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Fibre Inflation in Large Volume Compactifications

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GREECE

PRELUDE

- The main objectives of this talk: discuss moduli stabilisation and implement the scenario of inflation in string theories
- After some introductory concepts I will talk about fiber inflation in large volume compactifications

- In string theory, inflation can be driven by specific scalars, which are called moduli fields. These are associated with the compactification of the extra dimensions.

- Low energy effective models require such fields to be stabilised, otherwise gauge couplings and other SM parameters would not have a definite value.

- To explain what the issues are, I will devote a few slides to present some introductory concepts.

A few facts about Cosmology and de Sitter Vacua

▲ *Major Observational Discovery* ~ 25 *years ago is that:*

Expansion of the Universe is Accelerating

This phenomenon is explained through the concept of

Dark Energy

▲ **In General Relativity Equations, dark energy**
is incorporated through a positive cosmological constant:

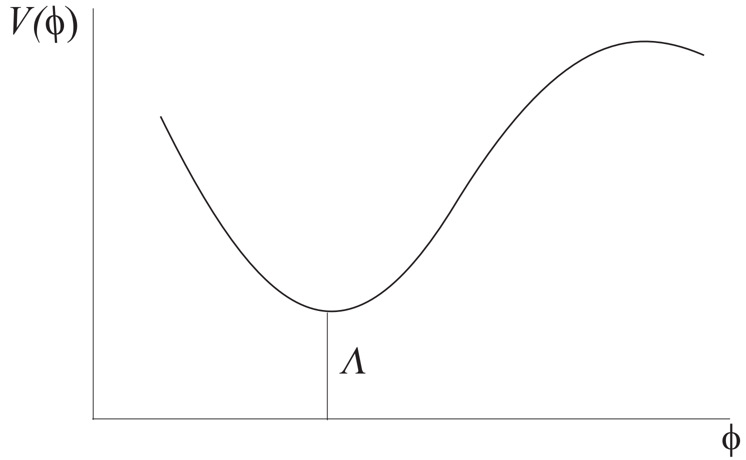
$$\mathbf{\Lambda} \approx 10^{-122} \quad (\textit{in 4-d } M_P^4 \textit{ Planck units})$$

$\mathbf{\Lambda}$ is interpreted as the **Vacuum Energy** which has a negative pressure, and leads to the accelerated expansion of the universe.

▲ From the Effective Field Theory point of view:

▲ ∃ a simple description in terms of:

Potential Energy $V(\phi)$ of a scalar field, ϕ

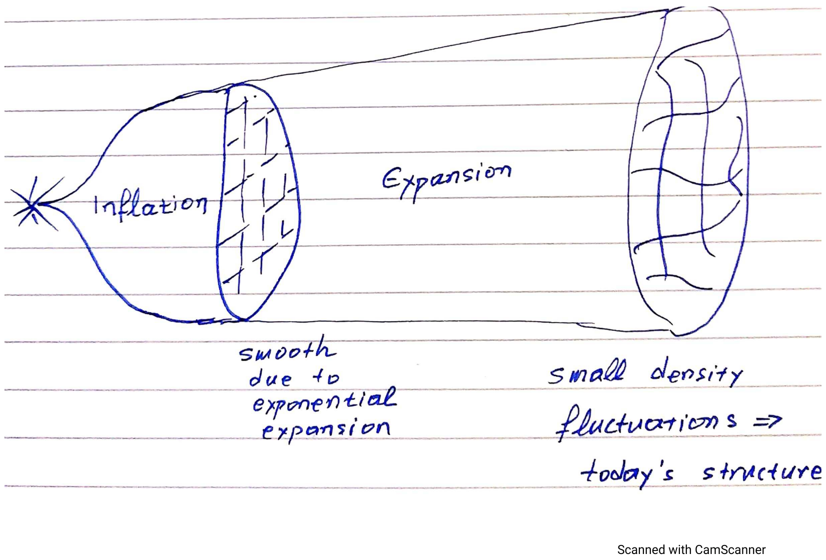


▲ $V(\phi)$ exhibits a (*possibly metastable*) positive minimum

corresponding to a so called:

▲ **de Sitter vacuum** ▲

With a few additional requirements the scalar potential:
$V(\phi)$ could be appropriate for cosmological inflation



Numerous EFT models have been constructed which successfully satisfy such constraints!

