

Hyperon nonleptonic decays in χ PT, revisited

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Transition amplitude

$$\mathcal{M}(B_i \rightarrow B_f \pi) = G_F m_{\pi^+}^2 \bar{u}_f (A^{(S)} + A^{(P)} \gamma_5) u_i$$

Dimensionless l -wave amplitudes

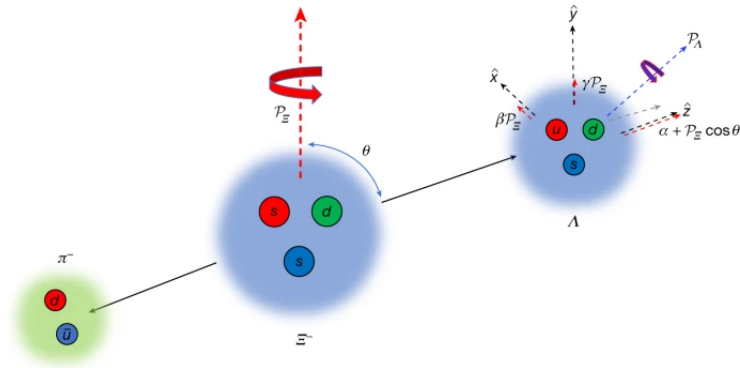
$$\text{parity-violating: } A^{(S)} \equiv S$$

$$\text{parity-conserving: } A^{(P)} \equiv \frac{|\vec{\mathbf{p}}_f|}{E_f + m_f} P$$

Decay observables

$$\alpha = \frac{2\Re(S^* P)}{|S|^2 + |P|^2} \quad \beta = \frac{2\Im(S^* P)}{|S|^2 + |P|^2} = \sqrt{1 - \alpha^2} \sin \phi$$

$$\Gamma = \text{kin}(|\vec{\mathbf{p}}_f|, E_f, m_f) (|S|^2 + |P|^2)$$



$\Xi^- \rightarrow \Lambda (\rightarrow p \pi^-) \pi^-$ decay [Nature 606, 6469 (2022)]

Produced $B(\bar{B})$ at e^+e^- colliders (e.g. BESIII) are **inherently** polarized.

$$\mathbf{P}_\Lambda \cdot \hat{\mathbf{z}} = \frac{\alpha_\Xi + \mathbf{P}_\Xi \cdot \hat{\mathbf{z}}}{1 + \alpha_\Xi \mathbf{P}_\Xi \cdot \hat{\mathbf{z}}}, \quad \mathbf{P}_\Lambda \times \hat{\mathbf{z}} = |\mathbf{P}_\Xi| \sqrt{1 - \alpha_\Xi^2} \frac{\sin \phi_\Xi \hat{\mathbf{x}} + \cos \phi_\Xi \hat{\mathbf{y}}}{1 + \alpha_\Xi \mathbf{P}_\Xi \cdot \hat{\mathbf{z}}},$$



Polarization and entanglement in baryon-antibaryon pair production in electron-positron annihilation

The BESIII Collaboration*

[Nature Phys. 15, 631 (2019)]

Article | [Open Access](#) | [Published: 01 June 2022](#)

Probing CP symmetry and weak phases with entangled double-strange baryons

[The BESIII Collaboration](#)

[Nature](#) 606, 64–69 (2022) | [Cite this article](#)

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[Nature 606, 6469 (2022)]

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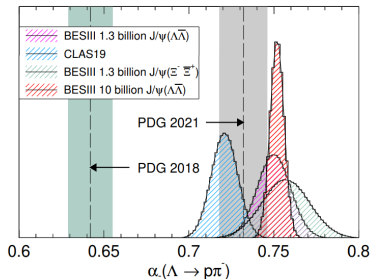
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Precise Measurements of Decay Parameters and CP Asymmetry with Entangled Λ - $\bar{\Lambda}$ Pairs

