

# Transverse momentum distribution of heavy quarkonium production at the EIC

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*thanks to my collaborators: Kyle Lee, Jian-Wei Qiu, George Sterman*

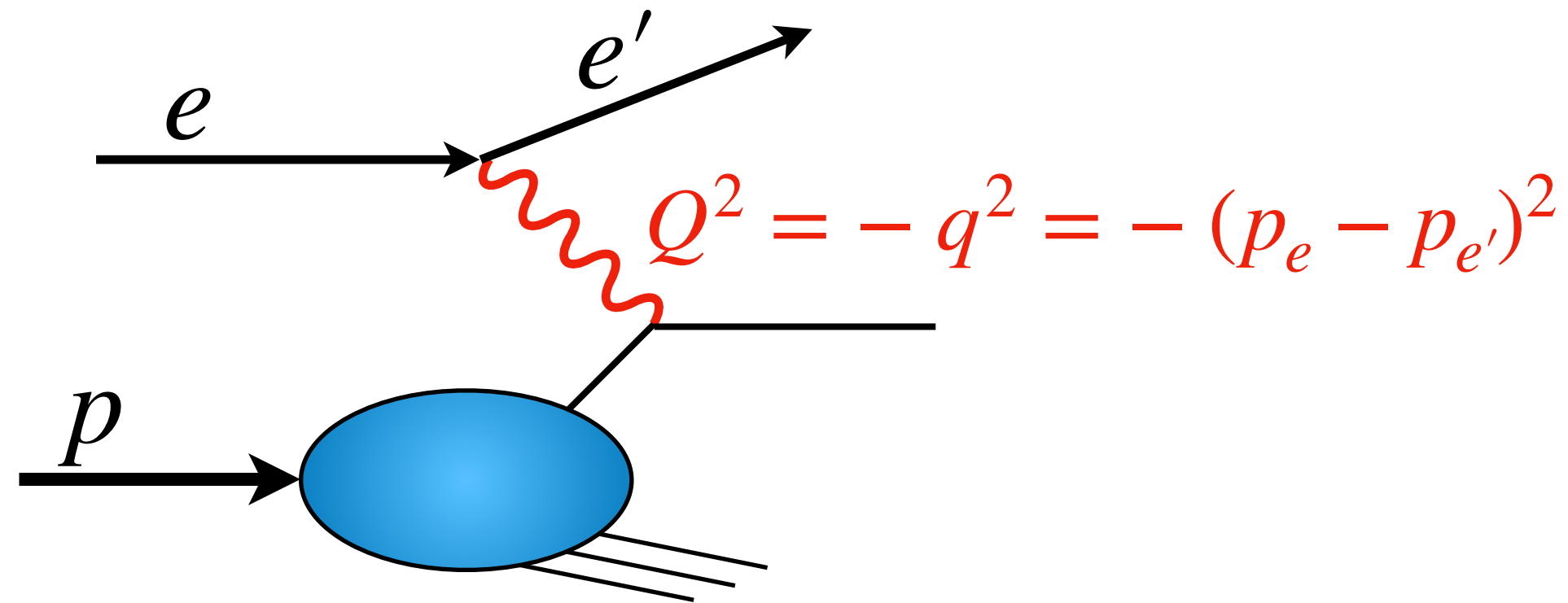
Plan:

1. Background: QED radiative corrections
2. Hadronic quarkonium production of high  $p_T$  revisited
3. Inclusive quarkonium production in ep collisions

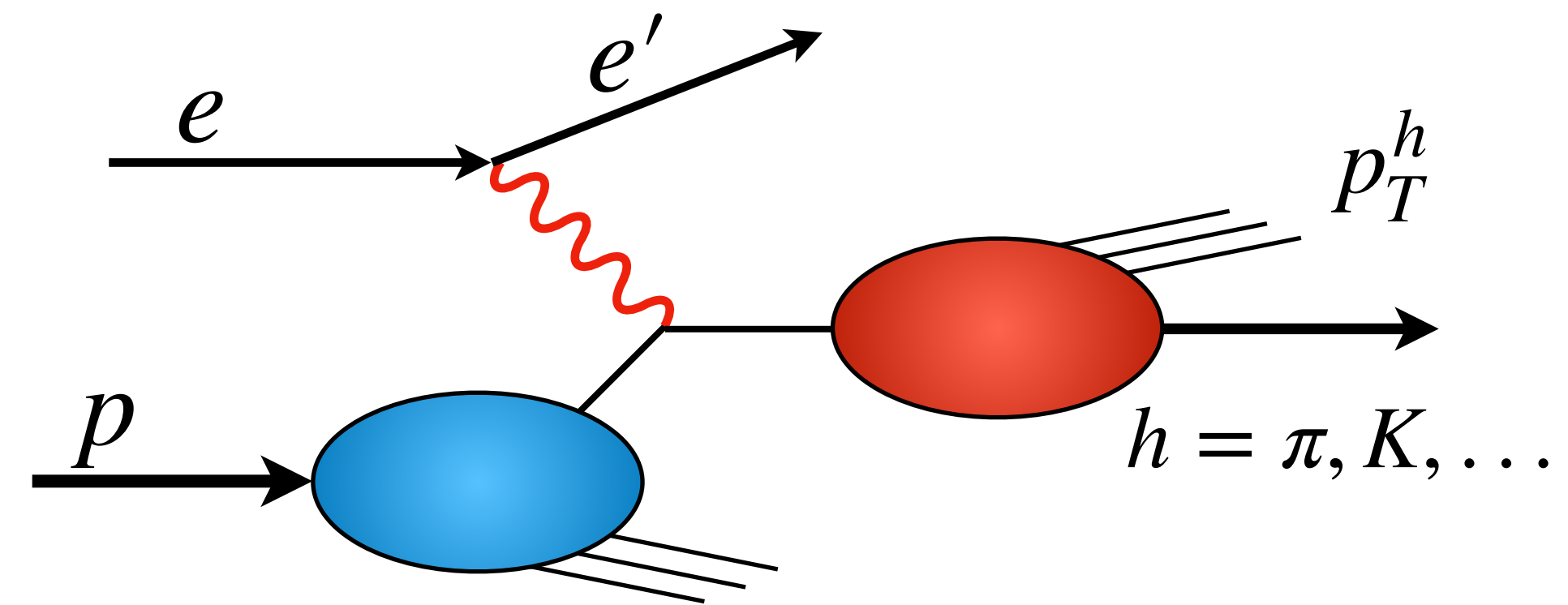


# Lepton-hadron scatterings with one-photon exchange

Inclusive DIS:  $e + p \rightarrow e' + X$



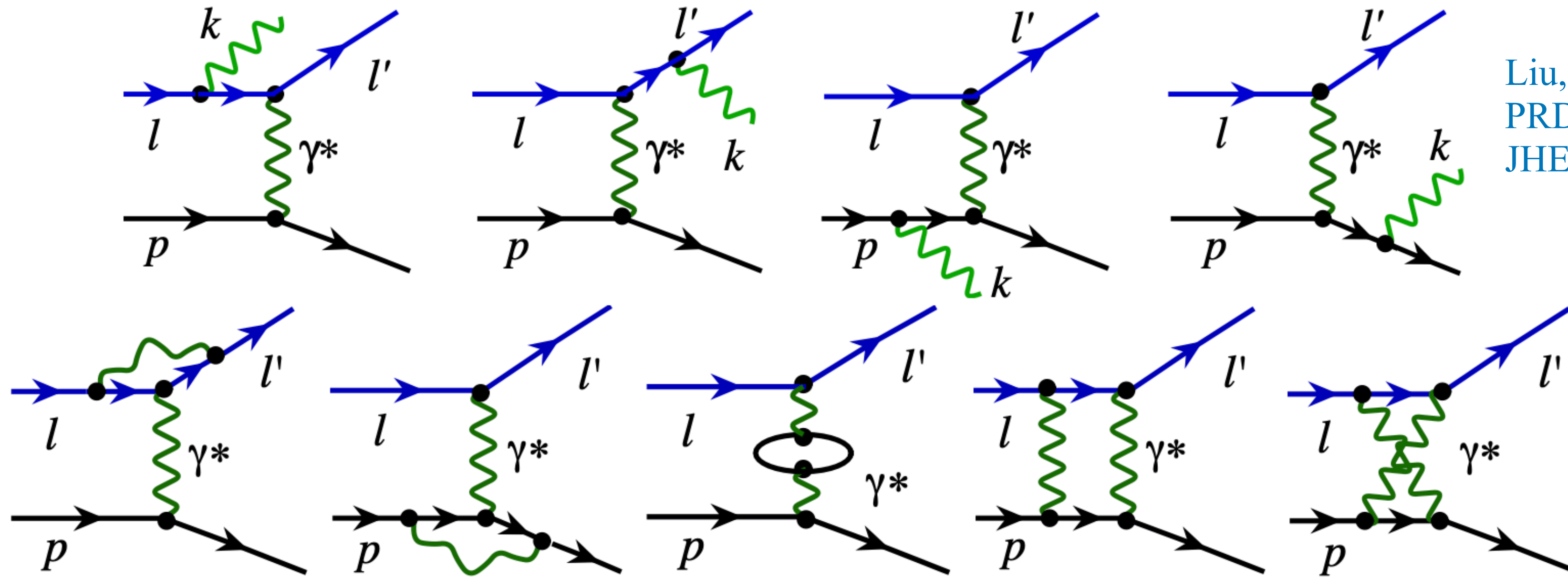
Semi-inclusive DIS:  $e + p \rightarrow e' + h + X$



- ❖ DIS explores PDFs inside the nucleon.
- ❖ SIDIS allows the extraction of transverse momentum-dependent distributions (TMDs);
  - ✓  $Q \gg p_T^h$  in a frame where the virtual photon collides with the nucleon moving along the z-axis.
  - ✓ Hard scale  $Q$  localizes the probe to resolve the  $x$ -dependence of PDFs.
  - ✓ Soft scale  $p_T^h \gtrsim 1/R_N$  allows us to study parton's confined transverse motion.

**Quarkonium in SIDIS is a crucial probe into hadron/nuclear structures.**

# Problem with QED radiative corrections (RCs)



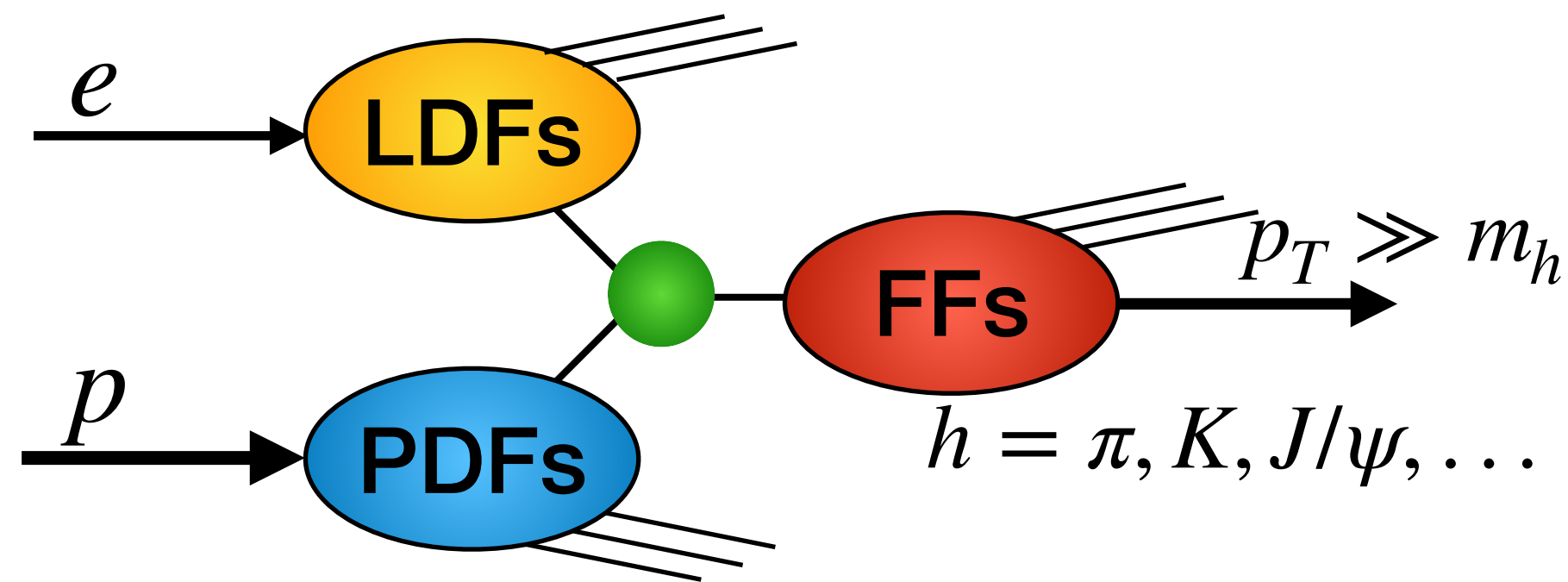
Liu, Melnitchouk, Qiu, Sato,  
PRD104, no.9, 094033 (2021),  
JHEP11, 157 (2021)

- ❖ QED RCs change the momentum transfer  $q$ , resulting in **trouble with the definition of a photon-hadron frame**.
- ❖ Change of  $q$  also modifies **the angular modulation between the leptonic and hadronic planes**.
- ❖ All perturbative collinear divergences with  $m \rightarrow 0$  along the direction of observed momenta can be factorized into PDFs, FFs, and lepton distribution functions (LDFs) of the incident lepton.

***Understanding RCs in hadron production in collinear factorization is a prerequisite!***

# Factorization beyond "one-photon exchange"

$$e + p \rightarrow h(p_T) + X$$



PDFs and FFs are common blocks in  $ep$  and  $pp$ .

Perturbatively calculable coefficients

$$\frac{d\sigma_{e+p \rightarrow h+X}}{dp_T} \approx \underbrace{f_{i/e}}_{\uparrow} \otimes \underbrace{f_{j/p}}_{\uparrow} \otimes \underbrace{D_k^h}_{\uparrow} \otimes \underbrace{C_{ij \rightarrow k}}_{\downarrow}$$

Universal functions: LDFs, PDFs, FFs

**LO:** Kang, Metz, Qiu and Zhou, PRD84, 034046 (2011)

**NLO:** Hinderer, Schlegel, Vogelsang, PRD92, no.1, 014001 (2015)

**NNLO:** Abelof, Boughezal, Liu, Petriello, PLB763, 52-59 (2016)

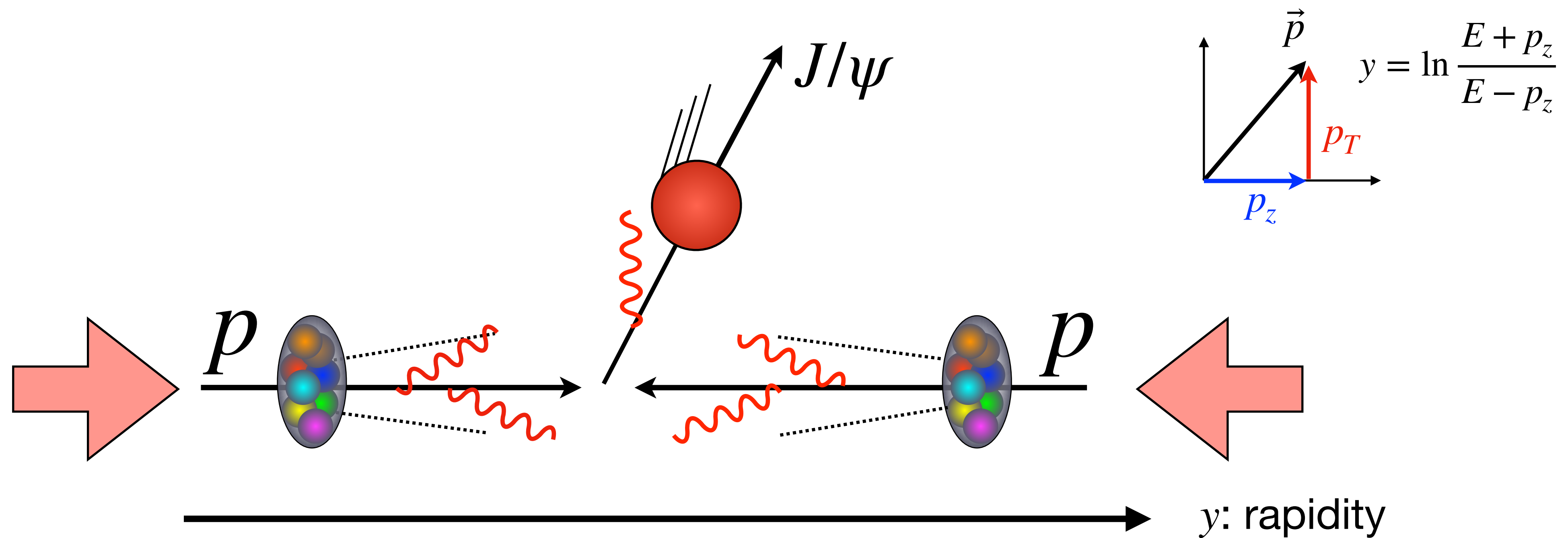
See also: J.W.Qiu, X.P.Wang and H.Xing, Chin. Phys. Lett. 38, no.4, 041201 (2021)

- ❖ The scattered lepton is **not observed**. (cf. SIDIS:  $e + p \rightarrow e' + h + X$ )
- ❖ LDFs: Probability densities for finding leptons, photons, and partons in the beam lepton (**Later**).
- ❖ **Remark:** DESY-HERA introduced an artificial cut on the transverse momentum of the scattered lepton:  $J/\psi$  lepto-production ( $Q^2 > 1\text{GeV}^2$ ) vs.  $J/\psi$  photo-production ( $Q^2 \lesssim 1\text{GeV}^2$ ). **In this study, we do not need to consider such an artificial separation!**



