The Standard Model prediction of the muon anomalous magnetic moment

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STRONG 2020

Vetenskapsrådet

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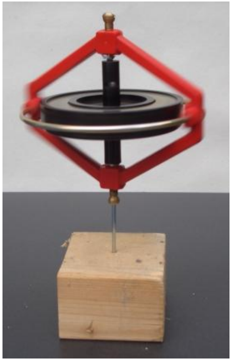
Introduction



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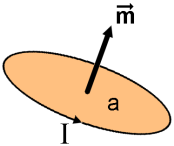
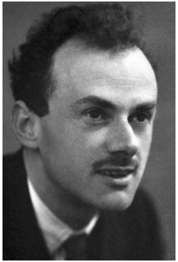
- Spin it around: angular momentum ($\vec{L}$)
- Put charges on the rim: current: will react with a magnetic field: magnetic moment $\Delta E = -\vec{\mu}\cdot\vec{B}$
- For a (single) rotating charge q: $\vec{\mu} = \frac{q}{2m}\vec{L}$
- More complicated system: $\vec{\mu} = {\color{red}g}\frac{Q}{2M}\vec{L}$

g: gyromagnetic or Landé factor



Introduction

- Now to fundamental particle physics: all fundamental particles are pointlike
- But they can have charge, angular momentum and magnetic moment
- Electron and muon have all
- Angular momentum = spin = $\vec{S} = \frac{1}{2}\hbar$
- Charge = $-e$ (basic unit of charge)
- $\vec{\mu} = -g\vec{S}$
-  Dirac (Nobel 1933): Dirac equation $g \equiv 2$

Source: Nobel Foundation Archive

Introduction

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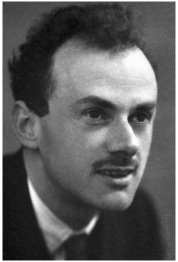
-  Dirac (Nobel 1933): Dirac equation $g \equiv 2$

`Source: Nobel Foundation Archive`

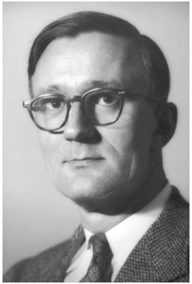
Introduction

- $g_p \neq 2$:  (Otto Stern, Nobel 1943) Proton has substructure

Source: Nobel Foundation archive

- After WWII: $a_e = 0.00119$ (Polykarp Kusch, Nobel 1955)

  (Lamb (shift) the other half)

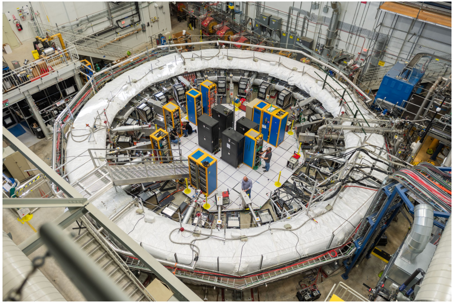


Source: Nobel Foundation archive

Introduction

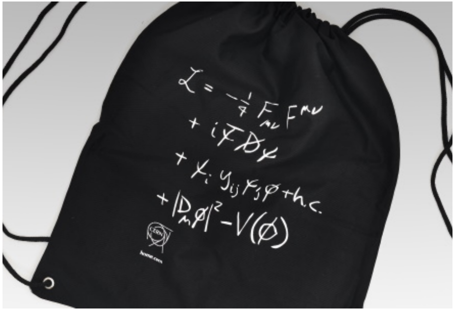
- Now known very precisely
  $g_e = 2.00231930436118(27)$ or $1.3 \cdot 10^{-13}$ (Fan, Gabrielse,. . . 2023)
- define the anomaly: $a_e = \dfrac{g_e - 2}{2}$
- $a_e = 0.00115965218062(12)$ or $1.0 \cdot 10^{-10}$ (PDG average)
- $a_\mu = 0.00116592059(22)$ or $1.9 \cdot 10^{-7}$ see previous talk
- $g_\mu = 2.00233184118(44)$ or $2.2 \cdot 10^{-10}$ (experimentalists are (too?) modest)
- Can we calculate this to the same precision?

Introduction



Source: Fermilab

$\Leftrightarrow$

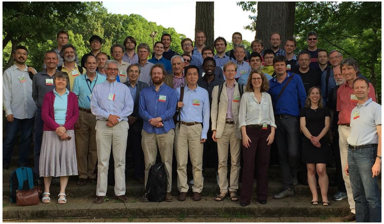


Source: CERN

Do these two agree?

How many people do we need?

