The boundary entropy function for interface conformal field theories

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Contents

- Review of interface conformal field theory (ICFT)
- Holographic setup
    - Non-universality of g_{eff}
- Constraints on g_{eff}
    - Holographic proof of g_{eff}-theorem

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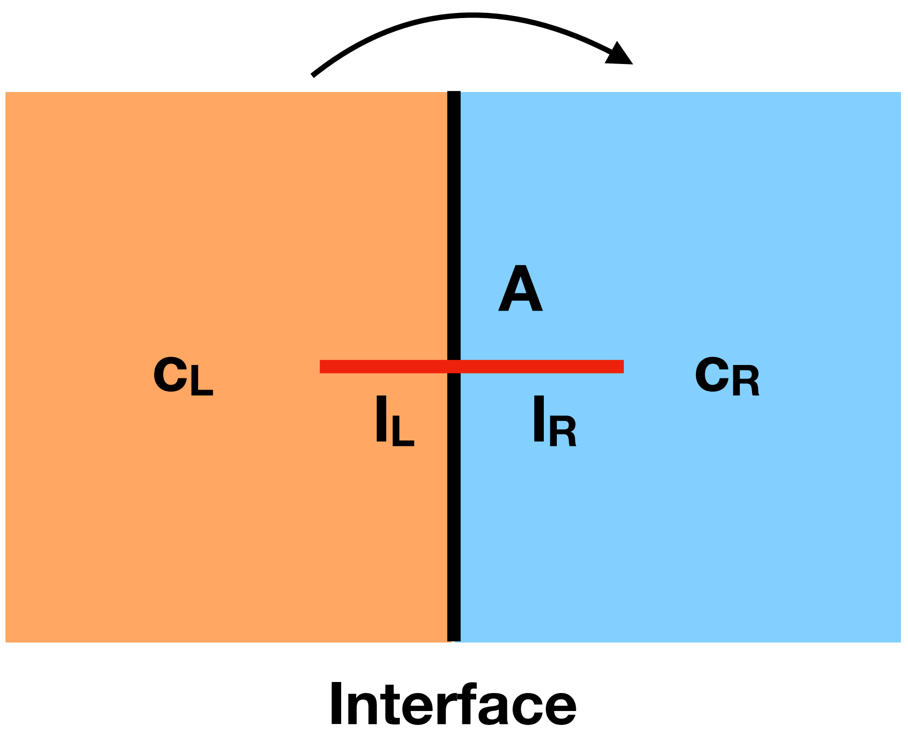
ICFT$_2$



Folding x→-x reduces to BCFT result

$$S_A = \frac{c_L + c_R}{6} \log(l/\varepsilon) + \log g$$

where *g* is the boundary entropy [Calabrese, Cardy]