**Fragmentation Functions for Heavy Quarkonium production**

# Hadronic quarkonium production of high $p_T$ revisited

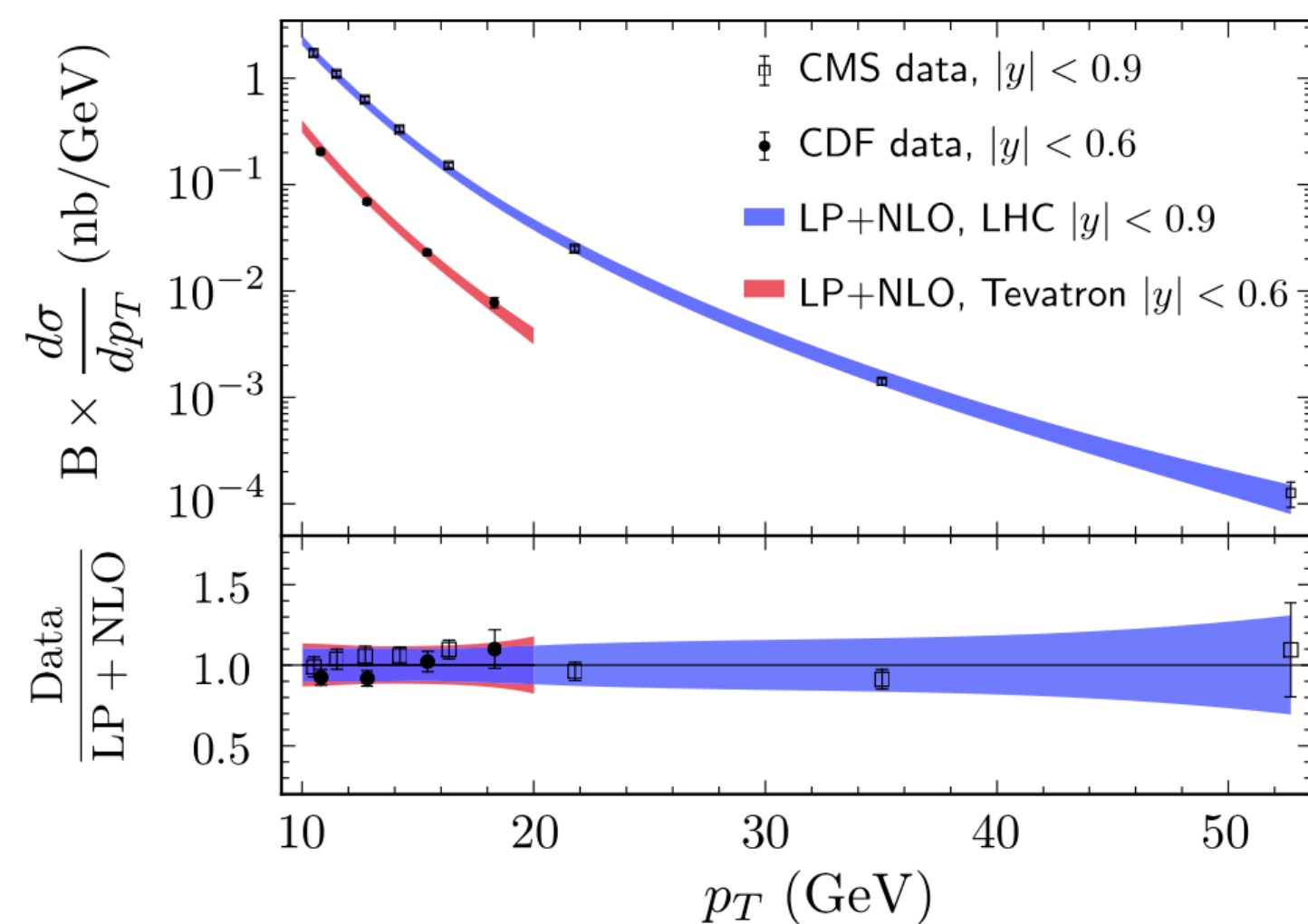
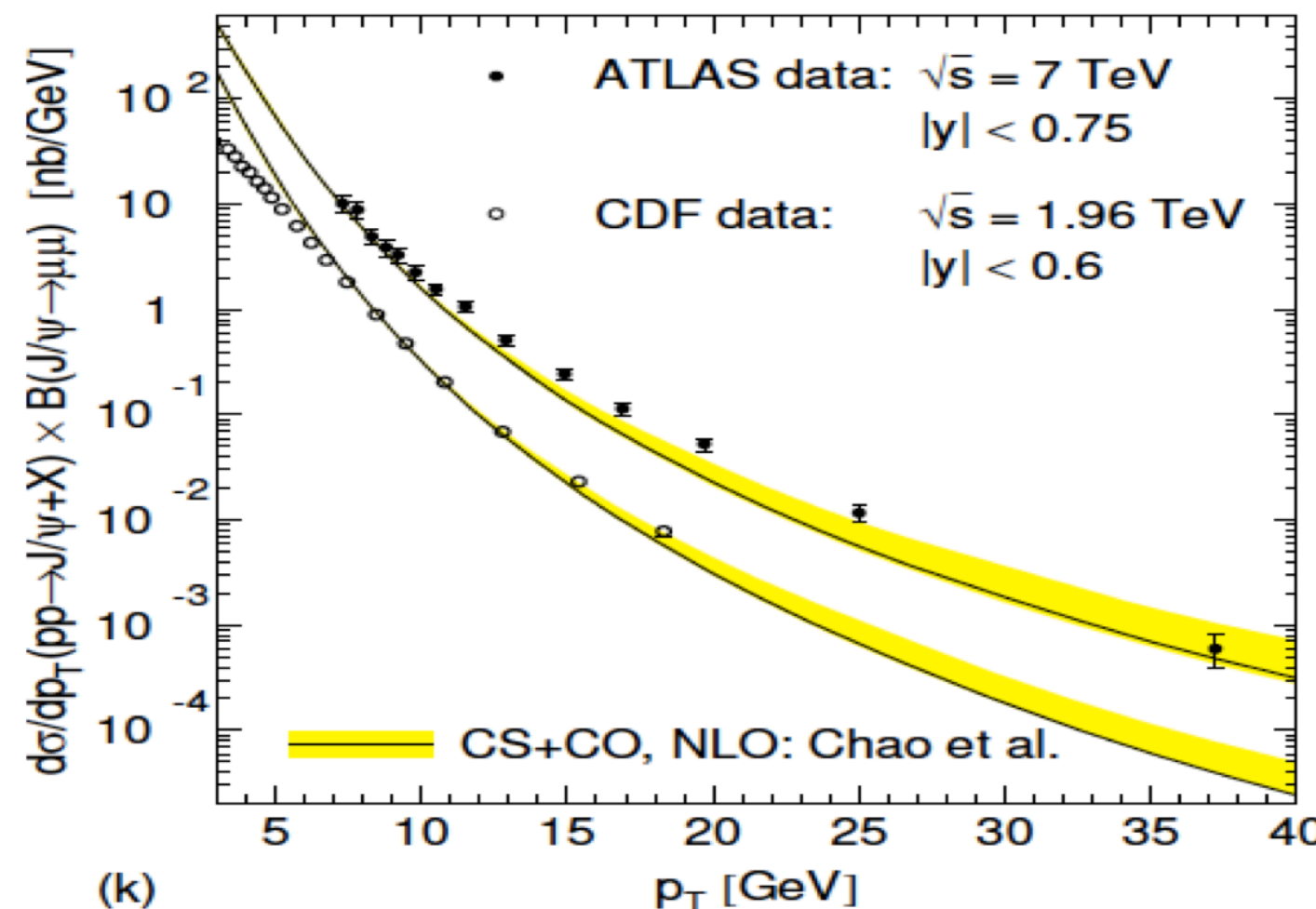
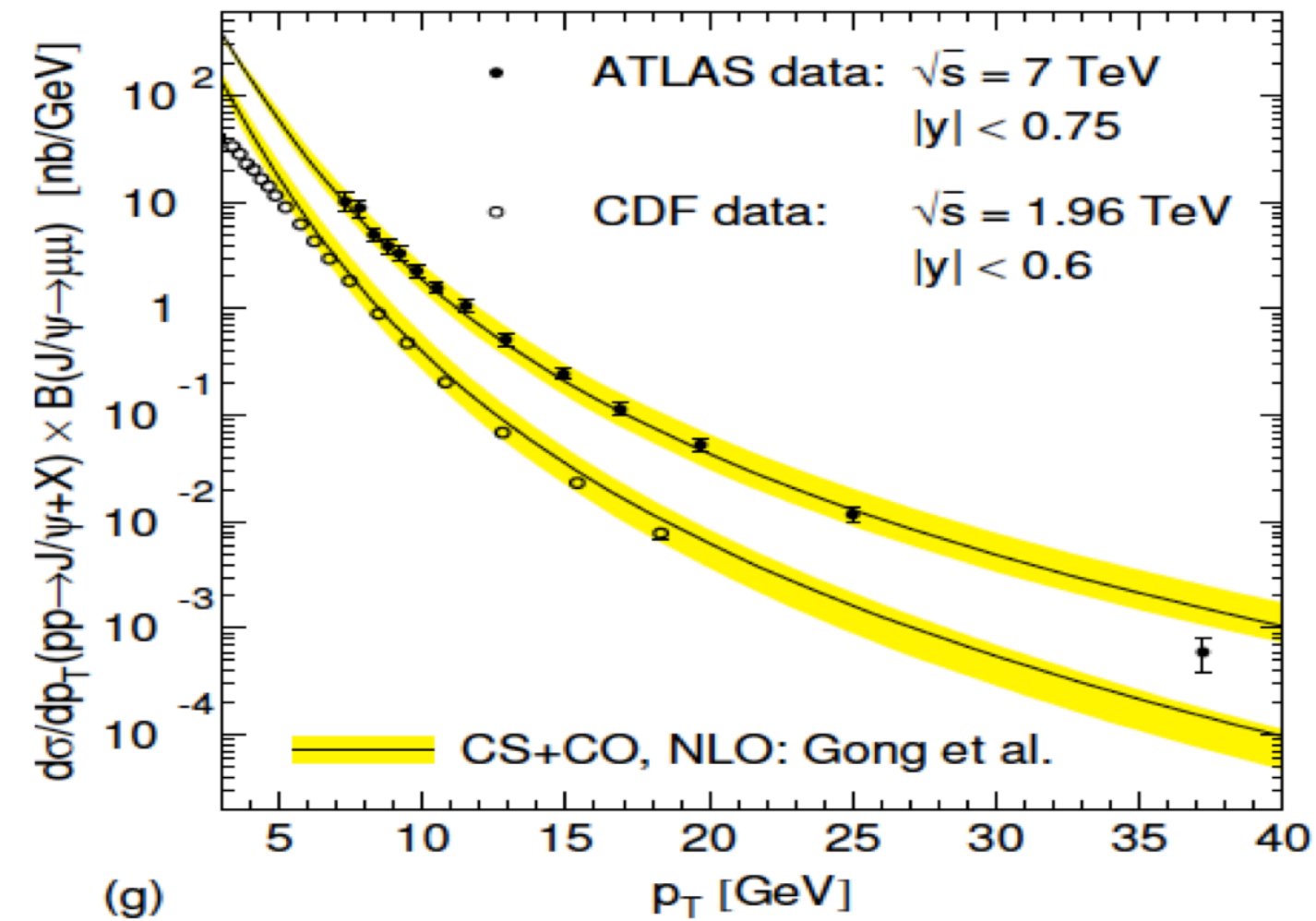
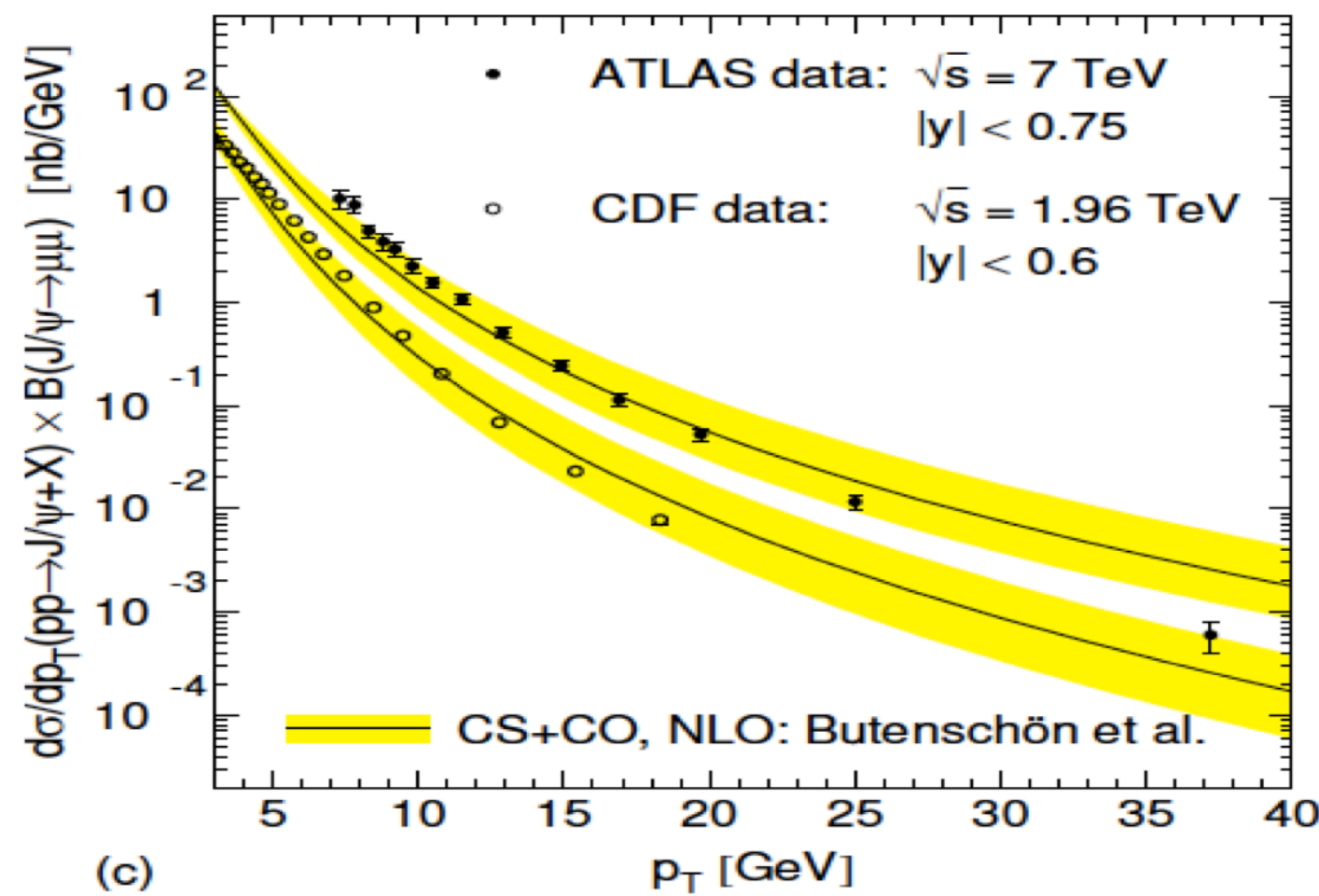


# NRQCD vs. data: lack of universality of LDMEs

$$d\sigma_\psi \approx \sum_{\mathcal{K}} d\hat{\sigma}_{Q\bar{Q}[\mathcal{K}]} \langle \mathcal{O}_{Q\bar{Q}[\mathcal{K}] \rightarrow \psi} \rangle \xrightarrow{\text{Global data fitting}}$$

**Long-Distance Matrix Elements**

	$\langle \mathcal{O}({}^3S_1^{[1]}) \rangle$ GeV <sup>3</sup>	$\langle \mathcal{O}({}^1S_0^{[8]}) \rangle$ 10 <sup>-2</sup> GeV <sup>3</sup>	$\langle \mathcal{O}({}^3S_1^{[8]}) \rangle$ 10 <sup>-2</sup> GeV <sup>3</sup>	$\langle \mathcal{O}({}^3P_0^{[8]}) \rangle$ 10 <sup>-2</sup> GeV <sup>5</sup>
Set I (Butenschoen <i>et al.</i> )	1.32	3.04	0.16	-0.91
Set II (Chao <i>et al.</i> )	1.16	8.9	0.30	1.26
Set III (Gong <i>et al.</i> )	1.16	9.7	-0.46	-2.14
Set IV (Bodwin <i>et al.</i> )	-	9.9	1.1	1.1



LDMEs should be universality, however:

- Numbers are not the same.
- Not even the sign.

**Much more work is needed!**

## Fits in NRQCD

Butenschoen, Kniehl, PRD84, 051501 (2011).

Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 (2012).

Gong, Wan, Wang, Zhang, PRL110, 042002 (2013).

Bodwin, Chung, Kim, Lee, PRL113, 022001 (2014).

...

## Fits in pNRQCD

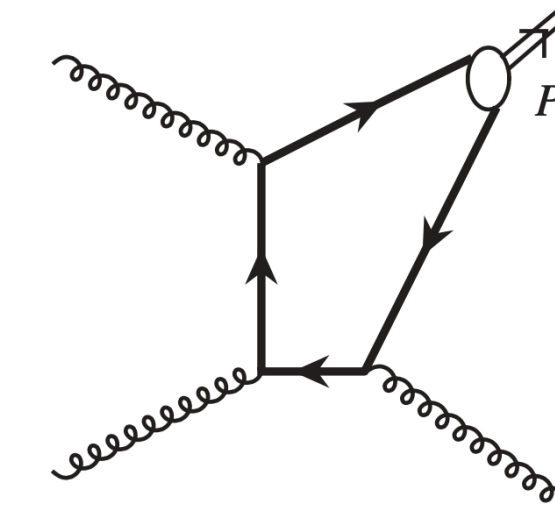
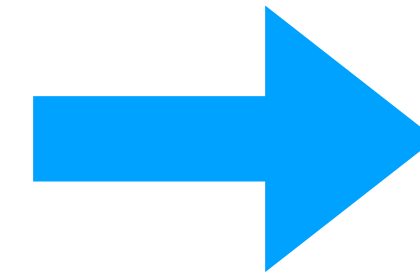
Brambilla, Chung, Vairo, Wang, PRD105, no.11, L111503 (2022).

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# Importance of higher order corrections at high $p_T$ (1/2)

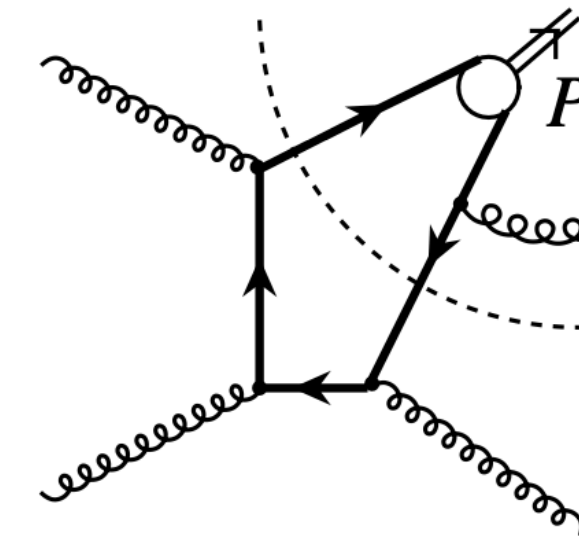
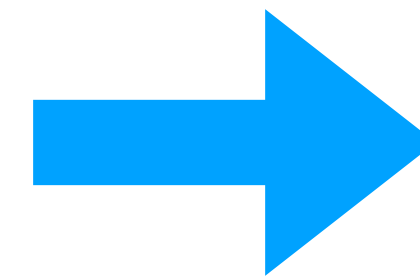
- At LO in CSM:

$$d\sigma(Q\bar{Q}[{}^3S_1^{[1]}]) \propto \frac{\alpha_s^3 m^4}{p_T^8}$$



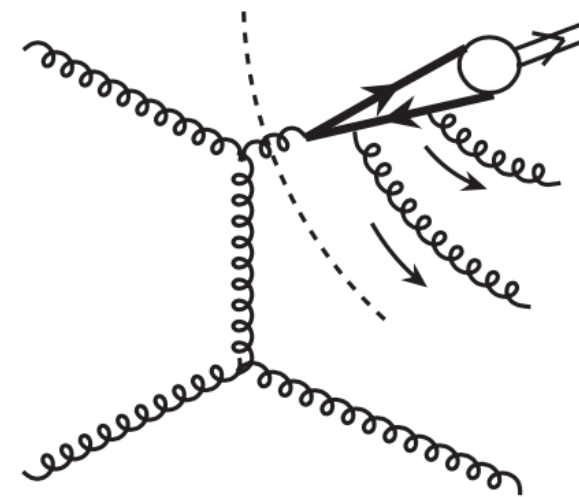
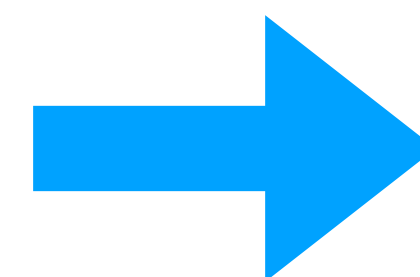
- At high  $p_T$  higher order corrections must be essential:

$$d\sigma(Q\bar{Q}[{}^3S_1^{[1]}]) \propto \frac{\alpha_s^3 m^4}{p_T^8} \times \frac{\alpha_s p_T^2}{m^2} = \frac{\alpha_s^4 m^2}{p_T^6}$$



- The gluon jet fragmentation at high  $p_T$ :

$$d\sigma \propto \frac{\alpha_s^5}{p_T^4} \quad \& \quad d\sigma \propto \frac{\alpha_s^2}{p_{\perp}^4} \times \alpha_s^3 \ln\left(\frac{p_T^2}{m^2}\right)$$



The latter is enhanced even if  $\alpha_s \ll 1$ ; we may not obtain reliable predictions by considering only diagrams in the naive  $\alpha_s$  expansion and  $v$  (quark velocity) expansion.

