Nonleptonic decays of heavy baryons

– an example from the practice –

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Measurement of decay asymmetry in $\Lambda_c^+ \to \Xi^0 K^+$

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M. Ablikim *et al.* [BESIII], "First Measurement of the Decay Asymmetry in the Pure W-Boson-Exchange Decay $\Lambda_c^+ \to \Xi^0 K^+$"

Main results:

- decay asymmetry $\alpha_{\Lambda_c^+ \to \Xi^0 K^+} = 0.01 \pm 0.16(\text{stat}) \pm 0.03(\text{syst})$
- phase shift $\delta_P - \delta_S = -1.55 \pm 0.25(\text{stat}) \pm 0.03(\text{syst})$ rad or $\delta_P - \delta_S = +1.59 \pm 0.25(\text{stat}) \pm 0.05(\text{syst})$ rad

Contrasting recent estimates (Geng 2019, Zou 2020, Zhong 2022), older estimates suggest a decay asymmetry of $\alpha_{\Lambda_c^+ \to \Xi^0 K^+} = 0$ (Körner 1992, Xu 1992, Źencaykowski 1994, Ivanov 1998, Sharma 1999) which is now confirmed by BESIII, together with $\delta_P - \delta_S = \pm\pi/2$.

Question:

Does this indicate vanishing mixing between S and P waves?

About the decay asymmetry parameter

$$\alpha(B_i \to B_f M) = \frac{|H_{+1/2,0}|^2 - |H_{-1/2,0}|^2}{|H_{+1/2,0}|^2 + |H_{-1/2,0}|^2}$$

With the transition element $\langle B_f, M|\mathcal{H}|B_i\rangle = \bar{u}_f(A - B\gamma_5)u_i$ of the non-leptonic decay $B_i \to B_f M$, the helicity amplitudes are given by a combination of parity violating (pv) S-wave (scalar) and parity conserving (pc) P-wave (pseudoscalar) contributions,

$$H_{\pm 1/2,0} = \bar{u}_f(\pm 1/2)(A - B\gamma_5)u_i(\pm 1/2) = \frac{1}{2}(H^{\rm pv}_{1/2,0} \pm H^{\rm pc}_{1/2,0}),$$

$$H^{\rm pv}_{1/2,0} = 2\sqrt{Q_+}A,\ H^{\rm pc}_{1/2,0} = 2\sqrt{Q_-}B,\ Q_\pm = (m_i \pm m_f)^2 - q^2.$$

Therefore,

$$\alpha(B_i \to B_f M) = \frac{2\,\mathrm{Re}(H^{\rm pv} H^{\rm pc*})}{|H^{\rm pv}|^2 + |H^{\rm pc}|^2} = \frac{2\sqrt{Q_+ Q_-}\,\mathrm{Re}(AB^*)}{Q_+|A|^2 + Q_-|B|^2}.$$

Tensor invariants - remembering the table

				I_1^-	I_2^-	I_3	I_4	$\hat{I}_3$	$\hat{I}_4$	I_5
CF	$12\,\Lambda_c^+$	$\to$	$\Lambda^0\pi^+$	-2	-2	-2	$+4$	-2	$+4$	$+1$
	$4\sqrt{3}\,\Lambda_c^+$	$\to$	$\Sigma^0\pi^+$	0	0	$+2$	0	-2	$+4$	$+1$
	$4\sqrt{3}\,\Lambda_c^+$	$\to$	$\Sigma^+\pi^0$	0	0	-2	0	$+2$	-4	-1
	$4\sqrt{3}\,\Lambda_c^+$	$\to$	$\Sigma^+\eta_\omega$	0	0	-2	0	-2	$+4$	-1
	$2\sqrt{6}\Lambda_c^+$	$\to$	$\Sigma^+\eta_\phi$	0	0	-2	$+2$	0	0	0
	$12\Lambda_c^+$	$\to$	$\Sigma^+\eta_8$	0	0	$+2$	-4	-2	$+4$	-1
	$6\sqrt{2}\Lambda_c^+$	$\to$	$\Sigma^+\eta_1$	0	0	-4	$+2$	-2	$+4$	-1
	$2\sqrt{6}\,\Lambda_c^+$	$\to$	$p\bar{K}^0$	$+1$	$+1$	$+2$	-2	0	0	0
	$2\sqrt{6}\,\Lambda_c^+$	$\to$	Ξ^0K^+	0	0	0	-2	0	0	-1

