

# Perspectives from lattice QCD with exascale supercomputers



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04/03/24 - Workshop, Warsaw



**EuroHPC**  
Joint Undertaking

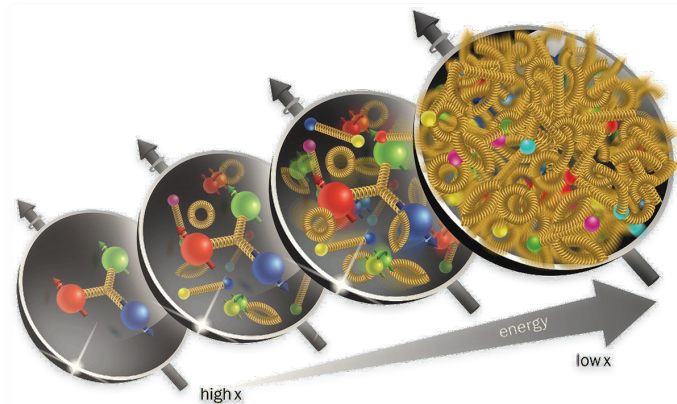
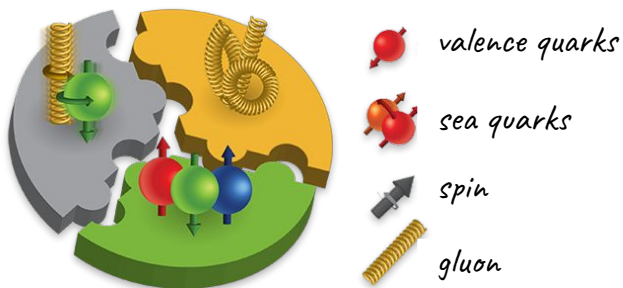


This project has received funding from the European High-Performance Computing Joint Undertaking (JU) under grant agreement No 101118139. The JU receives support from the European Union's Horizon Europe Programme.

# Overview



- State-of-the-art in Lattice QCD: A look at nucleon matrix elements
- State-of-the-art in Hyperons: A brief review of the available results
- Proposal #1: Diagonal and transition matrix elements of the baryons octet
- Proposal #2: Non-leptonic two-body decays



# Lattice QCD: Why? How?



Why?

The **Lattice Regularization** is the only-known, ab-initio, and model-independent approach for studying a quantum field theory non-perturbatively, such as QCD at low energies

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \sum_f \bar{\psi}_f (i\gamma^\mu D_\mu - m_f) \psi_f$$

How?



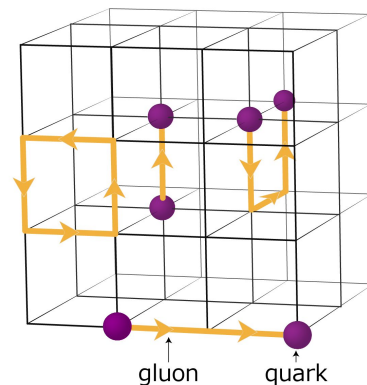
K. G. Wilson  
1974

## From QCD

- Continuous space-time
- Minkowski space-time
- Lie algebra  $A_\mu(x)$
- Lagrangian  $\mathcal{L}_{\text{QCD}}(g, \vec{m}_f)$

## To Lattice QCD

- 4D hypercubic lattice  $a, V$
- Euclidean space-time
- Lie group  $U_\mu(x) = e^{igaA_\mu(x)}$
- Lattice action  $S(a, \vec{\mu}_f)$



$$\langle \mathcal{O} \rangle_{a, V, \vec{\mu}_f} = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U]^V \mathcal{D}[\psi \bar{\psi}]^{V n_f} \mathcal{O}(U, \psi, \bar{\psi}) e^{-S(a, \vec{\mu}_f; U, \psi, \bar{\psi})}$$

# Lattice QCD: Why? How?



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K. G. Wilson  
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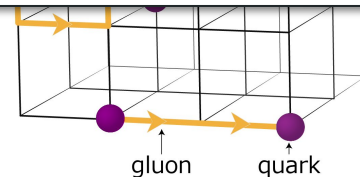
*From QCD*

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$$\langle \mathcal{O} \rangle = \lim_{\substack{\vec{\mu}_f \rightarrow \vec{m}_f \\ V \rightarrow \infty \\ a \rightarrow 0}} \langle \mathcal{O} \rangle_{a, V, \vec{\mu}_f}$$



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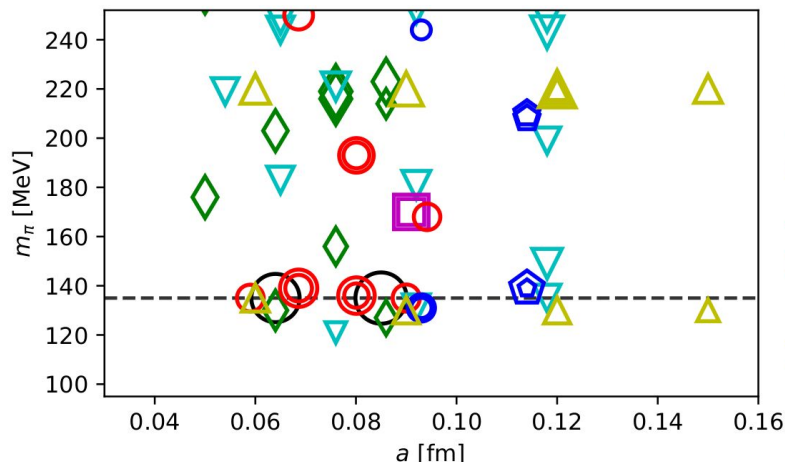


# State-of-the-art in Lattice QCD

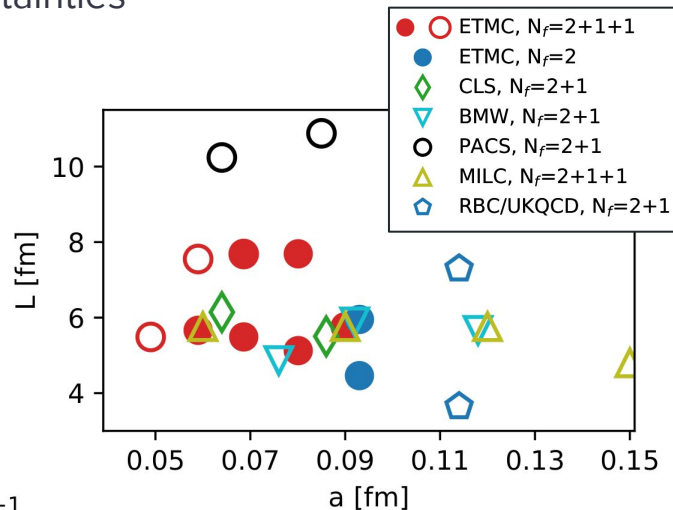
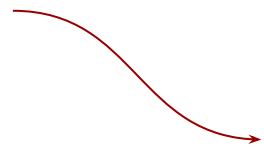


The study of nucleon matrix elements is one of the most advanced:

- Many collaborations are able to control all systematic uncertainties
  - Ensembles at physical quark masses
  - Continuum limit and infinite volume limit
  - Study of excited state contaminations



- ETMC,  $N_f=2+1+1$
- ETMC,  $N_f=2$
- ◇ CLS,  $N_f=2+1$
- ▽ BMW,  $N_f=2+1$
- JLab/W&M/LANL/MIT,  $N_f=2+1$
- PACS,  $N_f=2+1$
- △ MILC,  $N_f=2+1+1$
- ◇ RBC/UKQCD,  $N_f=2+1$

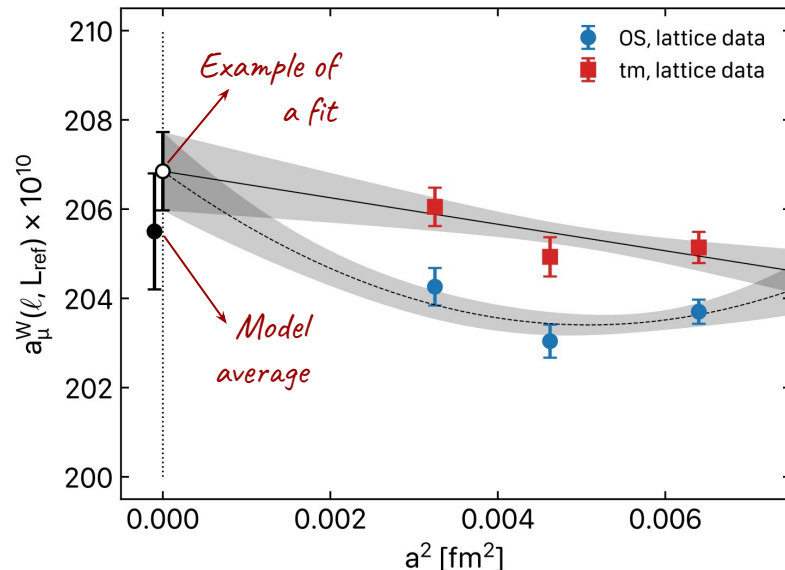
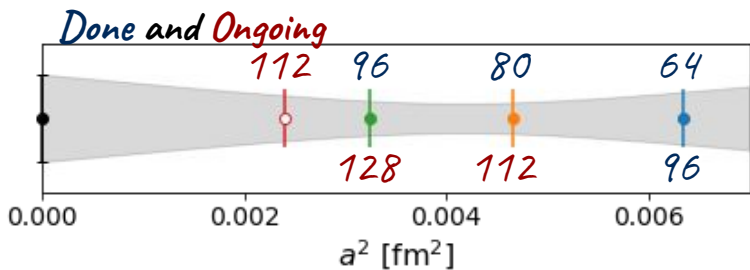
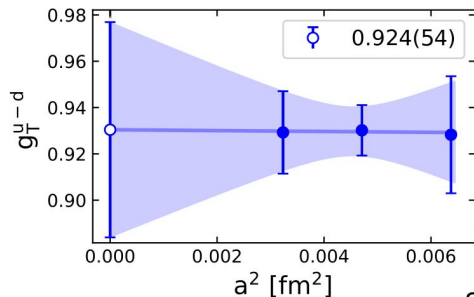