For ICFTs, *g* gets promoted to an effective $g_{eff}(l_L/l_R)$ function

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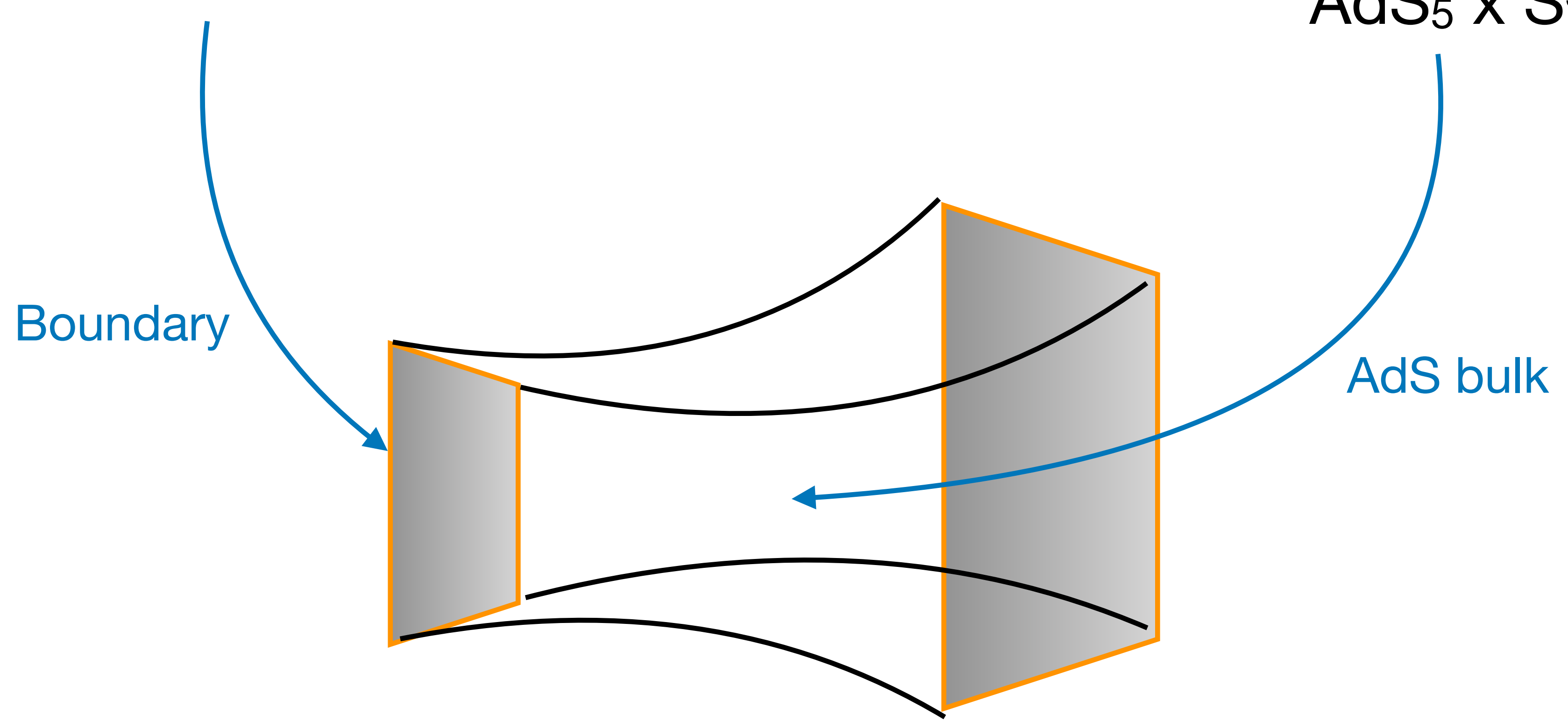
Holographic setup

Holography suggests:

Low dimensional non-gravitational theory ⟷ (Quantum) Gravity theory

AdS/CFT correspondence: [Maldacena]

