On the lack of scale-separated supersymmetric AdS_2 flux vacua

Nikolaos Liatsos

School of Applied Mathematical and Physical Sciences,
National Technical University of Athens

Based on: N. Cribiori, F. Farakos and N. Liatsos,
"On scale-separated supersymmetric AdS_2 flux vacua",
arXiv: 2411.04932 [hep-th]

Xmas Theoretical Physics Workshop

Athens, 18 December 2024

Outline

1. Introduction
2. Main argument
3. 2D supergravity analysis
4. Examples from flux compactifications
5. Discussion

Introduction

A given vacuum is characterized as scale-separated, if the dimensions of the internal compact space are much smaller than those of the external spacetime $\Rightarrow$ a lower-dimensional effective description makes sense.

- From a top-down perspective, an AdS vacuum is scale-separated if its radius, L_{AdS}, is much greater than the KK length scale:

$$\frac{L_{\mathrm{AdS}}}{L_{\mathrm{KK}}} \gg 1\,. \tag{1}$$

- From a bottom-up perspective, scale separation is the following hierarchy between the AdS radius and the ultraviolet cutoff of the effective theory, Λ_{UV}:

$$\Lambda_{\mathrm{UV}} L_{\mathrm{AdS}} \gg 1\,. \tag{2}$$

Vacua satisfying the last relation are scale-separated and can explain why only an AdS factor of spacetime is large enough to be visible at low energy.

An advantage of the bottom-up approach to scale separation is that Λ_{UV} does not necessarily need to have a geometric interpretation, while the KK scale is typically related to the geometry of the extra dimensions.

This approach has been employed by [Cribiori and Dall'Agata (2022), Cribiori and Montella (2023)] to prove that maximally supersymmetric AdS vacua of $\mathcal{N}=2$ and $\mathcal{N}=8$ supergravities in 4D and 5D with a residual abelian gauge symmetry cannot be scale-separated, since

$$|V_{\text{AdS}}| \gtrsim \Lambda_{\mathrm{UV}}^2 \,, \tag{3}$$

where Λ_{UV} is the UV cutoff postulated by the magnetic weak gravity conjecture [Arkani-Hamed, Motl, Nicolis and Vafa (2007)].

$\mathcal{N} = 1$ supersymmetric AdS_4 vacua that exhibit scale separation have been constructed by [DeWolfe, Giryavets, Kachru and Taylor (2005)] by compactification of massive type IIA supergravity on a CY_3 in the presence of fluxes and O6-planes.

Scale-separated AdS_3 vacua have been constructed by flux compactifications of IIA supergravity on 7D spaces with G2 holonomy [Farakos, Tringas and Van Riet (2020)].

No scale-separated AdS vacuum in 2D has been constructed so far.

We have provided a bottom-up explanation of why supersymmetric AdS_2 vacua cannot be scale-separated.

Main argument

For an AdS_2 vacuum the cosmological constant Λ is related to the radius, L, by

$$\Lambda = -\frac{2}{L^2}\,. \tag{4}$$

The 2D Planck scale M_{Pl} does not appear in the above equation.

An AdS_2 flux vacuum is supported by an (electric) gauge field strength with components along the AdS_2 directions:

$$-\frac{2}{L^2} = \frac{1}{2}|F_2|^2 = \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,. \tag{5}$$

In principle, we can have two different AdS_2 vacua supported by fluxes of the form

$$F_2^{(\alpha)} = \alpha\epsilon_2\,, \qquad F_2^{(\beta)} = \beta\epsilon_2\,, \tag{6}$$

separated by a domain wall particle.

The tension of such a particle, T, sets an upper bound on the UV cutoff of the 2D effective theory:

$$T > \Lambda_{\mathrm{UV}}\,. \tag{7}$$

If the domain wall is a BPS one,

$$T = |Q|\,, \tag{8}$$

thus

$$|Q| > \Lambda_{\mathrm{UV}}\,, \tag{9}$$

which is reminiscent of the magnetic weak gravity conjecture

The change in the flux F_2 induced by the domain wall is

$$\Delta F_2 = F_2^{(\beta)} - F_2^{(\alpha)} = (\beta - \alpha)\epsilon_2 = Q\epsilon_2\,. \tag{10}$$

If we impose flux quantization,

$$\alpha = N\,Q\,, \qquad \beta = (N+1)\,Q \tag{11}$$

for some integer N, the radius of the AdS_2 background with $F_2 = F_2^{(\alpha)}$ satisfies

$$\frac{1}{L_{(\alpha)}} = \frac{1}{2}|NQ| = \frac{1}{2}|N|\,T \gtrsim \Lambda_{\mathrm{UV}}\,. \tag{12}$$

The natural interpretation of (12) is that supersymmetric AdS_2 flux vacua are not scale-separated.