... and a lot more decays (in total: 196)

S. Groote, J.G. Körner, “Topological tensor invariants and the current algebra approach: analysis of 196 nonleptonic two-body decays of single and double charm baryons – a review,” Eur. Phys. J. C **82** (2022) 4, 297

Parity violating amplitude

Parity violating amplitude given by $A_{fki}=A_{fki}^{\rm fac}+A_{fki}^{\rm pole}+A_{fki}^{\rm com}$. According to the table, factorising contributions not important for the decay $\Lambda_c^+\to\Xi^0K^+$ considered here ($I_1^-=I_2^-=0$).

- pole contribution $A_{fki}^{\rm pole}$ much smaller than $B_{fki}^{\rm pole}$
- commutator contribution $A_{fki}^{\rm com}=\dfrac{\sqrt{2}}{f_k}\langle B_f|[M_k,\mathcal{H}_{\rm eff}^{\rm pc}]|B_i\rangle$

  $\mathcal{H}_{\rm eff}^{\rm pc}$: effective parity conserving Hamiltonian
  M_k: $SU(3)$ vector charge associated with meson k
  f_k: pseudoscalar coupling constant

  writing the commutator explicitly and using $\sum_\ell|B_\ell\rangle\langle B_\ell|=1$ to insert intermediate baryon states leads to s-channel and u-channel contributions:

$$A_{fki}^{\rm com}=A_{fki}^{\rm com}(s)-A_{fki}^{\rm com}(u)$$

s- and u-channel contributions

$$A^{\rm com}_{fki} = \frac{\sqrt{2}}{f_k}\left(\sum_\ell \langle B_f|M_k|B_\ell\rangle\langle B_\ell|\mathcal{H}^{\rm pc}_{\rm eff}|B_i\rangle - \sum_{\ell'}\langle B_f|\mathcal{H}^{\rm pc}_{\rm eff}|B_{\ell'}\rangle\langle B_{\ell'}|M_k|B_i\rangle\right)$$

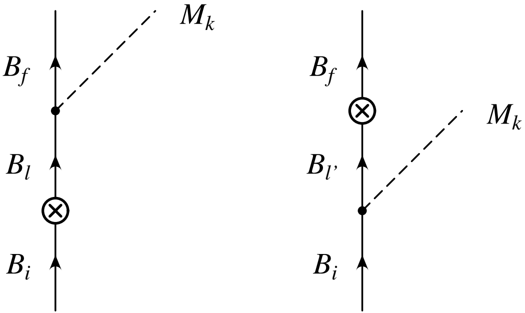

Figure: s-channel contribution – u-channel contribution

Completeness relation

Expressing transition elements by products of baryonic, mesonic and interaction matrices, one obtains the f-type baryon matrix element (for clarity, in the following I skip a couple of general factors)

$$(I^f)_{fk\ell} := \langle B_f | M_k | B_\ell \rangle = 4(\tilde{I}_1)_{fk\ell} + 2(\tilde{I}_2)_{fk\ell}$$

with $(\tilde{I}_1)_{fk\ell} = B_f^{a[bc]} B^\ell_{a[bc']} (M_k)^{c'}_c$, $(\tilde{I}_2)_{fk\ell} = B_f^{a[bc]} B^\ell_{b[c'a]} (M_k)^{c'}_c$, and

$$a_{\ell\ell'} := \langle B_\ell | \mathcal{H}^{\rm pc}_{\rm eff} | B_{\ell'} \rangle = B_\ell^{a[bc]} B^{\ell'}_{a[b'c']} H^{[b'c']}_{[bc]}$$

With completeness relation

$$\sum_\ell B^\ell_{k[mn]} B_\ell^{b[cd]} = \frac{2}{6}(\delta^b_k\delta^c_m\delta^d_n - \delta^b_k\delta^d_m\delta^c_n) + \\ -\frac{1}{6}(\delta^b_m\delta^c_n\delta^d_k - \delta^b_m\delta^d_n\delta^c_k) - \frac{1}{6}(\delta^b_n\delta^c_k\delta^d_m - \delta^b_n\delta^d_k\delta^c_m)$$

one can write the result in terms of tensor invariants.

