Superradiant interactions of Cosmic Noise

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Objective

- Propose macroscopic targets that interact with rates $\sim N^2$

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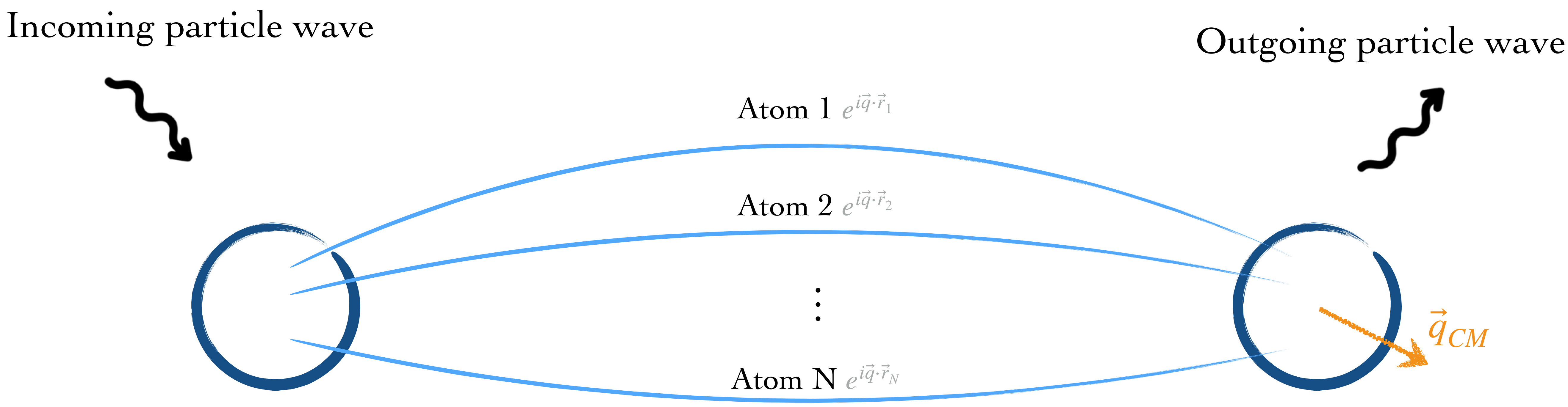
- Besides coherent elastic scattering, are there inelastic processes that are enhanced by N^2 ?

How does elastic scattering give you N^2?

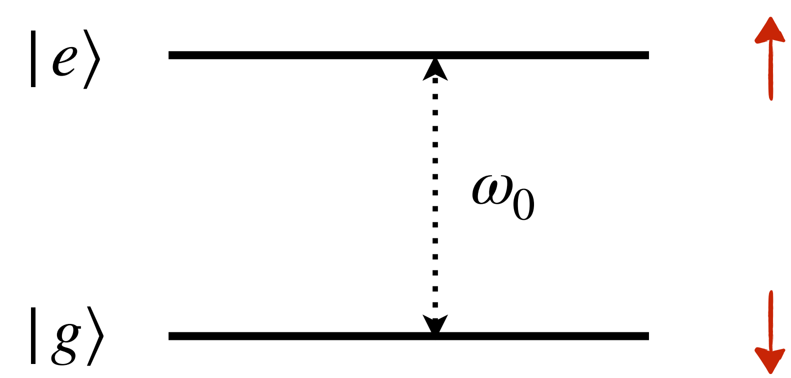


- For small enough momentum transfer, there are N paths all leading to the same final state
  - Scattering amplitude $\propto N$
  - Scattering rate $\propto N^2$
  - Momentum transfer $q_{CM} \sim 5 \times 10^{-6}$ eV
  - Energy transfer $E_{CM} \sim 10^{-49}$ eV

For $R = 10$ cm

Understanding N^2 scalings in inelastic scattering

- A target made of N two-level "atoms": nuclear and atomic transitions, spins in magnetic field, etc…

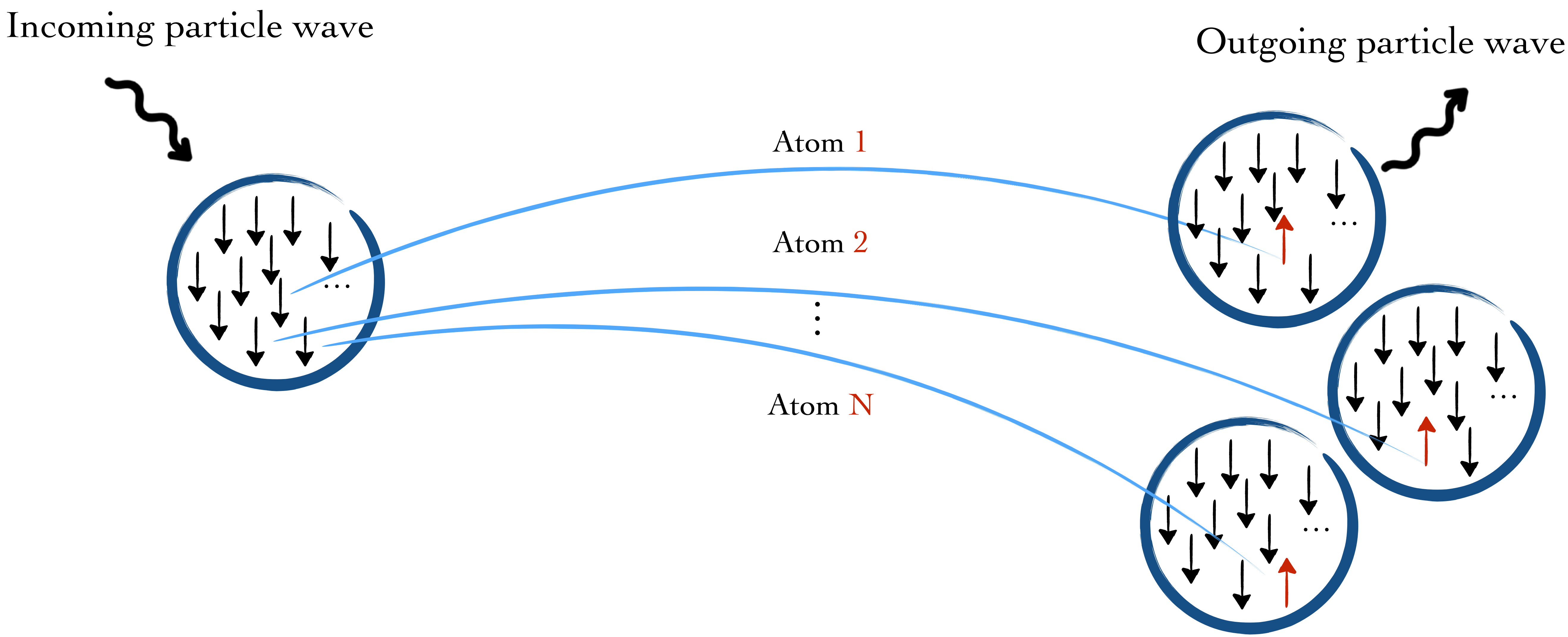


- Two extreme possibilities:
  - All atoms in the ground state $\prod |\downarrow\rangle$
  - All atoms in an equal superposition of ground and excited:

$$\prod\left(\frac{1}{\sqrt{2}}(|\downarrow\rangle + |\uparrow\rangle)\right) = \prod |\rightarrow\rangle \equiv \text{Product state}$$

Why does inelastic scattering normally $\propto N$?

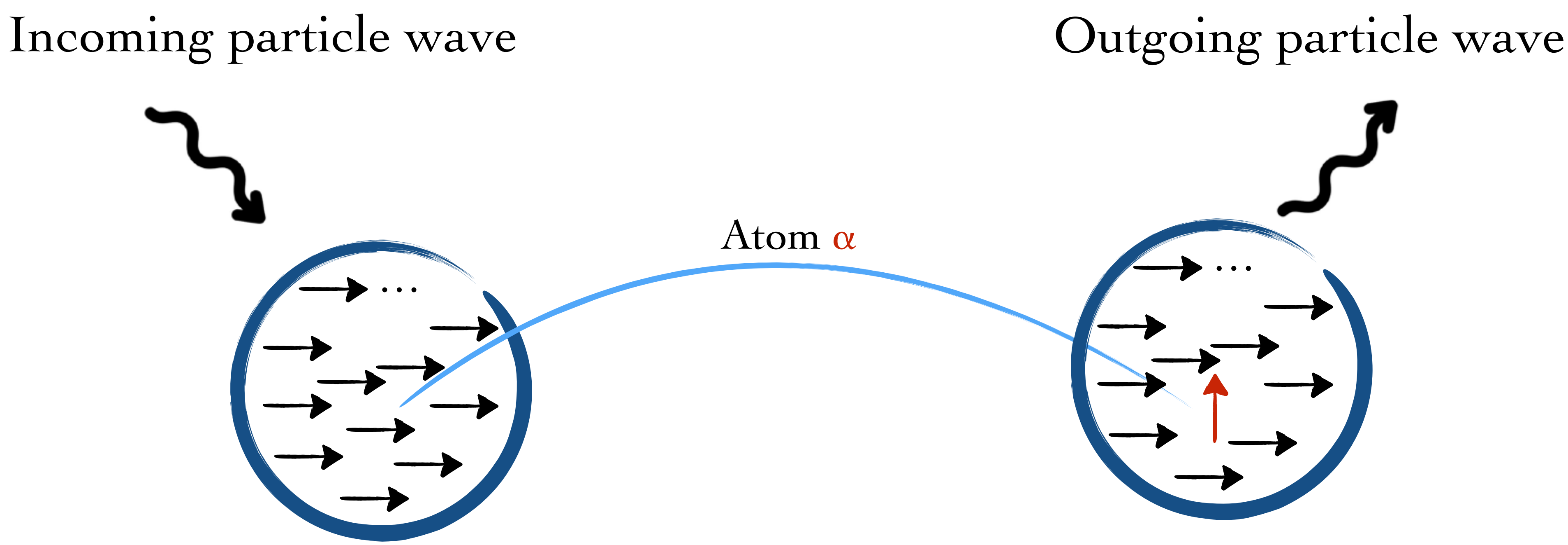
The rate of exciting a single atom



- N atoms all in the ground state $\Rightarrow$ N distinct orthogonal final states
    - Scattering amplitude $\propto 1$
    - Scattering rate $\propto N$
    - Momentum transfer $q \sim 5 \times 10^{-6}$ eV
    - Energy transfer $E \sim \omega_0 \gg 10^{-49}$ eV

For $R = 10$ cm

When does inelastic scattering $\propto N^2$?



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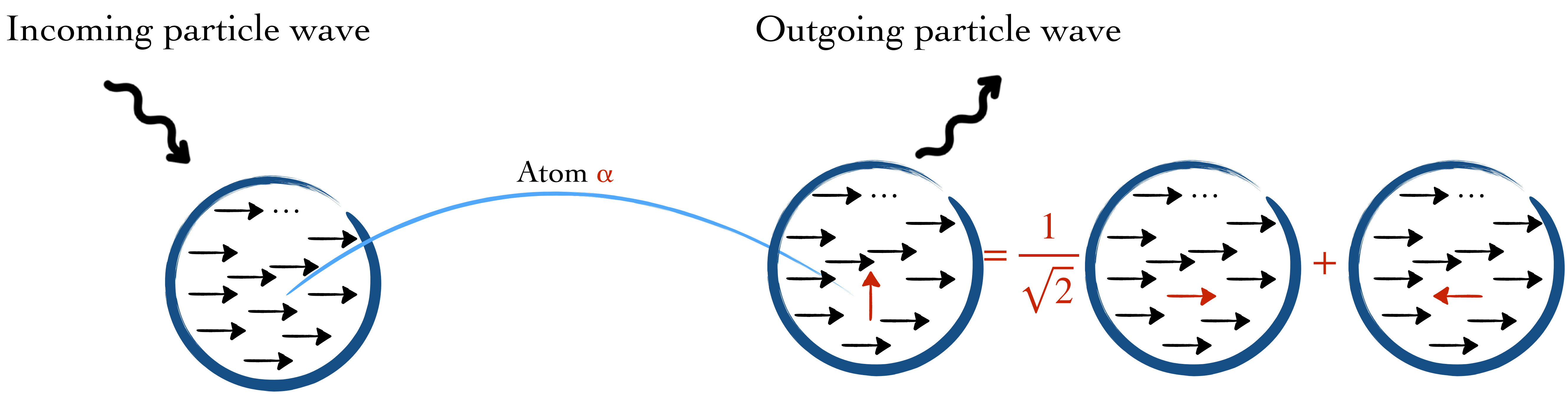
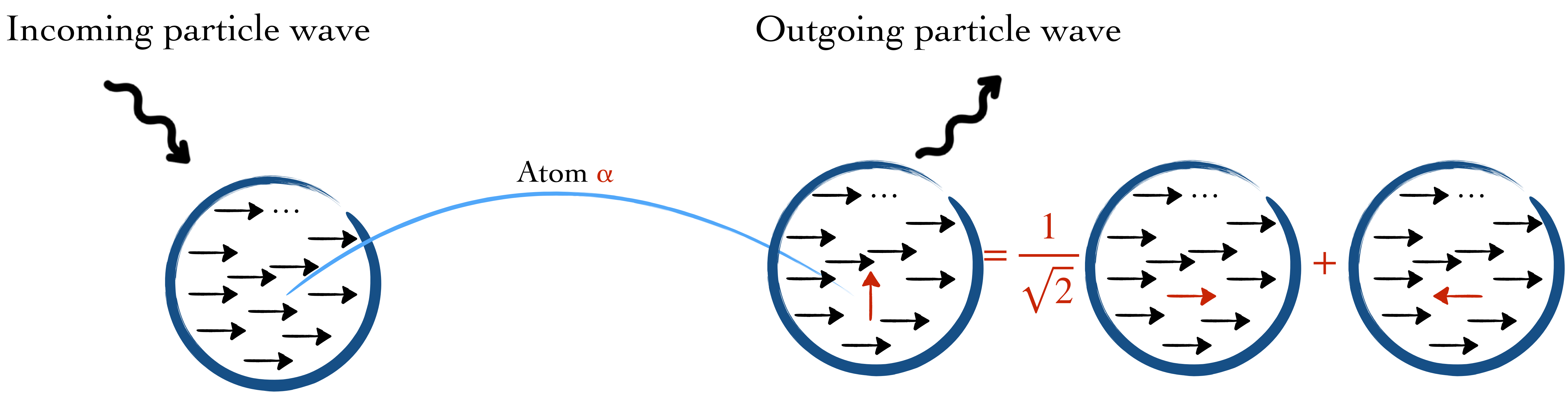


$$|\uparrow\rangle = \frac{1}{\sqrt{2}}\left(|\rightarrow\rangle + |\leftarrow\rangle\right)$$

When does inelastic scattering $\propto N^2$?



$$|\uparrow\rangle = \frac{1}{\sqrt{2}}\left(|\rightarrow\rangle + |\leftarrow\rangle\right)$$

- N atoms in an equal superposition of ground and excited, therefore there are $\sim \frac{N}{2}$ indistinguishable final states
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  - Energy transfer still large, $E = \omega_0$

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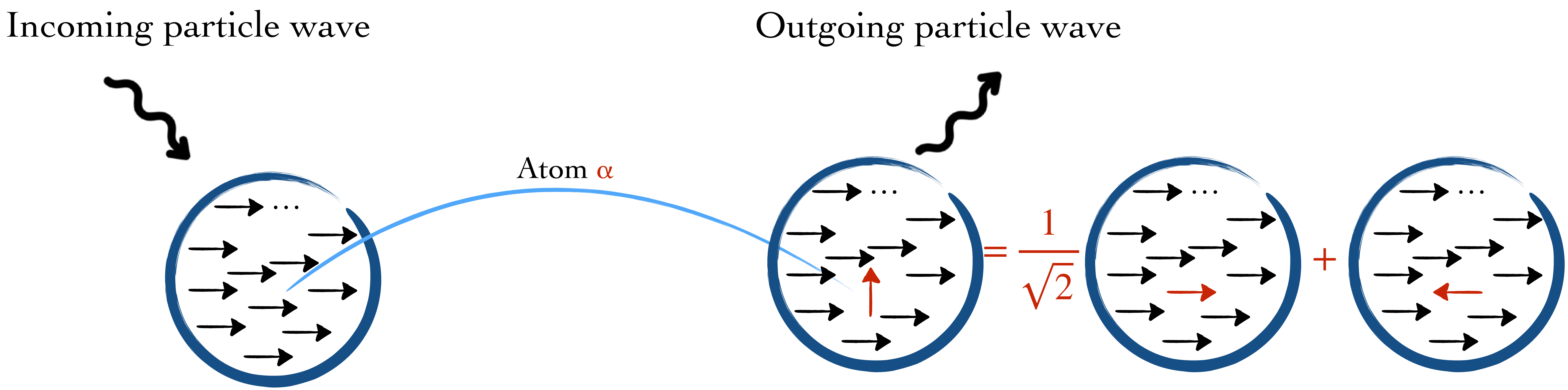