Many people involved: Fermilab (St. Charles) 2017:



Berne 2023:

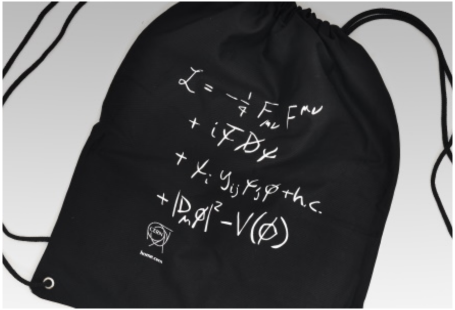
How many people do we need?

Muon g-2 Theory Initiative
`https://muon-gm2-theory.illinois.edu/`

Chaired by: Aida El-Khadra

University of Illinois
Urbana-Champaign

The Standard Model

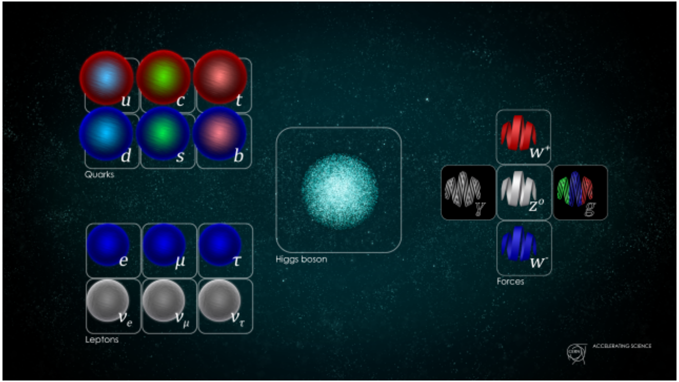


Source: CERN

- What is the Standard Model of particle physics?
- We have to go down in scale:
  - start at 1 meter
  - divide by ten a total of 18 times
  - Reach lengths scale of LHC experiments
    or alternatively about 1 TeV (fundamental) energy scale
- Matter and forces are unified: everything is a quantum field

These are what the symbols signify as well as how they interact

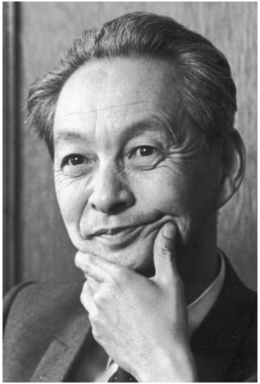
The Standard Model: particles and fields



Source: CERN

- Spin 1 fields
  - Bosons
  - Gauge fields
  - Interactions
- Spin 1/2 fields
  - Fermions
  - Quarks
  - Leptons
- Spin 0 field:
  - Higgs boson

The Standard Model: Quantum-Electro-Dynamics (Nobel 1965)



`Source: Nobel Foundation archive`

Sin-Itiro Tomonaga

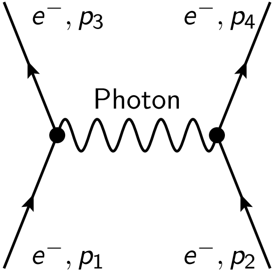
Julian Schwinger

Richard P. Feynman

$$\text{QED: } -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}_e\gamma^\mu\left(\partial_\mu - ieA_\mu\right)\psi_e - m_e\overline{\psi}_e\psi_e$$

The Standard Model: Feynman Diagrams

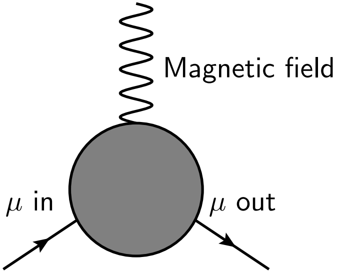
- QED: $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}_e\gamma^\mu\left(\partial_\mu - ieA_\mu\right)\psi_e - m_e\overline{\psi}_e\psi_e$
- Feynman diagrams: correspond to amplitude (quantum field theory)



$$\Longrightarrow \quad \overline{\psi}_e(p_3)\gamma_\mu\psi_e(p_1)\frac{1}{(p_1-p_3)^2}\overline{\psi}_e(p_4)\gamma_\mu\psi_e(p_2)$$

- More complicated diagrams: (much) more complicated expressions

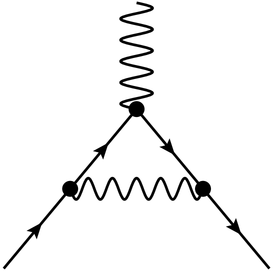
Contributions



- The blob can get very complicated
- QED contribution: only γ, e, μ, τ (photons and charged leptons)
- Electroweak: add W, Z,Higgs,neutrinos
- Hadronic: add quarks and gluons
  Two main parts: HVP and HLbL

QED contribution

- Main diagram: (four in total)

