta r^2 + \alpha' L_u L_v} \qquad \beta = \sqrt{p^2 + 4\alpha'^2 L_u L_v}$$

- classified linearized perturbations of this background: pure diffeos + propagating

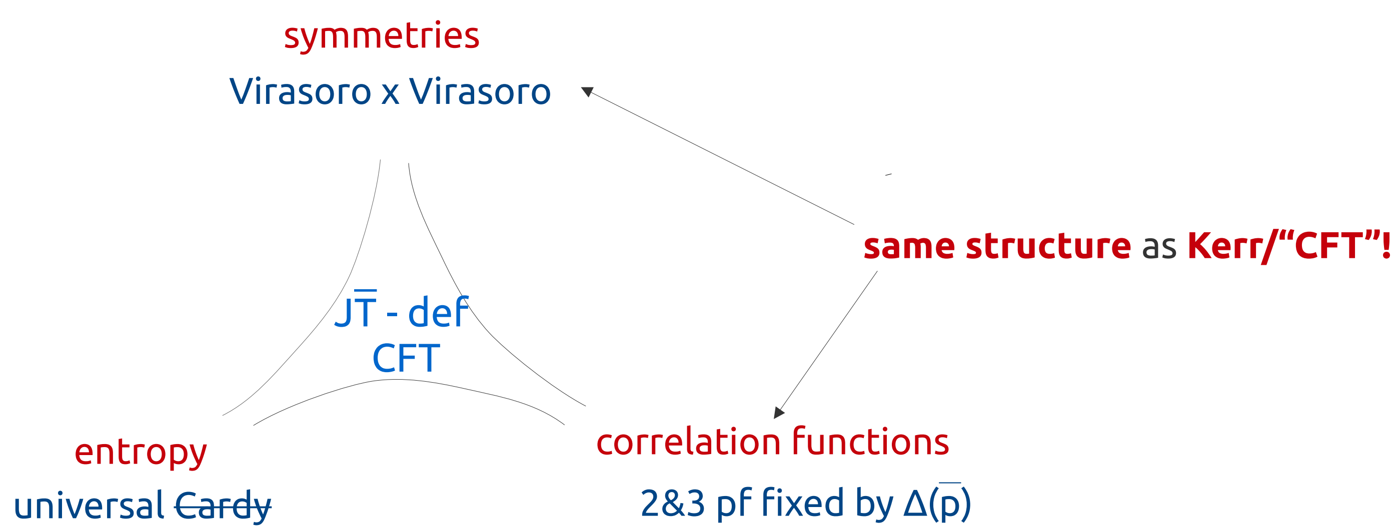
- allowed diffeos : their symplectic form with the allowed modes, notably $\delta L_{u,v}$ must vanish

  → charge conservation

The "QFT structure" of solvable irrelevant deformations

- study "QFT structure" explicitly for Smirnov-Zamolodchikov deformations ($T\bar{T}$, $J\bar{T}$) ← exactly solvable irrelevant def's

- e.g. for $J\bar{T}$ – deformed CFTs $\overbrace{SL(2,\mathbb{R})_L}^{\text{local \& conformal}} \times \overbrace{U(1)_R}^{\text{non-local}}$ (`half' non-local)

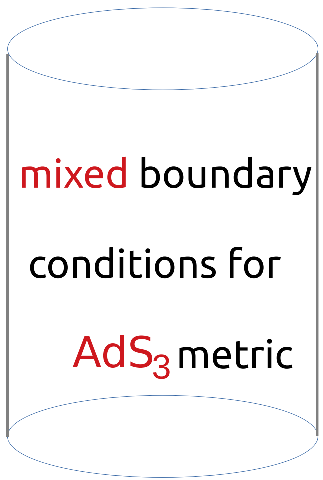


- ∃ non-local analogues of primary operators whose correlators are entirely determined by seed CFT

Holographic dual of $T\bar{T}$ - deformed CFTs

- $T\bar{T}$ deformation : **double trace**
- seed CFT : large c, large gap

  → Einstein gravity + low-lying matter fields



$$g^{(0)}_{\alpha\beta} - \mu g^{(2)}_{\alpha\beta} + \frac{\mu^2}{4} g^{(2)}_{\alpha\gamma} g^{(0)\gamma\delta} g^{(2)}_{\delta\beta} \;=\; \text{fixed}$$

