

Unification of Decoupling Limits in String and M-Theory

GenHET meeting in String Theory

CERN, April 30, 2024

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THE VELUX FOUNDATIONS
VILLUM FONDEN × VELUX FONDEN

based on work:

240y.xxxx, 2311.10564 (PRL) (Blair,Lahnsteiner,NO,Yan)

2309.14467 (JHEP) (Bidussi,Harmark,Hartong,NO,Oling)

2107.006542 (JHEP) (Bidussi,Harmark,Hartong,NO,Oling)

& earlier papers with Harmark,Hartong, and Menculini/Oling/Yan

Introduction/Motivation

- NRST as a simpler corner of (relativistic) ST

Gomis,Ooguri(2000),Danielsson,Guijosa,Kruczenski(2000)

Harmark,Hartong,NO(2017)/Bergshoeff,Gomis,Yan(2018) /& many papers since

- related corners (decoupled theories) via U-duality
 - simplification by removing part of spectrum
 - access to non-perturbative regime ?

e.g. AdS/CFT correspondence,

BFSS matrix theory conjecture (recent revival)

This talk:

- unification of decoupling (near-BPS) limits in ST/M-theory
- novel worldvolume actions/non-Lorentzian target spacetimes

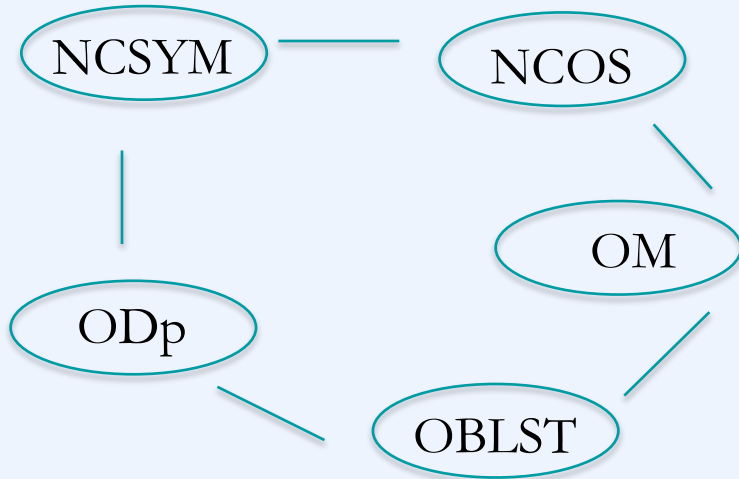
Blair,Lahnsteiner,NO,Yan (2023)

Bidussi,Harmark,Hartong,NO,Oling (2023), Gomis,Yan (2023)

Duality web of 'non-Lorentzian' string theories?

web of decoupled **non-gravitational theories**

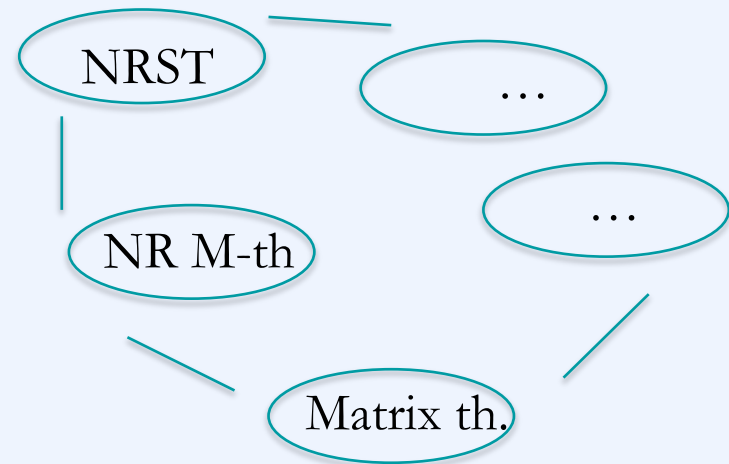
(`open string sector')



