Structure Constants from Integrability

A. Georgoudis

Xmas meeting 20/12/24, ATHENS

Queen Mary
University of London

Based on

[2207.01315] with B. Basso and A K-Sueiro

[2412.XXXXX] with B. Basso

Plan of the talk

- Introduce integrability, focus mostly on $\mathcal{N}=4$
- Present conjecture for Hexagons for short operators
- Discuss weak and holographic tests
- Small spin limit

What is Integrability?

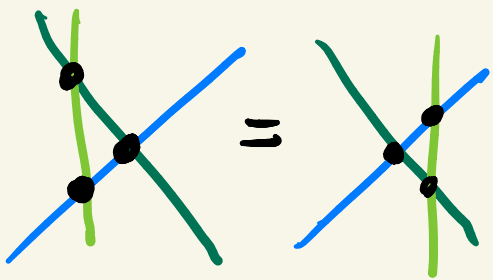
- As many charges as degrees of freedom

  - Keplerian motion! In 3d H, J^2, J_z $(\vec{A})$

  - QFT you need an infinite set of conserved charges

Most prominent example 2d theories: No particle production, factorization!

Yang-Baxter



$\bullet = S_{IJ}(u_I - u_J)$

$\hookrightarrow$ rapidity of the particles.

$\mathcal{N}=4$

- We will focus mainly on $\mathcal{N}=4$ SYM in $d=4$ in the large $N_c \to \infty$ limit
(results exist also for other holographic theories, deformations)

Full sym. is $PSU(2,2|4) \longrightarrow Sl(2) \leftarrow$ mostly work in this closed sub-sector (Z, D)

- $\mathcal{N}=4$ is a SCFT, in principle only $\{\gamma_i, C_{ijk}\}$ of primary operators are needed!

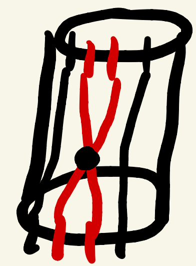
$$O_i \xrightarrow{x \to \lambda x} \lambda^{-\Delta-\gamma_i} O_i \qquad O_i(x)\, O_j(y) \simeq \sum_k C_{ijk}\, g(\Delta, x-y)\, O_k(x-y)$$

- $\mathcal{N}=4$ is conjectured to be integrable in the $N_c \to \infty$ limit

$\mathrm{Tr}[\ldots\ldots] \longleftrightarrow \uparrow\downarrow\uparrow\uparrow\uparrow$

↳ Operators of $\mathcal{N}=4$

Dilatation $\longleftrightarrow$ Energy



Heuristic 2d picture

Spectrum of $\mathcal{N}=4$

- For single trace operators obtaining anomalous dim. solved by QSC! We are able to obtain results in various regimes

weak

$$
\begin{aligned}
\gamma_{11} \;=\;& -242508705792 + 107663966208\zeta_3 + 70251466752\zeta_3^2 - 12468142080\zeta_3^3 \\
&+1463132160\zeta_3^4 - 71663616\zeta_3^5 + 180173002752\zeta_5 - 16655486976\zeta_3\zeta_5 \\
&-24628230144\zeta_3^2\zeta_5 - 2895575040\zeta_3^3\zeta_5 + 19278176256\zeta_5^2 - 9619845120\zeta_3\zeta_5^2 \\
&+2504494080\zeta_3^2\zeta_5^2 + \tfrac{882108048384}{175}\zeta_5^3 + 45602231040\zeta_7 + 14993482752\zeta_3\zeta_7 \\
&-12034759680\zeta_3^2\zeta_7 + 1406730240\zeta_3^3\zeta_7 + 30605033088\zeta_5\zeta_7 + 21217637376\zeta_3\zeta_5\zeta_7 \\
&-\tfrac{1309941061632}{275}\zeta_5^2\zeta_7 - 13215327552\zeta_7^2 - 4059901440\zeta_3\zeta_7^2 - 69762034944\zeta_9 \\
&+23284599552\zeta_3\zeta_9 - 3631889664\zeta_3^2\zeta_9 - 11032374528\zeta_5\zeta_9 - 6666706944\zeta_3\zeta_5\zeta_9 \\
&-23148129024\zeta_7\zeta_9 - 10024051968\zeta_9^2 - 54555179184\zeta_{11} + \tfrac{10048541184}{5}\zeta_3\zeta_{11} \\
&-726029568\zeta_3^2\zeta_{11} - 8975463552\zeta_5\zeta_{11} - 22529041920\zeta_7\zeta_{11} - \tfrac{1437993422496}{175}\zeta_{13} \\
&+\tfrac{1504385419392}{35}\zeta_3\zeta_{13} - 30324602880\zeta_5\zeta_{13} - \tfrac{151130039581392}{875}\zeta_{15} - 41375093760\zeta_3\zeta_{15} \\
&-\tfrac{196484147423712}{275}\zeta_{17} + 309361358592\zeta_{19} - 1729880064Z_{11}^{(2)} - \tfrac{1620393984}{5}\zeta_3 Z_{11}^{(2)} \\
&-131383296\zeta_5 Z_{11}^{(2)} + \tfrac{138107420928}{175}Z_{13}^{(2)} + \tfrac{3543865344}{35}\zeta_3 Z_{13}^{(2)} - \tfrac{5716780416}{7}Z_{13}^{(3)} \\
&-\tfrac{674832384}{7}\zeta_3 Z_{13}^{(3)} + \tfrac{48227088384}{175}Z_{15}^{(2)} + \tfrac{3581880576}{25}Z_{15}^{(3)} + 754974720Z_{15}^{(4)} \\
&-\tfrac{854924544}{11}Z_{17}^{(2)} + \tfrac{4963244544}{55}Z_{17}^{(3)} + \tfrac{818159616}{275}Z_{17}^{(4)} + \tfrac{175363688448}{1925}Z_{17}^{(5)}\,.
\end{aligned}
\tag{A.2}
$$

