An Inflationary Cosmology from (AdS) Wormholes

Panos Betzios

Work in collaboration with O. Papadoulaki
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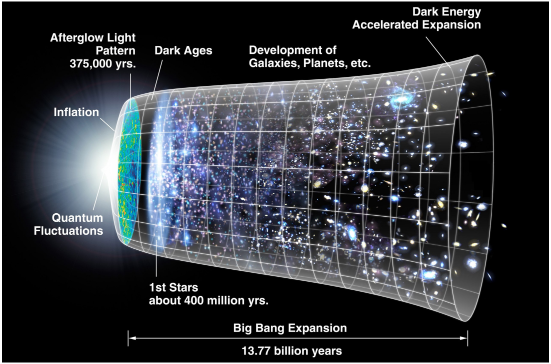
The History of our Universe

- Our Universe is currently expanding
- It is "Hot" ($T \simeq 2.73$ K)
- Extremely uniform at large scales $\delta T/T \sim 10^{-5}$



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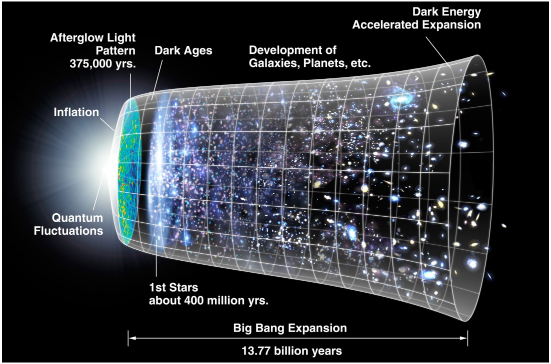
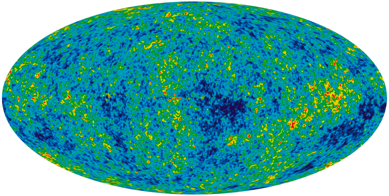
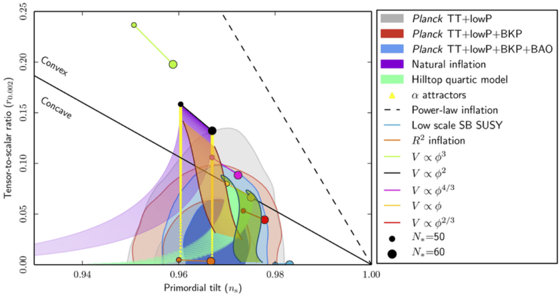


But how did it all start?



Features of the cosmic evolution

- Flatness "problem" - Universe is nearly flat, homogeneous and isotropic
- Horizon "problem" - causally disconnected regions of spacetime very similar
- Monopole "problem" - No exotic relics (ex: monopoles) around
- Production of primordial perturbations that are nearly scale invariant
- Inflation is a theory that can adequately explain these features (+more)



Pertinent Questions

- What gave rise to the initial conditions/state of inflation?
  i.e. Why to start high up in the inflaton potential?
  Understand physics before horizon crossing/exit $a_* = 1/H_*$
- Initial singularity/Planck scale - Our physical laws cease to work
- Do we really need a complete theory of quantum gravity to understand these problems?
- Is there any (approximate) way to compute (estimate) probabilities and features of the early universe Cosmology?

The Wheeler - DeWitt equation and "Quantum Cosmology"

- [Hartle and Hawking] gave one such appealing proposal for computing the "Wavefunction of the Universe"

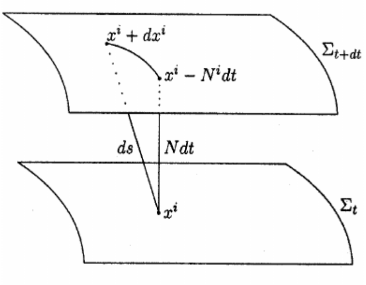
- Based on the so called [Wheeler DeWitt] (WDW) equation

- In this approach one uses the canonical (Hamiltonian) formalism of general relativity and promotes the constraints expressing diffeomorphism invariance to quantum operators annihilating the wavefunction

- Use the `[Arnowitt-Deser-Misner]` decomposition of the metric

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

N is called the "lapse", N^i is the "shift" vector and g_{ij} is the spatial metric on a slice Σ



- Start from the Einstein Hilbert (+ matter) action

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{|g|}\, R^{(4)} \; + \; S^{matter}$$

In ADM parametrization, the canonical Hamiltonian can be written in the form

$$H_c = \int_{\Sigma} d^3x \sqrt{g}\, (NH + N^i H_i)$$

$$H = 2\kappa g^{-1} \left(g_{ik} g_{jl} \pi^{kl} \pi^{ij} - \frac{1}{2} (g_{ij} \pi^{ij})^2 \right) - \frac{1}{2\kappa} R^{(3)} + H^{matter}$$

$$\pi^{ij} = \frac{\delta S}{\delta \dot{g}_{ij}}\,, \qquad H_i = -2 g_{ij} D_k \frac{\pi^{jk}}{\sqrt{g}} + H_i^{matter}$$

where D_i is the g_{ij} covariant derivative and we indicate possible additional matter contributions