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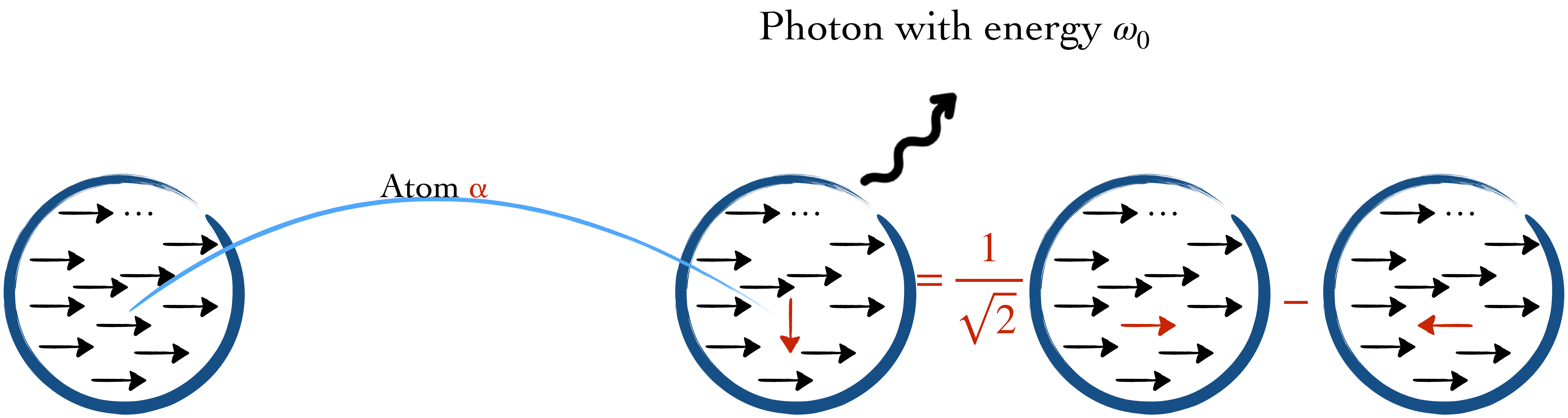
Analogous to Dicke Superradiance

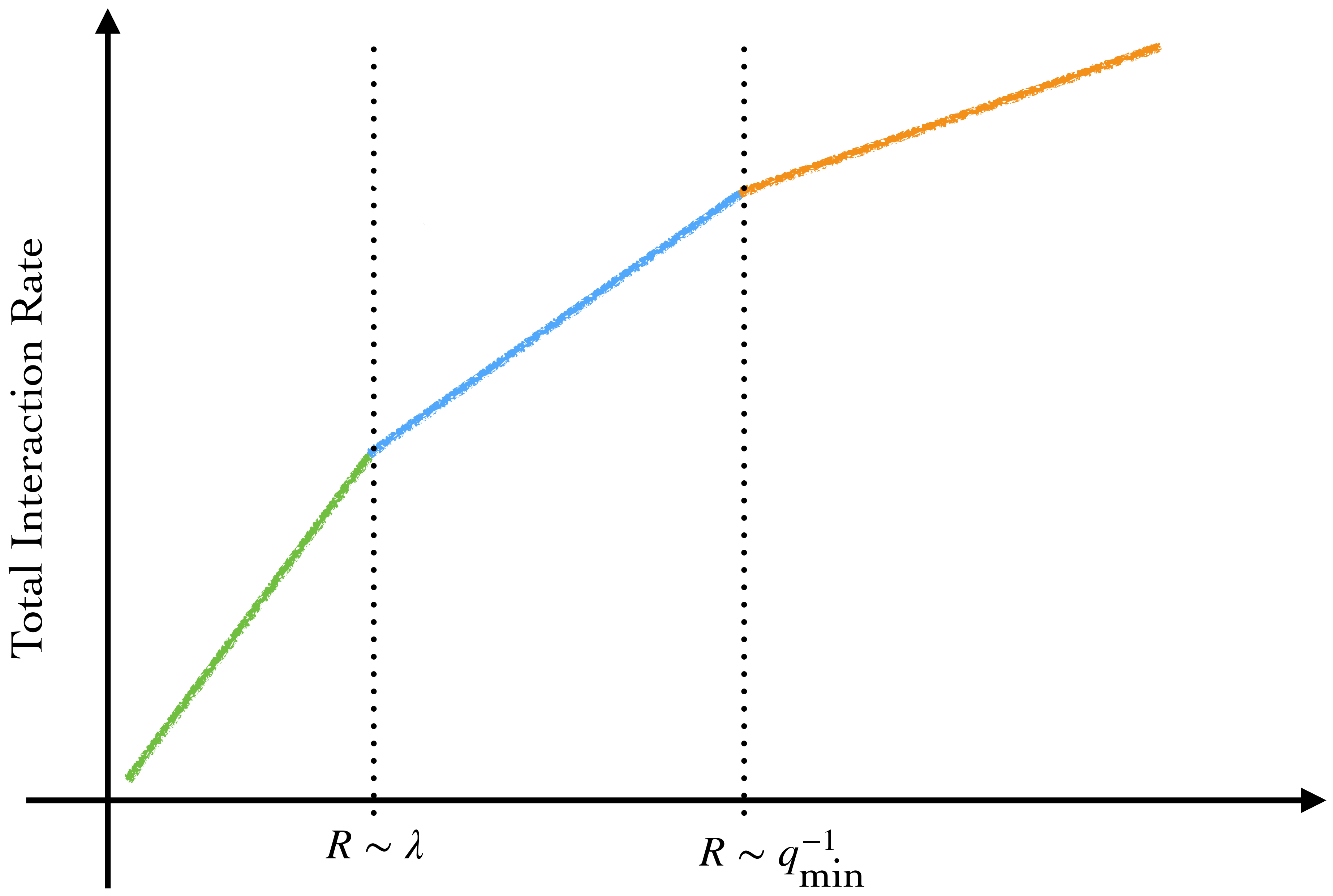
Dicke Superradiance (1954)



- Atoms in an equal superposition of ground and excited emitting photons, produce $\frac{N}{2}$ indistinguishable states
  - Amplitude $\propto N$
  - Photon emission rate $\propto N^2$
  - Similar effects in stimulated absorption and emission

Superradiant scattering from a target of size R

Scattering form factor: $\left| \sum_{\alpha=1}^{N} e^{i\vec{q}\cdot\vec{r}_a} \right|^2$



λ: particle deBroglie wavelength

$q_{\rm min}$: Minimum momentum transfer set by energy conservation

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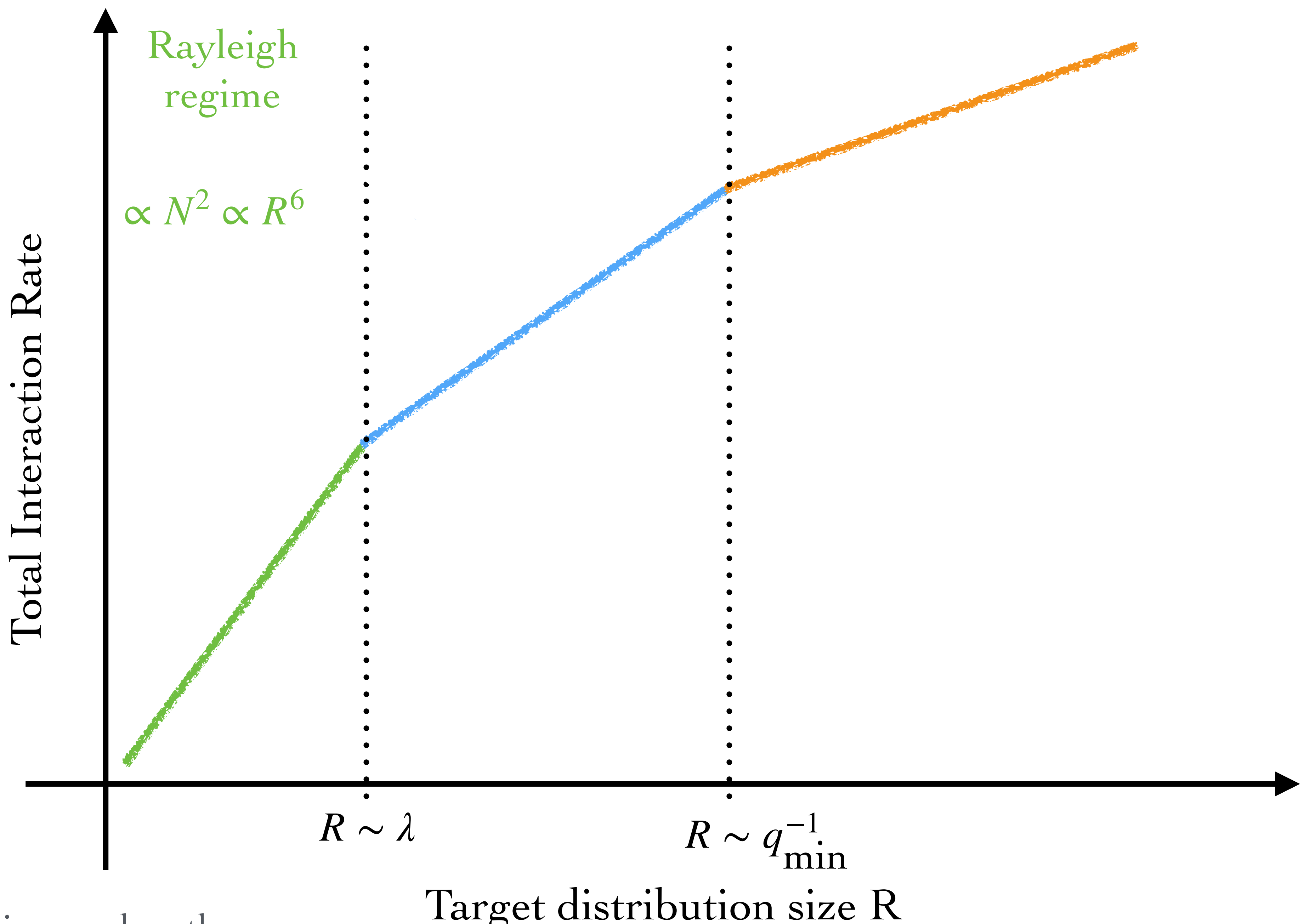

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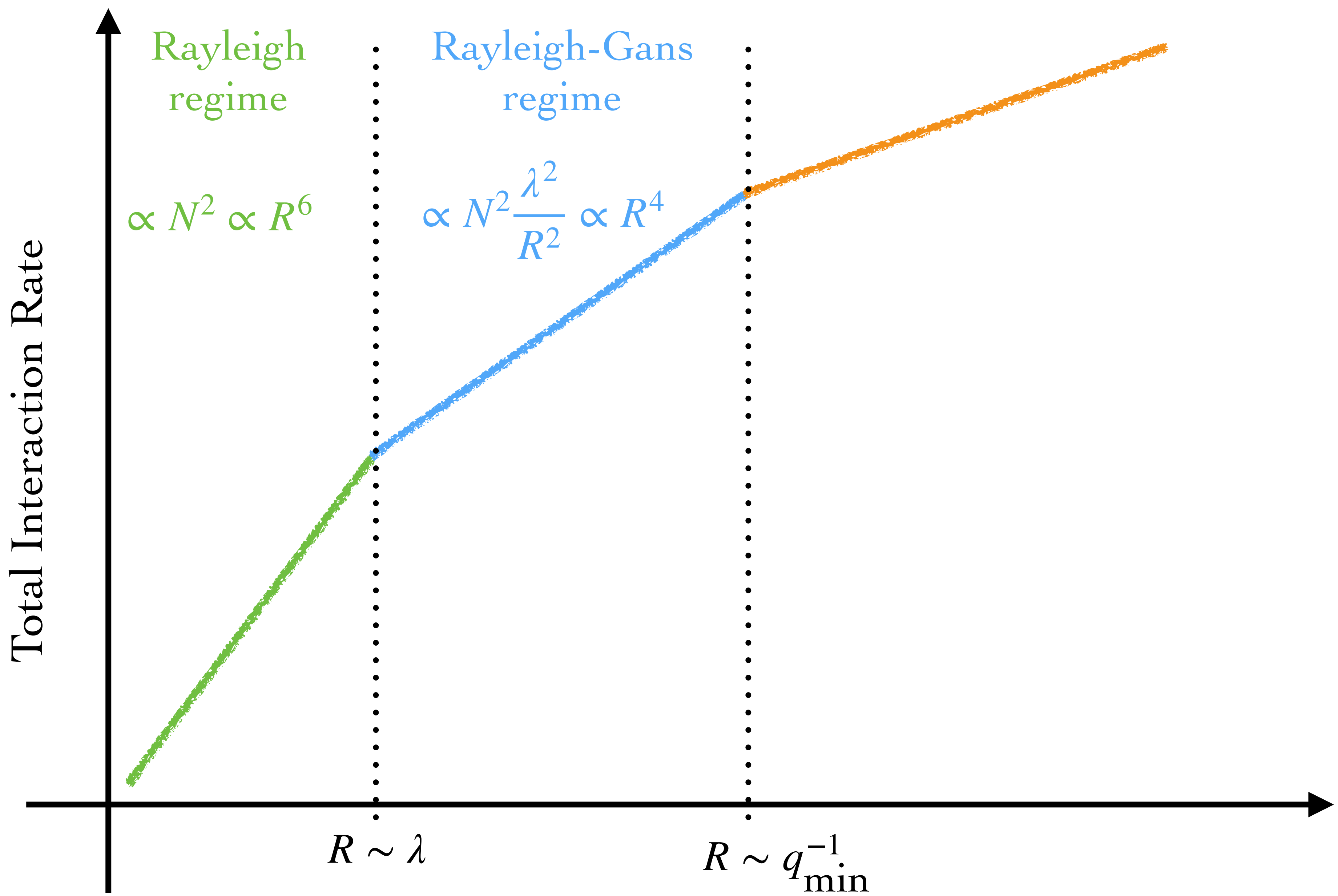

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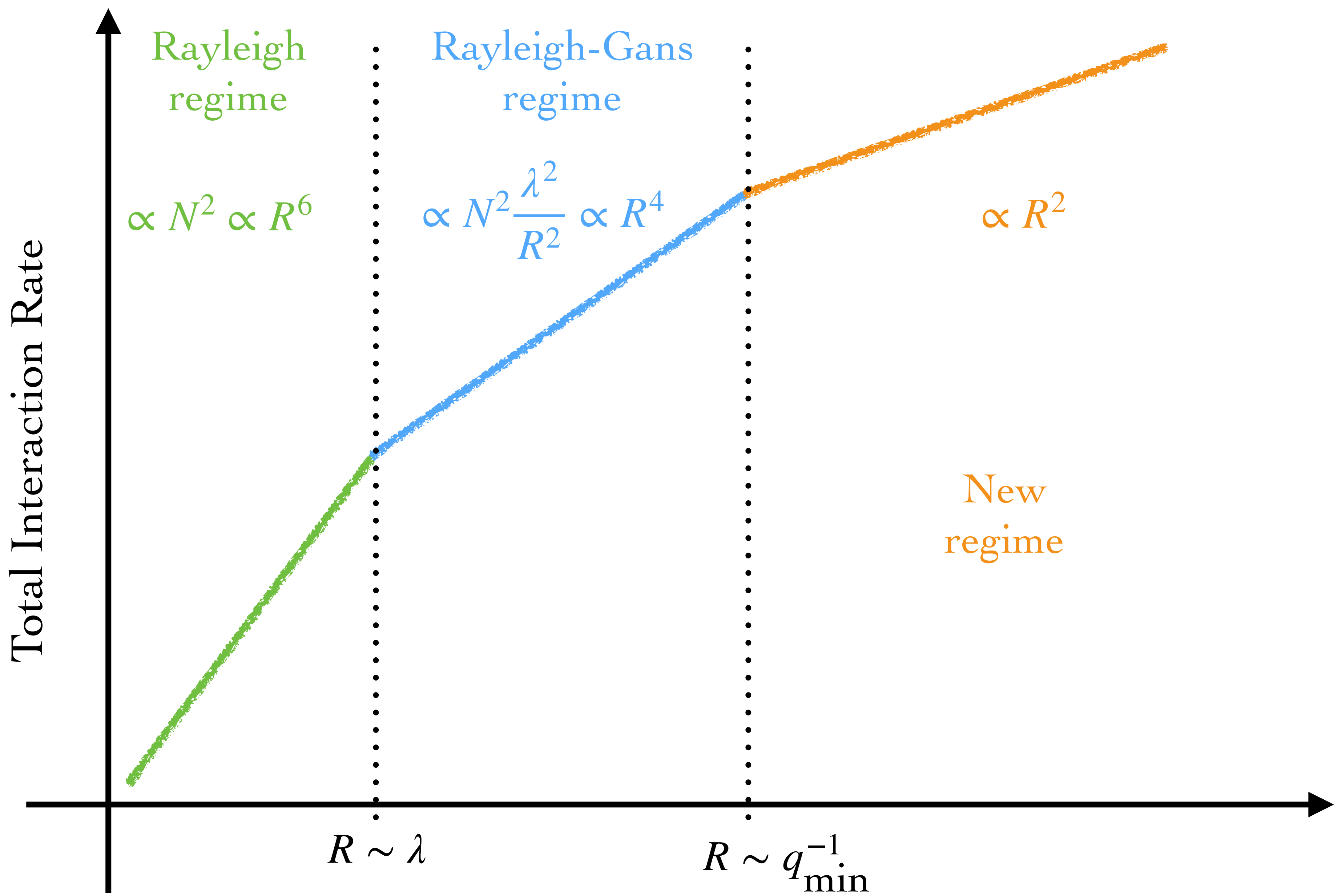