In terms of tensor invariants

Tensor invariants

$$I_1^-(\ell,\ell') = B_\ell^{a[bc]} B^{\ell'}_{a[bc']} M^d_{d'} H^{[c'd']}_{[cd]} \qquad I_2^-(\ell,\ell') = B_\ell^{a[bc]} B^{\ell'}_{b[c'a]} M^d_{d'} H^{[c'd']}_{[cd]}$$

$$I_3(\ell,\ell') = B_\ell^{a[bc]} B^{\ell'}_{a[b'c']} M^d_c H^{[c'b']}_{[db]} \qquad I_4(\ell,\ell') = B_\ell^{b[ca]} B^{\ell'}_{a[b'c']} M^d_c H^{[c'b']}_{[db]}$$

$$\hat{I}_3(\ell,\ell') = B_\ell^{a[bc]} B^{\ell'}_{a[b'c']} M^{c'}_d H^{[db']}_{[cb]} \qquad \hat{I}_4(\ell,\ell') = B_\ell^{a[bc]} B^{\ell'}_{b'[c'a]} M^{c'}_d H^{[db']}_{[cb]}$$

$$I_5(\ell,\ell') = B_\ell^{a[bc]} B^{\ell'}_{a'[b'c']} M^{c'}_c H^{[a'b']}_{[ab]}$$

$$\sum_\ell (\tilde{I}_1)_{fk\ell} a_{\ell i} = B_f^{a[bc]} (M_k)^{c'}_c \Big(\sum_\ell B^\ell_{a[bc']} B_\ell^{r[st]} \Big) B^i_{r[a'b']} H^{[a'b']}_{[st]} =$$

$$= \tfrac{2}{3} I_3 - \tfrac{1}{3} I_4 + \tfrac{2}{3} I_5 \quad (\text{note that } I^f = 4\tilde{I}_1 + 2\tilde{I}_2),$$

$$\sum_\ell (\tilde{I}_2)_{fk\ell} a_{\ell i} = B_f^{a[bc]} (M_k)^{c'}_c \Big(\sum_\ell B^\ell_{b[c'a]} B_\ell^{r[st]} \Big) B^i_{r[a'b']} H^{[a'b']}_{[st]} =$$

$$= -\tfrac{1}{3} I_3 + \tfrac{2}{3} I_4 + \tfrac{2}{3} I_5 \quad \Rightarrow \quad \sum_\ell (I^f)_{fk\ell} a_{\ell i} = 2I_3 + 4I_5.$$

Parity violating part in terms of tensor invariants

$$A^{\rm com}_{fki}(s) = \frac{\sqrt{2}}{f_k}\sum_{\ell}(I^f)_{fk\ell}a_{\ell i} = \frac{\sqrt{2}}{f_k}(2I_3+4I_5)$$

$$A^{\rm com}_{fki}(u) = \frac{\sqrt{2}}{f_k}\sum_{\ell'}a_{f\ell'}(I^f)_{\ell' ki} = \frac{\sqrt{2}}{f_k}(2\hat{I}_3+4I_5)$$

$$\Rightarrow\ A^{\rm com}_{fki} = A^{\rm com}_{fki}(s)-A^{\rm com}_{fki}(u) = \frac{2\sqrt{2}}{f_k}(I_3-\hat{I}_3)$$

depends only on the difference $I_3-\hat{I}_3$.
And if both $I_3=\hat{I}_3=0$ as in our case, it vanishes identically.

Benefit of tensor invariants

Calculating with tensor invariants, it is not necessary to dwell on the intermediate baryonic states B_ℓ and $B_{\ell'}$ (s and u channels).

