Gifts from infinite-dimensional current algebras

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based on
[2406.02662 w/ D.M. Hofman]
[2310.18391 w/ J.R. Fliss]
and thoughts in progress

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National and Kapodistrian University of Athens
18/12/2024

Outline

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Motivation

Free fields and current algebras

Gift 1: State-operator correspondence [2406.02662 w/ D.M. Hofman]

Gift 2: Topological entanglement entropy [2310.18391 w/ J.R. Fliss]

Gift ideas (outlook)

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Symmetry is a powerful organising principle

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IR constraints

Landau paradigm,

't Hooft anomaly matching

Selection rules

allowed transition, decays,

particles in the spectrum...

Topological protection

topological insulators,

topological superconductors,

fracton phases...

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fixes spectrum exactly

underpins (worldsheet of) string theory

further connections to quantum gravity

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more constraints: rational CFTs

even more power: connection to 3d TQFT and topological order

physical example: entanglement spectrum of fQHE states [Li, Haldane; 2008]

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Downside: Only in 2d$^?$ 😥

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Continuous symmetry: $\partial_\mu J^\mu = 0 \quad \Longleftrightarrow \quad \mathrm{d}\star J_{[1]} = 0$

$\Longrightarrow$ codimension-one topological operator $U(\Sigma_{d-1}) := \exp\left(\mathrm{i} \int_{\Sigma_{d-1}} \star J_{[1]} \right)$

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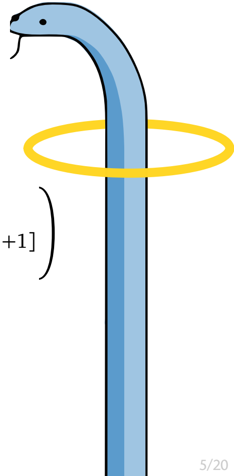
Act on p-dimensional extended operators by linking



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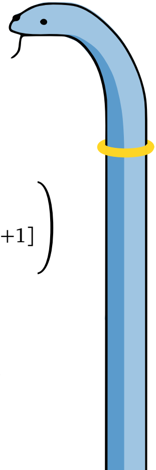
Other generalisations: higher-group, non-invertible, subsystem symmetries, and more.



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The star of the show is a free p-form field.

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$(d-p-2)$-form $\mathrm{U}(1)$ symmetry (magnetic)

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acts on: Wilson $= \exp\left(\mathrm{i}\int_{\gamma_p} A_{[p]}\right)$

$(d-p-2)$-form $\mathrm{U}(1)$ symmetry (magnetic)

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acts on:

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For any $\Lambda_{[p]}$, $\widetilde{\Lambda}_{[d-p-2]}$ satisfying:

$$\mathrm{d}\Lambda_{[p]} + \star\, \mathrm{d}\widetilde{\Lambda}_{[d-p-2]} = 0$$

(twisted self-duality)

Conserved currents: $\mathcal{J}_{\Lambda,\widetilde{\Lambda}} = \star\big(J_{[p+1]} \wedge \widetilde{\Lambda}_{[d-p-2]} + \widetilde{J}_{[p+1]} \wedge \Lambda_{[p]}\big)$

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act on *local* and *non-local* operators (wait for Gift 1)

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k = "level" ~ gauge coupling

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In $d = 2, p = 0 \rightsquigarrow$ free compact scalar $J(z)\alpha(z)$ and $\bar{J}(\bar{z})\bar{\alpha}(\bar{z})$ conserved

$\big[\mathcal{Q}(\alpha), \mathcal{Q}(\alpha')\big] = \mathrm{i}\, k \int \alpha\, \mathrm{d}\alpha' \implies [Q_n, Q_m] = k\, n\, \delta_{n+m,m} \implies$ *it's a Kac–Moody!*

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In higher d $[\ ,\ ] = \int \cdots$ is again a *spectrum-generating, infinite-dimensional current algebra*.