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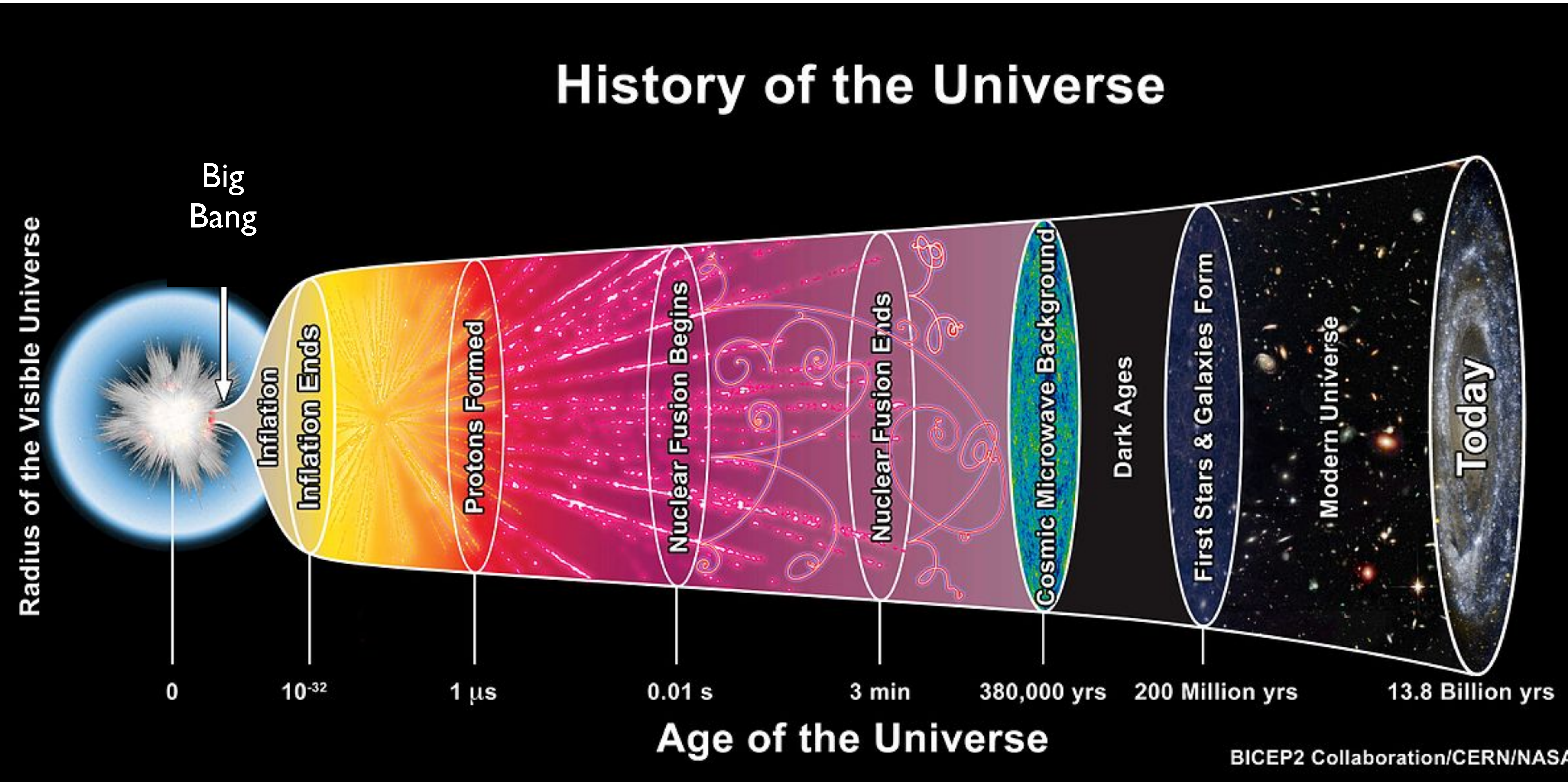
Outline

- Superradiant interactions: Inelastic processes with N^2 rates

- Sample superradiant interaction rate calculations
  - Cosmic Neutrino Background super-scattering
  - Axion and Dark Photon Dark Matter super-absorption and super-emission
  - Solar, Reactor, and Bomb Neutrino super-scattering

- Towards measuring the total interaction rate

A Brief History of the Universe



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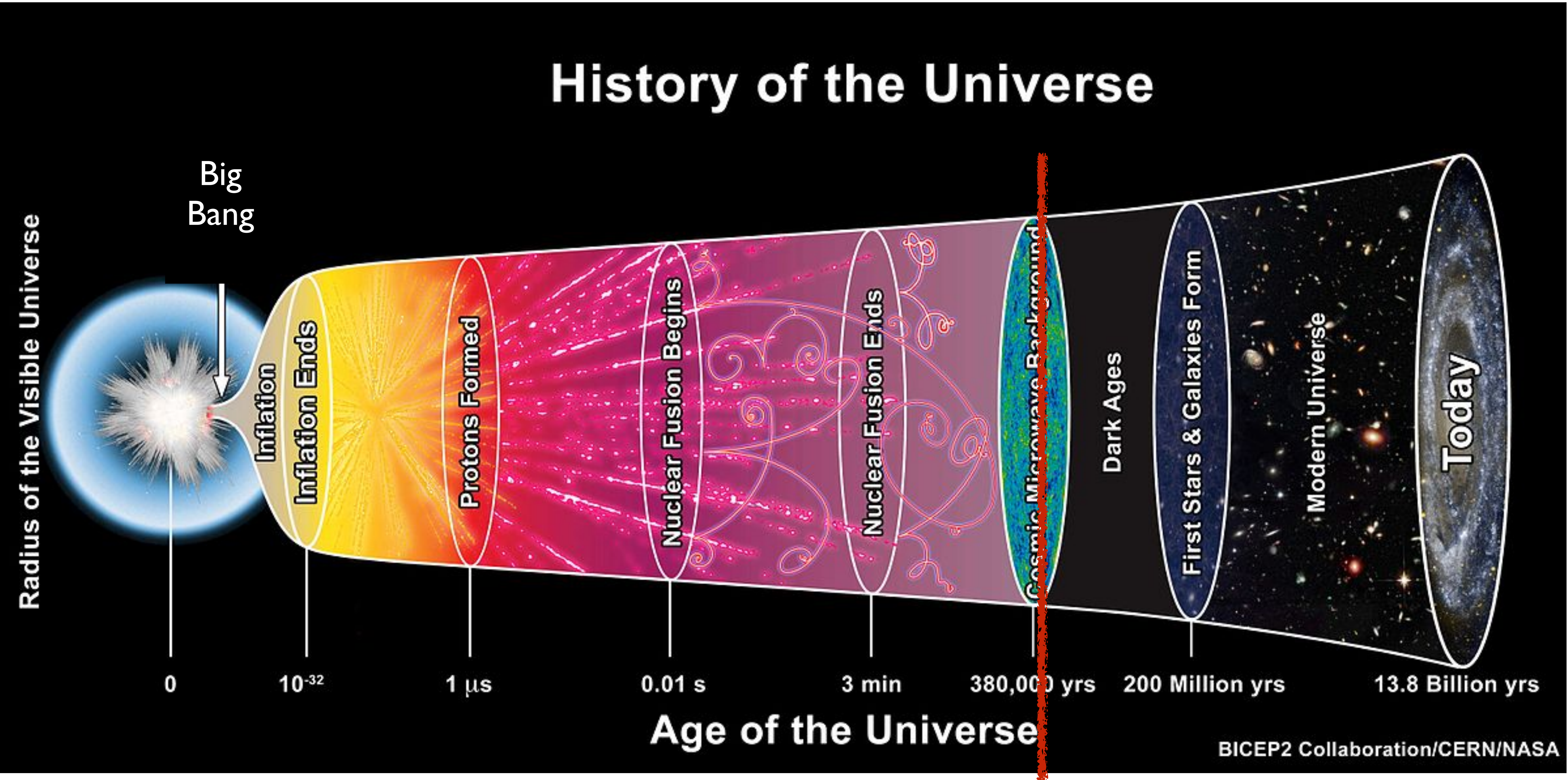
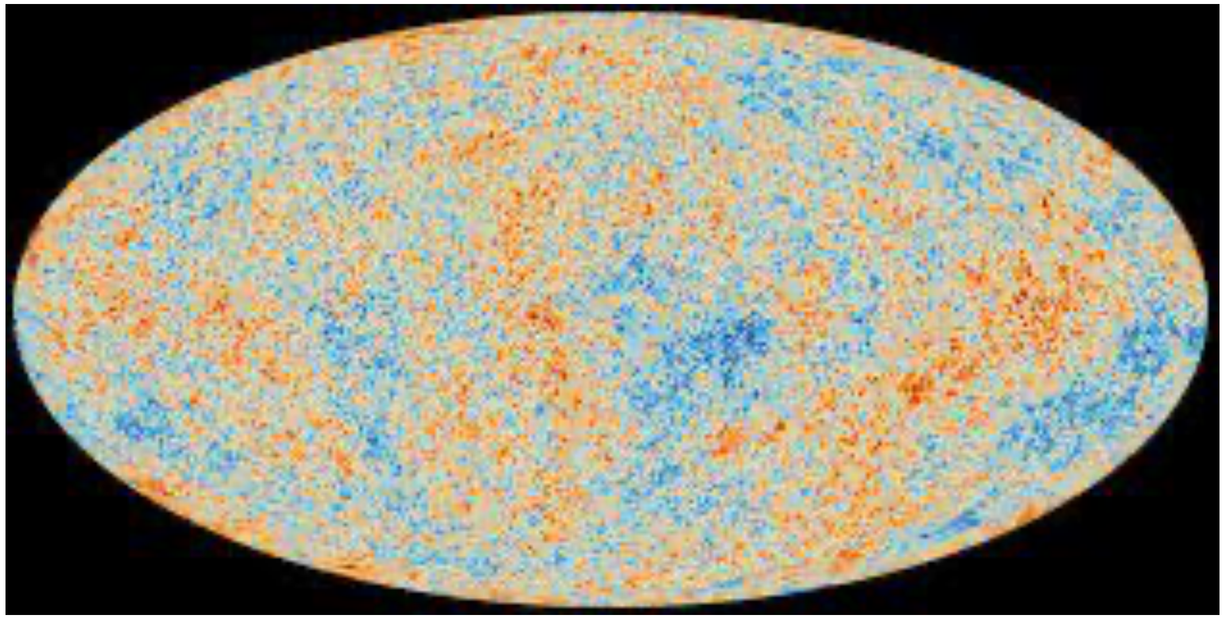


The Cosmic Microwave Background

The Cosmic Microwave Background (CMB)