Constraints and the Wheeler DeWitt equation

- Diffeomorphism invariance $\Rightarrow$ The physical states/configurations are independent of the choice of lapse and shift (N, N^i)
- This leads to constraints `[Dirac]` $\Rightarrow H,\ H_i = 0$
- Let us also consider as matter a scalar field ϕ (that will play the role of the inflaton)
- At the quantum level one has to impose the constraints, acting as operators on the wavefunctions

$$\widehat{H}_{WDW}(\pi_{ij}, g_{ij}; \pi_\phi, \phi)\,\Psi_\Sigma(g_{ij}, \phi) = 0\,, \quad \widehat{H}_i(\pi_{ij}, g_{ij}; \pi_\phi, \phi)\,\Psi_\Sigma(g_{ij}, \phi) = 0$$

$$\hat{\pi}_{ij}\Psi_\Sigma(g_{ij}, \phi) = -i\frac{\delta}{\delta g_{ij}}\Psi_\Sigma(g_{ij}, \phi)\,, \qquad \hat{\pi}_\phi\Psi_\Sigma(g_{ij}, \phi) = -i\frac{\delta}{\delta\phi}\Psi_\Sigma(g_{ij}, \phi)$$

- These (functional differential) equations are not really well defined
  $\Rightarrow$ There exists a "minisuperspace" ansatze/truncation that is better defined and leads to ODEs/PDEs

  Fortunately the isotropy and homogeneity of the universe makes this ansatze physically relevant

Minisuperspace and the No Boundary Proposal

- The WDW equation makes sense in the reduced minisuperspace ansatze

$$ds^2 = -N^2(t)dt^2 + a^2(t)d\Omega_\Sigma^2\,, \quad \phi = \phi(t)$$

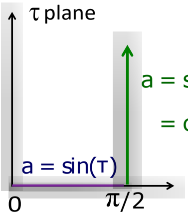
- In this case $\widehat{H}_i\Psi_\Sigma(a,\phi) = 0$ automatically and $\widehat{H}_{WDW}\Psi_\Sigma(a,\phi) = 0$ becomes a well defined PDE
- One has to supplement appropriate "boundary" conditions
- The [Hartle - Hawking] No Boundary (NB) proposal posits that one has to make an excursion to Euclidean signature and consider compact metrics with no boundary at early times
- The resulting state/wavefunction corresponds to the [Bunch - Davies] or Euclidean vacuum (the analogue of the Minkowski vacuum in a Cosmological setting i.e. $\Lambda > 0$)
- There is also an alternative [Linde - Vilenkin] Tunelling (T) proposal (defined via probability influx/outflux in the superspace boundaries), that we shall contrast it with

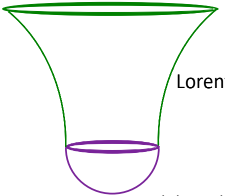
Consider the Einstein Hilbert action with positive cosmological constant

$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\,(R-2\Lambda)\,, \qquad \Lambda > 0$$

that admits an empty de Sitter solution

The [Hartle - Hawking] proposal classically describes a (complex) metric - half of Euclidean de-Sitter glued to half of Lorentzian de-Sitter -





$$ds^2 = d\tau^2 + \sin^2\tau\, d\Omega_3^2 \longrightarrow ds^2 = -dt^2 + \cosh^2 t\, d\Omega_3^2$$

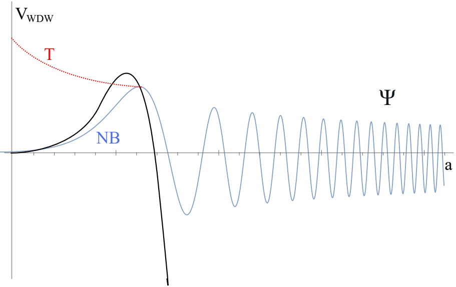
Semi-classics and WKB of minisuperspace WDW

- The minisuperspace WDW equation (positive cc./no matter) reads

$$\left(\widehat{\pi}_a^2 + a^2 - \frac{\Lambda}{3}a^4\right)\Psi_\Sigma(a) = 0 \qquad \widehat{\pi}_a = -i\kappa\frac{d}{da}$$

- To understand its semi-classical properties - convenient to employ a "WKB" ansatze ($\kappa = 8\pi G_N \hbar \to 0$) and matching

$$\Psi_\Sigma^L(a) = A_L e^{iS_L/\kappa} + B_L e^{-iS_L/\kappa}\,, \qquad \Psi_\Sigma^E(a) = A_E e^{S_E/\kappa} + B_E e^{-S_E/\kappa}$$