M. Ablikim et al. (BESIII Collaboration)

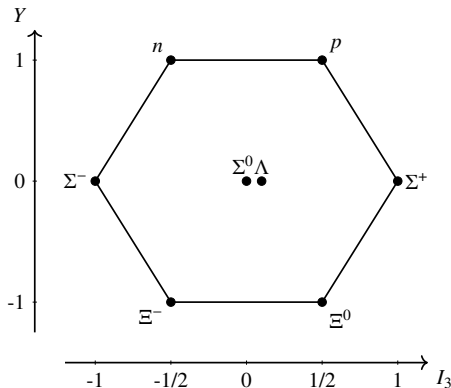
Phys. Rev. Lett. 129, 131801 – Published 22 September 2022

[Phys.Rev.Lett. 129, 131801 (2022)]



• $\Delta S = 1$ transitions:

- ▶ $\Sigma^+ \rightarrow n\pi^+$
- ▶ $\Sigma^- \rightarrow n\pi^-$
- ▶ $\Lambda \rightarrow p\pi^-$
- ▶ $\Xi^- \rightarrow \Lambda\pi^-$



$$\sqrt{2}A(\Sigma^+ \rightarrow p\pi^0) - A(\Sigma^+ \rightarrow n\pi^+) + A(\Sigma^- \rightarrow n\pi^-) = 0$$

$$A(\Lambda \rightarrow p\pi^-) + \sqrt{2}A(\Lambda \rightarrow n\pi^0) = 0$$

$$A(\Xi^- \rightarrow \Lambda\pi^-) + \sqrt{2}A(\Xi^0 \rightarrow \Lambda\pi^0) = 0$$

Summary of nonleptonic hyperon decays properties [PRD105, 116022 (2022)]

	\mathcal{B}	α	ϕ [rad]	Γ [$G_F^2 m_{\pi^+}^4$ GeV]
$\Sigma^+ \rightarrow n\pi^+$	48%	0.068(13)	2.91(35)	0.0769(02)
$\Sigma^+ \rightarrow p\pi^0$	52%	-0.994(04)	0.63(59)	0.0821(02)
$\Sigma^- \rightarrow n\pi^-$	100%	-0.068(08)	0.17(26)	0.0861(01)
$\Lambda \rightarrow p\pi^-$	64%	0.755(03)	-0.113(61)	0.0310(02)
$\Lambda \rightarrow n\pi^0$	36%	0.692(17)	--	0.0174(01)
$\Xi^- \rightarrow \Lambda\pi^-$	100%	-0.379(04)	-0.042(16)	0.0778(01)
$\Xi^0 \rightarrow \Lambda\pi^0$	100%	-0.345(08)	0.36(21)	0.0438(01)

Next step

Use α , γ , Γ data to extrapolate updated experimental L -wave amplitude values.

L -wave amplitude extraction: assuming CP conservation, $\Delta I = 1/2$

$$L = \sum_j L_j \exp(i\delta_j^L), \quad j \in \{2\Delta I, 2I\}$$

and final-interaction phase shifts [\[PRD105, 116022 \(2022\)\]](#)

	$ \mathbf{q} $ [MeV/c]	δ_1^S [°]	δ_3^S [°]	δ_1^P [°]	δ_3^P [°]
$\Lambda \rightarrow N\pi$	103	6.52(9)	-4.60(7)	-0.79(8)	-0.75(4)
$\Sigma \rightarrow N\pi$	190	9.98(23)	-10.70(13)	-0.04(33)	-3.27(15)

Relative sign between amplitudes fixed by Lee-Sugawara relation

$$\frac{3}{\sqrt{6}} A^{(S)}(\Sigma^- \rightarrow n\pi^-) + A^{(S)}(\Lambda \rightarrow p\pi^-) + 2A^{(S)}(\Xi^- \rightarrow \Lambda\pi^-) = 0$$

