

Aspects of gauge-strings duality

Carlos Nunez

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Outline

- 1 This is a talk about AdS/CFT. Using holography, the aim is to discuss UV conformal theories, leading to confinement, then leading to screening.
- 2 The plan is to deal with infinite families of $\mathcal{N} = 2$ SCFTs (eight Poincare SUSYS) in two, three, **four**, five and six dimensions. Then, break conformality leading to gapped systems preserving less SUSY. Focus on one example (in 4d) and trying to get Physics of it.
- 3 There are many technical aspects to this. I will try to focus on the ideas, showing few technical details.
- 4 The talk is based on different works in the last year with: **Dimitris Chatzis, Ali Fatemiabhari, Federico Castellani, Mauro Gilierti, Paul Merrikin, Ricardo Stuardo, Marcelo Oyarzo, Peter Weck, Niall Macpherson, Leonardo Santilli and Kostya Zarembo.**

In 1997, Maldacena conjectured the equivalence between CFTs ($\mathcal{N} = 4$ SYM) and strings on AdS-spaces ($\text{AdS}_5 \times S^5$).

Various developments followed (too many to be listed here!)

- Calculate correlation functions.
Witten, Bianchi-Freedman-Skenderis, Papadimitriou-Skenderis,....
- How to extend the conjectured duality to non-conformal QFTs, with varying amounts of SUSY.
Witten, Itzhaki-Maldacena-Sonnenschein-Yankielowicz, Boonstra-Skenderis-Townsend,....
- How to apply this conjecture to QFTs with immediate phenomenological relevance (QCD or systems in Condensed Matter), without so much emphasis on the stringy embedding. Karch-Katz-Son-Stephanov, Gürsoy-Kiritsis-Nitti, Hartnoll-Herzog-Horowitz, Gauntlett-Donos, etc.

In this talk, I focus on the second item, to learn about QFTs with different amounts of SUSY, in different dimensions, with various scales, etc.

Plan of this talk

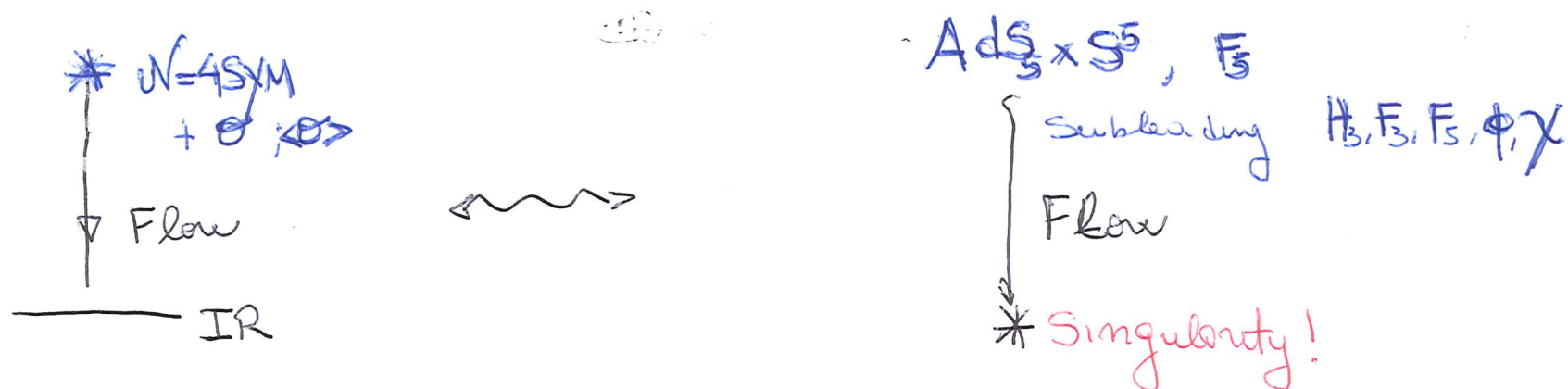
- Description of the Physics problem. Duals to gapped Yang-Mills or QCD-like QFTs. Un-welcomed features in models available: singularities in supergravity or stringy (non-field theoretic) behaviours of the dual description.
- **How to cure these un-welcomed features**
- One example, among many recently available. UV-CFT₄₊ deformation → gapped IR QFT₃.
- **Some observables: VEVs, number of degrees of freedom, screening of Wilson loops.**

To set the problem of Physics, let me start with some old things

The early 2000's saw the construction of holographic duals to Yang-Mills, QCD-like and its SUSY cousins.

- Starting from $\mathcal{N} = 4$ SYM and relevant-deforming it. The issue was that the IR ($r \rightarrow$ region of the geometry) was singular, invalidating some interesting calculations.

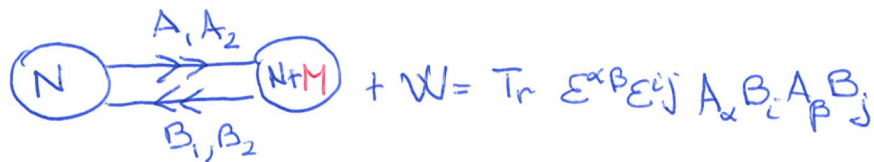
Girardello, Petrini, Porrati, Zaffaroni ($\times 3$, 1998, 1999), Polchinski-Strassler (2000), Brandhuber-Sfetsos (2000), Petrini, Samtleben, Schmidt, Skenderis (2018), Bobev, Gautason, Niehoff, van Muiden (2018).



- Consider a two node conformal quiver (strongly coupled), quasi-marginally deformed.

