Scattering amplitudes from holography

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Introduction

- Scattering amplitudes is one of the main QFT observables.
- They provide a link between theory and experiment.
- Their computation is part of textbook QFT.

Scattering amplitudes in $Mink_{d+1}$ from CFT_d

- In this talk I would argue for a new way to obtain scattering amplitudes by taking a limit of CFT_d correlations functions:

$$\lim \langle \mathcal{O}_{\Delta_1}(\vec{p}_1) \cdots \mathcal{O}_{\Delta_n}(\vec{p}_n) \rangle_{CFT_d} = \text{Scattering amplitude in } Mink_{(d+1)}$$

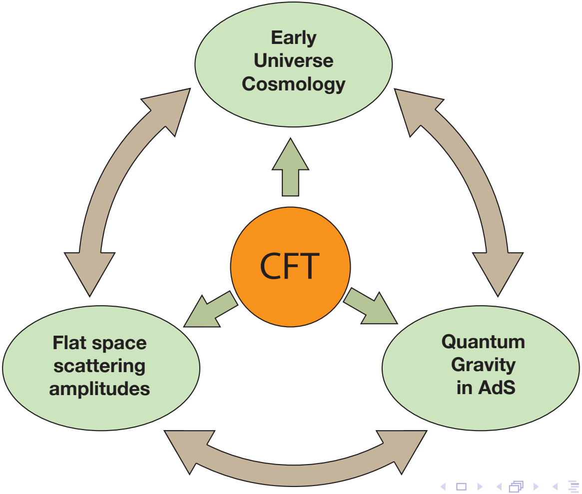
- $\mathcal{O}_\Delta$ are CFT primary operators of scaling dimension Δ
- I will explain in shortly which scattering amplitude of what theory such limit computes ...
- ... and most importantly what the limit is

Who ordered that?

- The relation originates from holography.
- Holography relates $(d+1)$ quantum gravity with d-dimensional quantum theory with no gravity, and it should hold for any spacetime asymptotics.
- In particular, for asymptotically flat gravity.

- Observables in asymptotically flat gravity are scattering matrices
- ... and holography suggest that they should have a description in terms of a quantum theory in one dimension less.

The ubiqiutous CFT



Flat-space holography

- The holographic duality we understand the best is that between AdS_{d+1} and CFT_d.
- The idea is now to take a 'flat-space' limit of AdS and follow this through the duality. [Polchinski (1999) Susskind (1999)].
- The underlying physical picture is compelling: the physics in a small region in the centre of AdS should be the same as that of flat space.
- However, this limit is *very subtle*.
- The purpose of this work is to provide a complete understanding of this limit.

References

- This is based on:

  R. Marotta, KS, M. Verma, Flat space spinning massive amplitudes from momentum space CFT, JHEP(2024), 2406.06447

- The topic has a long history:

  Giddings, Penedones, Maldacena, Simmons-Duffin, Ziboedov, Komatsu, Paulos, van Rees, Vieira, Fitzpatrick, Raju, Hijano, Neuenfield, Lipstein, McFadden, Gadde, Sharma, Pimentel, Li, de Gioia, Raclariu, Cassali, Gibbons, Strominger, Sleight, Taronna, Bagchi, Barnich, Donnay, Ciampanelli, Petropolulos, Petkou, Siampos, Mason, Ruzziconi, Yellshpur Srikant ....

- Most earlier works are done in bottom-up approaches:
  - Celestial holography: co-dimension two CFT in celestial sphere
  - Carrollian holography: co-dimension one CFT at null infinity
- Here we aim to provide a top-down approach.

Outline

Outline

Which theories?

- Suppose we want to compute the scattering amplitudes for a quantum field theory with Lagrangian

$$L_{\rm QFT} = L_{\rm QFT}(m_i^2, g_j)$$

where the m_j^2 are masses of the fields and g_j are coupling constants.

- This can be any model one may be interested in:
the standard model of particle physics or part of it, a beyond the SM model, maximally supersymmetric YM theories, with or without dynamical gravity.