▲ The main **Challenge** is therefore to successfully implement the Inflationary Scenario in a viable String Theory Model

▲ *We shall see that there are novel ways to do this by virtue of the appearance of new scalar (moduli) fields.*

However: we are now confronted with:

▲ **New Issues in String derived EFT** ▲

The main challenge is that **compactifications** are characterised by **large numbers** of massless moduli;

▲ ... *In general:*

Deformations of the **Compactifications**,
correspond to
massless scalar fields
at the **Effective Field Theory** level

▲ In four dimensions this might create problems with fifth forces and other cosmological issues, thus there are two main...

▲ Tasks ▲

▲ Generate a potential and assure positive mass-squared for all moduli fields, a project usually refer to as:

$\Rightarrow$ *$\mathcal{M}$oduli $\mathcal{S}$tabilisation* $\Leftarrow$

▲▲ Look for possible **Inflaton** candidates among the **moduli**

★ **Viable inflationary scenarios in** ★

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Type-IIB String Theory

▲ Some Moduli in Type IIB String Theory ▲

1. ▲ Dilaton $e^{\phi} = \frac{1}{g_s}$, (g_s : *string coupling*)
   Controls the worldsheet perturbative expansion of the theory

2. ▲ C_p: p-form potentials, KB-field B_2 and field strengths:

$$F_{p+1} = dC_p,\ F_3 = dB_2$$

▲ Scalars C_0, ϕ $\rightarrow$ *combined to* **axion-dilaton** *modulus:*

$$S = C_0 + i\, e^{\phi} \rightarrow C_0 + i{g_s}^{-1}$$

3. U^i, *Complex Structure (CS)* moduli$\cdots$*related to shape* $\rightarrow$
   ... analogous to the complex structure τ of the 2-torus $\mathcal{T}^2$

4. T_i : Kähler (*size*) moduli analogous to the overall size of $\mathcal{T}^2$.

$$T_i = c_k - i\tau_k$$

The Potential(s)

▲ Low energy dynamics can be captured by a holomorphic superpotential W, and a real Kähler potential K

The Gugov-Vafa-Witten superpotantial:

$$\mathcal{W}_0 = \int \mathbf{G_3} \wedge \Omega(U_a), \qquad (\mathbf{G_3} := F_3 - SH_3) \tag{1}$$

The Kähler potential:

$$\mathcal{K}_0 = -\log[-i(S-\bar{S})] - 2\log\left(\mathcal{V}(\tau_k)\right) - \log[-i\int \Omega \wedge \bar{\Omega}] \tag{2}$$

▲ The F-term contributions to the scalar potential of 4D $\mathcal{N}=1$ from the type IIB encoded in

$$\boxed{V = e^{\mathcal{K}}(K^{\mathcal{A}\overline{\mathcal{B}}}(D_{\mathcal{A}}W)(D_{\overline{\mathcal{B}}}\overline{W}) - 3|W|^2)}$$

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★★

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FIBRE INFLATION (FI)

▲ Two basic approaches will be analysed:

Non Perturbative

&

Perturbative

★ $\mathcal{A}$ ★

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Non-Perturbative Moduli Stabilisation

& Slow Roll Inflation

Moduli stabilisation in 4D type IIB effective supergravity models follows a **two-step procedure**.

▲ First, one fixes the CS moduli U^i and the axio-dilaton S by the leading order $W_0 \equiv W_{\rm flux}$ induced by the 3-form fluxes (F_3, H_3)

⋆ **No-scale** structure protects the Kähler moduli T_α
→ remain massless at tree-level.

▲▲ At a second step T_α can be stabilised by non-perturbative corrections in W and α' and string-loop (g_s) corrections in K:

$$W = W_0 + W_{\rm np}(S, T_\alpha),$$

$$K = K_{\rm cs} - \ln\left[-i\,(S - \bar{S})\right] - 2\ln\mathcal{U}, \quad (\mathcal{U} = \mathcal{U}(\mathcal{V}, \alpha', \cdots)) \quad (3)$$

★ **Fibre Inflation** models

have the following characteristics:

(*see refs 0808.0691, ..., 1709.01518*)

▲ The generic geometric set up includes $D3/D7$ branes and $O(3)/O(7)$ planes

▲ The internal (CY) volume is of the generic form

$$\mathcal{V} = f_{\frac{3}{2}}(\tau_i) - \sum_{j=1}^{N_s} \lambda_j {\tau_j}^{3/2}$$

with $\tau_k = -\mathrm{Im} T_k$

- $f_{\frac{3}{2}}$: degree $\frac{3}{2}$ homogeneous function of τ_i
- τ_i: "large" Kähler moduli (divisors) $i = 1, 2, \ldots N_l$.
- τ_j: "small" blow-up rigid divisors $j = 1, 2, \ldots N_s$.
- $N_l + N_s = h^{1,1}$.