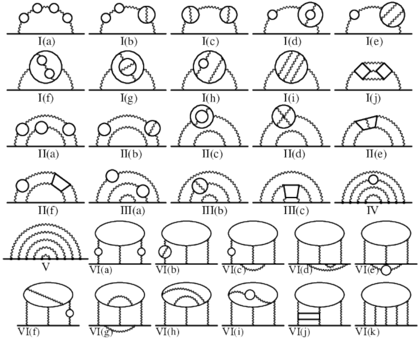


- Schwinger (1947): $a_e (= a_\mu) = \dfrac{\alpha}{2\pi}$

Jacob

QED contribution

- one-loop: 1 diagram
- Two-loop: 4 diagrams Petermann 1957, Sommerfield 1958 a_e, a_μ start to differ
  - $a_e = 0.00115965218062(12)$
  - $a_\mu = 0.00116592059(22)$
- Heroic effort at higher orders:
  - 3-loop: 72 diagrams (known analytically)
  - 4-loop: 891 diagrams (essentially known analytically)
  - 5-loop: 12672 diagrams (only known numerically)

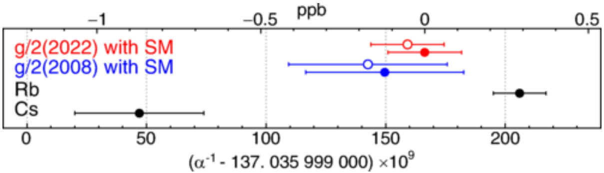


`Source: Cornell`

Toichiro Kinoshita

QED contribution: people

- Kinoshita, Nio, Aoyama, Cvitanovic, Lindquist, Remiddi, Laporta, Hayakawa, Volkov, Passera,…
- $a_e = C_1 \frac{\alpha}{\pi} + C_2 \left(\frac{\alpha}{\pi}\right)^2 + C_3 \left(\frac{\alpha}{\pi}\right)^3 + C_4 \left(\frac{\alpha}{\pi}\right)^4 + C_5 \left(\frac{\alpha}{\pi}\right)^5 + \cdots$
- C_i known to sufficient precision (some discrepancies in C_5)
- Problem: need a value for α to sufficient precision (atomic physics)
-
    - $1/\alpha = 137.035999206(11)$ (Rubidium, Paris 2020)
    - $1/\alpha = 137.035999046(27)$ (Cesium, Berkeley 2018)
    - $1/\alpha = 137.035999166(15)$ (from a_e)



Source: Phys. Rev. Lett. 130, 071801

- Sometimes in theory experiment is the problem

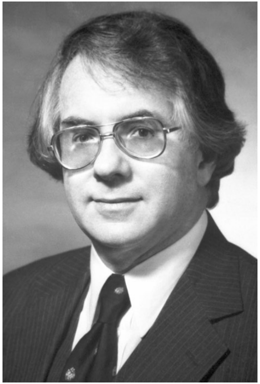
QED contribution

- No problem for $QED\ a_\mu$
- $a_\mu = 0.00116584718931(30)$ (QED, White paper)
- $a_\mu = 0.00116592059(22)$ (experiment)
- $a_\mu = 0.00000007340(22)$ (experiment-QED)

- Largest part explained by QED
- From now on will work in 10^{-11} units
- The $7340 \cdot 10^{-11}$ to be explained from elsewhere

Electroweak

Electroweak theory introduced by (Nobel 1979)



Sheldon Lee Glashow

Abdus Salam

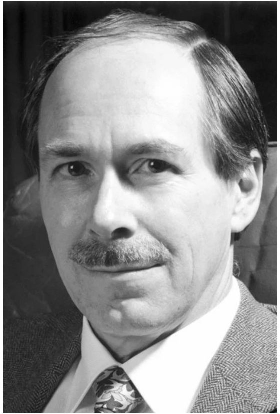
Steven Weinberg

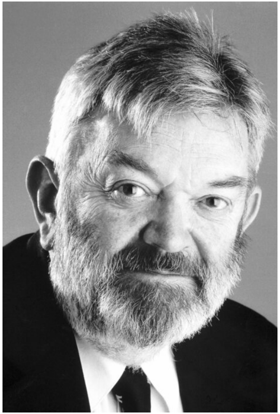
`Source: Nobel Foundation archive`

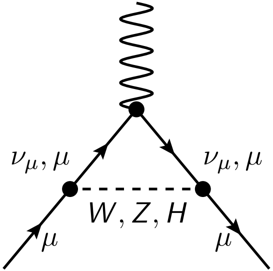
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad \rightarrow \quad W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - ig_W \left(W_\mu W_\nu - W_\nu W_\mu \right)$$

Electroweak

Electroweak theory valid at loop level (Nobel 1999)





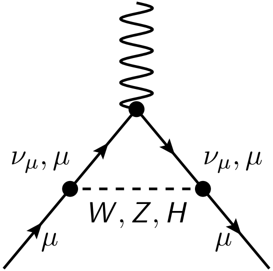


`Source: Nobel Foundation archive`

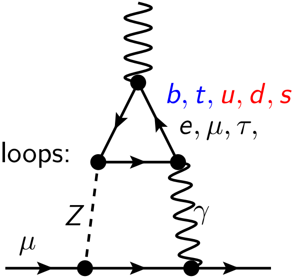
Gerardus 't Hooft

Martinus J.G. Veltman

Electroweak

- 

- Calulated by many people 1972 and on
- Jackiw, Weinberg, Fujikawa, Lee, Sanda, Bardeen, Gastmans, Lautrup
- Modern input: $a_\mu = 194.79(1) \cdot 10^{-11}$