# Ongoing challenge: the continuum limit

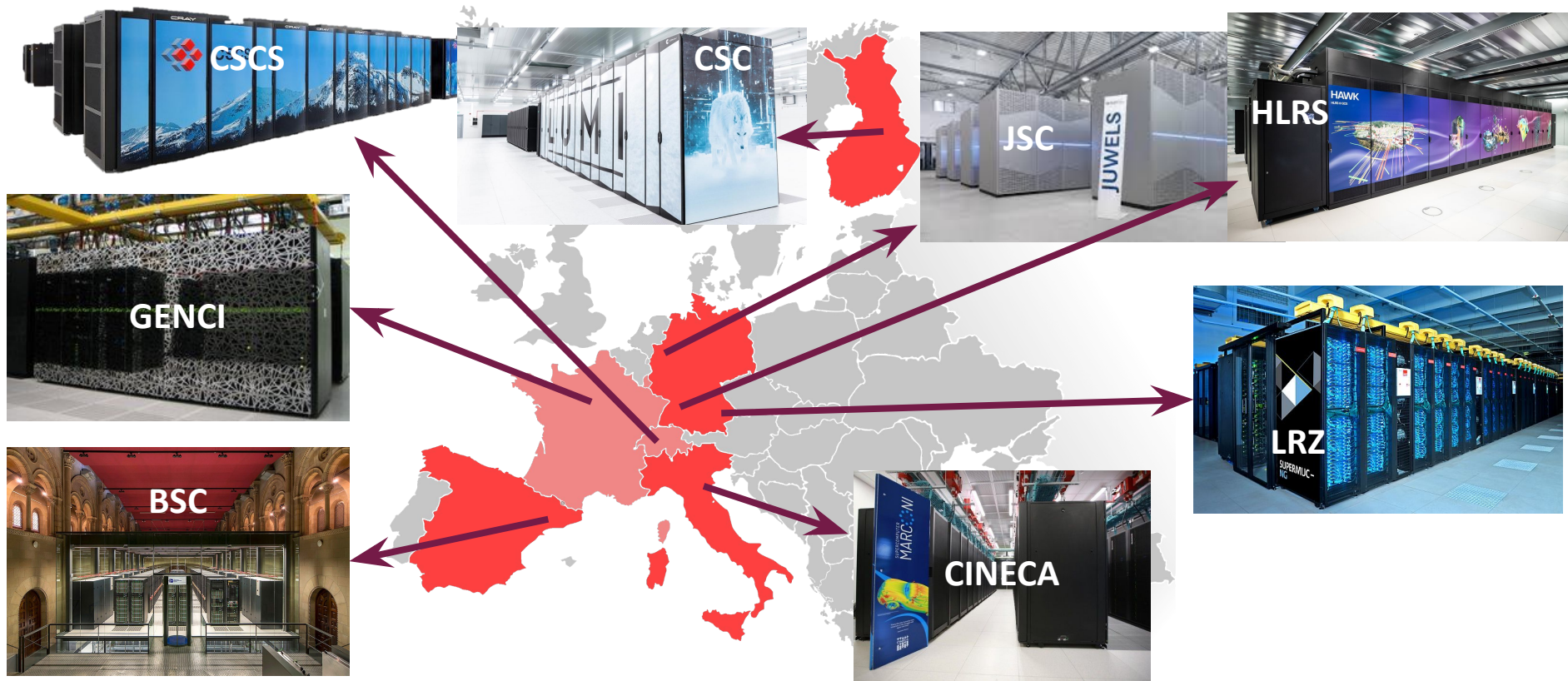
$$\langle \mathcal{O} \rangle = \lim_{\substack{\vec{\mu}_f \rightarrow \vec{m}_f \\ V \rightarrow \infty \\ a \rightarrow 0}} \langle \mathcal{O} \rangle_{a, V, \vec{\mu}_f}$$

$\checkmark \vec{\mu}_f \rightarrow \vec{m}_f$   
 $! V \rightarrow \infty \quad t \rightarrow \infty$   
 $a \rightarrow 0$

- Currently errors are dominated by the continuum extrapolation
- Ongoing new ensemble at  $a=0.05 \text{ fm}$
- Due to critical slowing down, it is probably the finest we can go with current algorithms!



# Supercomputers in Europe



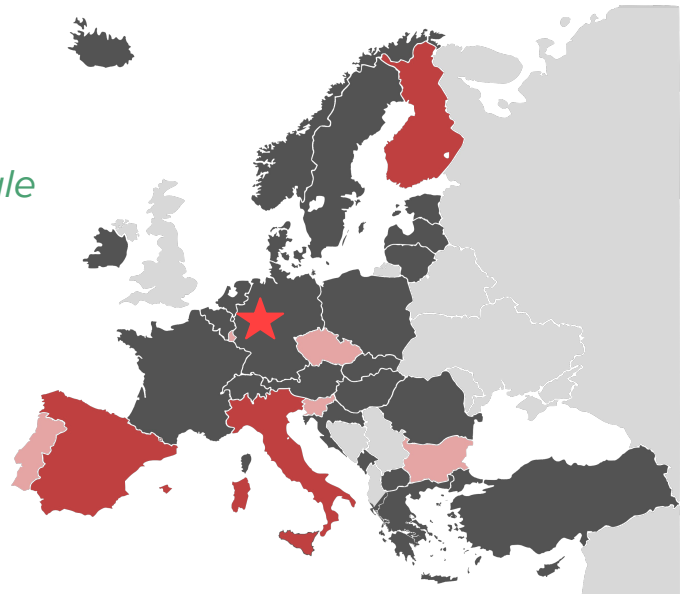
# EuroHPC Joint Undertaking

*The European High Performance Computing Joint Undertaking (EuroHPC JU) is pooling European resources to buy and deploy top-of-the-range supercomputers and develop innovative exascale supercomputing technologies and applications.*

The JU is currently supporting two main activities:

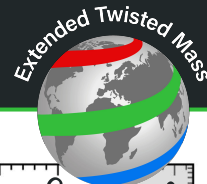
- Developing a pan-European supercomputing infrastructure:
  - **5 PetaFlop machines** in Bulgaria, Czech, Luxembourg, Slovenia, Portugal
  - **3 Pre-Exascale machines** with over 200 PetaFlops: Lumi in Finland, Leonardo in Italy and Marenostrom 5 in Spain
  - **2 Exascale machines:** JSC in Germany, TBD

Upcoming Budget (2021 - 2033): 8 billion Euro



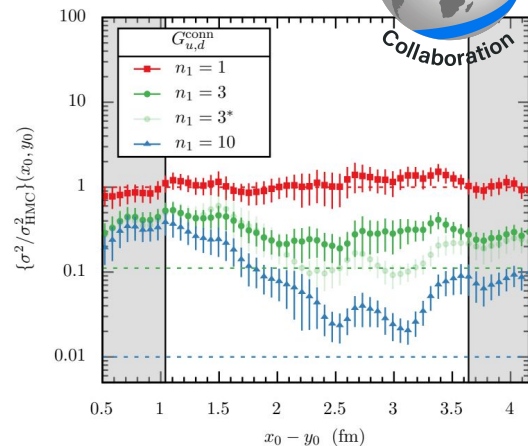
**EuroHPC**  
Joint Undertaking

# Upcoming algorithm developments

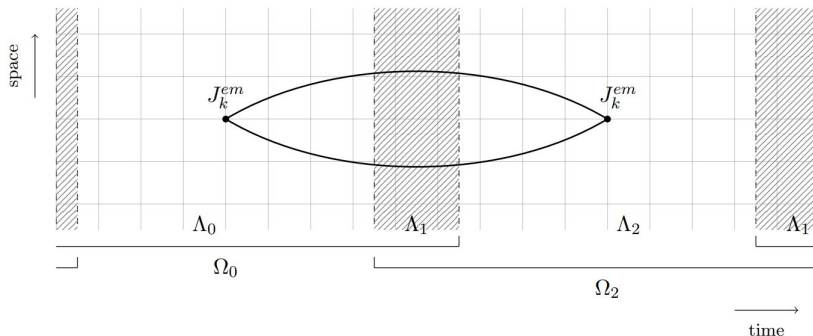


Multilevel algorithms are novel noise-reduction techniques

- M. Luscher and P. Weisz, “Locality and exponential error reduction in numerical lattice gauge theory”, JHEP09 (2001), 010 [arXiv:hep-lat/0108014 [hep-lat]].
- M. Ce’, L. Giusti and S. Schaefer, “Domain decomposition, multi-level integration and exponential noise reduction in lattice QCD,” Phys. Rev. D 93 (2016) no.9, 094507, [arXiv:1601.04587 [hep-lat]].
- M. Dalla Brida, L. Giusti, T. Harris, and M. Pepe, “Multi-level Monte Carlo computation of the hadronic vacuum polarization contribution to  $(g \mu^{-2})$ ,” Phys. Lett. B 816 (2021), 136191, [arXiv:2007.02973 [hep-lat]].