Quantum Corrections

The GVW superpotential $\mathcal{W}_0$

$$\mathcal{W}_0 = \int G_3 \wedge \Omega(U_a)\,, \tag{4}$$

is corrected by **non-perturbative** (NP) contributions.

▲ NP contributions can be generated by divisors which are stable under perturbations and have fixed complex structures, i.e., **rigid** ones, such as del Pezzo (dP) divisors. Thus, generically

$$\mathcal{W} = \mathcal{W}_0 + \sum_{k}^{N_s} \mathcal{A}_k e^{-a_k T_k}, \tag{5}$$

generated by D-brane instantons and gaugino condensation.

The coefficients $\mathcal{A}_k$ may depend on complex structure moduli, and after CS stabilisation they are considered constants.

The Kähler potential

Leading ${\alpha'}^3$ corrections in the Kähler potential depend on χ:

$$\xi \;=\; -\frac{\zeta(3)}{4(2\pi)^3}\chi$$

The ${\alpha'}^3$ corrections are incorporated into the Kähler potential through the shift:

$$\hat{\mathcal{V}} \to \mathcal{U} = \hat{\mathcal{V}} + \frac{\xi}{2}\frac{1}{{g_s}^{3/2}} \;.$$

Then, the α' corrected Kähler potential acquires the form:

$$\mathcal{K}_{\alpha'} \;=\; -\log(-i(S-\bar{S})) - 2\log(\mathcal{U}) - \log(-i\int \Omega\wedge\bar{\Omega}), \quad (6)$$

Procedure and Conditions

Step 1: Overall Volume $\mathcal{V}$, and volumes of N_s small blow-up divisors τ_j are stabilised by corrections described above.

$$\Downarrow$$

Then $\exists\ N_l - 1 \equiv h^{1,1} - N_s - 1$ directions remain flat.

$\Rightarrow$ $(N_l - 1)$-natural inflaton candidates

Step 2: Subleading $\mathcal{O}(g_s)$ corrections due to KK exchange and winding modes fix the remaining d.o.f.

★ The potential for these moduli is flatter and thus suitable for slow roll inflation.

★ **A simple model with $h^{1,1} = 3$** (see e.g. 1801.05434)

In suitable divisor basis $\hat{D}_b, \hat{D}_f, \hat{D}_s$ with D_s 'diagonal' (i.e. only $k_{sss} \neq 0$, while $k_{ijs} = 0, \forall i \neq s \neq j$), the internal volume is:

$$\mathcal{V} = \lambda_1 \tau_b \sqrt{\tau_f} - \lambda_j {\tau_s}^{3/2}$$

▲ As previously α'^3 corrections for K and NP in

$$W = W_0 + A_s e^{-ia_s T_s},$$

fix two (out of three) Káhler moduli.

String Loop Effects

(*hep-th/0507131,...,0704.0737*)

String-loop effects known as KK and winding types generate new $V_{g_s}^{KK} + V_{g_s}^{W}$ subleading potential terms for τ_f.

Kähler Cone Constraints

★ The Kähler moduli space must be such that ensures a positive definite Kähler form: ⋆

$$\int_{C_i} J > 0$$

This Kähler Cone Condition (KCC) concerns all topologically **non-trivial effective curves** C_i in the internal manifold (*Mori Cone*).

⋆ Thus, whilst at leading order the would be inflaton τ_f remains flat, fixing of $\mathcal{V}$ and τ_s puts bounds on field range of τ_f.

