# Energy and Pressure: Analogies Linking Hadrons, Superconductor Vortices, and Cosmological Constant

PRD 104,076010 (2021)
[arXiv:2103.15768],

PLB 849, 138418 (2023)
[arXiv:2302.11600]

- Cosmology – Friedmann equations and dark energy. Energy density and pressure.
- Analogy in hadrons: Energy-pressure relation of trace anomaly and confinement
- Analogy in vortices of type II superconductors
- Origin of the cosmological constant
- Pion mass and trace anomaly

Jul. 2, 2024
EIC-Asia, Shanghai

# Cosmology

- A constant cosmological constant is introduced by Einstein in the general relativity equation for a static universe.

$$R_{\mu\nu} + \frac{1}{2} R\, g_{\mu\nu} - \Lambda\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu} \qquad \Lambda = 4\pi G \rho$$

- Friedmann equations of Friedmann-Robertson-Walker scale parameter

$$(\frac{\dot{a}}{a})^2 = \frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \Omega_\Lambda + \frac{1-\Omega_0}{a^2}$$

radiation matter CC curvature

$$H(t) \equiv \dot{a}/a$$

$$\Omega_{r,m,\Lambda} \equiv \epsilon_{r,m,\Lambda}/\epsilon_c$$

$$\epsilon_c \equiv \frac{3c^2}{8\pi G} H^2$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\sum_i \epsilon_i + \epsilon_\Lambda + 3(\sum_i P_i + P_\Lambda))$$

$$\epsilon_\Lambda = \frac{\Lambda}{8\pi G} \qquad P_\Lambda = -\frac{\Lambda}{8\pi G}$$

- $P_\Lambda < 0$ anti-gravitates → acceleration

# Cosmological constant

- Equations of state: $P = \omega\epsilon$

  - Non-relativistic matter: $\omega \approx 0$
  - Radiation ($\gamma, \upsilon$): $\omega = 1/3$
  - Dark energy: $\omega < -1/3$
  - Cosmological constant: $\boxed{\omega = -1}$

*N.B.* $\Lambda\, g_{\mu\nu}$ in the GR equation - a salient feature

$$\omega = -1.00 \pm 0.04$$

- Puzzles:
  - Why is Λ > 0?
  - Identifying Λ as the vacuum energy, why so small compared to the plack energy density?

$$\epsilon_{\text{Plank}} \sim \frac{E_P}{l_P^3} \sim 3 \times 10^{132}\,\text{eVm}^{-3}$$

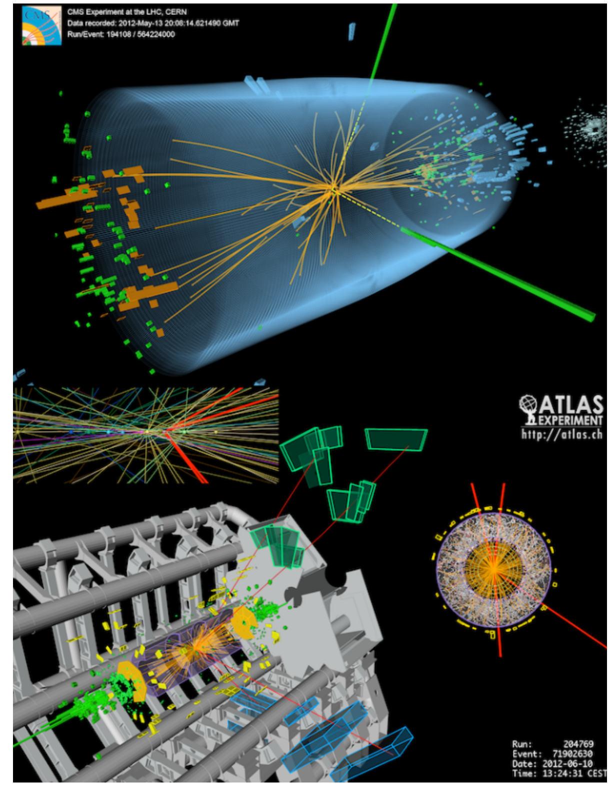
$$\frac{\epsilon_c}{\epsilon_{\text{Plank}}} \sim 10^{-123} \;\;!!!$$

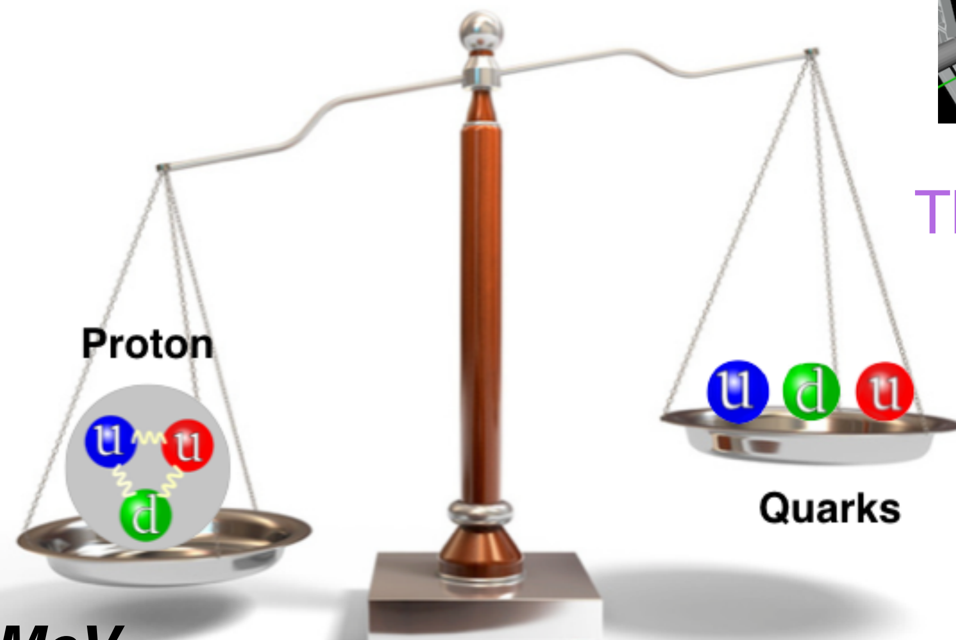

$$\Omega_{\text{baryon}} = 0.048, \Omega_{dm} = 0.262,$$

$$\Omega_r = 9.0 \times 10^{-5}, \Omega_\Lambda \approx 0.69$$

# Motivation

**Where does the proton mass come from, and how ?**




But the mass of the proton is ***938.272046(21) MeV.***

**~100 times of the sum of the quark masses!**

The Higgs boson make the u/d quark having masses (2GeV MS-bar):