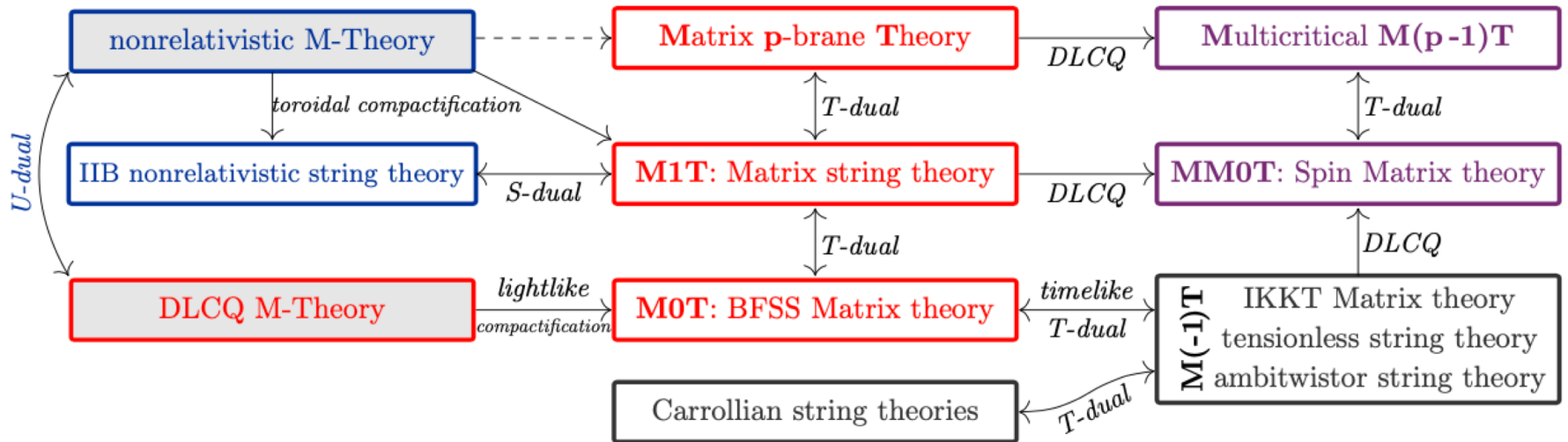
back to 2000s...

web of (non-Lorentzian) gravitational string theories

(`closed string sector')



Web of decoupling limits



obtained from insights using

- non-relativistic string theory on curved spacetimes
- applying string theory U-dualities (including light-like compactifications)
- anatomy of decoupling limits from BPS states

each node is its 'own' decoupled theory including actions for:

- fundamental (light) degrees of freedom
 - coupling to an appropriate (non-Lorentzian) target spacetime
- other (heavy) probe objects in the theory

further lessons

- near-BPS limits can be viewed as natural generalizations of non-relativistic point particle limit
- novel perspective on Matrix theory via non-Lorentzian backgrounds
cf. recent revival [Herderschee, Maldacena \(2023\)/Komatsu et al \(2024\)](#)
- new corners of the duality maze (via solution generating techniques):
IKKT, Carrollian string theory,..
- multi-critical limits & relation to spin-Matrix theory
- limits and geometries can also be understood from novel IW
contractions of the M-theory superalgebra

Outline

- anatomy of near-BPS limits from BPS mass formulae
- illustration of the duality web via example nodes
 - NR particle from BPS particle and torsional Newton-Cartan
 - BFSS/Matrix theory & Matrix 0-brane theory (M0T) and MpT
 - NR string theory and M-theory
- generalizations:
 - IKKT & Carrollian string theories
 - multi-critical string theories/M-theory & spin Matrix theory
- outlook

Anatomy of near-BPS limits

Limits are natural generalizations of non-relativistic point particle limit:

$$E = \sqrt{(mc^2)^2 + (pc)^2} - \Phi q = \sqrt{(mc^2)^2 + (pc)^2} - 1 \cdot mc^2 \simeq \frac{p^2}{2m}$$

general BPS bound states same pattern (DBI + WZ):

BPS & critical field

$$E = \sqrt{M_1^2 + M^2} - A_1 M_1 \simeq \frac{M^2}{2M_1}$$

$$M_1 \rightarrow \omega^2, M \rightarrow \omega, \omega \rightarrow \infty, A_1 \rightarrow 1$$

DLCQ orbit: scale a 1/2 BPS state (M1) to infinity
and cancel divergence with critical electric field

$$E = \sqrt{(M_1 + M_2)^2 + M^2} - (A_1 M_1 + A_2 M_2) \simeq \frac{M^2}{2(M_1 + M_2)}$$

$$M_1, M_2 \rightarrow \omega^2, M \rightarrow \omega, A_1, A_2 \rightarrow 1$$

DLCQ² orbit: scale a combination of 1/4 BPS bound state (M1+M2)
and cancel divergence with multi critical electric field

Limit of charged point particle action

action of charged particle in d dimensions

$$S = -m \int d\tau \sqrt{-\eta_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} + q \int d\tau A_\mu \dot{x}^\mu$$

$$\eta_{\mu\nu} \rightarrow (-c^2, \delta_{ij})$$

NR limit: BPS particle $q=m$ & near-critical gauge field, cancels divergence

$$S = \frac{m}{2} \int d\tau \dot{x}^i \dot{x}^i$$

- important example of BPS particles: KK states

NR limit can be incorporated naturally in form of higher-dim geometry

$$ds^2 = \frac{R^2}{c^2} \left(dy + \frac{c^2}{R} dx^0 \right)^2 - c^2 (dx^0)^2 + dx^i dx^i$$

in large c limit: compact direction becomes lightlike (DLCQ)