For the canonical field $\varphi \sim \sqrt{2}/3 \log(\tau_f)$, these bounds imply:

$$\varphi \lesssim 2.5$$

Notice however, that for a successful slow roll inflation we need

$$\varphi \sim \mathcal{O}(10) M_{Pl}$$

★ $\mathcal{B}$ ★

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PERTURBATIVE FIBRE INFLATION

in collaboration with

S. Bera, D. Chakraborty, P. Shukla,

Phys.Rev.D 110 (2024) 10, 106009, 2406.01694

The **perturbative LVS**

(*Antoniadis, Chen, GLK 1909.10525, JHEP 2020*)

provides a new way to realise

Fibre Inflation

without implementing non-perturbative effects.

▲ Hence use of **rigid** divisors can be circumvented,
and,
Kähler Cone Conditions do not put strong bound on the inflaton's range.

We will demonstrate this feature by considering a compact connected manifold with smooth geometry, more concretely a K3-fibred CY orientifold with toroidal-like volume.

A Global Model

We consider a CY_3 with $h^{1,1} = 3$
(*polytope Id: 249 in the CY database of KS/hep-th 0002240*)

- Hodge numbers $(h^{2,1}, h^{1,1}) = (115, 3)$,
- Euler number $\chi = -224$.
- In the divisor basis $\{\hat{D}_1, \hat{D}_2, \hat{D}_3\}$, the Kähler form is

$$J = t^1 \hat{D}_1 + t^2 \hat{D}_2 + t^3 \hat{D}_3$$

- The only non-zero intersection is $k_{123} = 2$ so the volume $\mathcal{V} \propto \int J \wedge J \wedge J$ is

$$\mathcal{V} = \frac{1}{3!} k_{ijk} t^i t^j t^k = 2\, t^1\, t^2\, t^3 = \frac{1}{\sqrt{2}} \sqrt{\tau_1\, \tau_2\, \tau_3}$$

- The Kähler cone conditions are:

$$\text{KCC:} \qquad t^1 > 0, \quad t^2 > 0, \quad t^3 > 0. \tag{7}$$

Global Model:

Subleading Corrections

Among other things, the divisor intersection analysis shows

- ▲ There are three $D7$-brane stacks which intersect at $\mathbb{T}^2$
- ▲ Because $D7$-brane stacks intersect on non-shrinkable two-torii:

$$\Downarrow$$

$\exists$ string-loop effects of the winding-type:

$$V_{g_s}^{\mathrm{W}} = -\frac{\kappa|W|^2}{\mathcal{V}^3}\sum_a \frac{C_a^w}{t^a}$$

All contributions give rise to the following scalar potential:

$$V_{\rm eff} \approx V_{\rm up} + \frac{\mathcal{C}_1}{\mathcal{V}^3}\left(\hat{\xi} - 4\,\hat{\eta} + 2\,\hat{\eta}\,\ln\mathcal{V}\right) \tag{8}$$

$$+\frac{\mathcal{C}_2}{\mathcal{V}^4}\left(\mathcal{C}_{w_1}\tau_1 + \mathcal{C}_{w_2}\tau_2 + \mathcal{C}_{w_3}\tau_3 + \frac{\mathcal{C}_{w_4}\,\tau_1\tau_2}{2(\tau_1+\tau_2)}\right. \tag{9}$$

$$\left.+\frac{\mathcal{C}_{w_5}\,\tau_2\tau_3}{2(\tau_2+\tau_3)} + \frac{\mathcal{C}_{w_6}\,\tau_3\tau_1}{2(\tau_3+\tau_1)}\right) + \frac{\mathcal{C}_3}{\mathcal{V}^3}\left(\frac{1}{\tau_1}+\frac{1}{\tau_2}+\frac{1}{\tau_3}\right) \tag{10}$$

- Part (8) fixes the volume $\mathcal{V}$ (*Antoniadis,Chen, GKL 2018*).
- Parts (9) and (10) fix one more modulus τ_k at large value. Hence:
- two τ_i are integrated out, and $V_{\rm eff}$ only depends on one light modulus, $V_{\rm eff} = V(\tau_f) \Rightarrow \tau_f$ drives inflation

Inflationary dynamics:

Define the canonically normalised fields,

$$\varphi^{\alpha} = \frac{1}{\sqrt{2}} \ln \tau_{\alpha}, \; \alpha \in \{1, 2, 3\}, \quad \text{so that}$$

$$\mathcal{V} \propto e^{\frac{1}{\sqrt{2}}(\varphi^1 + \varphi^2 + \varphi^3)}$$