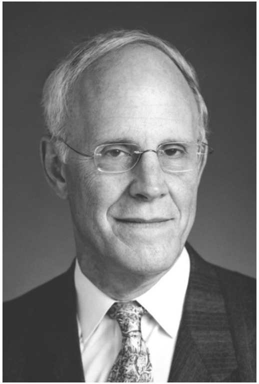
- Two loops: 

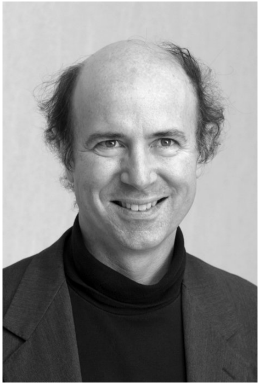
- Large logarithms (many mass scales)
- Quarks at low energy (nonperturbative)
- Axial anomaly
- Czarnecki, Vainshtein, de Rafael, Knecht,...
- Modern input: $a_\mu = -41.2(1.0) \cdot 10^{-11}$

- Add some more small (White paper): $a_\mu = 153.6(1.0) \cdot 10^{-11}$

Why are quarks difficult?

- Quantum Chromodynamics: very similar to QED and Electroweak at gauge level: Gell-Mann, Fritzsch, Leutwyler
- But asymptotic freedom and infrared slavery (confinement) (Nobel 2004)



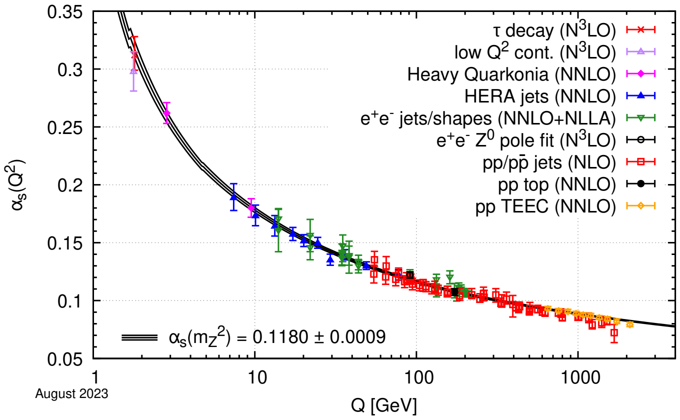


`Source: Nobel Foundation archive`

David J. Gross H. David Politzer Frank Wilczek

Why are quarks difficult?

- But asymptotic freedom and infrared slavery



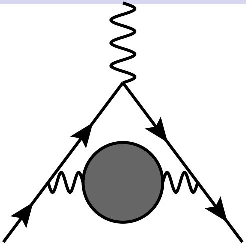
Source: pdg.lbl.gov

- Can use perturbation theoy at larger Q, but not at the lower Q

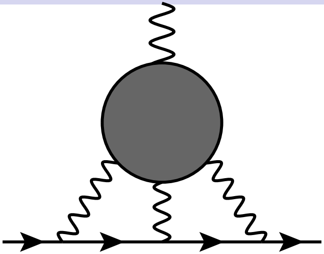
What to do then?

- “Standard” theory
  - Use other theoretical methods: dispersion theory, with (lots of) experimental input
  - All possible QCD based constraints one can think of
  - In the worst case: use models (but in a smart way)
- Brute force: lattice QCD (but needs a lot of thinking too)

Hadronic contributions



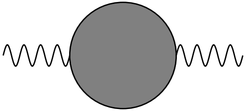
LO-HVP



HLbL

- Muon and photon lines, representative diagrams
- The blobs are hadronic contributions
- There are higher order contributions of both types: known accurately enough
- $a_\mu^{HVP} = 6845(40)\ 10^{-11}$ (LO+NLO+NNLO)(White paper; error has increased)
- $a_\mu^{HLbL} = 92(18)\ 10^{-11}$ (LO+NLO)(White paper)
- $a_\mu^{exp} - a_\mu^{QED} - a_\mu^{EW} = 7186(22) \cdot 10^{-11}$
- Difference: $\boxed{\Delta a_\mu = 249(49) \cdot 10^{-11}}$

HVP

-  $=$ Two-point function of two electro-magnetic currents Π
- Integrate over a weight function
- Can do that in:
  - Minkowski momentum space (dispersive approach)
  - Euclidean momentum space (early lattice QCD and MUonE)
  - Euclidean space (in principle lattice QCD)
  - Time-momentum representation (mixed; present lattice QCD)
- These are all related due to the analyticity property of two-point functions
- Simple: only one variable
- Problem: need 0.3% precision to match experimental a_μ