finite

$\frac{\sqrt{\lambda}}{4\pi}$	$\Delta_{S=2}(\lambda)$	$\frac{\sqrt{\lambda}}{4\pi}$	$\Delta_{S=2}(\lambda)$
0.1	4.115506377945221056840042671851	0.2	4.418859880802350962250362876243
0.3	4.826948662284842304671283425271	0.4	5.271565182595898008221528540034
0.5	5.712723424787739030626966875973	0.6	6.133862814488691819595425762346
0.7	6.531606077852440195886557953690	0.8	6.907504206024567515828872789717
0.9	7.2641695874391127748396398539	1	7.60407071704738848334286555
1.1	7.9292942641568451632186264	1.2	8.241563441147703542676050
1.3	8.54230287229506674486342	1.4	8.8326999393163090494514
1.5	9.11375404891588560886	1.6	9.386314656368554140399
1.7	9.65111042653013781471	1.8	9.9087717085593508789
1.9	10.1598480131615473641	2	10.4048217434405061127
2.1	10.6441190951617575972	2.2	10.878118797537726796
2.3	11.107159189584305149	2.4	11.331544000504529107
2.5	11.551547111042160297	2.6	11.76741650605722239
2.7	11.97937757952067741	2.8	12.18763591669137588
2.9	12.3923796509149519	3	12.5937814717988565
3.1	12.7920003457144898	3.2	12.9871829973986392
3.3	13.1794651919629055	3.4	13.368972849208144
3.5	13.555823016292914	3.6	13.740124720157966
3.7	13.921979717391474	3.8	14.101483156227149
3.9	14.278724162943763	4	14.45378636296056
4.1	14.62674834530641	4.2	14.79768407780976
4.3	14.96666327925592	4.4	15.13375175384302
4.5	15.29901169250472	4.6	15.4625019450274
4.7	15.6242782663505	4.8	15.7843935399844
4.9	15.942897981092	5	16.099839321454

Table 1: Conformal dimension of Konishi operator

strong

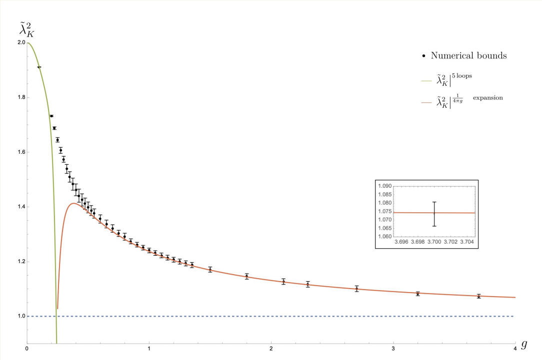
$$
\Delta_{\text{Konishi}} = 2\sqrt[4]{\lambda} - 2 + 2\sqrt[4]{\frac{1}{\lambda}} + \left(\frac{1}{2} - 3\zeta_3\right)\left(\frac{1}{\lambda}\right)^{3/4} + \left(6\zeta_3 + \frac{15\zeta_5}{2} + \frac{1}{2}\right)\left(\frac{1}{\lambda}\right)^{5/4}
$$
$$
+ \left(-\frac{81\zeta_3^2}{4} + \frac{\zeta_3}{4} - 40\zeta_5 - \frac{315\zeta_7}{16} - \frac{27}{16}\right)\left(\frac{1}{\lambda}\right)^{7/4} + \dots,
$$

