The Geometry of Black Hole Microstructure

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Based on
arXiv:2211.14326, arXiv:2212.06158, arXiv:2312.02286,
arXiv:2404.14477 and arXiv:2407.01665

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Motivation

- Quantum Mechanics and General Relativity provide extremely good descriptions of our world at respectively small and large scales. However, they are incompatible with each other.

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- The black-hole entropy puzzle:

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        - However, black holes have a thermodynamic behavior :

$$S = \frac{k_B c^3}{4G\hbar} A \qquad \text{for Sgr A*:} \quad S \sim 10^{90} \quad \rightarrow \sim e^{10^{90}} \text{ microstates}$$

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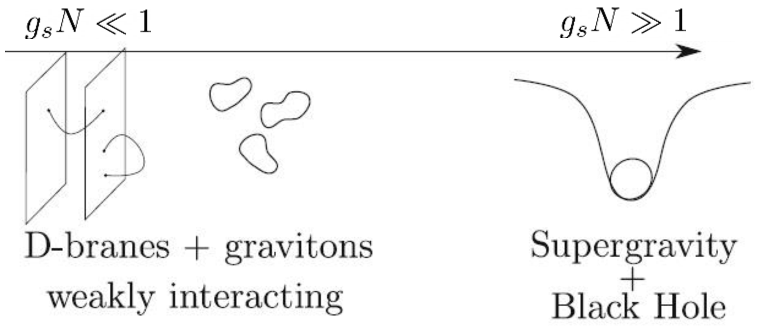
  ➢ What are the microstates?

Microscopic degrees of freedom

- An amazing achievement of String Theory:
  - Counting of microstates of supersymmetric black holes
    At $g_s N \ll 1$ as bound states of strings and branes
    [Strominger,Vafa '96]



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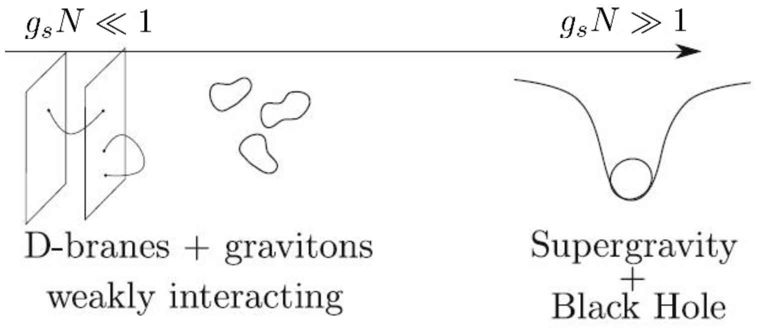

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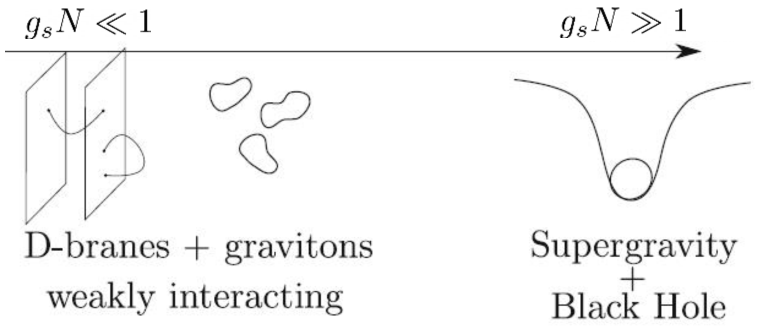

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- Standard lore:
  - Gravitational attraction is universal and gets stronger with G_N
  - Black hole horizon grows with G_N: $r_H = 2 G_N M$
    - ➢ Microstates are engulfed by the horizon
    - ➢ Standard black hole is recovered

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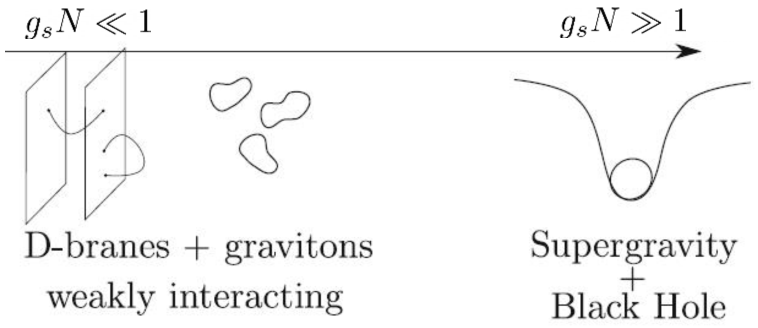

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    ➢ There are brane configurations that expand and never form a horizon!

The Fuzzball Proposal

- In the classical regime, the microstates resemble a black hole from afar, but differ in the vicinity of the horizon

- Fuzzballs have no horizon and no singularity
  - There are e^S of them
  - No horizons, no information paradox

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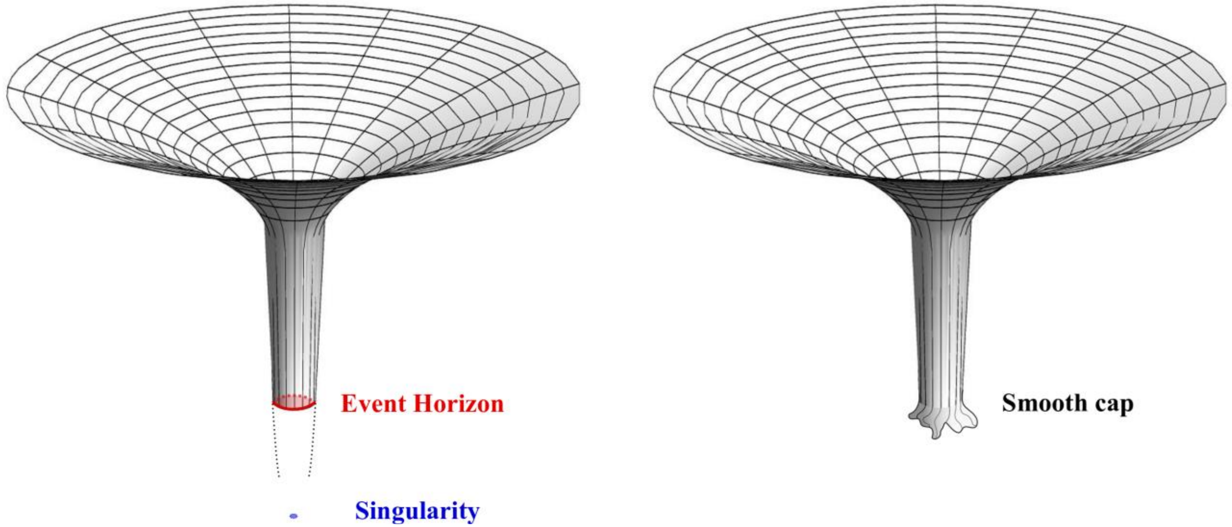
- Microstate geometries: smooth, horizonless solutions in supergravity with the same black hole charges



Microscopic degrees of freedom in the black hole regime

- Historically:
  - Construct horizonless solutions with black hole charges in the regime of the classical black hole solution
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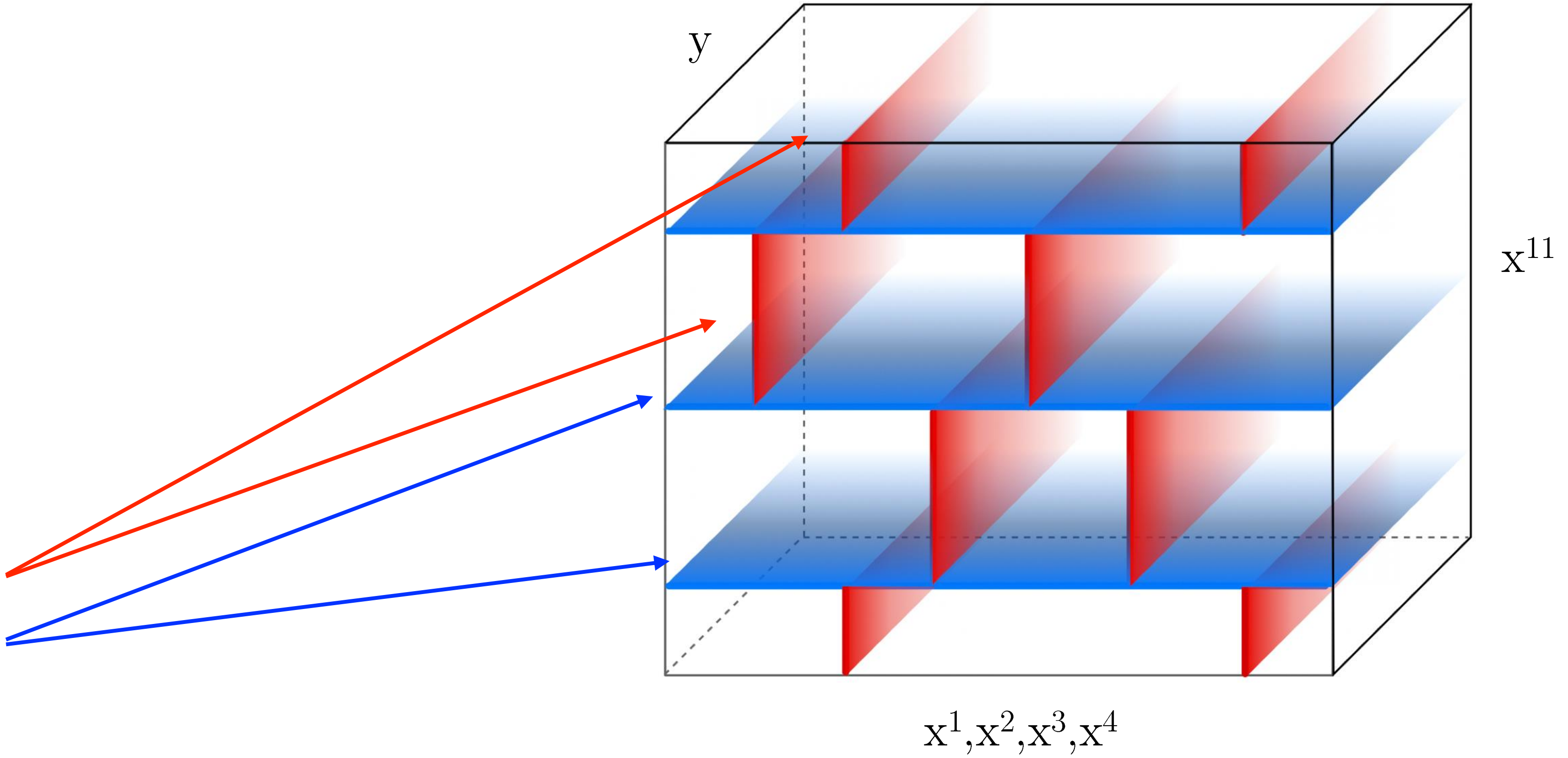
  - Construct horizonless solutions with black hole charges in the regime of the classical black hole solution
  - Count the number of solutions
- The endeavor has been successful, but it has not been possible to account for the total entropy
- For the D1-D5-P black-hole:
  - Superstrata: largest family of microstate geometries with the same charges
  - However, $S \sim \sqrt{N_1 N_5} N_P^{1/4} \ll \sqrt{N_1 N_5 N_P}$

