Extreme Mass Ratio Inspirals: theoretical expectations and challenges

Georgios Lukes-Gerakopoulos

Astronomical institute,Czech Academy of Sciences

@ Xmas Theoretical Physics Workshop @ Athens 2024, 20.12.2024

Astronomický ústav AV ČR

Table of Contents

Gravitational wave detectors and sources

- Gravitational waves are disturbances in the fabric of spacetime.

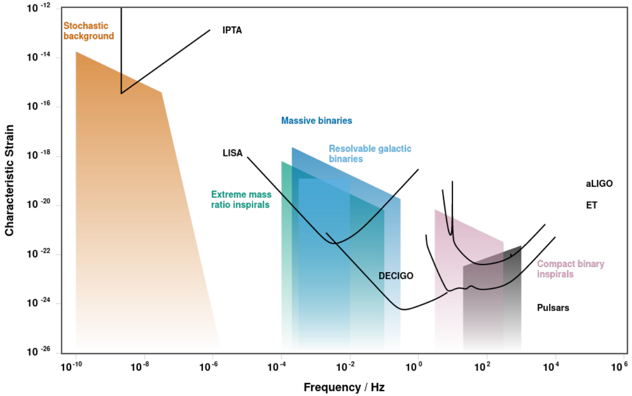


Figure: Moore, Cole and Berry from the Gravitational Wave Group at the Institute of Astronomy, University of Cambridge (http://gwplotter.com/)

What are EMRIs?

- An Extreme Mass Ratio Inspiral (EMRI): inspiral of a stellar compact object of mass μ into a SuperMassive Black Hole SMBH $M \geq 10^5 M_\odot$, the mass ratio $q = \mu/M \lesssim 10^{-4}$
- The inspiraling body traces the SMBHs spacetime background and emits information through gravitational waves.

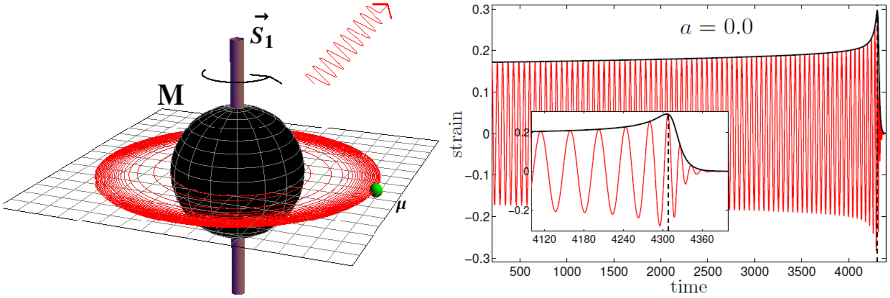


Figure: An EMRI on the equatorial plane on the left and on the right the respective waveform (Harms , 2016).

Why are EMRIs interesting?

- EMRIs scan thoroughly the spacetime background.
- By analyzing the GWs we can check the Kerrness of the primary black hole.
- Any deviation of the GW from the standard paradigm reveals deviation from GR.
- The rate of EMRIs has a direct impact on our population modeling.
- The perturbation modelling of EMRIs produces tools useful for other theoretical fields (Barack & Pound , 2019).
- For more see e.g. (Babak et al. , 2017; Barausse et al. , 2020)

Table of Contents

Distance and mass ranges of two body systems

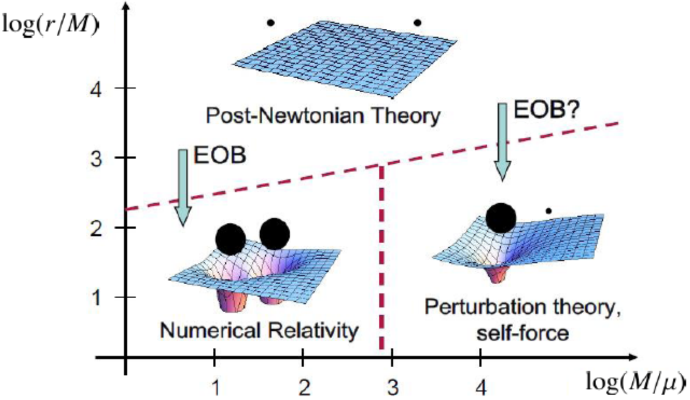


Figure: Leor Barack slide. M is the total mass, μ is the reduced mass, r is the distance between the bodies.

A very simple first approach to an EMRI

Assumptions:

- The inspiraling object is not spinning.
- The spacetime background is Kerr.
- The EMRI system is isolated from its surrounding environment.
- The loss of energy and angular momentum is adiabatic.