There can be a contribution to the cosmological constant not related to any 2-form flux:

$$-\frac{2}{L^2} = \frac{1}{2}|F_2|^2 \pm \lambda^2 \, . \tag{13}$$

- The positive sign is beyond the scope of our work, since a positive contribution to the cosmological constant means that there is at least one sector that spontaneously breaks supersymmetry.
- An additional negative contribution to the cosmological constant that is not induced by a flux can only make things worse as far as scale separation is concerned. Indeed, we have

$$\frac{2}{L^2} = -\frac{1}{2}|F_2|^2 + \lambda^2 \geq -\frac{1}{2}|F_2|^2 \gtrsim \Lambda_{\rm UV}^2 \, . \tag{14}$$

Thus, the presence of a flux-induced term in the vacuum energy is enough to exclude scale separation for supersymmetric AdS_2 vacua.

JT supergravity and one-form dilaton multiplet

$\mathcal{N} = (1,1)$ supersymmetric extension of Jackiw-Teitelboim (JT) gravity:

- field content: supergravity multiplet (e^a_μ, ψ_μ, A) + dilaton multiplet (ϕ, λ, F)
- off-shell action [Chamseddine (1991)]:

$$S = \int d^2x\, e \left(\phi R - 2\left(A + \frac{1}{L}\right) F - \frac{2}{L}\phi A + \frac{1}{2L}\phi \epsilon^{\mu\nu} \bar{\psi}_\mu \gamma_3 \psi_\nu - 2\epsilon^{\mu\nu} \bar{\lambda} \gamma_3 D_\mu \psi_\nu + \frac{1}{L} \bar{\lambda} \gamma^\mu \psi_\mu \right), \tag{15}$$

where L is a real constant with dimensions of length.

Euler-Lagrange equations for component fields of dilaton multiplet:

$$\delta F : \quad A = -\frac{1}{L}\,, \tag{16}$$

$$\delta\lambda : \quad \epsilon^{\mu\nu}\hat{D}_\mu\psi_\nu = 0\,, \tag{17}$$

$$\delta\phi : \quad R = -\frac{2}{L^2} - \frac{1}{2L}\epsilon^{\mu\nu}\bar{\psi}_\mu\gamma_3\psi_\nu\,, \tag{18}$$

where

$$\hat{D}_\mu\psi_\nu \equiv D_\mu\psi_\nu - \frac{1}{2}A\gamma_\nu\psi_\mu\,. \tag{19}$$

Equation (18) indicates the presence of an AdS_2 vacuum with length scale L.

Dualization of the auxiliary scalar F to a 2-form $H_2 = dB_1$:

$$\star H_2 = \frac{1}{2}\epsilon^{\mu\nu} H_{\mu\nu} = \epsilon^{\mu\nu}\partial_\mu B_\nu = F + \phi A - \frac{1}{2}\bar{\lambda}\gamma^\mu\psi_\mu - \frac{1}{4}\phi\epsilon^{\mu\nu}\bar{\psi}_\mu\gamma_3\psi_\nu\,. \quad (20)$$

Resulting action:

$$S_{\text{1-form}} = \int d^2x\, e \Big(\phi R - 2A\star H_2 + 2\phi A^2 - A\bar{\lambda}\gamma^\mu\psi_\mu - \frac{1}{2}\phi A\epsilon^{\mu\nu}\bar{\psi}_\mu\gamma_3\psi_\nu - 2\epsilon^{\mu\nu}\bar{\lambda}\gamma_3 D_\mu\psi_\nu\Big)\,. \quad (21)$$