Our approach

[Bena, Hampton, Houppe, Li, DT '22]

- We study the M2-M5-P black hole

[Dijkgraaf, Verlinde, Verlinde '97]



- Brane system
  - M2-branes wrapped along y, x^{11}
  - M5-branes wrapped along y, T^4
  - P along y

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- Entropy comes from the fractionation of the M2 branes:
  - Each M2-brane can break into N_5 strips
  - Total $N_2 N_5$ independent momentum carriers
  - Each has 4 oscillation directions (T^4) + 4 fermionic partners

$$S = 2\pi\sqrt{\frac{4+2}{6} N_2 N_5 N_P}$$

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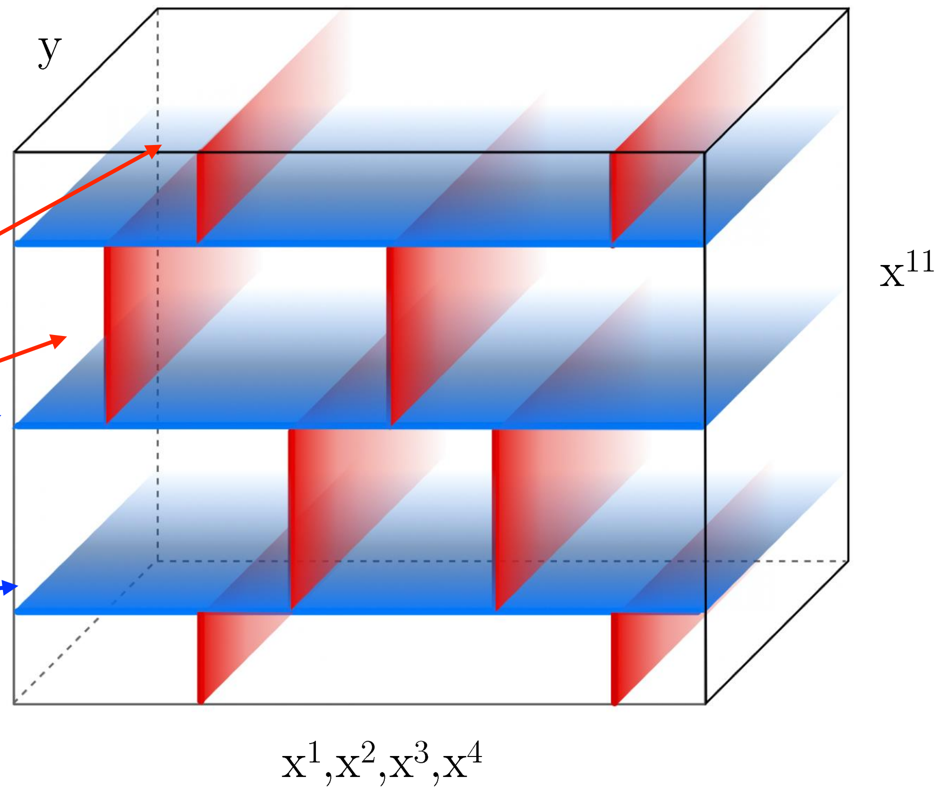

Finite coupling

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- A D3-brane is pulled by a D1 ending on it

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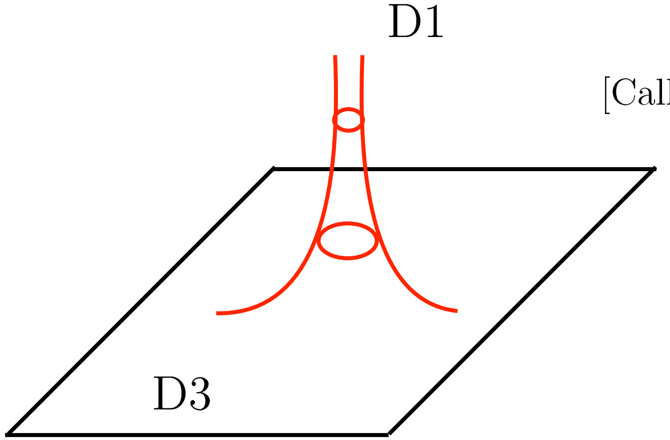

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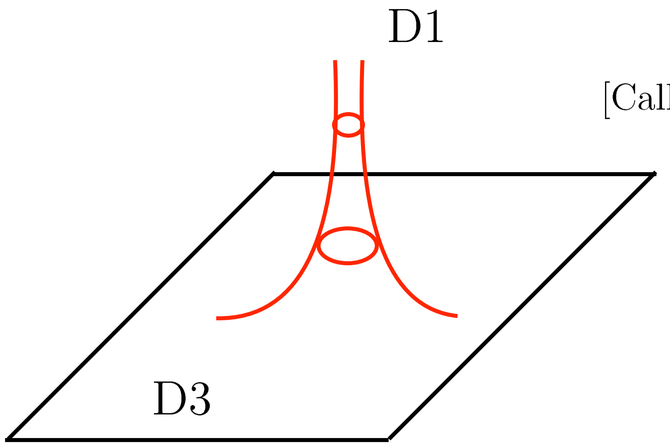

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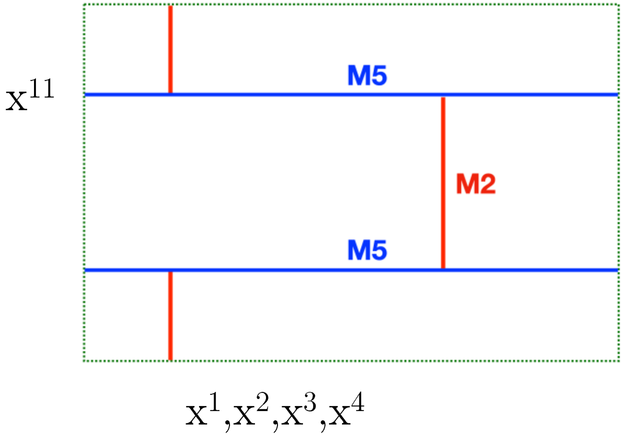

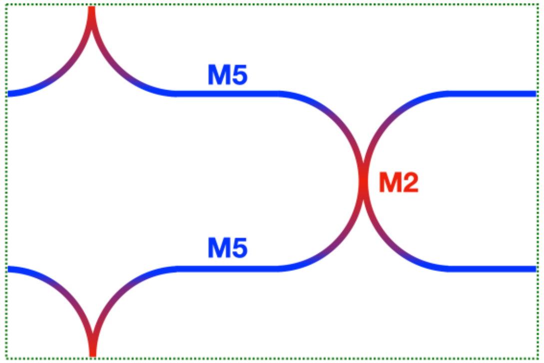

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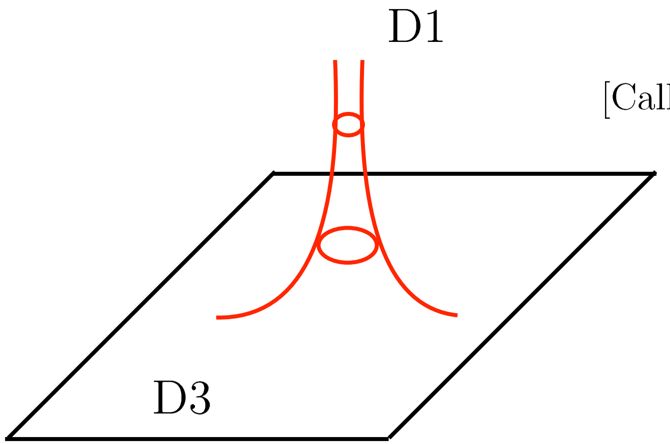