The above assumptions lead to

- a model in which the inspiraling object is shifting from geodesic to geodesic on a Kerr background.
- motion that the non-dissipative limit corresponds to is geodesic in Kerr, thus integrable (Carter, 1968).

Gravitational self-force: proper EMRI modelling

Basic Idea: The mass of the secondary inspiraling body μ is very small in comparison to the mass of the primary M, so it introduces only a perturbation to the background of the primary source.

We can calculate the gravitational self-force by expanding the spacetime metric of the binary system $\tilde{g}_{\mu\nu}$ in terms of the mass ratio $q = \mu/M$ around the unperturbed background of the primary black hole $g_{\mu\nu}$.

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + q\, h^{(1)}_{\mu\nu} + q^2\, h^{(2)}_{\mu\nu} + \ldots$$

Barack & Pound (2019) provide a review on the topic.

Two time scale approximation

Basic Idea: The dissipation due to radiation reaction in an EMRI is very *slow* in comparison to the orbital motion of the secondary around the primary.

The **slow time scale** is concerned with the evolution of the constants of motion (*action variables*).

The **fast time scale** is concerned with the orbital phases (*angle variables*).

Action angle variables for EMRIs (Hinderer & Flanagan , 2008).

Teukolsky equation: an efficient way

Basic Idea: Linearly perturb a field ${}_s\psi$ around Kerr black hole.

$$ {}_s\mathcal{O}\,{}_s\psi(x^\mu) = 4\pi\Sigma T \ , $$

where ${}_s\mathcal{O}$ is a second-order partial differential operator and T is a source term (Teukolsky , 1973).

For $s = -2$ we have a gravitational perturbation.

${}_{-2}\psi$ encodes the gravitational radiation emitted to infinity and infers the gravitational fluxes at the horizon of the primary as well.

The Teukolsky equation computed gravitational fluxes are equal to the averaged dissipative part of the first order self-force (adiabatic order).

GW phase splitting

$$\varphi = \varphi_{\mathrm{A}} + \varphi_{\mathrm{res}} + \varphi_{\mathrm{PA1}} + \dots$$

- φ_{A} the adiabatic term contributes a phase $\mathcal{O}(q^{-1})$ to the inspiral.
- φ_{PA1} the post adiabatic term contributes $\mathcal{O}(1)$.
- $\varphi_{\mathrm{res}} \sim \dfrac{\sqrt{\epsilon}}{q}$.

For quasi-circular inspirals the GW phase φ is twice the ϕ orbital phase.

To have a proper detection of an EMRI, we need GW templates with phase accurate up to one degree after ten of thousands of cycles!

Table of Contents

EMRIs into black holes surrounded by matter

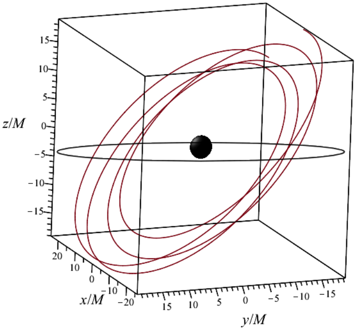
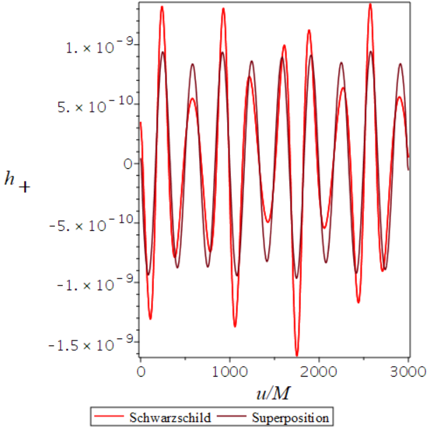


Figure: A geodesic orbit in a spacetime of Schwarzschild black hole surrounded by a ring-like structure (Polcar, LG, Witzany , 2022).



Growth of resonances

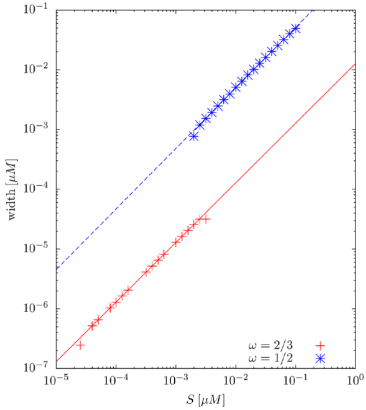


Figure: Growth of resonances as function of the measure of the secondary's spin implying $\epsilon \sim S^2 \sim q^2$ and hence phase shift $O(1)$ (Zelenka, LG, Witzany & Kopáček , 2020).