$m_u = 2.08(9)\ MeV$

$m_d = 4.73(12)\ MeV$

Laiho, Lunghi, & Van de Water, Phys.Rev.D81:034503,2010

# Mass of Hadrons – Energy Momentum Tensor

- Energy momentum tensor

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2$$

- Mass from trace of EMT – scalar, frame independent, scale invariant

$$\langle P|T^{\mu\nu}|P\rangle = 2P^\mu P^\nu$$

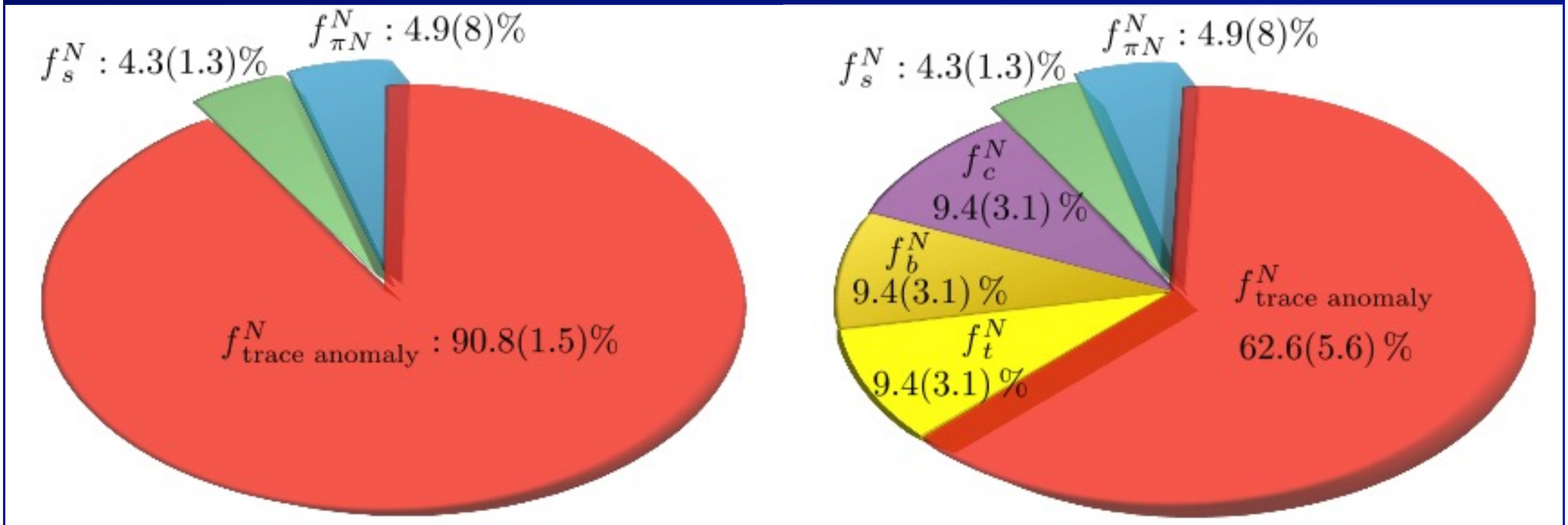
$$\frac{\langle P|\int d^3\vec{x}\,\gamma\,T^\mu_\mu(x)|P\rangle}{\langle P|P\rangle}| = M_N$$

$$T^\mu_\mu = \sum_f m_f\bar{\psi}_f\psi_f + \left[\frac{\beta(g)}{2g}G^{\alpha\beta}G_{\alpha\beta} + \sum_f m_f\gamma_m(g)\bar{\psi}_f\psi_f\right]$$

Chanowitz, Ellis, Crewther, Collin, Duncan, Joglekar

# Mass from Trace of EMT

- Lattice calculation of of quark condensate
  - Y.B. Yang et al ($\chi$ QCD) [arXiv: 1511.15089]; M. Gong et al, ($\chi$ QCD) [arXiv:1304.1194]
  - Overlap fermion ($Z_m$ $Z_s$ =1), 3 lattices (one at physical $m_\pi$)




$$m_h \langle N|\bar{\psi}_h \psi_h|N\rangle \sim -\frac{n_f}{3}\frac{\alpha_s}{4\pi}\langle N|G^2|N\rangle + \mathcal{O}(1/m_h)$$

$$f_f^N = \frac{m_f \langle N|\bar{\psi}_f \psi_f|N\rangle}{M_N}$$

$$\beta_0 = 11 - \frac{2}{3} n_f$$

Shifman, et al., Phys.Lett. B 78, 443 (1978)

Decoupling theorem: $$f_c^N + f_b^N + f_t^N + f_a^N \sim \sum_H \mathcal{O}_H(1/m_H)$$

# Rest Energy Decomposition from Hamiltonian

- Separate the EMT into traceless part and trace part (Ji, 1995)

$$T^{\mu\nu} = \overline{T}^{\mu\nu} + \frac{1}{4} g^{\mu\nu} (T^{\rho}_{\rho})$$

- Hamiltonian -- $H = \int d^3\vec{x}\, T^{00}(x)$

$$H_q(\mu) = \int d^3\vec{x}\, \Big(\frac{i}{4} \sum_f \bar{\psi}_f \gamma^{\{0} \overleftrightarrow{D}^{0\}} \psi_f - \frac{1}{4} T^{\mu}_{q\,\mu}\Big),$$

Quark energy (scale dependent)

$$H_g(\mu) = \int d^3\vec{x}\, \frac{1}{2}(B^2 + E^2),$$

Glue field energy (scale dependent)