Structure constants

- Less is Known for structure constants. Manly for Konishi Operator!
  - Perturbative computations up to 5-loops

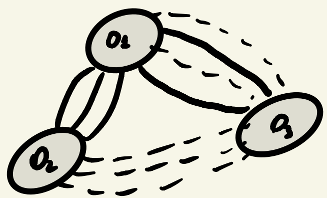
$$(c^2_{20'20'\mathcal{K}})^{(5)} = -64\,(7364 + 1812\,\zeta_3 - 414\,\zeta_3^2 + 2688\,\zeta_5 + 864\,\zeta_3\zeta_5 + 3717\,\zeta_7 + 5292\,\zeta_9)\,.$$

  - New numerical results at finite coupling
  - Strong coupling results from Virasoro-Shapiro Amplitude



Tree level

- In principle need to compute all Wick contractions



⚠ Grows very fast for large enough operators...

- We can do better using integrability!

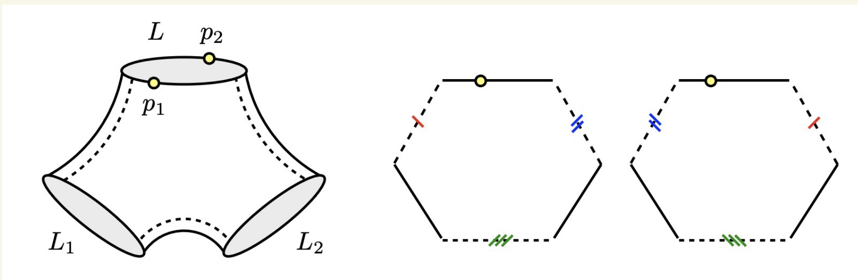
  - cutting $\quad |\{u_i\}\rangle = \sum_{\alpha \cup \bar{\alpha} = \{u_i\}} H(\alpha, \bar{\alpha}) |\alpha\rangle_\ell \otimes |\bar{\alpha}\rangle_r$
  - flipping $\quad \langle \{v_j\}|$
  - sewing $\quad \langle \{v_i\} | \{u_i\} \rangle$

For 1 non BPS and 2 BPS:

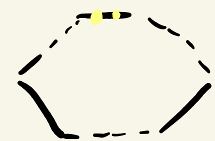
$$C_{123} = \frac{1}{N} \sum_{\beta \cup \bar{\beta}} (-1)^{|\bar{\beta}|} \# \underbrace{H(\beta) H(\bar{\beta})}_{\text{Hexagons}}$$

C_{IJK} From Integrability?

- Integrability based approach using Hexagons.



Fundamental building block is the Hexagon form factor



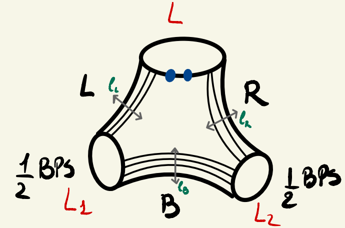
can also be used for higher point functions!

- We will be mostly interested in C_{IJK} where we have two protected and one non-protected operator

$$\langle \mathrm{Tr}[Z_1^{L_1}]\, \mathrm{Tr}[Z_2^{L_2}]\, \mathrm{Tr}[D^S Z^L] \rangle$$

Asymptotic Hexagons

- Let us first focus on large Op!



$$l_L = \frac{L_1 + L - L_2}{2}$$

$$l_R = L - l_L \qquad l_B = \frac{L_1 + L_2 - L}{2}$$

Structure constant can be obtained by gluing back the seams.
This entails to inserting an infinite set of mirror states.

$$|\mathcal{H}|^2 = \prod_{i,j,k}^{N_L N_R N_B} \frac{W^{L}_{(u_i)}\, W^{R}_{(u_j)}\, W^{B}_{(w_k)}}{P_{a_i b_j}(u_i, v_j)}$$

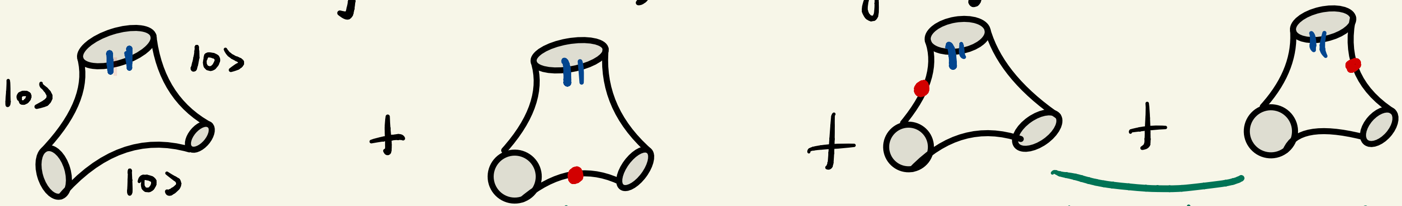
$$\boxed{C^{\bullet\circ\circ} = \underbrace{\mathcal{N}}_{\text{Normalization}} \times \underbrace{\sum\!\!\!\!\!\!\int_L \sum\!\!\!\!\!\!\int_R \sum\!\!\!\!\!\!\int_B e^{-l_L \mathcal{E}_L}\, e^{-l_R \mathcal{E}_R}\, e^{-l_B \mathcal{E}_B}}_{\text{Sum + int, gluing back}}\, |\mathcal{H}|^2}$$

