Generalised symmetries: from hydrodynamics to entanglement resolution

Based on - arXiv [hep-th]:

- 2205.03619, 2212.09787, 2309.14438, 2403.16957 (holography & EFT for chiral MHD)
- 2402.06322 (entanglement resolution)

Collaborators: Adrien Florio, Ruth Gregory, Nabil Iqbal, Napat Poovuttikul, Pablo-Saura Bastida, German Sierra and Javier Molina-Vilaplana

GenHET meeting in String Theory @ CERN

Arpit Das

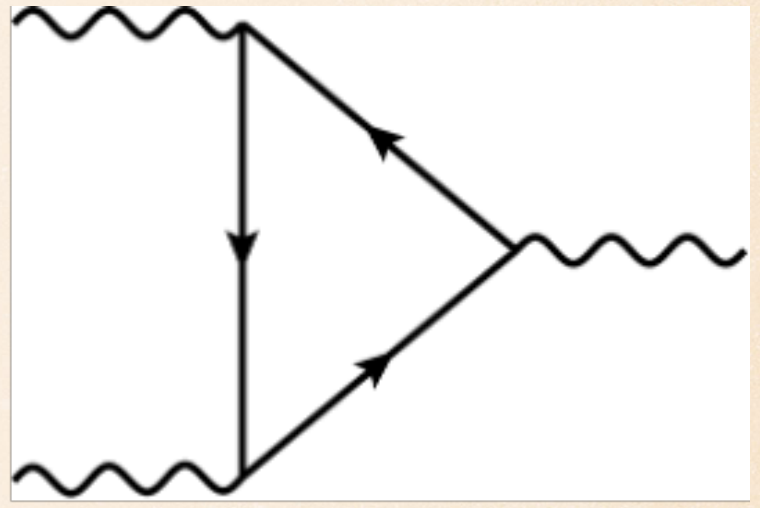
University of Edinburgh

Hydrodynamics

- Say we have a strongly interacting QFT at $T > 0$ and we want to understand the time-dependent, long-distance and late-time physics; d.o.f.s to keep track of : conserved quantities: $\omega_{conserved} \sim \mathcal{O}(L^{-1})$

- Given the conserved quantities, there is (usually) a well-established framework to construct the low-energy effective field theory: hydrodynamics $\rightarrow$ (conservation eqns → EoMs)

- Hydrodynamics is a genuine EFT, entirely dictated by symmetries and is still evolving!



$$d \star j_A = k\, F \wedge F$$

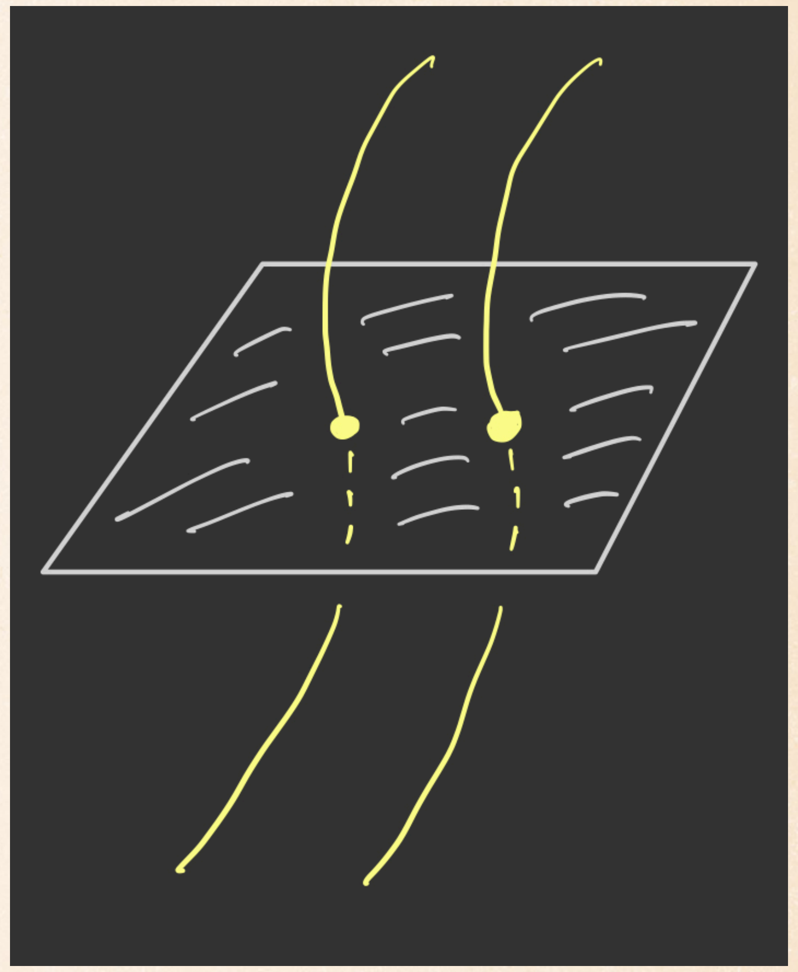
(Δ - anomaly)

Ordinary symmetries	⟷	Navier-Stokes equations
Systems with 't Hooft anomalies	⟷	[Sun, Surowka; Nieman, Oz; ...]
Systems with ABJ anomalies (applications: astrophysics, plasma physics, condensed matter, ...)	⟷	attempts - [Figueroa et. al.; Hattori et. al.; ...; Das et. al.]

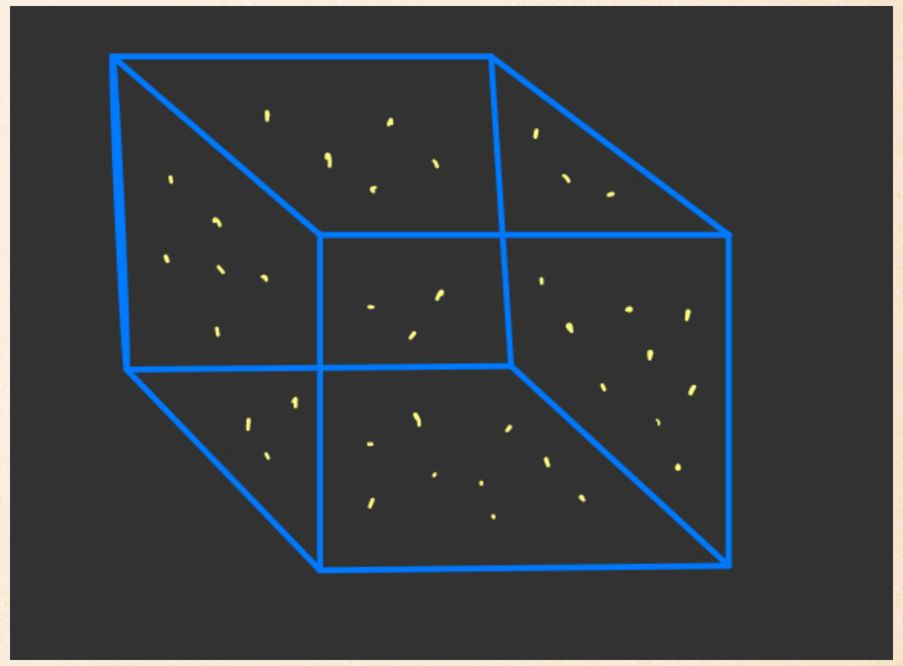
Symmetries

- Global symmetries imply conservation laws [Noether]
- Conserved operators are topological [Gaiotto, Kapustin, Seiberg, Willet]
- Since global symmetries are preserved along RG flows, they help in constructing EFTs
- Let us review ordinary symmetries (1-index) currents first:

$$\nabla_\mu j^\mu = 0 \qquad d \star j = 0$$



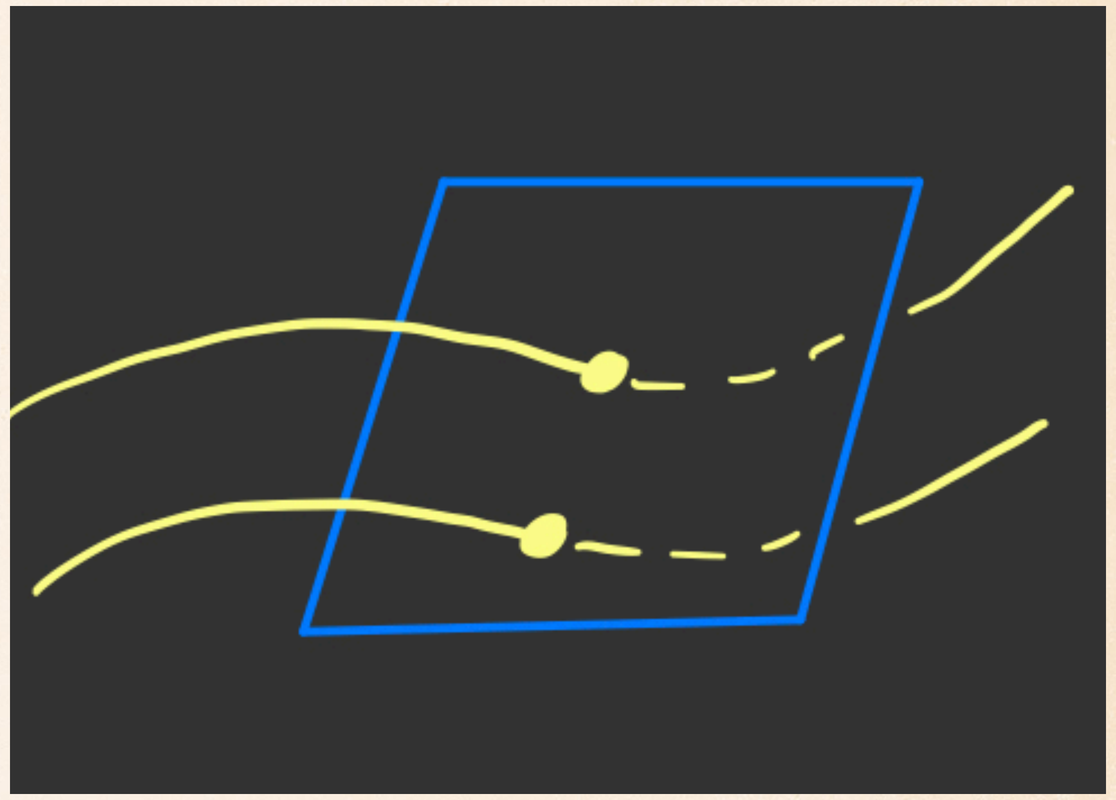
- An ordinary current counts particles, "catch them all" by integrating on a co-dimension 1 subspace:



$$Q = \int_{\mathscr{M}_{d-1}} \star j \quad \rightarrow$$ defines a $U(1)$-valued topological co-dim. 1 surface operator $$\rightarrow \quad U\left(\mathscr{M}_{d-1}\right) = \exp\left(i\alpha Q\left(\mathscr{M}_{d-1}\right)\right)$$

(fancy way to talk about "conserved charge" → call this a 0-form symmetry)

Higher-form symmetries



- Now consider a 2-index current:

$$\nabla_\mu J^{\mu\nu} = 0 \qquad d \star J = 0$$

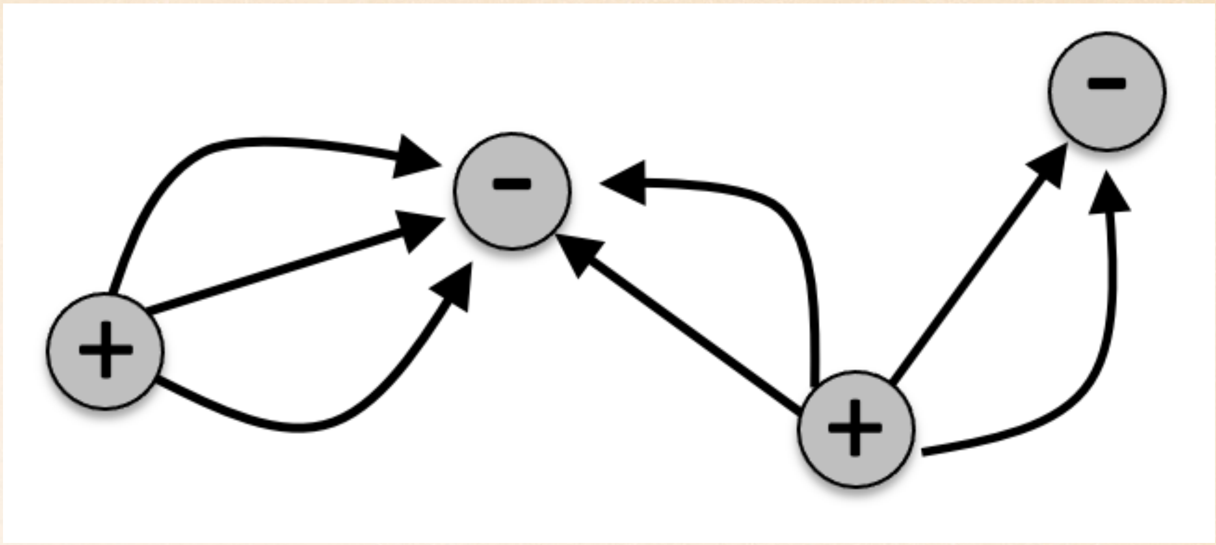
- 2-index current counts strings, as they don't end in space or time: "catch them all" by integrating on a co-dimension 2 subspace:

$$Q = \int_{\mathscr{M}_{d-2}} \star J \quad \rightarrow \quad \text{defines a } U(1)\text{-valued topological co-dim. 2 surface operator} \quad \rightarrow \quad U\left(\mathscr{M}_{d-2}\right) = \exp\left(i\alpha Q\left(\mathscr{M}_{d-2}\right)\right)$$

This is called a 1-form symmetry: counts conserved "string number"

[Gaiotto, Kapustin, Seiberg, Willet]

Chiral Plasma (QED at T>0)



- IR dynamics of chiral plasma: dynamical E&M coupled to massless Dirac fermion,

$$S_{EM}[A_\mu, \psi] = \int d^4x \left(-\frac{1}{g^2} F^2 + \bar{\psi}\gamma^\mu \left(\partial_\mu - iA_\mu\right)\psi \right)$$