MG, Monten '19

pure gravity $\approx$ Dirichlet at $\rho = -\mu$

McGough, Mezei & Verlinde '16

- holographic dictionary derived from field theory using Hubbard-Stratonovich trick
- 1st instance of mixed bnd. cond. on AdS_3 metric

  → bulk & boundary have independent definitions

  → contrast standard situation where properties of the boundary theory are inferred from the bulk
- change bnd. conditions on AdS_3 metric → radical modification of the bnd. theory: local → non-local
- precision holography

  → perfect match of bulk/boundary spectrum ✓

  → symmetries ✓

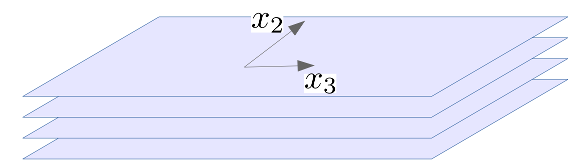
  → other observables?

Example 1: Non-commutative N=4 SYM

T-duality, shift, T-duality

- D3 branes in a spatial B - field ← induced by **TsT** + decoupling

$$\alpha' \to 0$$
$$B \to \infty$$



- ➡ non-commutative $\mathcal{N}=4$ SYM $\;\cdot \to \star\;$ Moyal star product

$$[x^i, x^j] = i\theta^{ij}$$

$$f(x) \star g(x) = e^{\frac{i}{2}\theta^{ij}\frac{\partial}{\partial \xi^i}\frac{\partial}{\partial \zeta^j}}\, f(x+\xi) g(x+\zeta)|_{\xi=\zeta=0}$$

Seiberg, Witten '99

- planar diagrams: same as in $\mathcal{N}=4$ SYM up to phase factors involving the external momenta

  → free energy (thermodynamics) same as in $\mathcal{N}=4$ SYM

Filk '96

- field redefinition NC $\mathcal{N}=4$ SYM → ordinary $\mathcal{N}=4$ SYM + infinite # of irrelevant operators

  with finely - tuned coefficients $\sim \theta^n \mathcal{O}_{4+2n}$

  → UV – completeness, properties of diagrams and thermodynamics mysterious in this picture

Holographic dual

- dual background: obtained via TsT + decoupling

Maldacena, Russo '99

$$ds^2 = gN\alpha' \left[r^2(-dt^2 + dx_1^2) + \frac{r^2}{1+b^2r^4}(dx_2^2 + dx_3^2) + \frac{dr^2}{r^2} + d\Omega_5^2 \right] \qquad e^{2\phi} = \frac{g^2}{1+b^2r^4}$$

- interpolates between $AdS_5 \times S^5$ in the IR → funny asymptotics in the UV (NC SYM)

  → leading irrelevant deformation: dim 6

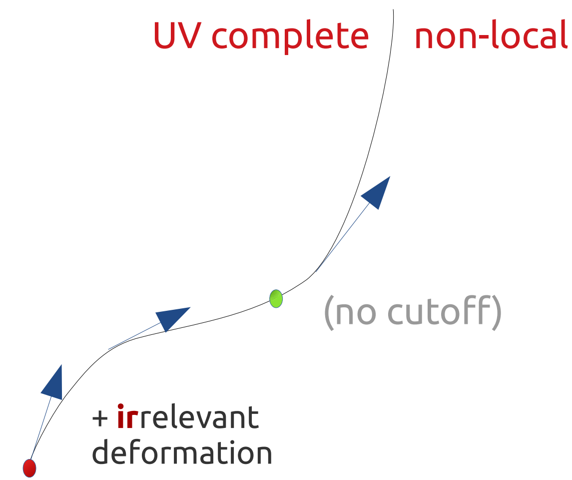
  → can argue for UV completeness from decoupling limit

- thermodynamics same as $\mathcal{N} = 4$

- correlation functions : non-locality → momentum-space

  → gauge-invariant operators : "open Wilson lines" $\Delta x^\mu = \theta^{\mu\nu} k_\nu$