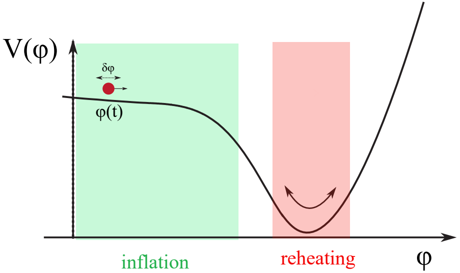


- For large a the wavefunction is oscillatory (Lorentzian), while for small a it has an exponential increasing/decreasing behaviour (Euclidean)
- The No Boundary proposal selects the increasing branch and the wavefunction vanishes at zero a - The Tunneling/[Vilenkin] proposal selects the decreasing branch

WDW and slow roll inflation

- One can include the presence of the scalar inflaton field ϕ
- We assume a slow roll approximation for the potential $V(\phi)$ in the inflationary region

$$\epsilon_V \equiv \frac{M_P^2}{16\pi}\left(\frac{V_\phi}{V}\right)^2 \ll 1\,, \quad \eta_V \equiv \frac{M_P^2}{8\pi}\frac{V_{\phi\phi}}{V} \ll 1$$



- The WDW wavefunction now depends on two arguments i.e. $\Psi_\Sigma(a,\phi)$
- Given the wavefunction, we can compute the probability for a specific "history"/realisation of the inflating Universe, via its norm $P = |\Psi|^2$

No Boundary/Tunneling and slow roll inflation

- In the slow roll approximation for the potential $V(\phi)$ one finds the semi-classical (WKB) No Boundary/Tunneling wavefunctions $(\kappa = 8\pi G_N)$

$$\Psi_{NB}(a,\phi) \simeq P_{NB}^{1/2}\,\Re\left(e^{iS_L(a,\phi)}\right)\,, \quad P_{NB} = e^{-S_E(\phi)}$$

$$\Psi_{T}(A,\phi) \simeq P_{T}^{1/2}\left(e^{-iS_L(a,\phi)}\right)\,, \quad P_{T} = e^{+S_E(\phi)}\,,$$

$$S_E(\phi) = -\frac{24\pi^2}{\kappa^2 V(\phi)}\,, \quad S_L(a,\phi) \simeq \frac{24\pi^2(a^2\kappa V(\phi)/3-1)^{3/2}}{\kappa^2 V(\phi)}$$

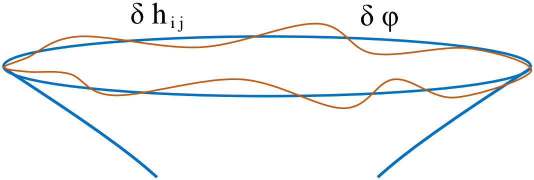
- S_E is the on-shell action of Euclidean de-Sitter (sphere)
  S_L is the on-shell action in the Lorentzian-oscillatory region when the scale factor is large $a^2 > 3/\kappa V(\phi)$

- The Euclidean/Lorentzian WKB matching for the value of the inflaton/size of the sphere is typically performed at horizon crossing during inflation $(\phi_*, a_*)\,, H(\phi_*)a_*(\phi_*) = 1$
  i.e. "beginning of inflation"

No Boundary and slow roll inflation: Fluctuations

[Halliwell - Hawking ...]

- It is also possible to describe (inhomogeneous) fluctuations of the fields $\phi(\Omega) = \phi_* + \delta\phi(\Omega)\,,\; g_{ij}(\Omega) = g^*_{ij} + \delta h_{ij}(\Omega)$ etc.



- The No Boundary proposal predicts the correct spectrum of primordial perturbations with a Gaussian suppression factor

$$|\Psi_{NB}(\phi_* + \delta\phi)|^2 \sim e^{-S_E(\phi_*)} \prod_{modes} \exp\left(-\delta\phi_{mode}\, C_{mode}\, \delta\phi_{mode}\right)$$

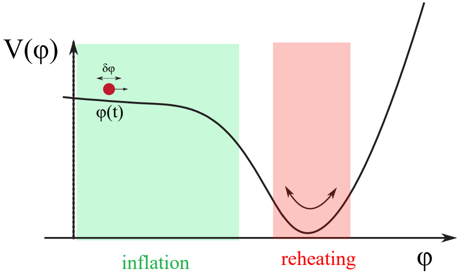
(it describes a Cosmological analogue of the "vacuum")

- In the Tunneling proposal such fluctuations are unsuppressed $(- \leftrightarrow +)$...