# QCD factorization approach

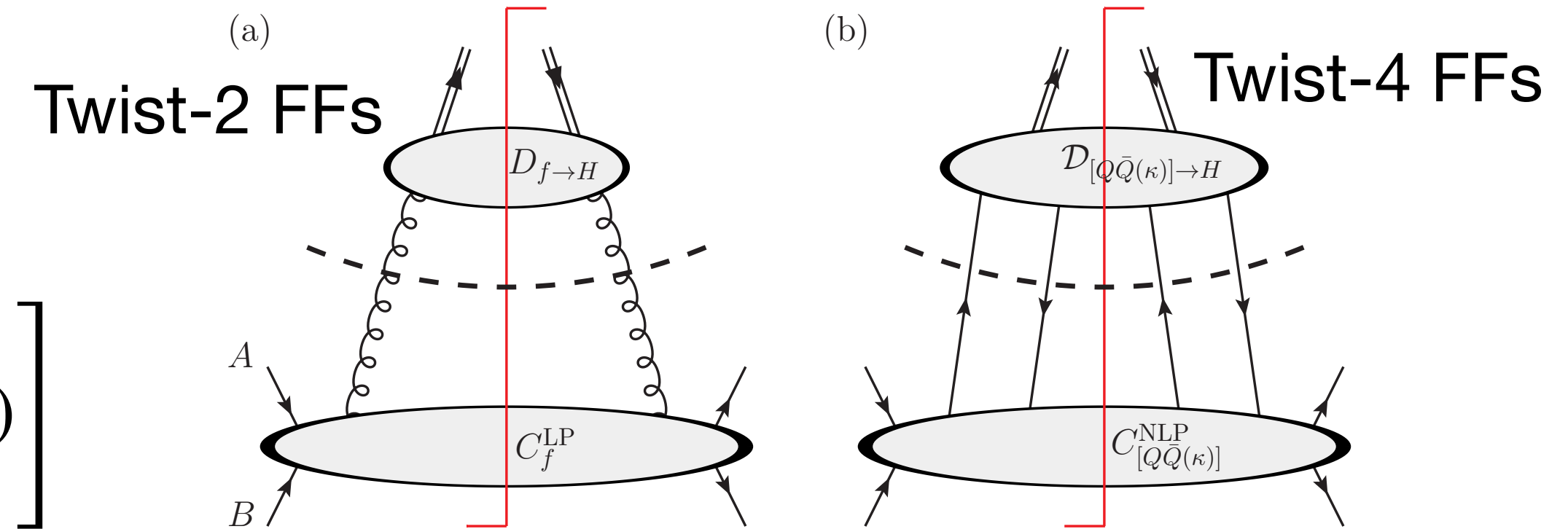
Nayak, Qiu, Sterman, PRD72 (2005) 114012

Kang, Qiu, Sterman, PRL108 (2012) 102002

Kang, Ma, Qiu, Sterman, PRD90 (2014) 3, 034006, PRD91 (2015) 1, 014030

Leading power (LP) up to NLO

$$d\sigma_{A+B \rightarrow [f, Q\bar{Q}] \rightarrow H+X}^{\text{QCD-Res}}(\mu) = \sum_{f=q, \bar{q}, g} C_{A+B \rightarrow [f]+X}^{\text{LP}}(\mu) \otimes D_{[f] \rightarrow H}(\mu) + \frac{1}{p_T^2} \left[ \sum_n C_{A+B \rightarrow [Q\bar{Q}(n)]+X}^{\text{NLP}}(\mu) \otimes \mathcal{D}_{[Q\bar{Q}(n)] \rightarrow H}(\mu) \right]$$



Subleading power (NLP) at LO

pQCD projection operators

$$n = (v, a, t)^{[1,8]} = (\gamma^+, \gamma^+ \gamma^5, \gamma^+ \gamma_\perp^i)^{[1,8]}$$

$v$ : vector

$a$ : axial vector

$t$ : tensor

} important at high  $p_T$   
suppressed at high  $p_T$

subtract double counting

Matching condition

$$d\sigma_{A+B \rightarrow H+X}(m \neq 0) = d\sigma_{A+B \rightarrow H+X}^{\text{QCD-Evol}}(m = 0) + d\sigma_{A+B \rightarrow H+X}^{\text{NRQCD-(n)}}(m \neq 0) - d\sigma_{A+B \rightarrow H+X}^{\text{QCD-(n)}}(m = 0)$$

$$\Rightarrow \begin{cases} d\sigma_{A+B \rightarrow H+X}^{\text{QCD-Evol}} & \text{when } p_T \gg m; d\sigma^{\text{NRQCD-(n)}} \approx d\sigma^{\text{QCD-(n)}} \\ d\sigma_{A+B \rightarrow H+X}^{\text{NRQCD-(n)}} & \text{when } p_T \rightarrow m; d\sigma^{\text{QCD-Evol}} \approx d\sigma^{\text{QCD-(n)}} \end{cases}$$



# Renormalization group improvement

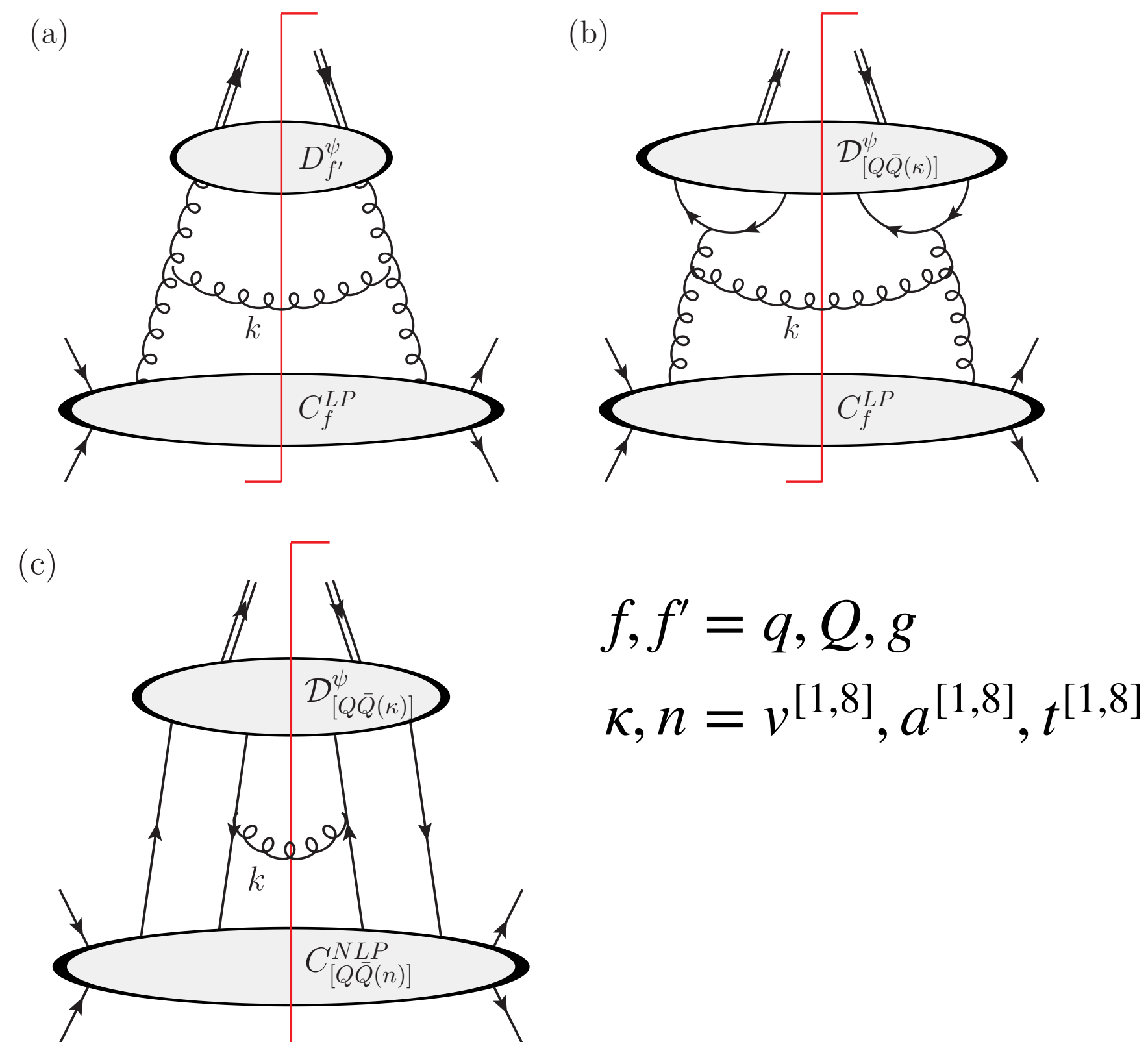
- **Twist-2 evolution equation: DGLAP + quark pair power corrections:**

$$\frac{\partial D_{[f] \rightarrow H}}{\partial \ln \mu^2} = \gamma_{[f] \rightarrow [f]} \otimes D_{[f] \rightarrow H} + \frac{1}{\mu^2} \gamma_{[f] \rightarrow [Q\bar{Q}(\kappa)]} \otimes \mathcal{D}_{[Q\bar{Q}(\kappa)] \rightarrow H}$$

The inhomogeneous term is added to the **slope**, not to the FF itself.

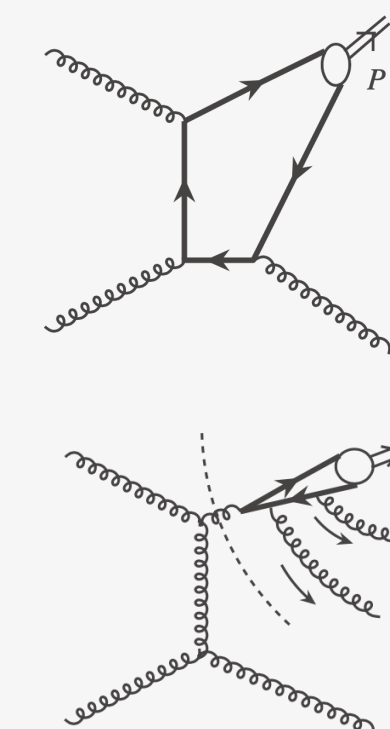
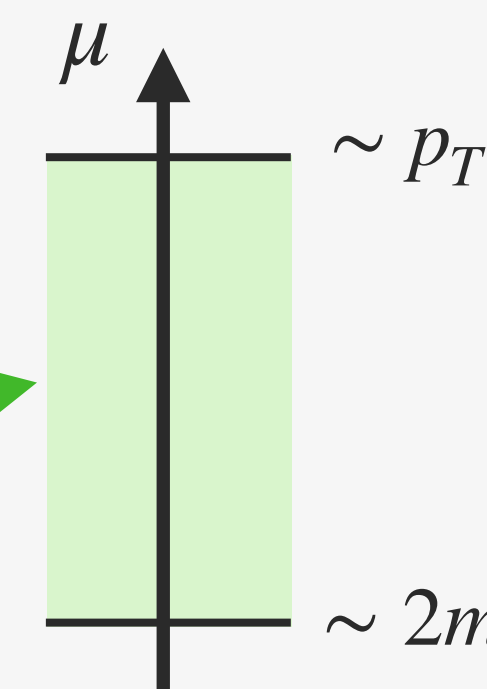
- **Twist-4 “DGLAP like” evolution equation:**

$$\frac{\partial \mathcal{D}_{[Q\bar{Q}(n)] \rightarrow H}}{\partial \ln \mu^2} = \Gamma_{[Q\bar{Q}(n)] \rightarrow [Q\bar{Q}(\kappa)]} \otimes \mathcal{D}_{[Q\bar{Q}(\kappa)] \rightarrow H}$$



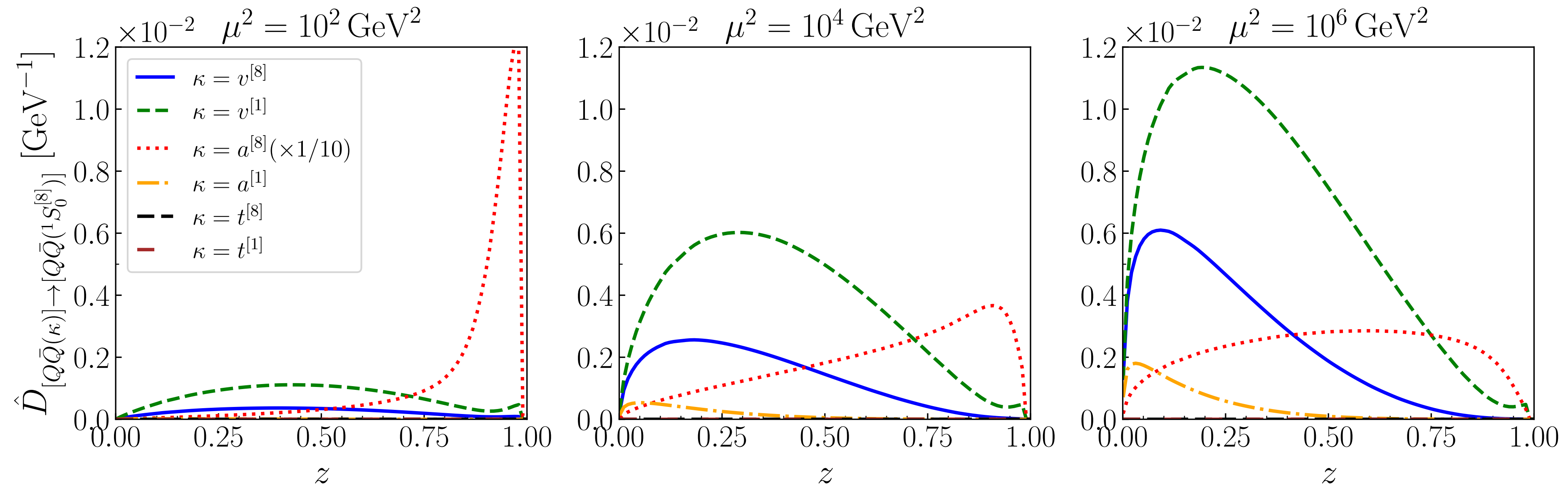
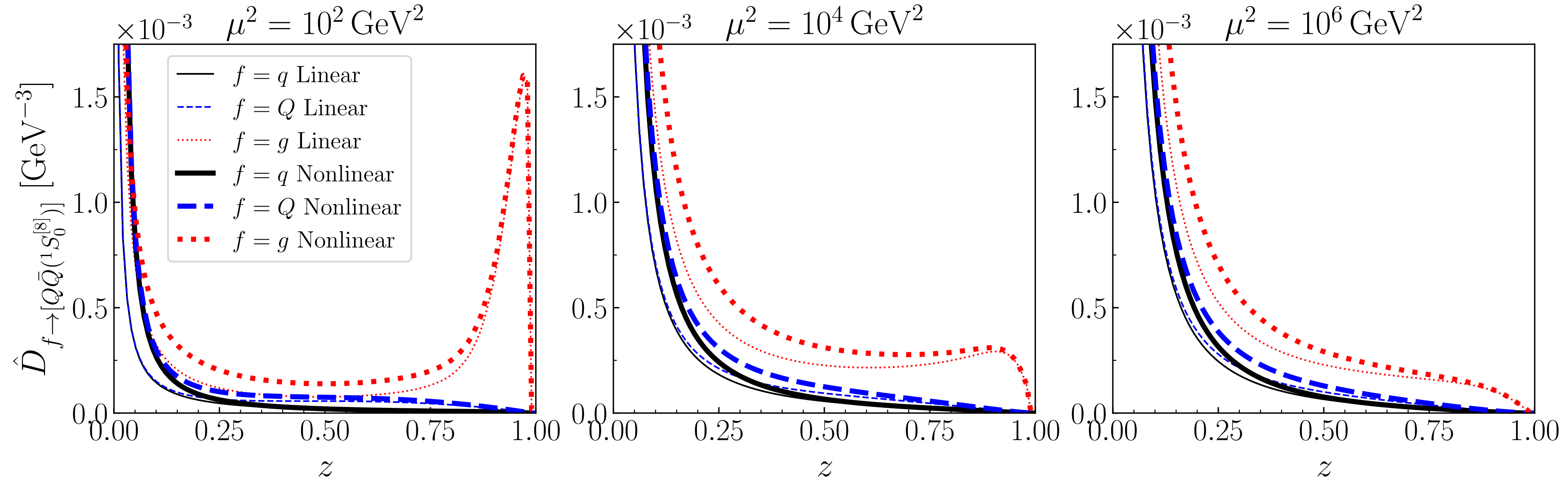
The RG improved factorized cross section covers all events in which the heavy quark pair can be produced:

1. at the short-distance ( $p_T$ ): early stage (**NLP**)
2. at the input scale ( $2m$ ): later stage (**LP**)
3. in-between (**Quark pair power correction**)