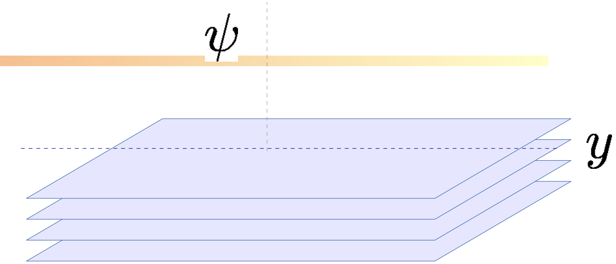
  → match between field theory and gravity



Gross, Hashimoto, Itzhaki '00

Rozali & van Raamsdonk '00

Example 2: Dipole-deformed N=4 SYM



- **TsT** along || and $\perp$ direction to the branes

- star-product deformation of dual field theory : dipole star product

Bergman, Ganor '00

$$\Phi \;\rightarrow\; L^\mu_\Phi = q_\Phi \lambda^\mu \qquad\qquad (\Phi_1 \star \Phi_2)(x^\mu) = \Phi_1(x^\mu - \frac{1}{2}L^\mu_2)\,\Phi_2(x^\mu + \frac{1}{2}L^\mu_1)$$

non-local

dipole length

- planar diagrams unaffected up to a phase → large N free energy unaffected

- Seiberg-Witten map: dipole theory = $\mathcal{N}=4\ \ SYM$ + infinite number of irrelevant ~~Lorentz~~ operators

- dual gravity background: TsT of original background + decoupling limit

Applications

- holographic modeling of strongly-coupled systems with non-relativistic conformal invariance, a.k.a.

  AdS/cold atom correspondence

  D.T. Son' 08

  Adams & Balasubramanian '08

$$ds^2 = -\lambda^2 r^4 (dx^+)^2 + r^2 \left(dx^+ dx^- + \sum_{i=1}^{d-2} dx_i^2 \right) + \frac{dr^2}{r^2}$$

$$x^+ \to c^2\, x^+ \,, \quad x^- \to x^-$$

$$x^i \to c\, x^i$$

- Schrödinger$_{d+1}$ backgrounds $\Longleftrightarrow$ NR CFT$_{d-1}$ codimension 2 holography (symmetries)

- however, certain Schrödinger$_5 \times S^5$ backgrounds $\Longleftrightarrow$ null dipole–deformed $\mathcal{N}=4$ SYM $\lambda^\mu || \hat{x}^-$

  → $\mathcal{N}=4$ + infinite # of Schrödinger-invariant irrelevant operators $\left\{ \begin{array}{l} \text{non-local along } x^- \\ \text{local and Schröd.-invar. along } x^+, x^i \end{array} \right.$

- NR CFT : compactify x^- DLCQ of null dipole theory (additional reduction along non-local direction

  Maldacena, Martelli, Tachikawa '08

- this type of NR CFT has a very special structure

Holographic perspective

MG, Monten '19

- holographic dual to double-trace $T\bar{T}$ – deformed CFTs: AdS_3 with mixed bnd. cond.

$$g_{\alpha\beta}(\rho, x^\alpha) = \frac{g^{(0)}_{\alpha\beta}}{\rho} + g^{(2)}_{\alpha\beta} + \ldots \qquad\qquad g^{(0)}_{\alpha\beta} - \mu g^{(2)}_{\alpha\beta} + \frac{\mu^2}{4} g^{(2)}_{\alpha\gamma} g^{(0)\gamma\delta} g^{(2)}_{\delta\beta} \;=\; \text{fixed} \;=\; \eta_{\alpha\beta}$$

- most general allowed backgrounds parametrized by two functions $\mathcal{L}(u), \bar{\mathcal{L}}(v)$

where the field-dependent coordinates u, v :=
$$\begin{cases} U = u - \mu \int^v \bar{\mathcal{L}}(v')\, dv' \\ V = v - \mu \int^u \mathcal{L}(u')\, du' \end{cases}$$

zero modes
$c_{\mathcal{L}}, c_{\bar{\mathcal{L}}}$

- large diffeomorphisms that preserve the mixed bnd. conditions $\longleftrightarrow$ symmetries of dual field theory

- under these diffeomorphisms, the $T\bar{T}$ coordinates change as

$$\xi^\rho = \rho(f'(u) + \bar{f}'(v))$$

$$U \to U + f(u) + \mu \int^v \bar{\mathcal{L}} \bar{f}' \qquad\qquad V \to V + \bar{f}(v) + \mu \int^u \mathcal{L} f'$$