→ DLCQ of M-theory via closely related limit

Curved space generalization

action of **charged relativistic particle in curved spacetime**

$$S = -mc \int \sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} d\lambda + q \int A_\mu \dot{x}^\mu d\lambda$$

- time-space split in metric: $g_{\mu\nu} = -c^2 T_\mu T_\nu + h_{\mu\nu}$

expand for large c:

$$S = -mc^2 \int \left[T_\mu - \frac{q}{mc^2} A_\mu \right] \dot{x}^\mu d\lambda + \frac{m}{2} \int \frac{h_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}{T_\rho \dot{x}^\rho} d\lambda + \mathcal{O}(c^{-2})$$

extremal particle

$$q = mc^2.$$

divergent term. cancels
with:

$$T_\mu = \tau_\mu + \frac{1}{2c^2} m_\mu,$$
$$A_\mu = \tau_\mu - \frac{1}{2c^2} m_\mu,$$

type II string theory: Apply this to D0 brane = BPS particle

-> *precisely* the D0-brane decoupling limit leading to Matrix Theory

NR particles couple to torsional Newton-Cartan

- action after limit:

$$S = \frac{m}{2} \int \frac{h_{\mu\nu} \dot{X}^\mu \dot{X}^\nu}{\tau_\rho \dot{X}^\rho} d\lambda - m \int m_\mu \dot{X}^\mu d\lambda$$

kinetic term

potential term:

coupling to

m_μ

$m_0 \sim$ Newtonian potential

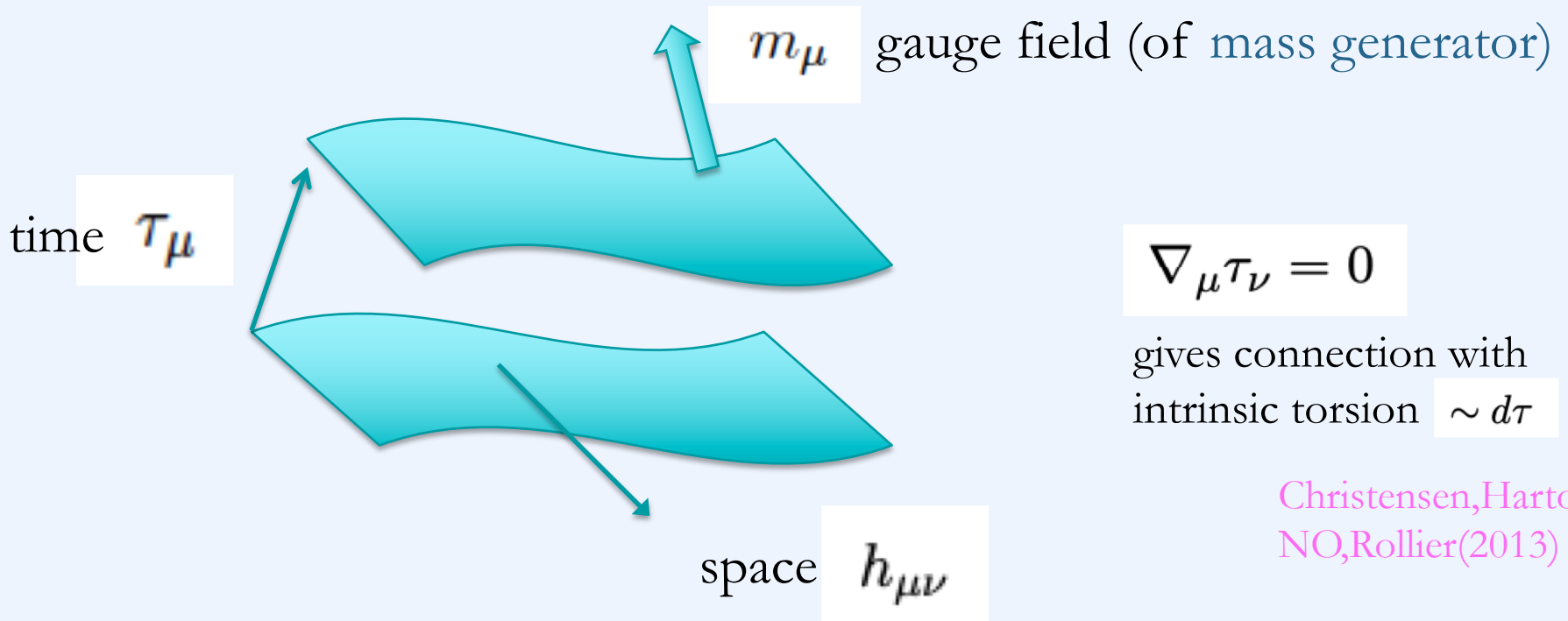
$$T^\mu = m \int d\tau \partial_\tau X^\mu \delta(x - X(\tau))$$

mass current

- curved backgrounds spacetime = torsional Newton-Cartan geometry:

$$\tau_\mu \quad h_{\mu\nu} \quad m_\mu$$

torsional Newton-Cartan geometry



Christensen, Hartong,
NO, Rollier (2013)