Coupling to AdS

- The first step is to consider this theory in AdS:

$$S_{AdS} = \int d^{d+1}x\sqrt{g}\left(\frac{1}{16\pi G}R + \Lambda + L_{\rm QFT}(m_i^2, g_j)\right)$$

- $\Lambda \sim \frac{1}{\ell^2}$ is the cosmological constant, and ℓ is the AdS radius:

$$ds^2_{AdS} = \ell^2\left(\frac{dr^2}{r^2} + \frac{1}{r^2}d\vec{x}^2\right)$$

  where we consider Euclidean AdS.
- The flat-space limit includes:

$$\Lambda \to 0 \quad \Rightarrow \quad \ell \to \infty$$

- When taking this limit we want to kept fixed the parameters of the original theory:

$$m_i^2, g_j \ \text{fixed}$$

From AdS to flat space

- We reparamerize the radial coordinate

$$r = \ell e^{\tau/\ell}$$

- Then from Euclidean AdS.

$$ds^2 = \ell^2 \left(\frac{dr^2}{r^2} + \frac{1}{r^2} d\vec{x}^2 \right) \quad \xrightarrow{\ell \to 0} \quad ds^2 = d\tau^2 + d\vec{x}^2 + O(1/\ell)$$

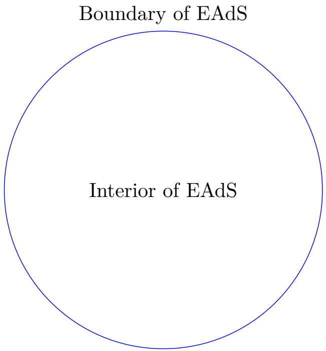
  Upon Wick rotation we get Minkowski.

- One can check that the AdS isometries become the flat space isometries.
- AdS_{d+1} isometries are also the conformal group in d dimensions.
- Boosts and time translations originate from dilations and special conformal transformations.

AdS/CFT correspondence

- The AdS/CFT correspondence relates AdS observables in $(d+1)$ dimensions to CFT observable in d dimensions.
- The CFT leaves at the boundary of AdS, in a space with metric:

$$ds^2_{Mink} = \ell^2 d\vec{x}^2$$



- We want to take $\ell \to \infty$, so we need to focus on
  - deep IR of the CFT
  - deep interior of AdS

CFT: a primer

A CFT consists of

- set of operators $\mathcal{O}^i_{\Delta_i}$ of scaling dimensions Δ_i
  - Examples of such operators are the energy momentum tensor and symmetry currents.
- their operator product coefficients (OPE) C^i_{jk}
- With this data one can (in principle) compute correlation functions:

$$\langle \mathcal{O}^{i_1}_{\Delta_1}(\vec{x}_1) \ldots \mathcal{O}^{i_n}_{\Delta_n}(\vec{x}_n) \rangle$$

- Holography computes the same object via a gravitational computation.

Holographic dictionary

- For every operator of the CFT $\mathcal{O}^i_{\Delta_i}$ there is a bulk field, Φ_i.
  - E.g., the energy-momentum tensor T_{ij} corresponds to the bulk metric g_{ij} and symmetry currents J_i correspond to bulk gauge fields, A_i.
- The dimension Δ_i are related to bulk masses m_i^2 via a relation of the form:

$$m_i^2 \ell^2 = \Delta(\Delta - d)$$

- As $\ell \to \infty$
  - $m_i^2 \to 0$, if Δ_i does not scale with ℓ.
  - to have $m_i^2 \neq 0$ one needs to have $\frac{\Delta_i}{\ell}$=fixed
- The CFT OPE coefficients C^i_{jk} are related with the bulk couplings g_j.
  - C^i_{jk} should be kept fixed as $\ell \to \infty$.

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Flat-space limit from the CFT perspective

- We need to focus on the deep interior of CFT.
- To capture massive fields, we need the CFT to contain operators of dimension Δ_i that scale to infinity with a fixed slope, $\frac{\Delta_i}{\ell}$=fixed.
- OPE coefficients are kept fixed.
- Operators should be renormalized, $\mathcal{O}_{\Delta_i} \to \sqrt{Z}_i \mathcal{O}_{\Delta_i}$, with

$$Z_i \to \infty \qquad \text{if } \Delta_i \sim O(1) \quad \text{(corresponding to massless fields)}$$
$$Z_i \to 0 \qquad \text{if } \Delta_i \sim \infty \quad \text{(corresponding to massive fields)}$$

Outline

Scattering amplitudes

Scattering amplitudes:

- Depend on on-shell (d+1)-momenta, p_i, $p_i p^i = m^2$ and polarisation vectors, ε_i.
- They are proportional to momentum preserving δ-function:

$$\mathcal{A}(p_i, \varepsilon_j) = \delta^{d+1}(\sum p_i)\hat{\mathcal{A}}(p_i, \varepsilon_j)$$

- Part of the difficulty in setting up the flat-space limit is the presence of the delta function.