$$H_{tr} = \int d^3\vec{x}\, \frac{1}{4}(T^{\mu}_{\mu}).$$

Caracciolo:1989pt, Makino:2014taa, DallaBrida:2020gux

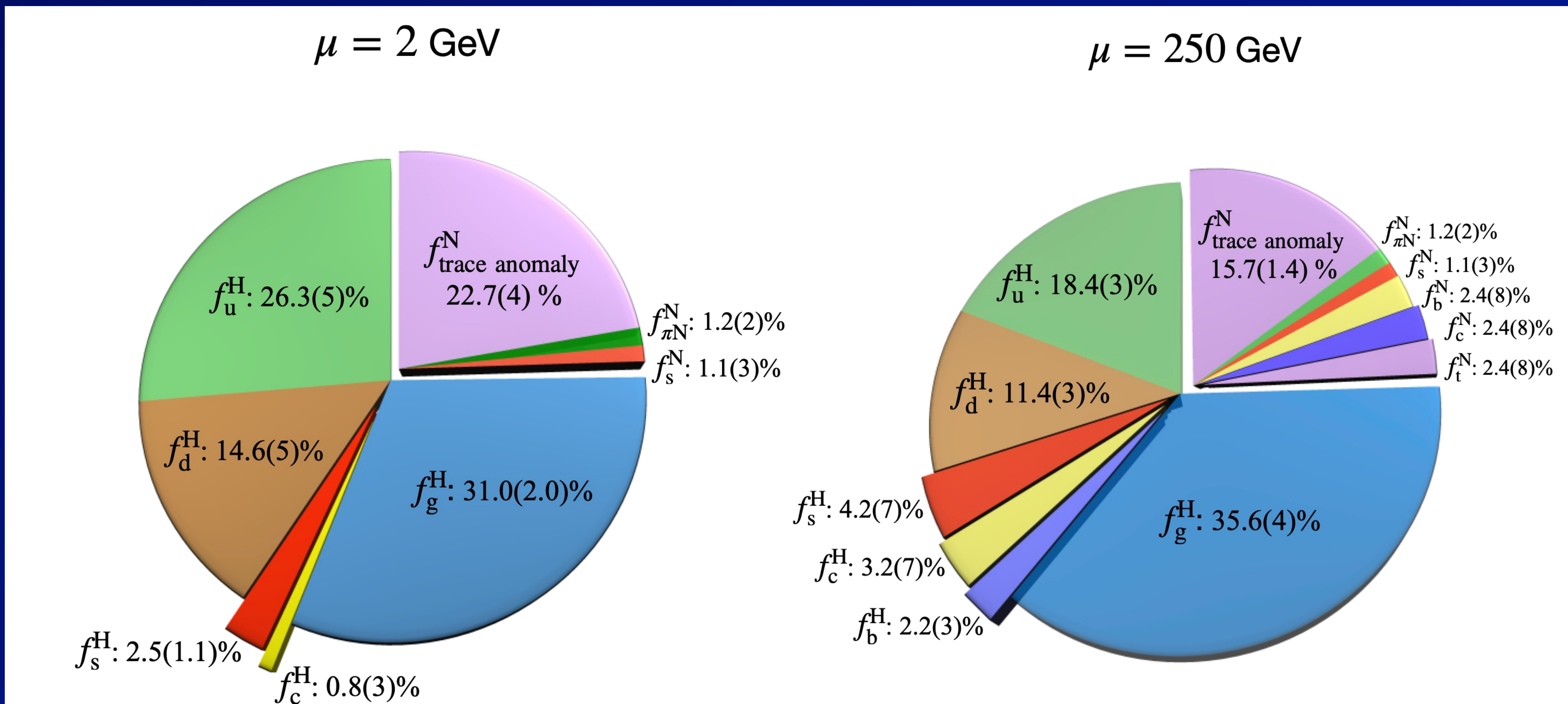
- Rest energy -- $E_0 = \langle H_{q_f}(\mu)\rangle + \langle H_g(\mu)\rangle + \langle H_{tr}\rangle,$

$$\langle H_{q_f}(\mu)\rangle = \frac{3}{4} \sum_f \langle x\rangle_f(\mu) M, \qquad \langle H_g(\mu)\rangle = \frac{3}{4}\langle x\rangle_g(\mu) M,$$

$$\langle H_{tr}\rangle = \frac{1}{4}\langle T^{\mu}_{\mu}\rangle = \frac{1}{4} M.$$

<x> - momentum fraction experimentally measurable

# Rest Energy Decomposition from Hamiltonian




$$f_f^H \quad = \quad \langle H_q \rangle / M = \frac{3}{4} \langle x \rangle_f(\mu), \qquad f_g^H = \langle H_g \rangle / M = \frac{3}{4} \langle x \rangle_g(\mu),$$

$$f_{\pi N}^N = \frac{1}{4} \frac{\sigma_{\pi N}}{M}, \qquad f_s^N = \frac{1}{4} \frac{\sigma_s}{M}, \qquad f_{\text{trace anomaly}}^N = \frac{1}{4} \frac{\langle H_{\text{ta}} \rangle}{M}$$

Y.B. Yang et al ($\chi$ QCD) [arXiv: 1511.15089]

Momentum fractions from CT18 (T.J. Hou et al, PRD, arXiv:1912.10053) at $\mu = 2$ GeV and 250 GeV.

# Rest Energy/Mass from Gravitational FF

- Gravitational Form factors from the EMT matrix elements

$$\langle P'|(T^{\mu\nu}_{q,g})(\mu)|P\rangle/2M_N = \bar{u}(P')[A_{q,g}(q^2,\mu)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}(q^2,\mu)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}q_\alpha}{2M_N} + D_{q,g}(q^2,\mu)\frac{q^\mu q^\nu - g^{\mu\nu}q^2}{M_N} + \bar{C}_{q,g}(q^2,\mu)M_N\, g^{\mu\nu}]u(P)$$

  - A(0) and A(0)+ B(0) : momentum and angular momentum - [Ji]
  - D(0): D term (deformation of space = elastic property) - [Polyakov]
  - C-bar term: pressure-volume work - [Lorce, Liu]

$$E_0 = \langle P|T^{00}_{q+g}|P\rangle|_{\vec{P}=0}/2M_N = A_{q+g}(0)M_N + \bar{C}_{q+g}(0)M_N$$

$$M_N = \langle P|T^{\mu}_{\mu}|P\rangle|_{\vec{P}=0}/2M_N = A_{q+g}(0)M_N + 4\bar{C}_{q+g}(0)M_N$$

$$A_{q,g}(0,\mu) = \langle x\rangle_{q,g}(\mu)$$

# Rest Energy/Mass from Gravitational FF

- Gravitational Form factors from the EMT matrix elements

$$3\bar{C}_{q,g}(0,\mu)M_N = -\langle P|(T^{ii}_{q,g})(\mu)|P\rangle|_{\vec{P}=0}/2M_N$$
$$= \langle P|(T_{q,g})^{\mu}_{\mu}(\mu) - (T^{00}_{q,g})(\mu)|P\rangle|_{\vec{P}=0}/2M_N$$

$$\bar{C}_q + \bar{C}_g = \frac{1}{4}(\sum_f f^N_f + f^N_a) - \frac{1}{4}(\langle x\rangle_q + \langle x\rangle_q) = 0$$

$\partial_\nu T^{\mu\nu} = 0$

$$E_0 = \langle P|T^{00}_{q+g}|P\rangle|_{\vec{P}=0}/2M_N = A_{q+g}(0)M_N + \bar{C}_{q+g}(0)M_N$$

$$= \frac{3}{4}(\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N + \frac{1}{4}(\sum_f f^N_f + f^N_a)M_N$$