Dispersive: idea

- $a_\mu^{LO-HVP} = \frac{\alpha^2}{3\pi^2}\int_0^\infty ds\,\frac{m_\mu^2}{3s^2}\hat{K}(s)R(s)$
- $\hat{K}(s)$ goes smoothly from 0.63 at threshold to 1 at large s
- $R(s) = \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)_{LO}}$ (R-ratio)
- large s: use perturbative QCD (from which s on?)
- low-to-medium s: just use data

Dispersive: some of the main people involved

`Source: IN2P3`

Michel Davier

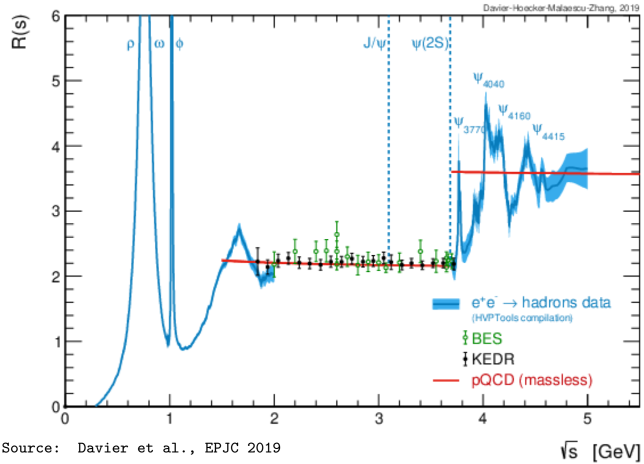
`Source: Humboldt universität`

Fred Jegerlehner

`Source: University of Liverpool`

Thomas Teubner

$R(s)$: overview of data

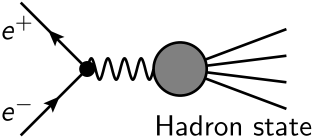


Source: Davier et al., EPJC 2019

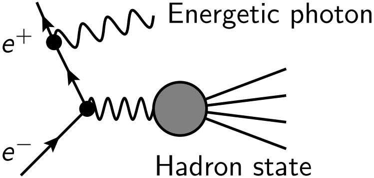
- Above 5 GeV: perturbative QCD, and integrate over Υ resonances
- 3-5 GeV: Integrate over ψ resonances, data and perturbative QCD
- 2-3 GeV: data and pQCD
- Below 2 GeV: data in exclusive channels

Exclusive data: two main methods and an older revival

- Direct measurements

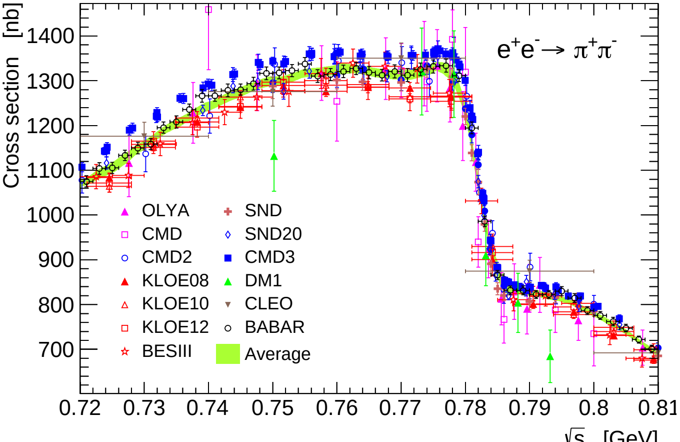


  - SND,CMD-2 at Novosibirsk
  - Luminosity and efficiency determination at each value of s
- Radiative return



  - KLOE/KLOE-2 Frascati (1.02 GeV), Babar SLAC (10 GeV), BES-III Beijing (4 GeV), Belle II KEK (10 GeV)
  - Luminosity and efficiency determination to be done once
  - Modelling radiative corrections and interference of photon from hadrons or e^+e^-
- $\tau \to \nu_\tau$+hadrons
  - Modelling isospin corrections to $e^+e^- \to$hadrons