**It will be critical in reaching large source-sink separation in the calculation of matrix elements and to suppress the excited state contamination, a major systemic effect!**



Funded by  
the European Union



EuroHPC  
Joint Undertaking



# Example: the weak axial-vector matrix element



The transition matrix element of the neutron  $\beta$ -decay is

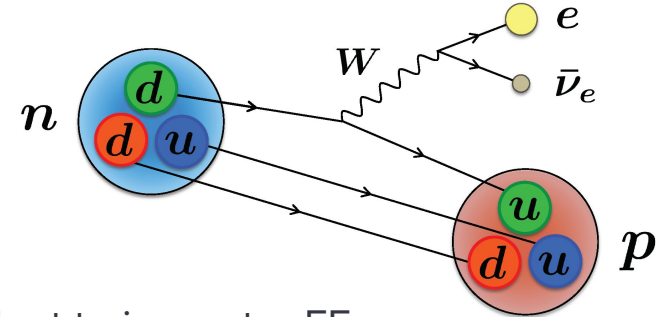
$$\mathcal{M}(n \rightarrow p e^- \bar{\nu}_e) = \frac{G_F}{\sqrt{2}} V_{ud} \sum_{\mu} \langle p(p') | W_{\mu} | n(p) \rangle L_{\mu}$$

with

$$W_{\mu} = V_{\mu} - A_{\mu}$$

$$V_{\mu} = \bar{u} \gamma_{\mu} d$$

$$A_{\mu} = \bar{u} \gamma_{\mu} \gamma_5 d$$



Neglecting isospin-breaking effects, transition FFs are equivalent to isovector FFs

$$\langle p(p') | A_{\mu} | n(p) \rangle$$

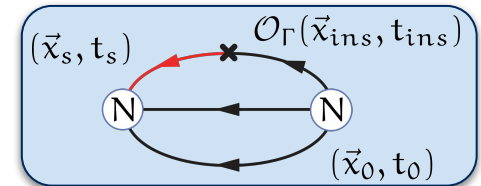


$$\langle N(p') | A_{\mu}^{\text{isov}} | N(p) \rangle$$

$$A_{\mu} = \bar{u} \gamma_{\mu} \gamma_5 d$$

$$u = d$$

$$A_{\mu}^{\text{isov}} = \bar{u} \gamma_{\mu} \gamma_5 u - \bar{d} \gamma_{\mu} \gamma_5 d$$



# Nucleon three-point functions



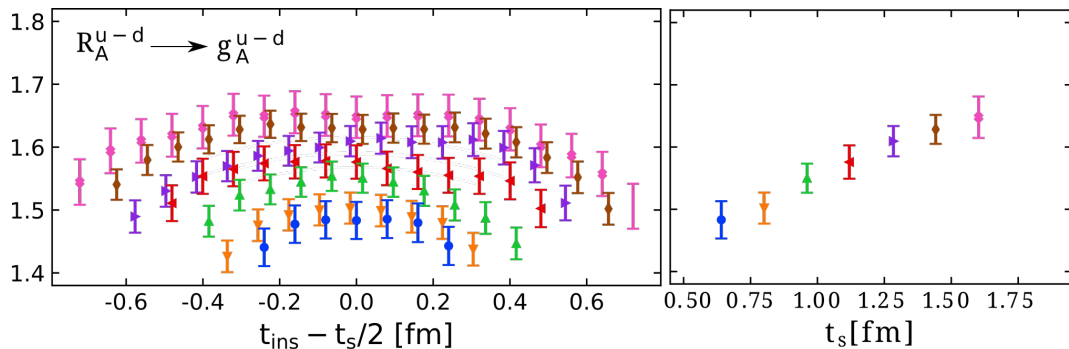
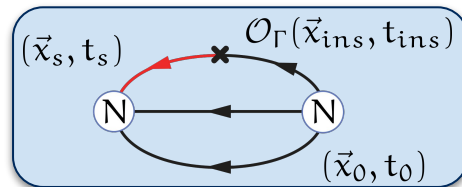
$$G_{\Gamma}(P; \vec{q}; t_s, t_{\text{ins}}) = \sum_{\vec{x}_s, \vec{x}_{\text{ins}}} e^{-i\vec{q} \cdot \vec{x}_{\text{ins}}} P^{\alpha\beta} \langle \bar{\chi}_N^{\beta}(\vec{x}_s, t_s) | \mathcal{O}_{\Gamma}(\vec{x}_{\text{ins}}, t_{\text{ins}}) | \chi_N^{\alpha}(\vec{0}, 0) \rangle$$

*suitable* ↑  
*projector*

e.g.  $\mathcal{O}_A(x) = \bar{\psi}(x) \gamma_5 \gamma^{\mu} \psi(x)$

- Three-point functions

- Ground state at  $t_s \rightarrow \infty, (t_s - t_{\text{ins}}) \rightarrow \infty$
- Error increases exponentially with  $t_s$
- Statistics increased to keep errors constant



×750 configurations

$t_s/a$	$t_s$ [fm]	$n_{src}$
8	0.64	1
10	0.80	2
12	0.96	5
14	1.12	10
16	1.28	32
18	1.44	112
20	1.60	128
Nucleon 2pt		477

~30M inversions!



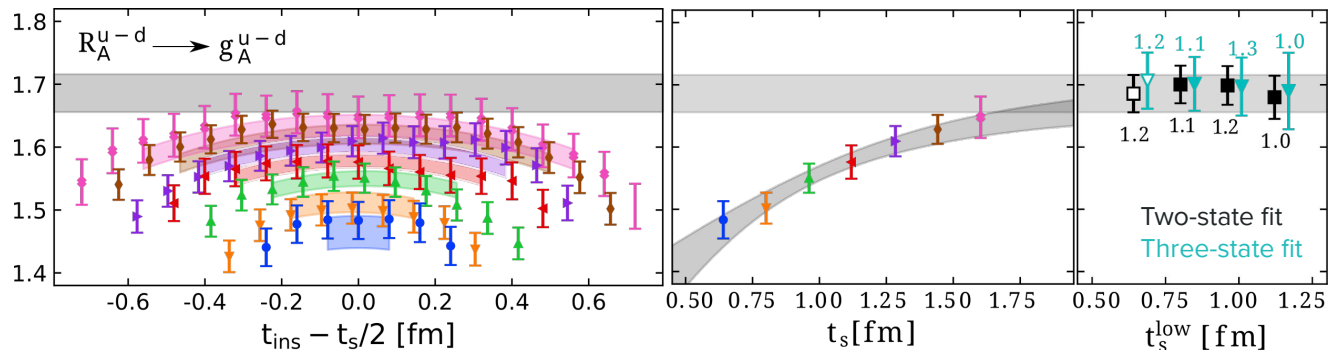
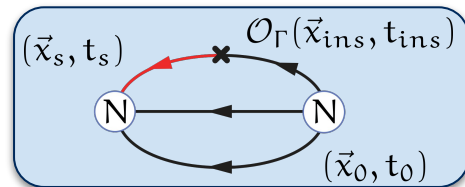
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$$G_{\Gamma}(P; \vec{q}; t_s, t_{\text{ins}}) = \sum_{\vec{x}_s, \vec{x}_{\text{ins}}} e^{-i\vec{q} \cdot \vec{x}_{\text{ins}}} P^{\alpha\beta} \langle \bar{\chi}_N^{\beta}(\vec{x}_s, t_s) | \mathcal{O}_{\Gamma}(\vec{x}_{\text{ins}}, t_{\text{ins}}) | \chi_N^{\alpha}(\vec{0}, 0) \rangle$$

$$G_{\Gamma}(t_s, t_{\text{ins}}) \simeq A_{00} e^{-m_N t_s} + A_{01} (e^{-E_1 t_{\text{ins}}} + e^{-E_1 t_s + (E_1 - m_N) t_{\text{ins}}}) + A_{11} e^{-E_1 t_s}$$

$$G(t) \simeq c_0 e^{-m_N t_s} + c_1 e^{-E_1 t_s} \quad \text{Desired matrix element: } \mathcal{M} = \frac{A_{00}}{c_0}$$