$d\tau = 0$ = absolute time (corresponds to original Newton-Cartan)

Review: Hartong, NO, Oling (2022)

Matrix Theory (a la late 90s)

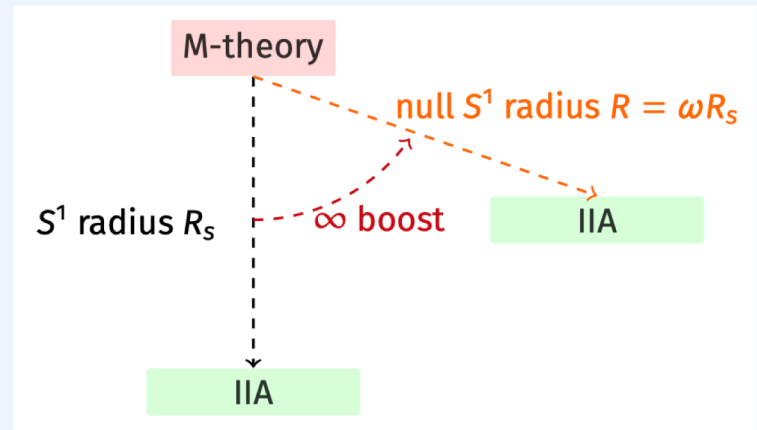
BFSS (1996) conjectured: “equivalence between uncompactified 11d M-theory and $N = \text{infinity}$ limit of the supersymmetric matrix quantum mechanics describing D0 branes”

$$S = \int d\tau \text{Tr} \left(\dot{X}_i^2 + \frac{1}{2} [X_i, X_j]^2 \right) \quad \begin{array}{l} i, j = 1, \dots, 9 \\ X_i \text{ in adjoint } SU(N) \end{array}$$

Susskind (1997): conjectured that finite N matrix QM is equivalent to DLCQ of M theory with N units of compact momentum

$$x^+ \sim x^+ + 2\pi R \quad p_+ = \frac{N}{R} \quad H = R \frac{\mathbf{p}^2}{2N} + \dots$$

Seiberg/Sen (1997) prescription that justified use of low-energy D0 brane dynamics consider light like circle as an infinite boost of spacelike circle



Matrix 0-brane theory (M0T)

DLCQ M-Theory

lightlike
compactification

M0T: BFSS Matrix theory

can go beyond the limit in flat space:

curved space
of M0T

$$\omega \rightarrow \infty.$$

$$\begin{aligned} ds^2 &= (\omega\tau_{\mu\nu} + \omega^{-1}H_{\mu\nu})dx^\mu dx^\nu, \\ C_{(1)} &= \omega^2 e^{-\varphi}\tau + c_{(1)}, \\ e^\phi &= \omega^{-3/2}e^\varphi, \\ C_{(n)} &= c_{(n)} \quad n \neq 1, \\ B_{(2)} &= b_{(2)}, \end{aligned}$$

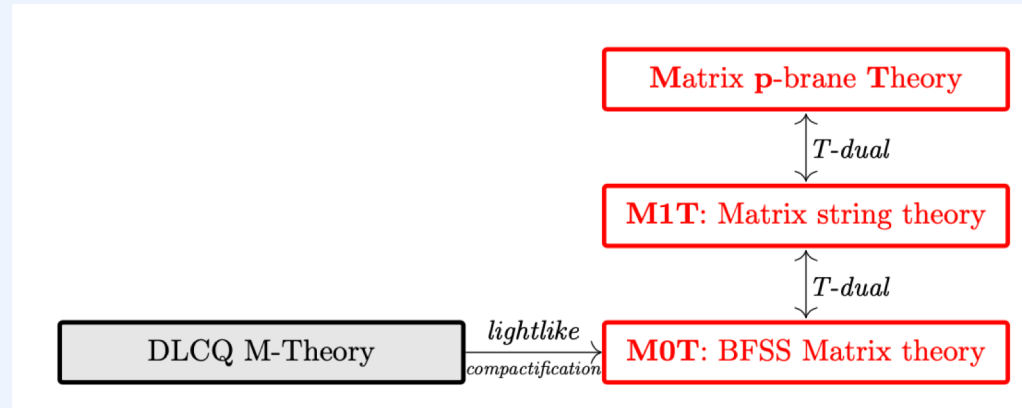
(TNC + other string fields)

- as opposed to earlier work:
now applied to the full type II string theory containing all possible extended objects in general backgrounds:

- D0-branes: curved bgr)
- D2-branes: NCYSMBFSS theory (on
- F1-string: novel action (target space Galilean/world-sheet Carrollian boost)

