A state-operator correspondence for nonlocal operators

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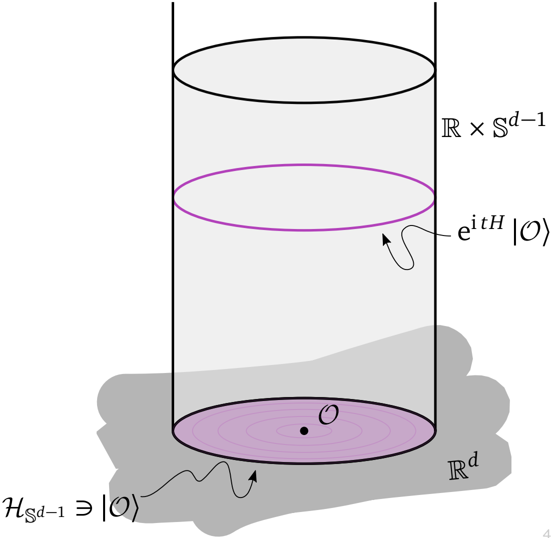
📜 based on upcoming work with D. M. Hofman

📍 GenHET meeting in String Theory
CERN – 30/04/2024

Outline

State-operator correspondence in CFT

local operators on $\mathbb{R}^d$ $\Longleftrightarrow$ states on $\mathcal{H}(\mathbb{S}^{d-1})$



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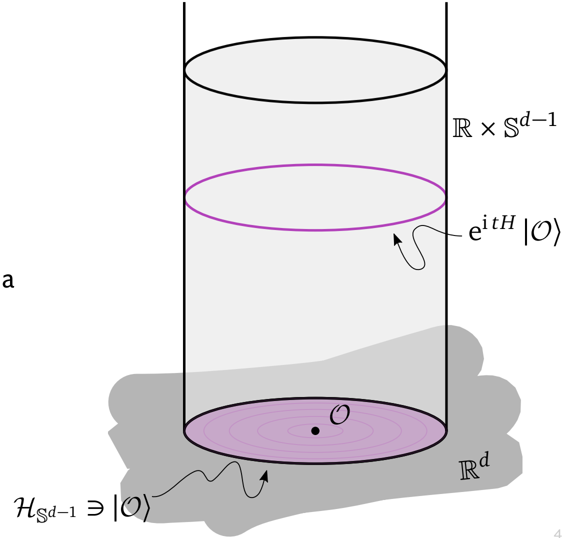

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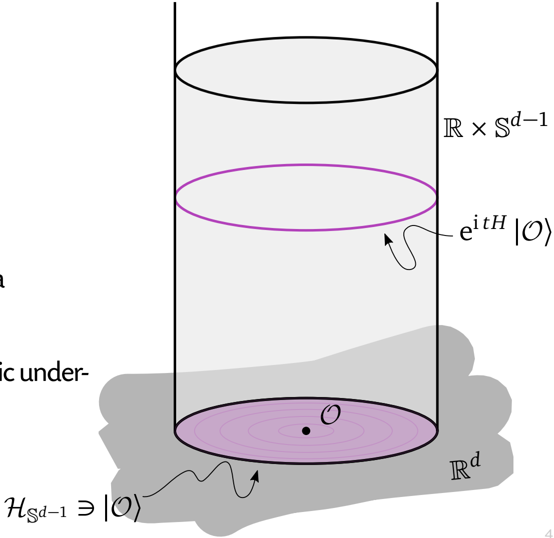
Lies at the heart of many developments of systematic understanding of CFTs: conformal bootstrap, VOAs, etc.



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In $d = 2$: cut and sew, resolution of the identity on $\mathbb{S}^1 \implies$ done[Friedan, Shenker 1987; Moore, Seiberg 1989]

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What if I have more symmetries at my disposal?

Recently vast generalisation of the notion of symmetry

see e.g. snowmass white paper [Córdova,Dumitrescu,Intrilligator,Shao '22] for references

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Continuous symmetry: $\partial_\mu J^\mu = 0 \quad \Longleftrightarrow \quad \mathrm{d}\star J_{[1]} = 0$

$\Longrightarrow$ codimension-one topological operator $U(\Sigma_{d-1}) := \exp\left(\mathrm{i} \int_{\Sigma_{d-1}} \star J_{[1]} \right)$

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Zero-form continuous symmetry: $\partial_\mu J^\mu = 0 \quad \Longleftrightarrow \quad \mathrm{d}\star J_{[1]} = 0$

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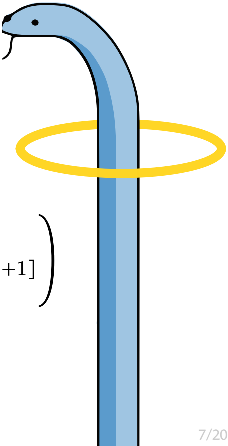

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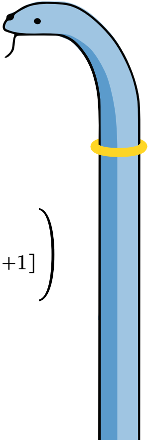