$c_{\mathcal{L}_f}$ $\qquad$ $c_{\bar{\mathcal{L}}_{\bar{f}}}$

winding! $\propto \oint \mathcal{L} f'_p \to Q_{f'_p}$

- these symmetries are identical to the result of the Lagrangian analysis

MG, Georgescu '22

- charge algebra : $Virasoro \times Virasoro$ or non-linear modification, depending on chosen basis

Smirnov-Zamolodchikov deformations

- irrelevant deformations of 2d QFTs → bilinears of two (higher spin) conserved currents $J^A,\ J^B$

- define

Zamolodchikov '04

SZ '16

$$\mathcal{O}_{J^A J^B}\ :\quad \boxed{\lim_{y\to x} \epsilon^{\alpha\beta} J^A_\alpha(x) J^B_\beta(y) = \mathcal{O}_{J^A J^B} +}\ \text{derivative terms}$$

nice factorization properties

- deformation :

$$\boxed{\frac{\partial S(\mu)}{\partial \mu} = \int d^2x\, \mathcal{O}_{J^A J^B}(\mu)}$$

- examples:

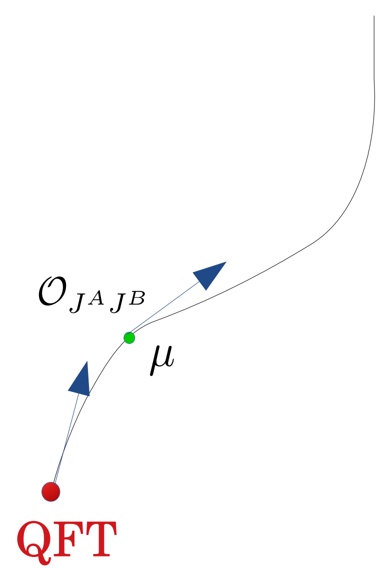
$$T\bar{T}:\quad J^A_\alpha = T_\alpha{}^A\,,\quad J^B_\beta = T_\beta{}^B\ \ (\times \epsilon_{AB}) \qquad (2,2)$$

universal

$$J\bar{T}:\quad J^A_\alpha = J_\alpha\,,\quad J^B_\beta = T_{\beta\bar{z}} \qquad (1,2)$$

~~Lorentz~~

$$\underbrace{SL(2,\mathbb{R})_L}_{\text{local \& conformal}} \times \underbrace{U(1)_R}_{\text{non-local!}}$$



- highly tractable : exact finite -size spectrum, S-matrix, preserves integrability

- deformed theory non-local (scale $\mu^{\#}$) but argued UV complete

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universal

$J\bar{T}:\quad J^A_\alpha = J_\alpha\ ,\quad J^B_\beta = T_{\beta\bar{z}}$ ~~Lorentz~~ (1,2)

same as extremal black holes ← $\boxed{\underbrace{SL(2,\mathbb{R})_L}_{\text{local \& conformal}} \times \underbrace{U(1)_R}_{\text{non-local!}}}$

- highly tractable : exact finite -size spectrum, S-matrix, preserves integrability
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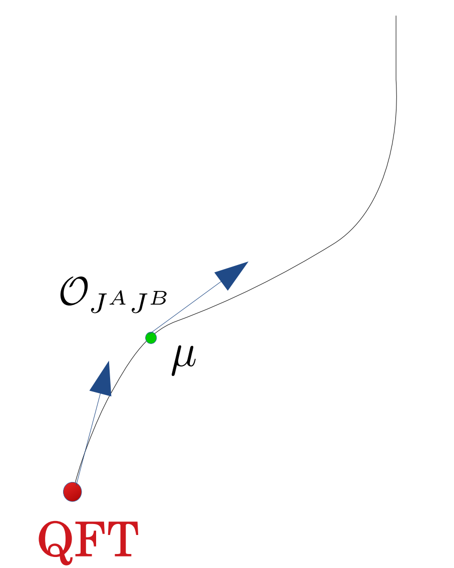