A SUSY two gauge nodes quiver and the holographic realisation of the dynamics towards the IR. A very powerful matching between $\mathcal{N} = 1$ QFT and Holography

Klebanov+ [Witten, Gubser, Nekrasov, Tseytlin, Strassler, Herzog](1999-2004), Butti, Graña, Minasian, Petrini, Zaffaroni (2004).



$$N = k M$$

flow



IR



*goldstone

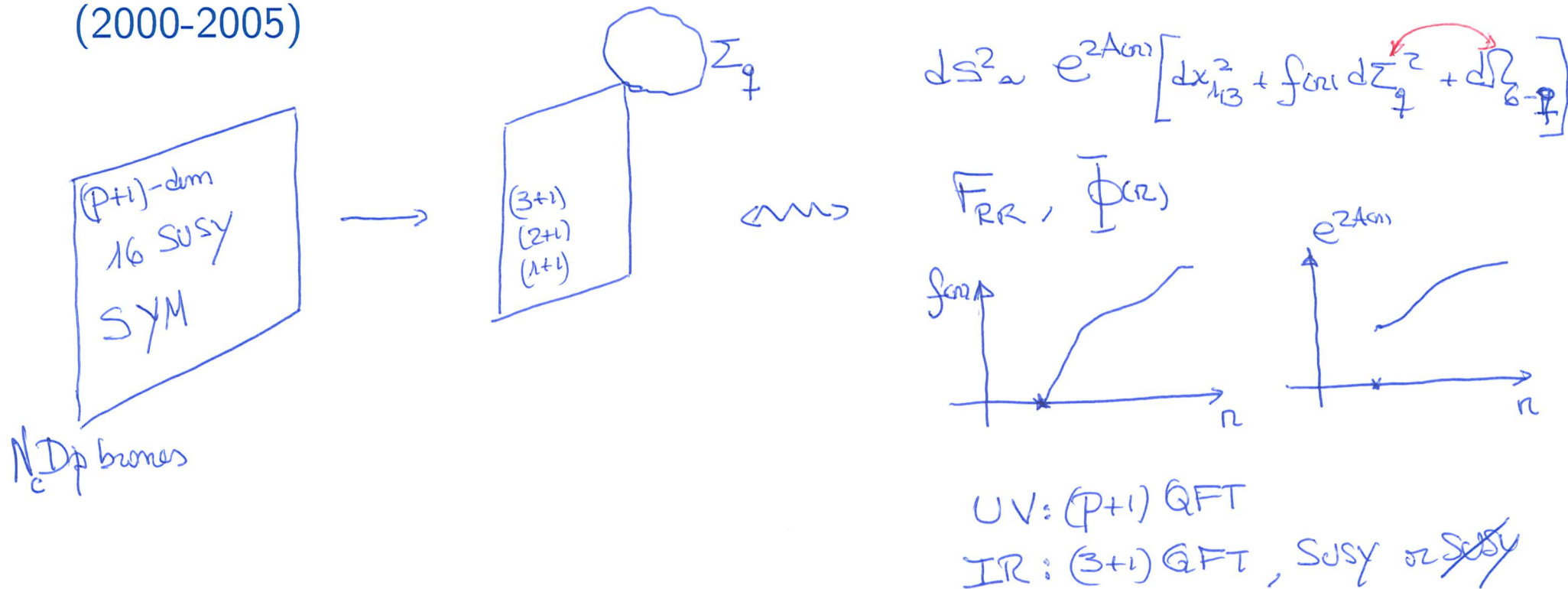
$$ds^2 \sim h^{-\frac{1}{2}}(r) dx_{1,3}^2 + h^{\frac{1}{2}}(r) \left\{ e^{2k} dr^2 + e^{2h} d\Omega_2^2(\theta, \phi) + \frac{e^{2g}}{4} \left[(\omega_1 - a d\theta)^2 + (\omega_2 + a \sin\theta d\phi)^2 \right] + \frac{e^{2k}}{4} (\omega_3 + a \cos\theta d\phi)^2 \right\}$$

$$F_3, F_5, H_3, \phi$$

$$h(r) \sim \begin{cases} h_0 & r \rightarrow 0 \quad \text{smooth!} \\ \frac{\log r}{r^4} & r \rightarrow \infty \quad \text{AdS}_3 \end{cases}$$

- Study Dp branes ($p = 4, 5, 6$) and compactified it to obtain duals to theories in $(3 + 1)$, $(2 + 1)$, $(1 + 1)$ with different SUSY preserved.

Witten (1998), Maldacena-CN (2000), Maldacena-Nastase (2001), Schvellinger, Gursoy, Ramallo, Paredes, Gomis, Russo, Gauntlett, Waldram, Kim, Martelli, Bigazzi, Cotrone, Petrini, Zaffaroni, Bertolini, di Vecchia, Lerda, Aharony, Sonnenschein,.... (2000-2005)



Features of these holographic duals

If the UV was well defined, the IR was singular.

If the IR was smooth, the UV was not (strictly) Asymptotically AdS. In some cases, the UV was non-field theoretical.

The addition of a large amount of matter transforming in the fundamental of the gauge group (quarks), was technically challenging. It had effects on the UV and IR of the geometries.

Aside from this, there were numerous things that were calculated with these holographic duals. A lot of important lessons were learned!

Question: Would it be possible to keep the virtues (like the smooth IR), repair the UV (making it AAdS) and also add (localised) sources?