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I will stick to $p = 1$ and free Maxwell theory in this talk

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...there's (much) more

$\mathrm{d}\Lambda^{\pm} = \pm \star \mathrm{d}\Lambda^{\pm}$

$J^{\pm} = \pm \star J^{\pm}$

For *any* chiral one-form $\Lambda^{\mp}$

$$Q^{\pm}_{\Lambda}[\Sigma_3] := \int_{\Sigma_3} J^{\pm} \wedge \Lambda^{\mp} \quad \text{is a conserved charge}$$

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Botomline:

unitary CFT_4 + continuous one-form symmetry $\implies$ infinitely many zero-form symmetries + spectrum generating algebra

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The Hamiltonian becomes a collection of oscillators:

half of the $J_n^{\pm}$'s

the other half

$$H_\Sigma = \frac{1}{2\mathrm{k}}\left(\|E\|^2_\Sigma + \|B\|^2_\Sigma\right) = \frac{1}{\mathrm{k}}\sum_{\mathrm{i}=1}^{b_2(\Sigma)} J^+_{0\mathrm{i}} J^-_{0\mathrm{i}} + \frac{1}{\mathrm{k}}\sum_{\mathrm{n}\in\mathcal{N}'} \mathcal{A}^\dagger_{\mathrm{n}}\mathcal{A}_{\mathrm{n}} + E_0$$

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Flux quantisation $\implies |j\rangle \rightsquigarrow |\boldsymbol{r}, \boldsymbol{s}\rangle$ with energy $\Delta_{\boldsymbol{r},\boldsymbol{s}} = \dfrac{(\boldsymbol{r} + \mathrm{t}\boldsymbol{s})^\dagger \mathbb{E} (\boldsymbol{r} + \mathrm{t}\boldsymbol{s})}{2\,\mathrm{Im}\,\mathrm{t}}$

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Dress with oscillators:

Generic state: $\quad |\boldsymbol{r},\boldsymbol{s};\{N_{\mathrm{n}}\}\rangle := \prod_{\mathrm{n}\in\mathcal{N}'}\left(\mathcal{A}^\dagger_{\mathrm{n}}\right)^{N_{\mathrm{n}}}|\boldsymbol{r},\boldsymbol{s}\rangle$ with energy $\Delta_{\boldsymbol{r},\boldsymbol{s}} + \sum_{\mathrm{n}\in\mathcal{N}'} N_{\mathrm{n}}\sqrt{\lambda_{\mathrm{n}}}$

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supports unique flux

I will now make a choice: $\Sigma_3 = \mathbb{S}^2 \times \mathbb{S}^1$

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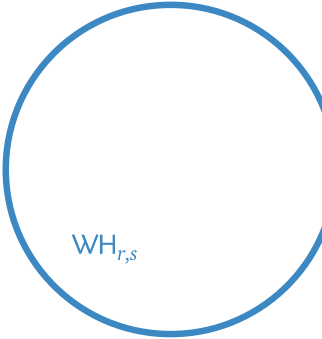

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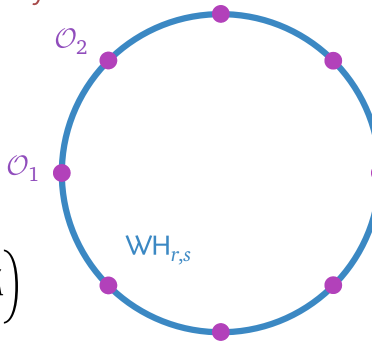

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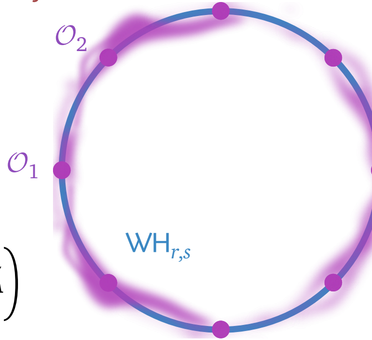

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Let's connect to the states

Path integral on $\mathbb{B}^3 \times \mathbb{S}^1$ with $\mathcal{L}$ insertion $\Longrightarrow$ state on $\mathbb{S}^2 \times \mathbb{S}^1$

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$$\Psi_{\mathcal{L}}[A_*] = \langle A_* | \mathcal{L} \rangle := \int_{\mathcal{C}[A_*]} \mathrm{D}A\, \mathrm{e}^{-S[A]}\, \mathcal{L}\big(\{\mathbf{0}\} \times \mathbb{S}^1\big)$$

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Interesting subtlety: time evolution $\equiv$ radial evolution on $\mathbb{B}^3$ mixes ladder operators:

$$\mathcal{A}_{\mathsf{n}}(r) = \mathbb{U}\big(r, r'\big)\, \mathcal{A}_{\mathsf{n}}\big(r'\big) + \mathbb{V}\big(r, r'\big)\, \mathcal{A}^{\dagger}_{\mathsf{n}}\big(r'\big)$$

squeezing operator $\sim \prod_{\mathrm{n}} \exp\left(\mathcal{A}_{\mathrm{n}}^2 + \left(\mathcal{A}_{\mathrm{n}}^\dagger\right)^2\right)$
takes care of Bogoliubov transformation

(Squeezed) Primary states

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Can check

$$J_0^\pm \left|\mathrm{WH}_{r,s}\right\rangle = j_{r,s}^\pm \left|\mathrm{WH}_{r,s}\right\rangle \text{ from above}$$

$$\implies \boldsymbol{S}^{-1}\left|\mathrm{WH}_{r,s}\right\rangle \text{ has energy } \Delta_{r,s} = \frac{|r+\mathrm{t}s|^2}{\operatorname{Im}\mathrm{t}}$$

scaling dimension for $\mathrm{WH}_{r,s}$.