New reference values extracted from data compared to [NPB 375 (1992)]:

additional form =

	S_{comp}	S_{real}	S_{Jenkins}	P_{comp}	P_{real}	P_{Jenkins}
$\Sigma^+ \rightarrow \pi\pi^+$	$0.09 + 0.24 i$	0.06	0.06	$1.70 + 0.00 i$	1.81	1.81
$\Sigma^+ \rightarrow \rho\pi^0$	$-1.37 + 0.08 i$	-1.38	-1.43	$1.25 - 0.00 i$	1.24	1.17
$\Sigma^- \rightarrow \pi\pi^-$	$1.85 - 0.35 i$	1.88	1.88	$-0.06 + 0.00 i$	-0.06	-0.06
$\Lambda \rightarrow \rho\pi^-$	$1.36 + 0.16 i$	1.38	1.42	$0.63 - 0.01 i$	0.63	0.52
$\Lambda \rightarrow \pi\pi^0$	$-1.02 - 0.12 i$	-1.03	-1.04	$-0.42 + 0.01 i$	-0.41	-0.39
$\Xi^- \rightarrow \Lambda\pi^-$	-1.99	-1.99	-1.98	0.39	0.39	0.48
$\Xi^0 \rightarrow \Lambda\pi^0$	1.52	1.52	1.52	-0.27	-0.27	-0.33

Previous values extracted on the assumption of real-valued amplitudes: comparison with complex-valued l -waves on most recent data.

- At low-energy regime, α_s too large for a perturbative description of hadron interactions from QCD.
- χ PT: EFT with hadrons as DF parametrizes meson-baryon interactions [*Physica A* 96 (1979) 1-2, 327-340], [*Annals Phys.* 158 (1984) 142].

Weak nonleptonic hyperon decays previously studied in

Nuclear Physics B261 (1985) 185–198
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**ON THE VALIDITY OF CHIRAL PERTURBATION THEORY
FOR WEAK HYPERON DECAYS***

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(Revised 14 June 1985)

**NUCLEAR
PHYSICS B**

Nuclear Physics B 375 (1992) 561–581
North-Holland

**Hyperon non-leptonic decays in chiral
perturbation theory**

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Non-leptonic hyperon decays in chiral perturbation theory

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Computing 1-loop corrections using *Heavy-Baryon* χ PT (nonrelativistic approach):

- ① [Nucl.Phys.B 261 (1985) 185-198]:
 - ▶ baryon decuplet not included as internal states; terms up to $O(M_K^2 \log M_K)$.
- ② [Nucl.Phys.B 375 (1992) 561-581]:
 - ▶ inclusion of decuplet, 3-meson vertex h_π ; terms up to $O(M_K^2 \log M_K)$.
- ③ [Eur.Phys.J.C 6 (1999) 85-107]
 - ▶ decuplet not included, 3-meson vertex h_π ; terms up to $M_K^2 (a + b \log M_K)$.
- ④ [Phys. Rev. D 61, 114014 (2000)]:
 - ▶ same as 2, contradicting results in h_π terms.

General conclusions

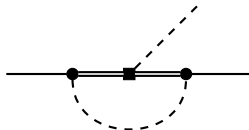
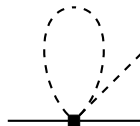
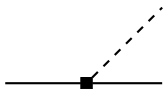
- LO chiral corrections to S -waves are in good agreement with experiment;
- P -waves are not well-described;
- Results from simultaneous fitting are unsatisfactory.