Matrix p-brane theory (MpT)

by further
T-dualities:



involves p-brane Newton-Cartan
geometry

$$\tau_{\mu\nu} = \tau_{\mu}{}^A \tau_{\nu}{}^B \eta_{AB}, \quad E_{\mu\nu} = E_{\mu}{}^{A'} E_{\nu}{}^{B'} \delta_{A'B'}$$

$$A=0 \dots p \quad / \quad A'=p+1 \dots D-1$$

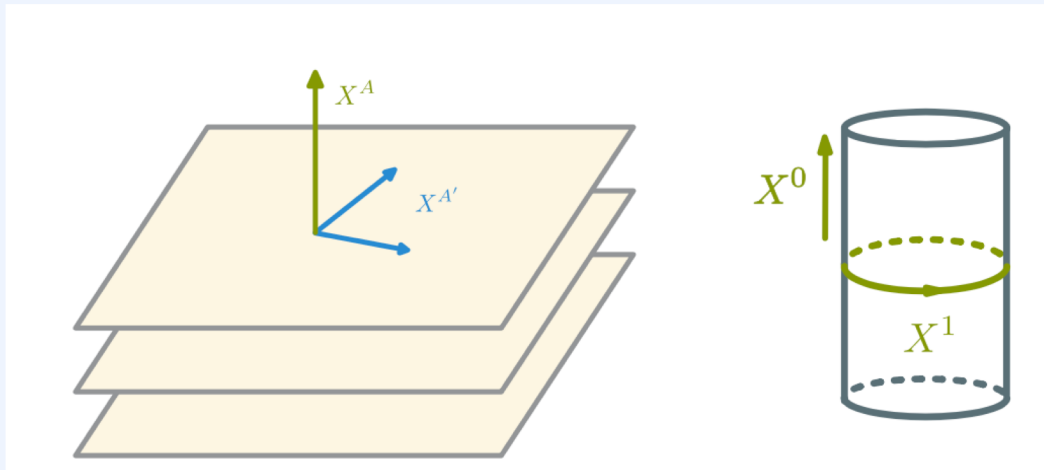
- action of the fundamental (light) excitations: N Dp-brane bound states

$$S_{Dp} = -\frac{T_p}{2} \int d^{p+1} \sigma e^{-\varphi} \text{Tr} \left[\sqrt{-\tau} \left(\tau^{\alpha\beta} D_{\alpha} \Phi^{\mu} D_{\beta} \Phi^{\nu} E_{\mu\nu} \right. \right. \\ \left. \left. + \frac{1}{2} \tau^{\alpha\gamma} \tau^{\beta\delta} F_{\alpha\beta} F_{\gamma\delta} - \frac{1}{2} [\Phi^{\mu}, \Phi^{\nu}] [\Phi^{\rho}, \Phi^{\sigma}] E_{\mu\rho} E_{\nu\sigma} \right) \right], \quad (5)$$

- in addition: action for other Dq-branes and F1-string

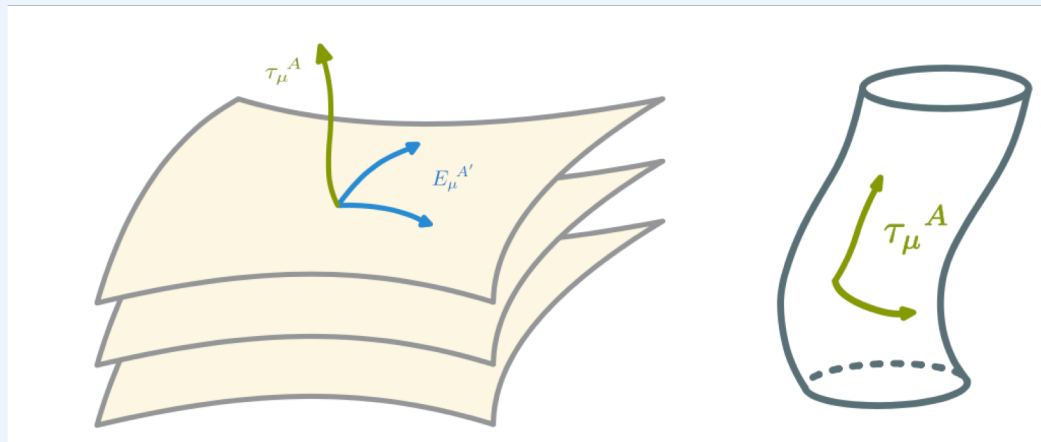
How does MpT relate to NR string theory (analogous decoupling limit) ?