×750 configurations

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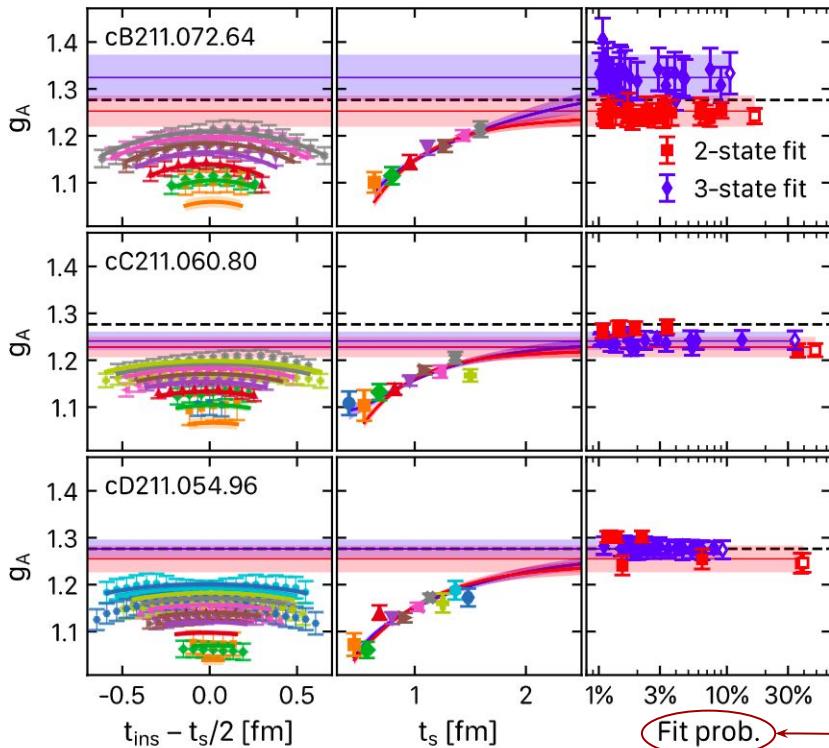
[C. Alexandrou, S. B., et al. "Nucleon axial, tensor, and scalar charges and  $\sigma$ -terms in lattice QCD". Phys. Rev., D102(5):054517, 2020]



# The three ensembles and model averaging



■ 6 ■ 8 ■ 10 ■ 12 ■ 14 ■ 16 ■ 18 ■ 20 ■ 22 ■ 24 ■ 26



Ensemble	$V/a^4$	$\beta$	$a$ [fm]	$m_\pi$ [MeV]	$m_\pi L$
cB211.072.64	$64^3 \times 128$	1.778	0.07957(13)	140.2(2)	3.62
cC211.060.80	$80^3 \times 160$	1.836	0.06821(13)	136.7(2)	3.78
cD211.054.96	$96^3 \times 192$	1.900	0.05692(12)	140.8(2)	3.90

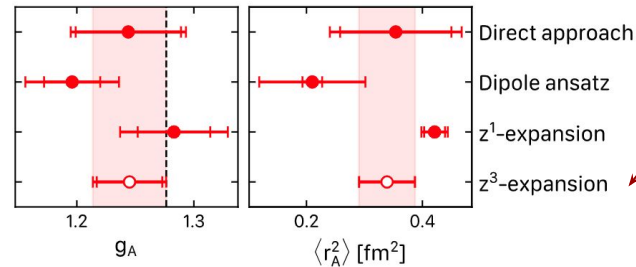
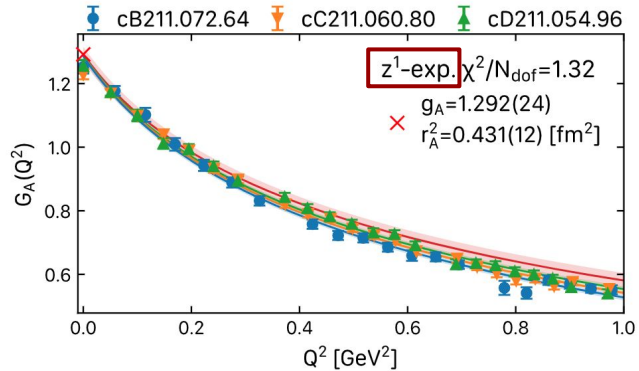
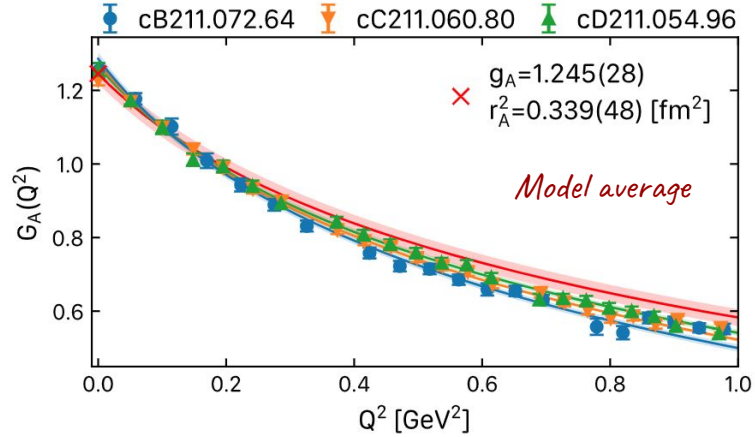
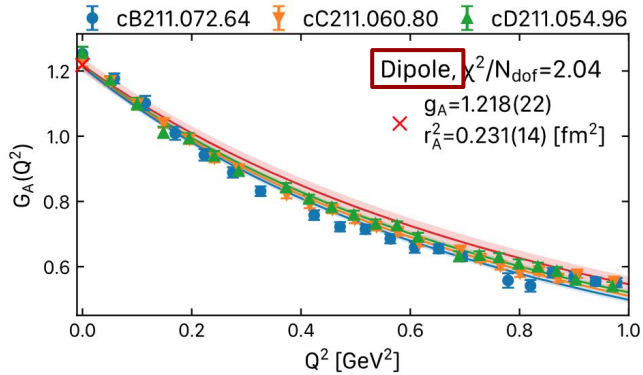
cB211.072.64			cC211.060.80			cD211.054.96		
750 configurations			400 configurations			500 configurations		
$t_s/a$	$t_s$ [fm]	$n_{src}$	$t_s/a$	$t_s$ [fm]	$n_{src}$	$t_s/a$	$t_s$ [fm]	$n_{src}$
8	0.64	1	6	0.41	1	8	0.46	1
10	0.80	2	8	0.55	2	10	0.57	2
12	0.96	5	10	0.69	4	12	0.68	4
14	1.12	10	12	0.82	10	14	0.80	8
16	1.28	32	14	0.96	22	16	0.91	16
18	1.44	112	16	1.10	48	18	1.03	32
20	1.60	128	18	1.24	45	20	1.14	64
Nucleon 2pt		477	20	1.37	116	22	1.25	16
			22	1.51	246	24	1.37	32
			Nucleon 2pt		650	26	1.48	64
						Nucleon 2pt		480

Up to 1.5fm for all ensembles

Model average over thousands of fits:  $\log(w_i) = -\frac{\chi_i^2}{2} + N_{\text{dof},i}$

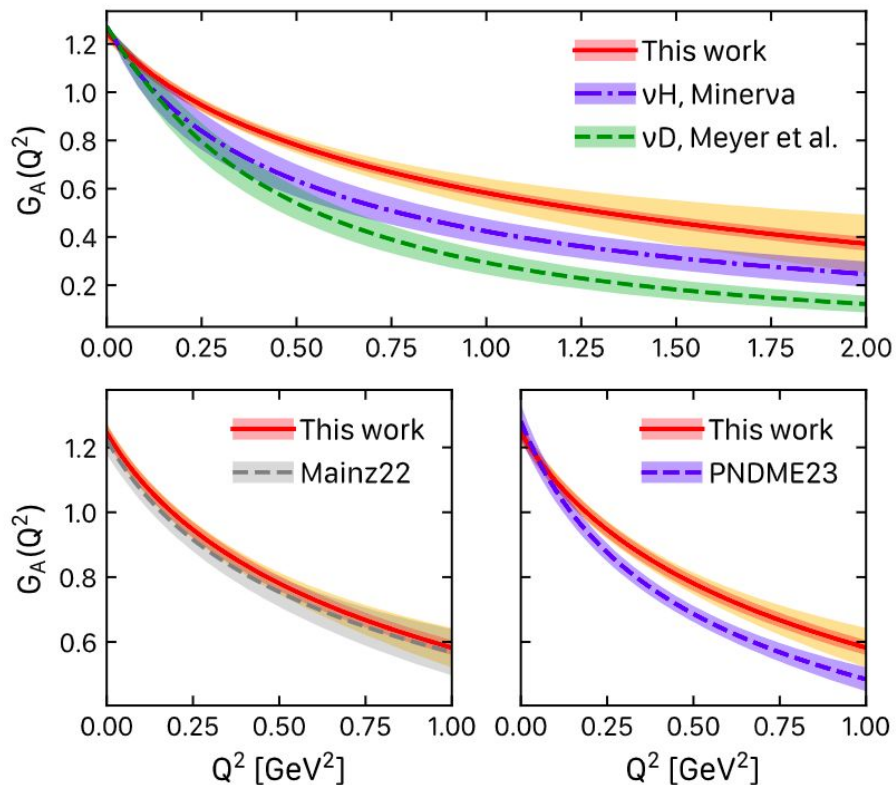
$$p_i = \frac{w_i}{Z} \quad \text{with} \quad Z = \sum_i w_i \quad [\text{E. T. Neil, J. W. Sitison, arXiv:2208.14983}]$$