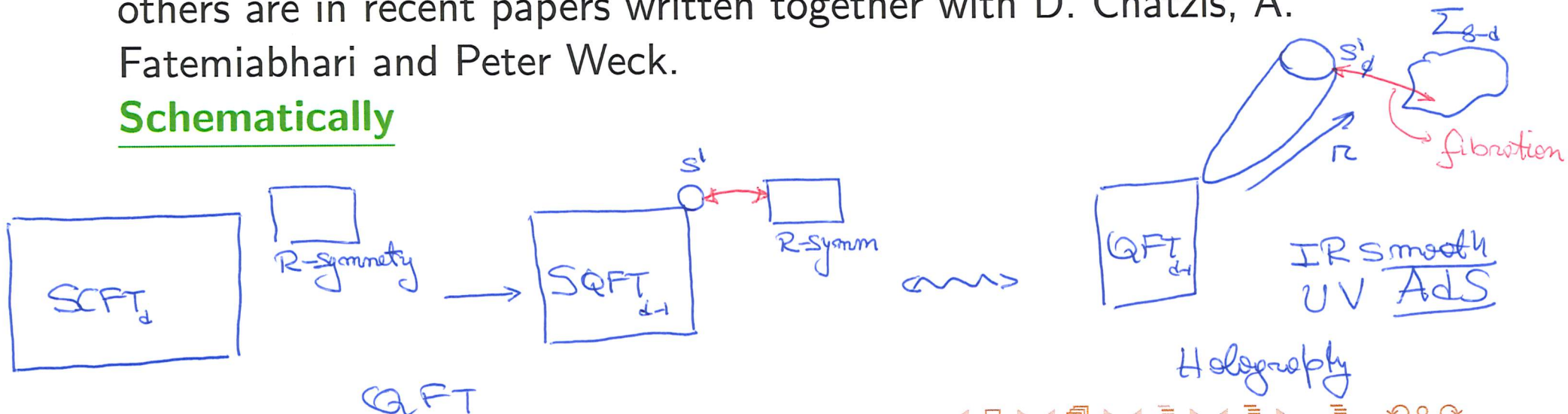
How to fix the issues (keeping the virtues)?

The strategy is the following:

- Consider a SCFT with fields in the adjoint, (bi)fundamental representations. Write the (known!) holographic dual
- Using these SCFTs (an infinite family of them) as UV fixed point, deform by a VEV or relevant operator, such that the IR is smooth.
- Preserve SUSY via a mixing (twisting) between the R-symmetry and the Lorentz group.

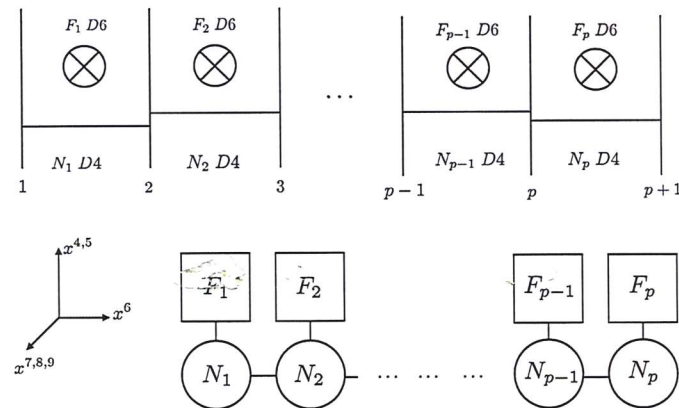
Let me present two examples (families of examples). Various others are in recent papers written together with D. Chatzidis, A. Fatemiabhari and Peter Weck.

Schematically



Consider 4d $\mathcal{N} = 2$ SCFTs.

For its historical and conceptual value, consider these 4d SCFTs. A quiver-like description and a Hanany-Witten set-up.



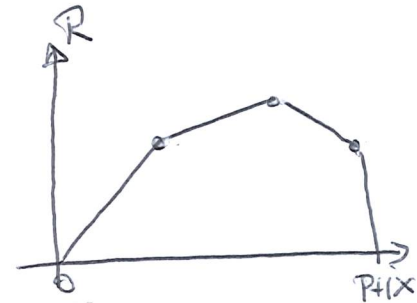
The global symmetries are $SO(2,4) \times SU(2)_R \times U(1)_r$. This will be reflected in the string dual containing $AdS_5 \times S^2 \times S^1$.

The beta function of these theories is $\beta \sim (2N_c - N_f)$. This implies

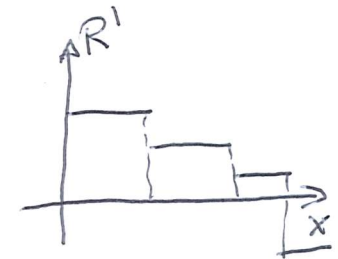
$$2N_1 = N_2 + F_1, \quad 2N_2 = N_1 + N_3 + F_2, \dots, \quad 2N_p = F_p + N_{p-1}.$$

One can define a 'Rank-function' $R(x)$, that is convex polygonal. For the quiver pictured above, we have

$$R(x) = \begin{cases} N_1 x & 0 \leq x \leq 1 \\ N_1 + (N_2 - N_1)(x - 1) & 1 \leq x \leq 2 \\ \dots & \\ N_k + (N_{k+1} - N_k)(x - k) & k \leq x \leq (k + 1) \\ N_P - N_P(x - P) & P \leq x \leq P + 1. \end{cases}$$



$$R'(x) = \begin{cases} N_1 & 0 \leq x \leq 1 \\ (N_2 - N_1) & 1 \leq x \leq 2 \\ \dots & \\ (N_{k+1} - N_k) & k \leq x \leq (k + 1) \\ -N_P & P \leq x \leq P + 1. \end{cases}$$



$$R'' = \underbrace{(2N_1 - N_2)}_{F_1} \delta(x-1) + \underbrace{(2N_2 - N_1 - N_3)}_{F_2} \delta(x-2) + \dots + \underbrace{(2N_P - N_{P-1})}_{F_{P-1}} \delta(x-P).$$