N=4 SYM_4 ⟷ Type IIB string theory on $AdS_5 \times S^5$

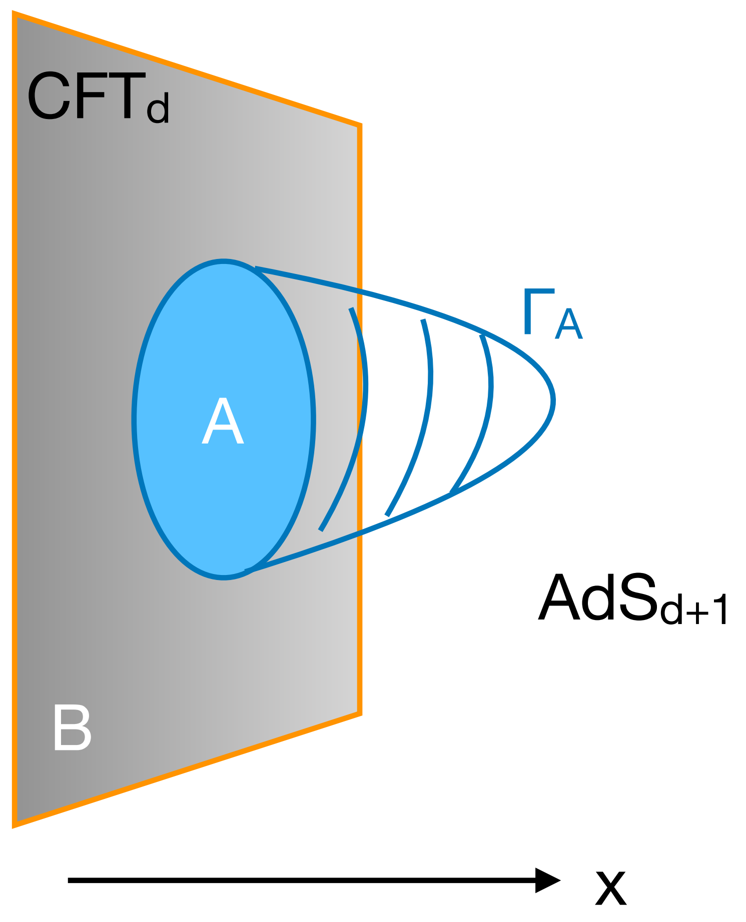


Holographic Entanglement Entropy

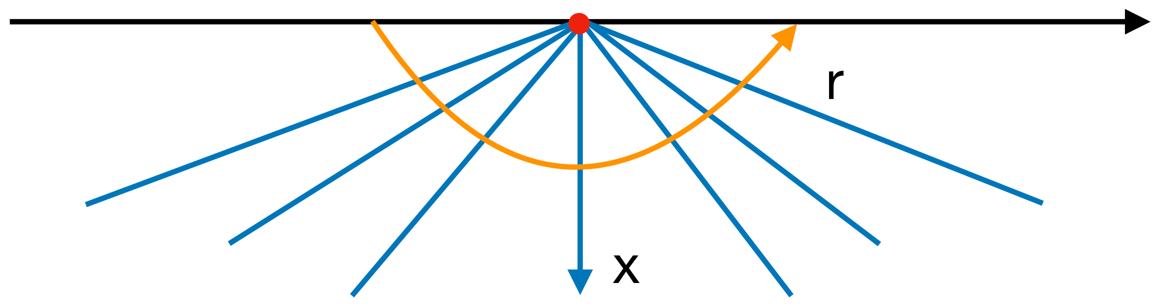
For static and asymptotically AdS spacetimes: [Ruy, Takayanagi]

$$S_A = \min_{\partial\Gamma_A = \partial A,\, \Gamma_A \approx A} \frac{\textbf{Area}(\Gamma_A)}{4G}$$

Γ_A is the minimal area surface



Holographic setup



Conformal defect spacetime in 2+1:

$$ds^2 = e^{2A(r)}\frac{dx^2 - dt^2}{x^2} + dr^2$$

pure AdS_3 $\quad e^{A(r)} = \cosh(r)$

Minimal RT-surface $\quad \mathcal{L} = \sqrt{\frac{e^{2A}}{x^2}(x')^2 + 1}$

Scale symmetry on AdS_2 slices yields

$$\frac{x'}{x} = \pm \frac{c_s e^{-A}}{\sqrt{e^{2A} - c_s^2}}$$

- $c_s=0$ → BCFT, $l_L=l_R=l/2$
- $0<c_s<e^{A^*}$ → non-trivial ICFT, $l_R>l_L$
- $c_s=e^{A^*}$ → $l_L=0$

Relation of c_s and l_L/l_R is highly non-trivial!