Holographic map

- Kinematics:

  We need a map from d-dimensional variables appropriate for CFT_d to the variables that scattering amplitudes depend on, p_i, ε_i.

- We will work with the CFT in momentum space:

  CFT correlators depend on (off-shell) d-momenta, $\vec{p}_i$:

$$\langle \mathcal{O}^{i_1}_{\Delta_1}(\vec{p}_1) \cdots \mathcal{O}^{i_n}_{\Delta_n}(\vec{p}_n) \rangle = \delta^d(\sum \vec{p}_i) \langle\langle \mathcal{O}^{i_1}_{\Delta_1}(\vec{p}_1) \cdots \mathcal{O}^{i_n}_{\Delta_n}(\vec{p}_n) \rangle\rangle$$

- From d-momenta to on-shell $(d+1)$-momenta:

$$p^i = (E, \vec{p}) \text{ with } E = \pm\sqrt{\vec{p}^2 + m^2}$$

  $\pm$ depends on whether particle is in-coming or out-going.

Flat-space limit

- The claim is then that:

$$\lim_{\ell\to\infty} \langle \mathcal{O}^{i_1}_{\Delta_1}(\vec{p}_1)\cdots\mathcal{O}^{i_n}_{\Delta_n}(\vec{p}_n) = \mathcal{A}^{i_1\ldots i_n}(p_i,\varepsilon_j)$$

- With the kinematical identification the spatial momentum conservation is manifest.
- Thus we need:

$$\lim_{\ell\to\infty} \langle\langle \mathcal{O}^{i_1}_{\Delta_1}(\vec{p}_1)\cdots\mathcal{O}^{i_n}_{\Delta_n}(\vec{p}_n)\rangle\rangle = \delta(E_1+\cdots+E_n)\hat{\mathcal{A}}^{i_1\ldots i_n}(p_i,\varepsilon_j)$$

- Before discussing the general case I will discuss a simple scattering amplitude that encodes electromagnetic form factors.

Outline

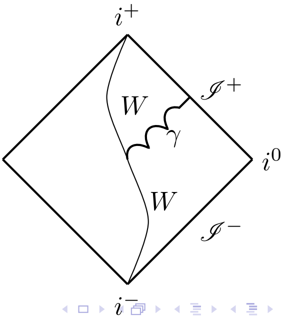
Electromagnetic form factors

- Electromagnetic form factors encode the electromagnetic properties of single-particle states:

$$\langle p', s | J^a(x) | p, s \rangle$$

$J^a(x)$ is the electromagnetic current
$|p, s\rangle$ is a single-particle state carrying momentum p and spin s

- By measuring the electromagnetic radiation one may extract the couplings of the photon with the massive field corresponding to the state from the electromagnetic multipoles.



Electromagnetic form factors

- A spin-s state has $(2s+1)$-electromagnetic form factors.
- For spin 1/2 particle (like the electron) the electromagnetic coupling is determined by the charge and the magnetic dipole coupling (which determines the gyromagnetic ratio).
- For massive spin-1 particle (like the W-boson) we have the charge, the magnetic dipole and the electric quadropole moment.

- The determination of the electromagnetic form factors of hadronic states is an active field of research.
- For example, part of the theoretical uncertainty in the determination of muon $g-2$ factor is due to uncertainties in the form factors.

Electromagnetic form factors from CFT

- We will now demonstrate how to compute the massive spin-1 electromagnetic form factor from a CFT.
- The CFT operator dual to a photon is a conserved current $\mathcal{J}^{\mu_2}$ and operator dual to a massive vector is a charged non-conversed vector operator, $\mathcal{O}_{\Delta}^{\mu_1}$.
- Thus we need compute:

$$\lim_{\ell \to \infty} \langle \mathcal{O}_{\Delta}^{\mu_1}(\vec{p}_1) \mathcal{J}^{\mu_2}(\vec{p}_2) \mathcal{O}_{\Delta}^{*\mu_3}(\vec{p}_3) \rangle = \delta(E_1 + E_2 + E_3) \hat{\mathcal{A}}^{\mu_1 \mu_2 \mu_3}(p_i, \varepsilon_j)$$

- To compute this limit, we need to know a little bit about CFT in momentum space.