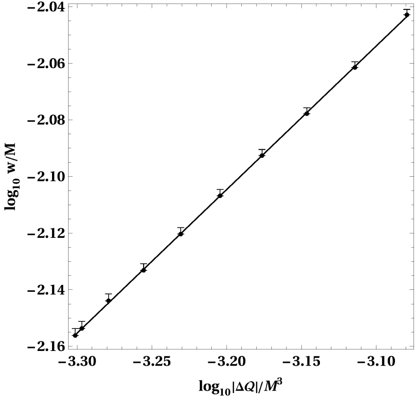


Figure: Growth of resonances as function of the measure of the quadrupole moment deviation from Kerr spacetime $\epsilon \sim \Delta \mathcal{Q}$ (LG & Witzany , 2021).

Crossing of prolonged resonances

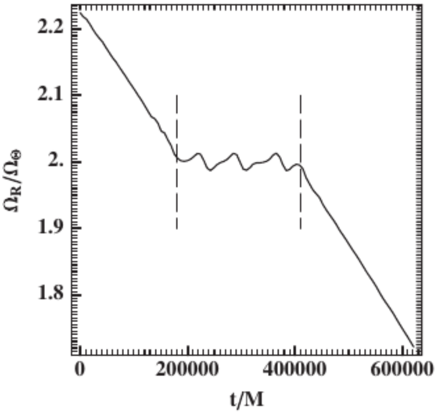


Figure: Frequency ratio when an inspiral crosses a prolonged resonance (Apostolatos, LG & Contopoulos , 2009).

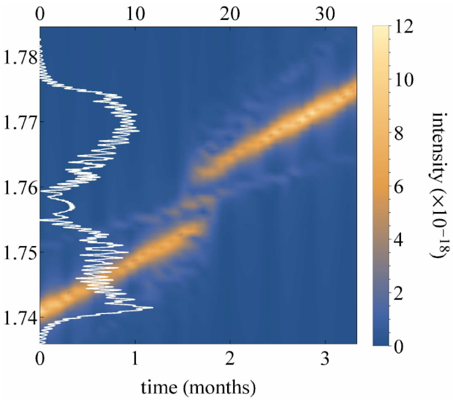


Figure: Crossing appearing as a glitch on a periodogram (Destounis et al., 2021)

Table of Contents

Summary

Theoretical expectations:

- EMRIs are GW sources promising to unveil the background of SuperMassive Black Holes.
- Deviations from the standard EMRI paradigm will provide evidence for deviations from GR.
- To model EMRI we have to invent new methods in perturbation theory.

Challenges:

- EMRIs signal will be weaker than other simultaneously detected GW sources.
- We have not yet achieved the necessary accuracy in waveform modeling of EMRIs.

References I

Carter B.,: *Phys. Rev. D* **174**, 1559 (1968)

Apostolatos T. A., Lukes-Gerakopoulos G., Contopoulos G.: *Phys. Rev. Lett.* 103, 111101 (2009)

Babak S., Gair J., Sesana A., Barausse E., Sopuerta C. F., Berry C. P. L., Berti E., et al., *Phys. Rev. D* **95**, 103012 (2017)

Barack, L. & Pound, A.: *Rep. Progr. Phys.* **82**, 016904 (2019)

Barausse E., Berti E., Hertog T., Hughes S. A., Jetzer P., Pani P., Sotiriou T. P., et al., *Gen. Rel. Grav.* **52**, 81 (2020)

Destounis, K., Suvorov, A. G., Kokkotas, K. D.: : *Phys. Rev. L* **126** 141102 (2021)

Hinderer, T. & Flanagan, E, E.: *Phys. Rev. D* **78**, 1550 (2008)

Harms H.: Ph.D. thesis (2016)

Lukes-Gerakopoulos, G., Witzany, V., *Handbook of Gravitational Wave Astronomy* A Living Reference Work. Springer, 42 (2021)

Polcar L., Lukes-Gerakopoulos G., Witzany V., *Phys. Rev. D* **106**, 044069 (2022)

Teukolsky, S: *Astrophys. J.* **185**, 635 (1973)

Zelenka O., Lukes-Gerakopoulos G., Witzany V., Kopáček O.: *Phys. Rev. D* **101**, 024037 (2020)

Thank you for your attention!

Figure: Tycho Brahe & Johannes Kepler @ Prague.