Same as from Hamiltonian

$$M = \langle P|T^{\mu}_{\mu}|P\rangle|_{\vec{P}=0}/2M_N = A_{q+g}(0)M_N + 4\bar{C}_{q+g}(0)M_N$$

$$= (\sum_f f^N_f + f^N_a)M_N$$

Same as from trace of EMT

# Energy - Equilibrium Correspondence (EEC)

- Rest energy

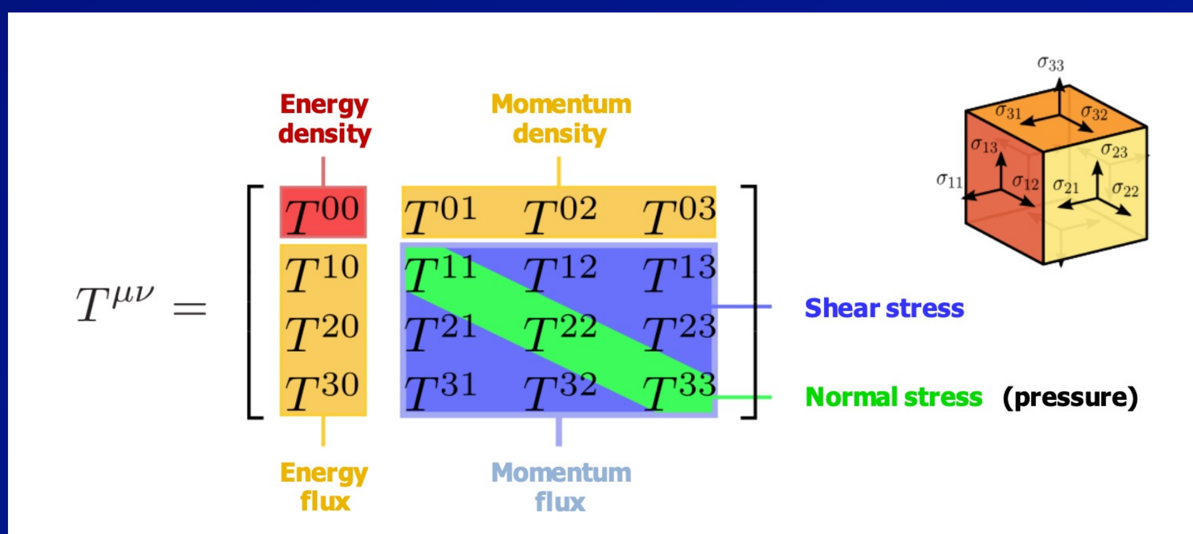
$$E_0 = \frac{3}{4}(\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N + \frac{1}{4}(\sum_f f_f^N + f_a^N)M_N$$

- Pressure balance – equilibrium

$$PV = \frac{1}{3}\langle P|(T^{ii}_{q,g})_{\mathrm{M}}(\mu)|P\rangle|_{\vec{P}=0}/2M_N = -\bar{C}_{q,g}(0,\mu)M_N$$

$$PV = -\frac{dE}{dV}V = -(\bar{C}_q + \bar{C}_g) = \frac{1}{4}(\langle x\rangle_q + \langle x\rangle_q) - \frac{1}{4}(\sum_f f_f^N + f_a^N) = 0$$

- Can one deduce the equation of state – E (V) ?



# Energy - Equilibrium Correspondence (EEC)

- Rest energy

$$E_0 = \frac{3}{4}(\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N + \frac{1}{4}(\sum_f f_f^N + f_a^N)M_N$$

- Pressure balance

$$PV = -\frac{dE}{dV}V = -(\bar{C}_q + \bar{C}_g) = \frac{1}{4}(\langle x\rangle_q + \langle x\rangle_q) - \frac{1}{4}(\sum_f f_f^N + f_a^N) = 0$$

- Inferred volume dependence $E = V^x, \;\; V dE/dV = dE/d(\log V) = x$

$$E_0 = E_T + E_S,$$

$$E_S = \frac{1}{4}[\langle H_m\rangle + \langle H_a\rangle] \propto V,$$

$$E_T = \langle H_{q_f}(\mu)\rangle + \langle H_g(\mu)\rangle \propto V^{-1/3}$$

$$PV = -\frac{d\,E_0}{d\,V}V = -E_S + \frac{1}{3}E_T = 0$$

# Trace Anomaly and Gluon Condensate

- Equation of state $E_0 = \epsilon V + \epsilon_K V^{-1/3}$, (cf. MIT Bag Model)

  where $\epsilon = \frac{E_S}{V}$, $\epsilon_K = E_T V^{1/3}$ are constants

- Picture: Nucleon is a bubble in the sea of gluon condensate, where

$$\epsilon = -\epsilon_{\rm vac}$$

*N.B.* $\langle OG_2\rangle_{\rm correlated} = \langle OG_2\rangle - \langle O\rangle\langle G_2\rangle$

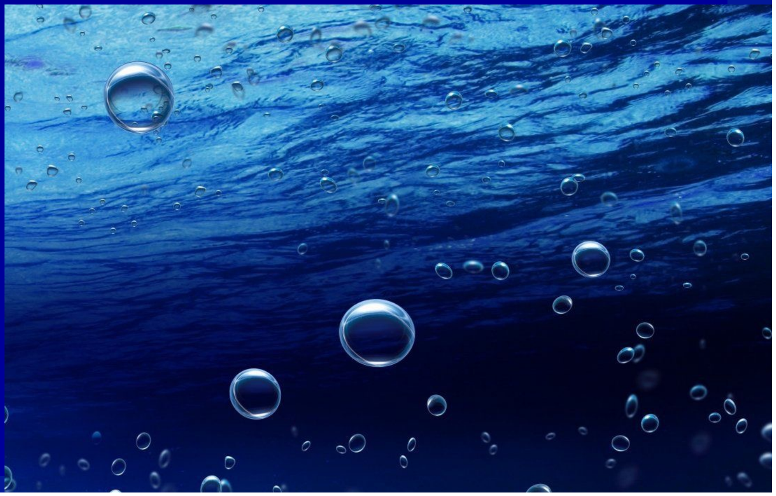
$$\epsilon_{vac} = \frac{\beta(g)}{2g}\langle 0|F^{\alpha\beta}F_{\alpha\beta}|0\rangle < 0$$