CFT in momentum space [Bzowski, McFadden, KS (2013-2023)]

- Lorentz invariance implies that the tensor structure is carried by tensors constructed from the momenta p_μ and the metric $\delta_{\mu\nu}$.

$$\langle\langle \mathcal{O}_\Delta^{\mu_1}(\vec{p}_1)\mathcal{J}^{\mu_2}(\vec{p}_2)\mathcal{O}_\Delta^{*\mu_3}(\vec{p}_3)\rangle\rangle = \sum_{j=1}^{6} A_j^{\mu_1\mu_2\mu_3}(\vec{p}_i, \delta_{\mu\nu}) F_j(p_1, p_2, p_3)$$

  In this case there are six such tensors. [Marotta, KS, Verma (2023)]

- Conformal invariance implies that the form factors F_j satisfy specific differential equations, which uniquely determine them.
- The correlator is determined by conformal invariance in terms of three constants.

Triple-K integrals and the flat-space limit

- All form factors are given in terms of triple-K integrals,

$$I_{\alpha\{\beta_1\beta_2\beta_3\}}(p_1,p_2,p_3) = \int_0^\infty dx\, x^\alpha \prod_{j=1}^{3} p_j^{\beta_j} K_{\beta_j}(p_j x),$$

  $K_{\beta_j}(p_j x)$ are modified Bessel functions and $\beta_i \sim \Delta_i$.
- In this expression the IR of the CFT correspond to $x \to \infty$ limit,
- Focusing on this region, by reparametrizing $x = \ell e^{\tau/\ell}$ and taking $\ell \to \infty$ while keeping p_1, p_2, p_3 and Δ/ℓ fixed, yields

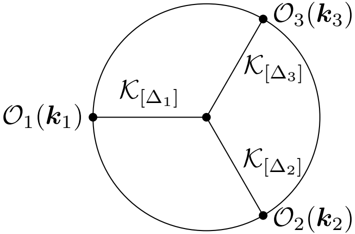
$$\lim_{\ell\to 0} I_{\alpha\{\beta_1\beta_2\beta_3\}}(p_1,p_2,p_3) \sim \frac{1}{\sqrt{Z_W Z_A Z_W}}\delta(E_1+E_2+E_3)$$

Scattering amplitude in the flat space limit

- Putting everything together, the tensors $A_j^{\mu_1\mu_2\mu_3}(\vec{p}_i, \delta_{\mu\nu})$ combine such that the reduced amplitude becomes a function of $(d+1)$-momenta and polarization vectors.
- The result reproduces in complete detail results obtained over 50 years ago [Kim, Tsai (1973)].
- The three constants that determine the CFT amplitude become the charge, magnetic and quadrupole moments.

The view from the bulk

- Instead of a CFT computation, one could have done an AdS computation.
- Then the triple-K integrals originate from "Witten diagrams":

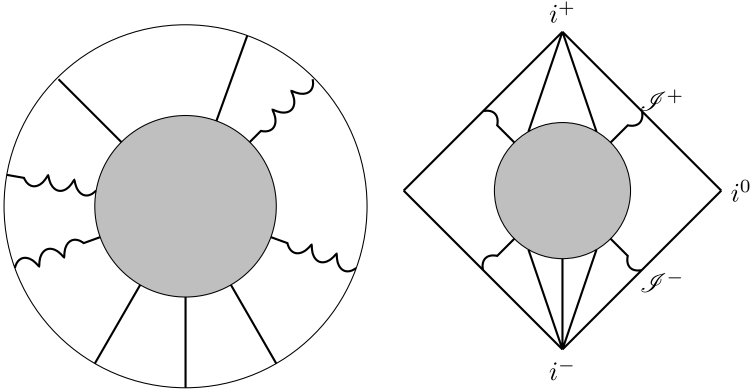


  where $\mathcal{K}_{[\Delta]} \sim K_{\Delta - \frac{d}{2}}(kz)$ is the scalar bulk-to-boundary propagator.
- As $\ell \to \infty$, the AdS metric limits to the flat space with the radial coordinate $r = \ell e^{\tau/\ell}$ becoming the time coordinate τ.

Outline

General case

- The general (perturbative) case works similarly:



- AdS propagators → flat space propagators in the flat space limit
- Interactions are kept fixed in the limit.

Outline

Conclusions/Outlook

- We discussed a new way to compute scattering amplitudes in $(d+1)$-dimensions via a limit of CFT correlators in d-dimensions.
- This (surprising) relation is rooted in deep ideas about quantum gravity (holography).
- There is still an enormous amount to be done:
  - Connect with bottom-up approaches.
  - Obtain non-perturbative results for scatttering amplitudes.
  - Connect with other approaches (generalised unitarity etc.)
  - ...
- CFTs satisfy powerful constraints (bootstrap equations) and one may hope to be able to solve them non-perturbatively.
- These should help delineate the landscape from the swampland.
- ...

A new unification principle?