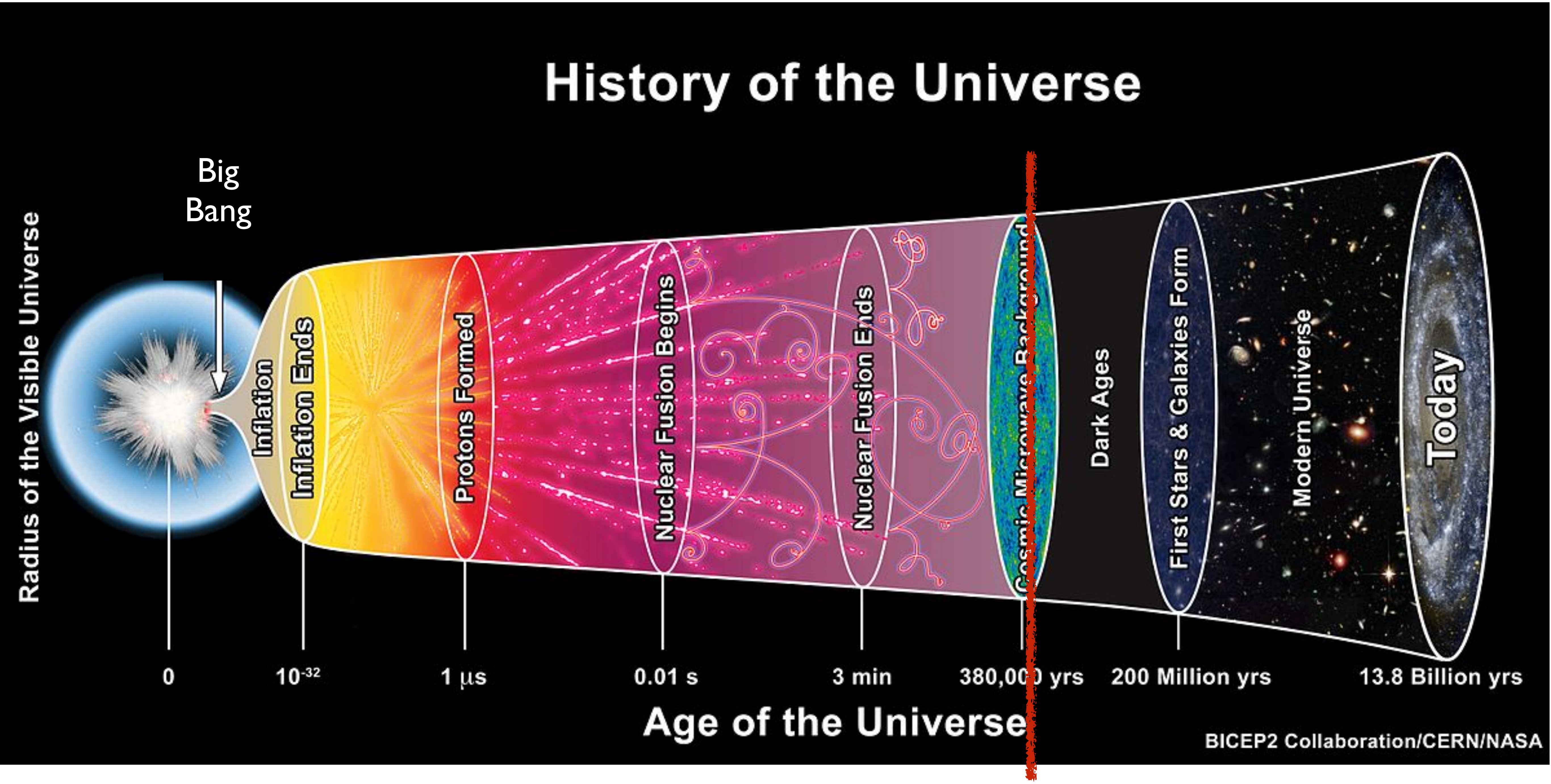


- Time when atoms formed and the universe became transparent to light

- The Universe was 400,000 years-old at the time

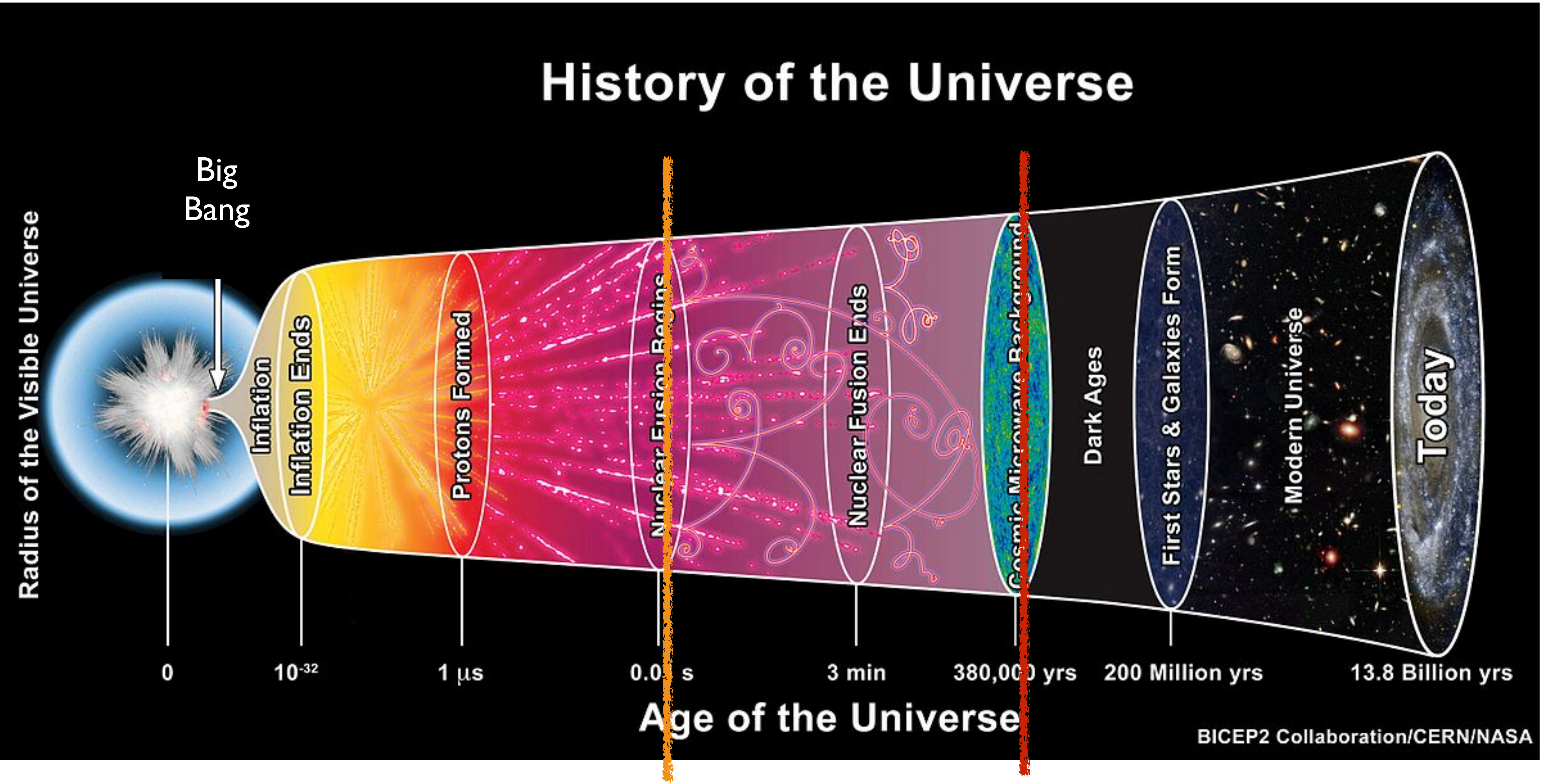
- Allowed to measure the composition of the Universe to 1%

A Brief History of the Universe



The CMB

A Brief History of the Universe



The Cosmic Neutrino Background (CνB)

The CMB

The Cosmic Neutrino Background (CνB)

- Relic neutrinos from the pre-BBN era $\tau_{\text{universe}} \sim 0.1$ sec

- They follow a Fermi-Dirac distribution with:
  - $\langle p_\nu \rangle = 6 \times 10^{-4}$ eV
  - $\langle E_\nu \rangle = 1.6 \times 10^{-6} \text{ eV} \left(\dfrac{0.1 \text{ eV}}{m_\nu} \right)$
  - $\langle \lambda_\nu \rangle = 2.1$ mm
  - $n_\nu = 56$ cm^{-3} per flavor, per helicity model

Why is the CνB important?

- Probes physics at a time much earlier than the CMB
- An entire sector of the Standard Model: 3 flavors and 7+ parameters
- Using non-relativistic particles for 3D tomography of the Universe

Superradiant CνB scattering

- CνB scattering from polarized nuclear or electron spins in a $\vec{B}_0$ magnetic field and prepared in the product state $\prod |\rightarrow\rangle$

$$\mathscr{L}_I = \frac{G_F}{\sqrt{2}} \bar{\nu}_f \gamma_\mu \gamma_5 \nu_f \bar{\psi} (g_L - g_R) \gamma^\mu \gamma_5 \psi$$, where ψ : any SM fermion

In the non-relativistic limit: $H_I = \frac{G_F}{\sqrt{2}} (g_L - g_R)\ \delta^{(3)}(\vec{x}_\nu - \vec{x}_\psi)\ \vec{\sigma}_\nu \cdot \vec{\sigma}_\psi$

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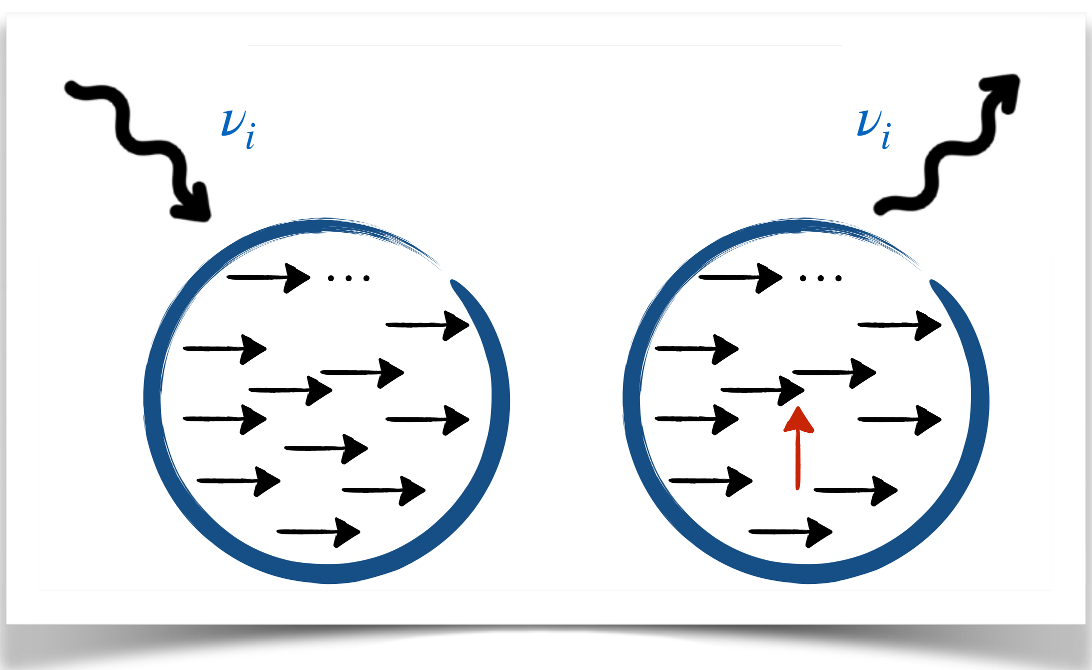
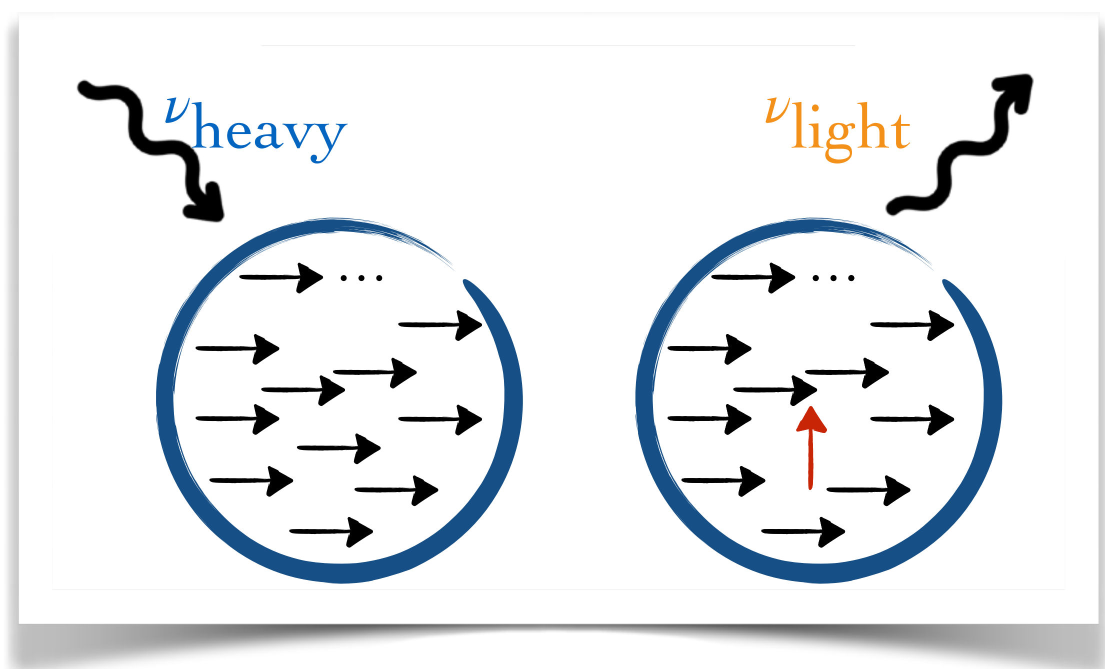
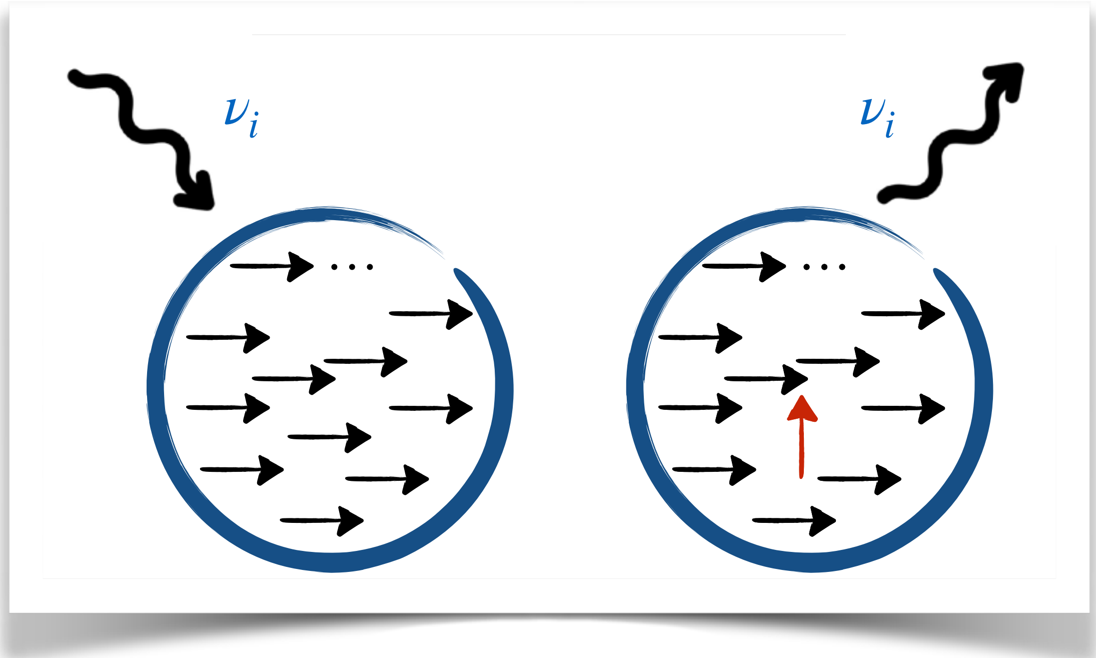
- Two possible processes:





Superradiant CνB scattering



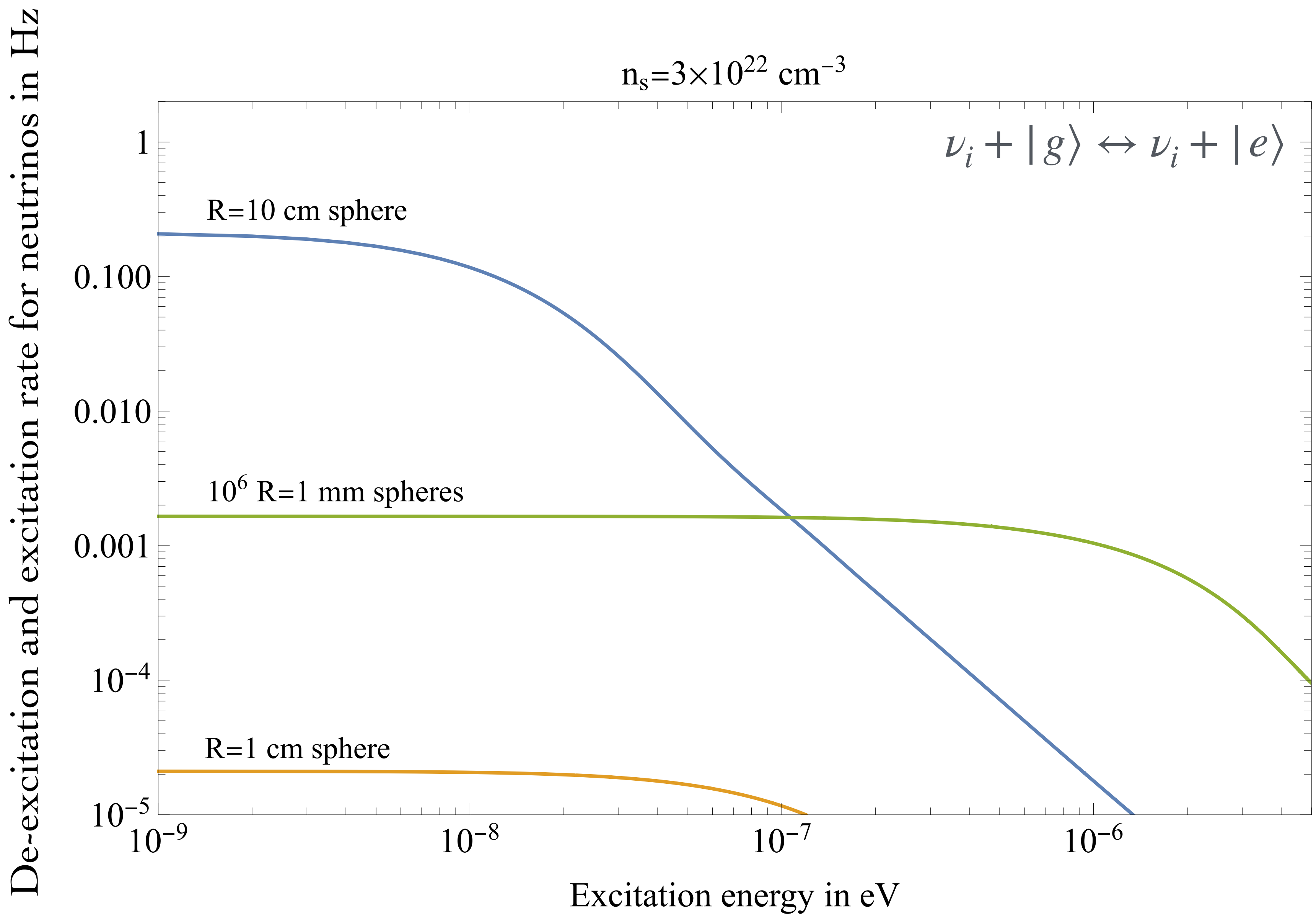
- Maximum available energy for excitation: $E_\nu \sim 10^{-6}\ eV$

- Need momentum transfer to be $\mathcal{O}(R^{-1})$ in order for coherence to be maintained across the entire distribution of size R

- Coherence imposes a maximum of energy transfer $\omega_0 \leq \dfrac{v_\nu}{R}$