An exponential (hierarchy) problem

- Remember the current cosmological constant problem

$$\frac{M_P^4}{V(\phi_{now})} \simeq 10^{120}$$



- There is an exponentially worse problem with the No Boundary proposal!

$$P_{NB} = |\Psi_{NB}(\phi_*)|^2 \simeq \exp\left(-S_E(\phi_*)\right) = \exp\left(\frac{M_P^4}{V(\phi_*)}\right)$$

- It gives an overwhelming probability ($P_{NB} \gg 1$) for an empty cold universe, with the smallest allowed number for the cosmological constant
- In the inflationary context it predicts the least number of e-folds
- The issue stems from the fact that the on-shell action for the positively curved Euclidean de-Sitter is negative

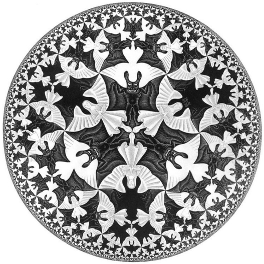
The No Boundary proposal and AdS/CFT

There is a case where the analogue of the No Boundary proposal works perfectly well: The AdS/CFT correspondence ($Z_{QGR}^{AdS} = Z_{CFT}^{\partial AdS}$)

- ex: Global $EAdS_4$ and the S^3 partition function (regular interior $\leftrightarrow$ N.B.)

$$ds^2_{H_4} = L^2_{AdS}(d\tau^2 + \sinh^2\tau d\Omega_3^2)$$

$$e^{-S_E(H_4)} \sim Z_{CFT}(S^3)\,, \quad S_E = \frac{L^2_{AdS}}{2G_N}$$



- Both sides can be computed and agree. For example in ABJM (finite-N) [Kapustin-Willet-Yaakov, Drukker-Marino-Putrov ...]
- Here it is crucial that the on-shell action of AdS is positive (after performing holographic renormalization) [Skenderis - Papadimitriou ...]
- No direct relation to Cosmology (as in the No Boundary proposal - with a simple $\tau = it$)

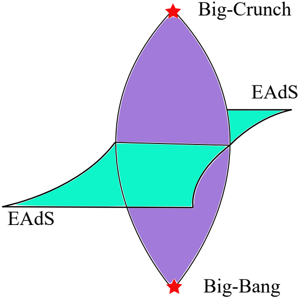
Euclidean Wormholes and Bang-Crunch Cosmologies

AdS/CFT context: [Maldacena-Maoz (04), PB-Gaddam-Papadoulaki (17) + Kiritsis (19-21), Van Raamsdonk et. al. (20-23) ...]

- In AdS/CFT there is an example that gives rise to FRW cosmologies: Two boundary Euclidean AdS wormholes ($' = d/d\tau$)

$$ds^2 = d\tau^2 + a^2(\tau)d\Omega_3^2\,, \quad a''(0) > 0\,,\, a'(0) = 0\,,\, a(\tau \to \pm\infty) \sim e^{H|\tau|}$$

- Euclidean Wormholes are NOT related to Black Holes (horizons) via a $\tau = it$ analytic continuation - Instead:



- Their analytic continuation ($\tau = it$) gives rise to Bang - Crunch Cosmologies (Remember that Λ is negative)

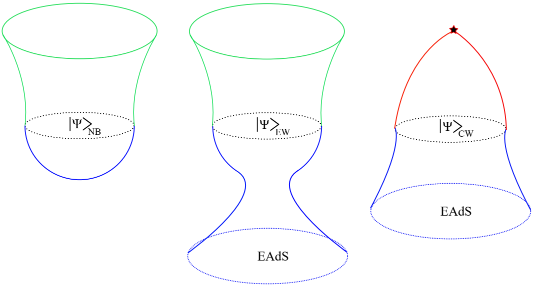
$$ds^2 = -dt^2 + a^2(t)d\Omega_3^2$$
$$\ddot{a}(0) < 0\,, \quad \dot{a}(0) = 0$$

A new proposal for the wavefunction of the Universe

- An issue with these geometries is that upon analytic continuation they inevitably crunch and do not allow for a period of inflation

- Our idea [PB - Papadoulaki (24)] : Combine features of both anti-de Sitter and de-Sitter - we need a Euclidean wormhole geometry that is asymptotically EAdS that transitions into EdS near its throat

- By cutting it in half we can "glue" to it an expanding Lorentzian Universe

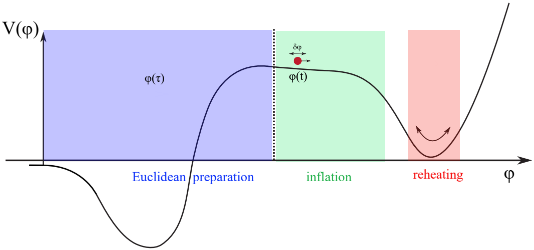


"Wineglass" AdS wormholes

- We shall call (half of) these geometries "wineglass" AdS (half) wormholes (Asym. Flat analogues: [Lavrelashvili-Rubakov-Tinyakov, Lehners])
- Their defining properties: They should asymptote to a EAdS space: $a(\tau \to \pm\infty) \sim \exp(H_{AdS}|\tau|)$ and in addition

$$a''(0) < 0\,, \quad a'(0) = 0\,, \quad a(0) = a_{\text{max}}\,, \quad \phi'(0) = 0$$

  so that a_{max} is a local maximum of the scale factor
- These are also good initial conditions for a subsequent inflationary evolution (since $\ddot{a}(0) > 0$)
- To support such solutions: A scalar potential that takes both positive and negative values (Ex1:)