Lin, Lunin and Maldacena wrote (2005) the Type IIA backgrounds ($\alpha' = g_s = 1$). Gaiotto-Maldacena (2010).

$$ds_{10}^2 = 4f_1 ds_{AdS_5}^2 + f_2(d\sigma^2 + d\eta^2) + f_3 d\Omega_2(\chi, \xi) + f_4 d\beta^2.$$
$$B_2 = f_5 d\Omega_2(\chi, \xi), \quad C_1 = f_6 d\beta, \quad A_3 = f_7 d\beta \wedge d\Omega_2, \quad e^{2\phi} = f_8.$$

The functions $f_i(\sigma, \eta)$ can be all written in terms of a function $V(\sigma, \eta)$ and its derivatives, $f_i \sim f_i(V, \partial_\sigma V, \partial_\eta V)$. For example

$$f_1(\sigma, \eta)^2 = \frac{2\dot{V} - \ddot{V}}{V''}, \quad f_2 = 2f_1 \frac{V''}{\dot{V}}, \dots$$

The function $V(\sigma, \eta)$ satisfies a Laplace-like equation with certain given boundary conditions to avoid nasty singularities,

$$\sigma^2 \partial_\sigma^2 V + \sigma \partial_\sigma V + \sigma^2 \partial_\eta^2 V = 0,$$

$$V(\sigma \rightarrow \infty, \eta) \rightarrow 0, \quad V(\sigma, \eta = 0) = V(\sigma, \eta = P + 1) = 0.$$

$$R(\eta) = \sigma \partial_\sigma V(\sigma, \eta)|_{\sigma=0}, \quad R(0) = R(P + 1) = 0.$$

Importantly, the rank function $R(\eta)$ appears as an initial condition for the Laplace problem.

Given $R(\eta)$, one can write the solution for $V(\sigma, \eta)$ as a Fourier series

$$V(\sigma, \eta) = - \sum_{n=1}^{\infty} \frac{c_n}{w_n} K_0(w_n \sigma) \sin(w_n \eta), \quad w_n = \frac{n\pi}{P+1}.$$

$$c_n = \frac{n\pi}{(P+1)^2} \int_{-(P+1)}^{P+1} R(\eta) \sin(w_n \eta) d\eta.$$

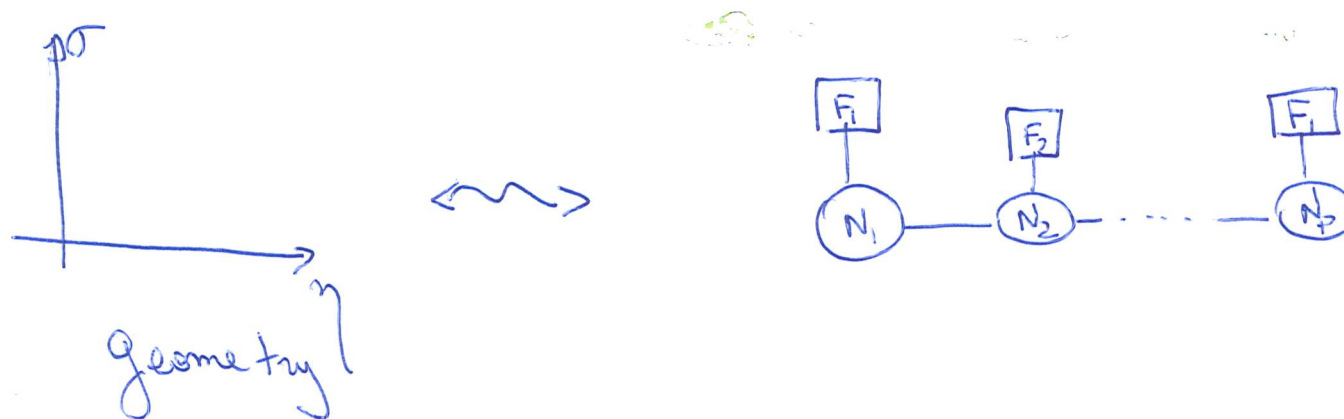
The backgrounds are trustable if the numbers P, N_k are large.

There are numerous checks that these supergravity backgrounds capture aspects of the SCFTs in question

These checks range from the calculation of free energies, central charges (number of degrees of freedom), Wilson loops, VEVs of certain operators, etc.

These calculations in String Theory are very precisely matched by localisation+matrix model computations. This works also for SCFTs in diverse dimensions.