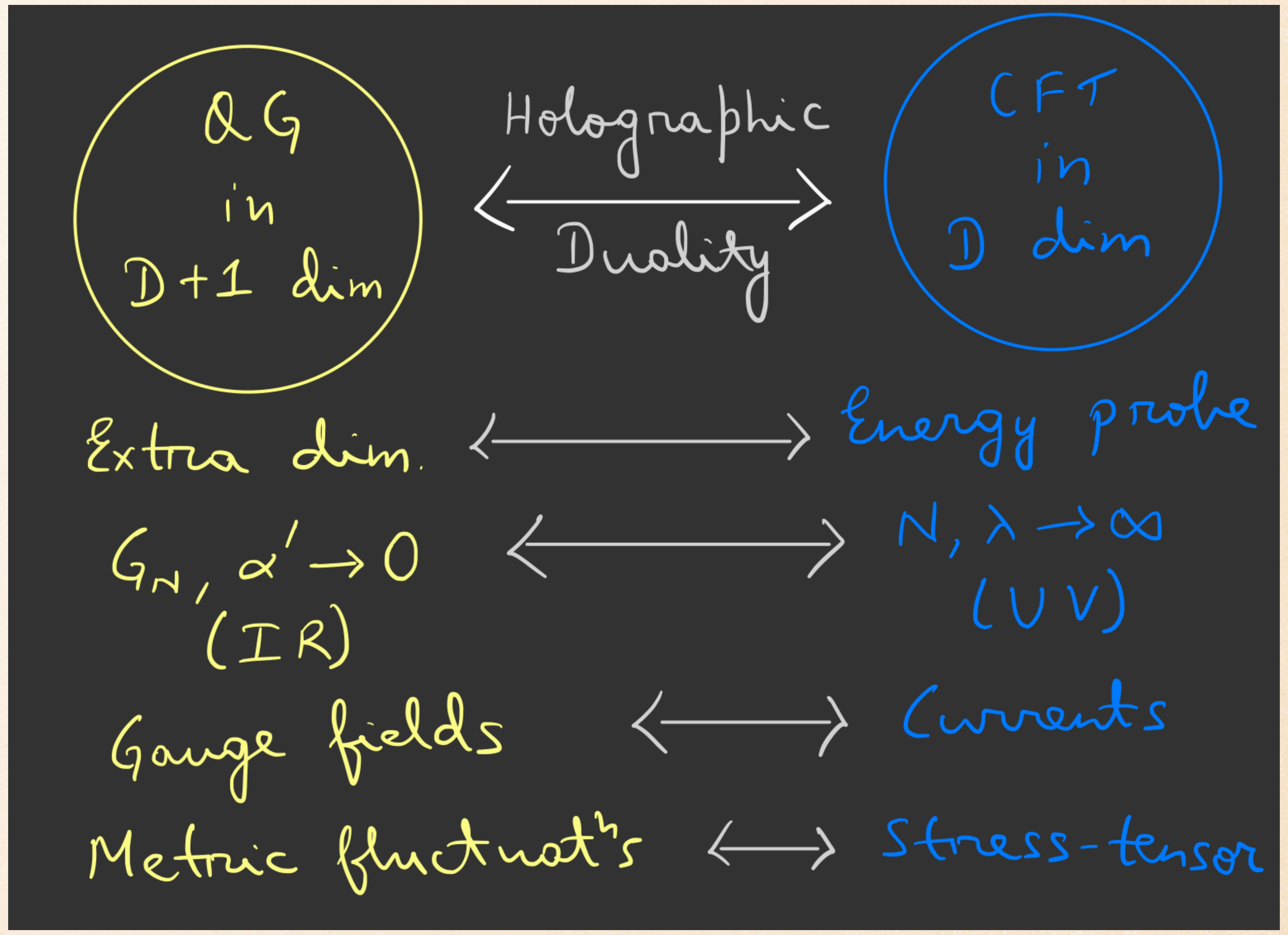
- Global symmetries:

$$\boxed{\partial_\mu J^{\mu\nu} = 0 \qquad \partial_\mu j^\mu_A = k\epsilon_{\mu\nu\rho\sigma} J^{\mu\nu} J^{\rho\sigma}} \qquad \left(\text{where} \quad J^{\mu\nu} := \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}\right)$$

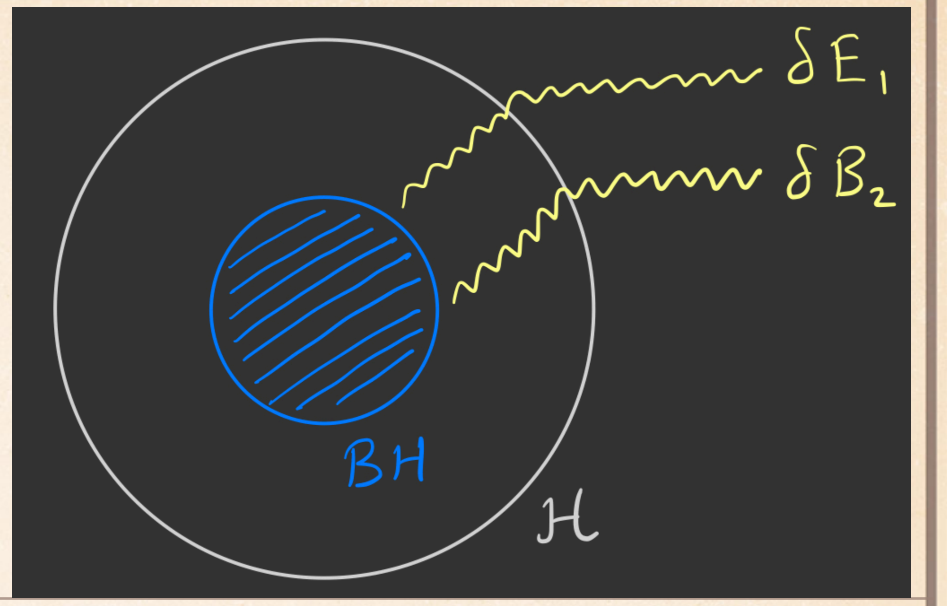
- We have two currents: j^μ_A and $J^{\mu\nu}$. So, $\longrightarrow$ $E_1 \leftrightarrow j^\mu_A$, $B_2 \leftrightarrow J^{\mu\nu}$ (bulk $\longleftrightarrow$ boundary)

- Gauge invariance of B_2: $B_2 \to B_2 + d\Lambda_1$ $\longleftrightarrow$ (conservation of $J^{\mu\nu}$)

- However, E_1 is a 1-form with mutilated gauge invariance to allow for the non-conservation of chiral current: j^μ_A $\longrightarrow$ $\boxed{d \star j_A = k \star J \wedge \star J}$