Models for "wineglass" AdS wormholes

- Consider a general GR-inflaton-radiation-matter system ($\kappa \equiv M_{Pl}^{-2}$)

$$S_E = \int d^4x \sqrt{g_E} \left(-\frac{1}{2\kappa} R + \frac{1}{2} \nabla^\mu \phi \nabla_\mu \phi + V(\phi) + \mathcal{L}_{rad.} + \mathcal{L}_{matter} \right)$$

and the spherically symmetric and homogeneous ansatze

$$ds^2 = d\tau^2 + a^2(\tau) d\Omega_3^2 \,, \quad \phi(\tau) \,,$$

The Einstein and inflaton EOMs reduce to

$$\frac{a'^2}{a^2} - \frac{1}{a^2} + \frac{\kappa}{3} \left(V(\phi) - \frac{\phi'^2}{2} \right) + \frac{\rho_{axion}}{a^6} - \frac{\rho_{rad.}}{a^4} - \frac{\rho_{matter}}{a^3} = 0 \,,$$

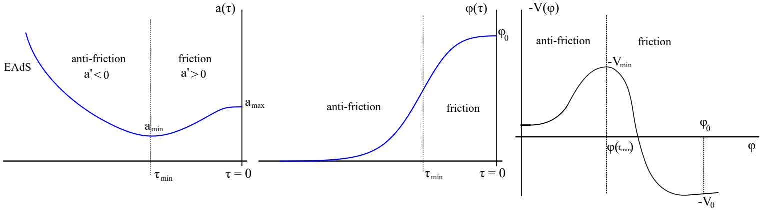
$$\phi'' + 3 \frac{a' \phi'}{a} - \frac{dV}{d\phi} = 0 \,,$$

- "Wineglass" Wormholes can be supported by axions [PB - Papadoulaki (23)] or magnetic radiation [PB - Papadoulaki - Gialamas 24]
- Magnetic radiation/fluxes lead to $\rho_{rad.} < 0$ (i.e. $T^E_{\tau\tau} \sim E^2 - B^2$)

- The EOM for the scalar field describes a particle moving in the potential $-V(\phi)$ with an (anti)-friction term $3a'\phi'/a$

- Ex1: Consider a potential $V(\phi)$ with a local maximum at $\phi = 0$
  i.e. $V(\phi) \sim -1 + m^2\phi^2/2$ with $m^2 < 0$
  (a dual RG flow driven by a relevant operator with conformal dimension $\Delta = 3/2 + \sqrt{9/4 + m^2} < 3$)

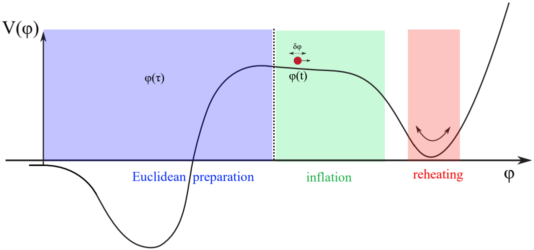
- The Euclidean evolution of the scale factor and the scalar field in $-V(\phi)$



- The Euclidean manifold initially shrinks ($a' < 0$/anti-friction) and then expands ($a' > 0$/friction) causing the ϕ particle to first accelerate and then stop at ϕ_0. (Desirable to stop as early as possible...)

Subsequent Lorentzian evolution

- The semi-classical Euclidean trajectory can describe the nucleation of the Universe at ϕ_0, high up in the potential with $a'(0) = \phi'(0) = 0$



- The potential should also contain a slow roll region for $\phi > \phi_0$, so that the Universe can subsequently inflate/expand ($\ddot{a} > 0$) in Lorentzian time
- Our proposal can accommodate various options consistent with the latest experimental constraints on inflation ex. `[Planck, BICEP]` etc.

Evading the issue of the No Boundary proposal

- To compute the semi-classical probability and compare with the No-Boundary proposal $(P = |\Psi|^2 \simeq e^{-S_E})$

  $\Rightarrow$ evaluate the Euclidean wormhole on-shell action

$$S_E^{\text{on-shell}} = 4\pi^2 \int_{UV}^{0} d\tau \left(\frac{\rho_{rad./axion}}{a^p} - a^3 V(\phi) \right) + S_{GH}^{UV} + S_{c.t.}^{UV} \, ,$$

  $(p = 3,\, 1$ for axion, radiation$)$

- The EAdS UV boundary contains the Gibbons-Hawking S_{GH}^{UV} as well as boundary counterterms $S_{c.t.}^{UV}$ that one needs to add in order to perform holographic renormalization

- Either numerically or analytically using thin/thick wall approximations one typically finds a positive on-shell action for the wormhole

- As in other Holographic examples, due to the AdS asymptotics we have a well defined probability $(P \simeq e^{-S_E} < 1)$ and the issue of the No Boundary proposal can be evaded : The Universe prefers to "nucleate" high up in the potential and then follows the slow roll trajectory

A model consistent with experimental data (SM + GR)

[P.B. - I. Gialamas - O. Papadoulaki (24)]

