Logarithmic Corrections in AdS/CFT

based on 2312.08909 with

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GenHET Meeting in String Theory

CERN

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Black Holes and Entropy Counting

- **Ultimate goal:** full quantum black hole entropy

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- entropy can depend on various continuous parameters of the theory/background

Entropy counting with some guidance...

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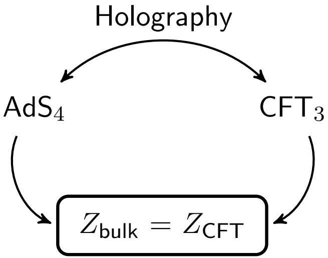

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  - learn about certain aspects of quantum gravity
  - develop tools/tricks of the trade

Holographic Gauge Field Theories

- large N limit of SCFT_3

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Operator $\mathcal{Q}$

find the entire spectrum

extract part that gives logs via the heat kernel

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- consider spectrum of massless fields + inf massive KK modes

- log piece comes **only from one part** of the 1-loop

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- from now on we focus on $AdS_4 \times S^7$

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- do we find a match? let's check...

KK supergravity on S^7

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Conclusion/Outlook

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