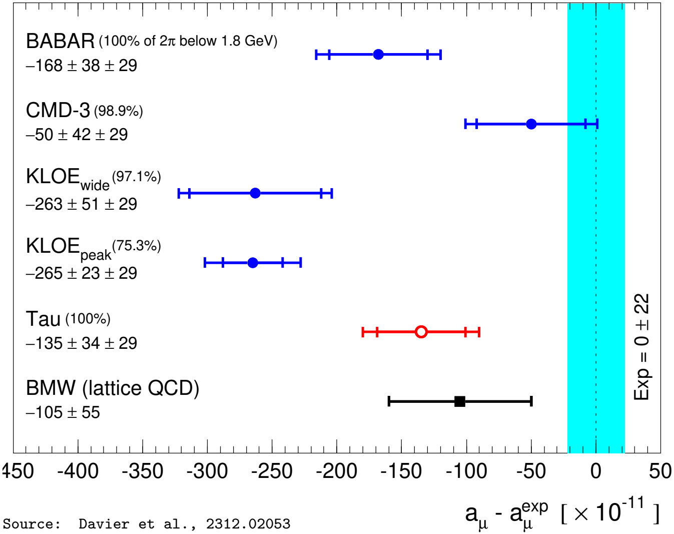
$e^+e^- \to \pi^+\pi^-$ largest contribution and error source



Source: Davier et al., 2312.02053

- 3 main experiments: KLOE, Babar, CMD-3
- Flat disagreement outside quoted errors
- No reason for the disagreement found despite many private/public discussions
- Dispersive error will need increasing

$e^+e^- \to \pi^+\pi^-$ largest contribution and error source

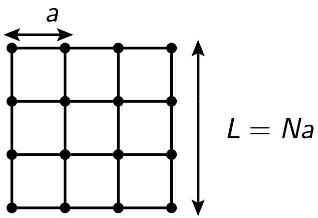


Source: Davier et al., 2312.02053

- Combining needs care
- χ^2 inflation needed
- CMD-3 disagrees with all previous measurements
- White paper update in progress
- White paper $-249 \pm 43 \pm 29$

Lattice QCD

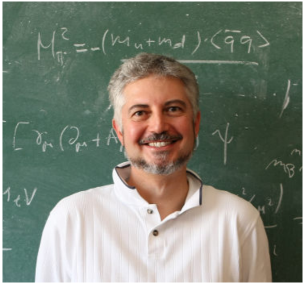
- Discretize and euclideanize space-time



- Quarks on the vertices, gluons and photons on the links
- Limits needed: $a \to 0$, $L \to \infty$, quark masses → physical
- determine a in physical units (scale setting)
- Largest lattices now $96^3 \times 144$ or about $2 \cdot 10^9$ degrees of freedom
- Do the Feynman path integral numerically
- BMW2020 (after White paper) 0.8% precision (Budapest-Marseille-Wuppertal)

A few of the people

Laurent Lellouch, Antoine Gerardin, Ruth Van de Water, Christine davies, Christoph Lehner, Harvey Meyer,. . .



Source: CPT

Laurent Lellouch

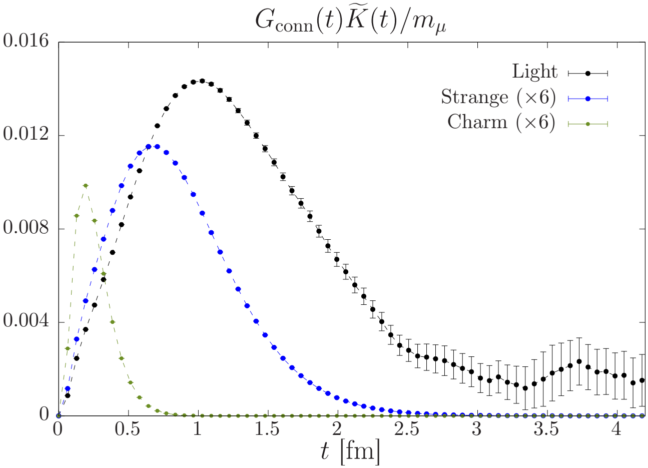
Source: LinkedIn

Ruth Van de Water

Source: University of Glasgow

Christine Davies

The difficulties



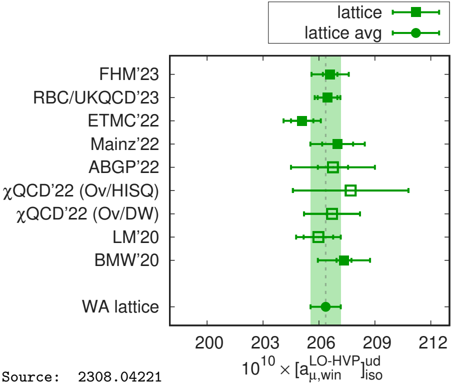
Source: Phys.Rev.D 100 (2019) 1, 014510

- $a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)\int_0^\infty dt K(t) G(t)$
- $G(t) = -\frac{1}{3}\sum_{k=1}^{3}\sum_{\vec{x}} J_k(t,\vec{x})J_k(0)$
- Long distance difficult noise/lattice size
- Short distance: lattice artefacts
- Intermediate: very precise