Gomis/Ooguri NR string lives in flat space



figures from
review on NRST
Gerben Oling & Ziqi Yan
(2202.12698)

General target space probed by NR strings



Non-relativistic String Theory

fundamental strings are **extremally charged** under B-field
(tension = charge)

similar limiting procedure as for particle gives NRST on curved spacetime

kinetic

potential

$$S_{\text{NR}} = -\frac{T}{2} \int d^2\sigma \left[\sqrt{-\tau} \eta^{AB} \tau_A^\alpha \tau_B^\beta h_{\alpha\beta} + \epsilon^{\alpha\beta} m_{\alpha\beta} \right],$$

torsional string Newton–Cartan geometry : $\tau_\mu^A, h_{\mu\nu}, m_{\mu\nu}$.

$$h_{\mu\nu} = e_\alpha^a e_\beta^b \delta_{ab}$$

$m_{\mu\nu}$ couples to

worldsheet tension current

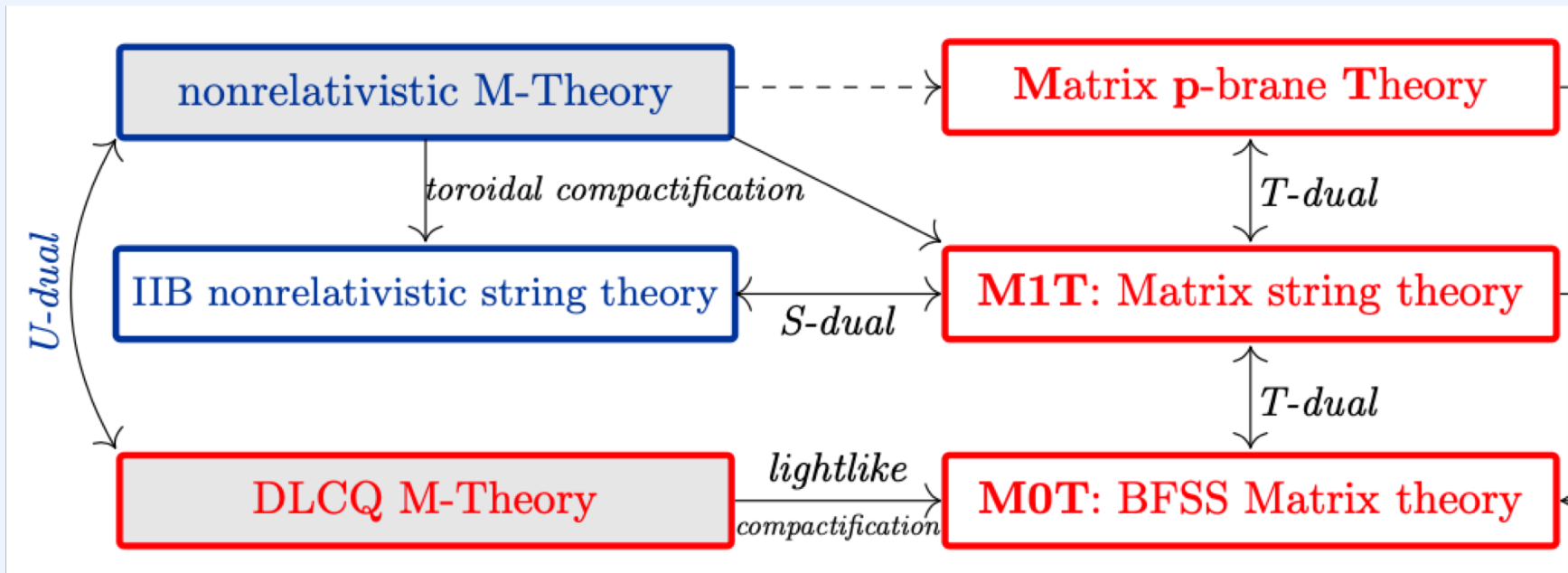
$$J_T^{\mu\nu} = T \int d^2\sigma \epsilon^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \delta(x - X(\sigma^\alpha)),$$

Bidussi et al (2021)

type II string theory: Apply this to F1-string = BPS object

-> type II NRST naturally emerges from BPS decoupling limit

Connections to NR string theory/M-theory



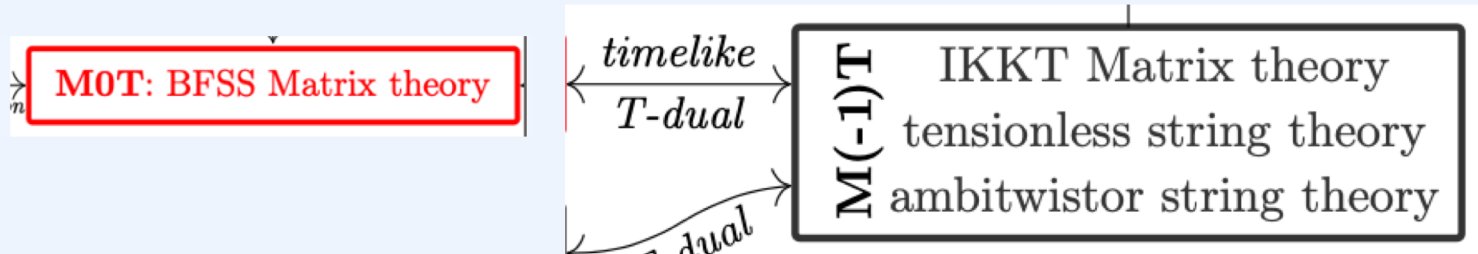
- near critical D1-string dual to near critical F1-string (=NRST)
- M-theory uplift (NR M-theory): near critical M2-brane