$$V = \frac{E_S}{|\epsilon_{\rm vac}|}$$

- Trace anomaly gives a negative constant pressure ⟹ confinement

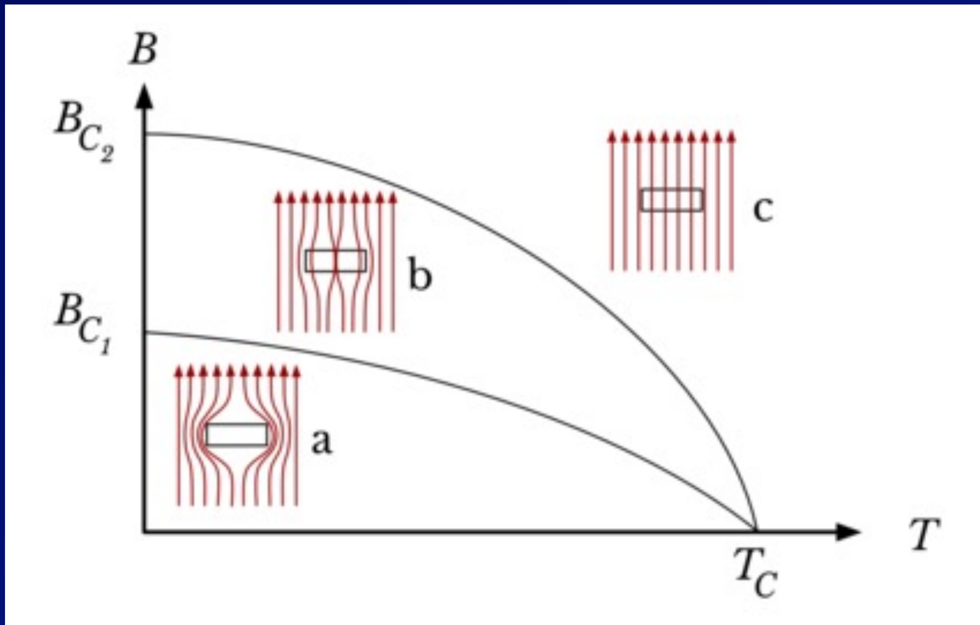
Same as in charmonium $V(r) = |\epsilon_{vac}|\,A\,r = \sigma\,r$

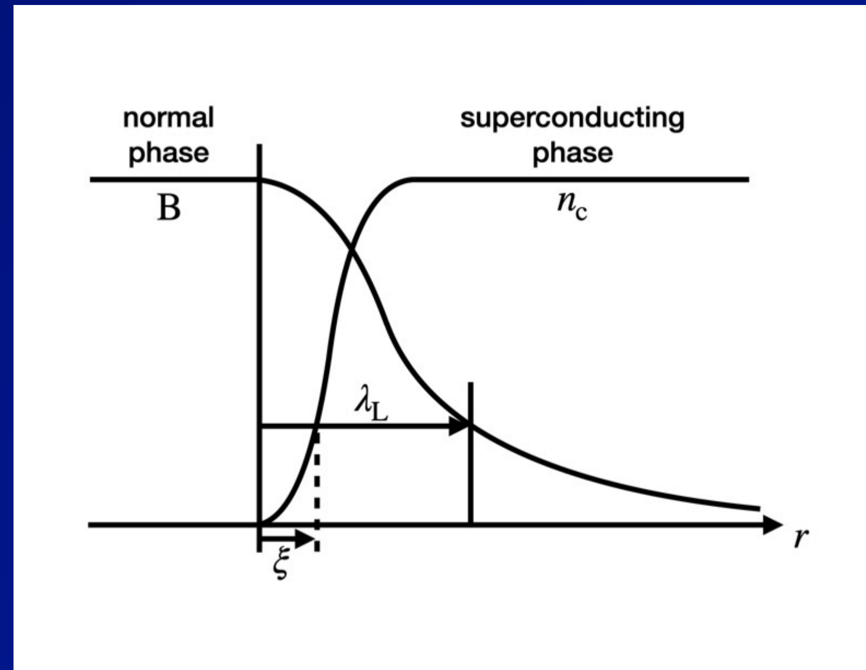
Bali ('97), Baker ('18)



- Many facets of color confinement
  - Dual superconductor
  - Magnetic monopole
  - Center vortices

# Type II Superconductor




**Ginzburg–Landau equations**

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m}(-i\hbar\nabla - 2e\mathbf{A})^2\psi = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} \ ; \ \mathbf{j} = \frac{2e}{m}\mathrm{Re}\{\psi^* (-i\hbar\nabla - 2e\mathbf{A})\psi\}$$

$$|\psi|^2 = n_s$$

**London penetration depth**

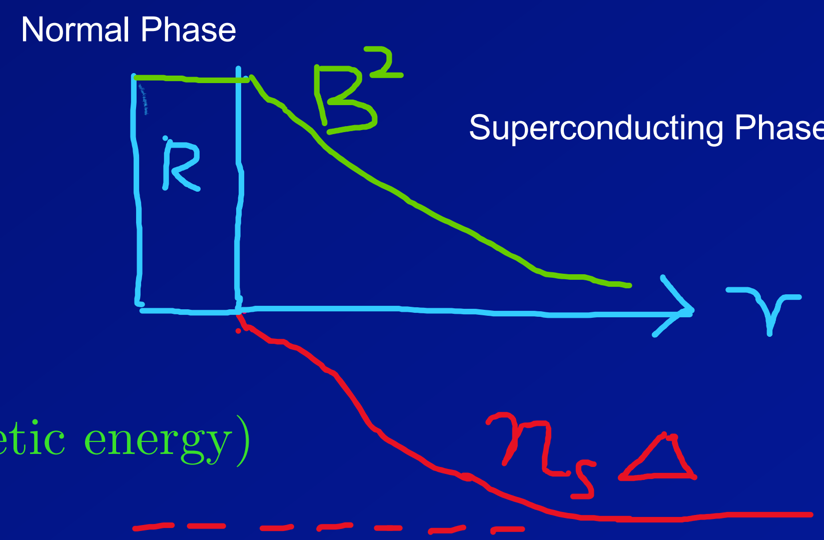
$$\lambda_L = \sqrt{\frac{m}{4\mu_0 e^2 \psi_0^2}}$$

Coherent length $\xi$

Type II: $\kappa = \lambda_L/\xi > 1/\sqrt{2}$

# Energetics and Equilibrium

- Type II superconductor




$$F = F_s + F_B + F_{sc}$$

$$F_s = \text{ cost of condensation energy}$$

$$F_B = \int dv\, B^2/2\mu_0 \ \text{(magnetic energy)}$$

$$F_{sc} = 1/2 \int dv\, \lambda_L^2 J_s \cdot J_s \ \text{(supercurrent kinetic energy)}$$

- Variational model (J.R. Clem, Jour. Low Temp. Phys. 18, 5/6 (1975))

$$\frac{|\psi|^2}{n_0} = \frac{n_s}{n_0} = \frac{\rho^2}{\rho^2 + R^2} \underset{\rho\to\infty}{\longrightarrow} 1$$

$$\frac{1}{\sqrt{2}H_c}\frac{E}{l} = \phi_0 H_c'/4\pi \ \text{ where } \phi_0 = hc/2e, \ \sqrt{2}H_c = \kappa\phi_0/2\pi\lambda_L^2$$

Equation of state

$$F'/l = \underbrace{\kappa R'^2/8}_{F_s} + \underbrace{1/8\kappa + K_0(R')/2\kappa R'\, K_1(R')}_{F_B + F_{sc}}, \ \text{ where } R' = R/\lambda_L$$

EEC $-\dfrac{dF'/l}{dA} A = 0$

## Superconductor Vortex

- $F_S$ – Cost of compensation energy
- $F_B$ -- Magnetic field energy
- $F_{SC}$ -- Supercurrent energy
- Total Electron mass

$$m_e\langle\bar{\psi}\psi\rangle \sim m_e\langle\psi^\dagger\psi\rangle$$

- Negative constant pressure from $F_S$
- Confinement due to the superconducting condensate