Compute 1-loop corrections from relativistic LO Lagrangian

$$\mathcal{L}_{\phi B}^S + \mathcal{L}_{\phi B}^W$$

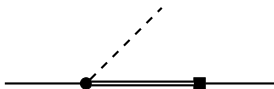
$$\mathcal{L}_{\phi B}^W = h_D \text{tr} \bar{B} \{ \xi^\dagger h \xi, B \} + h_F \text{tr} \bar{B} [\xi^\dagger h \xi, B] + h_C \text{tr} \bar{T}^\mu (\xi^\dagger h \xi) T_\mu$$

E.g. S -wave contributions:



Inclusion of lower-lying $\frac{1}{2}^{\pm}$ resonances [Phys. Rev. D 59, 094025 (1999)]

$$\mathcal{L}_{\text{res}}^{\text{w}} \propto d^* [\text{tr}(\bar{R}^+ \{\xi^\dagger h \xi, B\}) + \text{tr}(\bar{B} \{\xi^\dagger h \xi, R^+\})] + f^* [\text{tr}(\bar{R}^+ [\xi^\dagger h \xi, B]) + \text{tr}(\bar{B} [\xi^\dagger h \xi, R^+])] \\ + i w_d [\text{tr}(\bar{R}^- \{\xi^\dagger h \xi, B\}) - \text{tr}(\bar{B} \{\xi^\dagger h \xi, R^-\})] + i w_f [\text{tr}(\bar{R}^- [\xi^\dagger h \xi, B]) - \text{tr}(\bar{B} [\xi^\dagger h \xi, R^-])]$$



The resulting amplitudes:

$$S_{\text{theory}}, P_{\text{theory}} = l.c.(h_{D,F,C}, w_{d,f}, d^*, f^*)$$

Goal

To fit L_{theory} to L_{expt} using least squares method to obtain LEC's values.

TABLE 2

Predicted S-wave amplitudes for parameter values $h_D = -0.35$, $h_F = 0.86$, and $h_C = -0.36$

Decay	S_{expt}	S_{theory}	S_{tree}	ΔS_{loop}	ΔS_{octet}	ΔS_{decup}
$\Sigma^+ \rightarrow n\pi^+$	0.06	-0.09	0.00	-0.09	0.13	-0.22
$\Sigma^+ \rightarrow p\pi^0$	-1.43	-1.41	-0.85	-0.55	-0.04	-0.51
$\Sigma^- \rightarrow n\pi^-$	1.88	1.90	1.21	0.69	0.18	0.51
$\Lambda \rightarrow p\pi^-$	1.42	1.44	0.91	0.53	0.16	0.37
$\Lambda \rightarrow n\pi^0$	-1.04	-1.02	-0.64	-0.37	-0.11	-0.27
$\Xi^- \rightarrow \Lambda\pi^-$	-1.98	-2.04	-1.19	-0.84	-0.14	-0.71
$\Xi^0 \rightarrow \Lambda\pi^0$	1.52	1.44	0.84	0.60	0.10	0.50

[Nucl.Phys.B 375 (1992) 561-581]

- Good agreement with experiment.
- h_C not well determined by 1-loop fit ($h_C = -0.36(65)$). 3-meson h_π terms are negligible. Quark model prediction $h_D/h_F = -1/3$ is favored.
- Octet correction is small, while decuplet contribution is sizable.
- P -waves are poorly reproduced when using S -wave fit LEC's.

Resonance saturation S -wave fit ($h_D = -1/3 h_F$)

LEC	$[G_F m_\pi^2 \sqrt{2} f_\pi]$		s_{expt}	s_{theory}	s_{tree}	Δs_{loop}	Δs_{res}
h_F	0.994 ± 0.012	$\Sigma^+ \rightarrow n\pi^+$	0.09	--	--	--	--
h_C	-11.01 ± 0.11	$\Sigma^+ \rightarrow p\pi^0$	-1.37	-1.37	-0.94	-0.44	-2.15
w_f	11.00 ± 0.11	$\Sigma^- \rightarrow n\pi^-$	1.85	1.85	1.32	0.52	3.23
w_d	-22.77 ± 0.13	$\Lambda \rightarrow p\pi^-$	1.36	1.37	1.08	0.29	2.42
χ^2	123.5	$\Lambda \rightarrow n\pi^0$	-1.02	-1.00	-0.76	-0.23	-1.70
$\tilde{\chi}^2$	61.77	$\Xi^- \rightarrow \Lambda\pi^-$	-1.99	-2.02	-1.35	-0.66	-10.79
		$\Xi^0 \rightarrow \Lambda\pi^0$	1.52	1.27	0.96	0.31	7.40