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Non-universality of g_{eff}

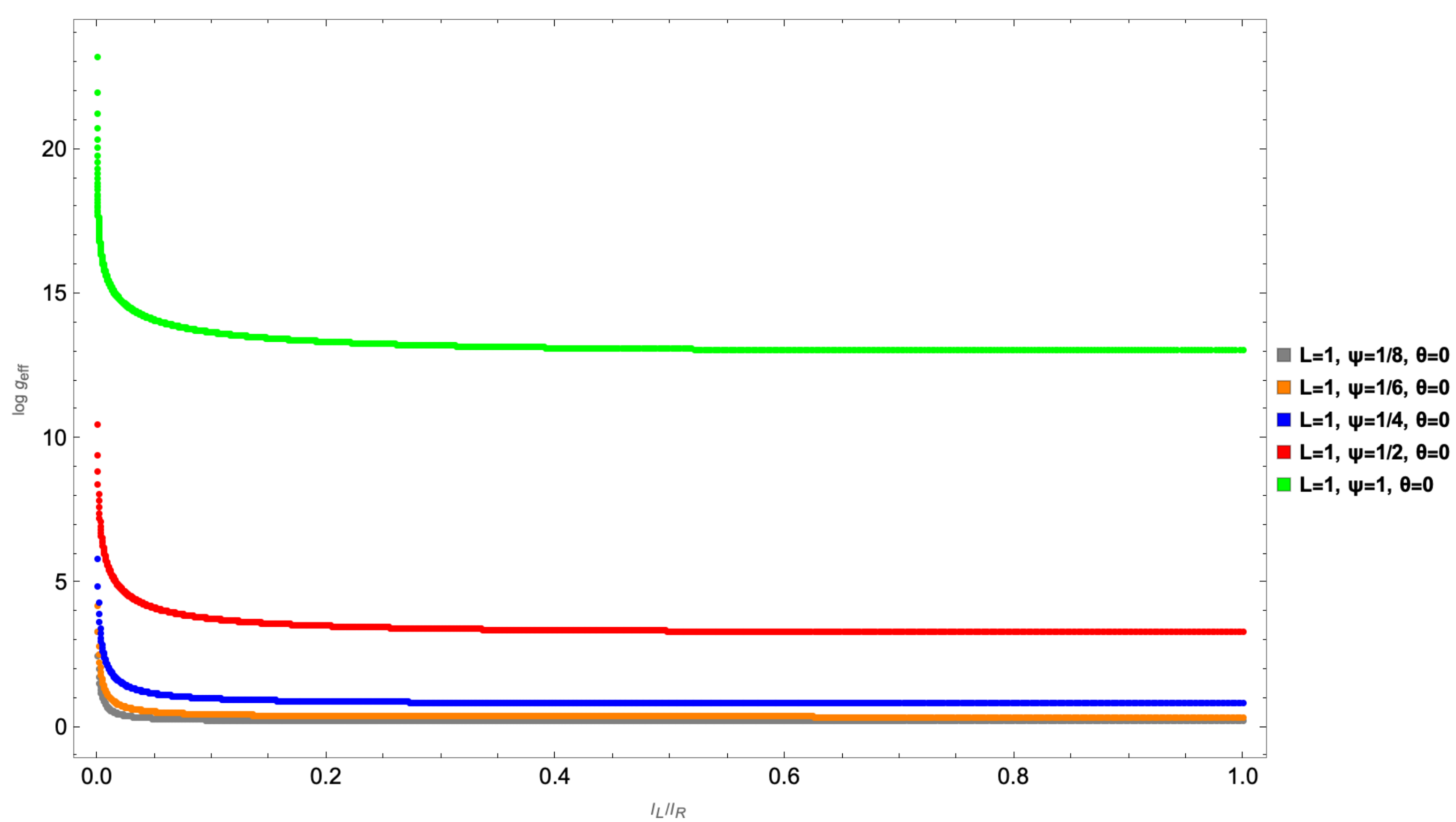
Super-Janus metric

$$ds^2 = R^2\left(\frac{\cosh^2(r+\psi)}{\cosh^2\psi\cosh^2\theta}ds^2_{AdS_2} + dr^2\right)$$