After integrating out A, the e.o.m. for the 1-form B_1 reads

$$\partial_\mu\left(\frac{1}{\phi}\star H_2\right) = 0\,, \quad (22)$$

so

$$\frac{1}{\phi}\star H_2 = c\,. \quad (23)$$

Then, varying $S_{\text{1-form}}$ with respect to ϕ, we find

$$R = -\frac{1}{2\phi^2}(\star H_2)^2 = -\frac{1}{2}c^2, \tag{24}$$

which for $c \neq 0$ describes an AdS_2 spacetime with length scale L related to the constant value of $\frac{1}{\phi} \star H_2$ by

$$c^2 = \frac{4}{L^2}\,. \tag{25}$$

Coupling to a domain wall particle

Action for a particle in 2D coupled to B_1:

$$S_{\mathrm{p}} = -|Q| \int_{\mathcal{W}_1} d\tau \sqrt{-g_{\mu\nu}(X^\rho)\dot{X}^\mu \dot{X}^\nu} |\phi(X^\rho)| + Q \int_{\mathcal{W}_1} d\tau B_\mu(X^\nu)\dot{X}^\mu \,, \tag{26}$$

where

- $\mathcal{W}_1$ is the worldline of the particle, parametrized by τ.
- $X^\mu(\tau)$ are the values of the spacetime coordinates on $\mathcal{W}_1$.
- Q is the real charge of the particle under B_1.
- $\dot{X}^\mu \equiv \frac{dX^\mu}{d\tau}$.

We consider the action

$$S = S_{\text{1-form}} + S_{\text{p}} \,. \tag{27}$$

- Euler-Lagrange equation for ϕ:

$$\begin{aligned} R = &- \frac{1}{2\phi^2}(\star H_2)^2 \\ &+ e^{-1}|Q|\frac{\phi}{|\phi|}\int_{\mathcal{W}_1} d\tau\, \delta^{(2)}(x^\rho - X^\rho(\tau))\sqrt{-g_{\mu\nu}\dot{X}^\mu(\tau)\dot{X}^\nu(\tau)}\,. \end{aligned} \tag{28}$$

- E.o.m. for B_1:

$$\partial_\mu\left(\frac{1}{\phi}\star H_2\right) = Qe^{-1}\epsilon_{\mu\nu}\int_{\mathcal{W}_1} d\tau\, \delta^{(2)}(x^\rho - X^\rho(\tau))\dot{X}^\nu(\tau)\,. \tag{29}$$

- E.o.m. for $g_{\mu\nu}$:

$$\begin{aligned} &\nabla^\mu\partial^\nu\phi - g^{\mu\nu}\nabla_\rho\partial^\rho\phi + \frac{1}{4\phi}(\star H_2)^2 g^{\mu\nu} \\ &+ \frac{1}{2}e^{-1}|Q\phi|\int_{\mathcal{W}_1} d\tau\, \delta^{(2)}(x^\lambda - X^\lambda(\tau))\frac{\dot{X}^\mu\dot{X}^\nu}{\sqrt{-g_{\rho\sigma}\dot{X}^\rho\dot{X}^\sigma}} = 0\,. \end{aligned} \tag{30}$$

Let $x^\mu = (t, x)$ be the coordinates of the 2D target spacetime. If the particle lies on the axis $x = 0$, then in the static gauge, in which $T = \tau$, the x component of (29) becomes

$$\partial_x(\phi^{-1} \star H_2) = -Q\delta(x)\,. \tag{31}$$

Integrating (31) over an interval $[-\epsilon, \epsilon]$, where $\epsilon > 0$, we find

$$\alpha_+ - \alpha_- = -Q\,, \tag{32}$$

where α_+ and α_- are the constant values of $\phi^{-1} \star H_2$ for $x > 0$ and $x < 0$ respectively.

If neither of $\alpha_\pm$ is zero, then the regions $x > 0$ and $x < 0$ are both AdS_2 spaces with length scales L_+ and L_- respectively, which are given by

$$\frac{1}{L_\pm} = \frac{1}{2}|\alpha_\pm| \,. \tag{33}$$

Therefore,

$$\begin{aligned} \max\left\{\frac{1}{L_+}, \frac{1}{L_-}\right\} &\geq \frac{1}{2}\left(\frac{1}{L_+} + \frac{1}{L_-}\right) = \frac{1}{4}(|\alpha_+| + |\alpha_-|) \\ &\geq \frac{1}{4}|\alpha_+ - \alpha_-| = \frac{1}{4}|Q| > \frac{1}{4}\Lambda_{\mathrm{UV}} \,, \end{aligned} \tag{34}$$

which implies the absence of scale separation.