- wrapped membranes WM2
- open M2-branes on M5 (OM)



see also:
 Blair, Gallegos, Zinnato (2021)
 Ebert, Sun, Yan (2021)
 Ebert, Yan (2023)

D-instanton limit and tensionless string



IIB* is time-like dual of IIA (solution generating)

Hull (1998)

→ IKKT describes light D(-1)-branes (=D-instantons)

limit with near-critical
RR 0-form

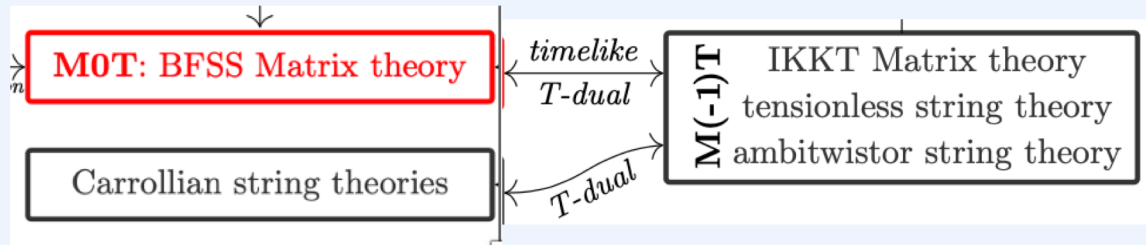
$$C_0 = \frac{\omega^2}{e^\varphi} + c_0, \quad G_{\mu\nu} = \frac{\tau_{\mu\nu}}{\omega}, \quad \Phi = \varphi + \frac{i\pi}{2} - 2 \ln \omega.$$

F1-string in this limit
becomes tensionless string

$$S_{F1} = -\frac{T}{\omega} \int d^2\sigma \sqrt{-\det(\partial_\alpha X^\mu \partial_\beta X^\nu \tau_{\mu\nu})}.$$

Isberg,Lindstroem,Sundborg,Theodoridis(1994)/Bagchi(2013)

thru the looking class: Carrollian strings



- perform q T-dualities on IKKT \rightarrow defines $M_p T$ for $p = -q-1 < -1$

geometry has $(9-q)$ -brane Carrollian boosts
(particle-like for $q=9$)

$$\delta_C \tau^A = \lambda^A_{A'} E^{A'} \quad \text{and} \quad \delta_C E^{A'} = 0.$$

- light excitations of $M_p T$ for $p < -1$: type II* Euclidean branes (S-branes)

Gutperle, Strominger (2002)

these are like tachyons Hull (1988)

--> natural connection to Carrollian physics

zero energy particles

deBoer, Hartong, NO, Sybesma, Vandoren (2021)

other work on Carrollian strings Cardona, Gomis, Pons (2016)

Bagchi, Banerjee, Chakraborty, Dutta, Parekh (2020) / Bagchi, Banerjee, Chakraborty (2020)

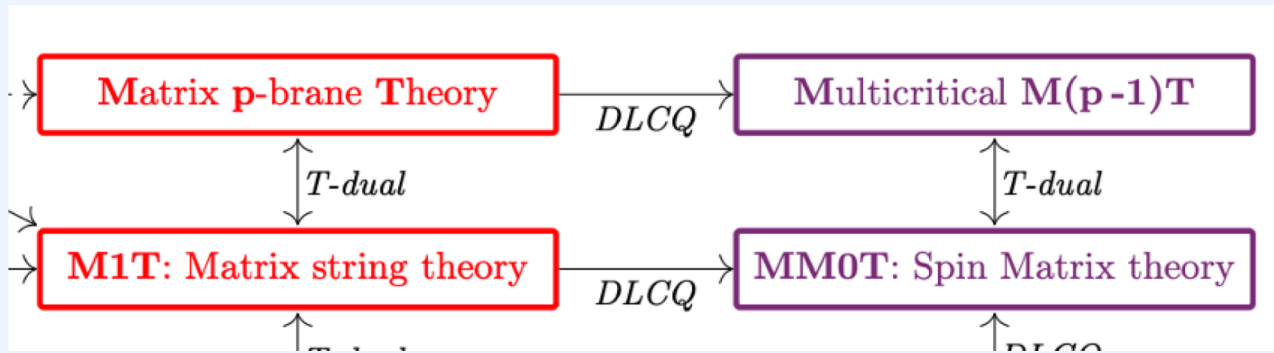
Bagchi, Banerjee, Hartong, Have, Kolekar, Mandlik (2023)

using expansions

Hartong, Have (2022) / Hartong, Hansen, NO (2019)