c_s takes values in

$$0 \le c_s \le e^{A_*} = \frac{1}{\cosh\psi\cosh\theta}$$

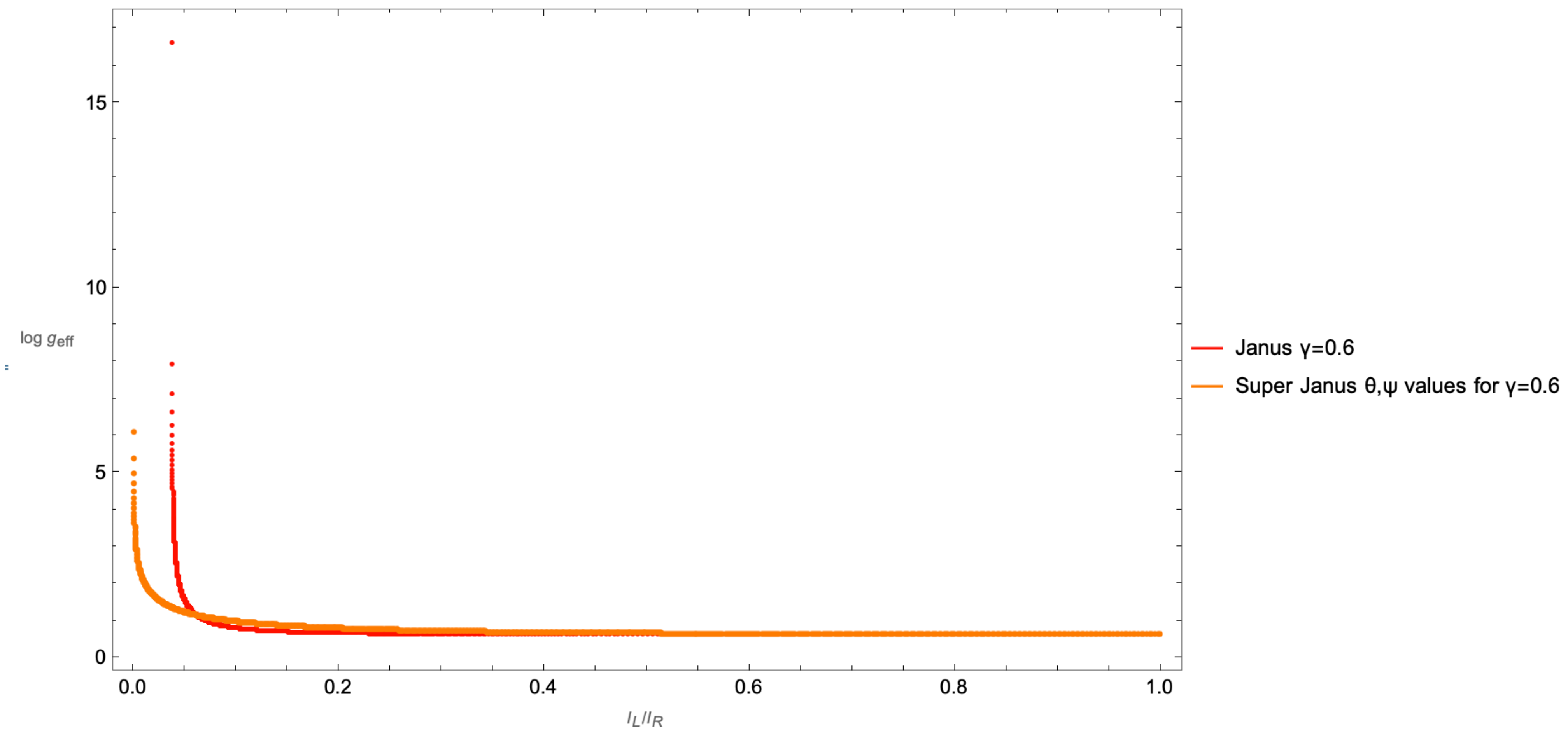
Solve for x(r), regularize intergation limits, the boundary entropy function is



- Curves blow at near the maximally asymmetric interval
- Attain the boundary entropy number *g* when ratio=1

Non-universality of g_{eff}

Is g_{eff} universal or not?



g_{eff} is **non-universal**, depends on the details of the underlying theory!