# Dipole vs z-expansion

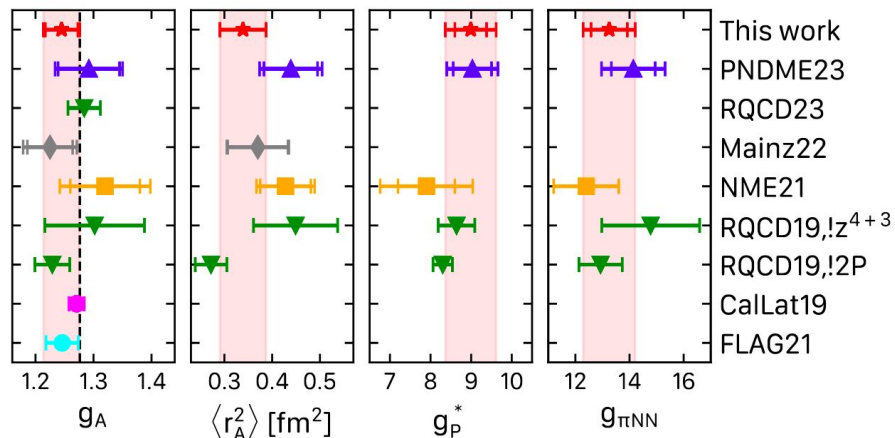


*Compatible with the direct approach but smaller error because all information used*

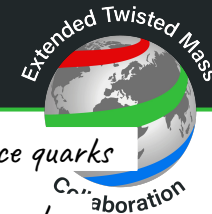
# Comparison with other works



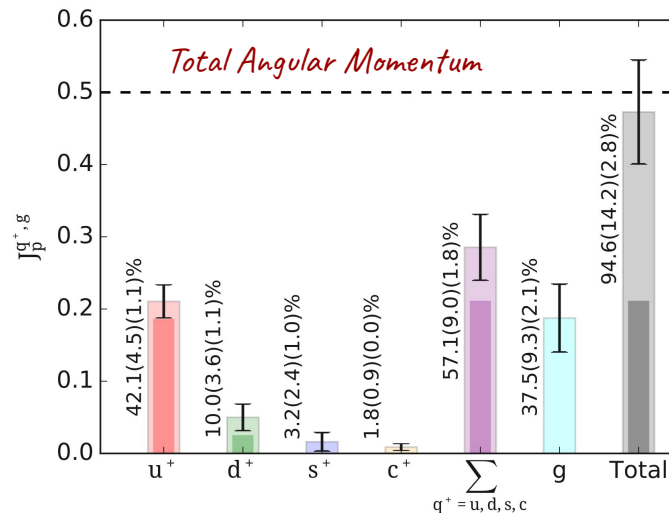
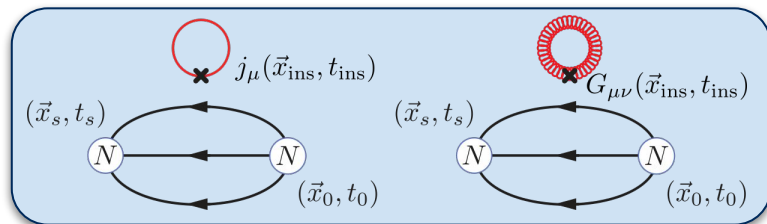
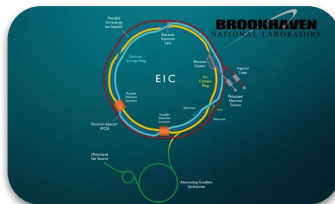
- Overall good agreement between recent lattice results and better agreement with the very recent results from Minerva



# And much more...



- Electromagnetic form factors
- Transversity form factors
- Gravitational form factors
- Second and higher Mellin moments
- PDFs and GPDs via LaMET
- + Single flavor decomposition
- + Gluon contributions



[C. Alexandrou, S. B., et al. “Complete flavor decomposition of the spin and momentum fraction of the proton using lattice QCD simulations at physical pion mass”. Phys. Rev., D101(9):094513, 2021]

# Why not more?

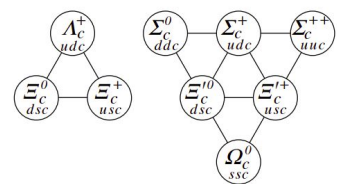
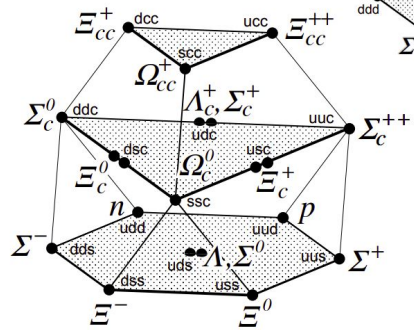
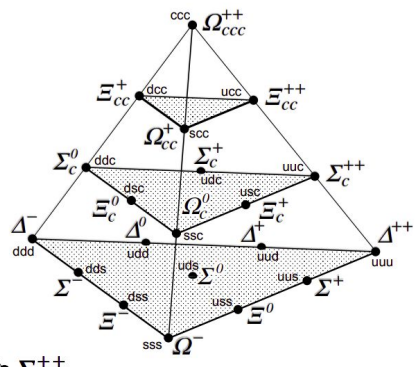
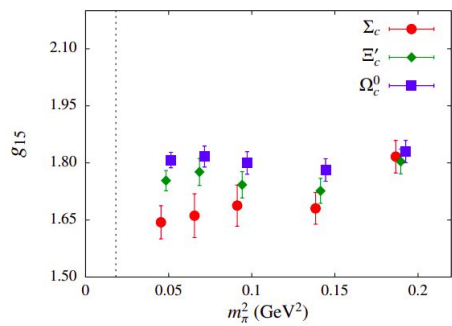


- Electromagnetic FFs
  - **N,  $\Sigma$ ,  $\Xi$ :** P. E. Shanahan *et al.* [[arXiv:1403.1965](#), [arXiv:1401.5862](#)]  
51 citations      61 citations
- Isovector charges
  - ★  **$g_A$  for 40 B.:** C. Alexandrou *et al.* [[arXiv:1606.01650](#)]      25 citations
  - **$g_A$  for  $\Sigma$ ,  $\Xi$ :** A. Savanur, H.-W. Lin [[arXiv:1901.00018](#)]      9 citations
  - ★  **$g_{A,S,T}$  for N,  $\Sigma$ ,  $\Xi$ :** G. Bali *et al.* 2023, [[arXiv:2305.04717](#)]      13 citations
- Transition form factors
  - **$f_1$  for  $\Sigma N$ ,  $\Xi\Sigma$ ,  $\Lambda p$ ,  $\Xi\Lambda$ :** P. E. Shanahan *et al.* [[arXiv:1508.06923](#)]      13 citations
  - ★ **Vector FF for  $\Sigma N$ ,  $\Xi\Sigma$ :** S. Sasaki [[arXiv:1708.04008](#)]      12 citations
  - ★ **V-A FF for  $\Lambda_c\Lambda$ :** S. Meinel [[arXiv:1611.09696](#)]      61 citations

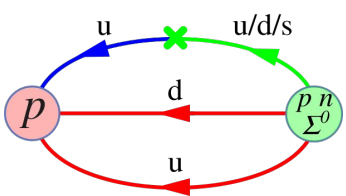


- “Axial charges of hyperons and charmed baryons using  $N_f=2+1+1$  twisted mass fermions”

- Isovector  $u-d$ ,  $u+d-2s$  and  $u+d+s-3c$  axial charges of 40 baryons
- One in a kind study:
  - Computed using fixed-insertion
  - i.e. fixed current and momentum transfer
  - Thus, got  $g_A$  for any possible baryon!

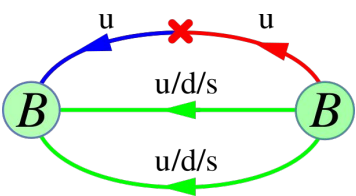


Fixed-sink



● Fixed ● Free

Fixed-insertion



● Sequential

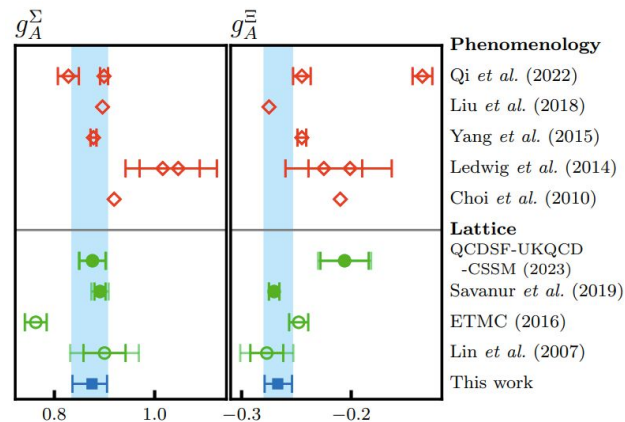
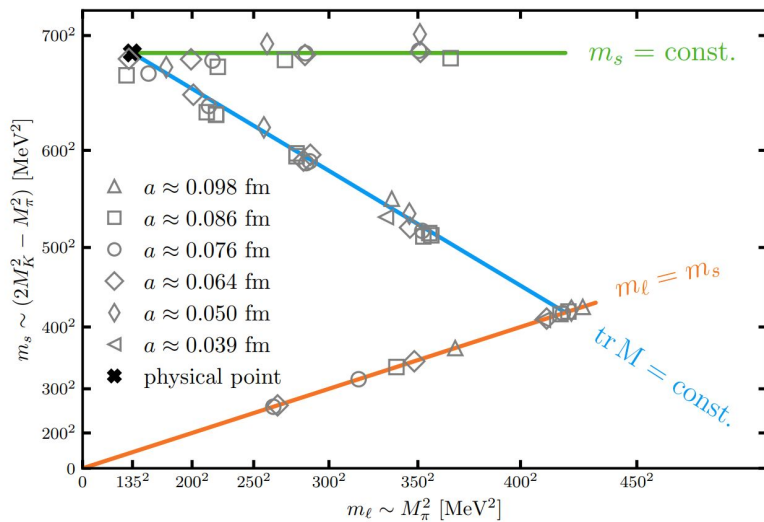