- The Higgs boson is the only experimentally observed scalar particle in nature and could perhaps also play the role of the inflaton
- A class of models of inflation that conform very well with experimental data : "Higgs Inflation" [Bezrukov - Shaposhnikov ...]
- These models include a non-minimal coupling term $\sim \xi\phi^2 R$ to the Einstein-Higgs action (Jordan-frame action)
(The [Starobinskii] R^2 model is a $\xi \to \infty$ limit of these models)
- Such terms typically appear when considering loop corrections to the effective action [Callan-Coleman-Jackiw ...]
- Current experimental data of the Higgs and Top mass [PDG ...] favor SM metastability $\Rightarrow$ the Higgs effective potential turns negative at high energies/field values (incl. loop corrections)

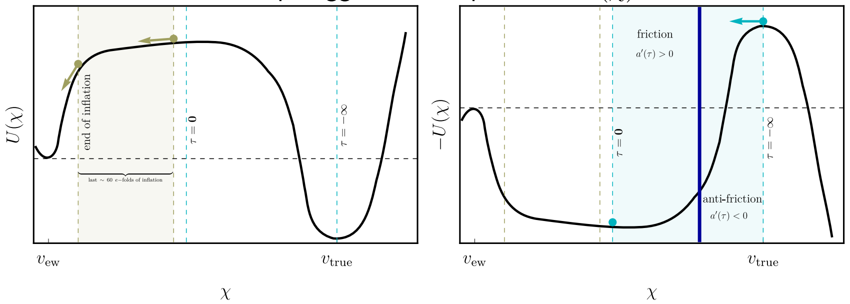
$$V_{tree} = \frac{\lambda}{4}(\phi^2 - v_{EW}^2)^2 \quad \Rightarrow \quad V_{eff.}^{RGI}(\phi, \lambda; \mu)\,, \quad \lambda(\mu) < 0\,,\, \mu \in [\mu_1, \mu_2]$$

A model consistent with experimental data (SM + GR)

[P.B. - I. Gialamas - O. Papadoulaki (24)]

- Going back to Einstein Frame ($g_{\mu\nu} = e^{2\Omega}\tilde{g}_{\mu\nu}$, $\phi(\chi)$) one finds a potential of the Higgs type at small χ, of the slow roll type at inflationary χ and with a negative true minimum at very high energies/field values

The one-loop Higgs effective potential $U(\chi)$ in EF



- We obtain a phenomenological model (consistent with current experimental data) that realises our proposal
  (for certain central values of the Higgs and Top quark masses)

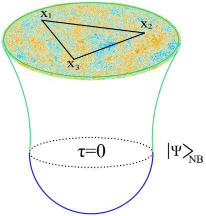
Future

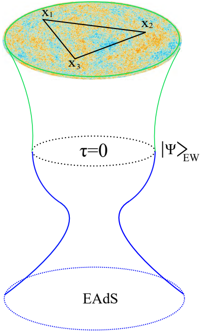
Cosmological Correlators

- Bulk correlators at $\tau = 0$ can be computed from the wavefunction using

$$\int D\phi \, |\Psi_{\tau=0}|^2 \, \phi(0, \vec{x}_1) ... \phi(0, \vec{x}_n)$$

Later time/Cosmological correlators are computed using the in/in formalism [Weinberg ...] or evolving the wavefunction in Lorentzian

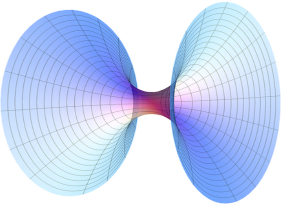




- We are currently studying Cosmological correlators in our setup and comparing them with the No-Boundary proposal [In progress + Pompey Leung (UBC) + Chris Waddell (Perimeter)]

- No leading deviations, since the metric resembles EdS near the throat, as long as one chooses the vacuum state in the $EAdS$ asymptotic regions

Holographic (AdS/CFT) embedding



- Our construction is amenable to a possible Holographic interpretation and embedding due to the $EAdS$ boundaries
- This relies on understanding the Holographic dual(s) of Euclidean wormholes

Pertinent Question

- Are there Microscopic UV complete models of Euclidean Wormholes? In AdS/CFT? (we want to understand string theory on target space wormhole backgrounds)
- This question is closely related to the factorization problem: [Maldacena Maoz (04)]

  Entanglement "holds up the throat" of a two sided eternal black hole, but it is not clear what is the analogue for Euclidean wormholes
  Proposals: (Statistical) Averaging [low-dim ...] vs. Interactions
  [PB - Kiritsis - Papadoulaki (19-21)] [Van Raamsdonk et. al. (20-22)]

Summary

- We proposed a new type of wavefunction for the universe computed from the gravitational path integral, with asymptotically $EAdS$ boundary conditions

- In the semiclassical limit, it describes a Euclidean AdS (half)-wormhole geometry. If the scale factor acquires a local maximum at the surface of reflection (Z_2) symmetry, it gives rise to an expanding universe upon analytic continuation to Lorentzian signature

- Our proposal can be realised with a non-trivial scalar potential $V(\phi)$ that takes both positive and negative values (i.e. in the SM + GR: $\phi \equiv$ Higgs)

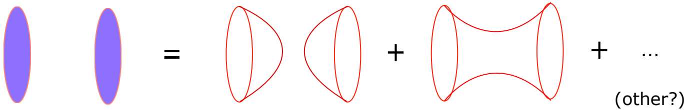
- Our proposal evades some issues of the No Boundary proposal, leading to a well defined probability $P \simeq e^{-S_E} < 1$. It can also favor a long-lasting period of inflation - (for certain scalar potentials)

- It also raises the interesting possibility of describing the physics of inflating cosmologies and their perturbations within the context of holography (Duals of EAdS Wormholes?)