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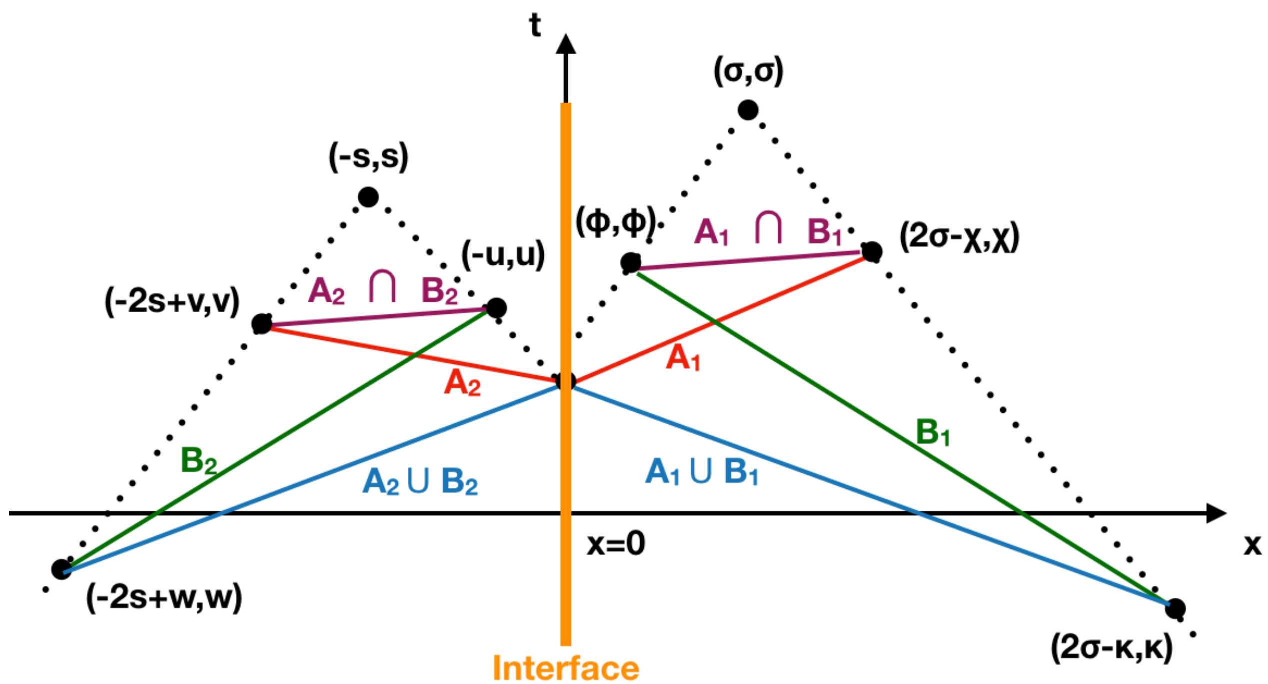
Constraints on g_{eff}

Strong subadditivity (SSA) inequality

$$S_A + S_B - S_{A\cap B} - S_{A\cup B} \geq 0$$

Proving entropic g_{eff}-theorem from SSA. Consider lightlike limits $u \to s$ and $\phi \to \sigma$:



$$S_A = S_{\rm int}(2\sigma - \chi) + S_{\rm int}(2s - v),$$
$$S_{A\cup B} = S_{\rm int}(2\sigma - \kappa) + S_{\rm int}(2s - w),$$
$$S_B = \frac{c}{3}\log\left[\frac{16}{\epsilon^4}(\sigma-\phi)(\sigma-\kappa)(s-u)(s-w)\right],$$
$$S_{A\cap B} = \frac{c}{3}\log\left[\frac{16}{\epsilon^4}(\sigma-\phi)(\sigma-\chi)(s-u)(s-v)\right],$$