$$\sum\!\!\!\!\!\!\int = \sum_{N=0}^{\infty} \prod_{i=1}^{N} \sum_{a_i=1}^{\infty} \int \frac{du_i}{2\pi}\, \mu_{a_i}(u_i) \prod_{k<j} P_{a_i a_j}(u_i, u_j)$$

Energy of the mirror magnon

$$\mathcal{E} = \log(x^{+a} x^{-a})$$

o Schematically (For Konishi) when gluing back we have:



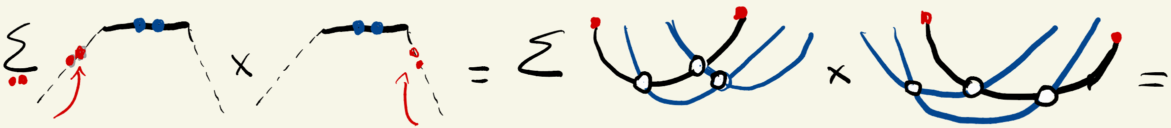
↳ $\boxed{\mathbb{W}_B \propto g^4}$

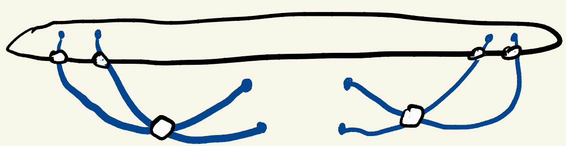
$\underbrace{\qquad}_{\mathbb{W}_L,\ \mathbb{W}_R \ \propto g^6}$

If $\ell_{L,R,B} \gg 1$ only contribution! $C_{000} = N$.

Distribute the physical excitations between the two hexagons

– Hexagon describes the creation of excitations over the different edges. When gluing back:

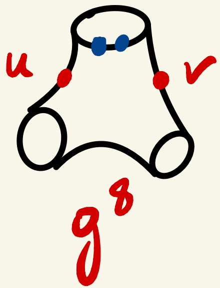




$\propto \ T(u^{+\gamma}) = \mathrm{str}\, S_{a_1}(u,z)$

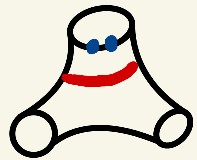
↳ SU(2|2) transfer matrices

Wrapping Effects

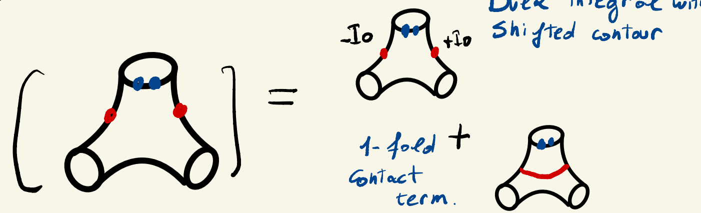


pole $u=v$ $\Longrightarrow$

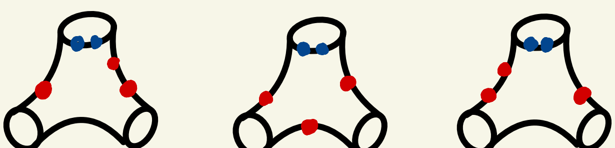
Badly divergent...



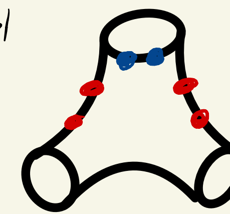
How can we fix this? Deform the contour and extract contact term (hard to do generally)



Many more terms at higher loops...