CνB superradiant scattering

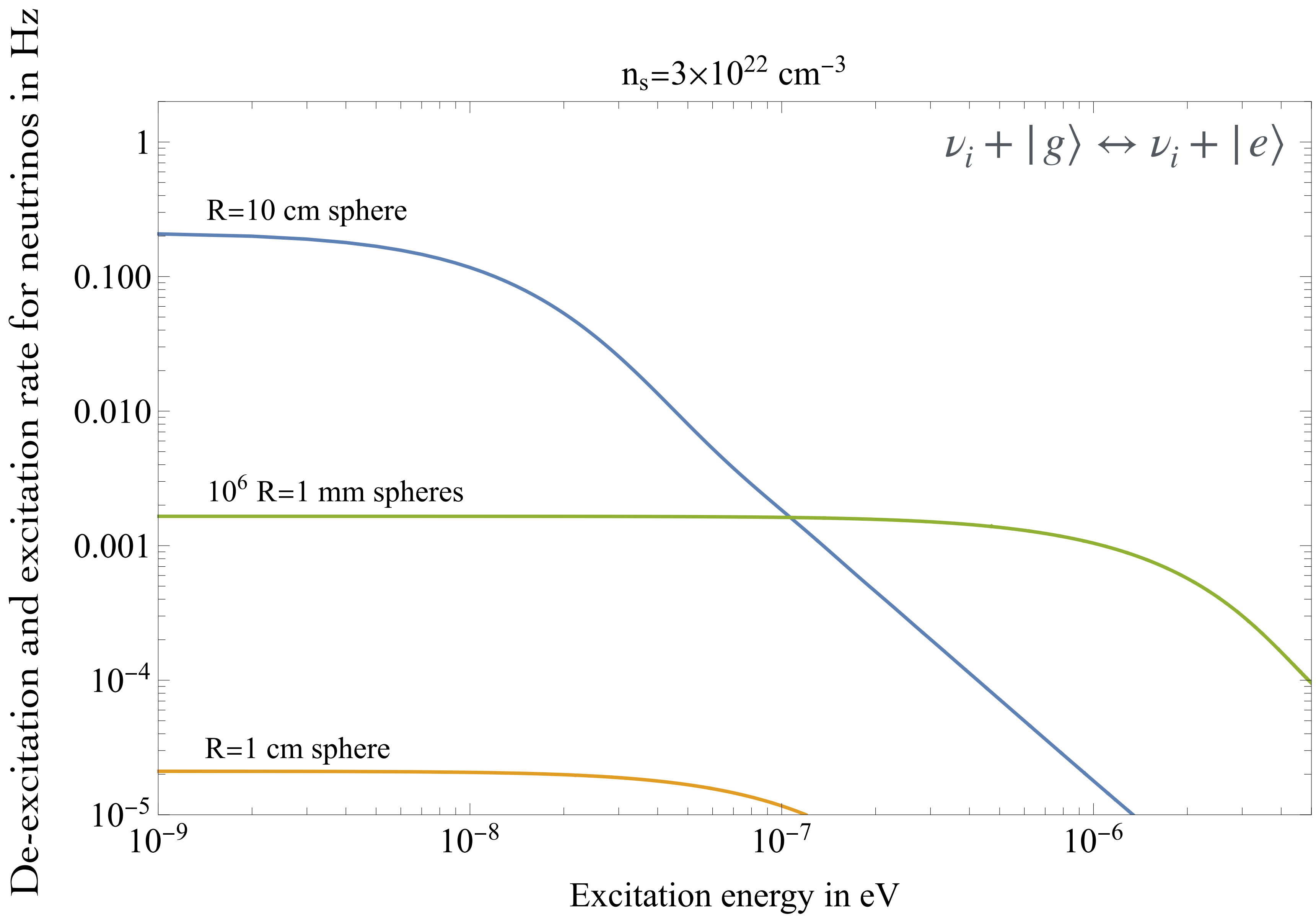
Total rate $\Gamma_+ + \Gamma_-$



To be compared with non-superradiant scattering rate $\sim 10^{-22}$ Hz

CνB superradiant scattering

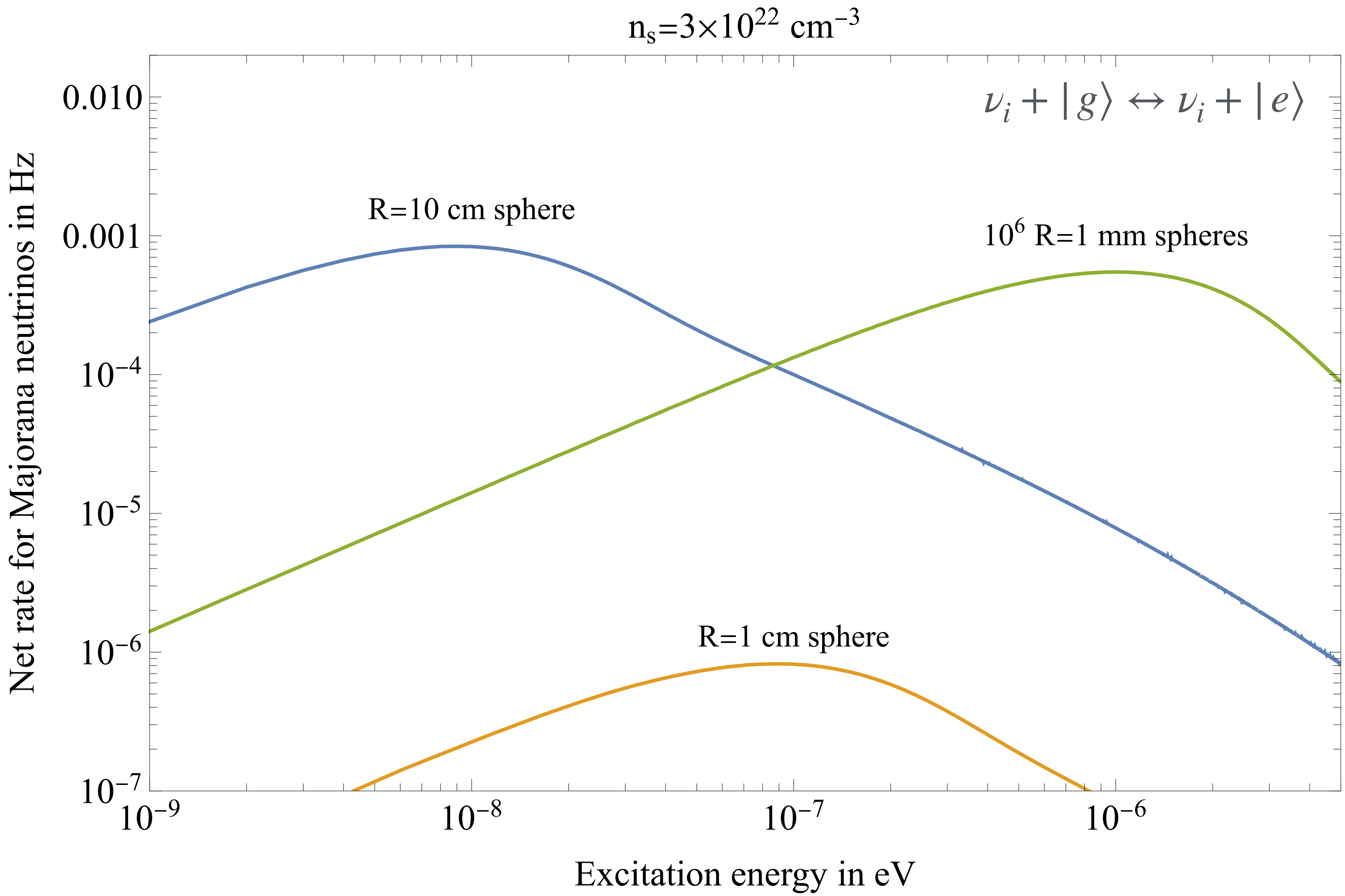
Total rate $\Gamma_+ + \Gamma_-$



Energy splitting ω_0 is a background discriminator

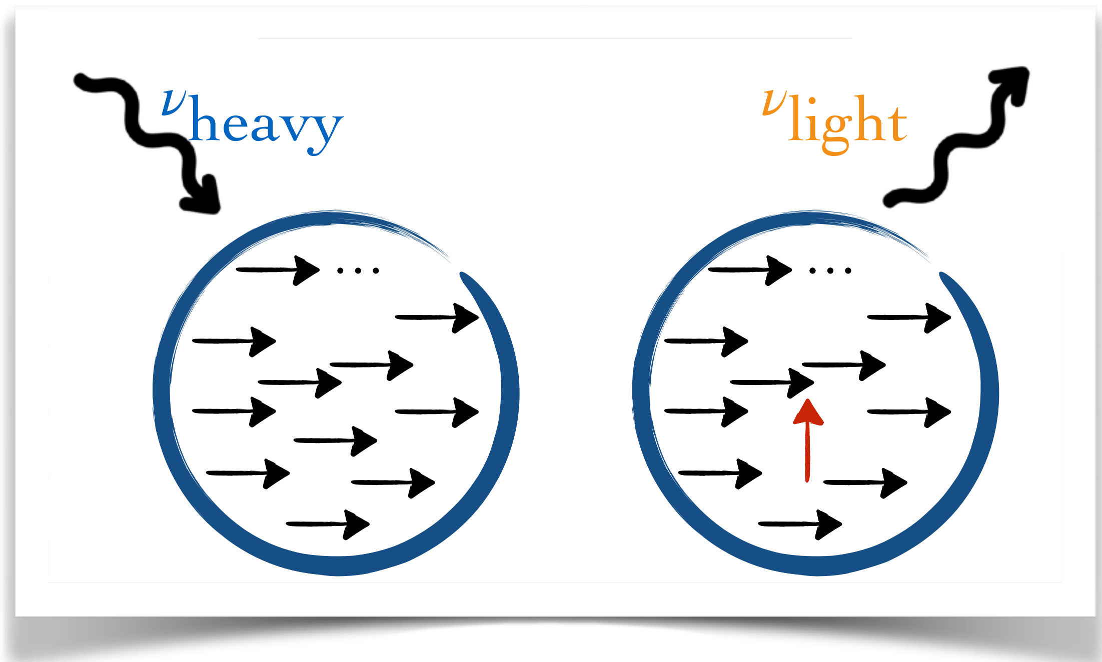
CνB superradiant scattering - Net rate

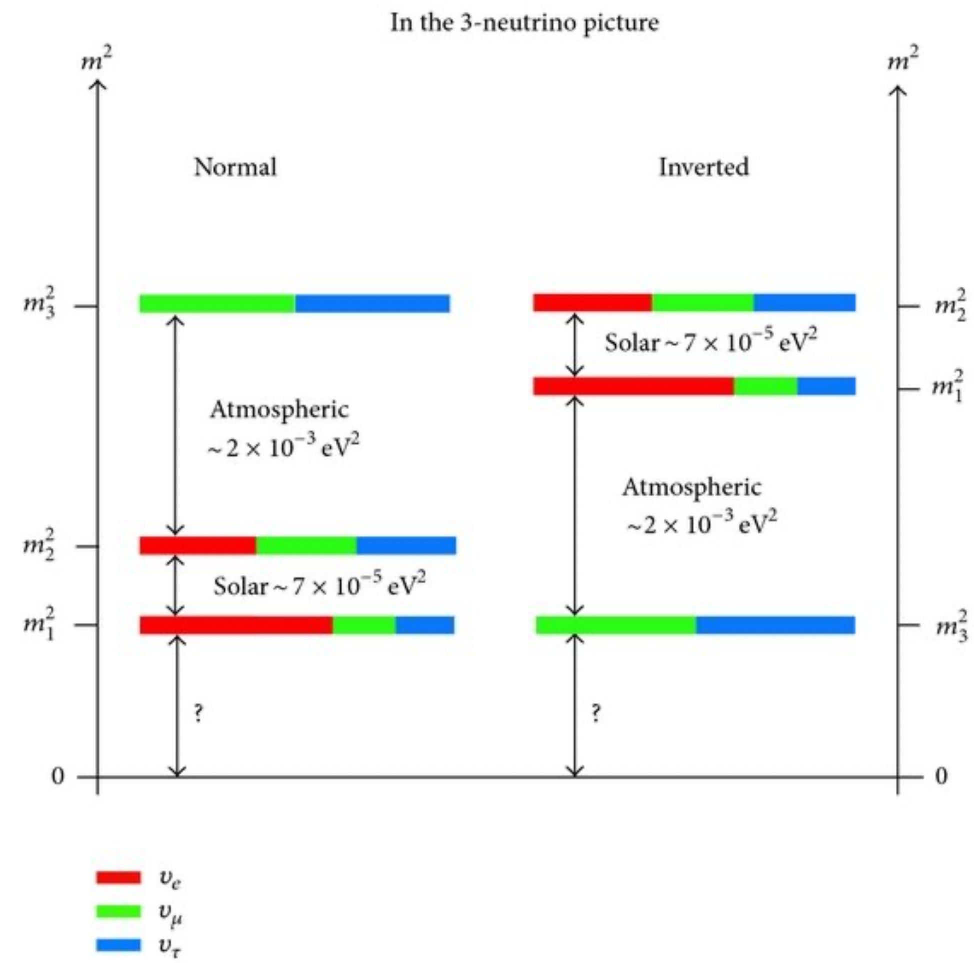
Net rate $|\Gamma_+ - \Gamma_-|$



Net superradiant interaction rate is non-zero and suppressed by $\mathcal{O}(\frac{\lambda_\nu}{R})$

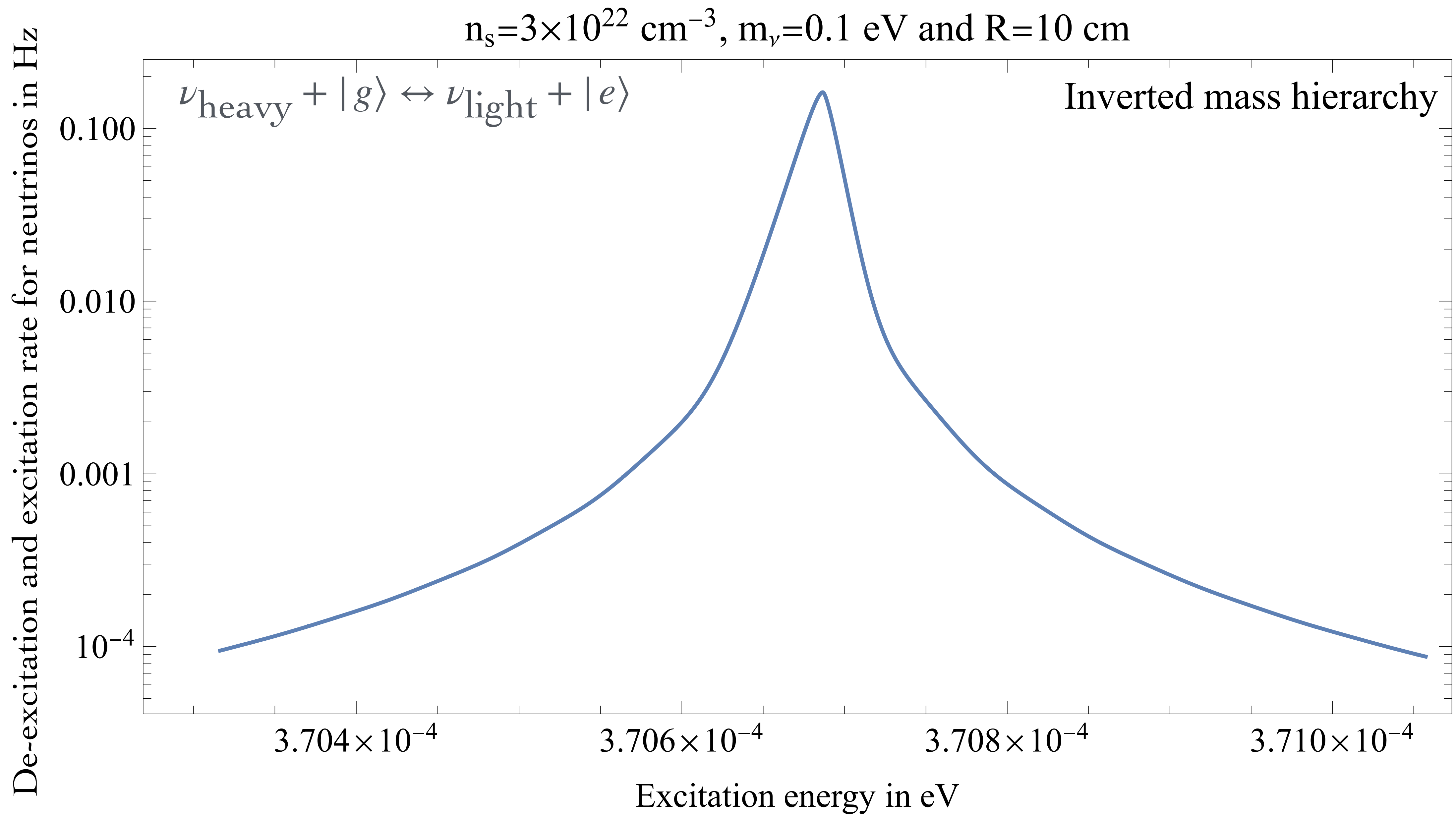
CνB superradiant and doubly inelastic scattering