# Quarkonium Fragmentation Functions

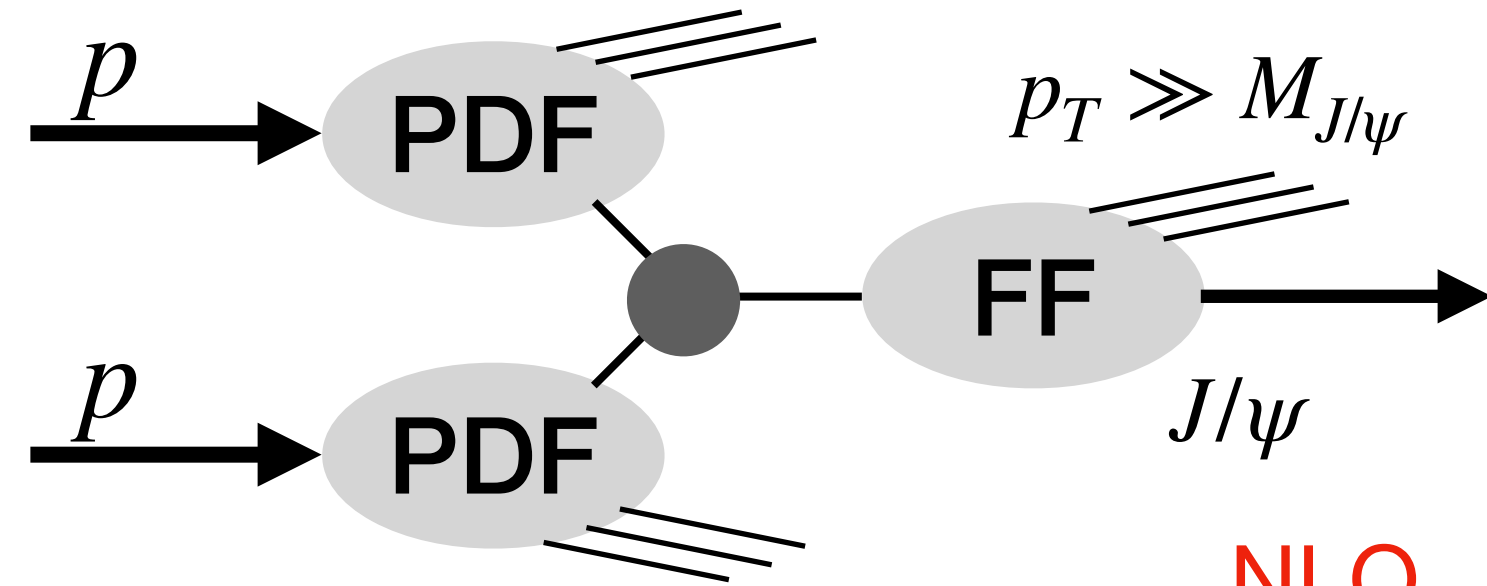
Lee, Qiu, Sterman, KW, arXiv:2211.12648 [hep-ph]



# Phenomenology

Lee, Qiu, Sterman, KW, SciPost Phys. Proc.8, 143 (2022)

$$p + p \rightarrow J/\psi + X \quad \text{Lee, Qiu, Sterman, KW, arXiv:2211.12648 [hep-ph]}$$



$$\frac{d\sigma_{p+p \rightarrow J/\psi+X}}{dp_T} \approx f_{i/p} \otimes f_{j/p} \otimes \left[ D_k^{J/\psi} \otimes C_{ij \rightarrow k} + D_{c\bar{c}}^{J/\psi} \otimes C_{ij \rightarrow c\bar{c}} \right]$$

NLO LO  $\times K_{\text{NLP}}$

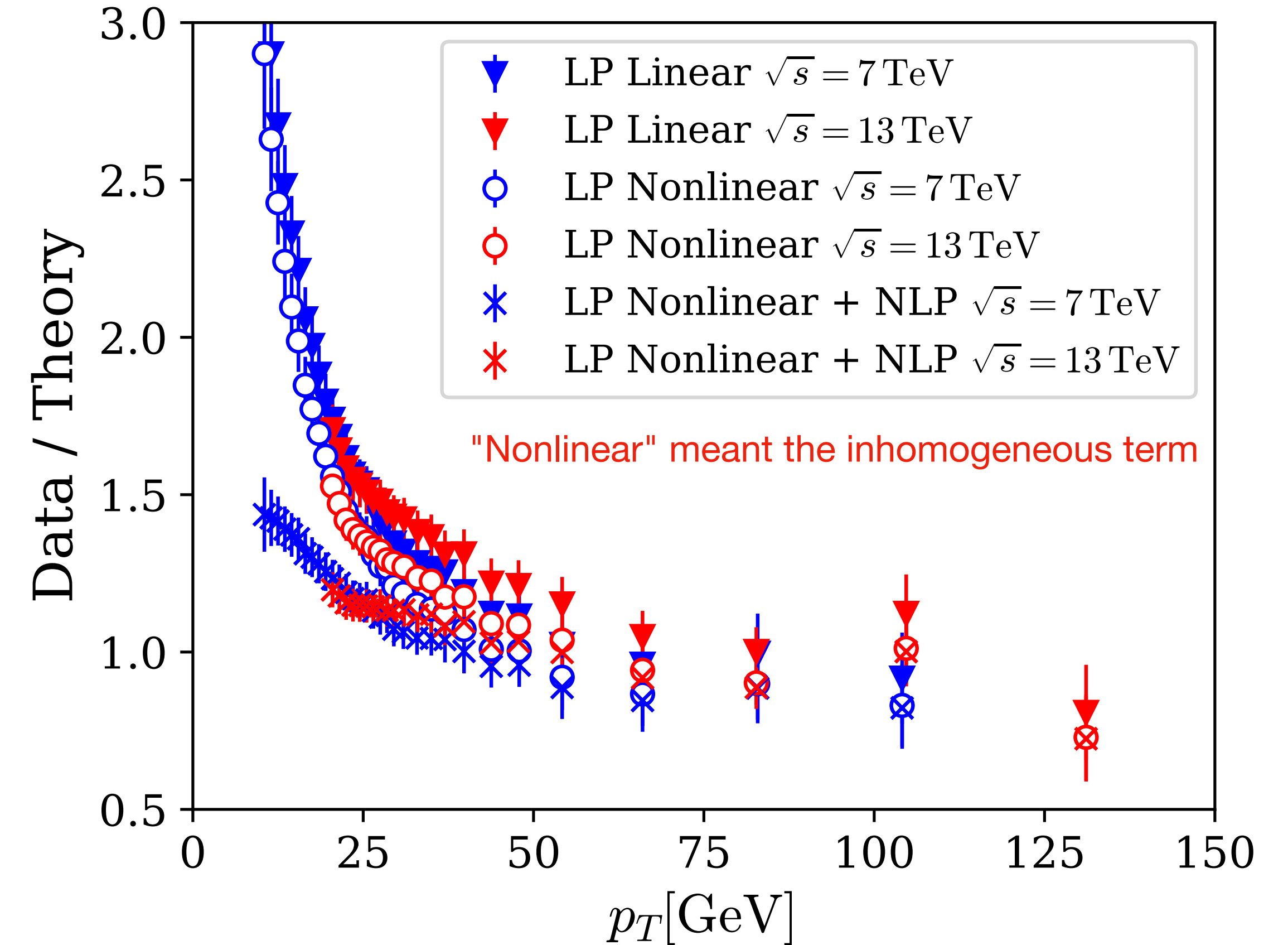
Input fragmentation functions:

$$D_{f \rightarrow H}(z; m, \mu_0) = \sum_{[Q\bar{Q}(n)]} \pi \alpha_s \left\{ \hat{d}_{f \rightarrow [Q\bar{Q}(n)]}^{(1)}(z; m, \mu_0, \mu_\Lambda) + \frac{\alpha_s}{\pi} \hat{d}_{f \rightarrow [Q\bar{Q}(n)]}^{(2)}(z; m, \mu_0, \mu_\Lambda) + \mathcal{O}(\alpha_s^2) \right\} \frac{\langle \mathcal{O}_{[Q\bar{Q}(n)]}^H(\mu_\Lambda) \rangle}{m^{2L+3}} \quad \text{LDMEs}$$

$$D_{[Q\bar{Q}(\kappa)] \rightarrow H}(z; m, \mu_0) = \sum_{[Q\bar{Q}(n)]} \left\{ \hat{d}_{[Q\bar{Q}(\kappa)] \rightarrow [Q\bar{Q}(n)]}^{(0)}(z; m, \mu_0, \mu_\Lambda) + \frac{\alpha_s}{\pi} \hat{d}_{[Q\bar{Q}(\kappa)] \rightarrow [Q\bar{Q}(n)]}^{(1)}(z; m, \mu_0, \mu_\Lambda) + \mathcal{O}(\alpha_s^2) \right\} \frac{\langle \mathcal{O}_{[Q\bar{Q}(n)]}^H(\mu_\Lambda) \rangle}{m^{2L+1}}$$

$\mu_0 = \mathcal{O}(2m)$  : input scale,  $\mu_\Lambda = \mathcal{O}(m)$  : NRQCD factorization scale

$$\kappa = v^{[c]}, a^{[c]}, t^{[c]}, \quad n = 2S+1 L_J^{[c]}$$

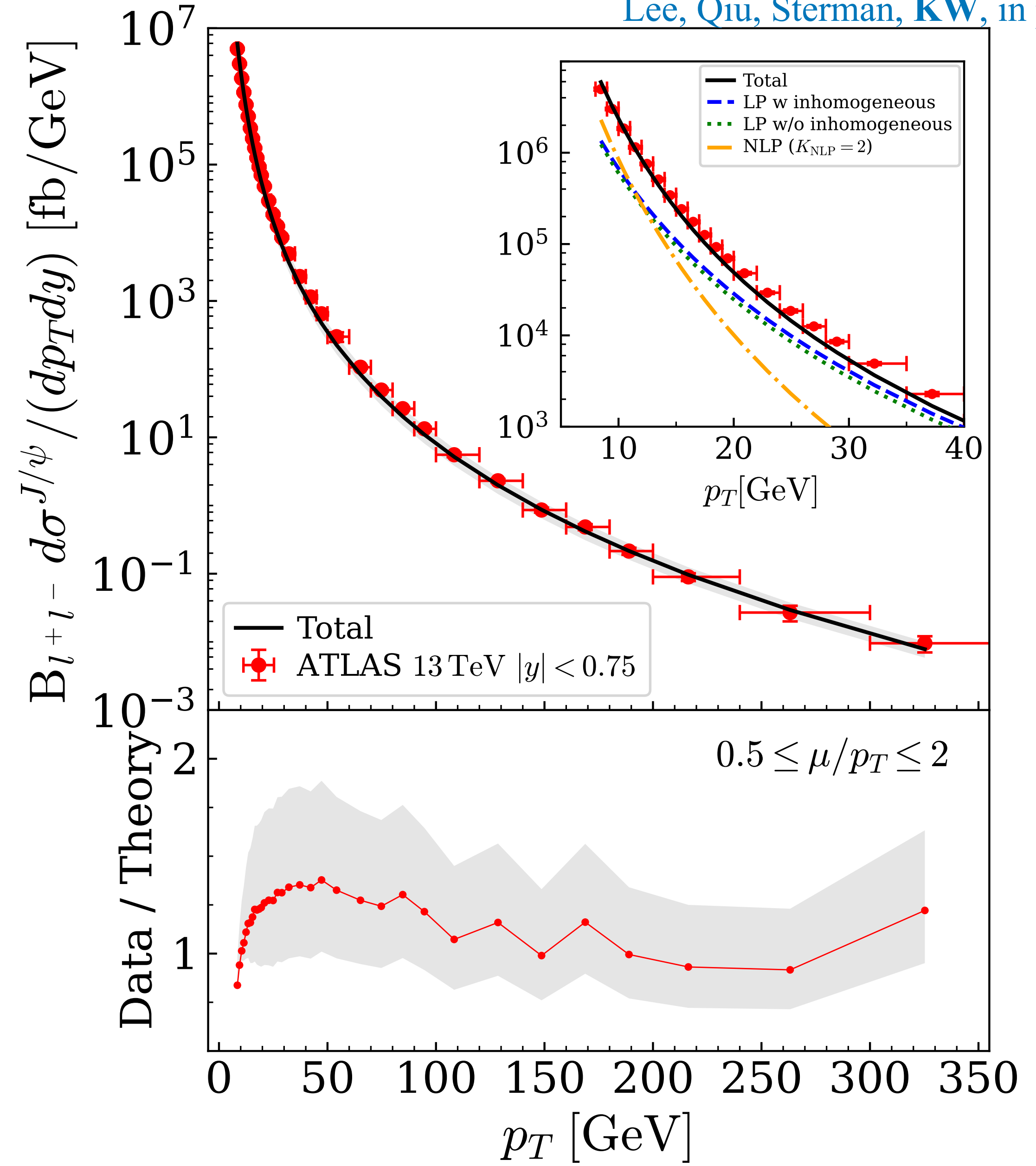
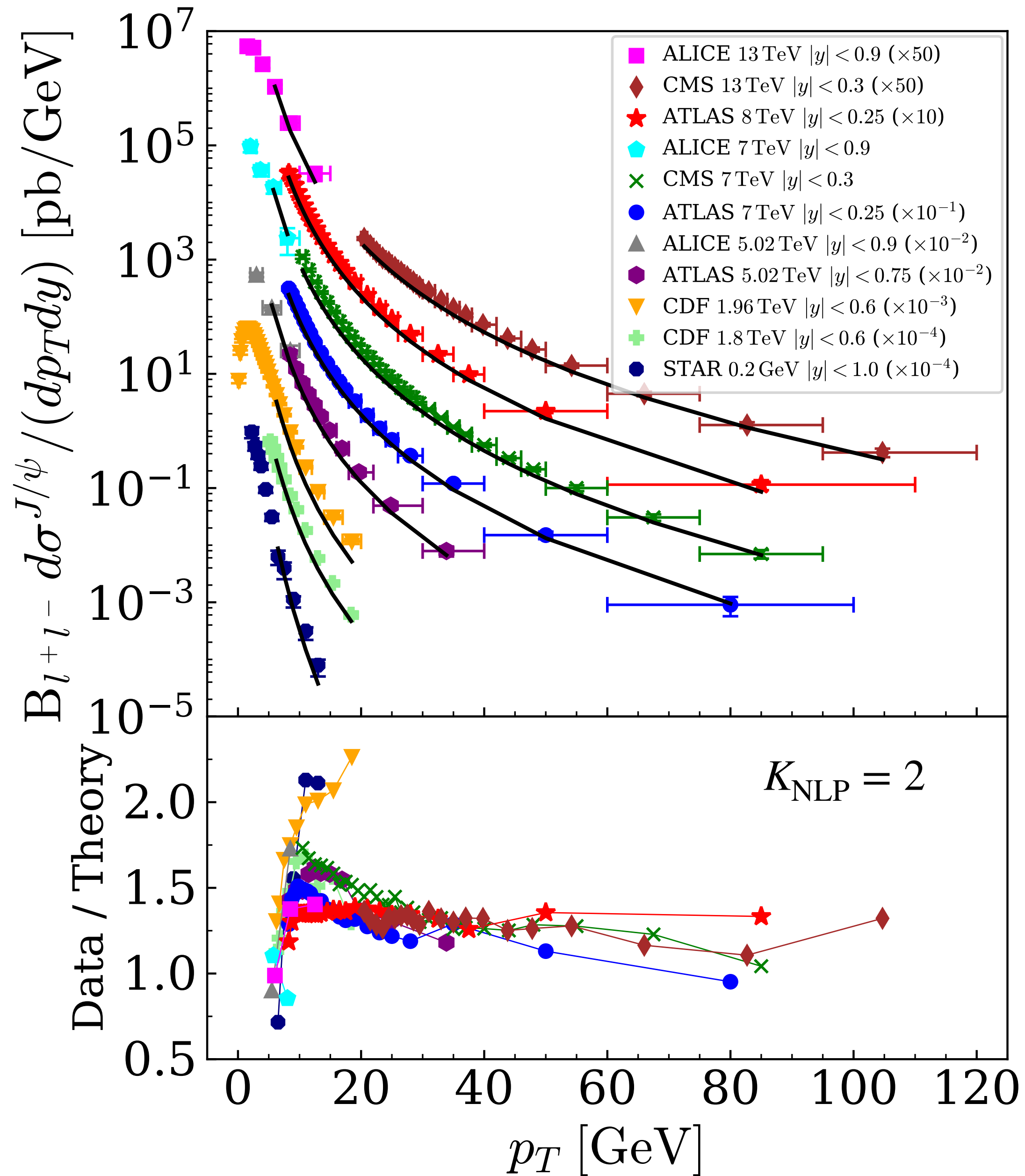


❖ We parametrized functional forms of the FFs if needed, and fitted the LDMEs to the high  $p_T$  data at CMS at 7, 13TeV.

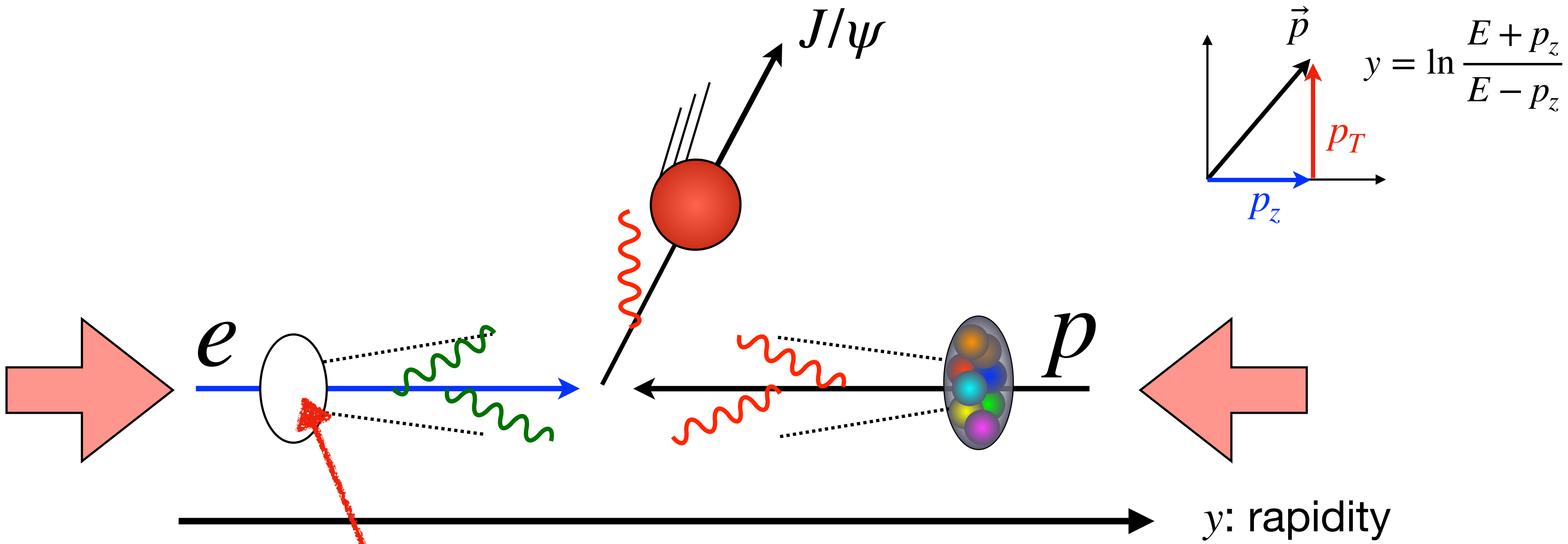
❖ Only the  $^1S_0^{[8]}$  channel is considered above, yielding unpolarized  $J/\psi$ .

# Green light to apply the FFs to ep collisions

Lee, Qiu, Sterman, KW, in preparation.