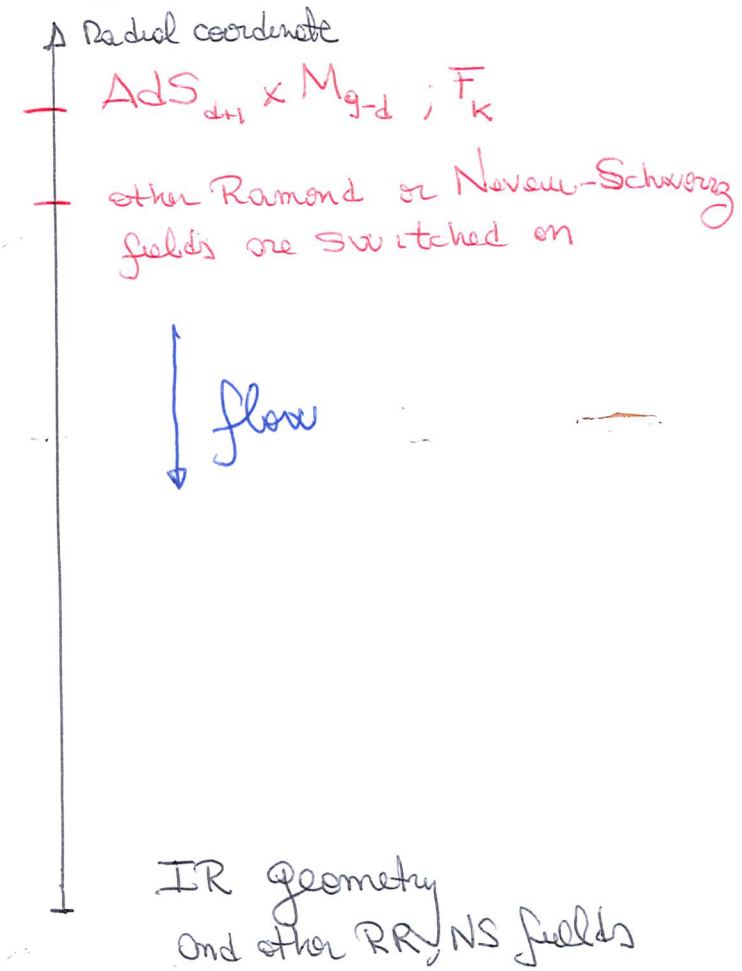
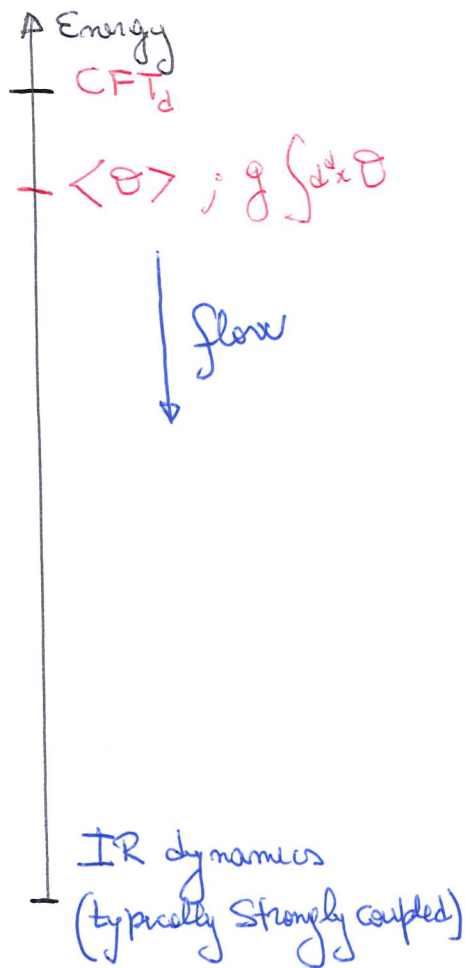
One feature to keep in mind is that there is a direction in supergravity (called η in here) that is associated with the "quiver tail". It encodes the SCFT, via the Rank function.



Let us now depart from the 'conformal world'.

I shall tell you about how to use these systems to learn about RG flows away from the fixed point. Less symmetries and less constrained dynamics are the features.

The main idea can be expressed with a drawing



The idea is to use metrics of the form $\text{AdS}_{d+1} \times M_{9-d}$,

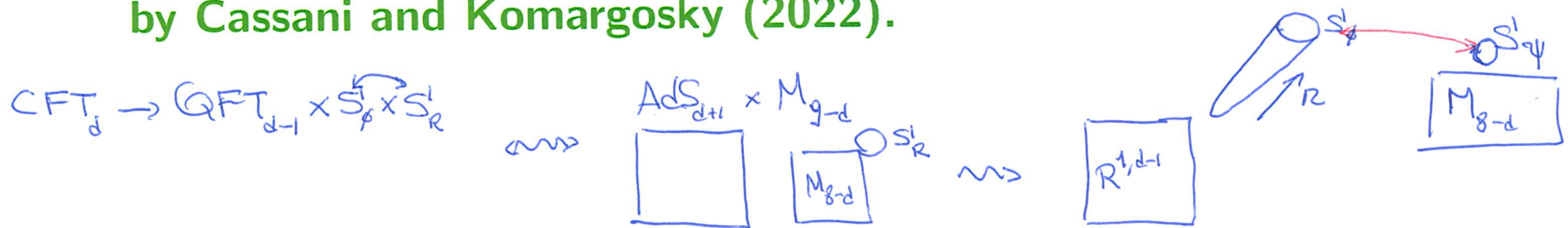
$$ds^2 \sim r^2 (-dt^2 + d\vec{x}_{d-2}^2 + d\phi^2) + \frac{dr^2}{r^2} + M_{8-d} + d\psi^2,$$

Perform a deformation, fibering the $U(1)_R$ with the Lorentz group

$$ds^2 \sim r^2 (-dt^2 + d\vec{x}_{d-2}^2 + f(r)d\phi^2) + \frac{dr^2}{r^2 f(r)} + M_{8-d} + (d\psi + A_\phi d\phi)^2,$$

$$f(r) = 1 - \frac{\mu}{r^d} - \frac{q}{r^{2d-2}}, \quad A_\phi = q \left(\frac{1}{r^2} - \frac{1}{r_+^2} \right) d\phi.$$

This is inspired on work by Anabalón and Ross (2021).
 Studied for $\mathcal{N} = 4$ SYM by Kumar and Stuardo (2024) and
 by Cassani and Komargosky (2022).



What is happening in the QFT?

A UV CFT has an R-symmetry. Pick a $U(1)_R$ and mix it with the circle direction on which we compactify the CFT. The background is smooth.

Thanks to this 'twisting', one can preserve some amount of SUSY. The gravity indicates the transition from a CFT_d to a $QFT_{(d-1)}$ that is gapped (smooth IR).