BPS domain wall solutions

Solution of the e.o.m. for a charged domain wall particle lying on the axis $x = 0$ in the static gauge:

- Ansatzes for metric, dilaton and 1-form:

$$ds^2 = -e^{2f(x)}dt^2 + dx^2, \tag{35}$$

$$B_1 = B(x)dt\,, \qquad \phi = \phi(x)\,. \tag{36}$$

- If $Q\phi(0) > 0$,

$$\phi(x) = \phi(0)\exp\left[\frac{1}{2}(\alpha + QH(-x))x\right], \tag{37}$$

$$f(x) = \ln|\phi(x)|\,, \tag{38}$$

$$B(x) = (\mathrm{sgn}\,Q)\phi^2(0)\exp[(\alpha + QH(-x))x]\,, \tag{39}$$

where α is a real constant and H is the Heaviside step function.

- If $Q\phi(0) < 0$,

$$\phi(x) = \phi(0)\exp\left[-\frac{1}{2}(\alpha + QH(-x))x\right], \tag{40}$$

$$f(x) = \ln|\phi(x)|, \tag{41}$$

$$B(x) = (\mathrm{sgn}\, Q)\phi^2(0)\exp[-(\alpha + QH(-x))x]. \tag{42}$$

Killing spinors associated with the above solutions:

- If $Q\phi(0) > 0$,

$$\epsilon(x) = \exp\left[\frac{1}{4}(\alpha + QH(-x))x\right]\eta\,, \tag{43}$$

where η is a constant Majorana spinor satisfying the condition

$$\eta = -\gamma_1\eta\,. \tag{44}$$

- If $Q\phi(0) < 0$,

$$\epsilon(x) = \exp\left[-\frac{1}{4}(\alpha + QH(-x))x\right]\eta\,, \tag{45}$$

where η is a constant Majorana spinor obeying

$$\eta = \gamma_1\eta\,. \tag{46}$$

Example in type IIA

Solutions of type IIA supergravity of the form [Lüst and Tsimpis (2020)]

$$\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{T}^6 \tag{47}$$

with

- electric RR flux F_2
- magnetic RR flux F_4 wrapping 4-cycles of the form $\mathrm{S}^2 \times \mathrm{T}^2$
- $H_3 = F_0 = 0$
- constant 10D dilaton, $\varphi = \varphi_0$
- no sources

E.o.m. in Einstein frame for constant φ:

$$R_{MN} = e^{3\varphi/2}\left(\frac{1}{2}|F_2|^2_{MN} - \frac{1}{16}g_{MN}|F_2|^2\right) + e^{\varphi/2}\left(\frac{1}{2}|F_4|^2_{MN} - \frac{3}{16}g_{MN}|F_4|^2\right), \tag{48}$$

$$0 = \frac{3}{4}e^{3\varphi/2}|F_2|^2 + \frac{1}{4}e^{\varphi/2}|F_4|^2\,, \tag{49}$$

$$0 = dF_p = d(\star F_p)\,, \qquad \text{with} \quad p = 2, 4\,, \tag{50}$$

where

$$|F_p|^2 \equiv \frac{1}{p!}F_{M_1\dots M_p}F^{M_1\dots M_p}, \tag{51}$$

$$|F_p|^2_{MN} \equiv \frac{1}{(p-1)!}F_{MM_2\dots M_p}F_N{}^{M_2\dots M_p}. \tag{52}$$

From the Einstein equations and the e.o.m. for the dilaton it follows that

$$R_{2D} = 2e^{3\varphi/2}|F_2|^2 = -2e^{-3\varphi/2}|F_8|^2, \tag{53}$$

where

$$F_8 = -e^{3\varphi/2} \star F_2 \,. \tag{54}$$

Flux quantization:

$$\int_{X_8} F_8 = (2\pi l_s)^7 f_8 \,, \tag{55}$$

where $X_8 = \mathrm{S}^2 \times \mathrm{T}^6$ and $f_8 \in \mathbb{Z}$, so

$$|F_8|^2 = \frac{\left((2\pi l_s)^7 f_8\right)^2}{(\mathrm{Vol}[X_8])^2} \,. \tag{56}$$