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To get the spectrum, turn $[\ ,\ ] = \int \cdots$ into a mode algebra $\Longrightarrow \left[\mathcal{A}_{\mathsf{n}}, \mathcal{A}^{\dagger}_{\mathsf{m}}\right] = E_{\mathsf{n}} \delta_{\mathsf{nm}}$

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Siegel-like Theta function

Non-trivial check: $\mathbf{ch}(q) = \sum_{\boldsymbol{r},\boldsymbol{s}} \mathrm{ch}_{\boldsymbol{r},\boldsymbol{s}}(q) = \sum_{\boldsymbol{r},\boldsymbol{s}} \mathrm{tr}\, q^{H_\Sigma} = \dfrac{\Theta_\Sigma(q;k)}{\eta_\Sigma(q)^2} = \mathcal{Z}\left(\mathbb{S}^1_\beta \times \Sigma\right)$

"spectral eta function" $\eta_\Sigma(q) = \prod_{\mathsf{n}} \left(1 - q^{E_{\mathsf{n}}}\right)^{-1/2}$

very reminiscent of 2d CFTs!

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Claim:[Hofman, Iqbal 2018] A unitary CFT in $d = 2p + 2$ with a p-form $\mathrm{U}(1)$ symmetry is realised by *free p-form fields.*

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Sketch of proof: $\langle J(x) J(y) \rangle = \mathrm{f}(x - y) \implies \langle \mathrm{d}J(x)\ \mathrm{d}J(y) \rangle = 0 \implies \mathrm{d}J = 0.$

very specific function [Costa, Hansen 2015]

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- $d = 4 \rightsquigarrow$ *free photon realisation (photonisation)*

$\rightsquigarrow$ precise one-to-one correspondence between *states on* $\mathcal{H}_\Sigma$ and *nonlocal operators*

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sets the stage for understanding higher-dimensional CFTs on compact manifolds

such a correspondence is not possible for generic CFTs [Belin, de Boer, Kruthoff 2018].

State-operator correspondence

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Recover the spectrum of states by path integrals on $\mathbb{B}^3 \times \mathbb{S}^1$ with insertions of line operators

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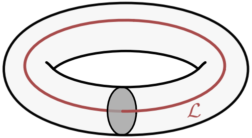

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For illustration, take $d = 4$ $(p = 1)$ and quantise on $\Sigma = \mathbb{S}^2 \times \mathbb{S}^1$

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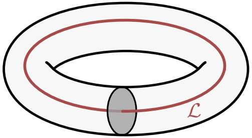

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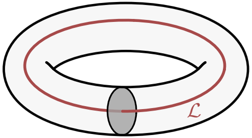

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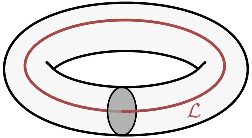

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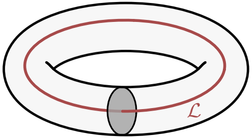

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Bogoliubov transformation
$\sim \prod_{\mathrm{n}} \exp\big(\mathcal{A}_{\mathrm{n}}^2 + (\mathcal{A}_{\mathrm{n}}^\dagger)^2\big)$

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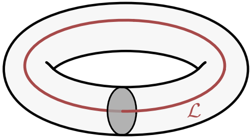

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$\rightsquigarrow$ Vacuum is prepared by *photons of all frequencies* smeared on $\mathbb{S}^1$ (consistent with [Belin, de Boer, Kruthoff 2018])

State-operator correspondence

The rest is straightforward

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Primary states

$$|r,s\rangle \leftrightsquigarrow \begin{array}{c}\text{Wilson–'t Hooft lines}\\ \text{dressed with squeezing operator}\end{array} = \mathfrak{S}^\dagger \times \exp\left(\mathrm{i}\, r \int_{\mathbb{S}^1} A + \mathrm{i}\, s \int_{\mathbb{S}^1} \widetilde{A}\right)$$

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Representation theory at work ⇝ sprinkle oscillators

$$\mathcal{A}^{\dagger}_{\mathsf{n}} |r,s\rangle \leftrightsquigarrow \mathfrak{S}^{\dagger} \times \left(\#^{*}_{\mathsf{n}} \mathcal{A}^{\dagger}_{\mathsf{n}} + \#^{*}_{\mathsf{n}} \mathcal{A}_{\mathsf{n}}\right) \times \exp\left(\mathrm{i}\, r \int_{\mathbb{S}^1} A + \mathrm{i}\, s \int_{\mathbb{S}^1} \widetilde{A}\right)$$