Holographic bulk action

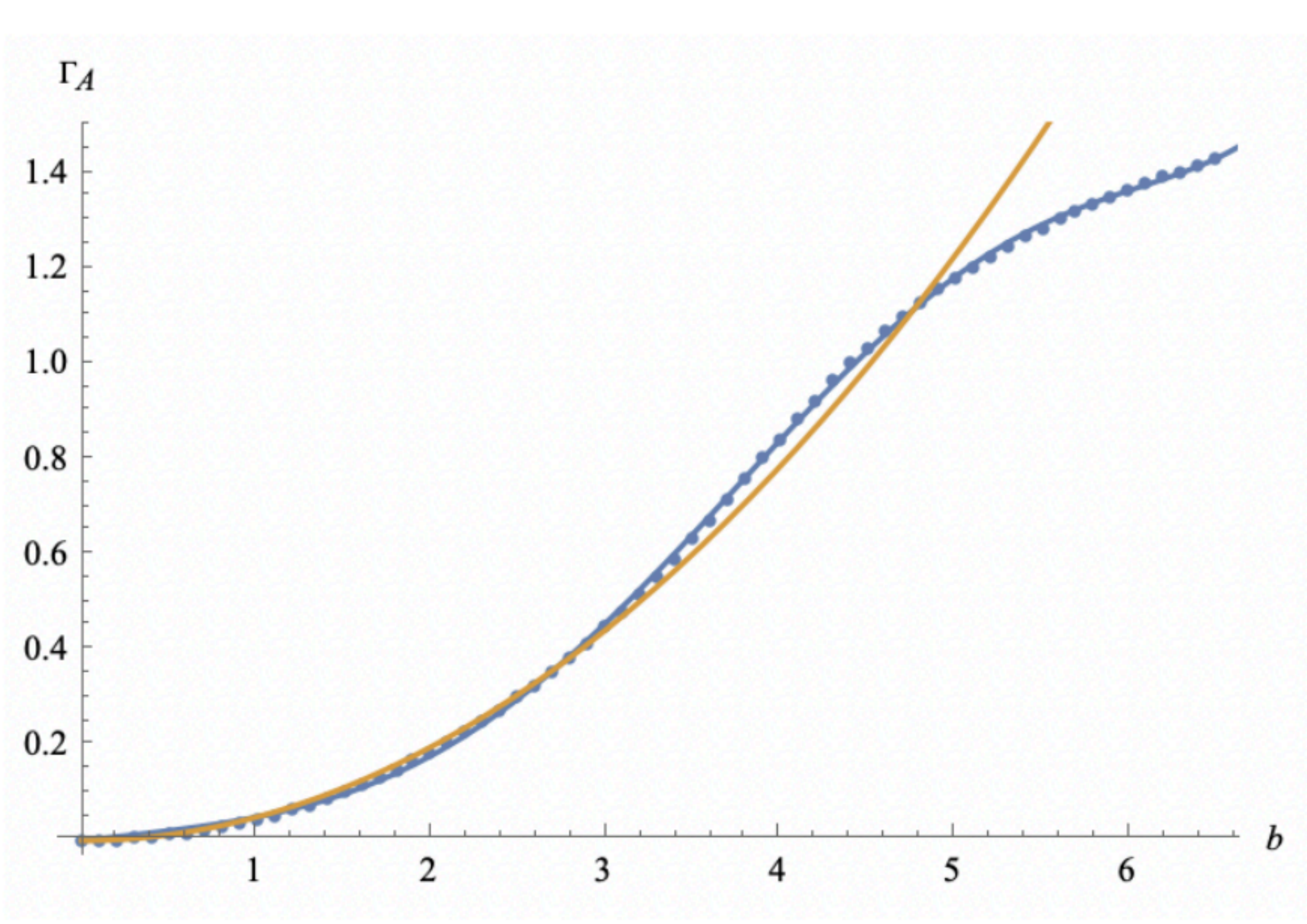


$(\mathscr{M}_5 = \mathrm{Sch} - AdS_5)$: probe-limit

$$S_{\text{bulk}} = \int_{\mathscr{M}^5} \left|dE_1\right|^2 + \left|dB_2\right|^2 + k\, E_1 \wedge \star dB_2 \wedge \star dB_2 + \cdots$$

(contains "mass" terms for E_1 like $(dB_2) \cdot E_1 \cdot (dB_2)$)

- Anomaly → decay of chiral charge: $n_A \sim e^{-\Gamma_A t}$; compute Γ_A which controls the chiral decay rate
- Heat up the system – put it on the black hole and study linear perturbations of fields: δE_1 and δB_2
- Compute quasi-normal modes (infalling modes at horizon) – lowest QNM: $\omega^l_{QNM} \longleftrightarrow \Gamma_A$
- Result: $\Gamma_A \sim \zeta\, b^2 \sim \dfrac{k^2 \rho\, b^2}{\chi_A}$ (for small b) and estimate of ζ matches with previous elementary hydro results, at small fields. However, deviation from quadratic behaviour at larger fields.



Hydrodynamic EFT

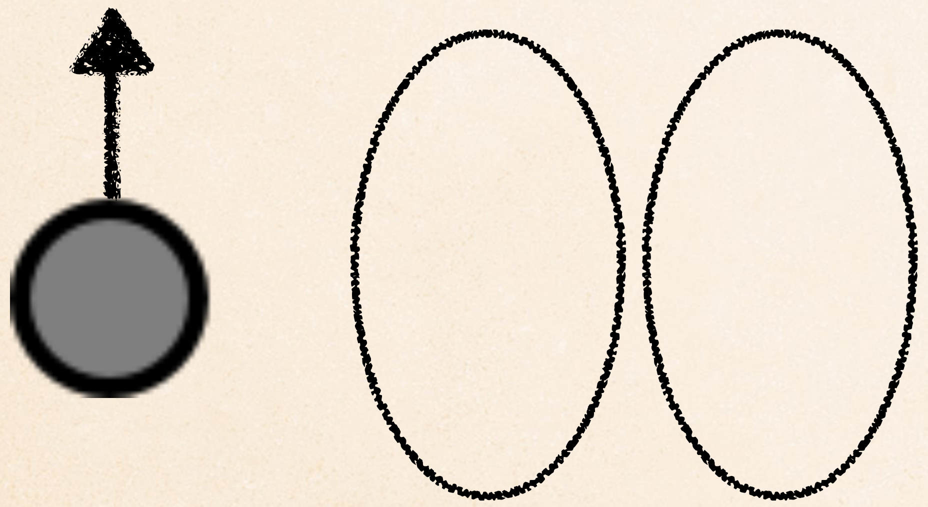
- Well-posed problem: what is the finite temperature hydro theory with the following symmetry structure?

$$\partial_\mu J^{\mu\nu} = 0 \qquad \partial_\mu j^\mu_A = k\, \epsilon_{\mu\nu\rho\sigma} J^{\mu\nu} J^{\rho\sigma}$$

→ true situation more subtle: "0-form non-invertible symmetry"
[Choi, Lam, Shao: Cordova, Ohmori; Karasik; Etxebarria, Iqbal]

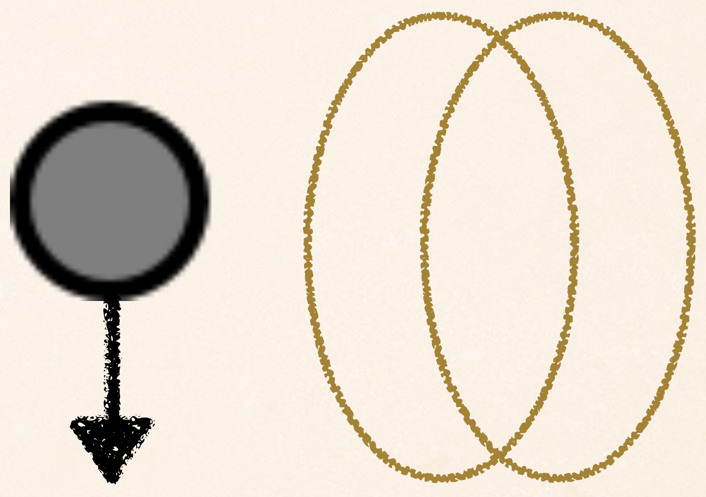
1-form symmetry for magnetic flux conservation

0-form "non-invertible" symmetry for axial charge "conservation"



"Defect operators exist even if there is no locally conserved current"

[Berger, Field]



$$\int_{\mathcal{M}_3} \star j_A - A \wedge dA$$

EFT: A hydro-action realising 1-form and non-invertible symmetry structure without any reference to QED!

→ reproduces known pheno. results: Chiral Separation Effect & Chiral Magnetic Effect

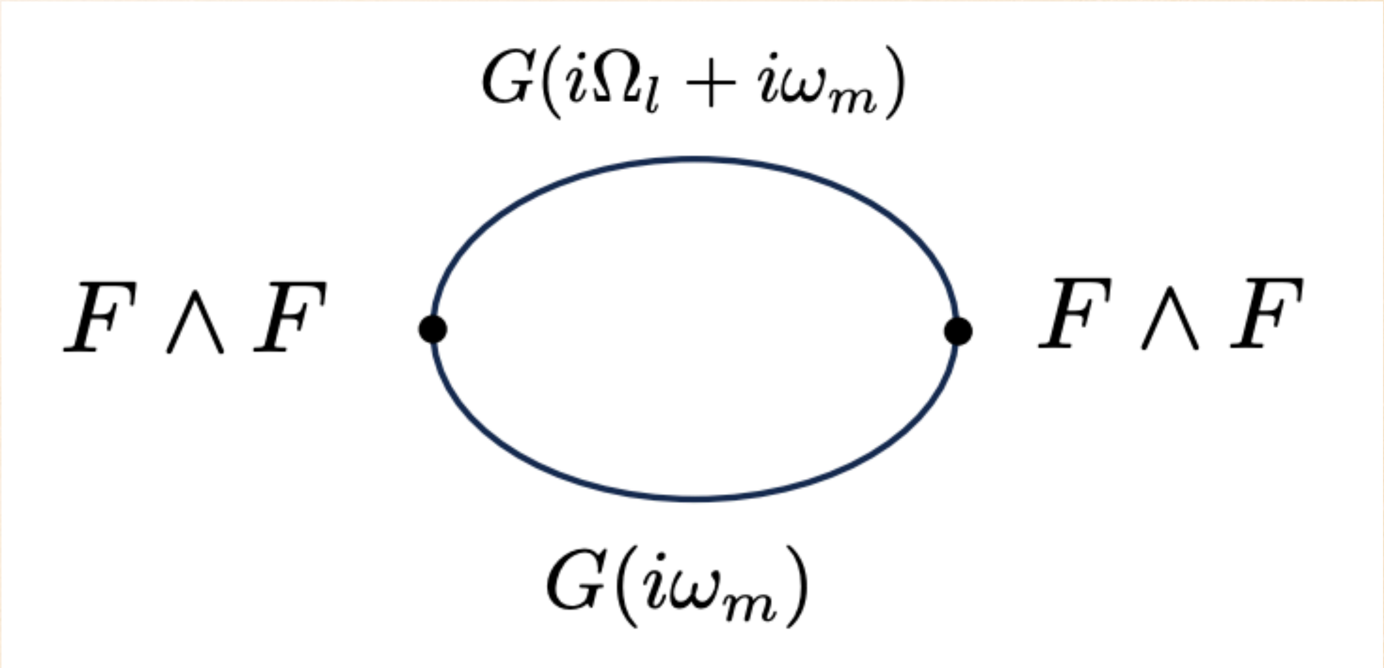
Results & Implications

- Universal transport coefficients: [Grozdanov et.al.,; Grozdanov et. al.; Das. et. al.]
  - ✦ Kubo formula (response-source eqns.):