## Hadron

- $H_{ta}$ -- Trace anomaly
- $H_g$ -- Glue field energy ($E^2+B^2$)
- $H_q$ -- Quark energy
- $H_\sigma$ -- Sigma terms
- Negative constant pressure from trace anomaly
- Confinement due to the glue condensate

# Trace Anomaly and Cosmological Constant

- Note the energy density-pressure relation of both hadrons and superconductor vortices satisfy $P = -\epsilon\,(\omega = -1)$ -- a unique feature arising from a vacuum condensate with constant energy density, much like the Archimedes principle. $P = -dE/dV = -d(\epsilon V)/dV = -\epsilon$
- Rewrite Einstein's equation -- the cosmological constant is an extra term in the energy-momentum tensor $\omega = -1.00 \pm 0.04$

$$R_{\mu\nu} + \frac{1}{2} R\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu} + \Lambda\, g_{\mu\nu}$$

$$T^{\mu\nu} = \overline{T}^{\mu\nu} + \frac{1}{4} g^{\mu\nu} (T^\rho_\rho)$$

- This suggests that, by analogy, the cosmological constant is a quantum trace anomaly which arises from gravitational conformal symmetry breaking like in QCD.

$$\Lambda = \frac{1}{4} \langle T^\mu_\mu \rangle_U / 8\pi G$$

# SC, Hadrons, Cosmos

- The common theme of hadrons and SC vortices is the existence of a condensate
- Hadrons: condensates from breaking of conformal and chiral symmetries. SC: Cooper pair condensate from gauge symmetry breaking.
- Cosmos – by analogy, the quantum gravity may have a condensate with negative energy density $\langle T^{\mu}_{\mu} \rangle_{\rm Gvac} < 0$

$$\langle T^{\mu}_{\mu} \rangle_{\rm U} = -\langle T^{\mu}_{\mu} \rangle_{\rm Gvac}$$

- Universe emerges as a bubble from the quantum gravity vacuum like the hadrons and vortices from their vacca.
- Cf. -- S. K. Blau. E. I. Guendelman. and A. H. Guth. Phys. Rev. 035. 1747 (1987)
- Cosmological constant

$$\Lambda = \frac{1}{32\pi G}\left[\langle T^{\mu}_{\mu} \rangle_{\rm U} - |\langle T^{\mu}_{\mu} \rangle_{\rm QCD}| - |\langle T^{\mu}_{\mu} \rangle|_{\rm EW} \cdots\right]$$

S.Weinberg, RMP 61 (1989)1

$$200\,{\rm MeV/fm^3} \sim 10^{44}\epsilon_c$$

# Emergent Expanding Universe


Explains why $\Lambda > 0$, but not why it is so small compared to the glue condensate

# Multiverse, Bubble Universe


Cappadocia, Turkey

Some ascending and some descending

# Pion Mass Puzzle (?)


- Pion mass in terms of trace of EMT

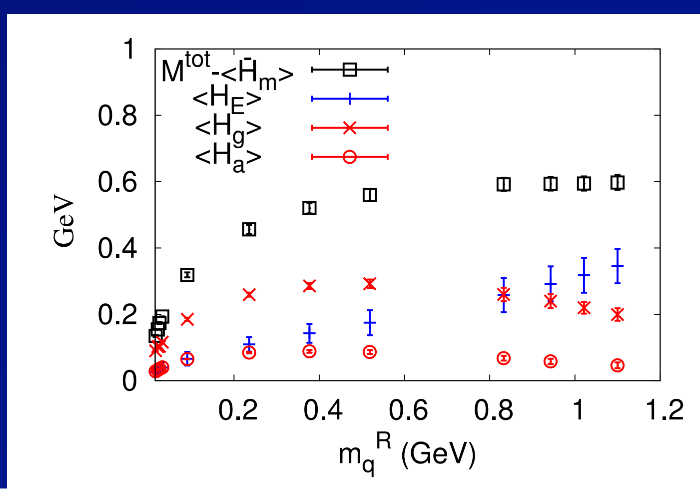
$$m_\pi = m\langle\pi|\bar{\psi}\psi|\pi\rangle + \langle\pi|\frac{\beta}{2g}G^2_{\mu\nu} + m\gamma_m\bar{\psi}\psi|\pi\rangle$$

- Gellmann-Oakes-Renner relation and Feynman-Hellman relation

$$m_\pi^2 = -2m\langle\bar{\psi}\psi\rangle/f_\pi^2,$$

$$\sum_f m_f \frac{\partial m_\pi}{\partial m_f} = \sum_f m_f\langle\pi|\bar{\psi}\psi|\pi\rangle_f \quad \longrightarrow \quad m\langle\pi|\bar{\psi}\psi|\pi\rangle = m_\pi/2$$

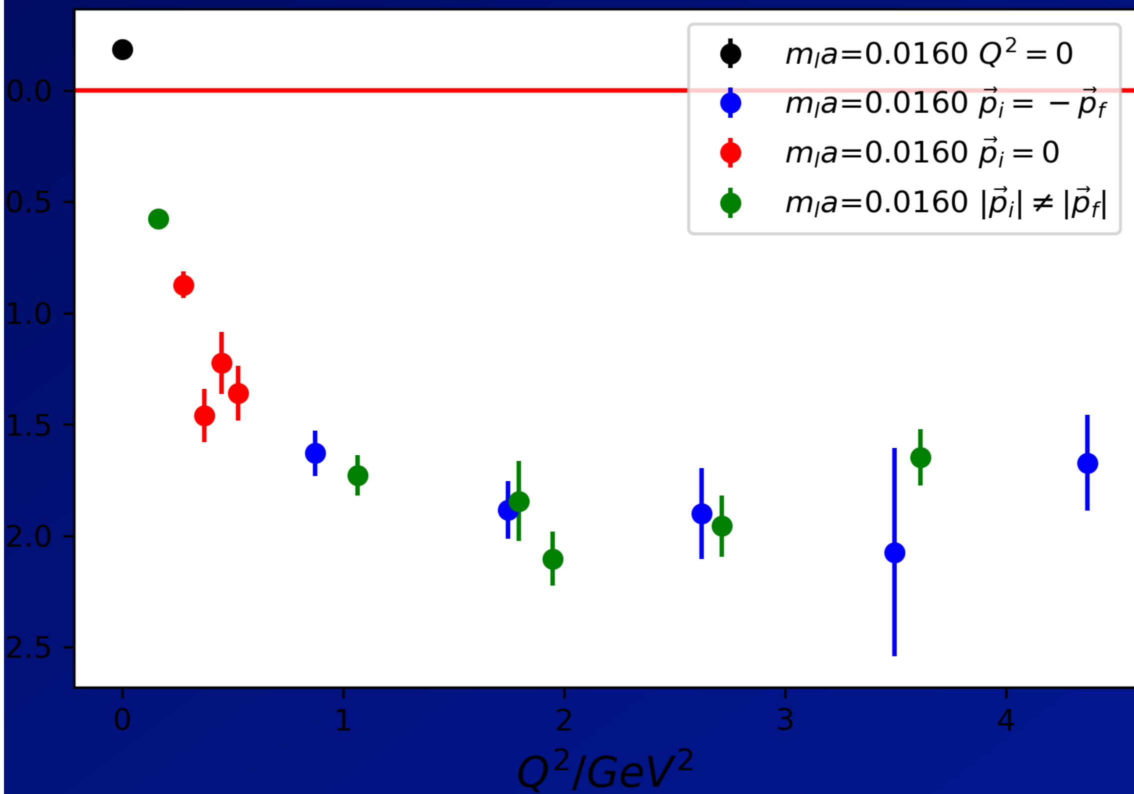
- Trace anomaly is proportional to quark mass.