# Quarkonium production in ep collisions



**NEXT**

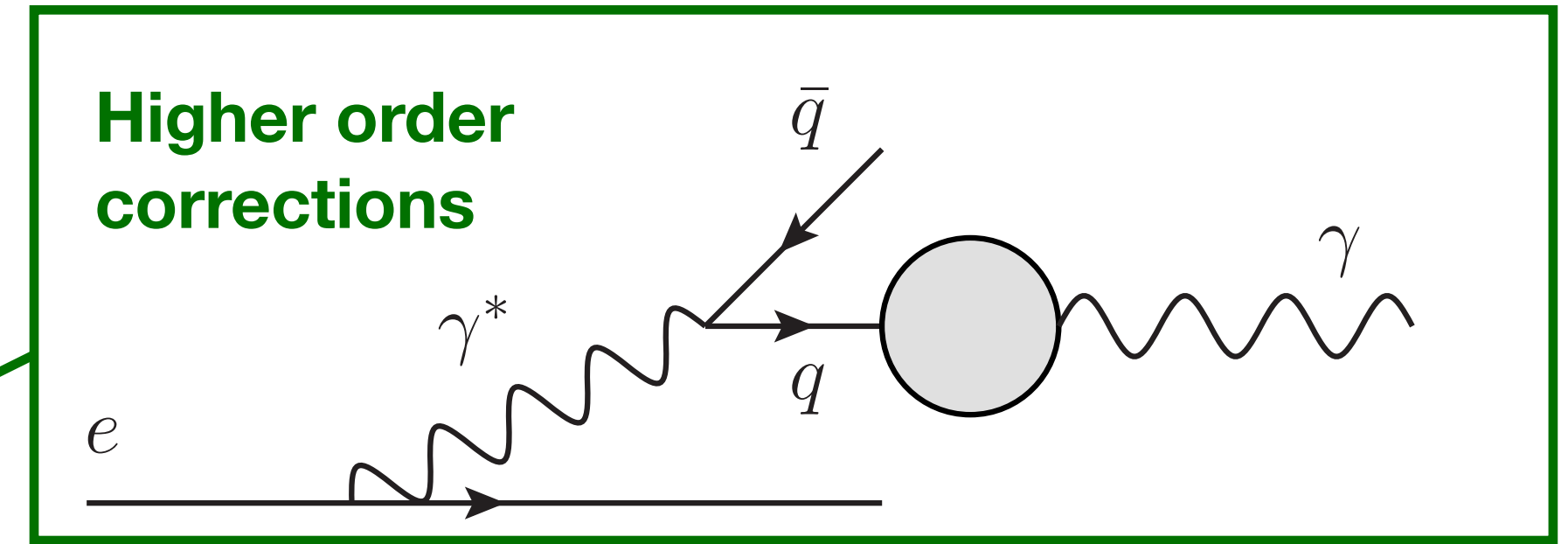
**Lepton Distribution Functions**

# Quantum evolution of LDFs

$$\xi^r = \frac{k_{\text{active lepton (parton)}}^+}{l_{\text{lepton}}^+} \quad e, \bar{e} = e^-, e^+$$

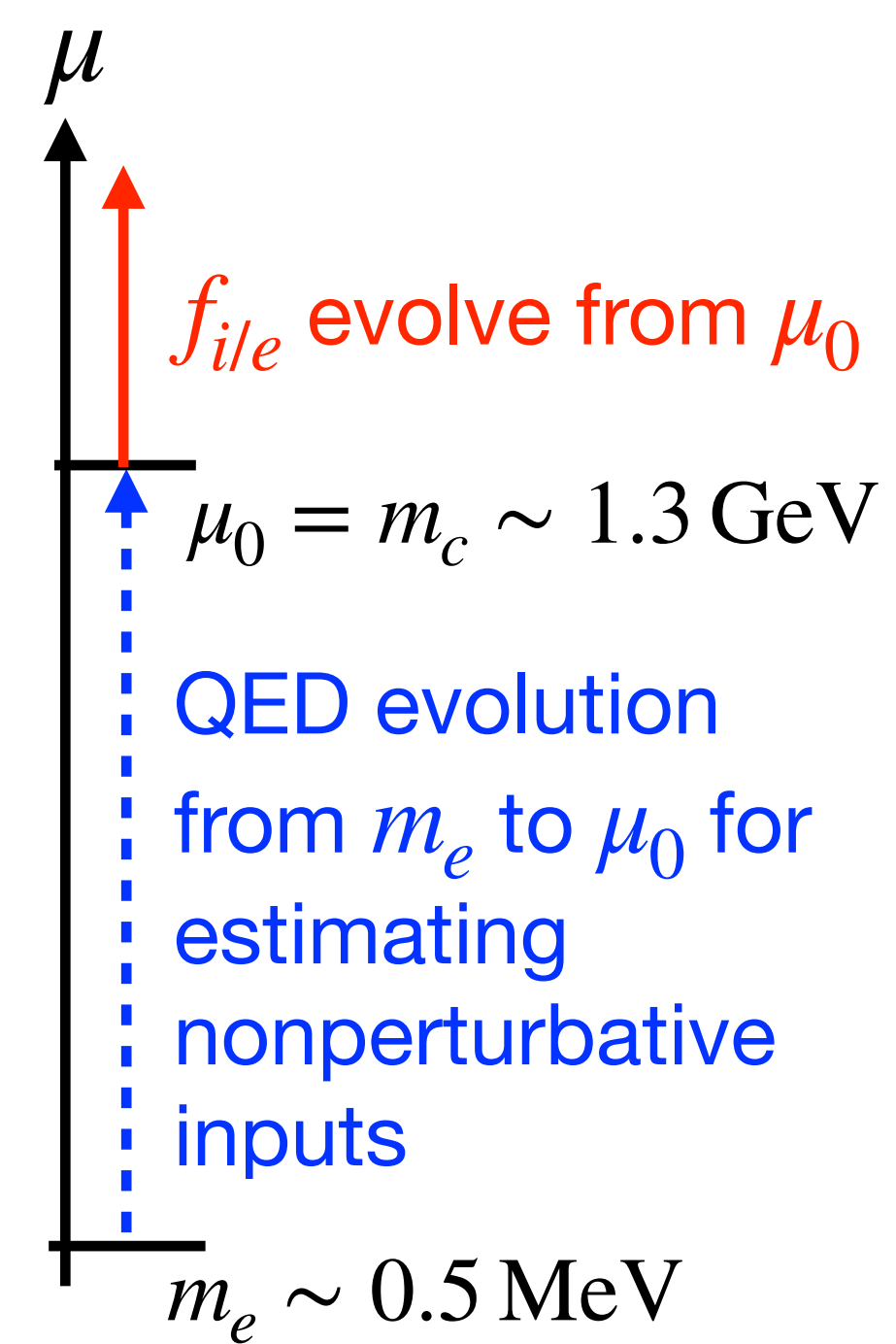
**QED part**

**Mixing part**



$$\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix} f_{ele}(\xi, \mu^2) \\ f_{\bar{e}le}(\xi, \mu^2) \\ f_{\gamma le}(\xi, \mu^2) \\ f_{qle}(\xi, \mu^2) \\ f_{\bar{q}le}(\xi, \mu^2) \\ f_{g le}(\xi, \mu^2) \end{pmatrix} = \begin{pmatrix} P_{ee}^{(1,0)} P_{e\bar{e}}^{(2,0)} P_{e\gamma}^{(1,0)} & P_{eq}^{(2,0)} P_{e\bar{q}}^{(2,0)} P_{eg}^{(2,1)} \\ P_{\bar{e}e}^{(2,0)} P_{\bar{e}\bar{e}}^{(1,0)} P_{\bar{e}\gamma}^{(1,0)} & P_{\bar{e}q}^{(2,0)} P_{\bar{e}\bar{q}}^{(2,0)} P_{\bar{e}g}^{(2,1)} \\ P_{\gamma e}^{(1,0)} P_{\gamma\bar{e}}^{(1,0)} P_{\gamma\gamma}^{(1,0)} & P_{\gamma q}^{(1,0)} P_{\gamma\bar{q}}^{(1,0)} P_{\gamma g}^{(1,1)} \\ P_{qe}^{(2,0)} P_{q\bar{e}}^{(2,0)} P_{q\gamma}^{(1,0)} & P_{qq}^{(0,1)} P_{q\bar{q}}^{(0,2)} P_{qg}^{(0,1)} \\ P_{\bar{q}e}^{(2,0)} P_{\bar{q}\bar{e}}^{(2,0)} P_{\bar{q}\gamma}^{(1,0)} & P_{\bar{q}q}^{(0,2)} P_{\bar{q}\bar{q}}^{(0,1)} P_{\bar{q}g}^{(0,1)} \\ P_{ge}^{(2,1)} P_{g\bar{e}}^{(2,1)} P_{g\gamma}^{(1,1)} & P_{gq}^{(0,1)} P_{g\bar{q}}^{(0,1)} P_{gg}^{(0,1)} \end{pmatrix} \otimes \begin{pmatrix} f_{ele}(\xi, \mu^2) \\ f_{\bar{e}le}(\xi, \mu^2) \\ f_{\gamma le}(\xi, \mu^2) \\ f_{qle}(\xi, \mu^2) \\ f_{\bar{q}le}(\xi, \mu^2) \\ f_{g le}(\xi, \mu^2) \end{pmatrix}$$