Multicritical Limits and Spin Matrix Theory

- can do one more DLCQ (near-critical fields corresponding to $\frac{1}{4}$ BPS bound states)



• multi-critical Matrix theory

e.g. MM0T: corresponding to D0-F1 light states

- generalizes the NR string that appears in Spin Matrix Theory (limit of AdS/CFT)

Harmark, Orsell (2014)

Harmark, Hartong, NO, (2017) / Harmark, Hartong, Mencilin, NO, Yan (2018) / Bidussi et al (2023)

- M-theory version connected to orthogonal M2-brane bound state

Nodal Riemann spheres

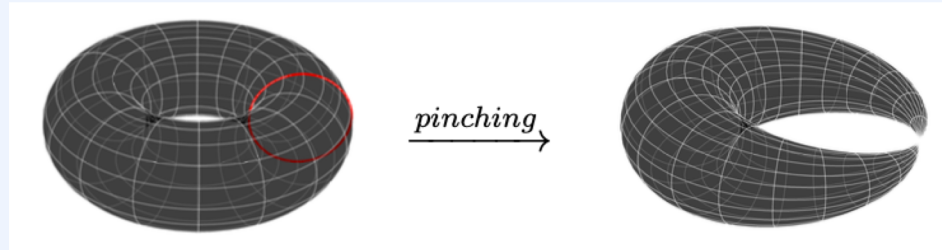
interesting degenerate topologies of 2D surfaces show up both on world-sheet and target space

Gomis, Yan (2023)

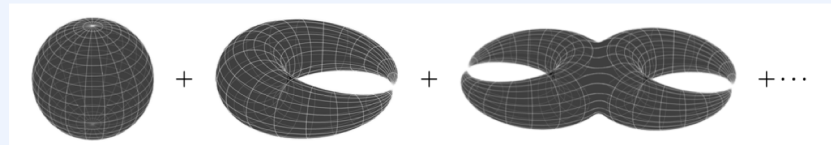
- F-string in M0T:

modular parameter

$$\tau \rightarrow i\infty.$$

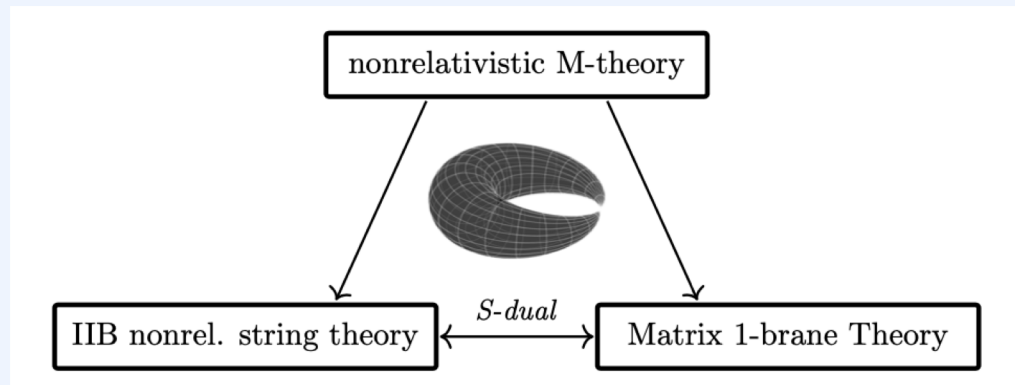


genus expansion
(cf. ambitwistor)



Geyer, Mason, Monteiro, Tourkine (2015)

- in target space:



Outlook

- obtained an archipelago of decoupling limits of ST with novel non-Lorentzian target spacetimes
 - understand role of background geometries/underlying algebras
 - strengthen (reject) underlying dualities at quantum level
- F1-string worldsheet description generically non-relativistic (except in NRST) and shows novel worldsheet topologies
- extensions:
 - 3rd DLCQ
 - NS5, KK5/KK6-brane (low co-dimension)
 - strong coupling behavior MpT for $p > 3$
 - heterotic/type I string theory versions
- reexamine Matrix Theory and its correspondence to supergravity
11D Lorentz invariance & Matrix Theory descriptions beyond flat space
- implications for AdS/CFT: BFSS on shrinking 3-torus: N=4 SYM
cf. M3T couples to non-Lorentzian 3-brane geometry ? NR AdS/CFT
Fontanella, Nieto Garcia (2024)
Guijosa et al (2023)
- Carrollian strings/insights into Carrollian FTs (relevance for flat space holo)

The end