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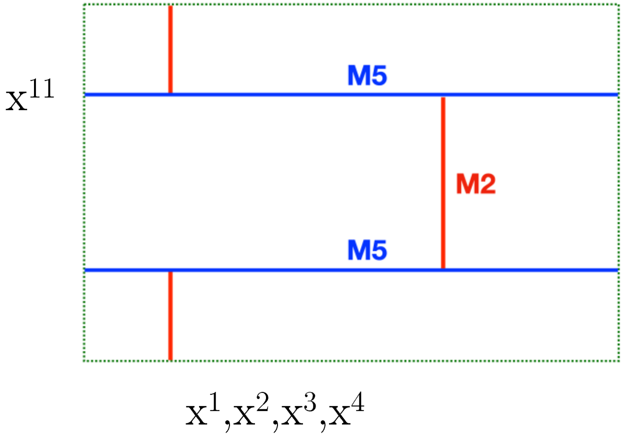
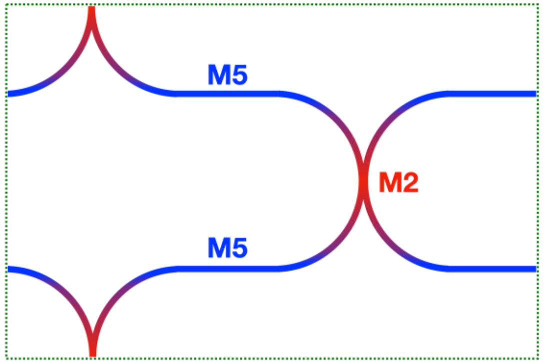
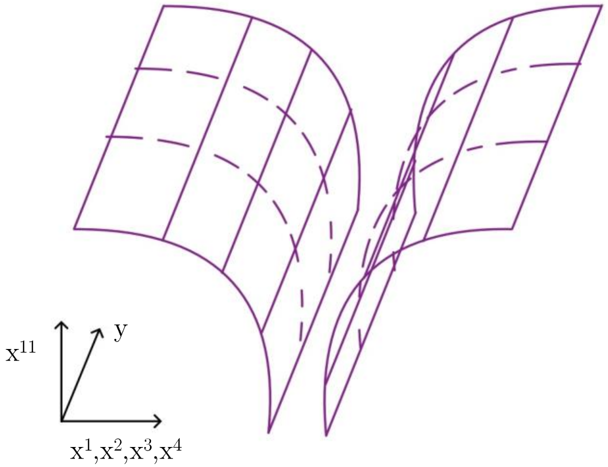
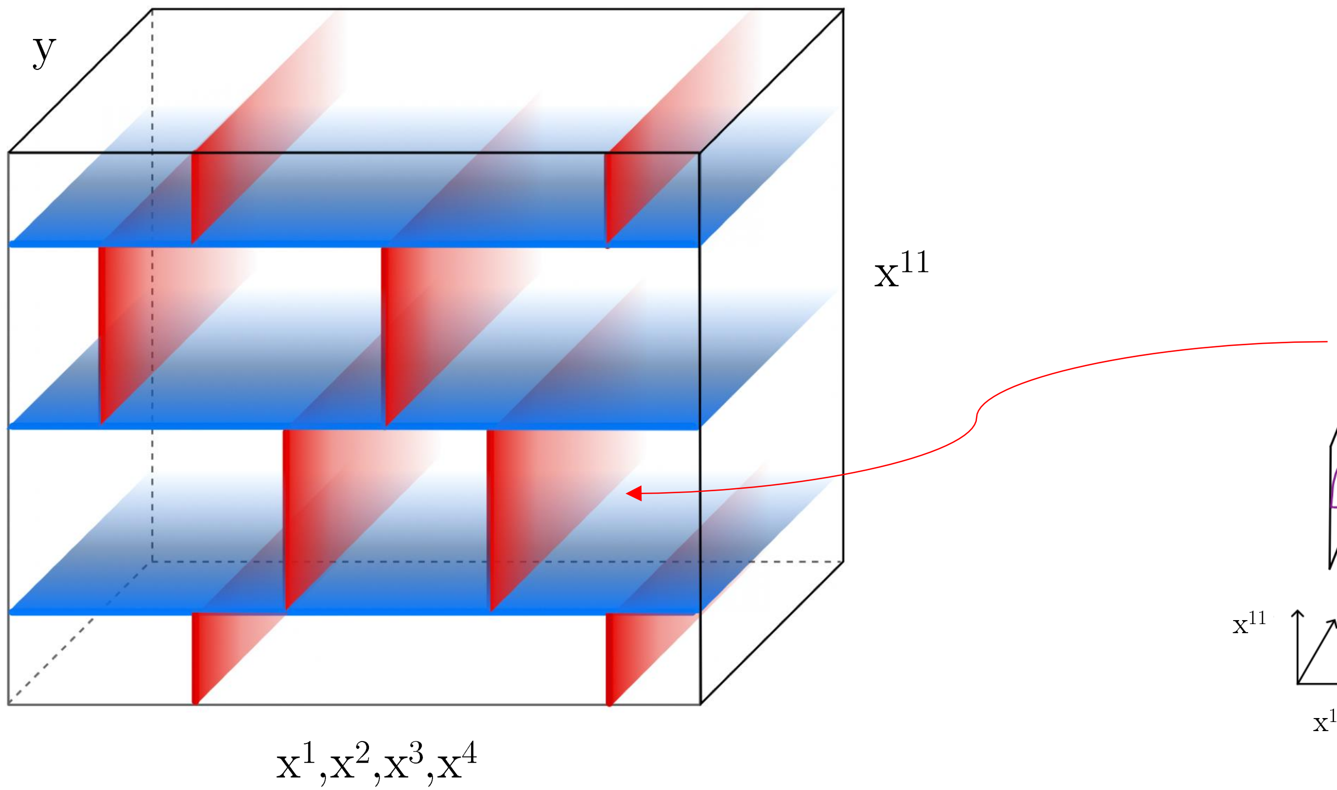


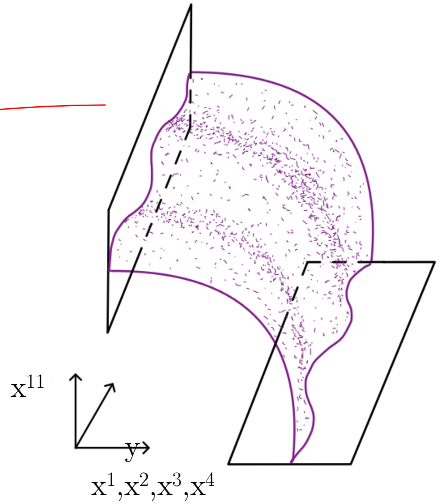


M5-M2 (y,x^{11},x^{1})



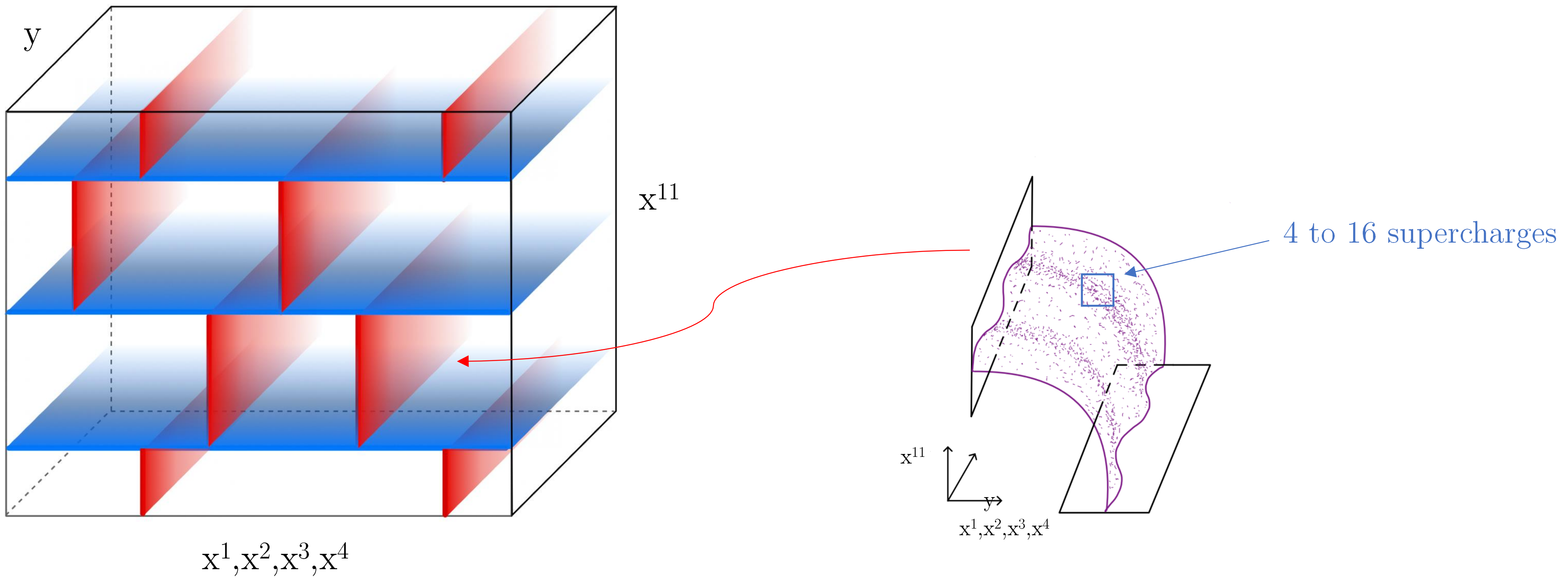
Finite coupling – The Supermaze



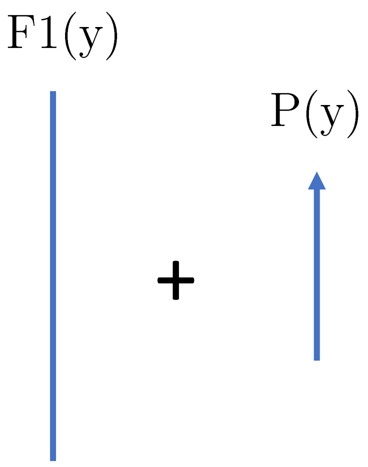


Finite coupling – The Supermaze

The Super-Maze preserves 4 supercharges globally, but if one zooms in at any location along it, it preserves locally 16 supercharges



Local Supersymmetry Enhancement: an example



- F1 and P preserve separately 16 supercharges
- Together, they preserve 8 supercharges