**Mixing part**                      **QCD part**

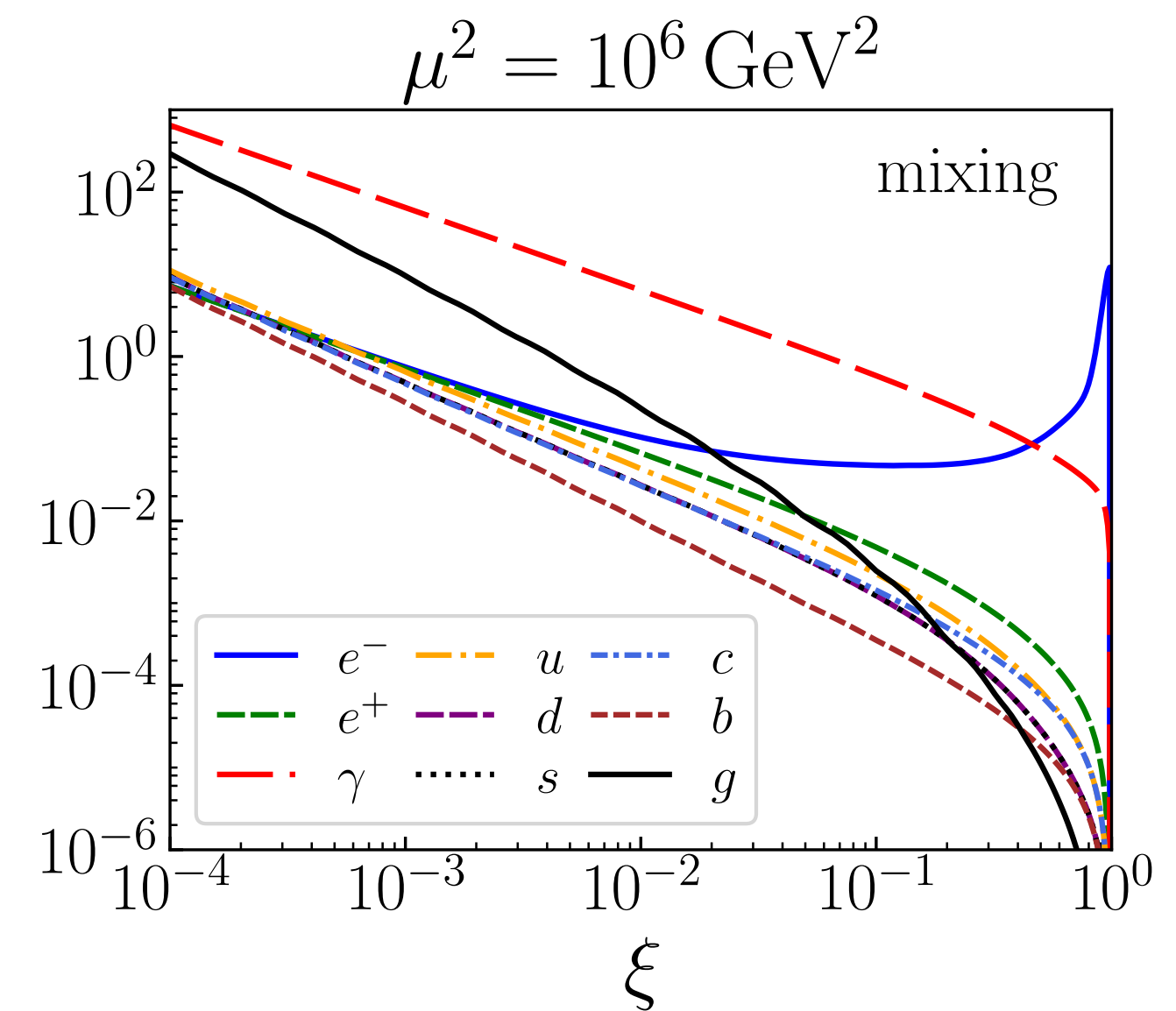
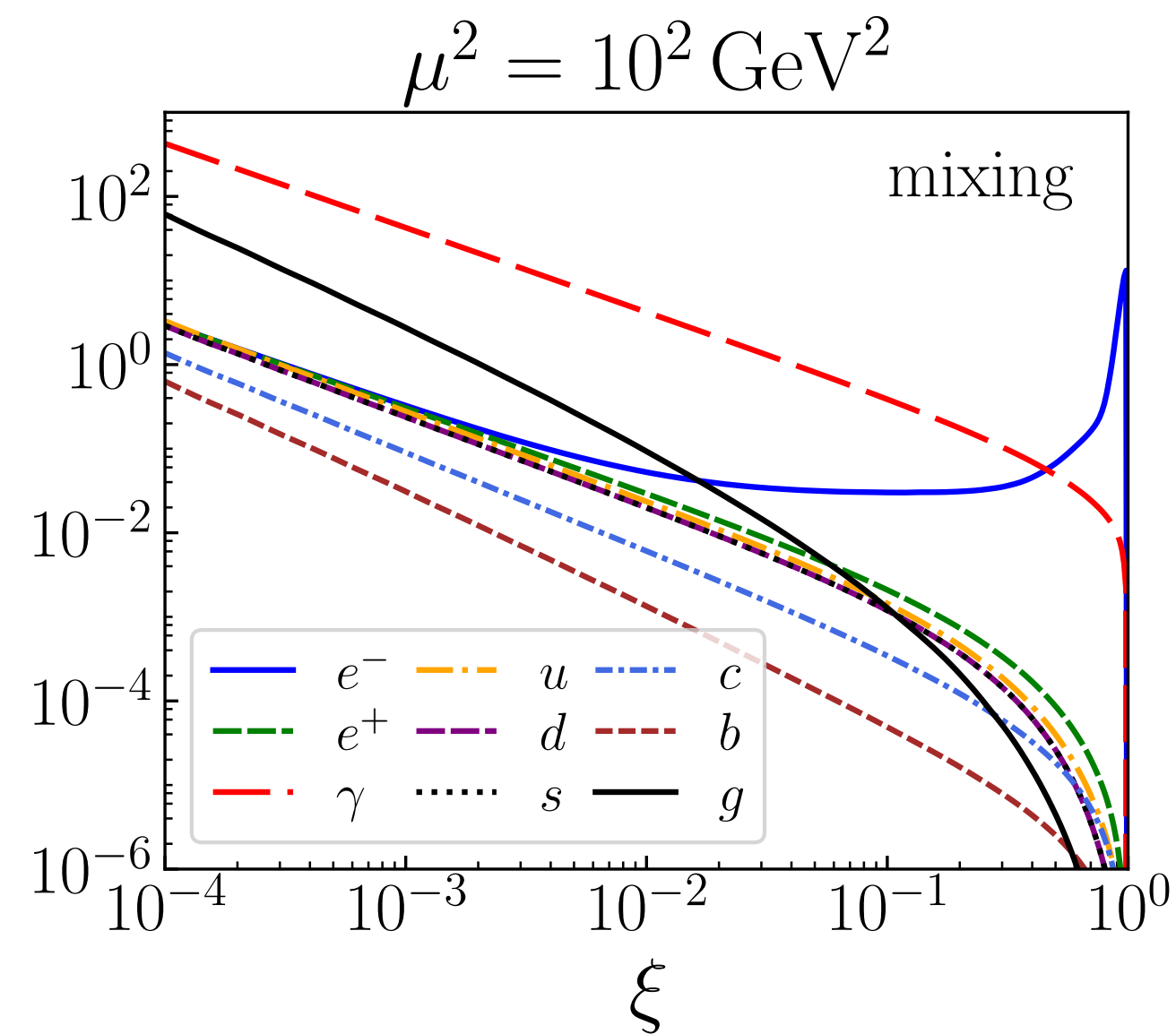
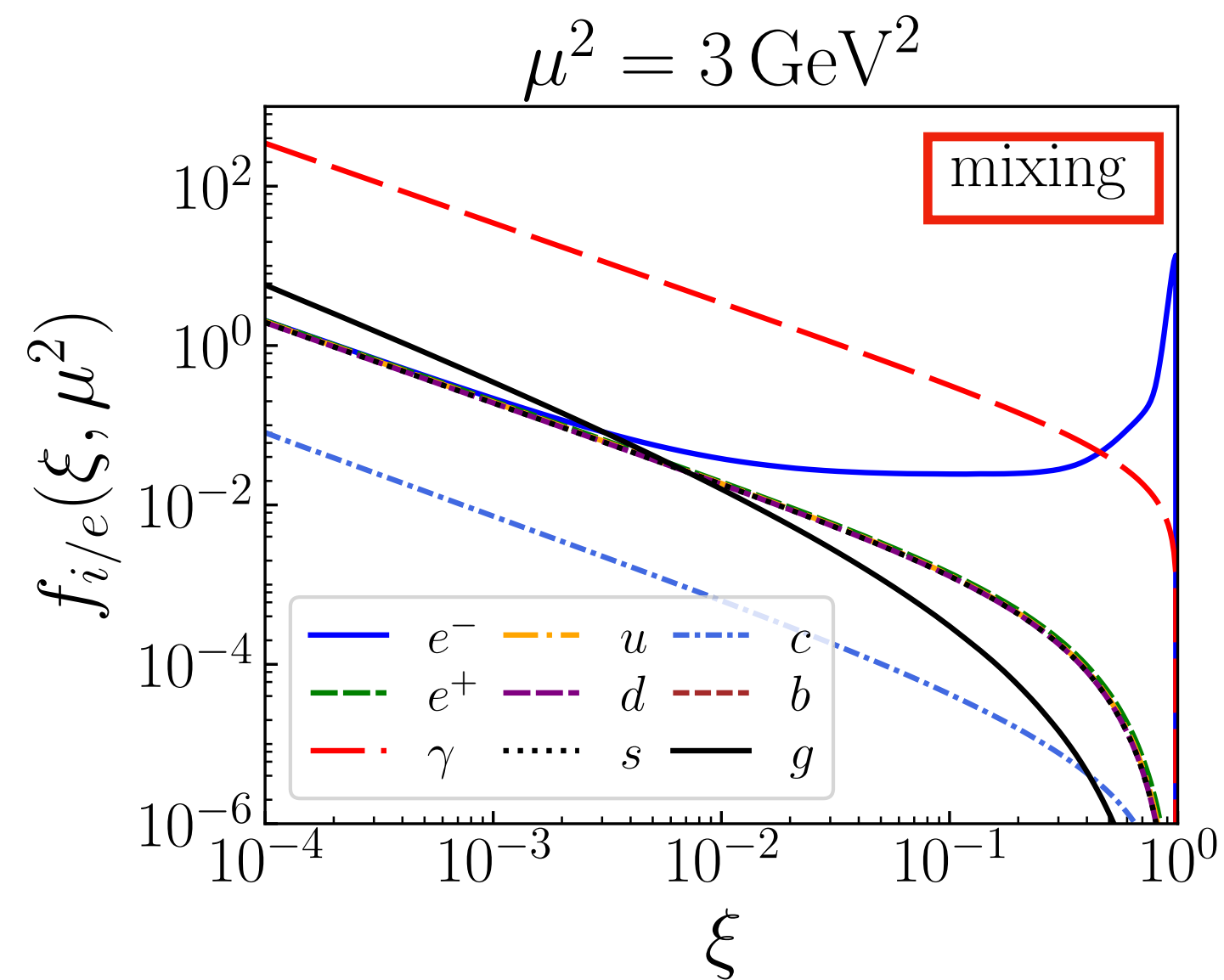
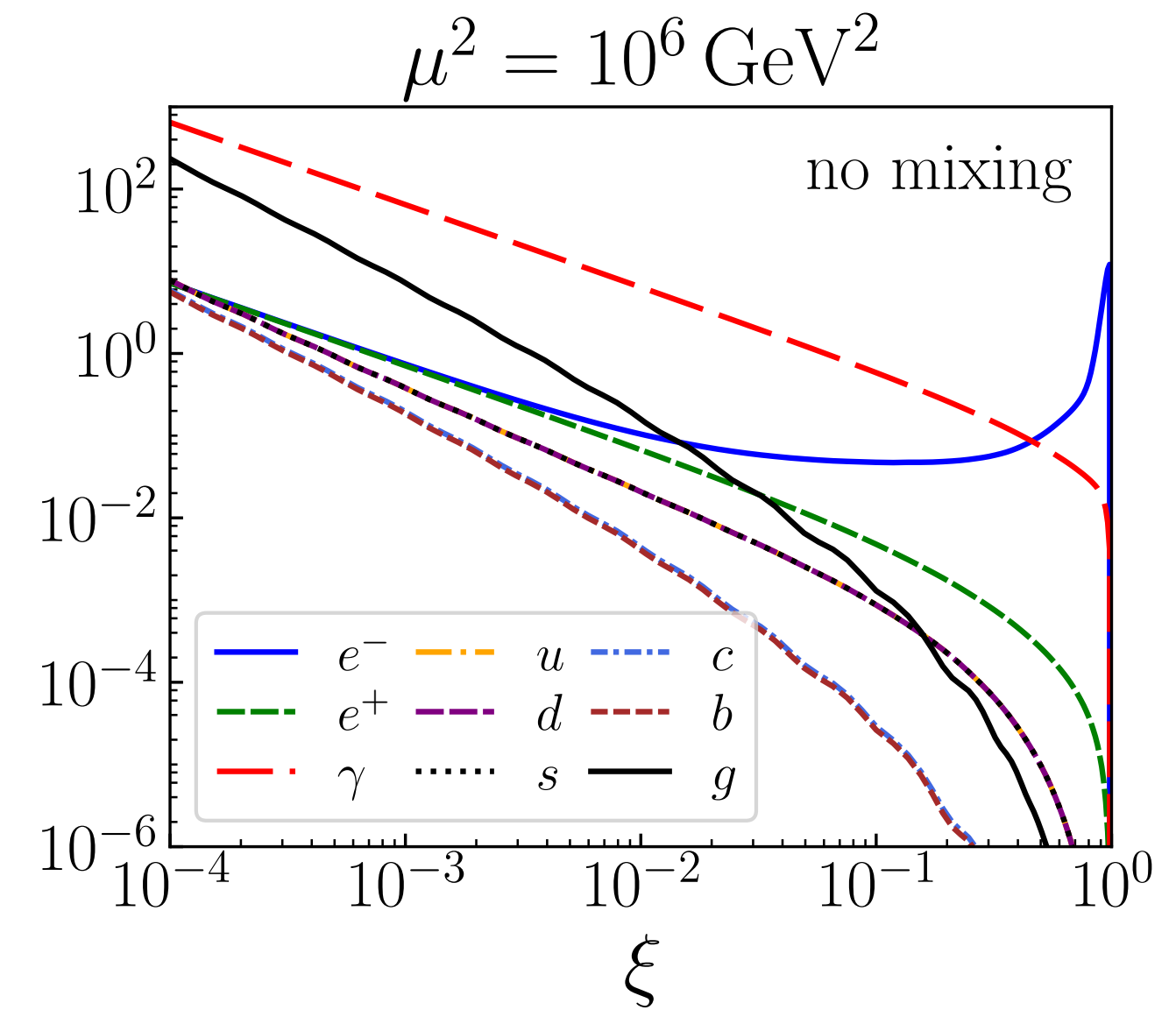
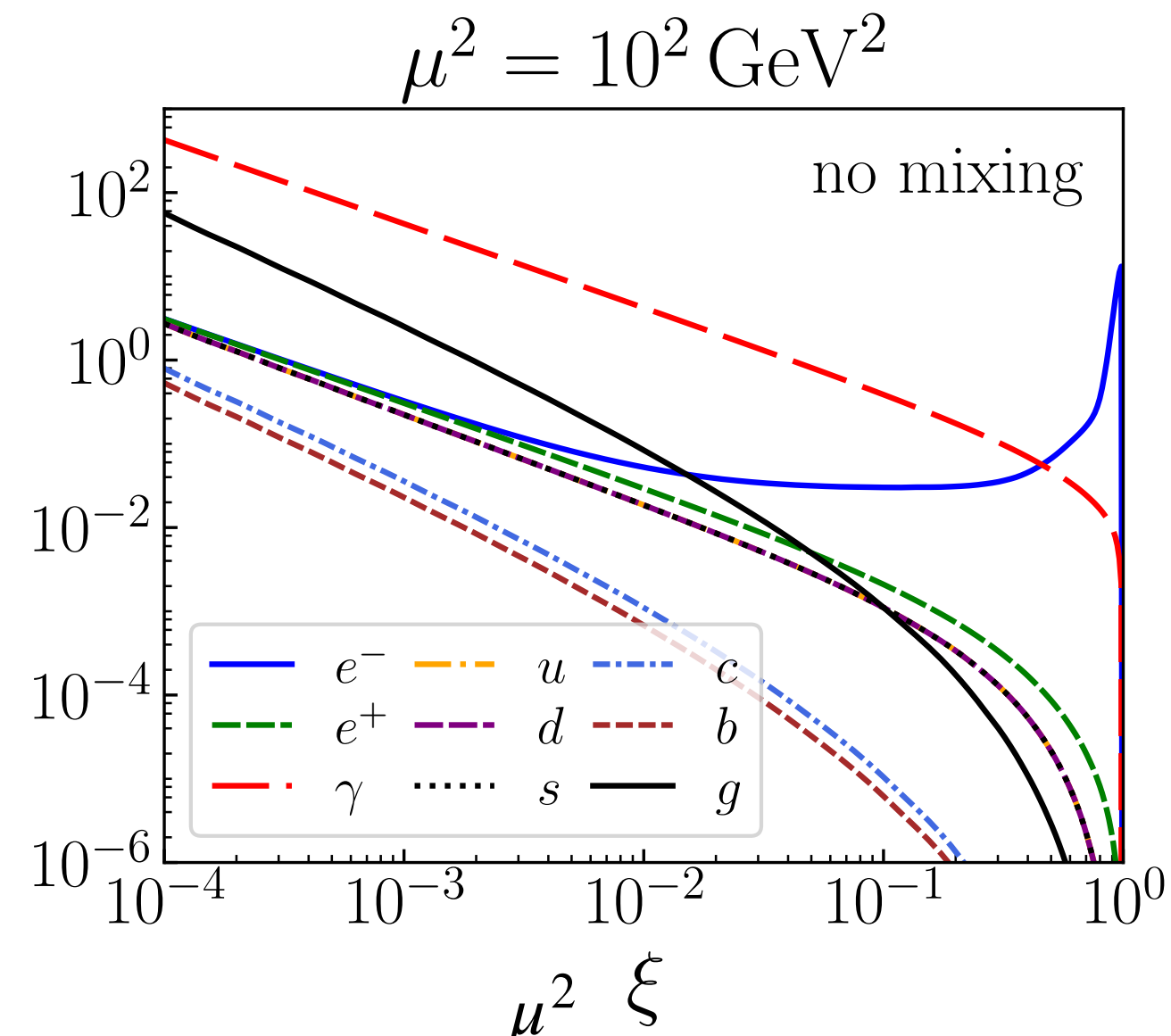
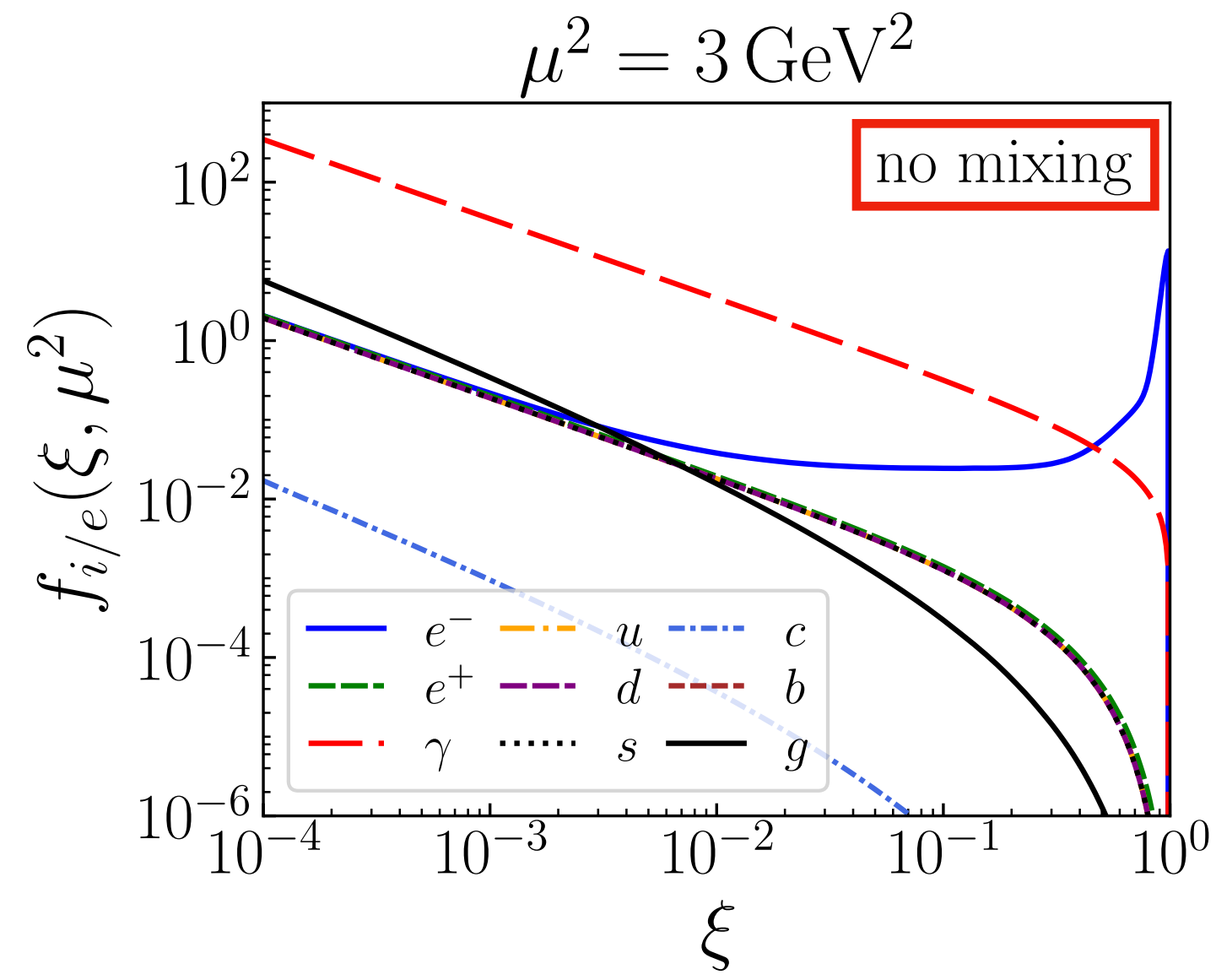


Splitting functions in QED+QCD:

$$P_{ij}(\xi, \mu^2) = \sum_{n,m=0}^{\infty} \left( \frac{\alpha_{em}(\mu^2)}{2\pi} \right)^n \left( \frac{\alpha_s(\mu^2)}{2\pi} \right)^m \hat{P}_{ij}^{(n,m)}(\xi) = \sum_{n,m=0}^{\infty} P_{ij}^{(n,m)}(\xi, \mu^2)$$

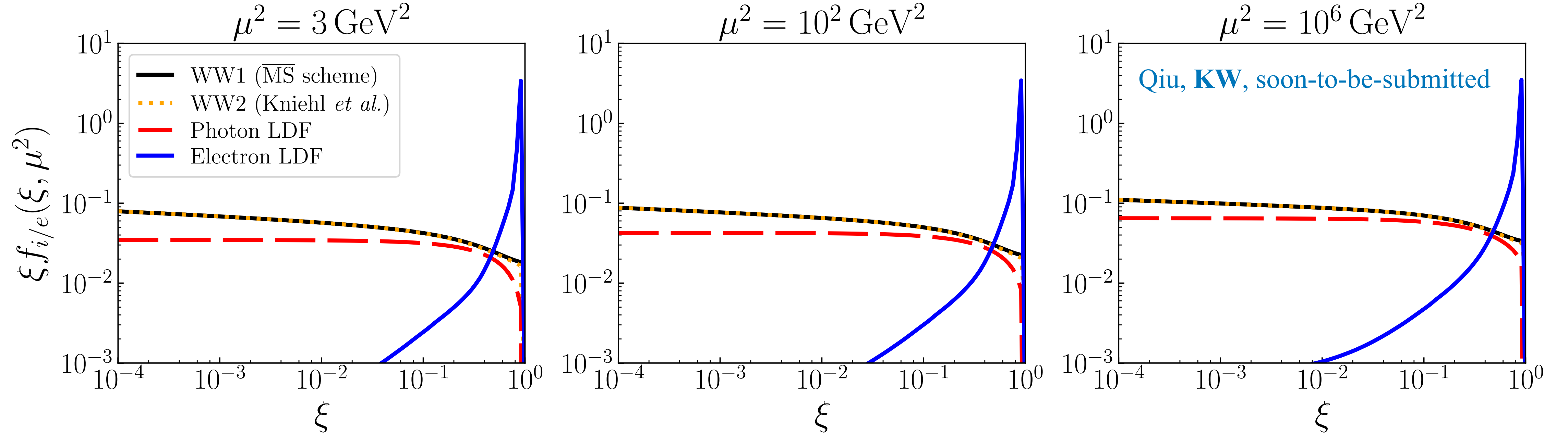
# LDFs after evolution

Qiu, KW, soon-to-be-submitted



QED (QCD) evolution is slow (fast) due to the weak (strong)  $\mu$ -dependence of  $\alpha_{em}$  ( $\alpha_s$ ).

# Photon LDF vs. Weizsäcker-Williams distribution



Weizsäcker-Williams (WW) distribution model at LO:

$$f_{\gamma le}^{WW1}(\xi, \mu^2) = \frac{\alpha_{em}(\mu^2)}{2\pi} P_{\gamma e}(\xi) \left[ \ln \left( \frac{\mu^2}{\xi^2 m_e^2} \right) - 1 \right]$$

$$f_{\gamma le}^{WW2}(\xi, \mu^2) = \frac{\alpha_{em}(\mu^2)}{2\pi} P_{\gamma e}(\xi) \left[ \ln \left( \frac{\mu^2}{\mu_{min}^2} \right) - 2m_e^2 \xi \left( \frac{1}{\mu^2} - \frac{1}{\mu_{min}^2} \right) \right] \quad \text{with } \mu_{min}^2 = (m_e^2 \xi^2) / (1 - \xi)$$

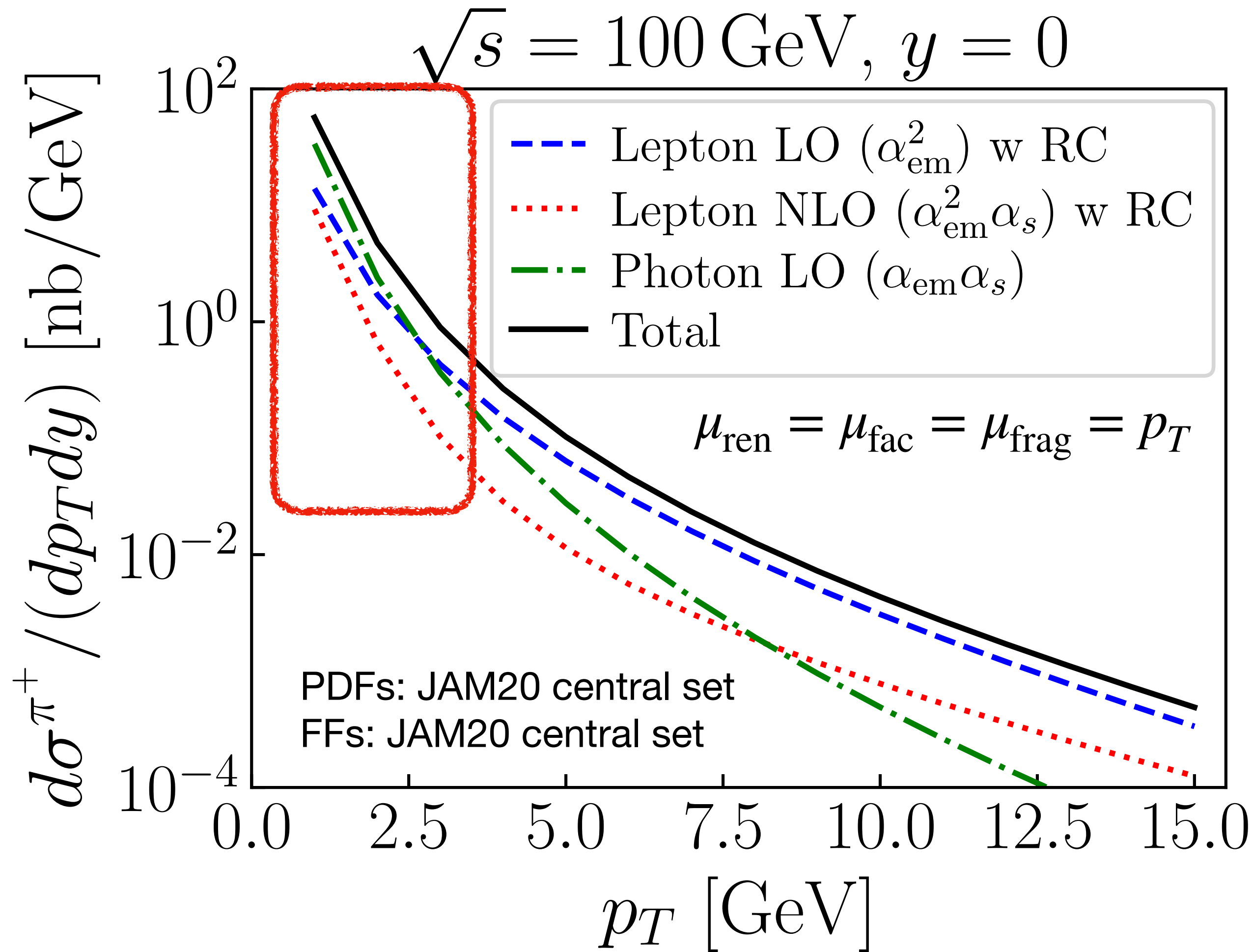
Hinderer, Schlegel, Vogelsang, PRD92, no.1, 014001 (2015)  
 B.A.Kniehl, G.Kramer and M.Spira, Z. Phys. C 76, 689 (1997)

- Photon LDF is smaller than the WW distribution because the large log is resummed, and there are higher-order corrections ( $\gamma \rightarrow e^+ e^-, q\bar{q}$ ).