$$j^i_{\text{ind}} \sim \sigma E^i_{\text{ext}} \text{ (weak-coupling)} \qquad E^k_{\text{ind}} \sim \epsilon_{ijk} J^{ij} \sim \rho\, j^k_{\text{ext}} \text{ (universal: } e \sim \mathcal{O}(1))$$

$$\sigma \sim \frac{G^R_{j^i j^i}(\omega, \vec{q}=0)}{i\omega}\Bigg|_{\omega\to 0} \qquad \rho \sim \frac{G^R_{J^{ij} J^{ij}}(\omega, \vec{q}=0)}{i\omega}\Bigg|_{\omega\to 0} \qquad \Rightarrow \qquad \sigma \neq \frac{1}{\rho}$$

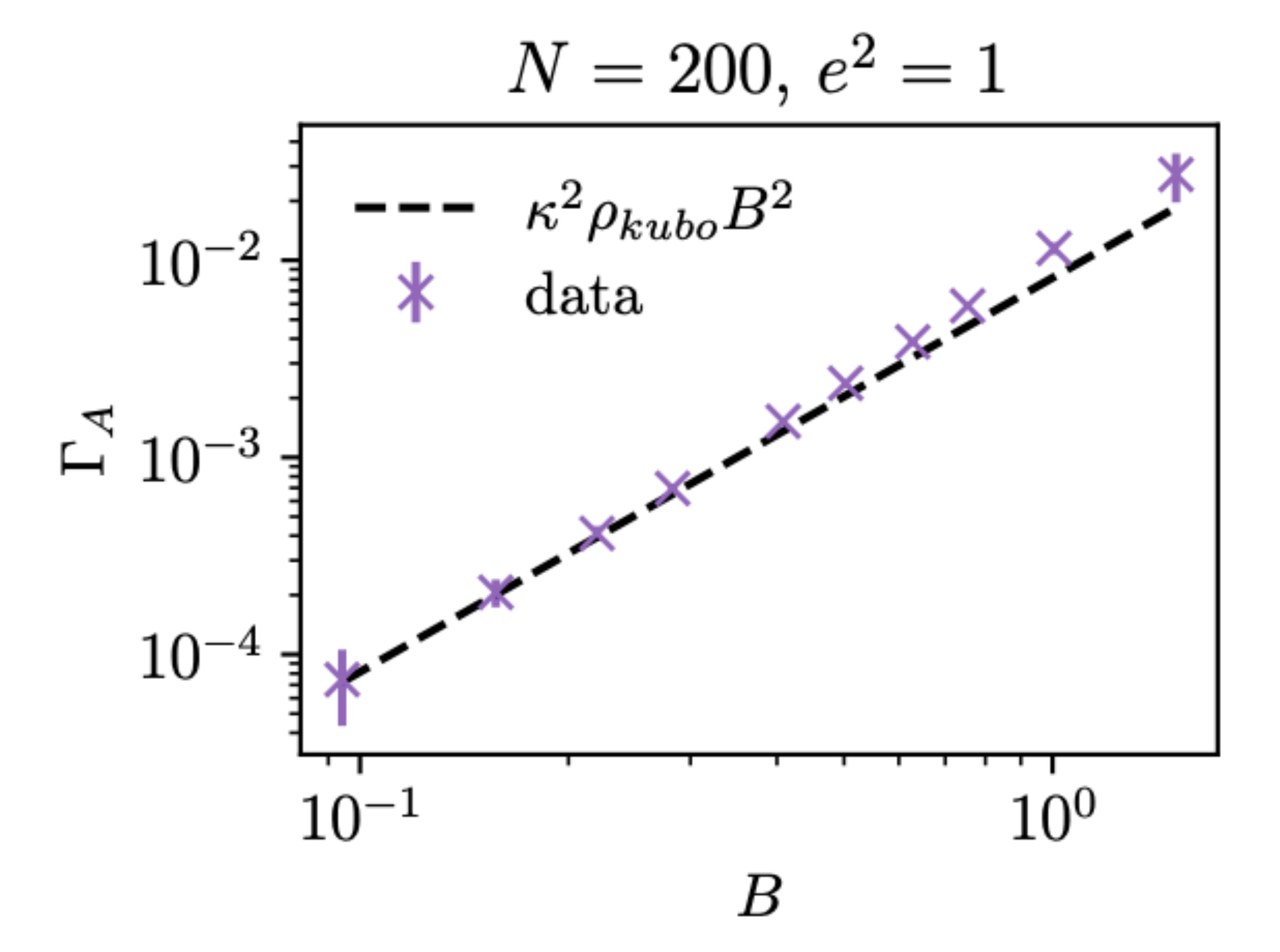
- We saw: $\Gamma_A \sim \zeta b^2 \sim \dfrac{k^2 \rho_A b^2}{\chi_A}$. What happens to $\Gamma_A(b \to 0)$? when hydrodynamic fluctuations are included. The leading (1-loop) fluctuation-driven contribution to the decay-rate still remains zero! Protected by the 0-form non-invertible symmetry which leads to chirality conservation. Contrasting to QCD: the CS diffusion rate – IR limit of the non-Abelian topological density $\mathrm{Tr}(F^a_{\mu\nu}\tilde{F}^{a\mu\nu})$ is non-zero!



Results (contd.)

- $\Gamma_A(\omega) \sim \frac{k^2\rho^2}{D^{\frac{5}{2}}\chi_A}\left(\frac{\pi}{2\sqrt{2}\beta}|\omega|^{\frac{3}{2}} + \mathcal{O}(\omega^2)\right)$. The numerical coefficients of the non-analytic pieces are universal in the sense that they do not receive UV corrections ↔ protected by the 0-form non-invertible symmetry

- Lattice results: classical real-time lattice simulations of scalar QED coupled to axion with axion shift giving the anomalous Ward identity



Symmetry resolution of entanglement

- Cardy's replica trick but with symmetry operator/ defect insertion:

[Calabrese, Cardy; Goldstein, Sela]

$$Z_{ab}(q^n, g) \equiv \mathrm{Tr}_{ab}[\hat{\mathscr{L}}_i(g)\, q^{n(L_0 - c/24)}]$$