S -waves

- Corrections to LO χ PT are large, for most channels coming from the resonances (also tree-level): confirmed importance of resonances.
- Good agreement with experiment and convergent behavior recovered.
- h_C is large, h_F is even close to Jenkins'. Large cancellation between resonance and loops.

Resonance saturation P -wave fit ($h_D = -1/3 h_F$)

LEC	$[G_F m_\pi^2 \sqrt{2} f_\pi]$		p_{expt}	p_{theory}	p_{tree}	Δp_{loop}	Δp_{res}
h_F	0.696 ± 0.005	$\Sigma^+ \rightarrow n\pi^+$	1.70	1.73	-0.03	1.76	1.88
h_C	0.527 ± 0.007	$\Sigma^+ \rightarrow p\pi^0$	1.25	1.22	-0.70	1.92	1.14
d^*	0.69 ± 0.04	$\Sigma^- \rightarrow n\pi^-$	-0.06	-0.10	0.97	-1.06	0.31
f^*	-4.190 ± 0.031	$\Lambda \rightarrow p\pi^-$	0.63	0.63	-0.33	0.95	0.76
χ^2	68.66	$\Lambda \rightarrow n\pi^0$	-0.42	-0.46	0.24	-0.70	-0.56
$\tilde{\chi}^2$	22.89	$\Xi^- \rightarrow \Lambda\pi^-$	0.39	0.39	0.12	0.27	-1.31
		$\Xi^0 \rightarrow \Lambda\pi^0$	-0.27	-0.29	-0.09	-0.20	0.89

P -waves

- Tree level is not dominant. “True” loops dominate in some decay channels: not a coherent picture.
- Good agreement with experiment, loss of convergent behavior.
- LEC’s size is closer to previous works.

Resonance saturation combined s - and p -wave fit

LEC	$[G_F m_\pi^2 \sqrt{2} f_\pi]$		α_{expt}	α_{theory}	Γ_{expt}	Γ_{theory}	s_{expt}	s_{theory}	p_{expt}	p_{theory}
h_D	0.3362 ± 0.0022	$\Sigma^+ \rightarrow n\pi^+$	0.07	0	0.08	0.07	0.09	0	1.70	-1.78
h_F	0.8887 ± 0.0028	$\Sigma^+ \rightarrow p\pi^0$	-0.98	-1.00	0.08	0.09	-1.37	1.34	1.25	-1.35
h_C	2.260 ± 0.006	$\Sigma^- \rightarrow n\pi^-$	-0.07	0.07	0.09	0.05	1.85	-1.37	-0.06	-0.05
w_d	17.60 ± 0.09	$\Lambda \rightarrow p\pi^-$	0.76	0.74	0.03	0.03	1.36	1.43	0.63	0.63
w_f	-1.022 ± 0.011	$\Lambda \rightarrow n\pi^0$	0.69	0.75	0.02	0.02	-1.02	-1.00	-0.42	-0.46
d^*	-12.480 ± 0.027	$\Xi^- \rightarrow \Lambda\pi^-$	-0.38	-0.40	0.08	0.08	-1.99	0.43	0.39	-2.07
f^*	-5.553 ± 0.016	$\Xi^0 \rightarrow \Lambda\pi^0$	-0.36	-0.42	0.04	0.04	1.52	-0.32	-0.27	1.43
χ^2	4971									
$\tilde{\chi}^2$	828.5									
h_D/h_F	0.378									

Combined fit

- Poor agreement with data, loss of convergent behavior for S -waves.
- LEC's size is inconsistent with other results.
- Quadratic relation in α and Γ disregards relative signs between amplitudes: too much freedom. Switch to fit separately to L -waves together (future).