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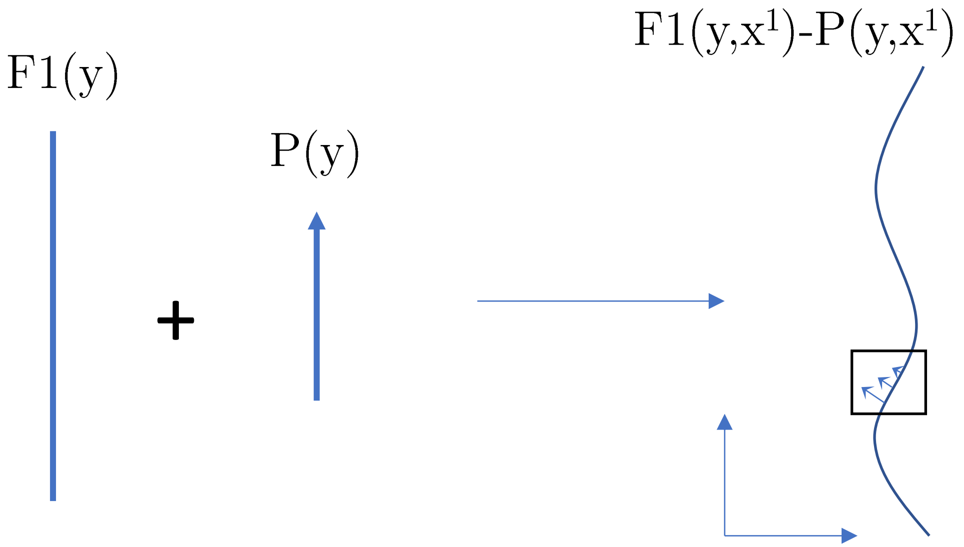

- F1 and P preserve separately 16 supercharges
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- It is possible to form a bound state possessing the same global charges, but only locally 16 supersymmetries.
- The F1-P profile is locally a transversely boosted F1.

Local Supersymmetry Enhancement

- General lessons:

  - For a charge system with # global supersymmetries, there may exist many configurations with the same global supersymmetries, but with an enhanced number of local supersymmetries.

Local Supersymmetry Enhancement

- General lessons:

  - For a charge system with # global supersymmetries, there may exist many configurations with the same global supersymmetries, but with an enhanced number of local supersymmetries.

  - The local enhancement of supersymmetry to 16 supercharges is the hallmark of the existence in certain duality frames of smooth supergravity solutions that result from the backreaction of these configurations and, more generally, of the absence of event horizons.

The Supermaze bound state

- Task: find the "glue" needed to transform the original brane system into a bound state

M5($y1234$)	M2(yz)	P(y)	M5($y234z$)	M2($y1$)	M5($1234z$)	P(z)	M2($1z$)	P(1)
$\otimes$	$\otimes$		$\times$	$\times$				
$\otimes$		$\otimes$			$\times$	$\times$		
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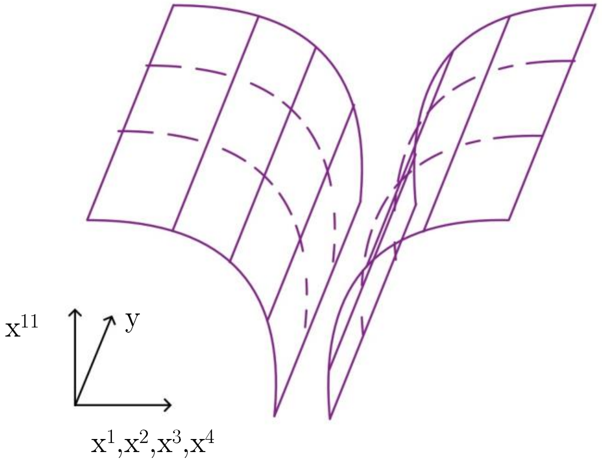
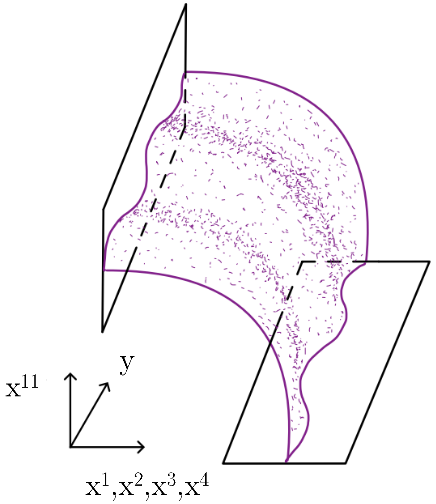
$$\Pi_{\text{M5-M2-P bound}} = \frac{1}{2}\Big[1 + a^2 P_{\text{M5}(y1234)} + b^2 P_{\text{M2}(yz)} + c^2 P_{\text{P}(y)} + ab\left(P_{\text{M5}(y234z)} + P_{\text{M2}(y1)}\right) + bc\left(P_{\text{P}(1)} - P_{\text{M2}(1z)}\right) - ac\left(P_{\text{M5}(1234z)} - P_{\text{P}(z)}\right)\Big]$$

$$a = \cos\alpha\cos\beta$$
$$b = \cos\alpha\sin\beta$$
$$c = \sin\alpha$$





$$\Pi_{\text{M5-M2-P bound}}\,\epsilon = 0$$
$$\Pi_{\text{M5}(y1234)}\,\epsilon = \Pi_{\text{M2}(yz)}\,\epsilon = \Pi_{\text{P}(y)}\,\epsilon = 0$$

Themelia

[Bena, Ceplak, Hampton, Houppe, DT, Warner '22]

- A Themelion is defined to be any object in String Theory that locally has 16 supersymmetries, but might have fewer, when considered globally.
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  - More generally, a themelion can carry multiple charges and preserve less SUSY when taken as a whole

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- We expect Themelia to be the fundamental constituents of black-hole microstructure:
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  - A fully back-reacted themelion can never give rise to a classical black-hole solution with an event horizon.
- Every known microstate geometry is a coherent collection of themelia
- Any such bound state of themelia should give rise to horizonless microstate geometries

Supergravity formulation of the Supermaze

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- There are two main technical hurdles to be overcome:

  - Construct the ¼-BPS momentum-less M2-M5 supermaze

  - Add momentum to this M2-M5 substrate

The most general ansatz describing M5-M2 intersections

- Brane system before backreaction:

[Lunin '07] [Bena, Houppe, DT, Warner '23]

	0	1	2	3	4	5	6	7	8	9	10
M2	$\star$	$\star$	$\star$								
M5	$\star$	$\star$		$\star$	$\star$	$\star$	$\star$				

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- We use the eight Killing spinors:

$$\Gamma^{012}\epsilon = -\epsilon\,, \qquad \Gamma^{013456}\epsilon = \epsilon$$

to solve the gravitino equation

$$\delta\psi_\mu \;\equiv\; \nabla_\mu\,\epsilon \;+\; \frac{1}{288}\left(\Gamma_\mu{}^{\nu\rho\lambda\sigma} \;-\; 8\,\delta^\nu_\mu\,\Gamma^{\rho\lambda\sigma}\right) F_{\nu\rho\lambda\sigma}\,\epsilon \;=\; 0$$

And then impose the Bianchi identities and equations of motion

The most general ansatz describing M5-M2 intersections

- Ultimately, we find that the eleven-dimensional metric has the form:

$$ds_{11}^2 = e^{2A_0}\Big[-dt^2 + dy^2 + e^{-3A_0}(-\partial_z w)^{-\frac{1}{2}}\, d\vec{u}\cdot d\vec{u} + e^{-3A_0}(-\partial_z w)^{\frac{1}{2}}\, d\vec{v}\cdot d\vec{v} + (-\partial_z w)\Big(dz + (\partial_z w)^{-1}\,(\vec{\nabla}_{\vec{u}}\, w)\cdot d\vec{u}\Big)^2\Big]$$

- The three-form potential is given by:

$$C^{(3)} = -e^{3A_0}(-\partial_z w)^{\frac{1}{2}} dt\wedge dy\wedge(-\partial_z w)^{\frac{1}{2}}\Big(dz + (\partial_z w)^{-1}\,(\vec{\nabla}_{\vec{u}}\, w)\cdot d\vec{u}\Big) + \frac{1}{3!}\,\epsilon_{ijk\ell}\Big((\partial_z w)^{-1}\,(\partial_{u_\ell} w)\, du^i\wedge du^j\wedge du^k - (\partial_{v_\ell} w)\, dv^i\wedge dv^j\wedge dv^k\Big)$$

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- The solution is completely determined by a single “maze” function G_0 that satisfies a non-linear Monge-Ampère-like equation:

$$\mathcal{L}_{\vec{v}} G_0 = (\mathcal{L}_{\vec{u}} G_0)\,(\partial_z^2 G_0) - (\nabla_{\vec{u}}\partial_z G_0)\cdot(\nabla_{\vec{u}}\partial_z G_0)$$

with $\quad w \equiv \partial_z G_0 \quad$ and $\quad e^{-3A_0}(-\partial_z w)^{\frac{1}{2}} = \mathcal{L}_{\vec{v}} G_0$