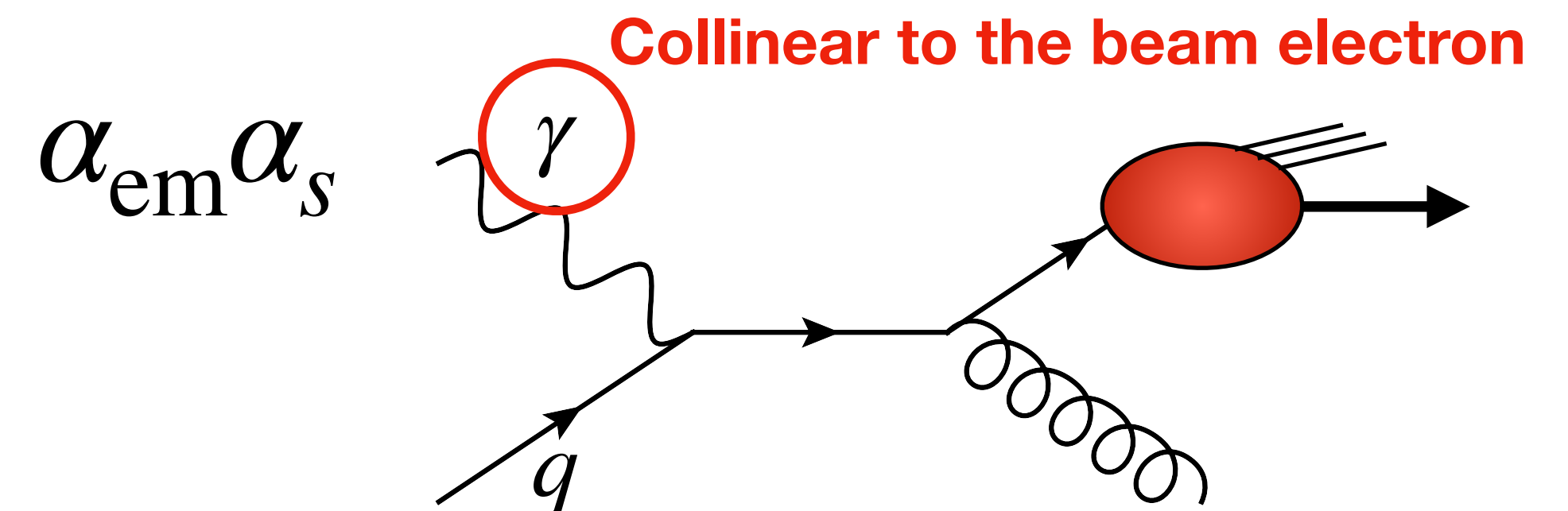
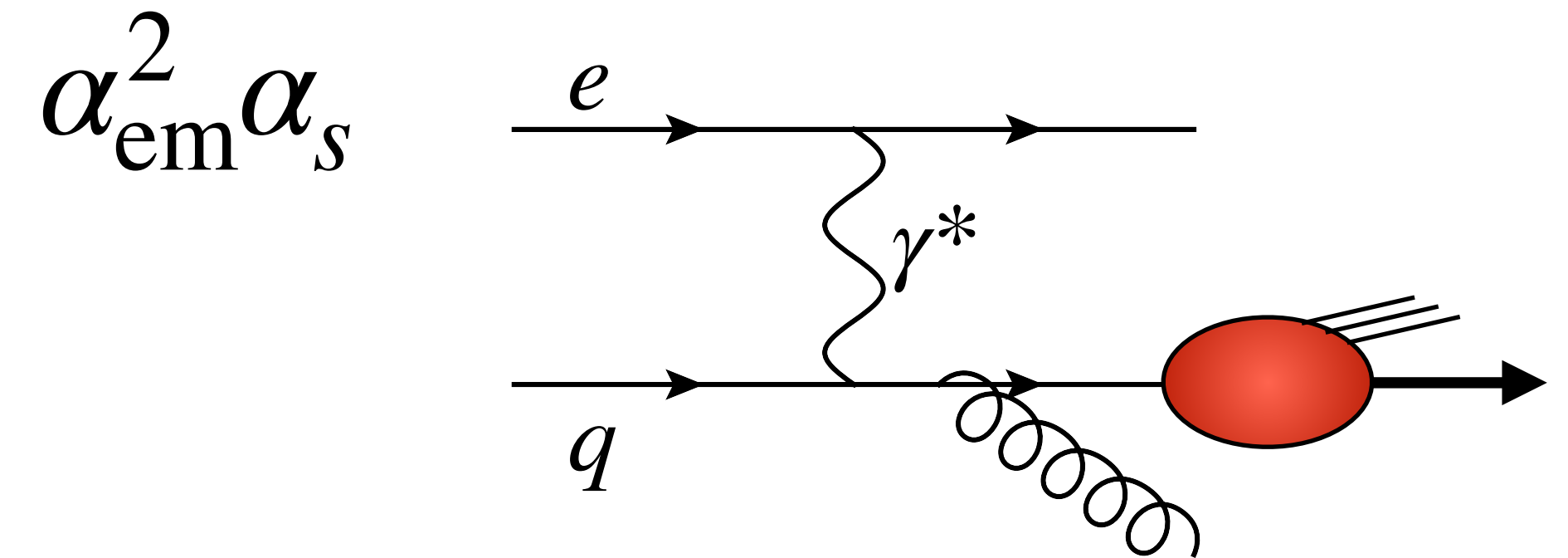
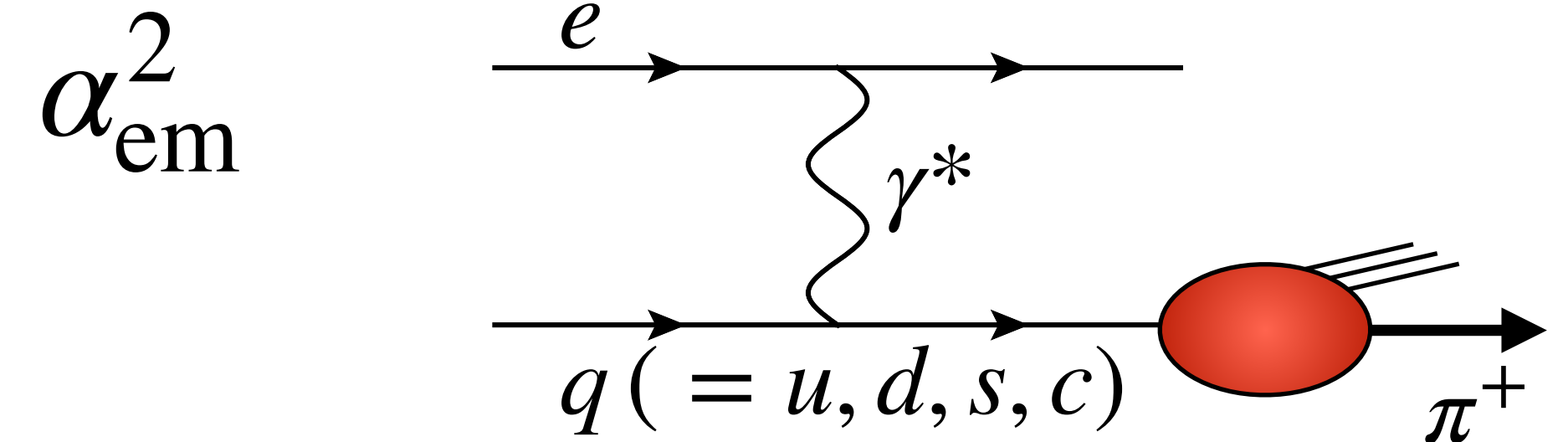


# Lepto- and photo-production of light-hadron at LP

Qiu, KW, soon-to-be-submitted



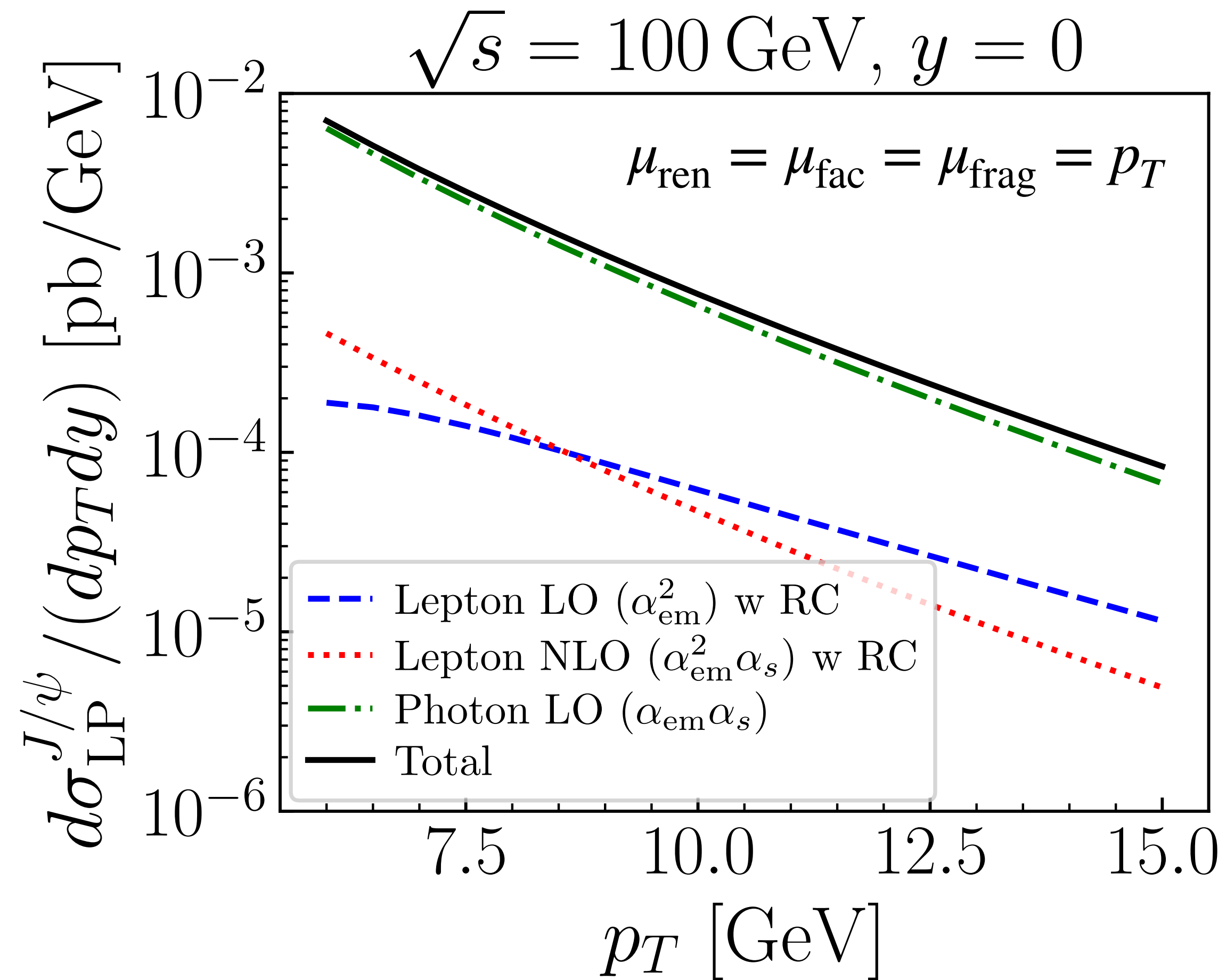
Leading-power (LP) in  $1/p_T$



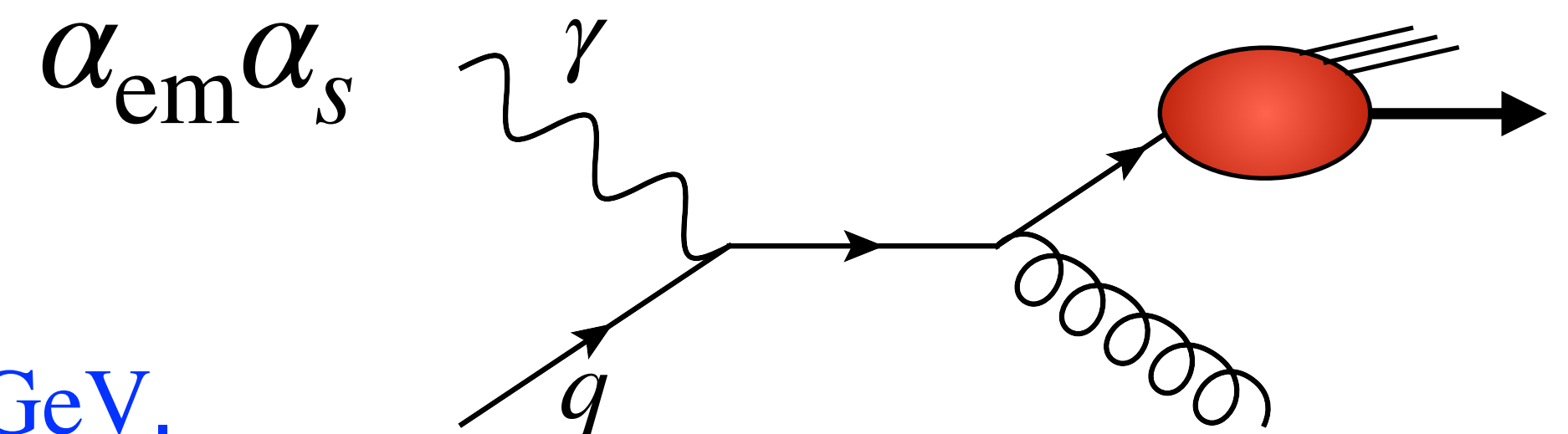
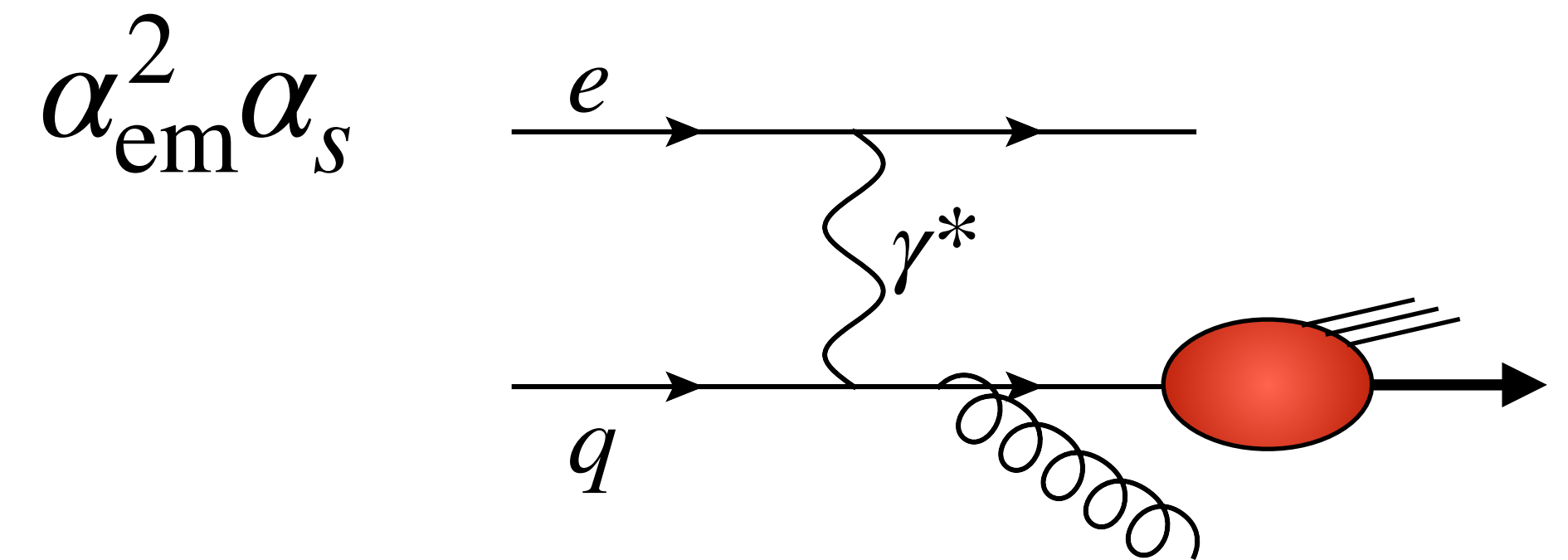
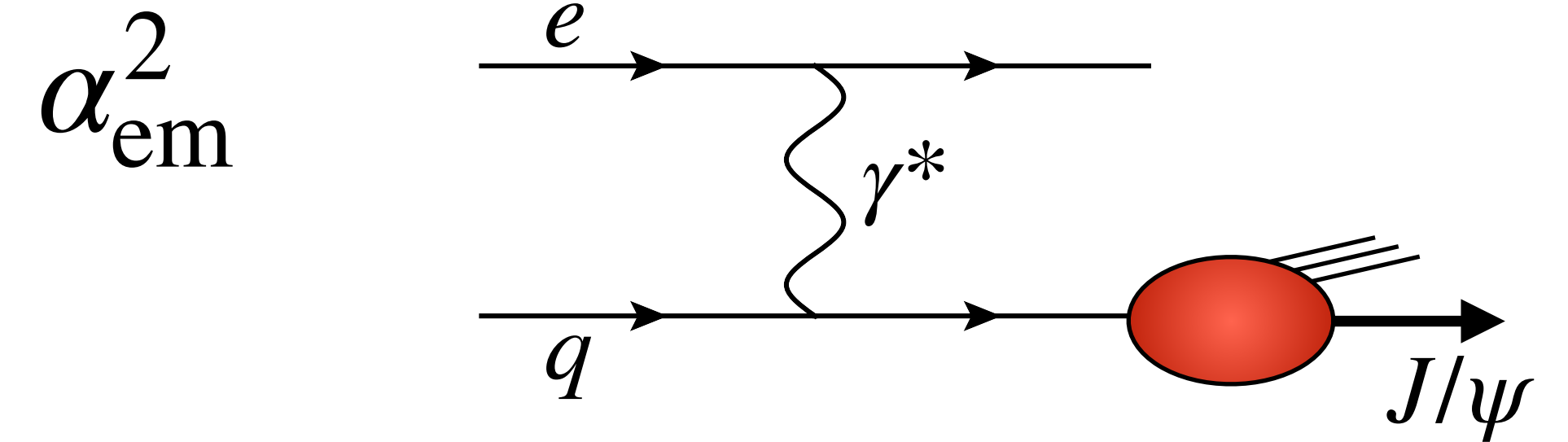
❖ The photoproduction overwhelms the leptoproduction at lower  $p_T$ .