One can play this trick in various scenarios. For example in the CFTs in $d = 4$, with eight SUSYs discussed before. Also, this works in other dimensions.

I will show the metric (there are accompanying RR and NS fields that I will not display). These are the holographic description of SCFTs deformed by VEVs that lead to a gapped system

I show the resulting metric in the case for which the UV is $AdS_5 \times S^2 \times S^1 \times \Sigma_2$ (d=4 N=2 SCFT). Other examples are available.

AdS₅ × S² × S¹ × Σ₂

These backgrounds represent the effect of performing the above procedure to any of the infinite backgrounds in the Gaiotto-Maldacena family of solutions. The metric reads

$$ds^2 = \tilde{f}_1 \left[4ds_5^2 + \tilde{f}_2 (d\theta^2 + \sin^2 \theta (d\varphi - B_1)^2) + \tilde{f}_4 (d\sigma^2 + d\eta^2) \right. \\ \left. + \tilde{f}_3 (d\chi + B_1)^2 \right], \quad f(r) = 1 - \frac{\mu}{r^4} - \frac{q}{r^6}, \quad B_1 = q \left(\frac{1}{r^2} - \frac{1}{r_+^2} \right) d\phi. \\ ds_5^2 = \frac{r^2}{L^2} (-dt^2 + dx_1^2 + dx_2^2 + f(r) d\phi^2) + \frac{L^2 dr^2}{f(r)r^2}. \\ \Phi, \quad F_2, \quad H_3, \quad F_4.$$

The functions $\tilde{f}_i(\sigma, \eta)$ are those in the Gaiotto-Maldacena system, written in terms of a potential (infinite family).

We are twisted-compactifying a 4d $N = 2$ SCFT linear quiver to three dimensions, preserving 4 SUSY, leading to a gapped QFT in the IR.

What can we calculate with these backgrounds?

Analysing asymptotics, one finds that (in terms the parameters of the solution μ, q)

$$\langle J_1 \rangle = -q, \quad - \langle T_{tt} \rangle = \langle T_{x_i x_i} \rangle = \langle \frac{T_{\phi\phi}}{4} \rangle = \mu.$$

A quantity that measures degrees of freedom

For backgrounds of the form (anisotropic like ours!)

$$ds^2 = -\alpha_0 dt^2 + \alpha_1 dy_1^2 + \alpha_2 dy_2^2 + \dots + \alpha_d dy_d^2 + \Pi_{i=1}^d (\alpha_1 \dots \alpha_d)^{\frac{1}{d}} b(r) dr^2 + g_{ij} (d\theta^i - A_1^i) (d\theta^j - A_1^j), \quad \Phi(r, \vec{\theta}).$$

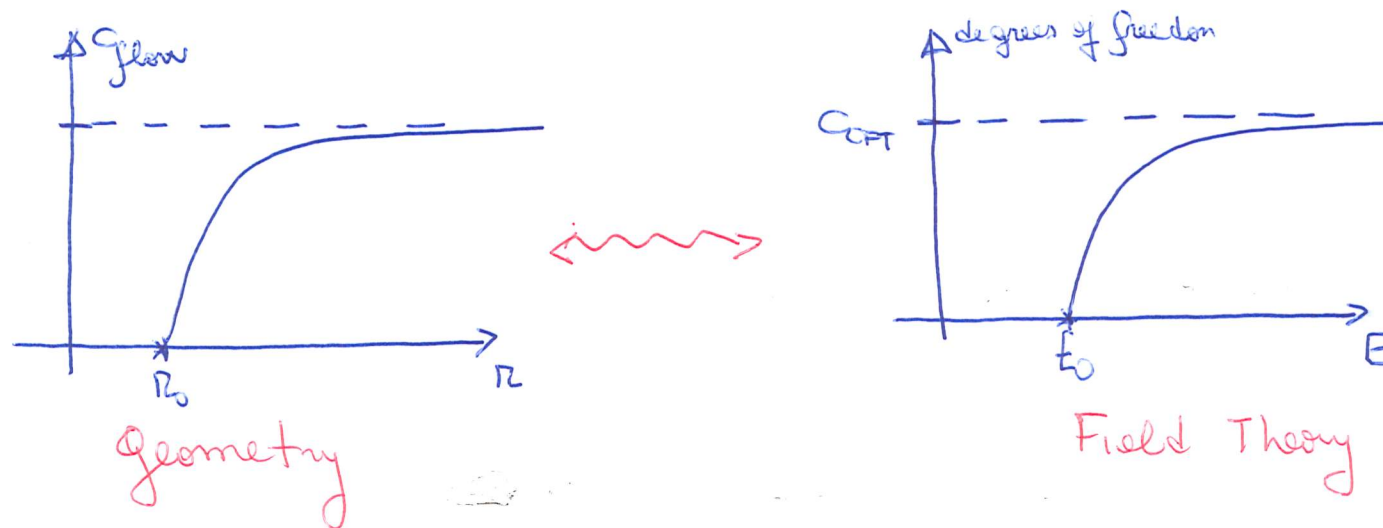
$$V_{\text{int}} = \int_X \sqrt{\det[g_{\text{int}}]} e^{-4\Phi}, \quad \hat{H} = V_{\text{int}}^2.$$

c_{flow} is a generalisation of the Brown-Henneaux expression.

$$c_{\text{BH}} \sim \frac{V_{\text{int}}}{G_{N,10}}, \quad c_{\text{flow}} = \frac{d^d}{G_N} b(r)^{d/2} \frac{\hat{H}^{\frac{2d+1}{d}}}{\hat{H}'^d}.$$

Using the functions in the backgrounds,

$$\frac{C_{\text{flow},(2+1)}}{C_{4\text{dSCFT}}} = \left(\frac{\sqrt{f(r)}}{1 + \frac{rf'(r)}{6}} \right)^3.$$



This quantity represents a flow across dimensions

It captures the low dimensional (d) gapped IR and the high dimensional ($d + 1$) CFT fixed point.