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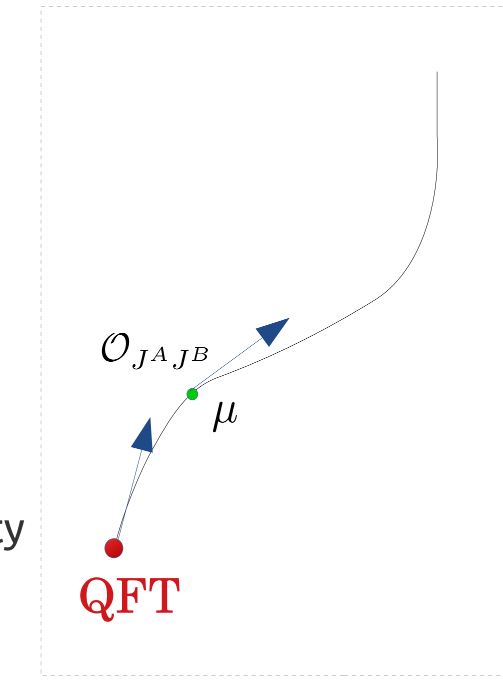


The "QFT properties'' of TT - deformed CFTs

$$\partial_\mu |n_\mu\rangle = \mathcal{X}_{T\bar{T}} |n_\mu\rangle \qquad \partial_\mu \tilde{L}^\mu_m = [\mathcal{X}_{T\bar{T}}, \tilde{L}^\mu_m]$$ conserved

MG '21

symmetries

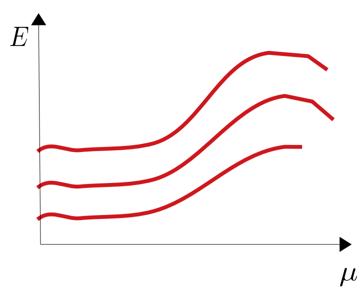
Virasoro x Virasoro

$T\bar{T}$ - def
CFT

entropy

Hagedorn

correlation functions



$$\langle \mathcal{O}(q)\mathcal{O}(-q)\rangle \sim \int d^2\sigma\, e^{iq\sigma} \frac{\#}{\sigma^{2\Delta+\mu q^2}}$$

Aharony, Barel '23

$$S(E) = S_{Cardy}(E_0) = \#\sqrt{c(E+\mu E^2)}$$

The “QFT properties’’ of TT – deformed CFTs

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non-standard

symmetries

Virasoro x Virasoro

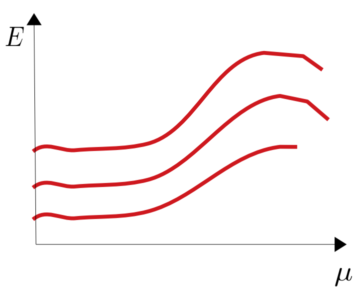
despite non-locality

$T\bar{T}$ - def
CFT

entropy

Hagedorn

correlation functions



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conserved

field-dependent coordinate

$$\tilde{L}^{cls}_m = (R + 2\mu H_R) \int d\sigma\, f(u) \mathcal{H}_L$$

rescaled

MG, Monten, Tsiares ‘22

symmetries

Virasoro x Virasoro

$T\bar{T}$ - def CFT

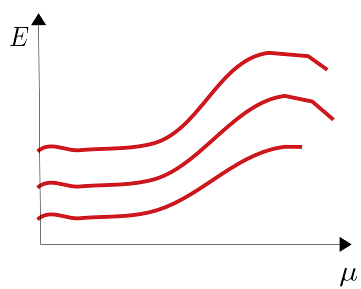
entropy

Hagedorn

correlation functions

$$\langle \mathcal{O}(q) \mathcal{O}(-q) \rangle \sim \int d^2\sigma\, e^{iq\sigma} \frac{\#}{\sigma^{2\Delta + \mu q^2}}$$

Aharony, Barel ‘23



$$S(E) = S_{Cardy}(E_0) = \# \sqrt{c(E + \mu E^2)}$$

The "QFT properties" of $J\bar{T}$ – deformed CFTs

- $J\bar{T}$ – deformed CFTs are dipole CFTs $\quad \overbrace{SL(2,\mathbb{R})_L}^{\text{local \& conformal}} \times \overbrace{U(1)_R}^{\text{non-local}}$

symmetries

Virasoro x Virasoro

non-standard

Bzowski, MG '18; MG, Monten '20, MG'20, MG'21

$J\bar{T}$ - def CFT

entropy

universal ~~Cardy~~

MG'17, Chakraborty, Giveon, Kutasov '18

correlation functions

CFT-like with $\Delta \to \Delta(\bar{p})$

MG '19, MG '21

- left: flowed Virasoro explicitly different from generator of left conformal transformations