Meson-baryon LO Lagrangian

$$\begin{aligned} \mathcal{L}_{\phi B}^s = & i \operatorname{tr} \bar{B} \not{D} B - m_B \operatorname{tr} \bar{B} B + D \operatorname{tr} \bar{B} \gamma^\mu \gamma_5 \{A_\mu, B\} + F \operatorname{tr} \bar{B} \gamma^\mu \gamma_5 [A_\mu, B] - i \bar{T}^\mu \not{D} T_\mu \\ & + m_T \bar{T}^\mu T_\mu + C (\bar{T}^\mu A_\mu B + \bar{B} A_\mu T^\mu) + \mathcal{H} \bar{T}^\mu \gamma_\nu \gamma_5 A^\nu T_\mu + \frac{f^2}{4} \operatorname{tr} \partial_\mu \Sigma \partial^\mu \Sigma^\dagger \end{aligned}$$

Inclusion of $\frac{1}{2}^\mp$ resonances [\[PRD59, 094025 \(1999\)\]](#)

$$\begin{aligned} \mathcal{L}_{RB}^s = & 2s_d [\operatorname{tr}(\bar{R} \gamma_\mu \{A_\mu, B\}) - \operatorname{tr}(\bar{B} \gamma_\mu \{A_\mu, R\})] \\ & + 2s_f [\operatorname{tr}(\bar{R} \gamma_\mu [A_\mu, B]) - \operatorname{tr}(\bar{B} \gamma_\mu [A_\mu, R])] \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{B^*B}^s = & \frac{D^*}{2} [\operatorname{tr}(\bar{B}^* \gamma_\mu \gamma_5 \{A_\mu, B\}) + \operatorname{tr}(\bar{B} \gamma_\mu \gamma_5 \{A_\mu, B^*\})] \\ & + \frac{F^*}{2} [\operatorname{tr}(\bar{B}^* \gamma_\mu \gamma_5 [A_\mu, B]) + \operatorname{tr}(\bar{B} \gamma_\mu \gamma_5 [A_\mu, B^*])] \end{aligned}$$

$$V^\mu = \frac{1}{2} (\xi \partial^\mu \xi^\dagger + \xi^\dagger \partial^\mu \xi), \quad A^\mu = \frac{i}{2} (\xi \partial^\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi)$$

$$\xi = \exp \frac{i\pi}{f}, \quad \Sigma = \xi^2 = \exp \frac{2i\pi}{f}$$

S-wave 1-loop corrections [Nucl. Phys. B 375 (1992)]



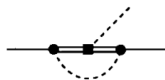
(a)



(b)



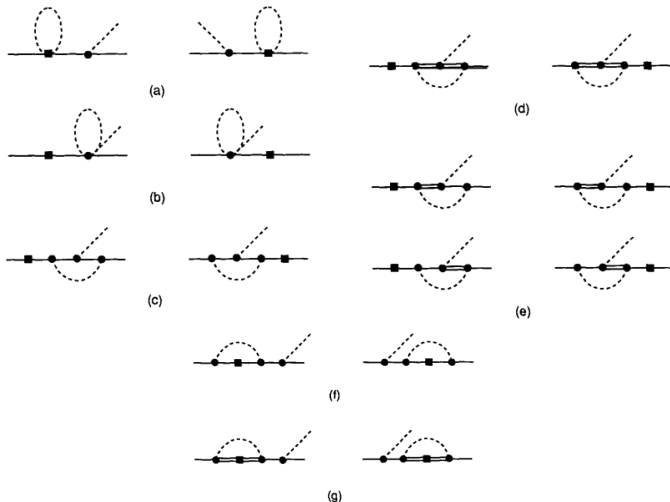
(c)



(d)