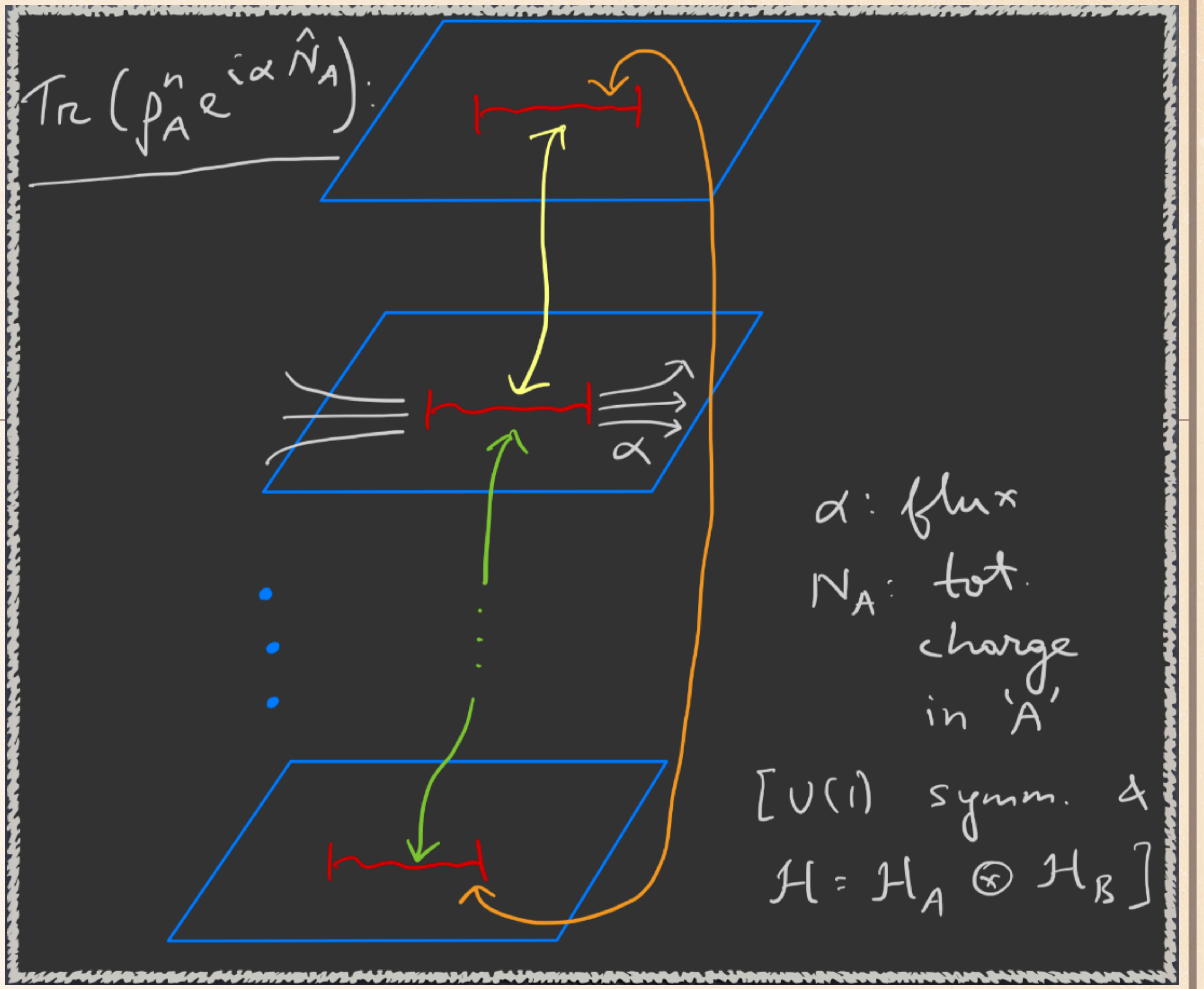
where, for group G-like symmetries, computing the above amounts to finding G-invariant conformal states or, Cardy states: $|a\rangle_g, |b\rangle_g$; such that:

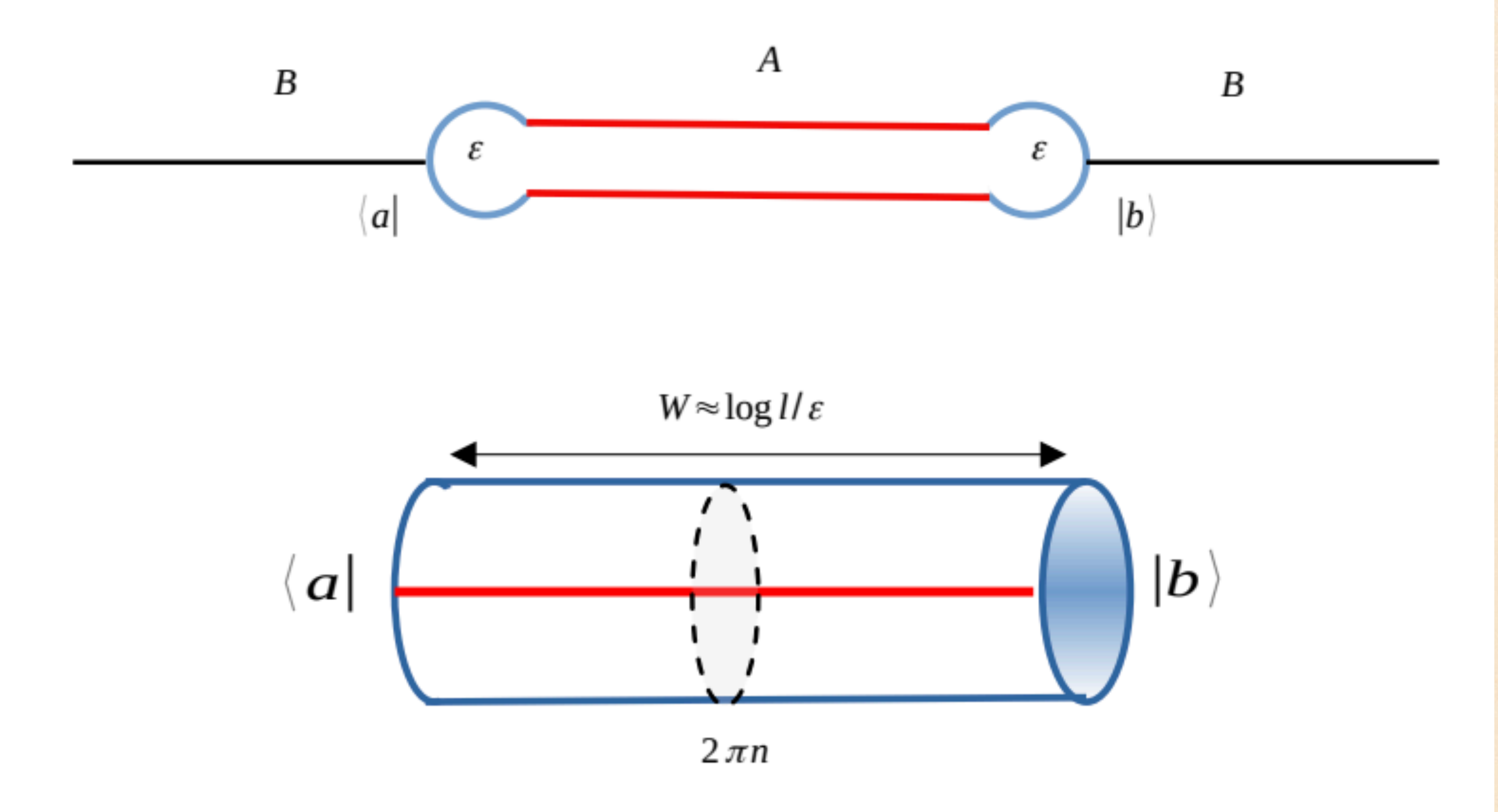
[Ohmori, Tachikawa; Northe]

$$\hat{\mathscr{L}}_i |a\rangle_g = |a\rangle_g \;\forall\; i \in G$$

- Entanglement is equipartitioned – at leading order w.r.t. ε_{UV} – among all charged sectors of the interval A, for group-like symmetries.

- What about when these operators are non-invertible?