- Out of equilibrium: All three neutrino mass eigenstates equally populated
- Mass range of heaviest neutrino: 0.05-0.1 eV
- Best case scenario: $\nu_2 \leftrightarrow \nu_1$ processes in the inverted hierarchy case

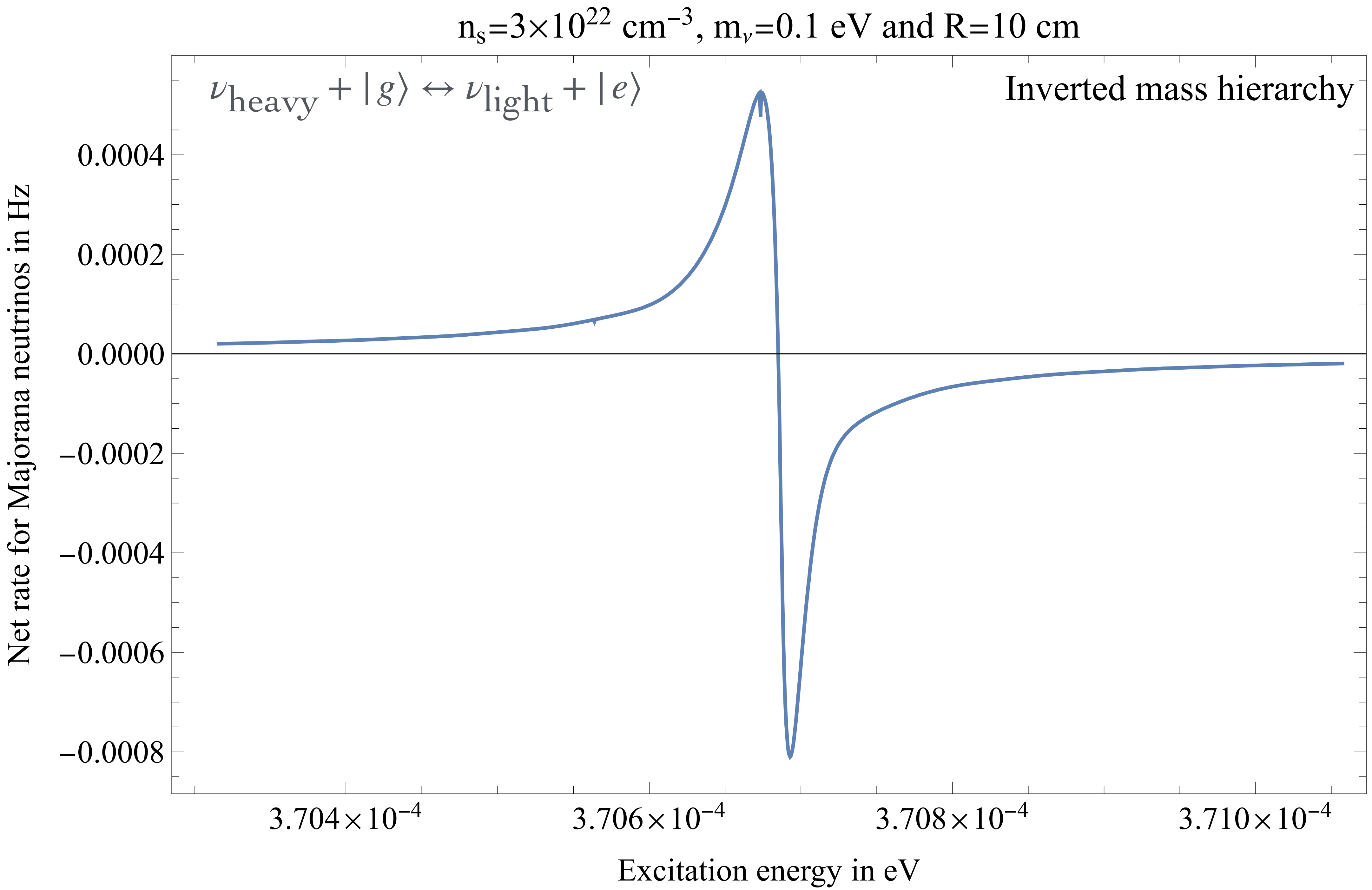
CνB superradiant scattering - total rate



Optimal excitation energy $\omega_0 = m_{\text{heavy}} - m_{\text{light}}$

Live width $\delta\omega_0 \sim \frac{v_\nu}{R}$

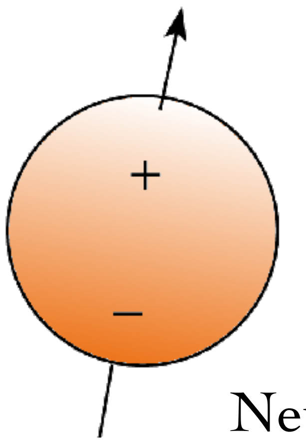
CνB superradiant scattering - Net Rate



Cosmological out-of-equilibrium population of heavy neutrinos allows for positive energy exchange

Why is the Electric Dipole Moment of the Neutron Small?

The Strong CP Problem and the QCD axion



Neutron
EDM

$$\frac{g_s^2}{32\pi^2}\theta_s \vec{E}_s \cdot \vec{B}_s$$

EDM ~ e fm θ_s

Experimental bound: $\theta_s < 10^{-10}$

Solution:
$\theta_s \sim a(x,t)$ is a dynamical field, an axion

Axion mass from QCD:

$$\mu_a \sim 6 \times 10^{-11}\ \text{eV}\ \frac{10^{17}\ \text{GeV}}{f_a} \sim (3\ \text{km})^{-1}\ \frac{10^{17}\ \text{GeV}}{\mathrm{f_a}}$$

$\mathrm{f_a}$: axion decay constant

A Plenitude of Particles from String Theory

AA, Dimopoulos, Dubovsky, Kaloper, March-Russell (2009)

AA, Craig, Dimopoulos, Dubovsky, March-Russell (2009)

- Extra dimensions
- Gauge fields
- Topology

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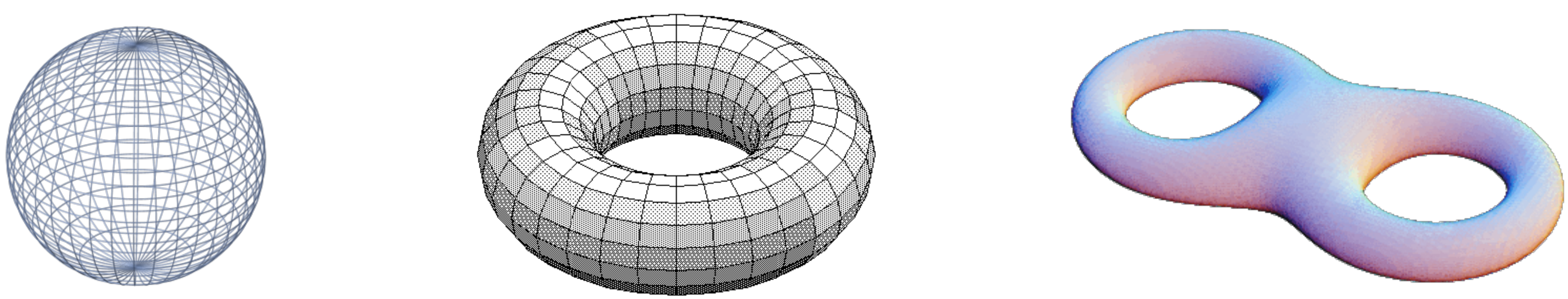

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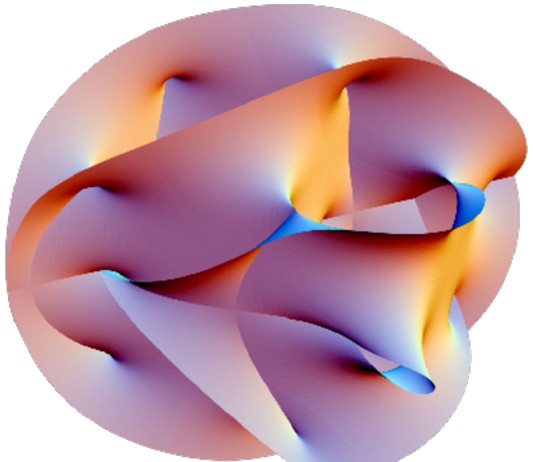

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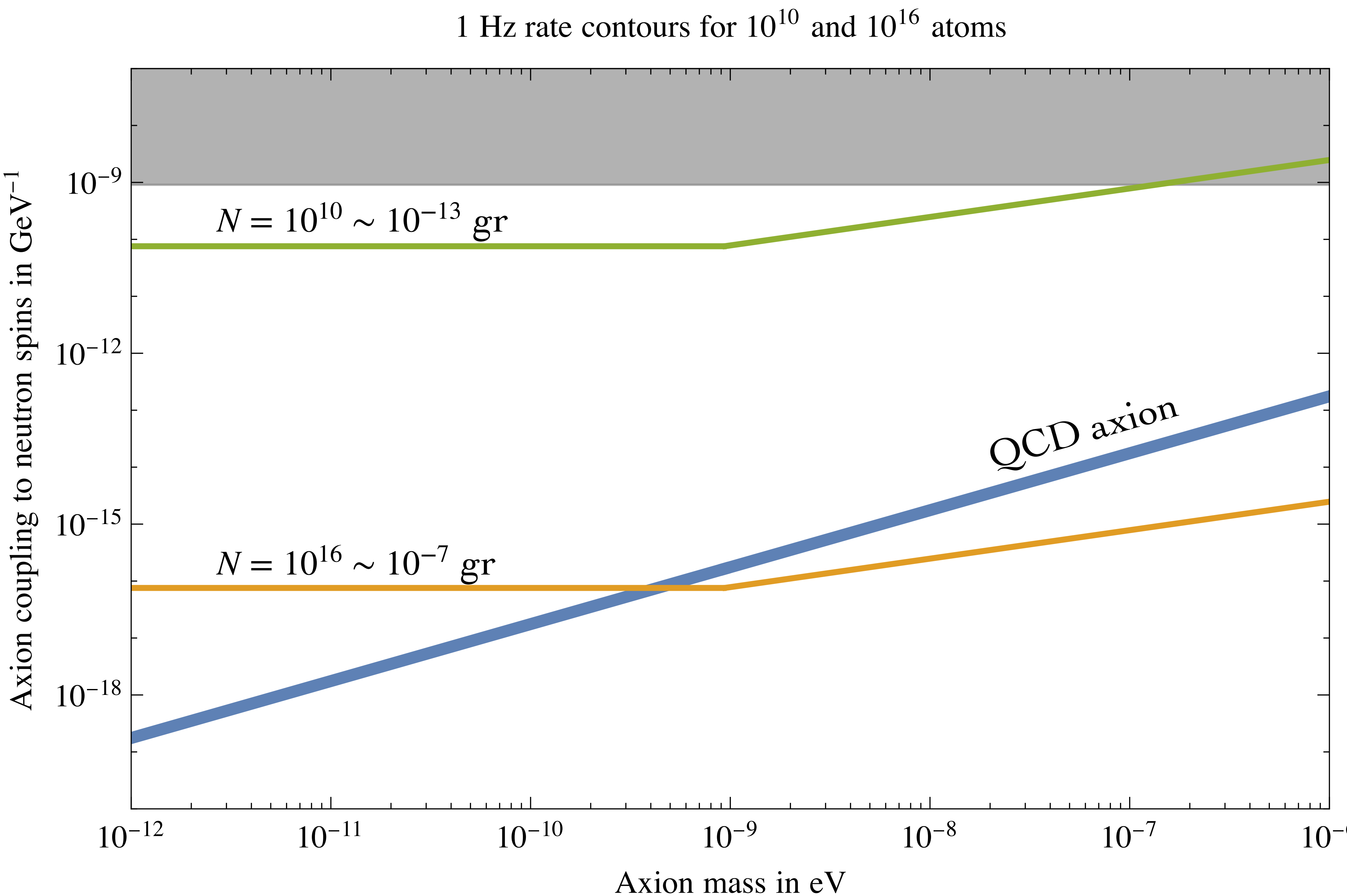
Give rise to a plenitude of massless particles in our Universe

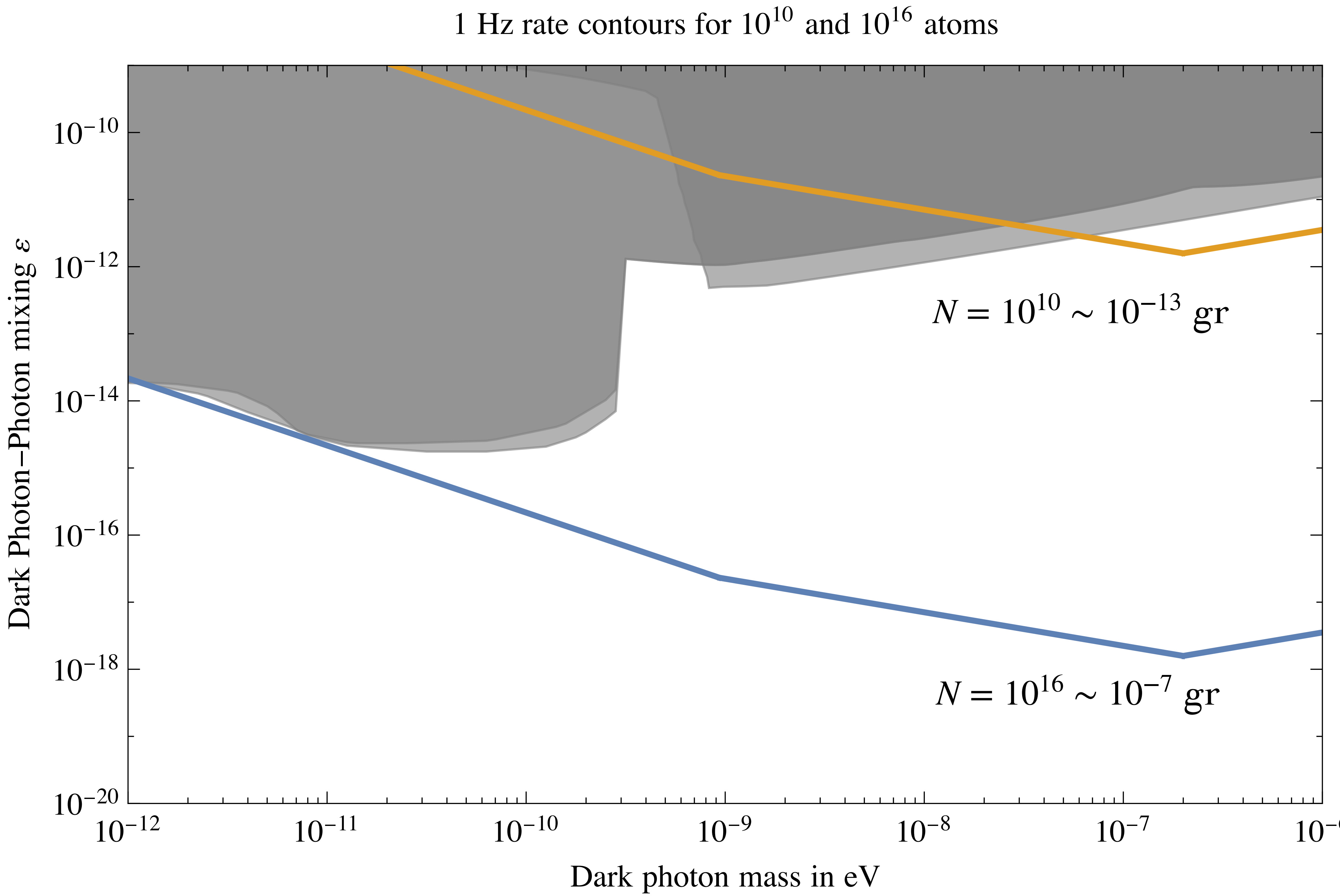
A Plenitude of (Almost*) Massless Particles

- Spin-0 non-trivial gauge field configurations: String Axiverse
- Spin-1 non-trivial gauge field configurations: String Photiverse
- Fields that determine the shape and size of extra dimensions as well as values of fundamental constants: Dilatons, Moduli, Radion