- “Octet baryon isovector charges from  $N_f = 2 + 1$  lattice QCD”
  - i.e.  $u$ - $d$  combination for  $N$ ,  $\Sigma$  and  $\Xi$
  - State-of-the-art calculation, 47 ensembles!

$$g_A^N = 1.284_{(27)}^{(28)}, \quad g_A^\Sigma = 0.875_{(39)}^{(30)}, \quad g_A^\Xi = -0.267_{(12)}^{(13)}$$

$$g_S^N = 1.11_{(16)}^{(14)}, \quad g_S^\Sigma = 3.98_{(24)}^{(22)}, \quad g_S^\Xi = 2.57_{(11)}^{(11)}$$

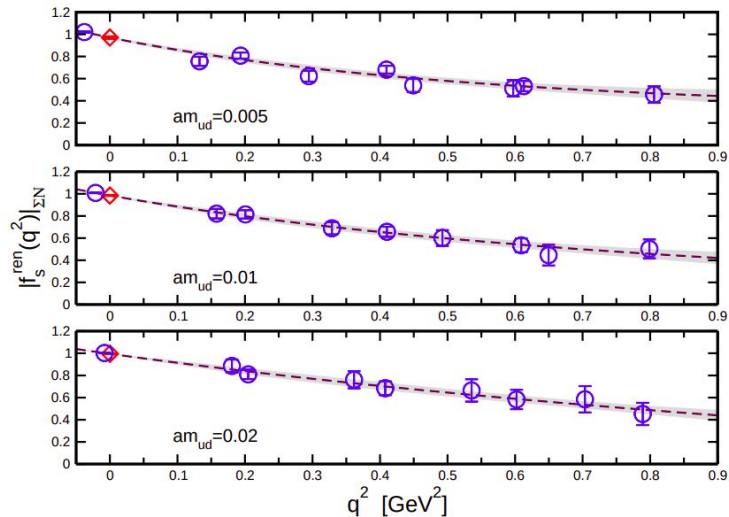
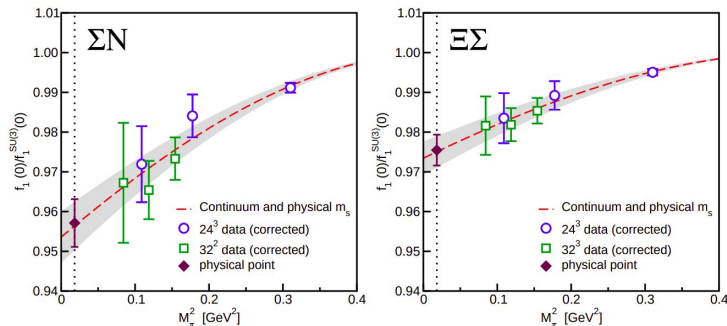
$$g_T^N = 0.984_{(29)}^{(19)}, \quad g_T^\Sigma = 0.798_{(21)}^{(15)}, \quad g_T^\Xi = -0.1872_{(41)}^{(59)}$$



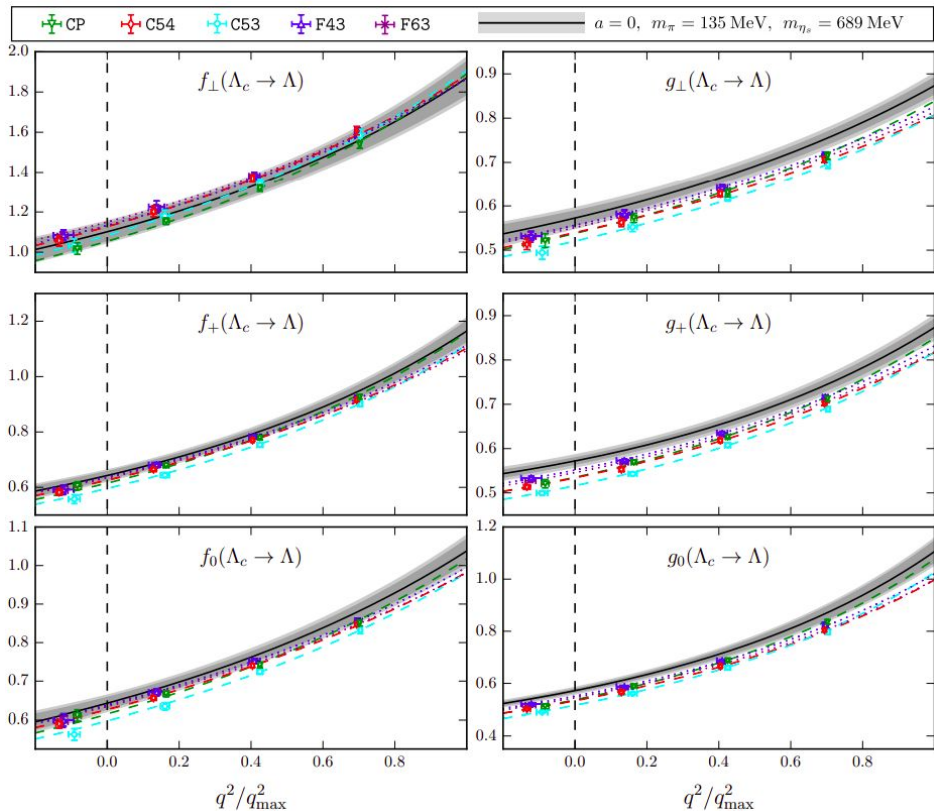
- “Continuum limit of hyperon vector coupling  $f_1(0)$  from 2+1 flavor domain wall QCD”
  - One of the few studies on transition FF

$$f_1(0) = \begin{cases} -0.9571(60)_{\text{stat}}(66)_{q^2}(37)_{\chi}(24)_{\text{scale}} & [\Sigma \rightarrow N] \\ +0.9755(39)_{\text{stat}}(16)_{q^2}(21)_{\chi}(24)_{\text{scale}} & [\Xi \rightarrow \Sigma], \end{cases}$$

$$= \begin{cases} -0.9571(99)_{\text{combined}} & [\Sigma \rightarrow N] \\ +0.9755(53)_{\text{combined}} & [\Xi \rightarrow \Sigma], \end{cases}$$







“  $\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell$  form factors and decay rates from lattice QCD with physical quark masses”

- State-of-the-art calculation
- There is actually quite few more literature on decays of  $\Lambda_c$  from lattice (e.g. [inspire-hep](#))

**$\Lambda_c \rightarrow N$  form factors from lattice QCD and phenomenology of  $\Lambda_c \rightarrow n \ell^+ \nu_\ell$  and  $\Lambda_c \rightarrow p \mu^+ \mu^-$  decays** #20  
 Stefan Meinel (Arizona U. and RIKEN BNL) (Dec 15, 2017)  
 Published in: *Phys.Rev.D* 97 (2018) 3, 034511 • e-Print: 1712.05783 [hep-lat]  
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [62 citations](#)

**First lattice QCD calculation of semileptonic decays of charmed-strange baryons  $\Xi_c^*$**  #15  
 Qi-An Zhang (Shanghai Jiao Tong U. and Tsung-Dao Lee Inst., Shanghai), Jun Hua (Shanghai Jiaotong U., INPAC and Tsung-Dao Lee Inst., Shanghai), Fei Huang (Shanghai Jiaotong U., INPAC and Tsung-Dao Lee Inst., Shanghai), Renbo Li (Nanjing Normal U.), Yuan Yuan Li (Nanjing Normal U.) et al. (Mar 11, 2021)  
 Published in: *Chin.Phys.C* 46 (2022) 1, 011002 • e-Print: 2103.07064 [hep-lat]  
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [37 citations](#)

**$\Lambda_c \rightarrow \Lambda^*(1520)$  form factors from lattice QCD and improved analysis of the  $\Lambda_b \rightarrow \Lambda^*(1520)$  and  $\Lambda_b \rightarrow \Lambda_c^*(2595,2625)$  form factors** #12  
 Stefan Meinel (Arizona U.), Gumaro Rendon (Brookhaven) (Jul 27, 2021)  
 Published in: *Phys.Rev.D* 105 (2022) 5, 054511 • e-Print: 2107.13140 [hep-lat]  
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [35 citations](#)

# What else can be done?



- Given the state-of-the-art on the nucleon, much more can be done!
  - Hyperons are not harder, actually should be even simpler
  - But they do not come for free and we require proper **motivation!**
- **Proposal #1:** Extend to the baryon octet
  - Diagonal and transition matrix elements
  - Same technology as used for the nucleon
- **Proposal #2:** Non-leptonic two-body decays
  - We are currently getting experience with  $\langle N | J | N \pi \rangle$
  - Where shall we start from?