Non-invertible symmetry in 2D rational CFTs

- Given an RCFT (finite #primaries), topological defect lines/Verlinde lines provide prototypical examples of non-invertible symmetries since these topological lines can be both invertible and non-invertible.
- For diagonal RCFTs: primaries ↔ TDLs ↔ Cardy states. Consider the 2D Ising model at criticality.
  - It has 3 TDLs:

    $\hat{\mathbf{1}}$ – line (identity) ⟷ $\mathbf{1}_{(0,0)}$

    $\hat{\eta}$ – line ($\mathbb{Z}_2$ symmetry) ⟷ $\varepsilon_{(\frac{1}{2},\frac{1}{2})}$

    $\hat{N}$ – line (KW-duality) ⟷ $\sigma_{(\frac{1}{16},\frac{1}{16})}$

  - Its fusion rules: $\hat{\eta}^2 = \hat{\mathbf{1}}$ $\quad \hat{N}^2 = \hat{\mathbf{1}} + \hat{\eta}$ $\quad \hat{\eta}\,\hat{N} = \hat{\mathbf{1}}$, implying that the N-line is non-invertible.
  - $\hat{\eta}$-line:there exists a $\mathbb{Z}_2$-symmetric Cardy state - the free boundary condition on the lattice - hence it is possible to symmetry resolve the $\hat{\eta}$-line. This is expected since this is a group-like symmetry. However, there doesn't exist any Cardy state invariant under the action of the N-line. So, we can't symmetry resolve this non-invertible line ⟷ N-line can't be gauged!

[Choi et. al.]

Result: tri-critical Ising

- Consider the tri-critical Ising model with central charge: c = 7/10. [Choi et. al.]
  - It has 6 TDLs out of which 2 are non-invertible: the *N*-line and the *W*-line.
  - There exists Cardy states which are invariant under the action of the *W*-line. In category theoretic language: the Fibonacci sub-category ($\{\mathbf{1}, \hat{W}\}$ with fusion: $\hat{\mathbf{W}}^2 = \mathbf{1} + \hat{\mathbf{W}}$) of the full tri-critical Ising category can be gauged. So, we can symmetry resolve this non-invertible line

- Entanglement is equipartitioned – at leading order w.r.t. ε_{UV} – among all the Fibonacci anyons charged sectors of the interval A! [Saura-Bastida et. al.]

Thank you for listening!!!

Extra slides...

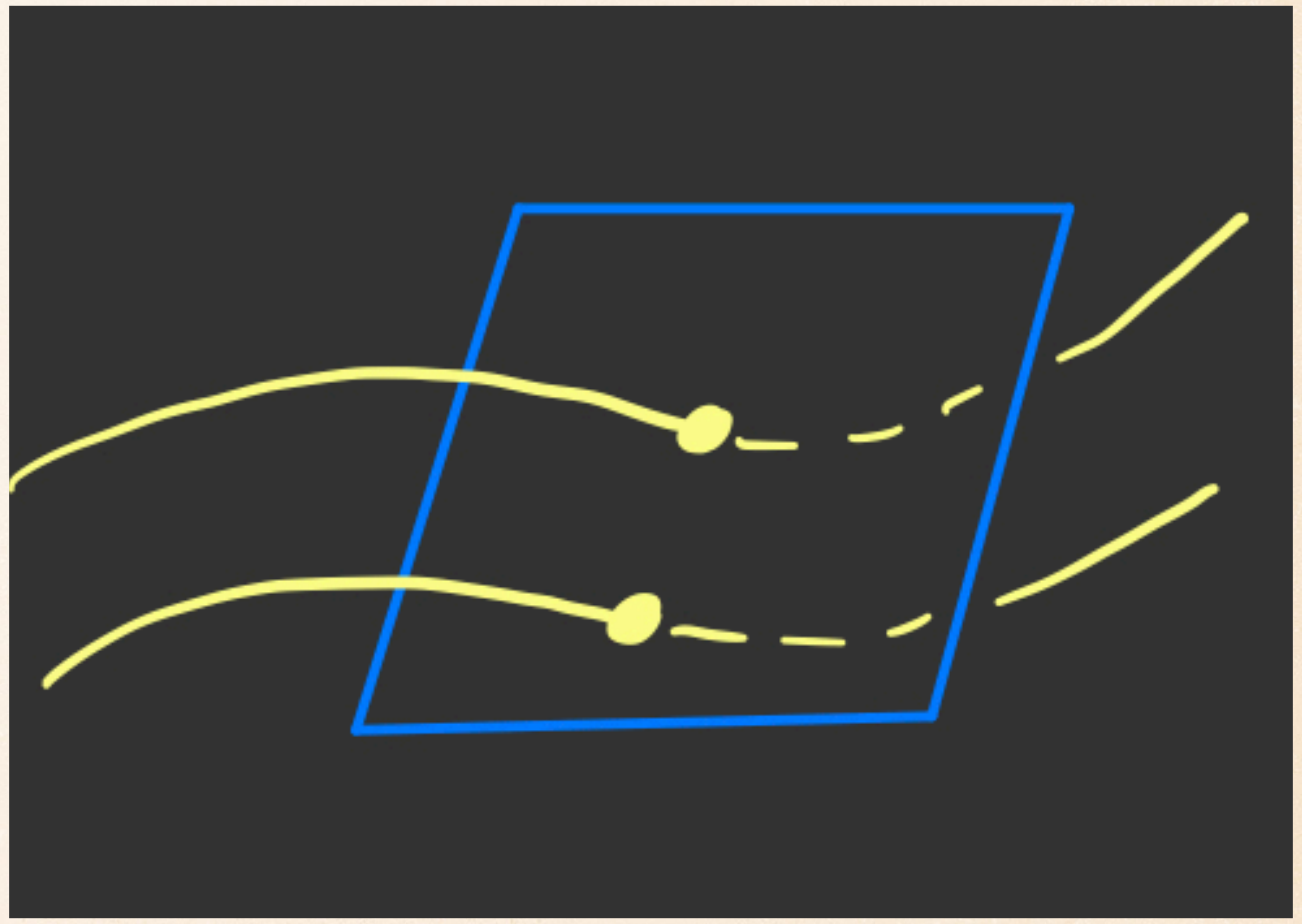
Non-invertible symmetry in QED I

- No conserved gauge-invariant current. Conserved charge?

$$Q(\mathscr{M}_3) = \int_{\mathscr{M}_3} \left(\star j_A - \frac{1}{4\pi^2} A \wedge dA \right)$$

- Try and make a topological surface operator like before:

$$U\left(\mathscr{M}_3\right) = \exp\left(\frac{i\alpha}{2} Q\left(\mathscr{M}_3\right) \right)$$



This is gauge-invariant under small gauge transformations but not under large ones, unless α is integer – but then the operator is always $\hat{1}$. Feels like no useful conserved charge…