Parity conserving amplitude

$$B_{fki} = B_{fki}^{\rm fac} + B_{fki}^{\rm pole} + B_{fki}^{\rm com}$$

- factorising contributions again not important for us
- commutator contribution

$$B_{fki}^{\rm com} = \frac{\sqrt{2}}{f_k}\Bigg(\sum_\ell \langle B_f|M_k|B_\ell\rangle\langle B_\ell|\mathcal{H}_{\rm eff}^{\rm pv}|B_i\rangle + - \sum_{\ell'}\langle B_f|\mathcal{H}_{\rm eff}^{\rm pv}|B_{\ell'}\rangle\langle B_{\ell'}|M_k|B_i\rangle\Bigg)$$

  of the parity conserving contribution skipped, as $b_{\ell\ell'} = \langle B_\ell|\mathcal{H}_{\rm eff}^{\rm pv}|B_{\ell'}\rangle$ much smaller than $a_{\ell\ell'}$

  D. Ebert, W. Kallies, Yad. Fiz. **40** (1984) 1250–1255,
  H. Y. Cheng, Z. Phys. C **29** (1985) 453–458.
- $\Rightarrow$ pole contribution the only relevant one

Parity conserving pole contribution

$$B_{fki}^{\rm pole} = \sum_\ell \frac{g_{fk\ell}\, a_{\ell\, i}}{m_i - m_\ell} + \sum_{\ell'} \frac{a_{f\ell'}\, g_{\ell' ki}}{m_f - m_{\ell'}} = B_{fki}^{\rm pole}(s) + B_{fki}^{\rm pole}(u) =$$

$$= \frac{\sqrt{2}}{f_k}\left(\sum_\ell g_{fk\ell}^A \frac{m_f + m_\ell}{m_i - m_\ell}\, a_{\ell\, i} + \sum_{\ell'} a_{f\ell'} \frac{m_i + m_{\ell'}}{m_f - m_{\ell'}}\, g_{\ell' ki}^A\right),$$

where the generalised Goldberger–Treiman relation

$$g_{fk\ell} = \frac{\sqrt{2}}{f_k}(m_f + m_\ell) g_{fk\ell}^A$$

has been used. Axial vector couplings in constituent quark model

$$g_{fki}^A = (4\tilde{I}_1 + 5\tilde{I}_2)_{fki}$$

The role of intermediate baryonic states

$$R_{fi} = \frac{m_f + m_\ell}{m_i - m_\ell}, \qquad R_{if} = \frac{m_i + m_{\ell'}}{m_f - m_{\ell'}}$$

depend on masses of intermediate states ℓ, ℓ'. Intermediate states are

- s channel: $\ell = \Sigma^+$
- u channel: $\ell' = \Xi_c$ for A, $\ell' = \Xi'_c$ for B

Taking the mean value $\bar{m}_\ell$ one obtains constant coefficients $R_{fi} = \bar{R}_{fi}$ and $R_{if} = \bar{R}_{if}$.

$$B_{fki}^{\rm pole} = \frac{\sqrt{2}}{f_k}\Big((I_3 + 2I_4 + 6I_5)\bar{R}_{fi} + (\hat{I}_3 + 2\hat{I}_4 + 6I_5)\bar{R}_{if}\Big).$$

Pure P wave

S-wave amplitude $A_{fki} = 0 \Rightarrow$ pure P-wave contribution.

About the phase shifts

In addition to the decay asymmetry

$$\alpha_{\Lambda_c^+\to\Xi^0K^+}=\frac{2\sqrt{Q_+Q_-}\,\mathrm{Re}(AB^*)}{Q_+|A|^2+Q_-|B|^2},$$

it is possible to measure in experiments also

$$\tan\Delta_{\Lambda_c^+\to\Xi^0K^+}=\frac{2\sqrt{Q_+Q_-}\,\mathrm{Im}(AB^*)}{Q_+|A|^2+Q_-|B|^2}.$$

Having the amplitudes $A=|A|e^{i\delta_P}$ and $B=|B|e^{i\delta_S}$ endowed with phases, one has $AB^*=|A||B|e^{i(\delta_P-\delta_S)}$. Because of this, $\alpha_{\Lambda_c^+\to\Xi^0K^+}=0$ leads automatically to $\delta_P-\delta_S=\pm\pi/2$.

Next steps:

- measure $\Delta_{\Lambda_c^+\to\Xi^0K^+}$
- take into account subdominant contribution to A