- $\exists$ non-local analogues of primary operators whose correlators are entirely determined by seed CFT

Holographic interpretation

- in AdS/CFT parlance, the Smirnov-Zamolodchikov deformations are double-trace → mixed boundary conditions for the dual fields

  $T\bar{T}$: mixed boundary conditions on the asymptotic metric (FG coefficients)

$$\gamma_{\alpha\beta}(\mu) = g^{(0)}_{\alpha\beta} - \mu g^{(2)}_{\alpha\beta} + \frac{\mu^2}{4} g^{(2)}_{\alpha\gamma} g^{(0)\gamma\delta} g^{(2)}_{\delta\beta} = fixed$$

pure gravity ~ Dirichlet at

$\rho = -\mu$

  $J\bar{T}$: mixed boundary conditions b/w the asymptotic metric and U(1) Chern-Simons gauge field

  ~ Compere-Song-Strominger bnd. cond. in metric sector, but ASG has different interpretation

- 1st instance of mixed bnd. cond. on AdS_3 metric → bulk & boundary have independent definitions

  → precision check of the holographic dictionary

- change bnd. conditions on AdS_3 metric → radical modification of the bnd. theory: local → non-local

- $T\bar{T}$, $J\bar{T}$ $\not\to$ non-AdS geometry b/c they are double-trace → need single-trace irrelevant deformations

Single-trace $T\bar{T}$ / $J\bar{T}$ - deformed CFTs

- AdS_3 /CFT_2 gauge group: S_p (permutations) → consider symmetric product orbifold CFTs $\mathcal{M}^p/S_p$
- standard $T\bar{T}$: double-trace $\sum_i T_i \sum_j \bar{T}_j$
- seed $\mathcal{M}^p/S_p$: single-trace $T\bar{T}$ deformation $\sum_{i=1}^p T_i\bar{T}_i \quad \Rightarrow \quad (T\bar{T}_{def.}\mathcal{M})^p/S_p$
- exact partition function, spectrum, thermodynamics, correlation functions

Apolo, Song ' 23
Chakraborty, Georgescu, MG '23

- can also show Virasoro & fractional Virasoro generators survive, as well as the flowed KdV charges
- the non-linear algebra of the unrescaled symmetry generators is (untwisted sector)

$$[Q_m, Q_n] = (m-n)\sum_i \frac{Q^i_{m+n}}{R+2\mu H^i_R} + (m-n)\sum_i \frac{4\mu^2 H^i_R Q^i_m Q^i_n}{R^i_u R^i_H} + \frac{c}{12}m(m^2-1)\sum_i \frac{1}{(R^i_u)^2}$$

$$R^i_u = R + 2\mu H^i_R$$

$$R^i_H = R + 2\mu H^i$$

- same as double-trace algebra, but with $\mu \to \mu/p$ inside expectation values
- dual to a stringy background

Status of the correspondence

"weak form" : the long string sector of string theory on this background ⟺ single-trace $T\bar{T}$

- spectrum of long string excitations exactly matches single-trace $T\bar{T}$ spectrum ✓ GIK '17
- correlation functions of long string vertex operators match $T\bar{T}$ answer (w=1) Cui, Shu, Song, Wang '23
- spectrum of deformed discrete states & correl. functions do not match

"stronger form" : UV theory shares certain universal features with single-trace $T\bar{T}$ – deformed CFTs

- black hole entropy $S(E)$ agrees with $T\bar{T}$ entropy (Cardy → Hagedorn) ✓ GIK '17
- the asymptotic symmetries of the ALD background are identical to those of single-trace $T\bar{T}$

  → same non-linear modification of Virasoro algebra in Fourier basis
- bnd. conditions on allowed diffeos dictated by black hole solutions $\omega(\mathcal{L}_{\xi^{ASG}}, \delta M) = \omega(\mathcal{L}_{\xi^{ASG}}, \delta J) = 0$