Putting momentum

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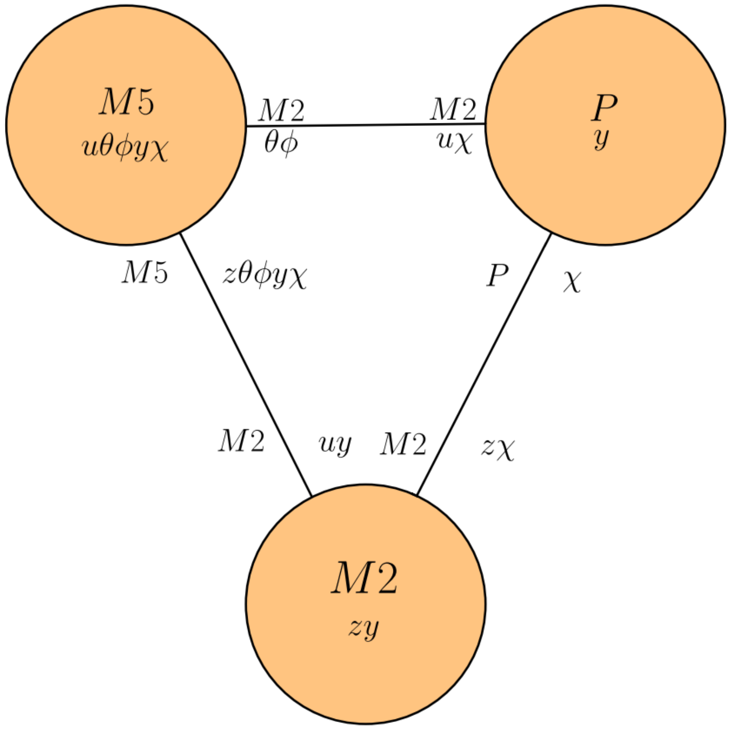
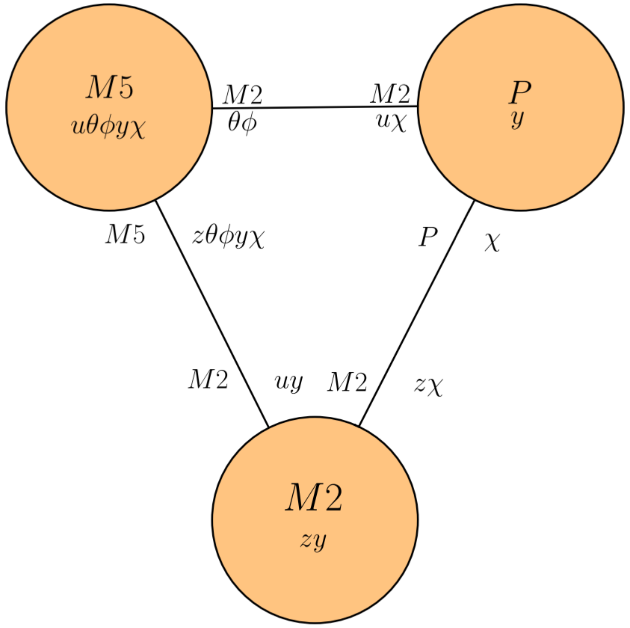


The 1/8-BPS M2-M5-P themelion

- We will use the most symmetric ¼-BPS supermaze solution as a substrate

$$ds_{11}^2 = e^{2A_0}\Big[-dt^2 + dy^2 + (-\partial_z w)\big(dz + (\partial_z w)^{-1}(\partial_u w)\,du\big)^2 + e^{-3A_0}(-\partial_z w)^{-\frac{1}{2}}\big(du^2 + u^2\,d\Omega_3^2\big) + e^{-3A_0}(-\partial_z w)^{\frac{1}{2}}\big(dv^2 + v^2\,d\Omega_3'^2\big)\Big]$$

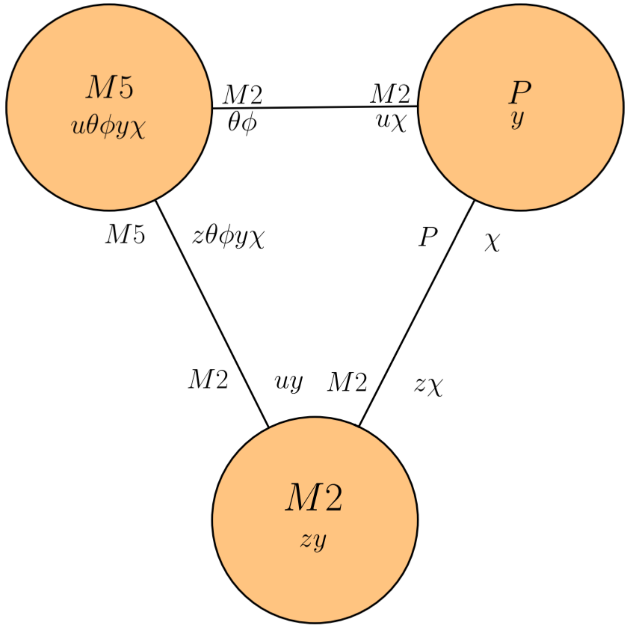


The 1/8-BPS M2-M5-P themelion

- Considering the fluxes and momenta indicated by the projector analysis we eventually find:

$$\begin{aligned} ds^2_{11} &= e^{2A_0}\,dx^-\left[2\,dx^+ + 2kf(x^-)\,d\chi + Pf(x^-)^2\,dx^-\right] \\ &\quad + e^{-A_0}(-\partial_z w)^{-\frac{1}{2}}\left(du^2 + u^2(d\theta^2+\sin^2\theta d\phi^2)+d\chi^2\right) \\ &\quad + e^{-A_0}(-\partial_z w)^{\frac{1}{2}}\,ds'^2_4 + e^{2A_0}(-\partial_z w)\left(dz + (\partial_z w)^{-1}(\partial_u w)\,du\right)^2 \end{aligned}$$

$$\begin{aligned} C^{(3)} &= e^{3A_0}(-\partial_z w)^{\frac{1}{2}}\,dx^-\wedge\left(dx^+ + kf(x^-)\,d\chi\right)\wedge\left(dz + (\partial_z w)^{-1}(\partial_u w)\,du\right) \\ &\quad - \left(\frac{\partial_u w}{\partial_z w}\right)u^2\sin\theta\,d\theta\wedge d\phi\wedge d\chi + \frac{1}{8}(\partial_v w)\,v^3\sin\varphi'_1\,d\varphi'_1\wedge d\varphi'_2\wedge d\varphi'_3 \\ &\quad + \frac{(\partial_z p)}{u^2}f(x^-)\,dx^-\wedge\left(du\wedge d\chi - u^2\sin\theta\,d\theta\wedge d\phi\right) \end{aligned}$$

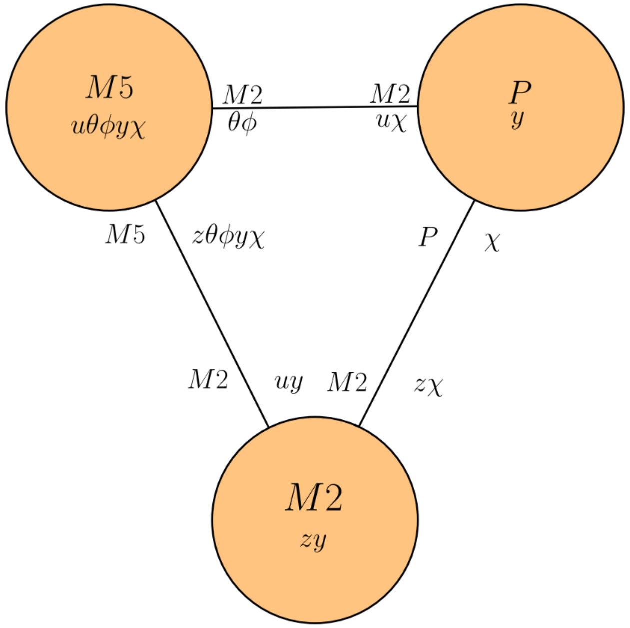


The 1/8-BPS M2-M5-P themelion

- Considering the fluxes and momenta indicated by the projector analysis we eventually find:

$$\begin{aligned} ds^2_{11} &= e^{2A_0}\, dx^- \left[2\, dx^+ \;+\; 2k f(x^-)\, d\chi \;+\; P f(x^-)^2\, dx^-\right] \\ &\quad + e^{-A_0}\, (-\partial_z w)^{-\frac{1}{2}} \left(du^2 + u^2(d\theta^2 + \sin^2\theta d\phi^2) + d\chi^2\right) \\ &\quad + e^{-A_0}\, (-\partial_z w)^{\frac{1}{2}}\, ds'^2_4 \;+\; e^{2A_0}\, (-\partial_z w) \left(dz \;+\; (\partial_z w)^{-1}\, (\partial_u w)\; du\right)^2 \end{aligned}$$

$$\begin{aligned} C^{(3)} &= e^{3A_0}\, (-\partial_z w)^{\frac{1}{2}}\, dx^- \wedge \left(dx^+ + k f(x^-)\, d\chi\right) \wedge \left(dz \;+\; (\partial_z w)^{-1}\, (\partial_u w)\; du\right) \\ &\quad - \left(\frac{\partial_u w}{\partial_z w}\right) u^2 \sin\theta\, d\theta \wedge d\phi \wedge d\chi + \frac{1}{8}\, (\partial_v w)\, v^3 \sin\varphi'_1\, d\varphi'_1 \wedge d\varphi'_2 \wedge d\varphi'_3 \\ &\quad + \frac{(\partial_z p)}{u^2}\, f(x^-)\, dx^- \wedge \left(du \wedge d\chi - u^2 \sin\theta\, d\theta \wedge d\phi\right) \end{aligned}$$



- The polarization function k is determined in terms of the prepotential p through

$$k \;=\; \frac{1}{u^2}\left(\partial_u p - \frac{(\partial_u w)}{(\partial_z w)}\, \partial_v p\right), \qquad \mathcal{L}\left(\frac{p}{u^\ell}\right) - \frac{2\,m}{u^2}\,\frac{p}{u^\ell} \;=\; 0\,, \qquad \mathcal{L}(H) \;\equiv\; e^{-A_0}\, (-\partial_z w)^{-\frac{1}{2}}\, \hat{\mathcal{L}}(H)$$

- The momentum density P is fixed by

$$\mathcal{L}(P) \;=\; -4\, e^{-A_0}\, (-\partial_z w)^{-\frac{1}{2}} \left[2\left(\left(\sqrt{P}\, b_1\right)^2 + \left(\sqrt{P}\, b_2\right)^2\right) - e^{2A_0} \left(\sqrt{P}\, b_1\right)\left((-\partial_z w)^{\frac{1}{2}}\, \partial_u k + (\partial_u w)\, (-\partial_z w)^{-\frac{1}{2}}\, \partial_z k\right)\right]$$

Putting momentum

- The "maze equation" is a daunting non-linear equation

- Strategy to add momentum:
  - Assume the ¼-BPS substrate geometry is given
  - Add momentum waves and fluxes to create the $^1/_8$-BPS solutions that are locally ½-BPS.