$$S_{int} = \frac{c}{3}\log l/\varepsilon + \log g_{eff}$$

Constraints on g_{eff}

We take the limit $v \to w$ and $\chi \to \kappa$ and the SSA takes the form

$$-\eta \frac{dS_{\text{int}}}{d\psi}\bigg|_{\psi=2\sigma-\kappa} - \delta \frac{dS_{\text{int}}}{d\xi}\bigg|_{\xi=2\sigma-\omega} + \frac{c}{3}\left(\frac{\eta}{\sigma-\kappa} + \frac{\delta}{s-w}\right) \geq 0.$$

where η and δ are infinitesimal, positive constants.

Finally, we assume $w,\kappa<0$ and s and σ very small. The SSA gives the tightest bound

$$\eta \frac{d\log g_{\text{eff}}}{d\psi} + \delta \frac{d\log g_{\text{eff}}}{d\xi} \leq 0.$$

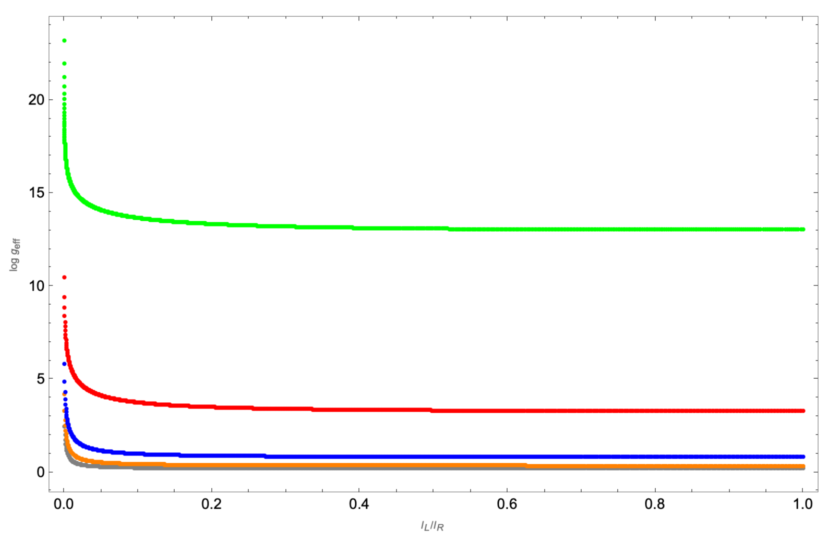
→

$$\frac{d\log g_{\text{eff}}(\rho)}{d\rho} \leq 0$$

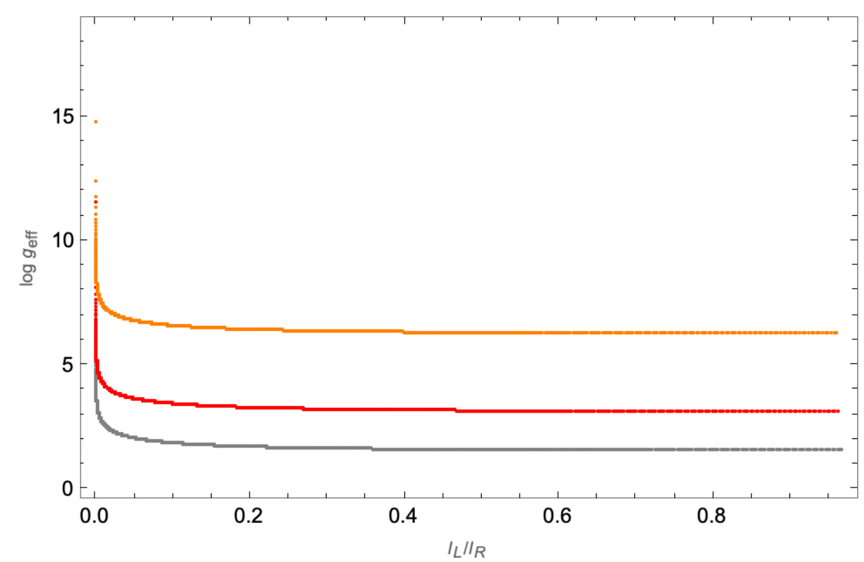
Constraints on g_{eff}

The monotonicity condition for g_{eff} : $$\frac{d \log g_{\text{eff}}(\rho)}{d\rho} \leq 0$$

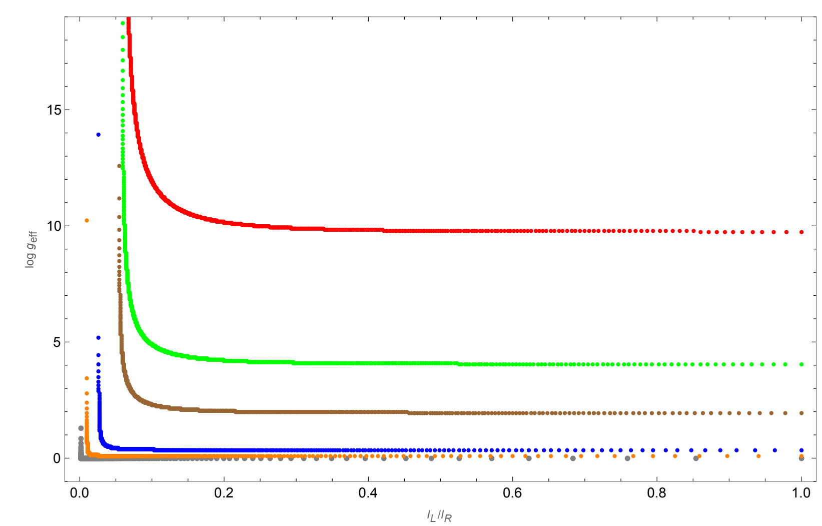
can be confirmed employing holography and in various examples:



Super-Janus



RS braneworlds



Janus