Then, the curvature scalar of the external AdS_2 spacetime is

$$R_{2D} = -2\left(\frac{(2\pi l_s)^7 f_8}{e^{3\varphi/4}\mathrm{Vol}[X_8]}\right)^2 . \qquad (57)$$

For $2\kappa_{10}^2 = (2\pi)^7 l_s^8 = 1 \Leftrightarrow l_s = (2\pi)^{-7/8}$, the AdS_2 radius, L, is given by

$$\frac{1}{L} = \frac{(2\pi)^{7/8}|f_8|}{\mathrm{Vol}[X_8]} e^{-3\varphi/4}. \qquad (58)$$

This scale is related to the tension of a D0-brane smeared along the compact directions.

We can deduce this tension by determining its backreaction on the 2D curvature.

Action for a D0-brane in 10D in the Einstein frame:

$$S_{\rm D0} = -T_0 \int_{\mathcal{W}_1} d\tau e^{-\frac{3}{4}\varphi} \sqrt{-g_{MN} \dot{X}^M \dot{X}^N} + \mu_0 \int_{\mathcal{W}_1} C_1 \, , \qquad (59)$$

where

- $T_0 = \frac{1}{l_s} = (2\pi)^{7/8}$
- μ_0 is the real charge of the brane under the RR 1-form C_1 .

In the presence of the D0-brane,

$$R_{MN} = \frac{1}{2} \left(T^{\rm D0}_{MN} - \frac{1}{8} g_{MN} T^{\rm D0} \right) + \dots \, , \qquad (60)$$

where $T^{\rm D0}_{MN} \equiv -\frac{2}{\sqrt{-g_{(10)}}} \frac{\delta S_{\rm D0}}{\delta g^{MN}}$ and $T^{\rm D0} \equiv g^{MN} T^{\rm D0}_{MN}$.

If the D0-brane lies at $x^1 = x^2 = \ldots = x^9 = 0$, then in the static gauge, in which $X^0 = \tau$, we have $T^{D0}_{\mu i} = T^{D0}_{ij} = 0$ and

$$T^{\rm D0} = g^{\mu\nu} T^{\rm D0}_{\mu\nu} = -\frac{\sqrt{-g_{00}}}{\sqrt{-g_{(10)}}} T_0 e^{-3\varphi/4} \delta(x^1) \delta^{(8)}(x^i) \,. \tag{61}$$

Contribution of D0-brane to the curvature of the 2D external spacetime:

$$R_{2D} = \frac{3}{8} T^{\rm D0} + \ldots \,. \tag{62}$$

Smearing of the D0-brane along the compact 8D internal space:

$$\delta^{(8)}(x^i) \to \frac{\sqrt{g_{(8)}}}{{\rm Vol}[X_8]} \,, \tag{63}$$

where $g_{(8)} = \det g_{ij}$.

In the smeared approximation,

$$R_{2D} = -\frac{3}{8}\frac{(2\pi)^{7/8}}{\mathrm{vol}[X_8]}\frac{\sqrt{-g_{00}}}{\sqrt{-g_{(2)}}}e^{-3\varphi/4}\delta(x^1) + \ldots\,, \tag{64}$$

where $g_{(2)} = \det g_{\mu\nu}$.

Tension of the smeared D0-brane:

$$T_{\text{smeared D0}} \sim \frac{1}{e^{3\varphi/4}\mathrm{Vol}[X_8]}\,, \tag{65}$$

so

$$\frac{1}{L} \sim |f_8|\, T_{\text{smeared D0}} \gtrsim T_{\text{smeared D0}}\,, \tag{66}$$

in accordance with our general argument against scale separation.