A behaviour that occurs for various other observables

The quantity is the product of two parts, one referring to the UV SCFT and a radial dependent part indicating the flow in energy.

This also occurs for other quantities (Entanglement Entropy, Complexity).

These 'universal' behaviours are actually a consequence of a development by Gauntlett and Varela in 2007! This is very non trivial from the QFT viewpoint.

Let me briefly comment on something that is non-universal. that follows from a question by Kostas Sfetsos.

Screening

We have systems with colour and flavour groups. Could the Wilson loop presented be screened?

The string probe we are using moves in the (r, η) -directions.

$$t = \tau, \quad x = \gamma, \quad r = r(\gamma), \quad \eta = \eta(\gamma).$$

$$S_{NG} = \frac{T}{2\pi} \int d\gamma \sqrt{F^2 + G^2 r'^2 + S^2 \eta'^2}.$$

$$F^2 = g_{tt} g_{xx}, \quad G^2 = g_{tt} g_{rr}, \quad S^2 = g_{tt} g_{\eta\eta}.$$

One try to numerically solve the Euler-Lagrange equations. In work with Fatemiabhari and Giliberti, a minimisation of the action is done in a system similar to the one discussed here.

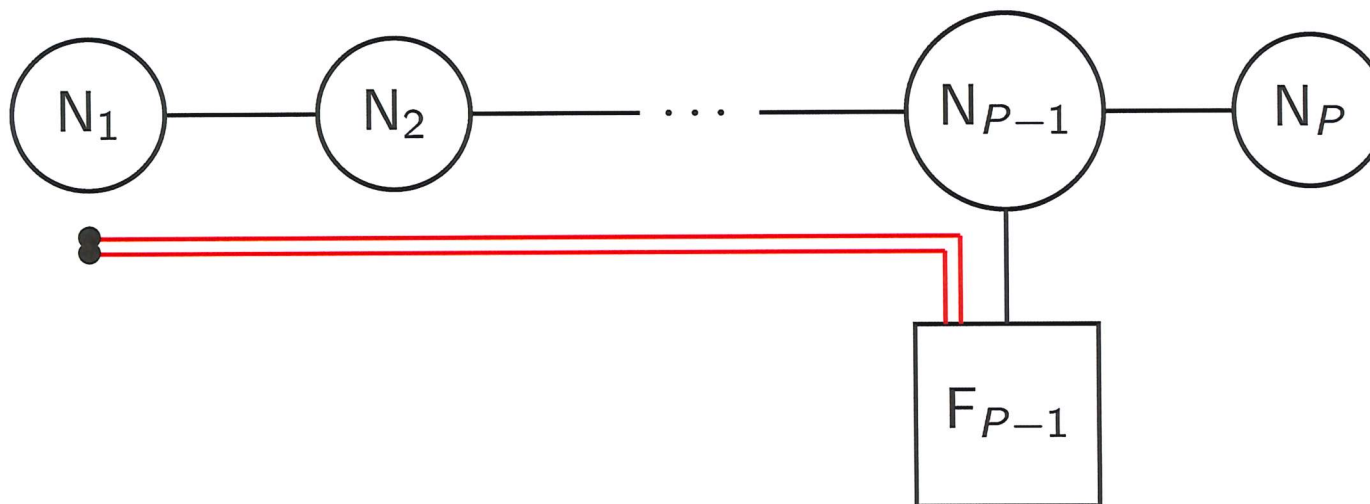
We find that the probe F1 "dives" into the flavour group. There is a form of screening, which is very non-universal.

Let me comment first of the kind of screening we observe. Then, I show some plots. There are still various things to understand and explore!

Screening? In QCD

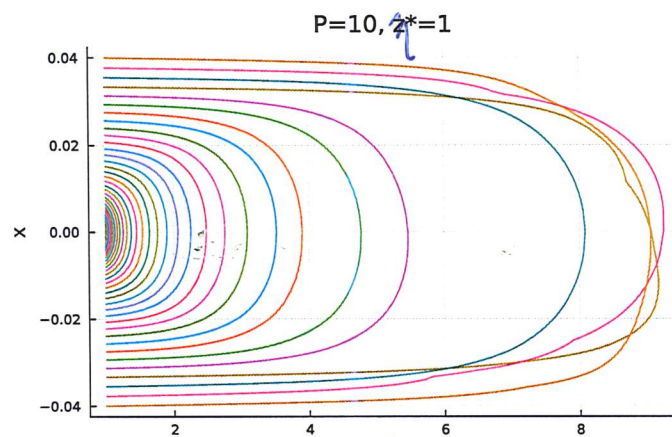
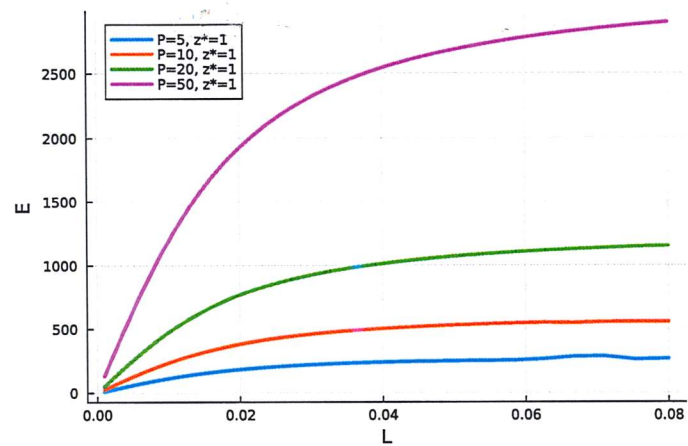
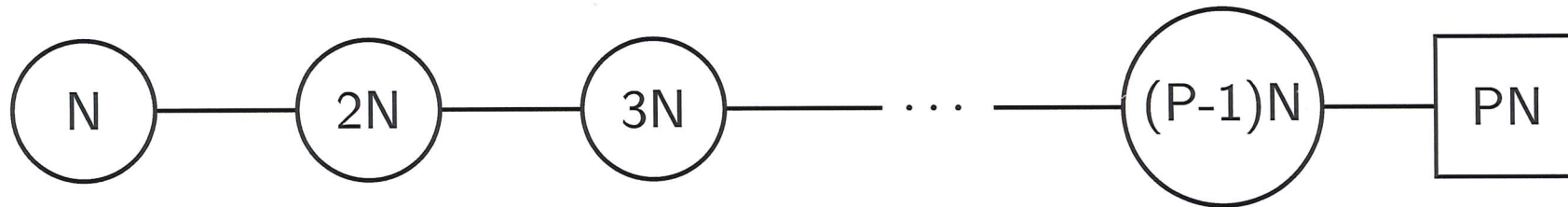