Thank you!

The factorisation problem: $Z(J_1, J_2) \neq Z_1(J_1) Z_2(J_2)$

[Maldacena - Maoz (2004) ...]



Possible resolutions:

- The QGR path integral corresponds to an average: $\langle Z(J_1) Z(J_2) \rangle$ $\Rightarrow$ Several options [...]
- Explicit averaging over ensembles of CFT's - (Unitarity crisis)
- In canonical AdS/CFT there is a single theory with fixed parameters
- Approximate statistical averaging ("ETH" - "Quantum Chaos") $\Rightarrow$ "Statistical wormholes" from complicated/almost random Hamiltonians [...]

Is this is what happens in our Cosmology?

No factorisation problem due to interactions?

[PB - Kiritsis - Papadoulaki (19 - 21)], see also related work by [Van Raamsdonk et. al. (20-22)] and [Bachas - Lavdas (18)]

A potentially microscopic understanding of wormhole saddles?:

- Interactions between holographic QFT's
- It is actually quite subtle!: "Why to have a disconnected pair of boundaries and not a single one?" $\Rightarrow$ UV soft - IR strong cross-interactions (reminiscent of confinement...)
- Wormhole cross correlators - no short distance singularities $\Rightarrow$ averages of lower point correlators in individual subsystems
- I.e. can the exact Schwinger functional acquire an "averaged" form

$$Z_{system}(J_1, J_2) = \sum_S e^{w(S)} Z_S^{(QFT1)}(J_1) Z_S^{(QFT2)}(J_2)$$

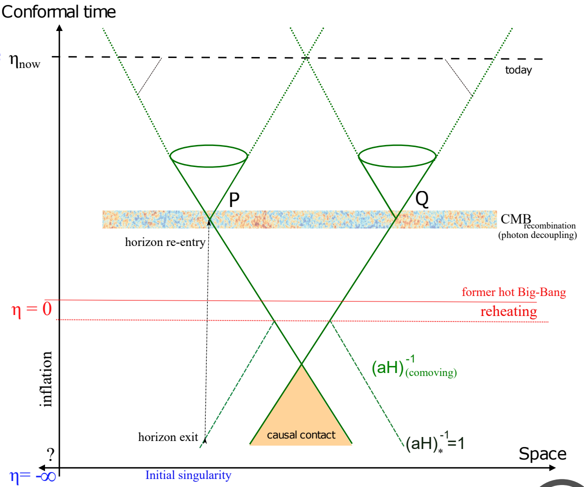
in a single unitary/reflection positive system? (S some "sector")
[PB - Kiritsis - Papadoulaki (21)] ($S \equiv R$ - $U(N)$ representations)

The inflationary paradigm

- Consider an FRW metric

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 = a^2(\eta)[-d\eta^2 + d\vec{x}^2]\,, \quad \eta = \int \frac{dt}{a(t)} = \int \frac{d\log a}{(aH)}$$

- The conformal time elapsed η depends on the comoving Hubble radius $(aH)^{-1} = (da/dt)^{-1}$
- Inflation: Comoving Hubble radius was initially decreasing $\equiv d^2a/dt^2 > 0$ (after reheating it increases $\equiv d^2a/dt^2 < 0$)
- This means that superhorizon scales entering the present universe, actually started from a small region where local microphysics established homogeneity and isotropy (thermal equilibrium)



The No Boundary proposal and Stochastic Inflation

[Starobinskii, Goncharov-Linde-Mukhanov ...]

- Assume a slow roll inflationary scenario and split the evolution of a scalar field into UV and IR modes (wtr Hubble scale H)

- The IR physics at scales $\Delta t \sim 1/H$, $\Delta L \gg 1/H$ is governed by an effective stochastic equation

$$\dot{\phi} = -\frac{V'}{3H} + \xi(t)\,, \qquad \langle \xi(t)\xi(t') \rangle = \frac{H^3}{4\pi^2}\delta(t-t')$$

  and a Fokker-Planck equation for the probability $P(t,\phi)$ that the field has the value ϕ at time t

$$\partial_t P + \partial_\phi J = 0\,, \quad J = -\frac{V'}{3H}P - \partial_\phi\left(\frac{H^3}{8\pi^2}P\right)$$

- For a potential bounded from below $V(\phi) \geq V_{min} > 0$, one finds an equilibrium $(J=0)$ distribution consistent with the No Boundary proposal

$$P_{eq.}(\phi) \sim \exp\left(\frac{24\pi^2}{\kappa^2 V(\phi)}\right) \sim P_{NB}(\phi) \qquad H^2 \sim \kappa V/3$$

Ideas to evade this problem

- The Tunneling wavefunction [Linde - Vilenkin] evades this issue ($P_T \simeq e^{+S_E}$), but does not describe correctly the cosmological fluctuations beyond minisuperspace (they get enhanced)

- Selection rule or anthropic reasoning
  [Linde, Hartle - Hawking - Hertog ...]