The scalar potential takes the form

$$V = \mathcal{C}_0 \left(\mathcal{C}_{\rm up} + \mathcal{R}_0 e^{-\gamma\phi} - e^{-\frac{\gamma}{2}\phi} + \mathcal{R}_1 e^{\frac{\gamma}{2}\phi} + \mathcal{R}_2 e^{\gamma\phi} \right), \qquad (11)$$

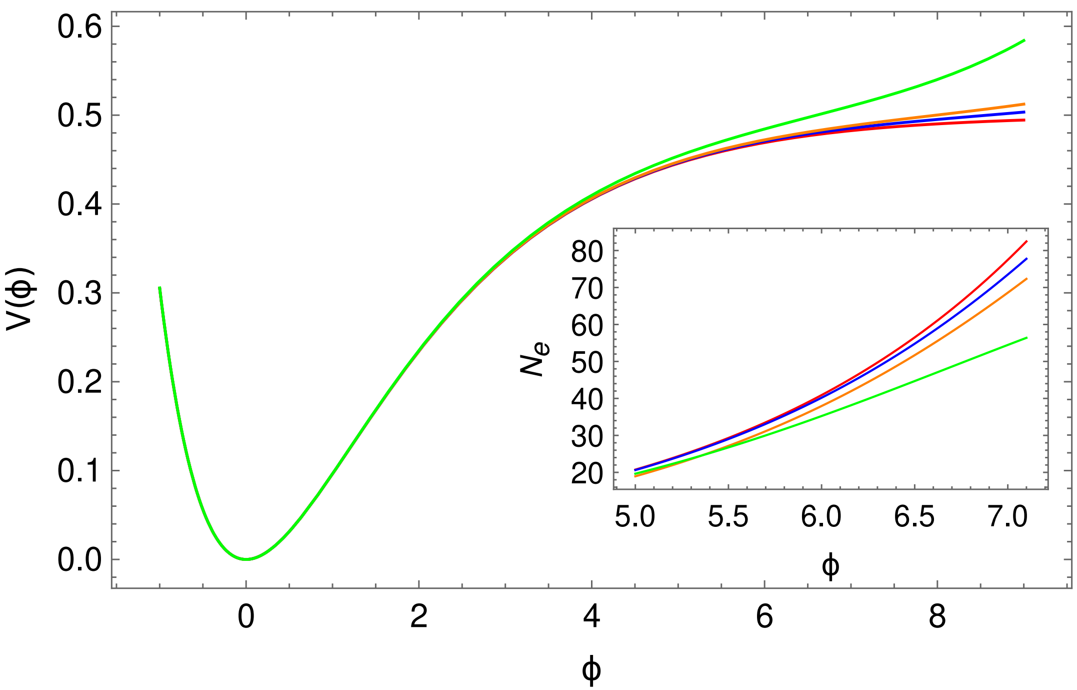
- The size of up-lift required for dS vacuum is $C_{up} = 1 - \mathcal{R}_0 - \mathcal{R}_1 - \mathcal{R}_2$
- $\overline{D3}$ up-lift not possible due to absence of $O(3)$-planes
- D7-brane or T-uplift (1512.04558) can be implemented.

A benchmark model:

$$\mathcal{C}_0 \sim 4 \times 10^{-10}, \;\; \mathcal{R}_1 \sim 10^{-6}, \;\; \mathcal{R}_2 \sim 10^{-7}$$

which correspond to string parameters:

$$|W_0| = 6, \;\; g_s = 0.28, \;\; \langle \mathcal{V} \rangle = 6 \times 10^3$$



Efolds, scalar perturbation amplitude, spectral index:

$$N_e{}^* = 58, \quad P_s = 2.1 \times 10^{-9}, \quad n_s{}^* = 0.966$$

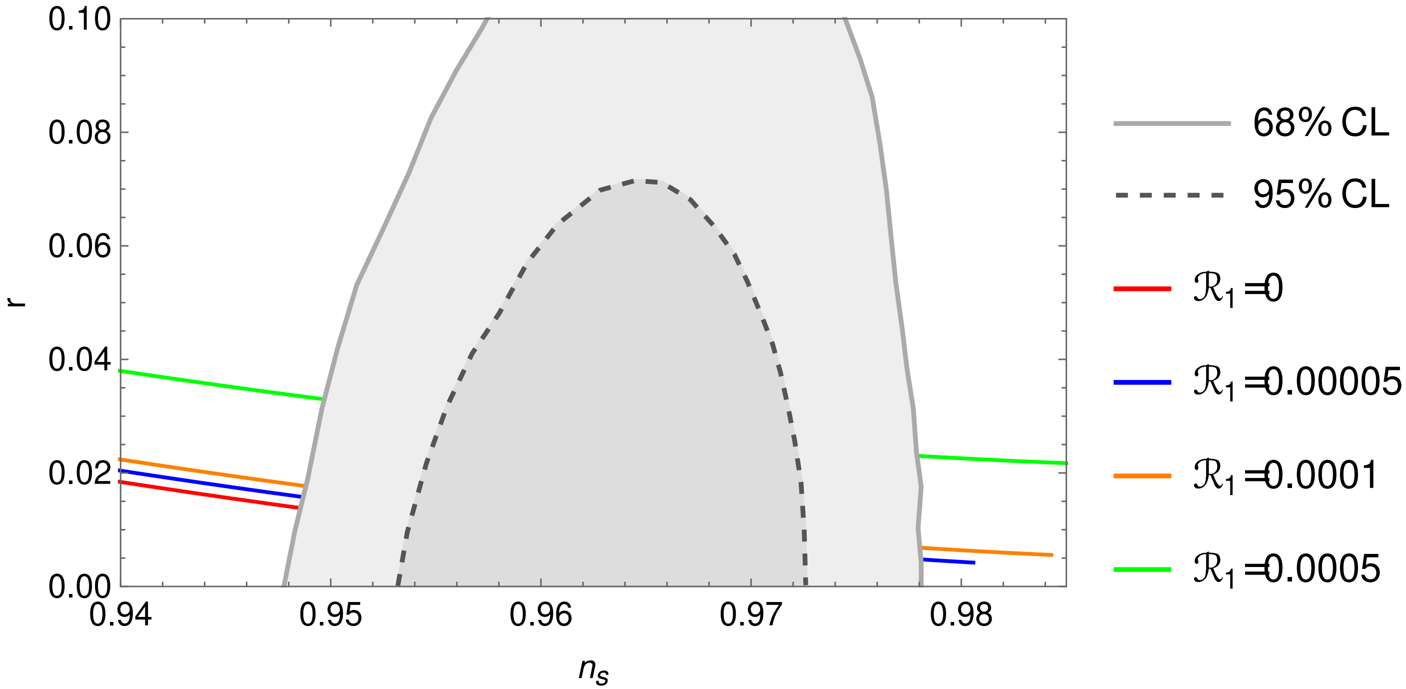


Figure 1: Plot of spectral index n_s vs tensor-to scalar ratio r.

CONCLUSIONS

In this talk, I have presented :

Fibre Inflation

▲ In Large Volume Compactifications

with **Perturbative Corrections** (PLVS)

• It was shown that Kähler Cone Conditions are milder and easy to satisfy in PLVS.

• This was instrumental for constructing a robust string scenario with Fibre Inflation

▲ The model has Global Embedding within simple CYs having:

• minimal number of Kähler moduli to accommodate inflation

• simple toroidal volume:

$$\mathcal{V} = \sqrt{\tau_1 \tau_2 \tau_3}.$$

THANK YOU FOR YOUR ATTENTION

APPENDIX

In String Theory:

multigraviton scattering generates higher derivative couplings in curvature

(Green et al, hep-th/9704145; Antoniadis, et al hep-th/9707013, Kiritsis, et al hep-th/9707018)

Leading correction term in type II-B action:

proportional to the fourth power of curvature:

$$\boxed{\propto R^4}$$

Reduction on $\mathcal{M}_4 \times \mathcal{X}_6$, (with $\mathcal{M}_4$ 4-d Minkowski) induces:

$$\Rightarrow \quad \underbrace{\frac{\alpha}{l_s^8} \int_{M_4 \times \mathcal{X}_6} e^{-2\phi} \mathcal{R}_{(10)}}_{\textit{standard } \mathcal{EH} \textit{ term}} + \underbrace{\frac{\beta}{l_s^2} \chi \int_{M_4} (\zeta(2) - \zeta(3) e^{-2\phi}) \mathcal{R}_{(4)}}_{\textit{induced } \mathcal{EH} \textit{ term}},$$

Induced Einstein Hilbert ($\mathcal{EH}$) term $\propto$ Euler characteristic:

$$\chi \propto \int R \wedge R \wedge R$$

▲▲ *this $\mathcal{EH}$ term possible in 4-dimensions only*!