➢ **This can be achieved:**
  - The underlying ¼-BPS system remains unaffected.
  - The momentum and extra fluxes are governed by a linear system on the ¼-BPS background!

An interesting near-brane limit

- To render the non-linear system of intersecting branes more manageable one can consider some form of "near-brane" limit.

An interesting near-brane limit

- To render the non-linear system of intersecting branes more manageable one can consider some form of "near-brane" limit.

[Bena, Houppe, DT, Warner '23]

- There exists a scaling limit that maps the asymptotically flat M2-M5 solutions to M-theory solutions consisting of a warped product of $AdS_3 \times S^3 \times S^3 \times \Sigma$, with Σ a Riemann surface. [Bachas, D'Hoker, Estes, Krym '13]
  - The BPS equations reduce to a linear system ⟶ Explicit solutions exist.
  - However, their brane interpretation is not clear.

$AdS_3 \times S^3 \times S^3 \times \Sigma$ solutions

- The metric and the fluxes have the form:

$$
\begin{aligned}
ds^2_{11} &= e^{2A}\left(\hat{f}_1^2\left(\frac{d\mu^2}{\mu^2}+\mu^2\left(-dt^2+dy^2\right)\right) + \hat{f}_2^2\, ds^2_{S^3} + \hat{f}_3^2\, ds^2_{S'^3} + \frac{\partial_w h \partial_{\bar{w}} h}{h^2}\,|dw|^2\right) \\
C^{(3)} &= b_1\,\hat{e}^{012} + b_2\,\hat{e}^{345} + b_3\,\hat{e}^{678}
\end{aligned}
$$

- The solutions are determined in terms of a real harmonic function h and a complex function G:

$$
\partial_w \partial_{\bar{w}} h = 0\,, \qquad \partial_w G = \frac{1}{2}\left(G+\bar{G}\right)\partial_w \log(h)
$$

$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \Sigma$ solutions

$$ds_{11}^2 = e^{2A}\left(\hat{f}_1^2\left(\frac{d\mu^2}{\mu^2} + \mu^2\left(-dt^2 + dy^2\right)\right) + \hat{f}_2^2\, ds_{S^3}^2 + \hat{f}_3^2\, ds_{S'^3}^2 + \frac{\partial_w h \partial_{\bar{w}} h}{h^2}|dw|^2\right)$$

$$ds_{11}^2 = e^{2A_0}\Big[-dt^2 + dy^2 + (-\partial_z w)\left(dz + (\partial_z w)^{-1}(\partial_u w)\, du\right)^2 + e^{-3A_0}(-\partial_z w)^{-\frac{1}{2}}\left(du^2 + u^2\, d\Omega_3^2\right) + e^{-3A_0}(-\partial_z w)^{\frac{1}{2}}\left(dv^2 + v^2\, d\Omega'^2_3\right)\Big]$$

[Bena, Houppe, DT, Warner '23]

- To map the asymptotically flat M2-M5 solutions to the AdS_3 ones we take:

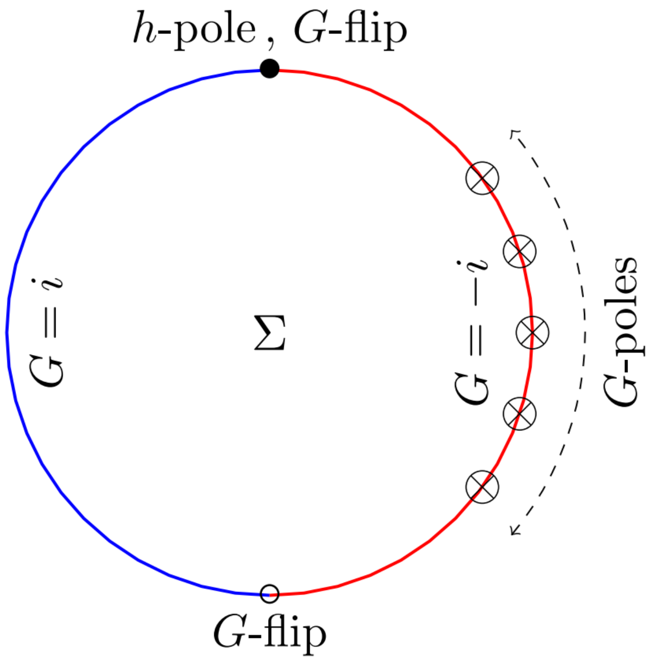
$$u = (\mu\rho)^{\frac{1}{2}}\, e^{-\frac{1}{4}\tilde{\Phi}}\,, \qquad v = (\mu\rho)^{\frac{1}{2}}\, e^{+\frac{1}{4}\tilde{\Phi}}\,, \qquad z = \frac{1}{2\rho\mu}\, e^{\frac{1}{2}\tilde{\Phi}}\left(\Phi + 2\,\xi\right), \qquad w = -\frac{1}{2\rho\mu}\, e^{-\frac{1}{2}\tilde{\Phi}}(\Phi - 2\xi)$$

where we wrote $w = \xi + i\rho$

Primary example

- We consider a solution defined by:

$$h = -i(w - \bar{w})\,, \qquad G = -\left(i\frac{w-\alpha}{|w-\alpha|} + \sum_{a=1}^{n+1} \frac{\zeta_a \mathrm{Im}(w)}{(\bar{w}-\xi_a)|w-\xi_a|} \right)$$



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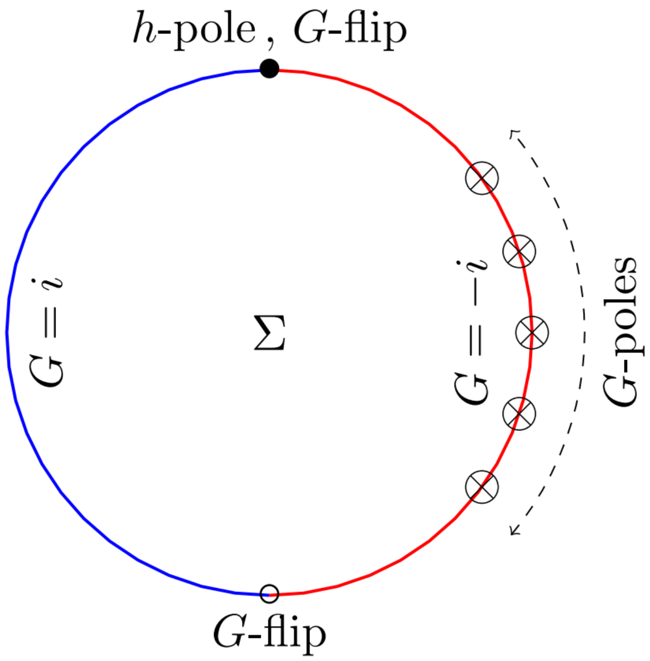
- In terms of the u, v, z coordinates:

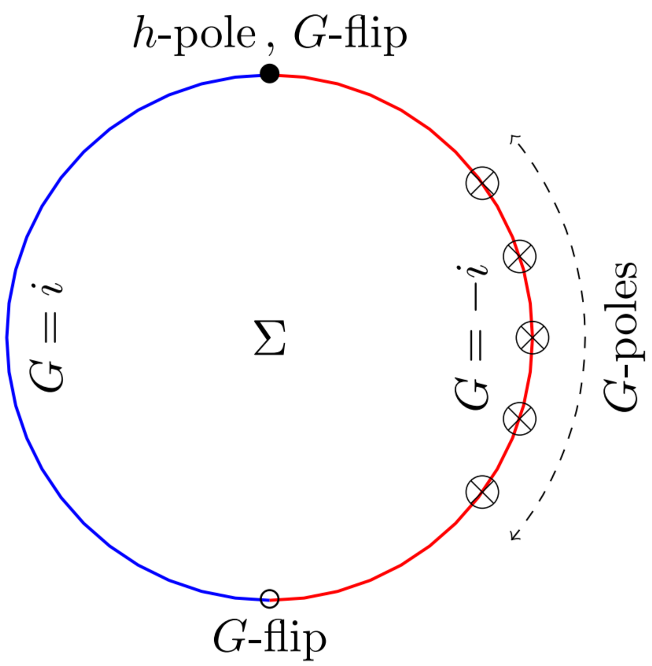
$$\begin{aligned} u &= \sqrt{\frac{\mu}{|\alpha|}} \left(\xi - \alpha + \sqrt{(\xi-\alpha)^2 + \rho^2} \right)^{1/2} e^{\frac{1}{2}\hat{\Phi}}, \\ v &= \rho\sqrt{\mu|\alpha|} \left(\xi - \alpha + \sqrt{(\xi-\alpha)^2 + \rho^2} \right)^{-1/2} e^{-\frac{1}{2}\hat{\Phi}}, \\ z &= \frac{|\alpha|}{\mu} \left(\xi - \alpha + \sqrt{(\xi-\alpha)^2 + \rho^2} \right)^{-1} e^{-\hat{\Phi}} \left(\xi + \alpha + \sqrt{(\xi-\alpha)^2+\rho^2} + \sum_{a=1}^{n+1} \frac{\zeta_a(\xi-\xi_a)}{\sqrt{(\xi-\xi_a)^2+\rho^2}} \right) \end{aligned}$$



[Bena, Chakraborty, DT, Warner '24]

- The M5 sources lie along $\rho = 0 \;\; \rightarrow \;\; v = 0$, the origin of the $\mathbb{R}^4$ transverse to the M2-M5 system

Primary example



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[Bena, Chakraborty, DT, Warner '24]

- The M5 sources lie along $\rho = 0 \;\rightarrow\; v = 0$, the origin of the $\mathbb{R}^4$ transverse to the M2-M5 system