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$$\prod_{\mathsf{n}} \left(\mathcal{A}_{\mathsf{n}}^\dagger\right)^{N_{\mathsf{n}}} |r,s\rangle \leftrightsquigarrow \mathfrak{S}^\dagger \times \prod_{\mathsf{n}} \left(\#_{\mathsf{n}}^* \mathcal{A}_{\mathsf{n}}^\dagger + \#_{\mathsf{n}}^* \mathcal{A}_{\mathsf{n}}\right)^{N_{\mathsf{n}}} \times \exp\left(\mathrm{i}\, r \int_{\mathbb{S}^1} A + \mathrm{i}\, s \int_{\mathbb{S}^1} \widetilde{A} \right)$$

That's it. That's the entire spectrum.

Outline

Motivation

Free fields and current algebras

Gift 1: State-operator correspondence [2406.02662 w/ D.M. Hofman]

Gift 2: Topological entanglement entropy [2310.18391 w/ J.R. Fliss]

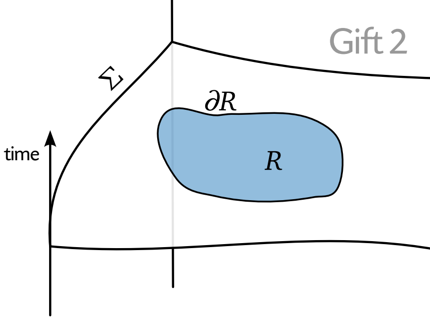
Gift ideas (outlook)

Gift 2

Topological order = patterns of long range entanglement

In (2+1)d: $\mathcal{S}_{\text{EE}} = \frac{|\partial R|}{\varepsilon} - \gamma$ [Kitaev, Preskill; Levin, Wen 2006]

γ: $\log\big($total quantum dimension$\big)$



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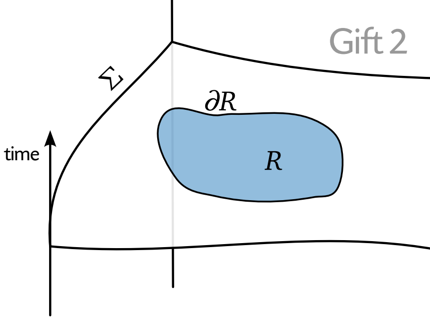

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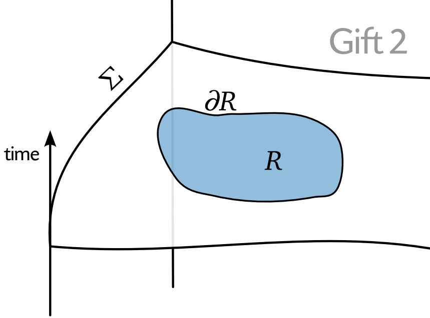

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bulk/edge correspondence

- anomaly/symmetry inflow *(cf. SPTs, SymTFTs)*
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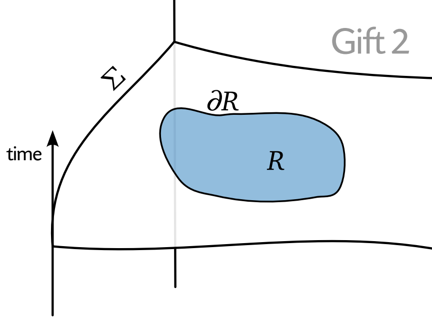

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Here: Consider higher-dimensional topological order and study its entanglement

The theory: a (d-*dim*) theory of (p-*dim*) surface-net condensates [Bombin, Martin-Delgado 2006]

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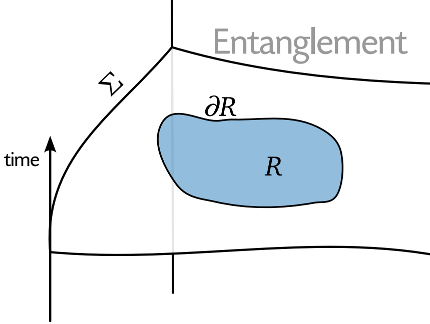
Entanglement

(replica trick)