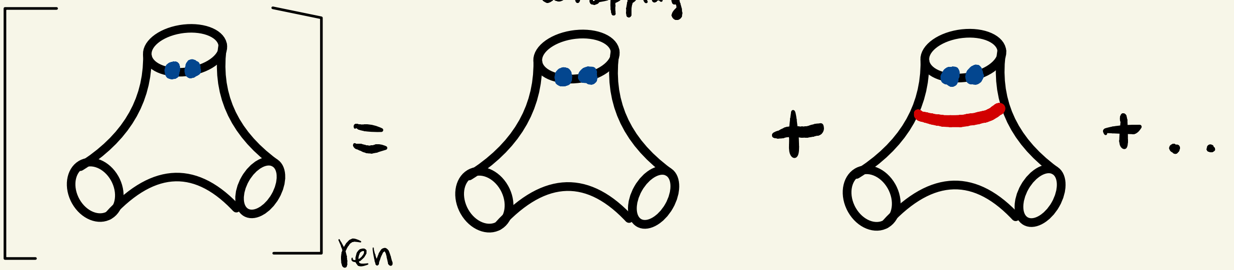
Or even more complicated double wrapping effects.



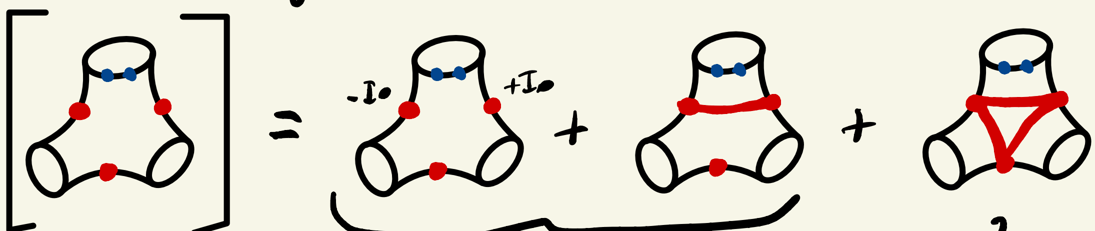
Idea: Compute as many as we can and try to extract some pattern.

∘ It seems that this localized terms modify the bare integrand

$$\mathbb{W}_{L,R} \Rightarrow \mathbb{W}_{asymp} + \underbrace{\delta\, \mathbb{W}}_{\text{wrapping}}$$



∘ By looking at the correction for the Bottom bridge

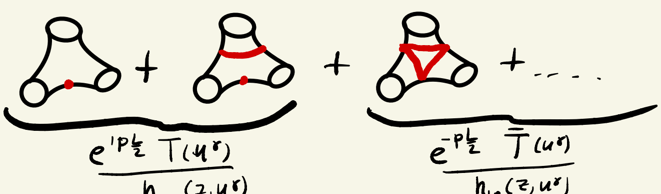


fit in the picture

?

$\Longrightarrow$ New effect uncovered @ 5-loops

$\mathbb{W}_B =$



$\Longrightarrow$ Second wave is suppressed! SU(4|4) transfer matrix.

Conjecture for Short Operators

- New building block is the non-perturbative T-matrices from the QSC.

$$\mathbb{W}_a^L(u) = e^{\frac{1}{2}L\mathcal{E}_a(u)}\frac{\mathbf{T}_{a,1}(u)}{\mathbf{T}_{a,0}^-(u)}$$

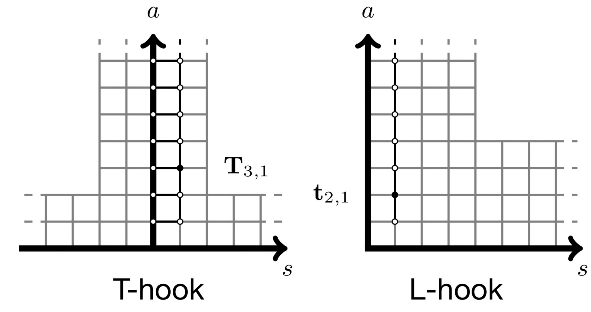
$$\mathbb{W}_a^R(u) = e^{\frac{1}{2}L\mathcal{E}_a(u)}\frac{\mathbf{T}_{a,1}(u)}{\mathbf{T}_{a,0}^+(u)}$$

$$\mathbb{W}_a^B(u) = e^{-\frac{1}{2}L\mathcal{E}_a(u)}\mathbf{t}_{a,1}(u)$$

$$\mathbf{T}_{a,s}^{\pm} = \mathbf{T}_{a,s}\left(u \pm \tfrac{i}{2}\right)$$

They are solutions of the hirota eq

$$T_{a,s}^+ T_{a,s}^- = T_{a+1,s}T_{a-1,s} + T_{a,s+1}T_{a,s-1}$$



Test @ Weak Coupling

- First non-trivial test is 5-loop. Still cumbersome to compute the integrals for the LR bridge. We can focus on the Bottom one

$$\mathcal{R} = \frac{C^{\circ\circ\circ}}{\lim_{\ell_B\to\infty} C^{\circ\circ\circ}} = 1+\sum_{a=1}^{\infty}\int\frac{du}{2\pi}\,\mu_a(u)\,e^{-\ell_B E_a(u)}\,\mathbb{W}_a^B(u) + \dots$$