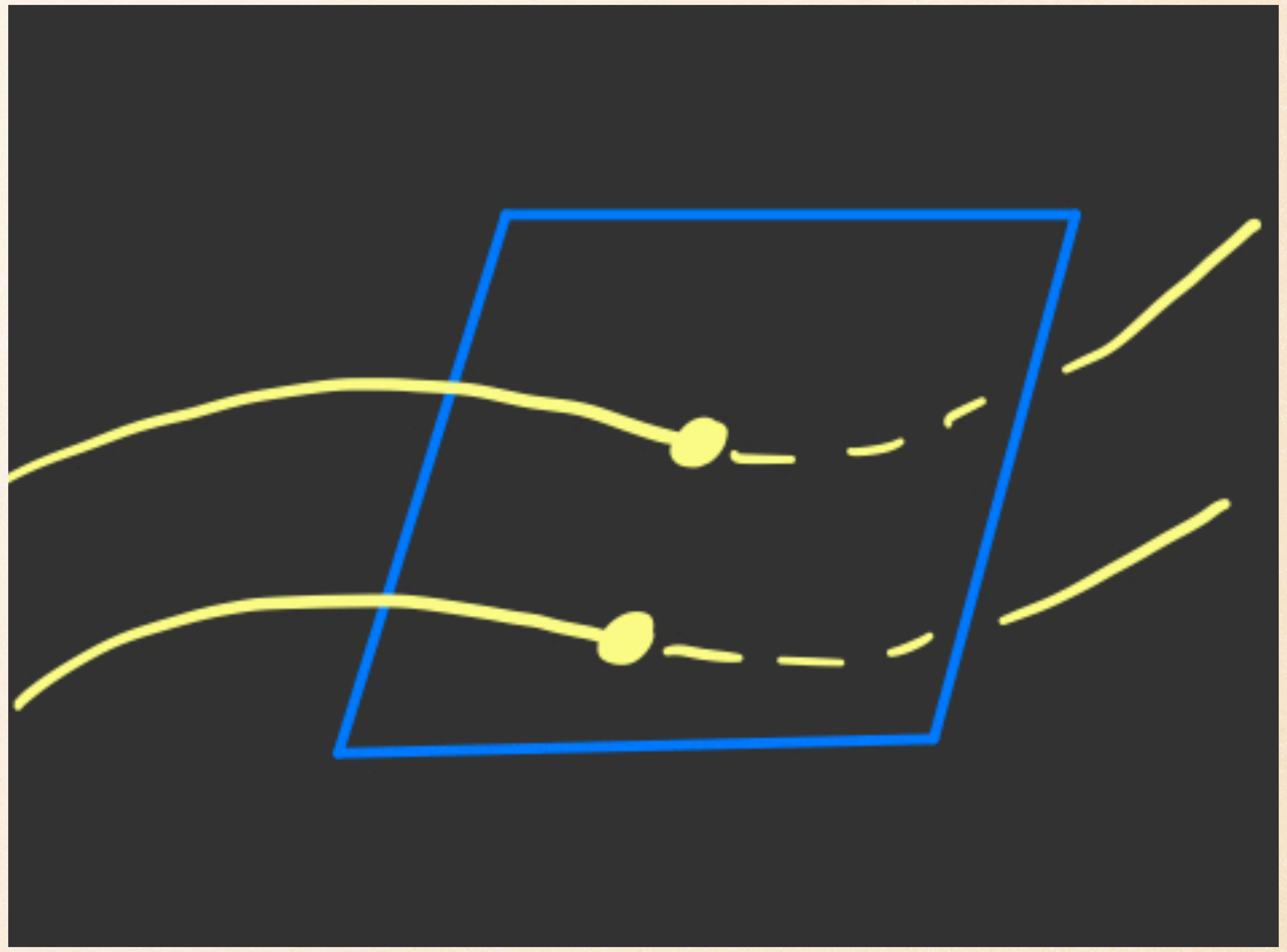
Non-invertible symmetry in QED II

- Try to make α fractional, $\alpha = 2\pi/N$:

$$\frac{2i\pi}{2N}Q(\mathscr{M}_3) = \frac{2i\pi}{2N}\int_{\mathscr{M}_3}\left(\star j_A - \frac{1}{4\pi^2}A\wedge dA\right) \qquad \ldots(5)$$

- Idea (Choi et. al., Cordova et. al.): Let us introduce a new dynamical field a only on the defect! Then we can write:

$$\frac{2i\pi}{2N}Q(\mathscr{M}_3) = \int_{\mathscr{M}_3} i\left(\frac{2\pi}{2N}\star j_A + \frac{N}{4\pi}a\wedge da + \frac{1}{2\pi}a\wedge dA\right)$$



"Integrating out a" -> results in Eq.(5): and now everything is gauge-invariant for $N \in \mathbb{Z}$!

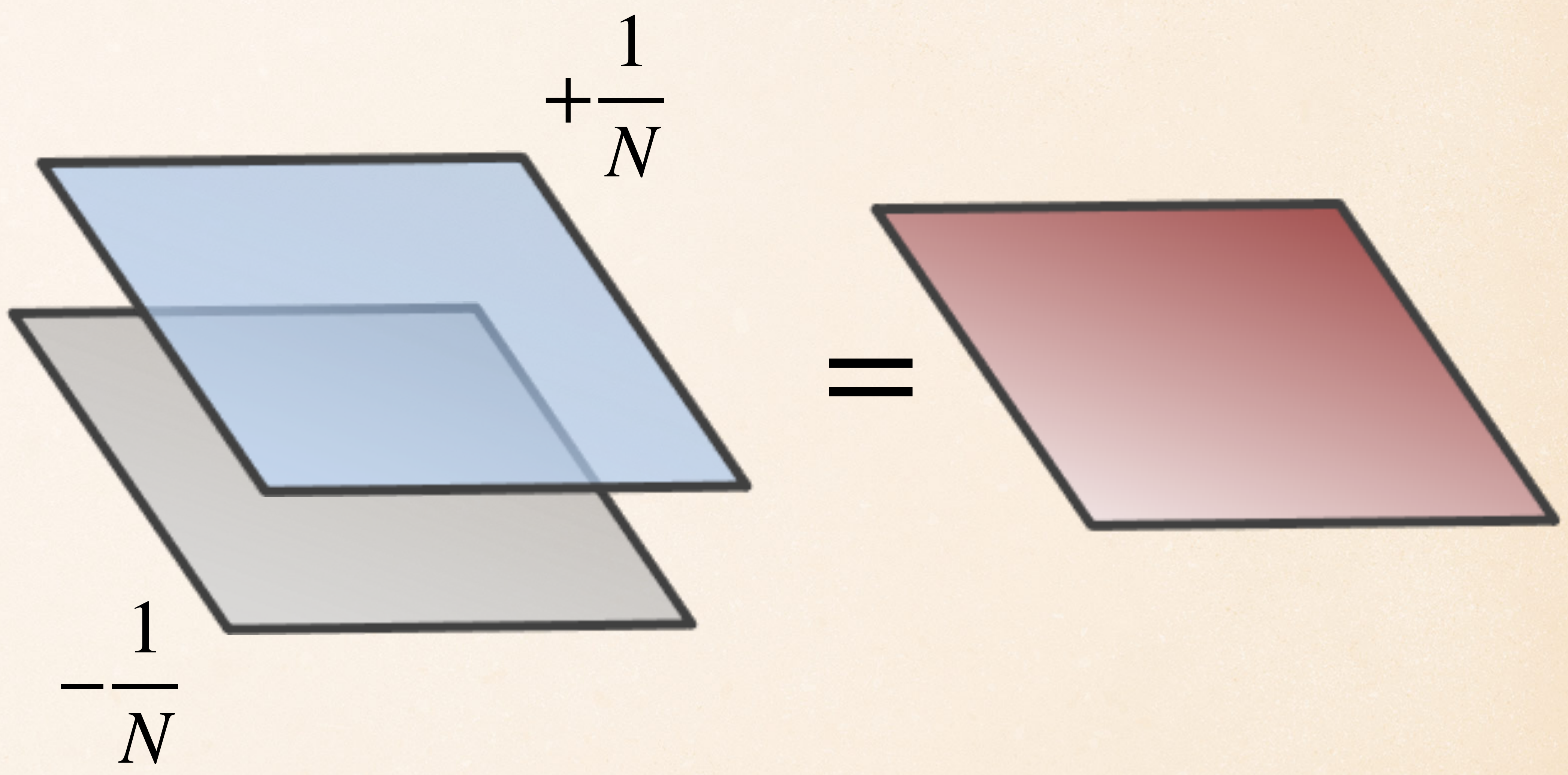
Constructed a topological charge operator, at the cost of introducing a new dynamical field a.

Properties of the defect operator

$$\frac{1}{N}Q(\mathcal{M}_3) = \int_{\mathcal{M}_3} \left(\frac{1}{N} \star j_A - \frac{N}{4\pi^2} a \wedge da + \frac{1}{2\pi} a \wedge dA \right) \qquad \psi \to e^{\frac{i}{N}\gamma^5}\psi$$

- Replace axial phase rotation $1/N$ with any rational number by using fancier TQFT. Correct way to interpret axial symmetry in QED.

- Called non-invertible symmetry: has no inverse -> acting with opposite charge doesn't give the identity operator.



No local conserved current! However, topological operators are useful – selection rules, constraints on effective actions, etc.