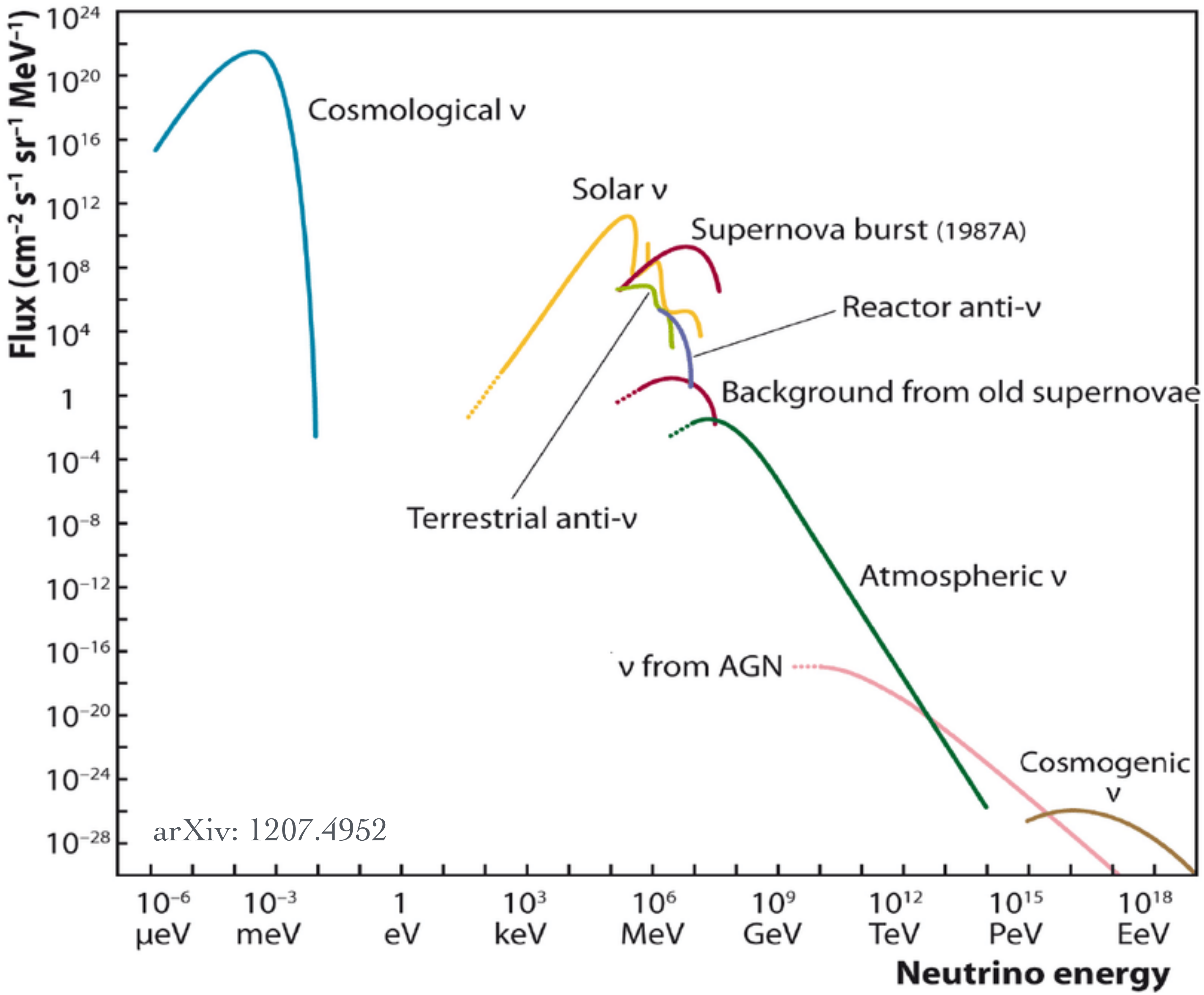
Axion DM superradiance and superabsorption



Dark Photon DM superradiance and superabsorption



Neutrino sources



Solar, Reactor, and Bomb Neutrinos

- Sample rates on a target with $n_s = 3 \times 10^{22} \dfrac{\text{spins}}{cm^3}$

  - Neutrinos with flux of $10^{11}\ cm^{-2}s^{-1}$

    $$\Gamma_{\text{solar}} = \frac{1}{2.5 \text{ hours}} \left(\frac{R}{10 \text{ cm}}\right)^4$$

  - 1 GW reactor produces roughly $10^{21}\ \dfrac{\bar{\nu}}{s}$

    $$\Gamma_{\text{reactor}} = \frac{1}{3 \text{ hours}} \left(\frac{R}{10 \text{ cm}}\right)^4 \left(\frac{100\ m}{d}\right)^2$$

  - $\mathcal{N}_{\text{Mton bomb}} = \mathcal{O}(1) \left(\dfrac{R}{10 \text{ cm}}\right)^4 \left(\dfrac{10\ km}{d}\right)^2$

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Evolution of the quantum state due to cosmic noise

Ex. the cosmic neutrino background density matrix

$$\rho_\nu = \prod_{\{\mathbf{k},s\}} \left[(1 - \langle n \rangle_{\mathbf{k},s}) \left|0\right\rangle_{\mathbf{k},s} \left\langle 0\right|_{\mathbf{k},s} + \langle n \rangle_{\mathbf{k},s} \left|1\right\rangle_{\mathbf{k},s} \left\langle 1\right|_{\mathbf{k},s} \right]$$

Integrating out the cosmic noise parameters:

$$\dot{\rho}_S \simeq \boxed{-i\delta\omega_S[J_z, \rho_S]} + \frac{\gamma_-}{2}\mathcal{L}_{J_-}[\rho_S(t)] + \frac{\gamma_+}{2}\mathcal{L}_{J_+}[\rho_S(t)] + \frac{\gamma_z}{2}\mathcal{L}_{J_z}[\rho_S(t)]$$

constant energy shift

Evolution of the quantum state due to cosmic noise

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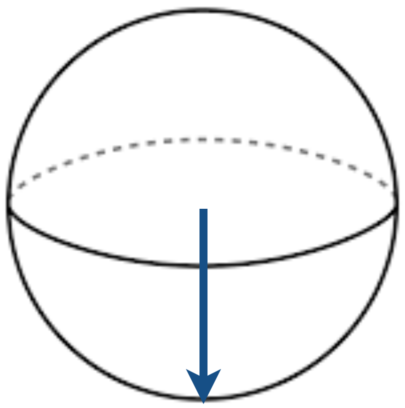
Cosmic noise

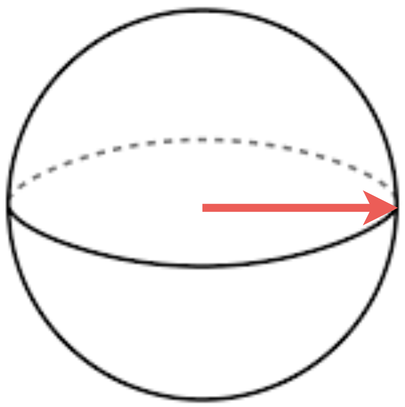
This is analogous to the Lindblad formalism for spins/atoms in a photon bath

How does one measure N^2 effects?

Net energy transfer, i.e $\langle J_z \rangle$



vs



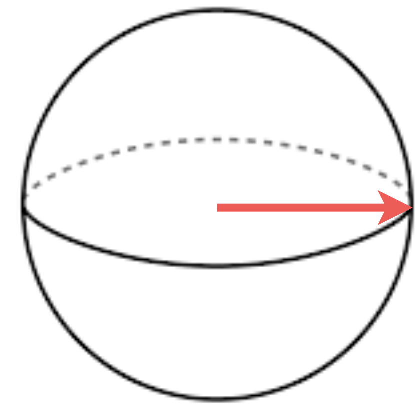
- Starting from $\prod |\downarrow\rangle$
  - $\langle J_z \rangle = -\dfrac{N}{2} + N\gamma_+ t$
  - $\delta J_z \approx \sqrt{N\gamma_+ t}$
  - $SNR \approx \sqrt{N\gamma_+ t}$

- Starting from $\prod |\rightarrow\rangle$
  - $\langle J_z \rangle = \dfrac{N^2}{4}(\gamma_+ - \gamma_-)t$
  - $\delta J_z \approx \sqrt{N}/2$
  - $SNR \approx N^{3/2}/2\,|\gamma_+ - \gamma_-|\,t$

Product state easily prepared and
may offer metrological advantage when the net rate is non-zero

Measuring net energy transfer for the CνB

vs

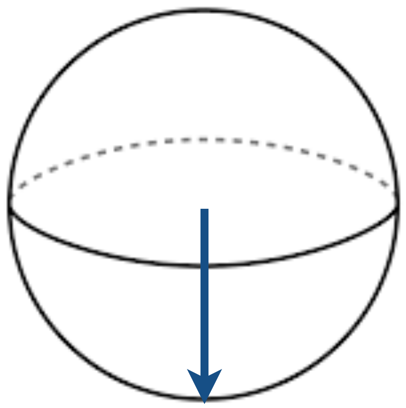


- The KATRIN experiment measures the end point of tritium decay to determine the electron neutrino mass
- Can look for the CνB being absorbed by the tritium
- Sensitivity of KATRIN comparable to a much smaller spin sample prepared in the product state*

$$C_{\text{boost}} \sim 2 \times 10^{11} \left(\frac{10\ \text{cm}}{R}\right)^{3/2} \left(\frac{3 \times 10^{22}\ \text{cm}^3}{n_s}\right)^{3/2} \left(\frac{1000\ \text{sec}}{t}\right) \left(\frac{10^3}{N_{\text{shots}}}\right)^{1/2}$$

How does one measure N^2 effects?

Diffusion on the bloch sphere, i.e $\langle J_z^2 \rangle$



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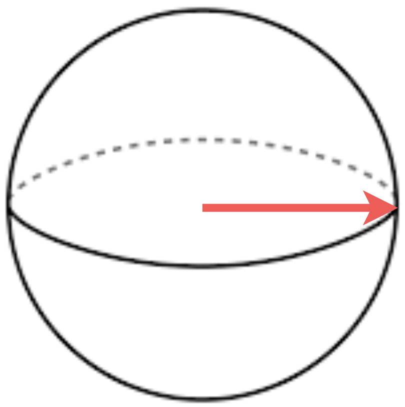


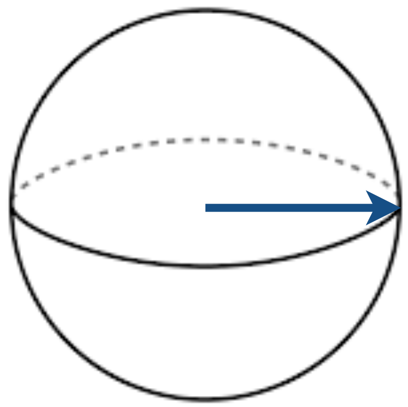
- Starting from $\prod |\downarrow\rangle$
- $\langle J_z^2 \rangle = \frac{N^2}{4} - (N^2 - N)\gamma_+ t$
- $\delta J_z^2 = \sqrt{N^3 \gamma_+ t}$
- $SNR \approx \sqrt{N\gamma_+ t}$

- Starting from $\prod |\rightarrow\rangle$
- $\langle J_z^2 \rangle = \frac{N}{4} + \frac{N^2}{4}(\gamma_+ + \gamma_-)t$
- $\delta J_z^2 \approx \frac{N}{2\sqrt{2}}$
- $SNR \approx \sqrt{2}N|\gamma_+ + \gamma_-|t$

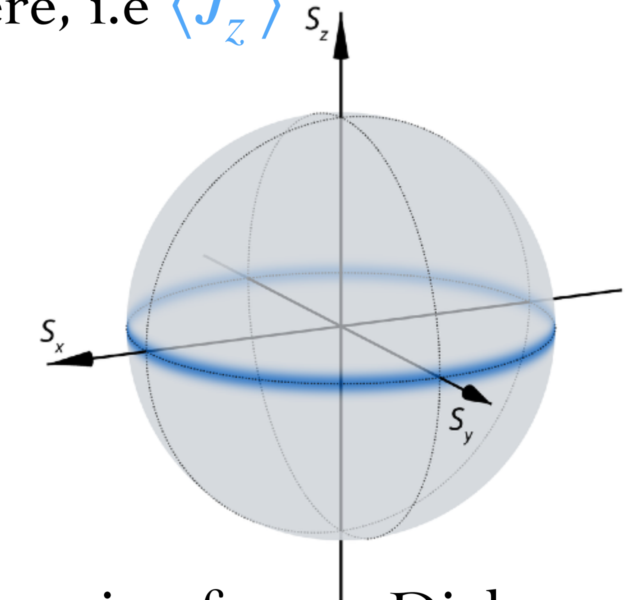
Product state sensitive to the sum of the rates when measuring $\langle J_z^2 \rangle$

How does one measure N^2 effects?

Diffusion on the bloch sphere, i.e $\langle J_z^2 \rangle$



vs



- Starting from $\prod |\rightarrow\rangle$
- $\langle J_z^2 \rangle = \frac{N}{4} + \frac{N^2}{4}(\gamma_+ + \gamma_-)t$
- $\delta J_z^2 \approx \frac{N}{2\sqrt{2}}$
- $SNR \approx \sqrt{2}N(\gamma_+ + \gamma_-)t$

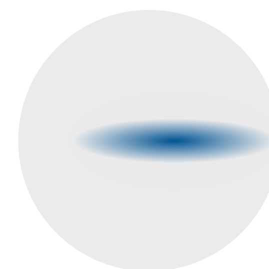
- Starting from a Dicke state
- $\langle J_z^2 \rangle = \frac{N^2}{4}(\gamma_+ + \gamma_-)t$
- $\delta J_z^2 \approx \frac{1}{2}\sqrt{N^2(\gamma_+ + \gamma_-)t}$
- $SNR \approx \sqrt{2}\sqrt{N^2(\gamma_+ + \gamma_-)t}$

Dicke state is hard to produce

Need a special quantum protocol to maximize the potential of these states

Final Thoughts

- Best bet: a squeezed state

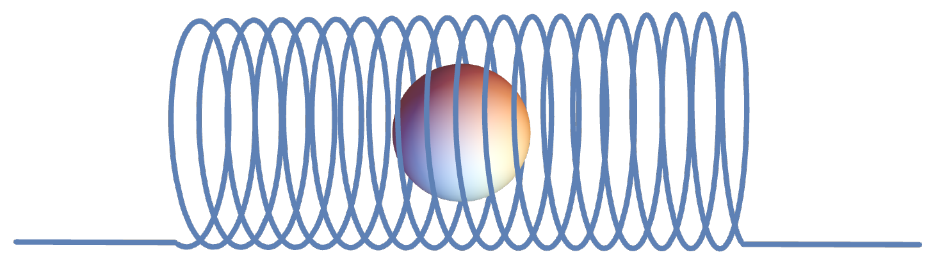


  - Somewhere in between a product state and a Dicke state
  - Preserves much of the good signal-to-noise ratio properties of the Dicke state

- Presently working on concrete protocol and experimental setup with S. Dimopoulos, M. Galanis, O. Hosten

Towards measuring superradiant interactions

Nuclear spin polarized sphere coupled to an LC circuit



Equivalent to the Tavis-Cummings Hamiltonian:

$$H = \omega_{LC} a^\dagger a + \omega_0 J_z + g a J_+ + g^* a^\dagger J_-$$

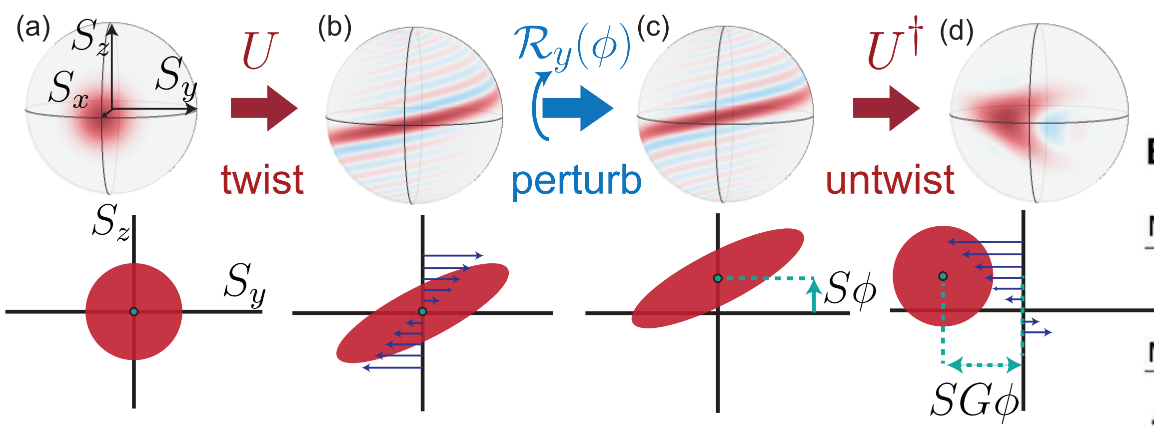
$g = \mu_f \sqrt{\frac{\omega_{LC}}{V_{solenoid}}}$ is the coupling between the spins and the circuit

Can we apply protocols that have worked at higher frequencies and different atomic systems to nuclear spins?