Interested in $\mathcal{S}_{\text{EE}} = -\,\mathrm{Tr}_{\mathcal{H}_R}\,(\rho_R \log \rho_R) \equiv \lim_{n\to 1} \frac{1}{1-n} \log \mathrm{Tr}_{\mathcal{H}_R}\, \rho_R^n$



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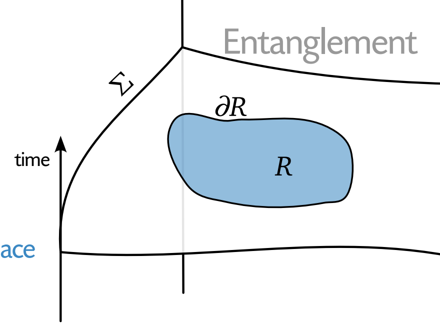

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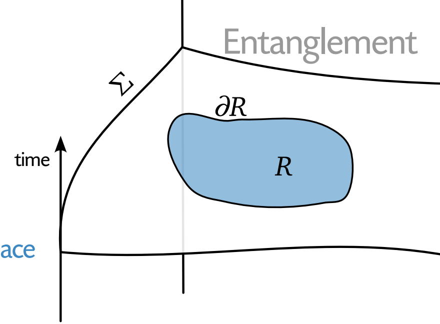

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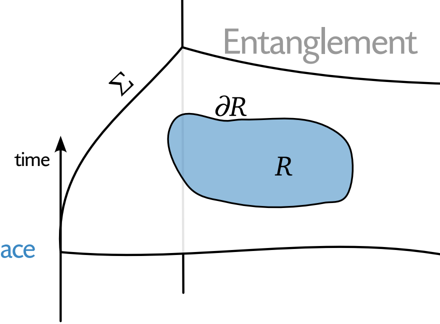

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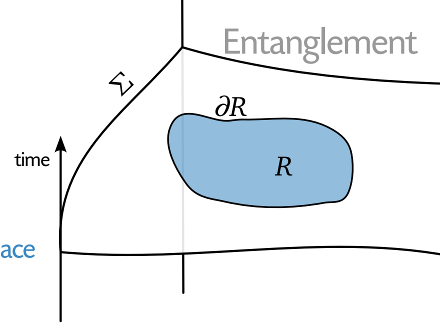

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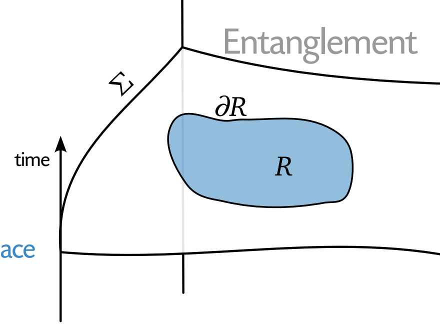
proportional to heat kernel coefficients

Entanglement



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proportional to heat kernel coefficients

subleading universal topological term

Outline

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Free fields and current algebras

Gift 1: State-operator correspondence [2406.02662 w/ D.M. Hofman]

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Gift ideas (outlook)

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Non-invertible current algebras

Gauge charge conjugation $J \sim -J \implies$ symmetry broken but resurrected as non-invertible
[{others}, Antinucci, Galati, Rizi 2022; Aguilera-Damia,Argurio,Chaudhuri 2023]

Current algebra also: $D(\Lambda) \otimes D(\Lambda') = \mathrm{e}^{\mathrm{i}k\int \Lambda \wedge \mathrm{d}\Lambda'} D(\Lambda + \Lambda') \oplus \mathrm{e}^{-\mathrm{i}k\int \Lambda \wedge \mathrm{d}\Lambda'} D(\Lambda - \Lambda')$

$\rightsquigarrow$ Should still fix the spectrum *TIP w/ Aguilera-Damia, Argurio, Chaudhuri*

Current algebras in gravity

Linearised gravity enjoys "biform" symmetries with charges $R -$(traces) and $\star R \star -$(traces)
[[Hinterbichler et al 2022, (Hull et al 2024)^2]]

Also leads to a current algebra (depends on more parameters 2 KY tensors, 2 one-forms)

$\rightsquigarrow$ Implications: linearised spectrum, asymptotic symmetries...? *TIP w/ Mathys*

Non-linear p-form electrodynamics

Thank you!