# Proposal #1: The Baryon octet

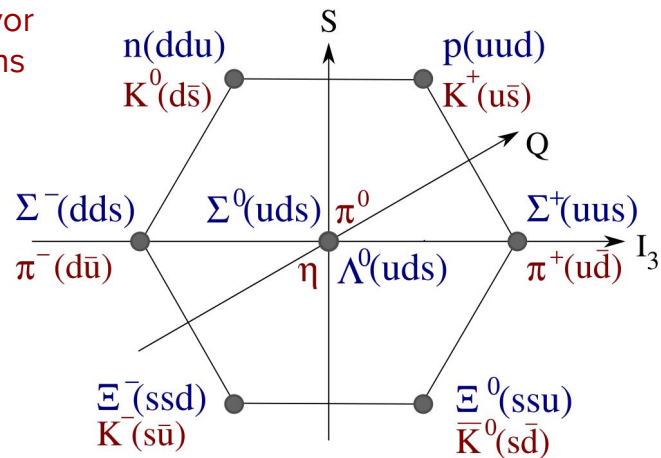


Assuming SU(2) isospin symmetry, there are 16 matrix elements non-zero

$$\begin{aligned}
 & \mathcal{J}^{\bar{N}\pi N}, \mathcal{J}^{\bar{\Sigma}\pi\Sigma}, \mathcal{J}^{\bar{\Lambda}\pi\Sigma}, \mathcal{J}^{\bar{\Xi}\pi\Xi}, \quad \text{and} \quad \mathcal{J}^{\bar{N}K\Sigma}, \mathcal{J}^{\bar{N}K\Lambda}, \mathcal{J}^{\bar{\Lambda}K\Xi}, \mathcal{J}^{\bar{\Sigma}K\Xi}, \\
 & \mathcal{J}^{\bar{N}\eta N}, \mathcal{J}^{\bar{\Sigma}\eta\Sigma}, \mathcal{J}^{\bar{\Lambda}\eta\Lambda}, \mathcal{J}^{\bar{\Xi}\eta\Xi}, \quad \text{and} \quad \mathcal{J}^{\bar{N}\eta' N}, \mathcal{J}^{\bar{\Sigma}\eta'\Sigma}, \mathcal{J}^{\bar{\Lambda}\eta'\Lambda}, \mathcal{J}^{\bar{\Xi}\eta'\Xi}.
 \end{aligned}$$

Notation: current flavor labelled using mesons

Baryons ( $B$ )	Mesons ( $M$ )	Currents ( $J_{\Gamma}^M$ )
$N \begin{cases} n & (ddu) \\ p & (uud) \end{cases}$	$K \begin{cases} K^0 & (d\bar{s}) \\ K^+ & (u\bar{s}) \end{cases}$	$\begin{aligned} & d\bar{\Gamma}s \\ & \bar{u}\Gamma s \end{aligned}$
$\Sigma \begin{cases} \Sigma^- & (dds) \\ \Sigma^0 & (uds) \\ \Sigma^+ & (uus) \end{cases}$	$\pi \begin{cases} \pi^- & (d\bar{u}) \\ \pi^0 & (u-d) \\ \pi^+ & (u\bar{d}) \end{cases}$	$\begin{aligned} & d\bar{\Gamma}u \\ & \bar{u}\Gamma u - \bar{d}\Gamma d \\ & \bar{u}\Gamma d \end{aligned}$
$\Lambda \quad \Lambda^0 \quad (uds)$	$\eta \quad (u+d-2s)$ $\eta' \quad (u+d+s)$	$\begin{aligned} & \bar{u}\Gamma u + \bar{d}\Gamma d - 2\bar{s}\Gamma s \\ & \bar{u}\Gamma u + \bar{d}\Gamma d + \bar{s}\Gamma s \end{aligned}$
$\Xi \begin{cases} \Xi^- & (ssd) \\ \Xi^0 & (ssu) \end{cases}$	$\bar{K} \begin{cases} K^- & (\bar{s}u) \\ \bar{K}^0 & (\bar{s}d) \end{cases}$	$\begin{aligned} & \bar{s}\Gamma u \\ & \bar{s}\Gamma d \end{aligned}$



# Proposal #1: The Baryon octet



- **Outcome #1: Charges and moments**
  - Comparison to available results, Ref. PDG
  - A wealth of predictions
    - Scalar charges
    - Tensor chargers
    - Second moments
    - etc...
  - Constraints to physics BSM

[Patrik Adlarson]

**BCSIII**

$\Lambda \rightarrow pl^- \bar{\nu}_\mu$

	$\Lambda \rightarrow p$	$\Sigma^- \rightarrow n$	$\Xi^0 \rightarrow \Sigma^+$	$\Xi^- \rightarrow \Lambda$
$g_1(0)/f_1(0)$	0.718(15)	-0.340(17)	1.210(50)	0.250(50)
$f_2(0)/f_1(0)$	1.90(10)	2.80(14)	1.36(7)	2.25(11)
$\tilde{f}_2(0)/f_1(0)$	0.72	-0.26	1.22	0.22
$r_3$	1.60	4.1	0.56	3.7
$r_T$	5.2	1.7	7.2	1.1

$R^{sc} = \frac{\Gamma(B_1 \rightarrow B_2 \mu^- \bar{\nu}_\mu)}{\Gamma(B_1 \rightarrow B_2 e^- \bar{\nu}_e)}$   
 $\epsilon_S = 0.003(40), \quad \epsilon_T = 0.017(34) \quad \text{at 90\% CL from SLWD}$

$R^{sc} = 1 + r_S \epsilon_S + r_T \epsilon_T$   
 $R_{EM}^{sc}$

Potential for  $|V_{ub}|$  determination and test of BSM searches from determination of Wilson coefficients  $\epsilon_S$  and  $\epsilon_T$

Nice example where low-energy precision experiments with direct searches in collider experiments

FIG. 1. (color online). 90% CL constraints on  $\epsilon_{i,T}$  at  $\mu = 2$  GeV from the measurements of  $pp \rightarrow \mu \mu$  in different channels (dashed lines) and combined (filled ellipse). LHC bounds obtained from CMS data at  $\sqrt{s} = 8$  TeV (7 TeV) are represented by the black solid (dashed) ellipse.

## Electromagnetic

Quantity	Value	
$\mu_\Lambda$	-0.613(4)	$\mu_N$
$\mu_{\Sigma^+}$	2.458(10)	$\mu_N$
$\mu_{\Sigma^-}$	-1.160(25)	$\mu_N$
$\mu_{\Xi^0}$	-1.250(14)	$\mu_N$
$\mu_{\Xi^-}$	-0.6507(25)	$\mu_N$
$ \mu_{\Sigma^0} \rightarrow \Lambda^0 $	1.61(8)	$\mu_N$
$\langle r_E^2 \rangle_{\Sigma^-}$	0.61(15)	fm

## Axial-vector

$B \rightarrow B'$	$g_A/f_1$
$\Lambda^0 \rightarrow p$	0.718(15)
$\Sigma^- \rightarrow n$	-0.340(17)
$\Xi^0 \rightarrow \Sigma^+$	1.22(5)
$\Xi^- \rightarrow \Lambda^0$	0.25(5)

---

$B \rightarrow B'$	$g_2/f_1$
$\Xi^0 \rightarrow \Sigma^+$	-1.7(2.1)

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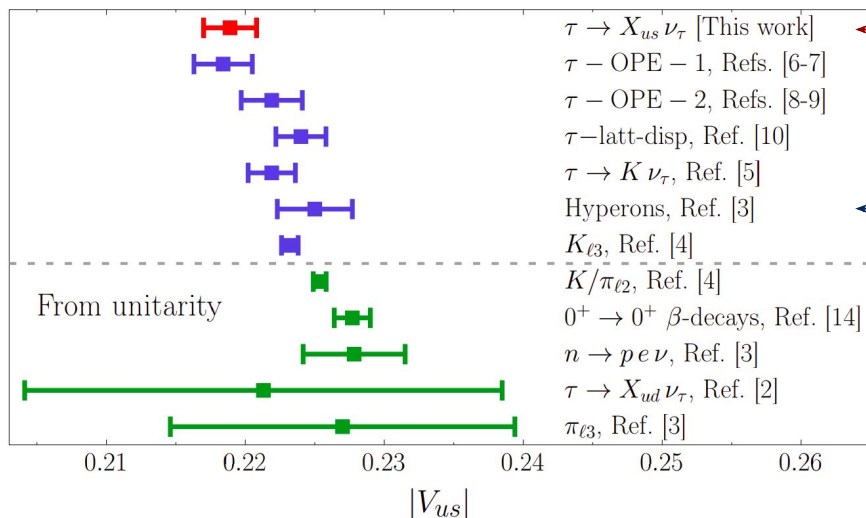
$B \rightarrow B'$	$f_2/f_1$
$\Sigma^- \rightarrow n$	0.97(14)
$\Xi^0 \rightarrow \Sigma^+$	2.0(9)

# Proposal #1: The Baryon octet



- **Outcome #2:** Baryonic determination of  $|V_{us}|$

$$\Gamma(B \rightarrow B' \ell \nu_\ell) = \frac{G_F^2 (1 + \Delta_{RC})}{60\pi^3} (m_B - m_{B'})^5 (1 - 3\delta_{BB'}) |V_{us}|^2 \left[ |f_1^{\bar{B}'KB}|^2 + 3 |g_A^{\bar{B}'KB}|^2 - 2\delta_{BB'} g_2^{\bar{B}'KB} g_A^{\bar{B}'KB} + O(\delta_{BB'}^2) \right]$$