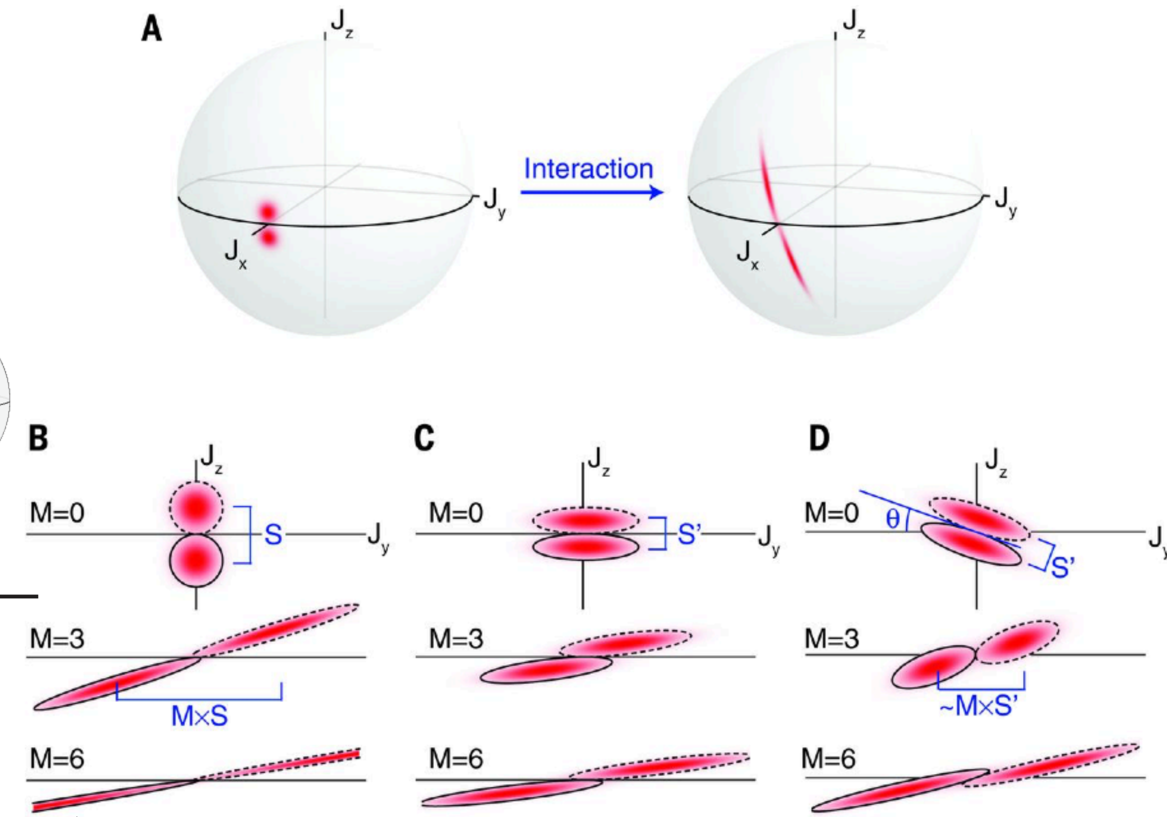
Some things are the same with cavities, some are not…

Towards measuring superradiant interactions

Phase magnification protocols



E. Davis, G. Bentsen, M. Schleier-Smith (2015)



O. Hosten, M. Kasevich (2016)

Let the state squeeze, unsqueeze, magnify
and measure when uncertainty is large

Conclusions

- Superradiant interactions can significantly boost interaction rates of cosmic relics like the CνB ($0.1\ Hz$ vs $10^{-22}\ Hz$)

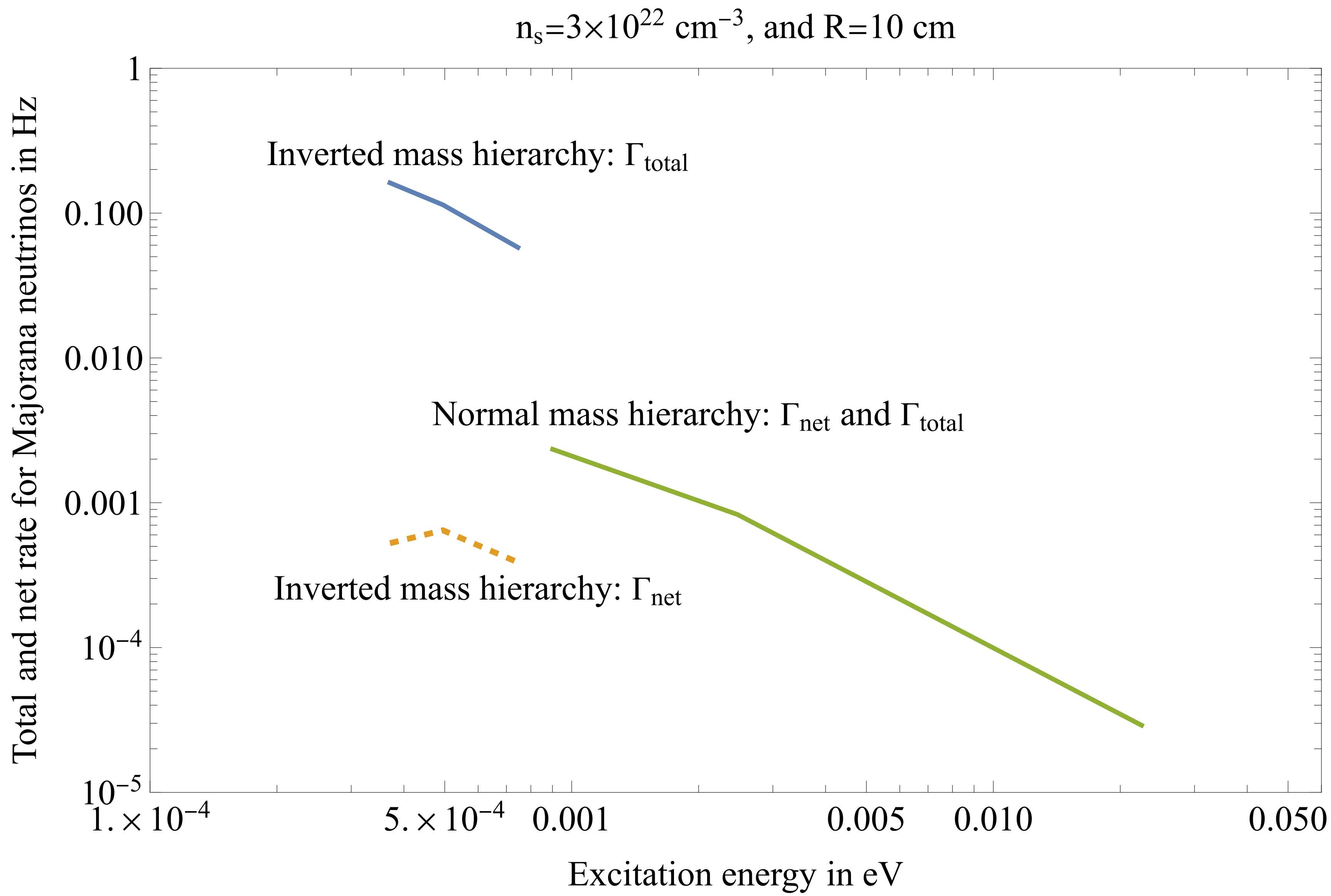
- Axion searches could be a stepping stone

- There are observables that are depend on excitation and de-excitation rates, not just energy transfer

- This is just the beginning

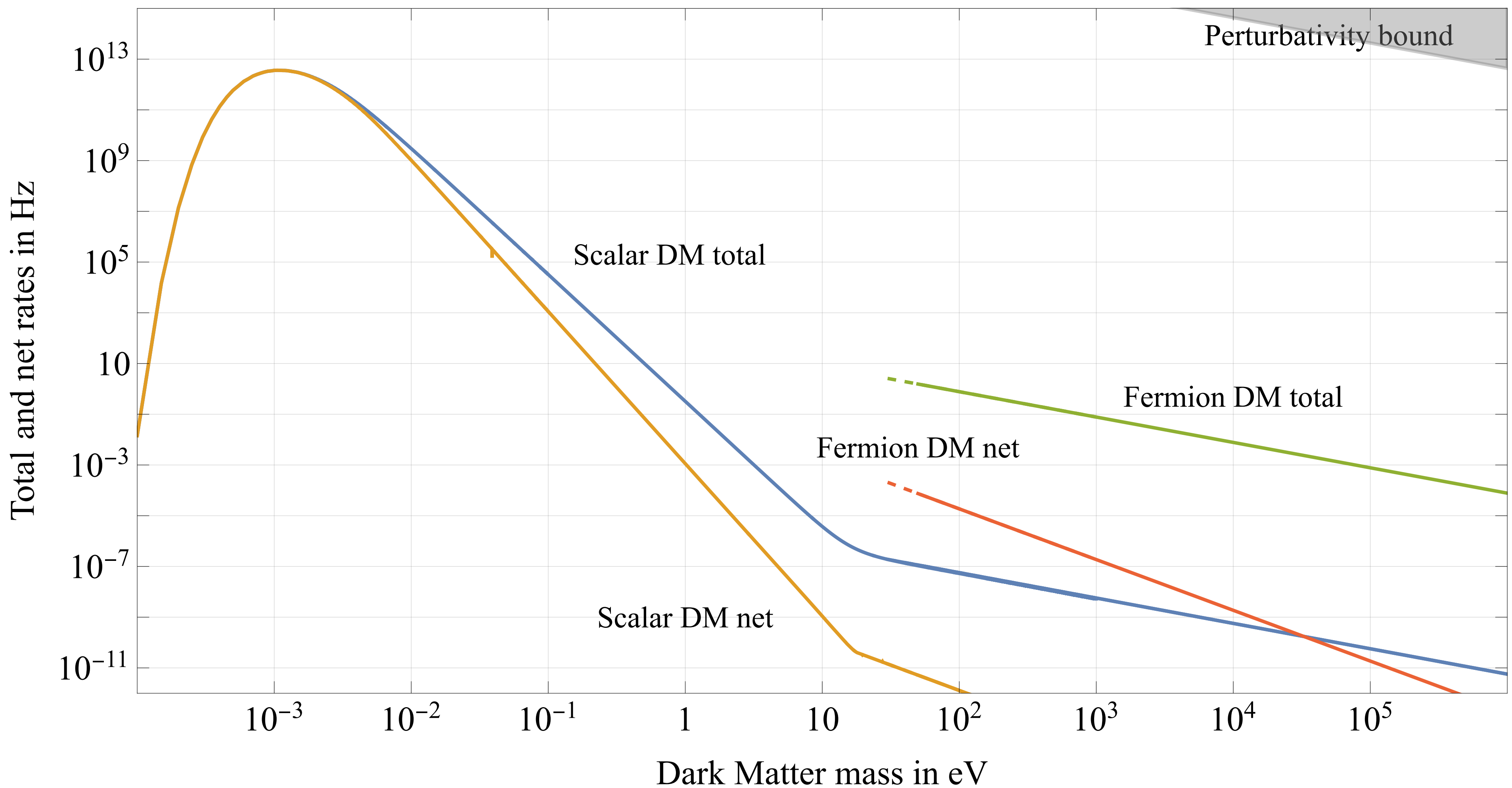
Back-up slides

Summary of neutrino interaction rates for doubly inelastic processes



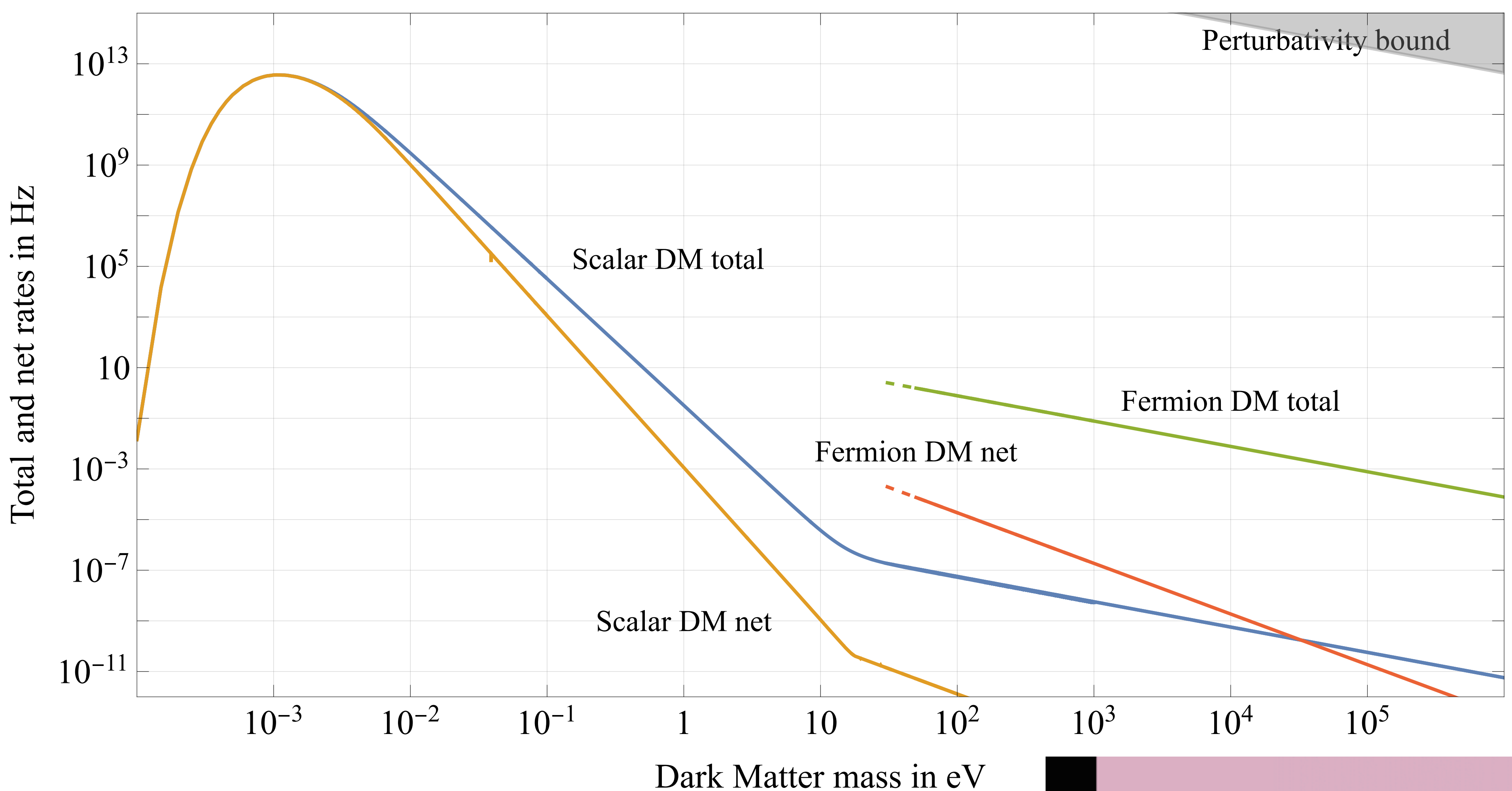
DM scattering

Inelastic DM scattering from R=10 cm sphere with $n_s=3\times10^{22}$ cm^{-3}



DM scattering

Inelastic DM scattering from R=10 cm sphere with n_s=3×10^{22} cm^{-3}



Effects of spin-spin relaxation

Local dephasing effects due to spin-spin dipole interactions

Dephasing timescale, T_2, can vary from *ms* to 1000 *s* in solids and liquids, respectively

$$\dot{\rho} \supset \frac{1}{2T_2} \sum_{\alpha} \left(\sigma_z^{(\alpha)} \rho(t) \sigma_z^{(\alpha)} - \rho(t) \right)$$

Can be shown that there is no cooperative enhancement for T_2

Dephasing affects the evolution of the system after T_2