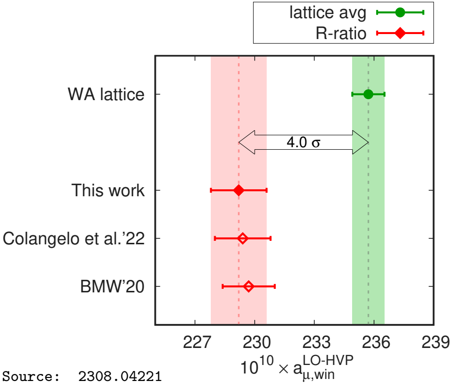
Results

- Finite size effects
- Continuum extrapolation ($a \to 0$)
- Electromagnetic corrections
- Quark masses (accurately and $m_u \neq m_d$)
- Noise at long distances
- Disconnected contributions
- Only BMW has it all at precision below 1%
- Others are compatible but larger errors
- Connected, light quarks, intermediate distances: can compare (window observables)

Comparison of intermediate window



`Source: 2308.04221`

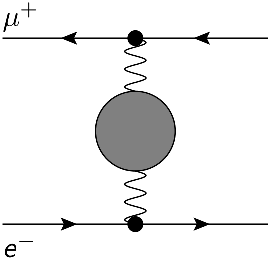


`Source: 2308.04221`

- Very good agreement on the light quark connected intermediate window (but it is where the lattice is best)
- 4σ disagreement with dispersive for this (not including CMD3 from 2023)

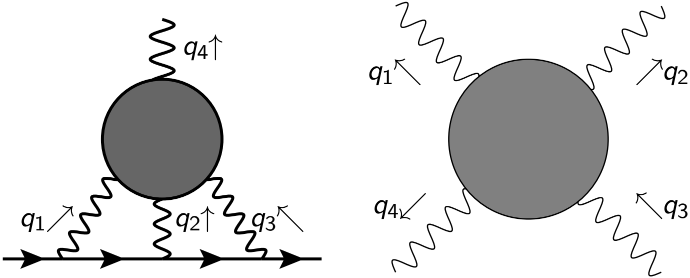
MUonE: measure $\mu e \to \mu e$

- MUonE: measure $\mu e \to \mu e$ very precisely



- Needs to be measure very precisely
- Need all other contributions calculated very precisely
- Test have been done at CERN

HLbL: Hadronic light-by-light

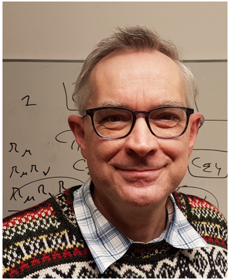


- $=\Pi^{\mu\nu\lambda\sigma}(q_1,q_2,q_3)$ of four vector currents (not two)
- 6 variables (not just one)
- Actually we really need $\left.\frac{\delta\Pi^{\mu\nu\lambda\sigma}(q_1,q_2,q_3)}{\delta q_{4\rho}}\right|_{q_4=0}$
- Mixed: q_4 at zero, q_1^2, q_2^2, q_3^2 so three-variables, or Q_1^2, Q_2^2, Q_3^2 $(q_i^2 = -Q_i^2)$
- Models, Dispersive methods, Lattice QCD

HLbL dispersive: some of the people

Source: Cornell

Toichiro Kinoshita



Source: Lund University

Johan Bijnens

University of Minnesota

Arkady Vainshtein

Source: Universität Bern

Gilberto Colangelo

and de Rafael, Prades, Hoferichter, Procura, Stoffer, Roig, Sanchez-Puertas, Rodriguez-Sanchez, Hermansson-Truedsson, Rebhan, Leutgeb, Holz,. . .

HLbL dispersive history

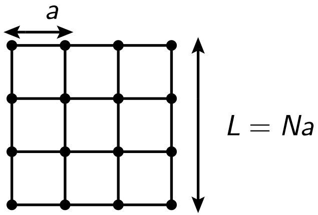
- late 1990s: two groups (Kinoshita, Bijnens); models and physics sense: $90(30) \cdot 10^{-11}$ after counting proposed by de Rafael
- Lots of work on the single pion exchange 2000-2015 (Knecht, Nyffeler,. . .)
- Start of connection with QCD (Melnikov, Vainshtein)
- Always a problem of separating contributions
- Breakthough in 2015: how to do dispersive consistently (Colangelo,. . .)
- Also connection to short-distance major progress (Bijnens, Hermansson-Truedsson, Rodriguez-Sanchez)
- Main remaining: 3 pion and medium mass resonances: much work in progress

Contributions HLbL White paper

- "Long distance": under good control
  - Dispersive method: Berne group around G. Colangelo
  - π^0 (and η, η') pole: $93.8(4.0) \cdot 10^{-11}$ S.Leupold
  - Pion and kaon box (pure): $-16.4(2) \cdot 10^{-11}$
  - $\pi\pi$-rescattering (include scalars below 1 GeV):$-8(1) \cdot 10^{-11}$
- Charm (beauty, top) loop: $3(1) \cdot 10^{-11}$
- "Short and medium distance"
  - Scalars, tensors: $-1(3) \cdot 10^{-11}$
  - Axial vector: $6(6) \cdot 10^{-11}$
  - Short-distance: $15(10) \cdot 10^{-11}$
- $a_\mu^{HLbL} = 92(19) \cdot 10^{-11}$
- Since then:
  - Short distance constraints improved
  - Axial vectors better understood
  - Work in progress to put all together better