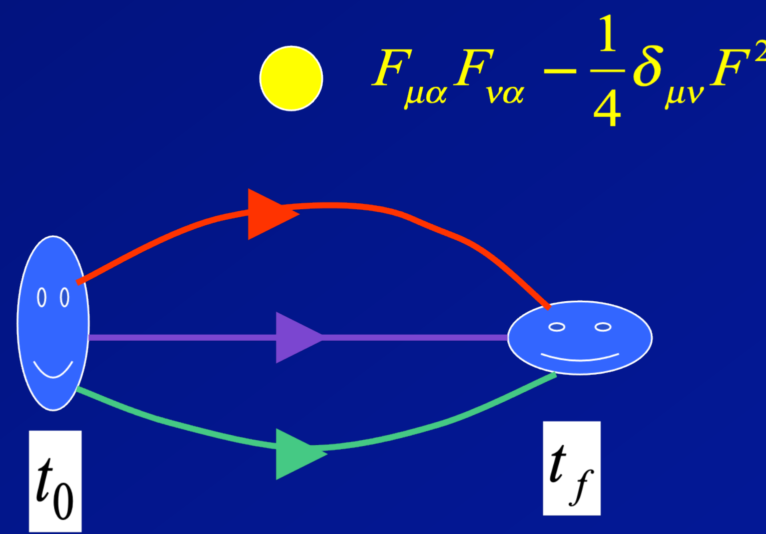



Since the trace anomaly is $\propto$ V. Does this imply that V → 0 at the chiral limit ?

Y.B. Yang et al. ($\chi$QCD), PRD (2015); 1405.4440

# Pion trace anomaly FF




$$\mathcal{F}^{\mathrm{ChPT}}_{\mathrm{ta},\pi}(Q^2)\sim\frac{1}{2}-\frac{1}{2m_\pi^2}Q^2.$$

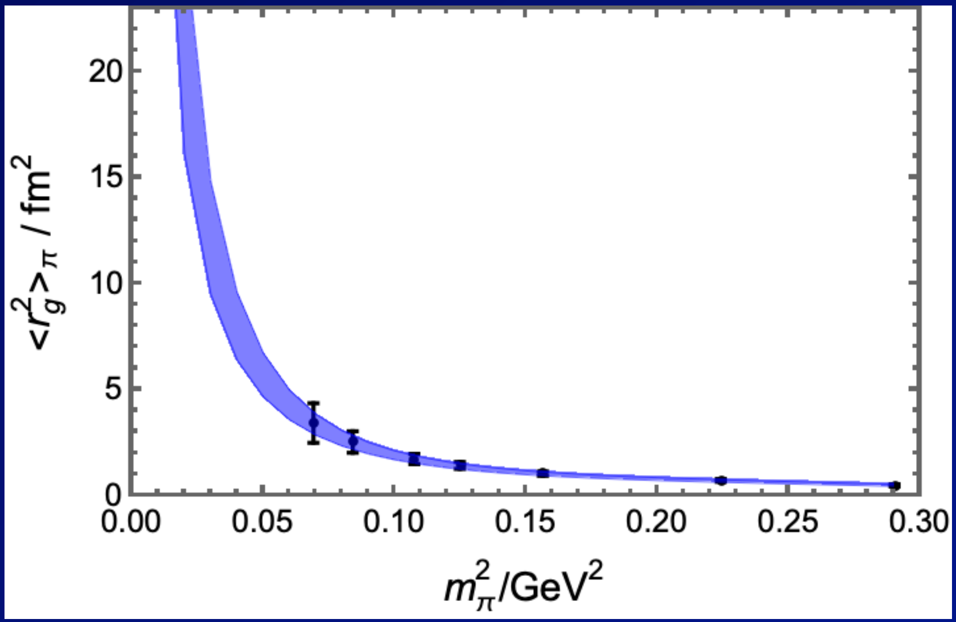
Novikov, Shifman;
Chen, Savage;
Hatta

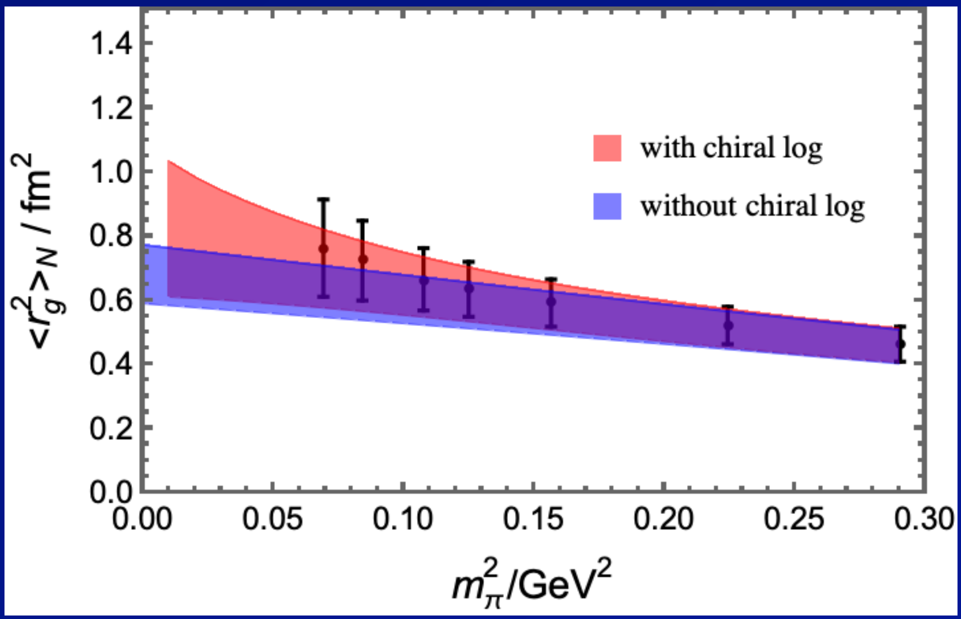
B. Wang, et al.,
arXiv:2401.05496
$m_\pi$ = 340 MeV