In these systems, it is more like this



Let us see this at work in a couple of examples

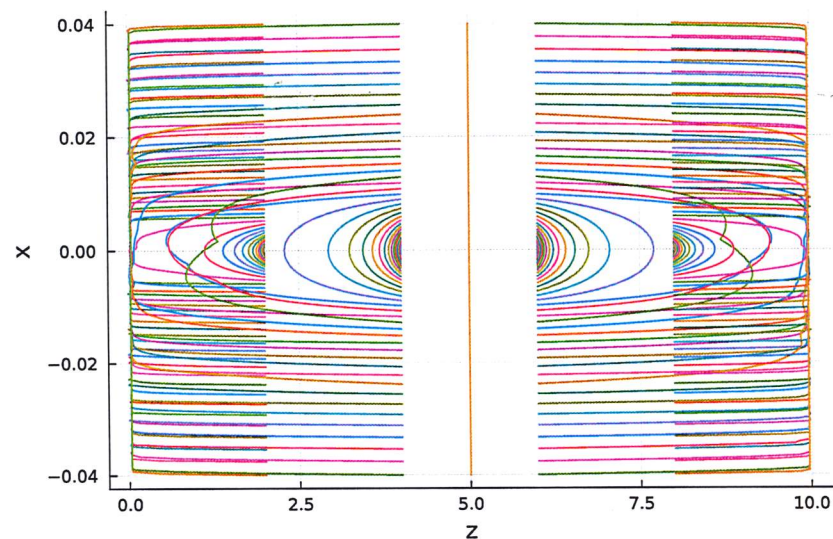
For the quiver



Consider the following interesting example



Running the Wilson loop calculation, placing the string in different colour groups, one find different behaviours



The closer flavour group "attracts" the probe string! The case of the middle gauge group is interesting!

A universal expression?

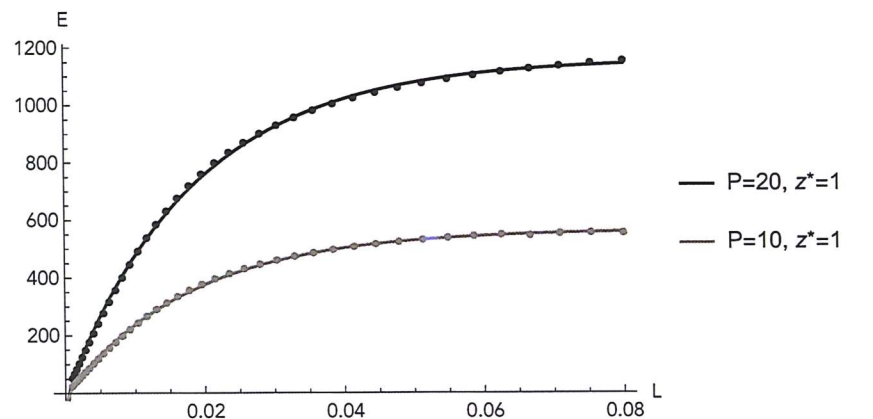
A fit function for the energy of the quark-antiquark pair as the function of the separation L .

$$E(L) = -\frac{a}{L} + \gamma \frac{1 - e^{-bL}}{b}.$$

For small L and for large L behaves as

$$E(L \rightarrow 0) = -\frac{a}{L} + \gamma L, \quad E(L \rightarrow \infty) = \frac{\gamma}{b}.$$

Contains the conformal behaviour at small L and transition towards confining with quark-antiquark string tension given by γ parameter. For large L we capture the screening behaviour as the energy function saturates.



Physics is hidden
in a, b, γ
functions of $\{q, \text{quiver}, P, \text{etc}\}$

Some closing comments and conclusions

- I have presented one example $d = 4$ of a generic formalism to write string duals to SCFTs in $d=1,2,3,4,5,6$ with eight Poincare SUSYs.
- I have shown how to use these SCFTs as UV-fixed point for an RG flow, ending in a gapped field theory. Ending in another fixed point is also possible.
- Various observables calculated in the SCFT are interesting along the flow. Some are universal, some are not. It is interesting to better understand the dynamics induced by the presence of flavour degrees of freedom.
- **I am presently exploring similar solutions, with or without SUSY. Calculating correlations functions and other observables.**