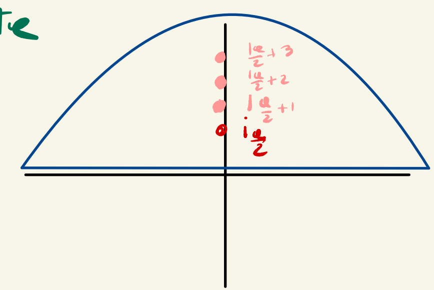
$$\mathbb{W}_a^B = t_{a,1} = -\sum_{i=1}^{4} Q_i^{[+a]}\,\tilde{Q}^{[-a]\,i}$$

- The integrand then takes the general form

$$\mu_a(u)\,e^{-\ell_B E_a(u)}\,\mathbb{W}^B = \sum c_i\, a\, u^{\nu}\,\frac{\eta^{[+a]}_{a_1\dots a_n}(u)\;\bar{\eta}^{[-a]}_{a_1\dots a_n}(u)}{(u-\frac{ia}{2})^{\ell}(u+\frac{ia}{2})^{\ell}}$$

$$\eta_{a_1\dots a_n}(u) = \sum_{0\le k_1<\dots<k_n}^{\infty}\frac{1}{(u+ik_1)^{a_1}\cdots(u+ik_n)^{a_n}} \qquad \eta_{a_1\dots a_n}(i) = (i)^{a}\,\zeta_{a_1\dots a_n}$$

We can compute the integral by Residues



We can now compute R for different values of ℓ_B

g^{10} Matches with PT

$$\boxed{\ell_B = 1} \to 8064\,\zeta_3 + 864\,\zeta_3^2 - 720\,\zeta_5 - 9936\,\zeta_3\zeta_5 + 8022\,\zeta_7 - 54432\,\zeta_9$$

$$\ell_B = 2 \quad - 1728\,\zeta_3^3 + 26880\,\zeta_6 - 36360\,\zeta_5^2 - 9360\,\zeta_3\zeta_5 - 64512\,\zeta_3\zeta_7 + 24738\,\zeta_7 + 64992\,\zeta_9 - 471240\,\zeta_{11}$$

g^{12}

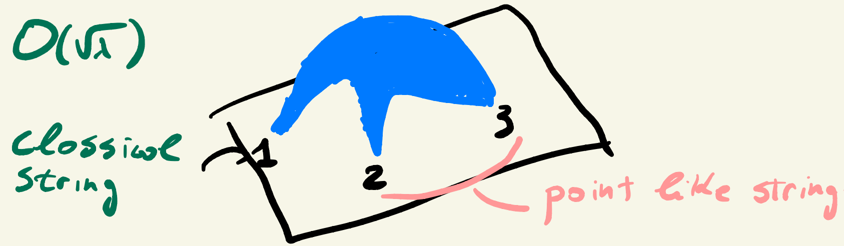
$$\ell_B = 3 \quad \frac{20086}{5}\zeta_{3,5,3} - 20088\,\zeta_{5,3}\zeta_3 - 5184\,\zeta_3^3 - 40392\,\zeta_3^2\zeta_5 + 33120\,\zeta_5^2 + 9480\,\zeta_7 - 82152\,\zeta_3\zeta_7 - 380736\,\zeta_5\zeta_7 + 159012\,\zeta_9 - 329680\,\zeta_3\zeta_9 + \frac{2158038}{5}\zeta_{11} + 3418272\,\zeta_{13}$$

g^{14}

Classical strings

- Expansion for $L, S, \Delta = O(\sqrt{\lambda})$

$$\log C^{\circ\circ\circ} \simeq -\text{Area}$$



- Several non-trivial checks already known. Let us focus on R.

$$\operatorname{Log} R_{\text{string}} = \int_{u_+} \frac{du(x)}{2\pi} \sum_{j=1}^{4} \left[\mathrm{Li}_2\left(\xi e^{i\tilde{p}_j}\right) - \mathrm{Li}_2\left(\xi e^{i\hat{p}_j}\right) \right]$$

$$\xi = e^{-(L_1+L_2)\,\varepsilon(x)/2}$$

$$u(x) = g\left(x + \frac{1}{x}\right)$$

- Using QSC we can obtain the same result from our conjecture. Important to have $SU(4|4)$ T-matrices otherwise missing the second Li_2 ...

Recap

- We have seen the hexagon conjecture and how it works for "large" Operators
- Wrapping can be accounted for case by case, this is complicated.
- Solution (Based on Known results) use QSC T-matrices as they encapsulate already the full tower of wrapping effects. Tested against known perturbative data and holography.