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Holographic proof of g_{eff}-theorem

Compact form of g_{eff}

$$\log g_{\text{eff}}(c_s) = \lim_{\epsilon \to 0} \left[\frac{1}{4G_N} \int_{-r_c^-}^{r_c^+} \frac{e^A}{\sqrt{e^{2A} - c_s^2}} dr - \log\left(\frac{2l_L}{\epsilon}\right) - \log\left(\frac{2l_R}{\epsilon}\right) \right]$$

$\longrightarrow$
$$\frac{d \log g_{\text{eff}}}{dc_s} = \frac{1}{4G_N} \int_{-r_c^-}^{r_c^+} \frac{c_s e^A}{(e^{2A} - c_s^2)^{3/2}} dr \geq 0$$

Scale symmetry

$$\frac{x'}{x} = \pm \frac{c_s e^{-A}}{\sqrt{e^{2A} - c_s^2}}$$

$\longrightarrow$
$$\frac{l_L}{l_R} = \exp\left(- \int_{-\infty}^{\infty} \frac{c_s e^{-A}}{\sqrt{e^{2A} - c_s^2}} dr\right)$$

$\longrightarrow$
$$\frac{d\,(l_L/l_R)}{dc_s} = -\frac{e^A e^{-\frac{c_s e^{-A}}{\sqrt{e^{2A} - c_s^2}}}}{(e^{2A} - c_s^2)^{3/2}} < 0$$

$\longrightarrow$
$$\frac{d \log g_{\text{eff}}}{d(l_L/l_R)} \leq 0$$

Things I find interesting

- Provide a c-theorem for the effective central charge
- Understand the replica trick for asymmetric intervals around the interface
- Higher dimensions?

Thank you very much !