HLbL Lattice QCD

- Discretize and euclideanize space-time

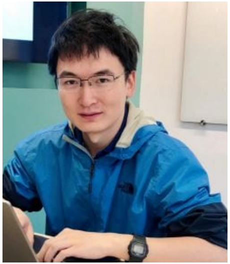


- Quarks on the vertices, gluons and photons on the links
- Limits needed: $a \to 0$, $L \to \infty$, quark masses → physical
- determine a in physical units (scale setting)
- Largest lattices now $96^3 \times 144$ or about $2 \cdot 10^9$ degrees of freedom
- Do the Feynman path integral numerically
- But now needs an integration over three variables and a four-point function
- Tour de force but it got done!!!

HLbL Lattice QCD: some of the people

`Source: UCONN`

Tom Blum



`Source: UCONN`

Luchang Jin

`Mainz University`

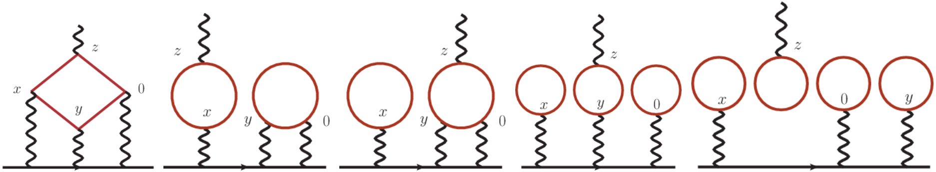
Harvey Meyer

`A. Gerardin`

Antoine Gerardin

But of course many more

HLbL Lattice QCD



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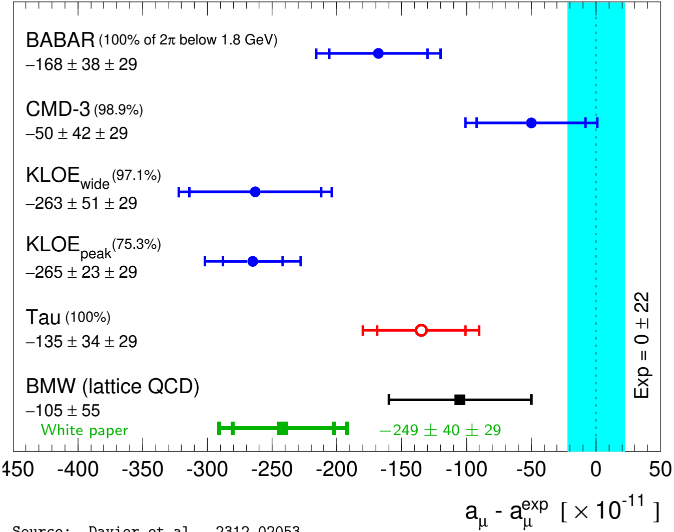
- Two independent groups (similar methods), latest results
- RBC/UKQCD $124.7(11.5) \cdot 10^{-11}$
- Mainz $106.8(14.7) \cdot 10^{-11}$
- Dispersive $92(19) \cdot 10^{-11}$

- Other lattice methods: calculate formfactors needed in the dispersive method $\pi^0, \eta \to \gamma^* \gamma^*$

Final plot



Source: Davier et al., 2312.02053

- One large source of uncertainty
- Situation unclear but SM prediction typically low

An order of magnitude estimate for heavy contributions

- Let's make an order of magnitude estimate for a_μ
- $|\vec{\mu}| \equiv 2(1+a_\mu)\frac{e\hbar}{2m_\mu}$ so $a_\mu \propto m_\mu$
- Magnetic interaction: spin flips $\Rightarrow$ (usually) one more factor of m_μ
- $1/(16\pi^2)$ Loop factor
- coupling constants: g_{BSM}^2
- make dimensionless $1/M_{BSM}^2$
- $a_\mu \approx \frac{g_{BSM}^2}{16\pi^2}\frac{m_\mu^2}{M_{BSM}^2}$
- Plug in m_W,g_W: $a_\mu^{EW} \approx 400 \cdot 10^{-11}$
- If discrepancy: new physics at few 100 GeV scale (but other options exist)

Conclusions

- The Standard Model prediction for a_μ is the work of $\gg 100$ people
- Large investment both in experiment and theory
- QED, EW and the higher order hadronic: under control
- HLbL: dispersive and lattice QCD in decent agreement and improvements to be expected
- HVP: one full Lattice QCD calculation only, dispersive has problems with experimental inputs but much work ongoing