Other Regimes?

- Having expressed everything in terms of QSC building blocks allows us to study other regimes, if solution of QSC exist.

Let us look at the small spin limit.

$$\mathrm{Tr}[D^S Z^L] \xrightarrow{S\to 0} ?$$ Near to BPS, but not clear field theory understanding.

- Focusing on the $\mathbb{W}_B$ contribution

$$t_{a,1} = -\sum_{i=1}^{4} Q_i^{+a} \tilde{Q}^{(-a)i}$$
$$= -\sum_{b=1}^{2} P_b(u^{+a}) P^b(u^{-a}) + O(S^2)$$

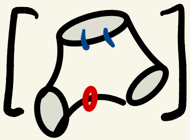
$$Q_i^+ \tilde{Q}^{i-} = P_b^+ P^{b-} + \dots$$

$\hookrightarrow P_b \propto \sqrt{S}$

At leading order in s, need to keep only quadratic terms.

$\mathbb{W}_B$ at Small Spin

We need to focus on a single mirror particle insertion

 all the rest is subleading in S.

$L = J =$ Length of un prot.

$$\sum_{a=1}^{\infty} \oint \frac{du}{2\pi}\mu_a(u)\, e^{-\ell_B \tilde{E}_a(u)}\, t^a(u) \longrightarrow S\int_0^{\infty} \frac{dt\, e^t}{(1-e^t)^2} \oint \frac{dx\,dy}{(2\pi)^2} \frac{t_J(x,y)}{(xy)^{\ell_B+\frac{1}{2}}(xy-1)}$$

Where: $x = x^{+a}$ $y = x^{-a}$ and

$$\mathbf{t}_J(x,y) = \frac{i}{(xy)^{J/2}}\left[\sum_{n=1}^{J/2} \mathrm{I}_{2n-1}\left(x^J\left(x^{1-2n}-y^{2n-1}\right)+y^J\left(x^{2n-1}-y^{1-2n}\right)\right)\right.$$
$$\left. + \sum_{n=1}^{\infty} \mathrm{I}_{2n-1}(x^{1-2n}-y^{2n-1})(y^J-x^J)\right],$$

$$\mathrm{I}_x = \frac{2\pi\, I_x(4\pi g)}{J\, I_J(4\pi g)}$$

I is Bessel function

Very simple to integrate

ℓ_B	$C_{123}^{\circ\circ\bullet}\|^B$ for $J=2$
1	$3g^4(4\zeta_2\zeta_3+5\zeta_5)-48g^6(\zeta_3\zeta_4+\zeta_2\zeta_5+7\zeta_7)+4g^8(15\zeta_4\zeta_5+63\zeta_3\zeta_6-56\zeta_2\zeta_7+1470\zeta_9)$
2	$4g^6(-3\zeta_3\zeta_4+9\zeta_2\zeta_5+14\zeta_7)+84g^8(\zeta_3\zeta_6-4\zeta_2\zeta_7-20\zeta_9)$
3	$2g^8(-30\zeta_4\zeta_5+56\zeta_2\zeta_7+105\zeta_9)$

Week Coupling check

- For $L=2$ i can interpolate between different values of S

$$C_{123}^{\circ\circ\bullet}|_{l_B=1}^{B} = g^4\left(c_{1|4} + c_{2|4}\zeta_3\right) + g^6\left(c_{1|6} + c_{2|6}\zeta_3 + c_{3|6}\zeta_5\right) + \mathcal{O}(g^8)\,,$$