P-wave diagrams

P-wave 1-loop corrections, [Nucl. Phys. B 375 (1992)]



Resonance saturation s -wave fit

LEC	$[G_F m_\pi^2 \sqrt{2} f_\pi]$		s_{expt}	s_{theory}	s_{tree}	Δs_{loop}	Δs_{res}
h_D	7.5 ± 0.9	$\Sigma^+ \rightarrow n\pi^+$	0.09	--	--	--	--
h_F	-2.7 ± 0.4	$\Sigma^+ \rightarrow p\pi^0$	-1.37	-1.37	7.20	-8.57	-7.39
h_C	-38.5 ± 3.2	$\Sigma^- \rightarrow n\pi^-$	1.85	1.85	-10.18	12.03	9.18
w_f	25.7 ± 1.7	$\Lambda \rightarrow p\pi^-$	1.36	1.35	-0.23	1.57	7.49
w_d	-91 ± 8	$\Lambda \rightarrow n\pi^0$	-1.02	-1.05	0.16	-1.21	-5.26
$\chi^2 = \tilde{\chi}^2$	82.86	$\Xi^- \rightarrow \Lambda\pi^-$	-1.99	-2.01	6.35	-8.36	-27.72
h_D/h_F	-2.792	$\Xi^0 \rightarrow \Lambda\pi^0$	1.52	1.33	-4.49	5.82	19.08

Resonance saturation p -wave fit

LEC	$[G_F m_\pi^2 \sqrt{2} f_\pi]$
h_D	-0.286 ± 0.008
h_F	0.10 ± 0.08
h_C	-0.79 ± 0.18
d^*	3.6 ± 0.4
f^*	-1.93 ± 0.32
χ^2	16.76
$\tilde{\chi}^2$	8.38
h_D/h_F	-2.91

	P_{expt}	P_{theory}	P_{tree}	Δp_{loop}	Δp_{res}
$\Sigma^+ \rightarrow n\pi^+$	1.70	1.70	-0.25	1.95	1.78
$\Sigma^+ \rightarrow p\pi^0$	1.25	1.25	-0.29	1.54	1.30
$\Sigma^- \rightarrow n\pi^-$	-0.06	-0.06	0.15	-0.21	-0.02
$\Lambda \rightarrow p\pi^-$	0.63	0.63	0.11	0.52	0.31
$\Lambda \rightarrow n\pi^0$	-0.42	-0.46	-0.08	-0.39	-0.23
$\Xi^- \rightarrow \Lambda\pi^-$	0.39	0.39	-0.20	0.59	-0.48
$\Xi^0 \rightarrow \Lambda\pi^0$	-0.27	-0.28	0.15	-0.43	0.33

Resonance saturation combined s - and p -wave fit

	s_{expt}	s_{theory}	s_{tree}	Δs_{loop}	Δs_{res}	p_{expt}	p_{theory}	p_{tree}
$\Sigma^+ \rightarrow n\pi^+$	0.09	--	--	--	--	1.70	-1.78	0.58
$\Sigma^+ \rightarrow p\pi^0$	-1.37	1.34	-0.39	1.73	1.18	1.25	-1.35	-0.42
$\Sigma^- \rightarrow n\pi^-$	1.85	-1.37	0.55	-1.92	-0.98	-0.06	-0.05	1.20
$\Lambda \rightarrow p\pi^-$	1.36	1.43	1.23	0.20	-0.99	0.63	0.63	-0.80
$\Lambda \rightarrow n\pi^0$	-1.02	-1.00	-0.87	-0.14	0.69	-0.42	-0.46	0.59
$\Xi^- \rightarrow \Lambda\pi^-$	-1.99	0.43	-0.95	1.38	2.05	0.39	-2.07	0.70
$\Xi^0 \rightarrow \Lambda\pi^0$	1.52	-0.32	0.67	-0.99	-1.43	-0.27	1.43	-0.51