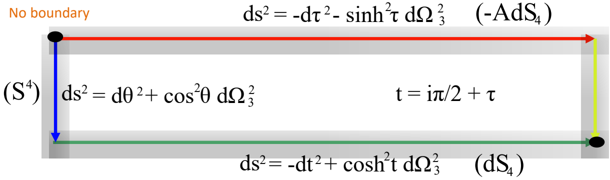
- The gravitational path integral is not very well defined - non-renormalizability and the conformal mode problem - Understand it in a Picard-Lefschetz fashion and define an appropriate (steepest descend) contour in field space.
  [Halliwell-Louko, Hartle-Hawking-Hertog, Lehners, ...]

- Quantum effects (loops) and secular terms in expanding cosmologies. It is possible that the (non-perturbative!?) wavefunction has a very different behaviour than its naive semi-classical expansion
  (seen in $2d$ models [PB-Papadoulaki (20), Anninos (24)])

- Change entirely the assumptions/setup giving rise to our Cosmology?
  [...]

Complexified metrics and contours

- Alternative contours to the [Hartle-Hawking] one?
- One of them seems to connect cosmology with a $(-)AdS$ space [Hartle-Hawking-Hertog (11), Maldacena-Turiaci-Yang (19), ... PB-Papadoulaki (20)]



Contour (in)-dependence?, Steepest descend?
- No physically transparent meaning...

- Another approach: Domain-wall/Cosmology correspondence [Skenderis - Townsend] (Multiple analytic continuations...)
- We wish to retain a clear understanding of the Lorentzian/Euclidean sections related with a simple $t = i\tau$ [PB - Papadoulaki]

Future Directions

- We would like to perform a thorough WKB analysis of the two-parameter (a, ϕ) WDW equation (turning points, caustics etc...)
- It is important to understand whether the resulting (half)-wormhole wavefunction is normalisable or not
- Analyse the spectrum of fluctuations around such wormholes
- Embed our setup in holography. A UV complete microscopic model of Euclidean wormholes? [PB - Papadoulaki - Kiritsis, Van Raamsdonk ...]
- Understand what our (half)-wormholes correspond to from a dual field theory perspective
- A related simpler question [PB - Gaddam - Papadoulaki ...]: What does opening up a hole in the center of EAdS and fixing bcs there mean for the holographic CFT?

WDW equation and normalizability of the wavefunction

- Issue II: The No-Boundary wavefunction is non-normalizable
- Our WDW equation is ($A = \log a$ avoids normal ordering issues)

$$\left[\frac{\partial^2}{\partial A^2} - \frac{\partial^2}{\partial \tilde{\phi}^2} + \left(\frac{12\pi^2}{\kappa}\right)^2 \left(e^{6A}\tilde{V}(\tilde{\phi}) - e^{4A} + \tilde{Q}^2\right)\right]\Psi = 0$$

with $\tilde{\phi} = \phi/M_{Pl}$, $\tilde{V} = \kappa V/3\,, \tilde{Q}^2 = \kappa Q^2/3$)

- Unfortunately we cannot solve this equation in closed form, but the work of [Hawking - Page] showed that a similar equation admits a discrete set of normalisable solutions/states
- Their idea is that semi-classical (half)-wormhole solutions are superpositions of these elementary states [Hawking - Page]
- If true this would mean that our (half)-wormholes would be described by a normalisable WDW wavefunction in contrast with the No Boundary wavefunction, but this remains to be checked

Issues with the No Boundary proposal

- Given the wavefunction, we can also compute the probability for a specific "history"/realisation of the Universe, via its norm $P = |\Psi|^2$

$$P_{NB} = |\Psi_{NB}(\phi)|^2 \simeq \exp\left(-S_E(\phi)\right) = \exp\left(\frac{M_P^4}{V(\phi)}\right)$$

- This comes from the leading semi-classical piece of the wavefunction and indicates that the wavefunction is non-normalizable

- Perhaps this is not a deep problem due to the minisuperspace and (WKB) approximations involved

- Since the stochastic description is just an effective description of the IR sector, which the No Boundary proposal seems to describe correctly, perhaps there is no fundamental reason to demand its normalizability

- Nevertheless, even using it in this restricted sense, there is a more acute problem for the No Boundary proposal in the context of inflation
(See the reviews by [Lehners, Maldacena])