▲▲ Introducing 7-branes ▲▲

Localised vertices can emit gravitons and KK-excitations in 6d

⇒ KK-exchange between graviton vertices and $D7$-branes

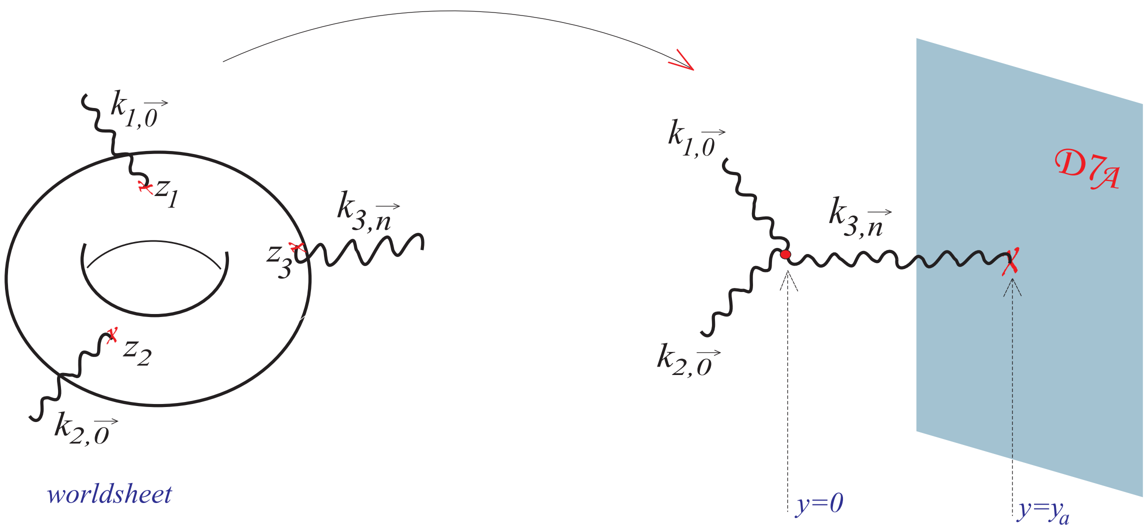


Figure: 3-graviton scattering (2 massless 1 KK) KK-propagating in 2-d towards $D7$

Corrections

(assuming 3 intersecting D7 branes)

$$\frac{1}{(2\pi)^3}\int_{M_4\times\mathcal{X}_6} e^{-2\phi}\mathcal{R}_{(10)} + \frac{4\zeta(2)\chi}{(2\pi)^3}\int_{M_4}(1+\sum_{i=1,2,3} e^{2\phi}T_i\log(R^i_\perp)\mathcal{R}_{(4)} \ ,$$

- ▲ T_i : D7-brane tension
- ▲ $R^i_\perp$: D7-transverse 2-dimension

Extracting the coefficients of the Kähler potential

$$\eta = -\frac{1}{2}g_s T_0 \xi \ \ ; \ \ \xi = -\frac{\chi}{4}\times\begin{cases}\frac{\pi^2}{3}g_s^2 & \text{for orbifolds}\\ \zeta(3) & \text{for smooth CY}\end{cases} \qquad (12)$$

Loop corrections $K_{g_s}^{KK}$ and $K_{g_s}^{W}$ to the Kähler potential arise respectively from exchanges of closed KK strings between parallel stacks of D-branes/O-planes or exchanges of winding strings between intersecting stacks of branes/O-planes

$$K_{g_s}^{KK} = g_s \sum \frac{C_i^{KK} t_i^{\perp}}{\mathcal{V}}, \qquad (13)$$

$$K_{g_s}^{W} = g_s \sum \frac{C_i^{W}}{\mathcal{V} t_i^{\cap}}, \qquad (14)$$

In our geometric configuration there are only intersecting D7-branes, hence only the second type of corrections are present.