General argument for flux compactifications

In a type II setup with an AdS_2 solution supported by a magnetic flux F_n,

$$R_{2D} \sim -e^{\frac{5-n}{2}\varphi}|F_n|^2\,, \tag{67}$$

Flux quantization:

$$\int_{\Sigma_n} F_n \sim f_n\,, \tag{68}$$

where $f_n \in \mathbb{Z}$ and Σ_n is the n-cycle threaded by F_n.

$$R_{2D} \sim -\left(e^{\frac{5-n}{4}\varphi}\frac{f_n}{\mathrm{Vol}[\Sigma_n]}\right)^2 \qquad \Rightarrow \qquad \frac{1}{L} \sim e^{\frac{5-n}{4}\varphi}\frac{|f_n|}{\mathrm{Vol}[\Sigma_n]}\,. \tag{69}$$

Let the electric flux $\star F_n$ be sourced by a Dp-brane with $p = 8 - n$ electrically coupled to the RR potential C_{9-n}.

We split the coordinates of the 8D internal space as $x^i = (x^{\hat{i}}, x^{\tilde{i}})$, where

- $x^{\hat{i}}$, $\hat{i} = 2, \ldots, n+1$, are the coordinates tangential to Σ_n,
- $x^{\tilde{i}}$, $\tilde{i} = n+2, \ldots, 9$ are the internal coordinates normal to Σ_n.

The spacetime coordinates $(x^0, x^{\tilde{i}})$ are parallel to the Dp-brane, while the coordinates $(x^1, x^{\hat{i}})$ are transverse to it.

Internal space metric:

$$ds_8^2 = g_{ij} dx^i dx^j = g_{\hat{i}\hat{j}} dx^{\hat{i}} dx^{\hat{j}} + g_{\tilde{i}\tilde{j}} dx^{\tilde{i}} dx^{\tilde{j}}. \tag{70}$$

If the $D(8-n)$-brane lies at $x^1 = x^{\hat{i}} = 0$, its backreaction on the 2D external curvature is

$$R_{2D} = \frac{1}{8}(5-n)(2\pi)^{\frac{n-1}{8}} e^{\frac{5-n}{4}\varphi} \frac{\sqrt{-g_{00}}\sqrt{\tilde{g}_{(8-n)}}}{\sqrt{-g_{(10)}}} \delta(x^1)\delta^{(n)}(x^{\hat{i}}) + \dots \,, \quad (71)$$

where $\tilde{g}_{(8-n)} = \det g_{\tilde{i}\tilde{j}}$.

Smearing of the $D(8-n)$-brane along Σ_n:

$$\delta^{(n)}(x^{\hat{i}}) \to \frac{\sqrt{\hat{g}_{(n)}}}{\mathrm{Vol}[\Sigma_n]} \,, \quad (72)$$

where $\hat{g}_{(n)} = \det g_{\hat{i}\hat{j}}$.

$$R_{2D} = \frac{1}{8}(5-n)\frac{(2\pi)^{\frac{n-1}{8}}}{\mathrm{Vol}[\Sigma_n]} \frac{\sqrt{-g_{00}}}{\sqrt{-g_{(2)}}} e^{\frac{5-n}{4}\varphi}\delta(x^1) + \dots \,. \quad (73)$$

Thus, the $D(8-n)$-brane behaves as an effective D0-brane with tension

$$T_{\text{eff. D0}} \sim \frac{e^{\frac{5-n}{4}\varphi}}{\text{Vol}[\Sigma_n]} \tag{74}$$

and

$$\frac{1}{L} \sim |f_n| T_{\text{eff. D0}} \gtrsim T_{\text{eff. D0}} \,, \tag{75}$$

so such flux-supported AdS_2 vacua in type II compactifications are not scale-separated.

Discussion

- We have provided a bottom-up argument excluding the existence of supersymmetric scale-separated AdS_2 vacua supported by fluxes.
- It relies on the existence of a fundamental BPS domain wall between two such vacua, whose tension is an upper bound for the UV cutoff of the 2D effective theory.
- If scale separation is possible at all in 2D, it is most likely to occur in setups with at most one supercharge.
- Such a model could be constructed by compactifying type II superstring theory on a manifold with $Spin(7)$ holonomy, which indeed preserves $1/16$ of the original supersymmetry, and then performing an appropriate orientifold projection to reduce it further by a factor of $1/2$.

Acknowledgements

Thank you for your attention!

The research work was supported by the Hellenic Foundation for Research and Innovation (HFRI) under the 3rd Call for HFRI PhD Fellowships (Fellowship Number: 6554).