X.B. Tong, J.P. Ma and F. Yuan
arXiv:2203.13493

# Trace anomaly Radii

- MSR Radii for pion (left) and proton (right)




$$\langle r^2\rangle_{\rm m}({\rm H}) = -6\left.\frac{d\mathcal{F}_{\rm m,H}(Q^2)}{dQ^2}\right|_{Q^2\to 0}.$$

$$\langle r^2\rangle_{\rm m}^{\rm ChPT}(\pi) = \frac{3}{m_\pi^2}.$$ ~ 6 fm$^2$

$$\langle r_g^2\rangle^{1/2}(\text{pion}) = 4.6(6)\text{fm}\ [3.1(1.4)\text{fm}]$$
$$\langle r_g^2\rangle^{1/2}(\text{proton}) = 0.89(10)(07)\text{fm}$$

- Photoproduction of J/Ψ at threshold and Drell-Yan process
- GPD moments (2302.11600)

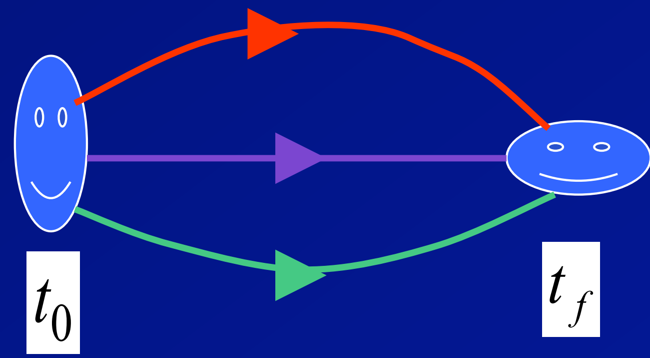
$$\mathcal{F}_{\rm ta}(Q^2) = \left[(A(Q^2) - B(Q^2)\frac{Q^2}{4m_{\rm N}^2} + 3D(Q^2)\frac{Q^2}{m_{\rm N}^2}\right] - \mathcal{F}_\sigma(Q^2),$$

# Trace anomaly Spatial Distribution

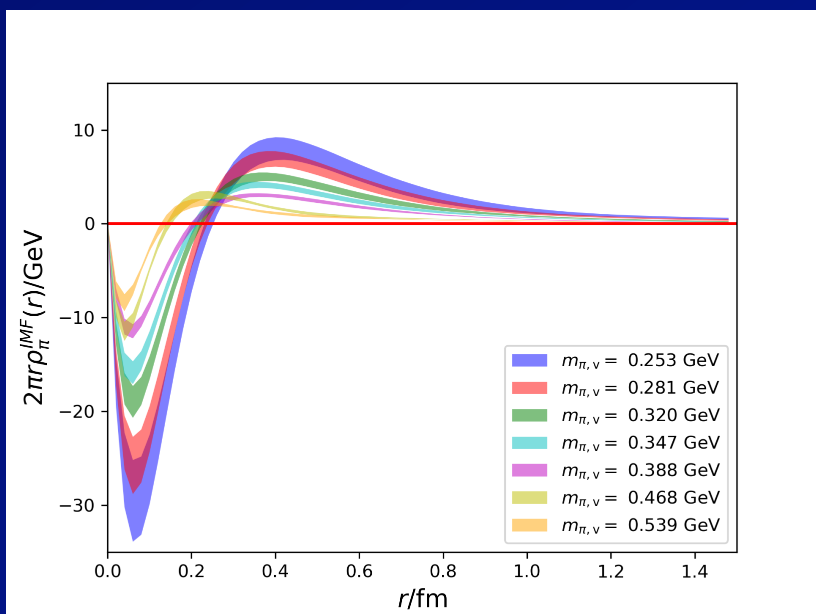
- Distribution function – 2 D Fourier transform in the elastic (Breit) infinite momentum frame

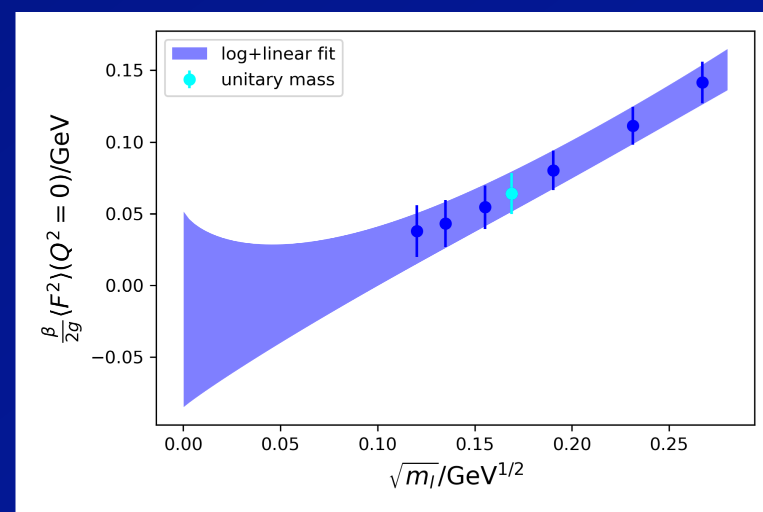
$F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$

$$\rho_{\mathrm{H}}^{\mathrm{IMF}}(\mathbf{r}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{r}_\perp}\, \tilde{G}_{\mathrm{H}}(Q^2)\Big|_{\mathbf{P}\cdot\mathbf{\Delta}=0}^{P_z\to\infty},$$




- B. Wang, et al. ($\chi$QCD) 2401.0546




- It changes sign in pion so that the integral approaches zero at the chiral limit.

# Summary and Challenges

- The confinement mechanisms of 1d heavy quarkoniums, 2d superconductor vortices, and 3d light hadrons, are the same – through respective condensates. $P = -\epsilon\,(\omega = -1)$
- Energy-pressure realtion is the same for the cosmological constant. This suggests that it originates from the guantum gravity trace anomaly, like in QCD. Thus, the Universe emerges as a bubble from the true vacuum, much like the hadrons.
- Glue part of trace anomaly is responsible for confinement, it sould be an order parameter for confinement – deconfinement transition.
- Quark condensate $\leftarrow$ chiral symmetry breaking (restoration at T and $\mu$)
- Trace anomaly (confinement) $\leftarrow$ conformal symmetry breaking (conformal phases with multi-flavors and SU(N); finite T > $T_c$)
- $\chi$SB and CSB are apparently linked in the case of the pion trace anomaly distribution and also in finite T cross over transition.
- Challenges for EIC and COMPASS is to measure the trace anomaly form factors for the proton and, particularly, the pion. Photoproduction of J/$\Psi$ at threshold is a possibility, GPD moments is another.