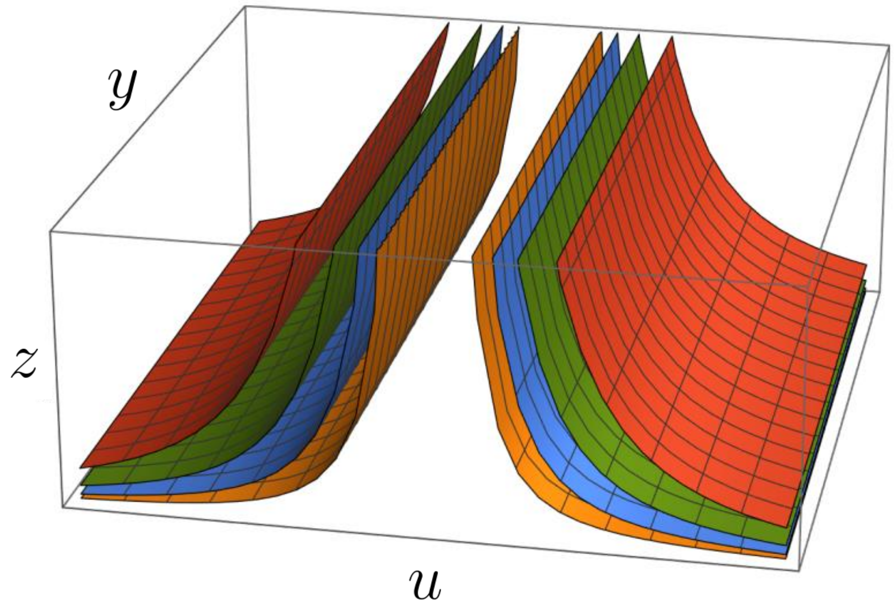
- At $(\xi = \xi_a, \rho = 0)$ one has $\hat{z}|_{\xi=\xi_a,\rho=0} \equiv u^2 z|_{\xi=\xi_a,\rho=0} = 2\xi_a - \sum_{b=1}^{a-1} \zeta_b + \sum_{b=a+1}^{n+1} \zeta_b = \text{costant}$

Moreover, $z \sim \frac{1}{\mu}$, $u \sim \sqrt{\mu}$

➢ The M5-brane is deformed into a spike in the M2-direction with the AdS coordinate μ sweeping the combined world-volume.

The M2-M5 Mohawk

- The M5-brane is deformed into a spike in the M2-direction with the AdS coordinate μ sweeping the combined world-volume.



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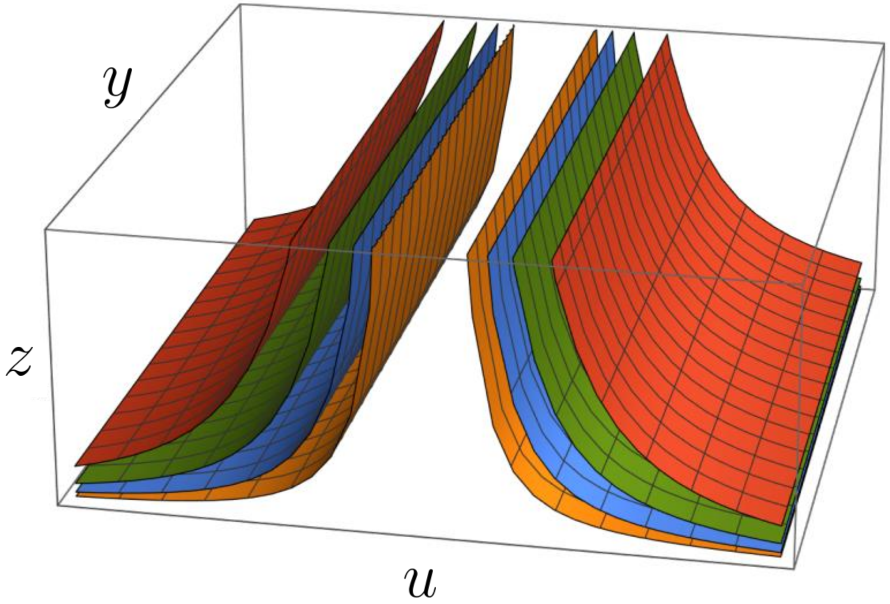

- To make this picture more precise, we can compute the M2 charge sourced at each singular point ξ_a:

$$Q_{M2,a} = 8\,\zeta_a \left(2\,(\xi_a - \alpha) + \zeta_a + 2 \sum_{b=1}^{a-1} \zeta_b \right) \quad \Rightarrow \quad \lim_{\rho \to 0} \hat{z} \,\Big|_{\xi=\xi_a} = \frac{Q_{M2,a}}{2\,Q_{M5,a}} \quad , \quad Q_{M5,a} = 4\zeta_a$$

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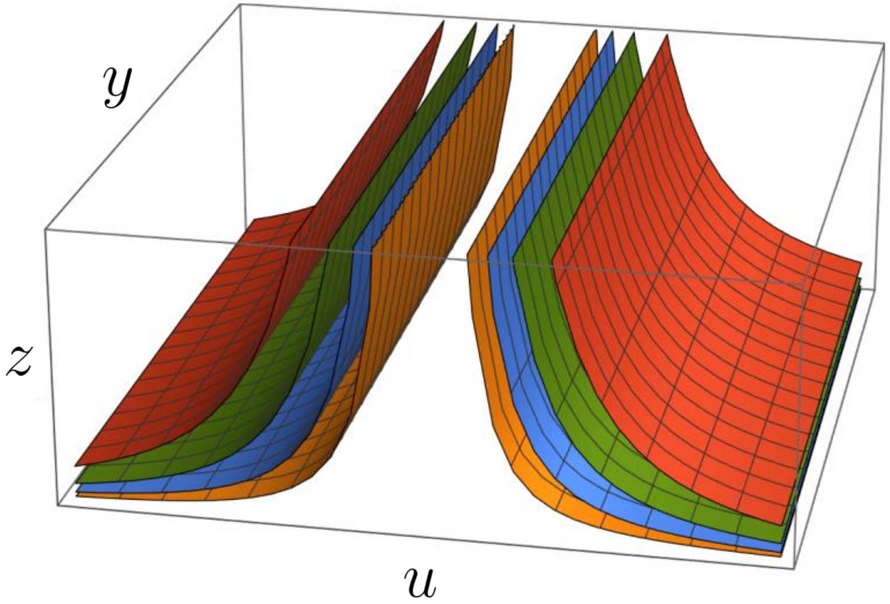

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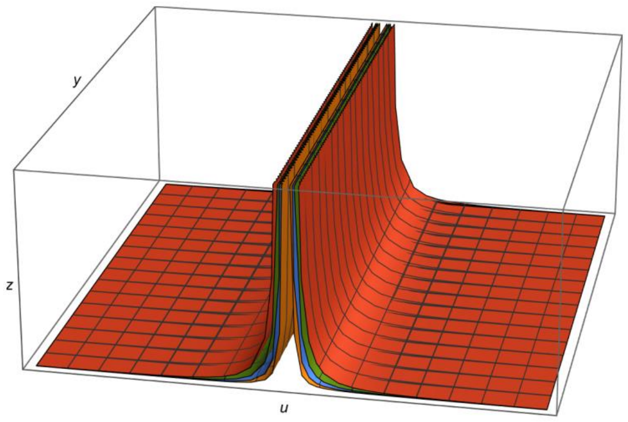
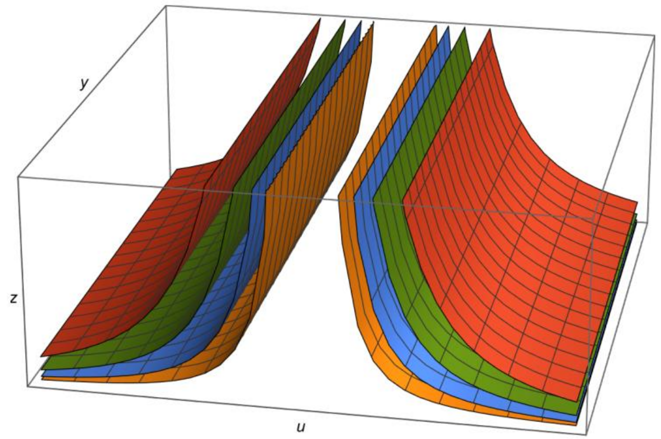
  - The steepness of the spike is determined by the number of M2's pulling divided by the number of M5's being pulled

Second Fractionation

- Brane interpretation
  - From infinity: single stack of semi-infinite M2's ending on and deforming a single stack of M5's.
  - As one zooms in: the back-reaction causes the stacks to resolve into physically separated spikes