Will be on the arXiv by next week

Can be improved via lattice QCD

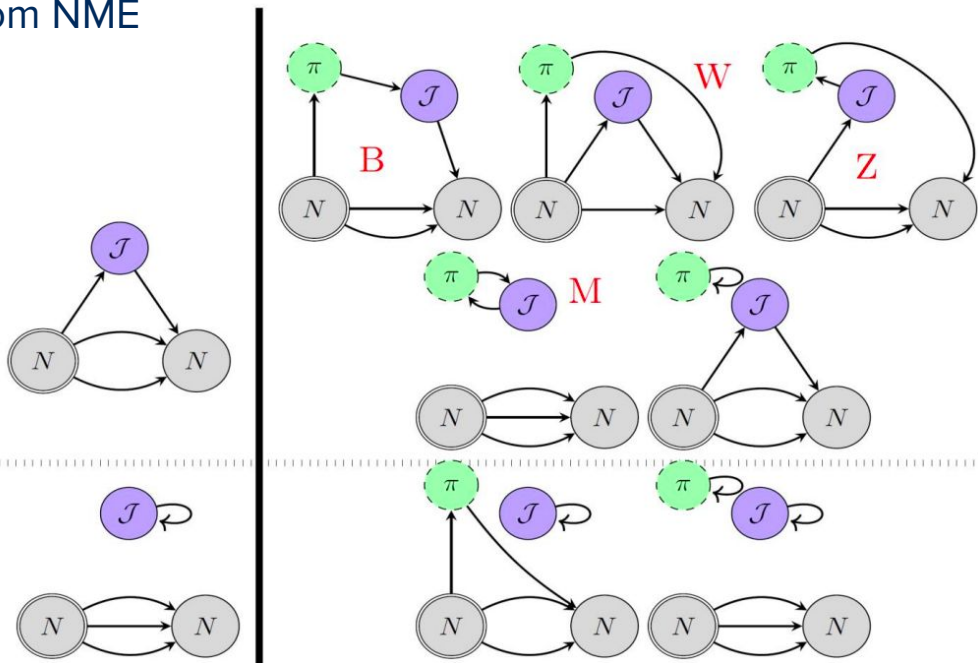
$B \rightarrow B'$	$R_{\text{exp.}}^{\mu e}$ [20]
$\Lambda^0 \rightarrow p$	0.189(41)
$\Sigma^- \rightarrow n$	0.442(39)
$\Xi^0 \rightarrow \Sigma^+$	0.0092(14)
$\Xi^- \rightarrow \Lambda^0$	0.6(5)

$B \rightarrow B'$	$\Gamma/\Gamma(\text{total})$	Err./ $ V_{us} $
$\Lambda^0 \rightarrow p$	$8.34(14) \cdot 10^{-4}$	0.8%
$\Sigma^- \rightarrow n$	$10.17(34) \cdot 10^{-4}$	1.7%
$\Xi^0 \rightarrow \Sigma^+$	$2.52(8) \cdot 10^{-4}$	1.6%
$\Xi^- \rightarrow \Lambda^0$	$1.27(23) \cdot 10^{-4}$	9.1%
$\Xi^- \rightarrow \Sigma^0$	$0.87(17) \cdot 10^{-4}$	9.8%

# Proposal #2: Non-leptonic two-body decay



- We are getting experience with  $\langle N | J | N \pi \rangle$ 
  - Useful to remove excited states from NME
- It could be extended to
  - $\langle \Lambda | H_w | N \pi \rangle$
  - $\langle \Sigma | H_w | N \pi \rangle$
  - $\langle \Xi | H_w | \Lambda \pi \rangle$
  - ...
- Clearly very very challenging...
- ... I could not find any existing study



# Why so difficult?

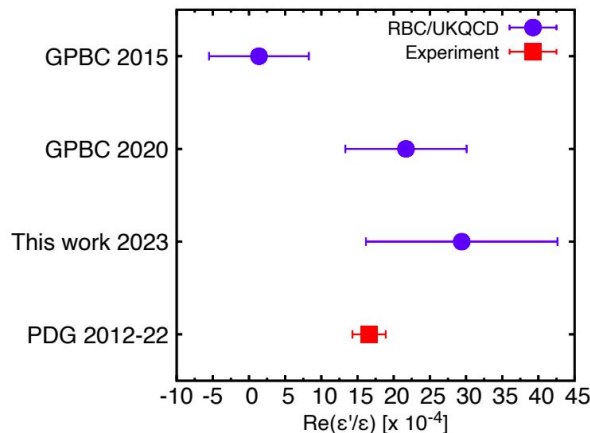


- The decay has to be on-shell, having  $E_{B_1} \simeq E_{B_2P}$  to compute  $\langle B_1 | H_w | B_2P \rangle$ 
  - Achieved using a properly-sized lattice [T. Blum *et al.* [arXiv:2306.06781](https://arxiv.org/abs/2306.06781)]
  - ... or G-boundary conditions [R. Abbott *et al.* [arXiv:2004.09440](https://arxiv.org/abs/2004.09440)]
  - ... or twisted-boundary conditions [G. M. de Divitiis, N. Tantalo [arXiv:hep-lat/0409154](https://arxiv.org/abs/hep-lat/0409154)]

- $E_{B_2P}$  is not a ground state
  - GEVP required

- Results available only for  $K \rightarrow \pi\pi$

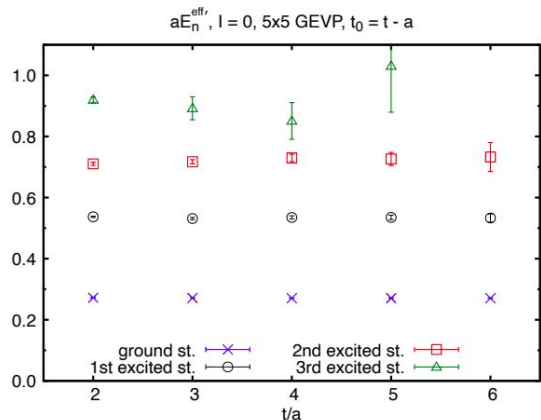
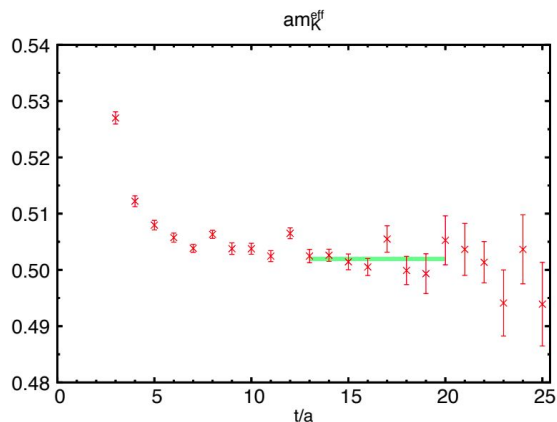
- **What is interesting & possible to look at??**



# Mesonic vs baryonic case



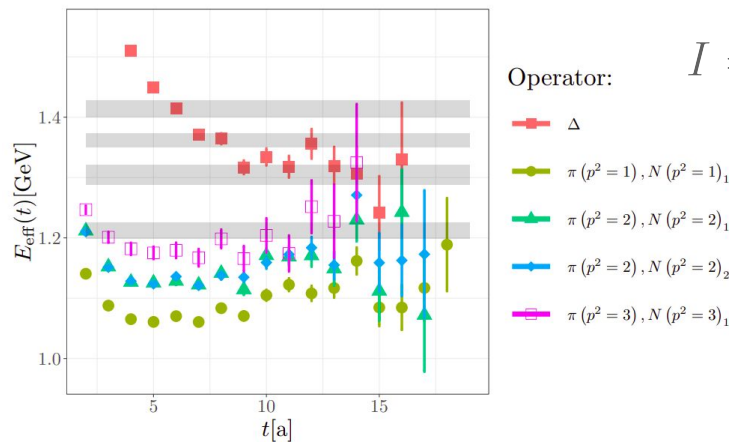
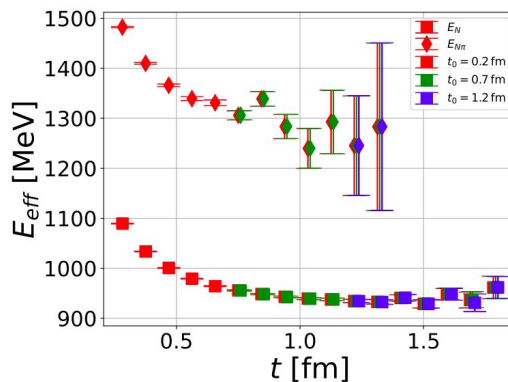
[T. Blum et al. [arXiv:2306.06781](https://arxiv.org/abs/2306.06781)]



[Preliminary]

[C. Alexandrou et al.

[arXiv:2307.12846](https://arxiv.org/abs/2307.12846)]



$$I = \frac{1}{2}$$

$$I = \frac{3}{2}$$



# Thank you for your attention and the organization!



Looking forward to discussion sessions:

- Proposal #1: Diagonal and transition matrix elements of the baryons octet.  
**We have the technology to provide results with controlled systematics.**
- Proposal #2: Two-body decays, **where is best to start from?**



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the European Union



**EuroHPC**  
Joint Undertaking



**INNO4**  
**SCALE**

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