The coefficent can be written in terms of Harmonic sums

$$\begin{aligned}
c_{1|4} =& 4\left(\mathrm{S}_{-2}^2 - 2\mathrm{S}_{-3}\mathrm{S}_1 - 2\mathrm{S}_{-2}\mathrm{S}_1^2 - 2\mathrm{S}_1\mathrm{S}_3 - \mathrm{S}_4 + 2\mathrm{S}_{-3,1} + 4\mathrm{S}_1\mathrm{S}_{-2,1} + 2\mathrm{S}_{-2,2} + 2\mathrm{S}_{3,1} - 4\mathrm{S}_{-2,1,1}\right)\\
c_{2|4} =& 24\mathrm{S}_1\\
c_{1|6} =& \frac{32}{3}\left(-6\mathrm{S}_{-6} + 3\mathrm{S}_{-3}^2 - 30\mathrm{S}_{-5}\mathrm{S}_1 - 6\mathrm{S}_{-4}(\mathrm{S}_{-2} - 3\mathrm{S}_1^2) + 5\mathrm{S}_3^2 + 6\mathrm{S}_6 + 30\mathrm{S}_{-5,1} - 12\mathrm{S}_{-4,2} - 24\mathrm{S}_{-3,3} + \right.\\
& \mathrm{S}_1^3(\mathrm{S}_3 - 6\mathrm{S}_{-2,1}) - 48\mathrm{S}_3\mathrm{S}_{-2,1} + 12\mathrm{S}_{-2,1}^2 - 3\mathrm{S}_{-3}(4\mathrm{S}_{-2}\mathrm{S}_1 - 3\mathrm{S}_1^3 + 11\mathrm{S}_1\mathrm{S}_2 + 4(-4\mathrm{S}_3 + \mathrm{S}_{-2,1})) +\\
& 54\mathrm{S}_{4,-2} - 6\mathrm{S}_{4,2} + 6\mathrm{S}_{5,1} - 48\mathrm{S}_{-4,1,1} + 6\mathrm{S}_{-2}(\mathrm{S}_1^2\mathrm{S}_2 - 9\mathrm{S}_4 - 10\mathrm{S}_1\mathrm{S}_{2,1} + 2(\mathrm{S}_{-3,1} + \mathrm{S}_{-2,2} + 4\mathrm{S}_{3,1} -\\
& 2\mathrm{S}_{-2,1,1})) + 12\mathrm{S}_2(3\mathrm{S}_{-3,1} + \mathrm{S}_{3,1} - 6\mathrm{S}_{-2,1,1}) + 6\mathrm{S}_1^2(2\mathrm{S}_4 - 5\mathrm{S}_{-3,1} - 4\mathrm{S}_{-2,2} - \mathrm{S}_{3,1} + 6\mathrm{S}_{-2,1,1}) +\\
& 36\mathrm{S}_{-2,2,2} - 48\mathrm{S}_{-2,3,1} - 36\mathrm{S}_{2,-3,1} - 12\mathrm{S}_{2,3,1} - 48\mathrm{S}_{3,1,-2} + 12\mathrm{S}_{4,1,1} - 72\mathrm{S}_{-3,1,1,1} - 3\mathrm{S}_1(3\mathrm{S}_2(\mathrm{S}_3 -\\
& 6\mathrm{S}_{-2,1}) + 2(\mathrm{S}_5 + 5\mathrm{S}_{-4,1} - 6\mathrm{S}_{-2,3} + 4\mathrm{S}_{2,-3} - 2\mathrm{S}_{2,3} + 3\mathrm{S}_{4,1} - 11\mathrm{S}_{-3,1,1} - 2\mathrm{S}_{-2,1,-2} - 10\mathrm{S}_{-2,2,1} -\\
& \left. 10\mathrm{S}_{2,1,-2} - 3\mathrm{S}_{3,1,1} + 18\mathrm{S}_{-2,1,1,1})) + 72\mathrm{S}_{2,-2,1,1} - 24\mathrm{S}_{3,1,1,1} + 144\mathrm{S}_{-2,1,1,1,1}\right)\\
c_{2|6} =& -\frac{32}{3}\left(6\mathrm{S}_{-3} + 15\mathrm{S}_{-2}\mathrm{S}_1 - 4\mathrm{S}_1^3 + 9\mathrm{S}_1\mathrm{S}_2 + 4\mathrm{S}_3 - 12\mathrm{S}_{-2,1}\right)\\
c_{3|6} =& -240\mathrm{S}_1
\end{aligned} \tag{A.6}$$

$$S_{\pm a}^{(x)} = \sum_{i=1}^{x} \frac{(\pm 1)^i}{i^a}$$

$$S_{\pm a,b,c\ldots}^{(x)} = \sum_{i=1}^{x} \frac{(\pm 1)^a}{i^a} S_{b,c}^{(i)}$$

For $S \to 0$

$$C_{123}^{\circ\circ\bullet}|^{B} = s\left[3g^4(4\zeta_2\zeta_3 + 5\zeta_5) - 48g^6(\zeta_3\zeta_4 + \zeta_2\zeta_5 + 7\zeta_7) + \mathcal{O}(g^8)\right] + \mathcal{O}(s^2)\,.$$

Matches!

Summary

- Small spin expansion can be easily implemented in our formulation. Lots of perturbative checks. Simple enough we can write all-loop result
- Strong coupling accessible via numerics, we could extract results up to g^{-3}
- Combining small spin expansion + classical strings we are able to extract C_{IJK} up to g^{-2}!

Conclusions and Outlook

- Presented conjecture for computing C_{IJK} using hexagons for short operators
- Our formalism easily extends to other regimes (small spin). We were able to extract C_{IJK} at strong coupling combining it with classical strings

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- Can we tackle other regimes? (Large spin)
- Can we simplify our formulation further!

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