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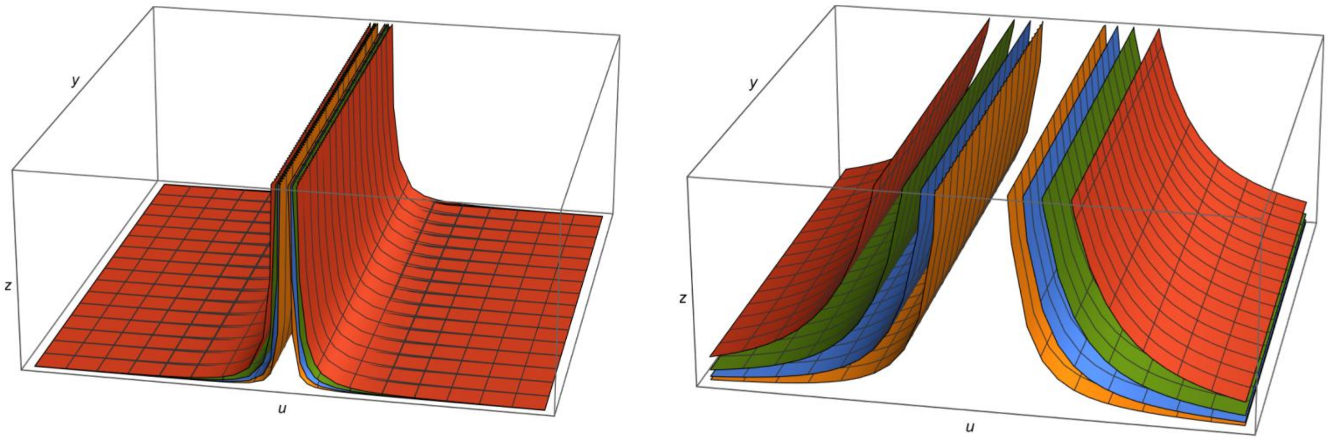
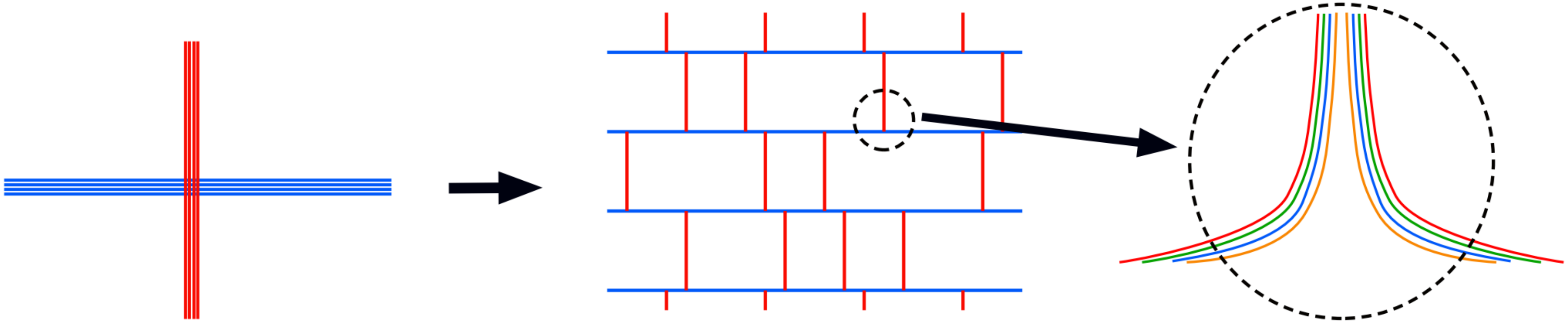
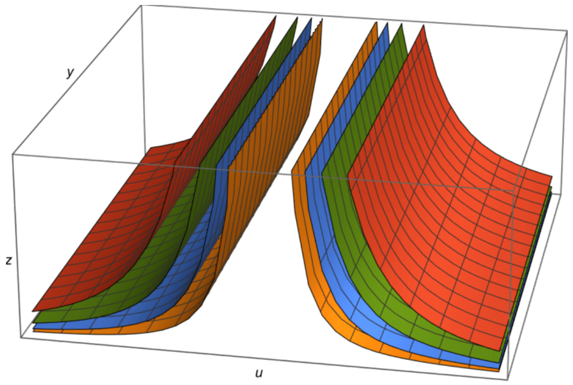


- There exists a second level of fractionation



Outlook

- Study in detail these solutions and put momentum on them



Geometric Transition

Bubbling Geometry

Put Momentum on it

Outlook

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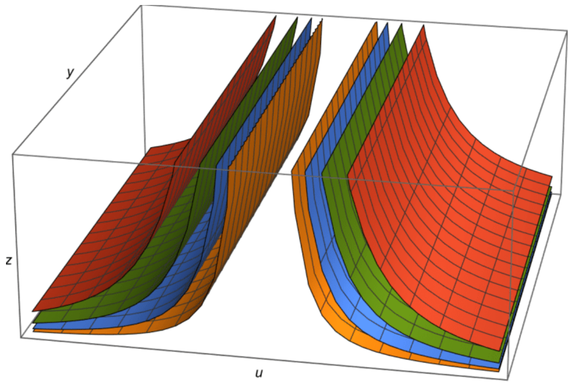


Geometric Transition

Bubbling Geometry

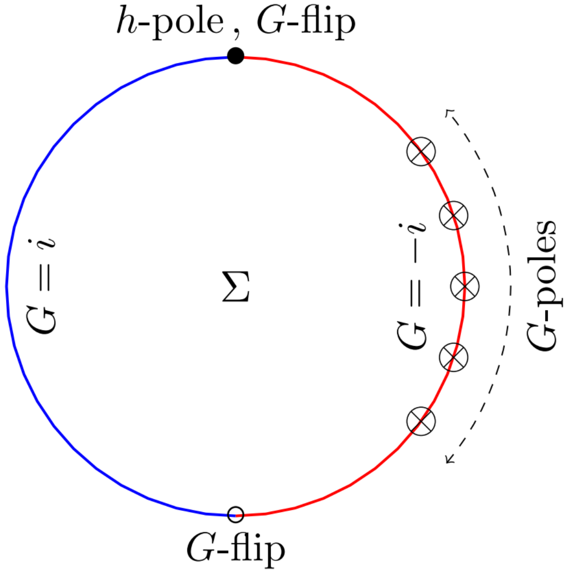
Put Momentum on it

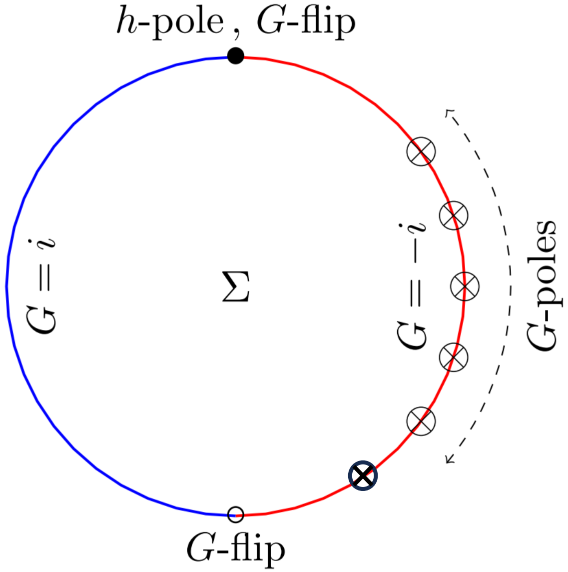
- **End goal:** Supergravity formulation of the supermaze
  - If it turns out as we expect it will, this would finally constitute proof that the microstates of three-charge supersymmetric black holes are horizonless geometries

Thank you!

Putting an M5-M2 probe

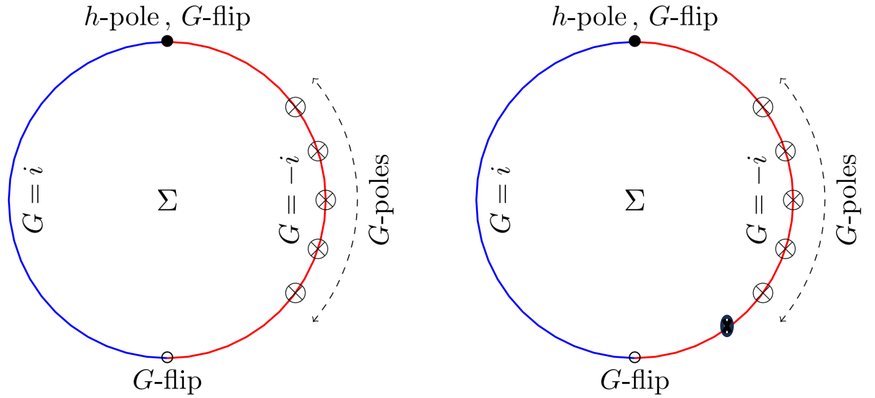
- We can confirm this picture by using probes that are M5 branes with an M2 spike



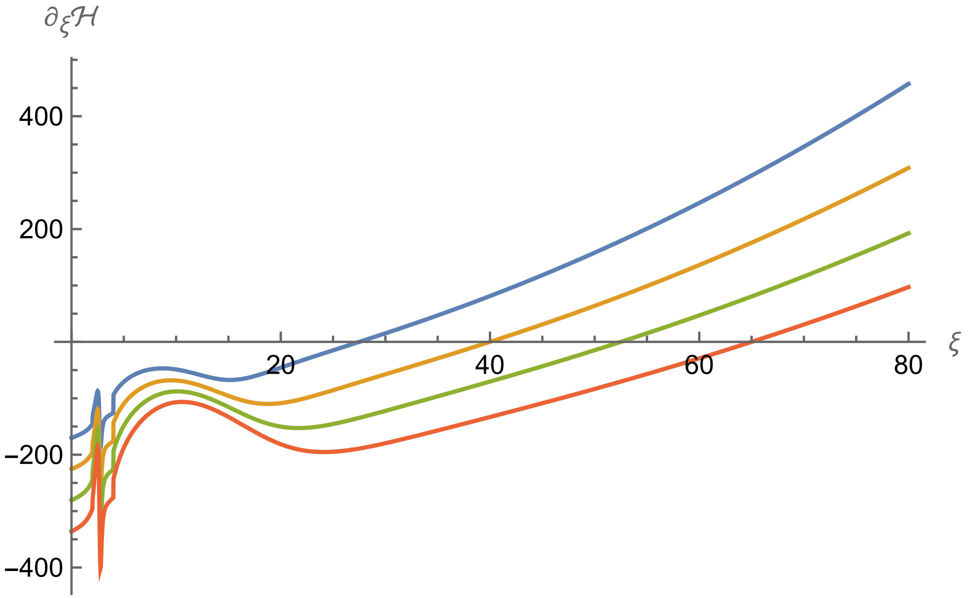


Putting an M5-M2 probe

- We consider a probe M5-brane extending along $AdS_3 \times S^3$ with worldvolume M2 flux on it.



- Our expectations are verified



$$\xi_0 = \frac{1}{4}\Pi + 4\alpha - 2 - 4\sum_{b=1}^{a-1}\zeta_b$$

$$Q_{M2,a} = 8\,\zeta_a\left(2\,(\xi_a - \alpha) + \zeta_a + 2\sum_{b=1}^{a-1}\zeta_b\right)$$

➢ Exactly what we expect from $Q_{M2,a}$!