[Verlinde 1995; Kapustin 2005]

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takes care of Bogoliubov transformation

(Squeezed) Primary states

$$\left|\mathsf{WH}_{r,s}\right\rangle = \int_{\mathcal{C}[\quad]} \mathrm{D}A\, \mathrm{e}^{-S[A]}\ \mathsf{WH}_{r,s}\left(\{\mathbf{0}\} \times \mathbb{S}^1\right) = \boldsymbol{S}\,|r,s\rangle$$

Can check

$$J_0^\pm \left|\mathsf{WH}_{r,s}\right\rangle = j_{r,s}^\pm \left|\mathsf{WH}_{r,s}\right\rangle \text{ from above}$$

$$\implies \boldsymbol{S}^{-1}\left|\mathsf{WH}_{r,s}\right\rangle \text{ has energy } \Delta_{r,s} = \frac{|r + \mathrm{t}s|^2}{\operatorname{Im}\mathrm{t}}$$

scaling dimension for $\mathsf{WH}_{r,s}$.

[Verlinde 1995; Kapustin 2005]

Descendants

$$\boldsymbol{S}\mathcal{A}_{\mathrm{n}}^\dagger \boldsymbol{S}^{-1}\left|\mathsf{WH}_{r,s}\right\rangle = \int_{\mathcal{C}[\quad]} \mathrm{D}A\, \mathrm{e}^{-S[A]}\left[\mathcal{A}_{\mathrm{n}}^\dagger \mathsf{WH}_{r,s}\right]\left(\{\mathbf{0}\} \times \mathbb{S}^1\right)$$

Bottomline:

$$\text{line operators on } \mathbb{R}^3 \times \mathbb{S}^1 \quad \Longleftrightarrow \quad \text{states on } \mathcal{H}\big(\mathbb{S}^2 \times \mathbb{S}^1\big)$$

$$\mathsf{WH}_{r,s}(\{\mathbf{0}\} \times \mathbb{S}^1) \quad \leftrightsquigarrow \quad \text{squeezed } |r, s\rangle$$

$$\text{“}J^{\pm}_{\mathsf{n}}\text{” } \mathsf{WH}_{r,s}(\{\mathbf{0}\} \times \mathbb{S}^1) \quad \leftrightsquigarrow \quad \text{squeezed } |r, s; \{1_{\mathsf{n}}\}\rangle$$

photons sprinkled
over Wilson–'t Hooft loops $\quad \leftrightsquigarrow \quad$ generic state

Unitary CFT in $d = 2p + 2$

$+$

$\mathrm{U}(1)^{[p]}$ symmetry

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$+$

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$$\Downarrow$$

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$\mathrm{U}(1) \times \widetilde{\mathrm{U}(1)}$ current algebra

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$(d = 4)$

classifies states:

primaries: definite charge

descendants: act with $\mathcal{A}_{\mathrm{n}}^{\dagger}$

classifies operators:

squeezed primaries: $\mathrm{WH}_{r,s}$

descendants: sprinkle photons

Unitary CFT in $d = 2p + 2$

$+$

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states on $\mathcal{H}(\mathbb{S}^2 \times \mathbb{S}^1)$

$\Updownarrow$

line operators on $\mathbb{R}^3 \times \mathbb{S}^1$

Non-invertible symmetries

Gauging charge-conjugation breaks $\mathrm{U}(1)^{[1]}$ $\rightsquigarrow$ gets restored as non-invertible symmetry

So does the Kac–Moody algebra

Representation theory of non-invertible Kac–Moody $\implies$ state-operator on the orbifold branch?

Non-abelian story?

No non-abelian higher-form symmetries

However $\left[J^{\mathrm{A}}_{\mathrm{m}}, J^{\mathrm{B}}_{\mathrm{n}}\right] = f^{\mathrm{AB}}_{\mathrm{C}} k^{\mathrm{r}}_{\mathrm{mn}} J^{\mathrm{C}}_{\mathrm{r}} + \mathrm{k}\sqrt{\lambda_{\mathrm{n}}}\delta_{\mathrm{mn}}$ may make sense

Higher-dimensional WZW?

$\mathcal{N} = 4$ SYM partition function: $\mathcal{Z} \sim \sum \alpha_r q^{\Delta_r}$ [Vafa, Witten 1994]

And [Kapustin, 2005] suggests scaling dimensions of $\frac{1}{2}$BPS operators.

Thank you!