# Lepto- and photo-production of $J/\psi$ at LP

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Leading-power (LP) in  $1/p_T$



PDFs: CT18ANLO central set

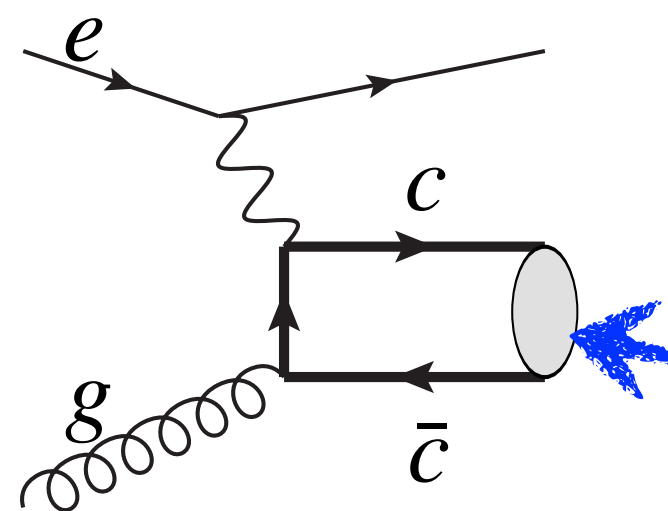
FFs: LQSW set [Lee, Qiu, Sterman, KW, 2108.00305, 2211.12648]

Input scale for the evolution of quarkonium FFs:  $\mu_0 = 2m_{J/\psi} \sim 6 \text{ GeV}$ .

# Power corrections: Parton production

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$$\alpha_{em}^2 \alpha_s \text{ (LE)}$$



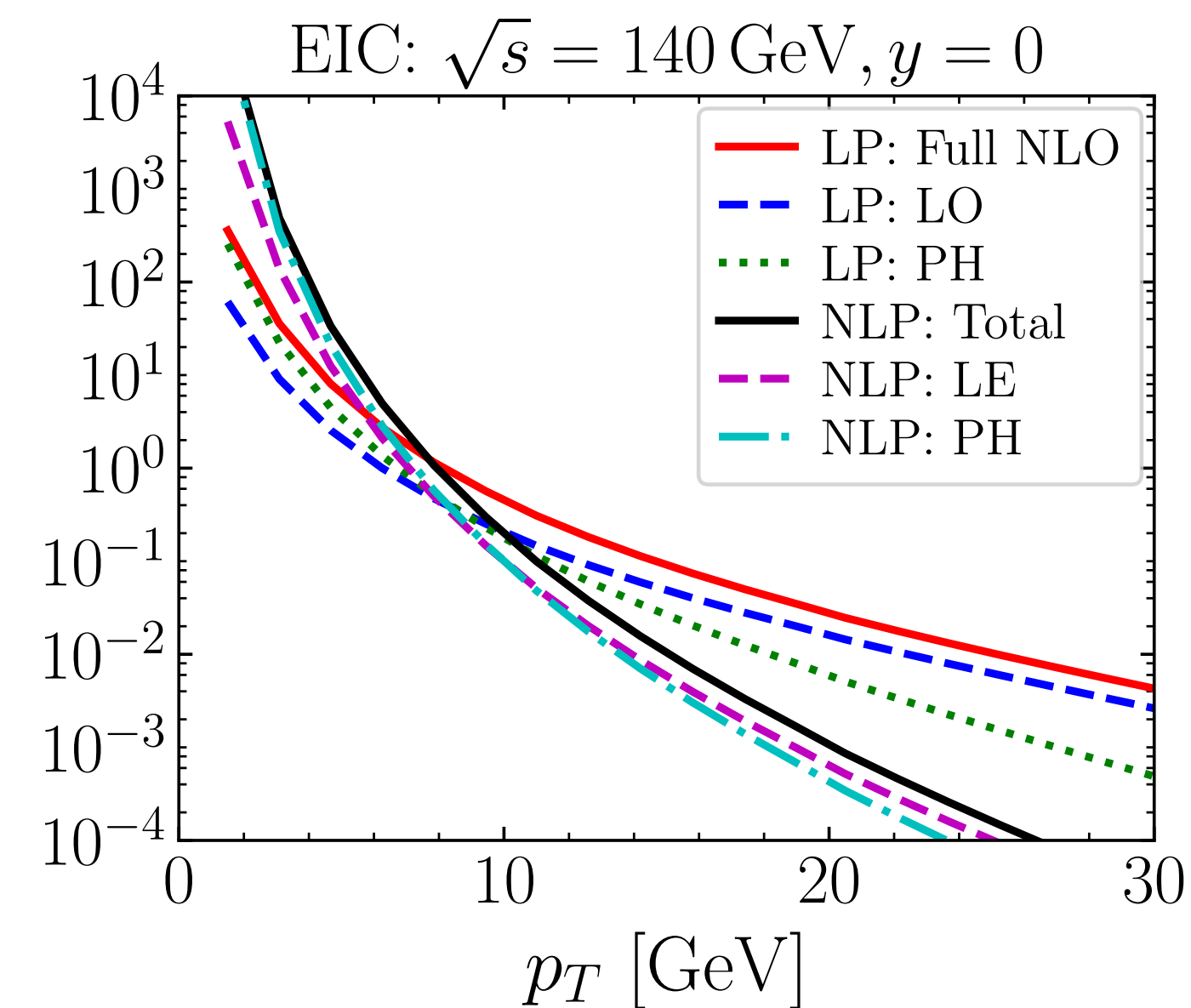
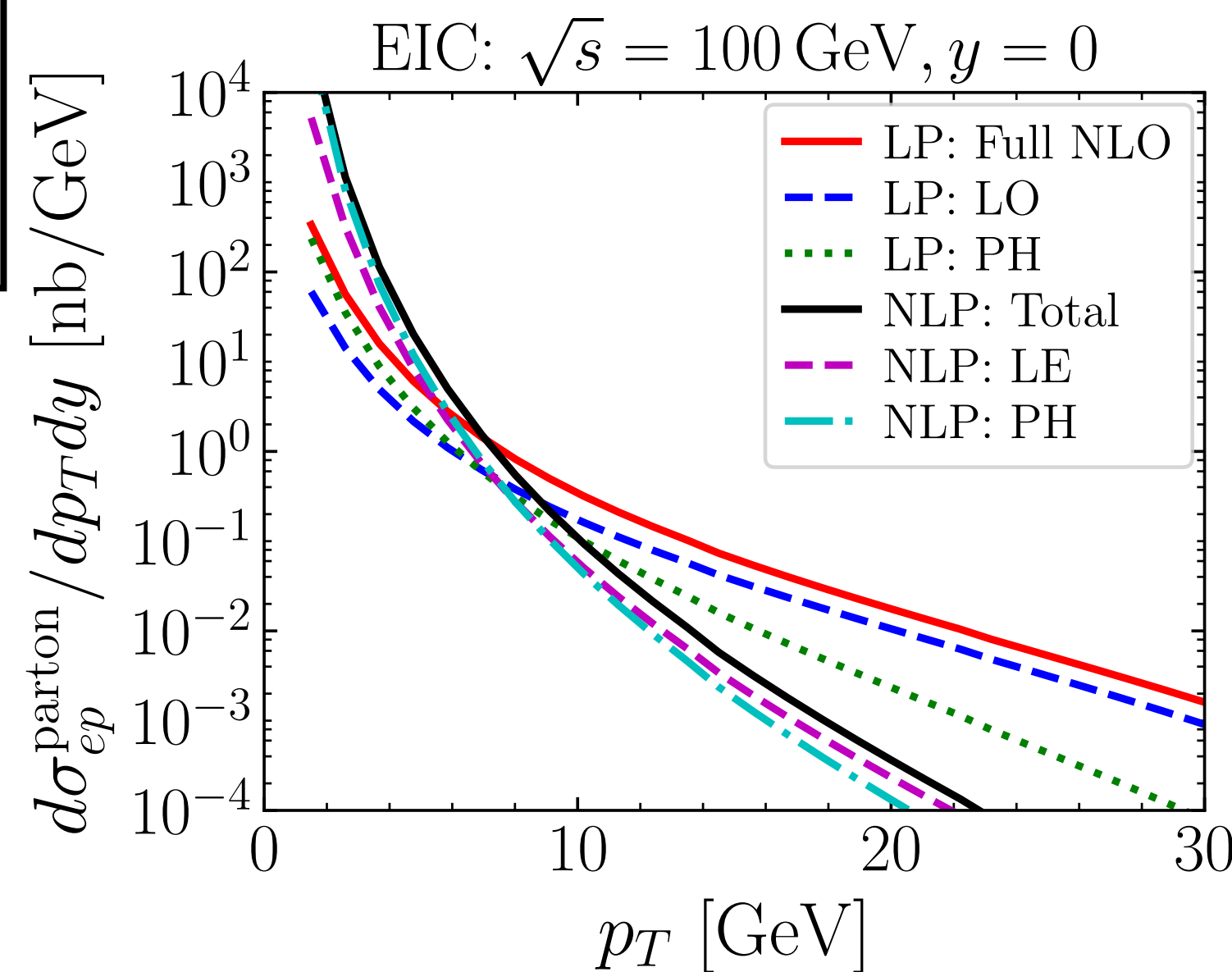
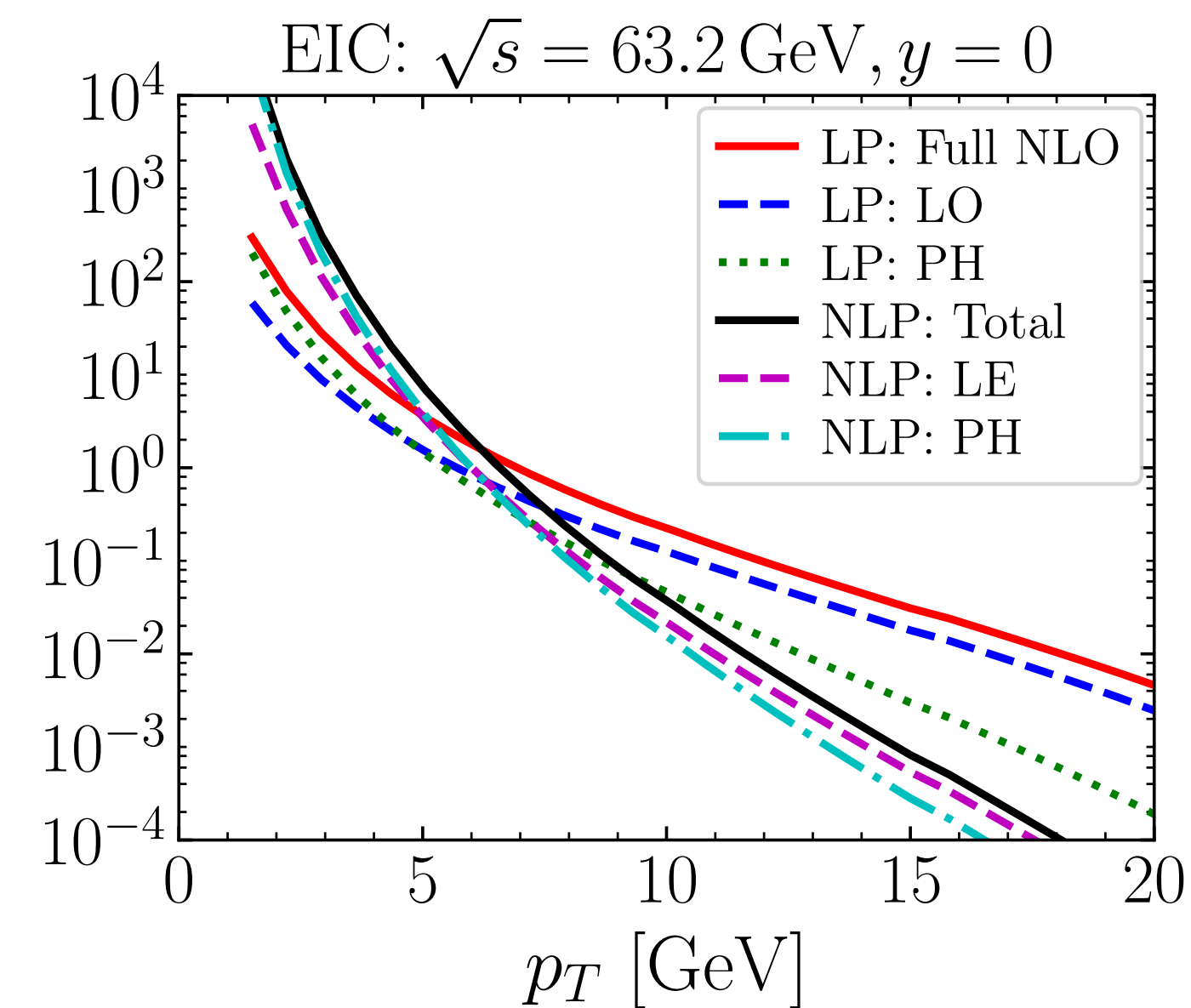
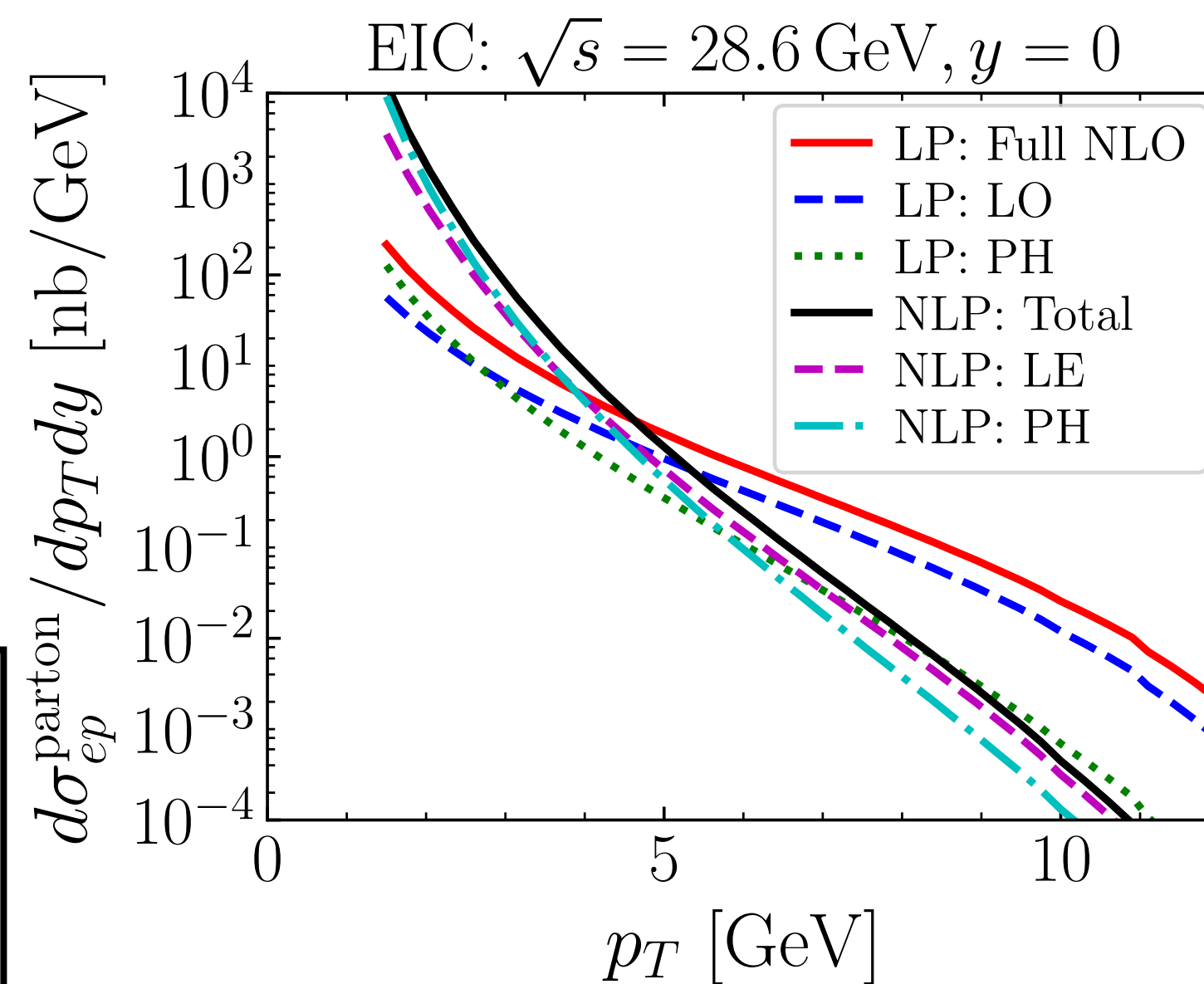
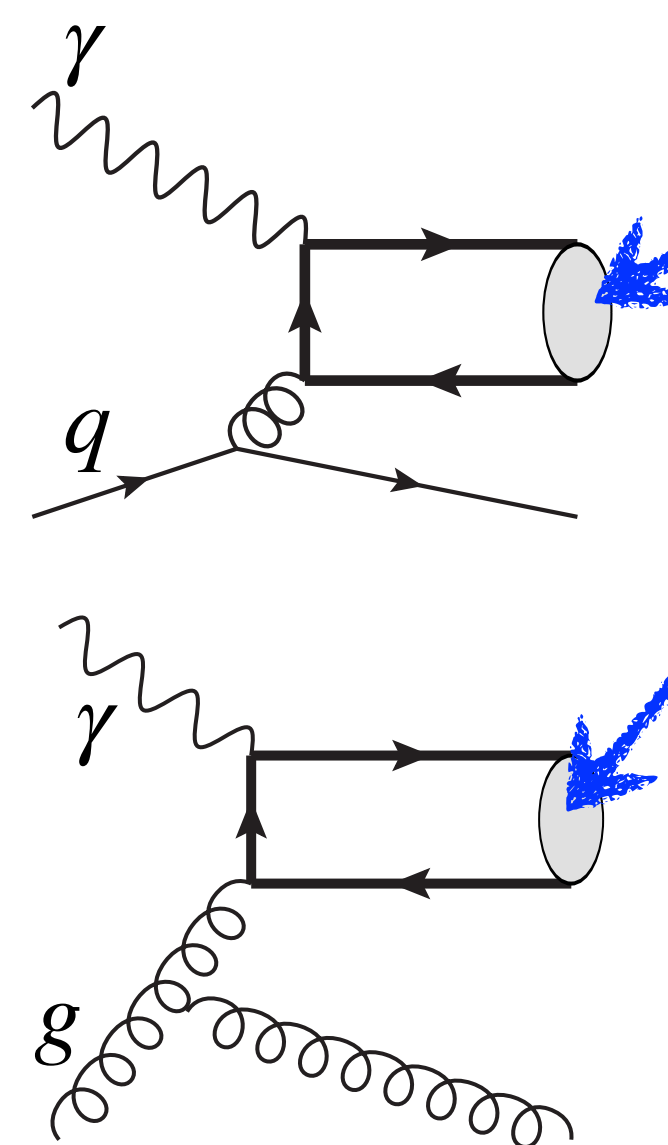
projection

$$\kappa = (v, a, t) [1,8]$$

$$= (\gamma^+, \gamma^+ \gamma^5, \gamma^+ \gamma_\perp^i) [1,8]$$

*v*: vector  
*a*: axial vector  
*t*: tensor (suppressed at high  $p_T$ )

$$\alpha_{em} \alpha_s^2 \text{ (PH)}$$

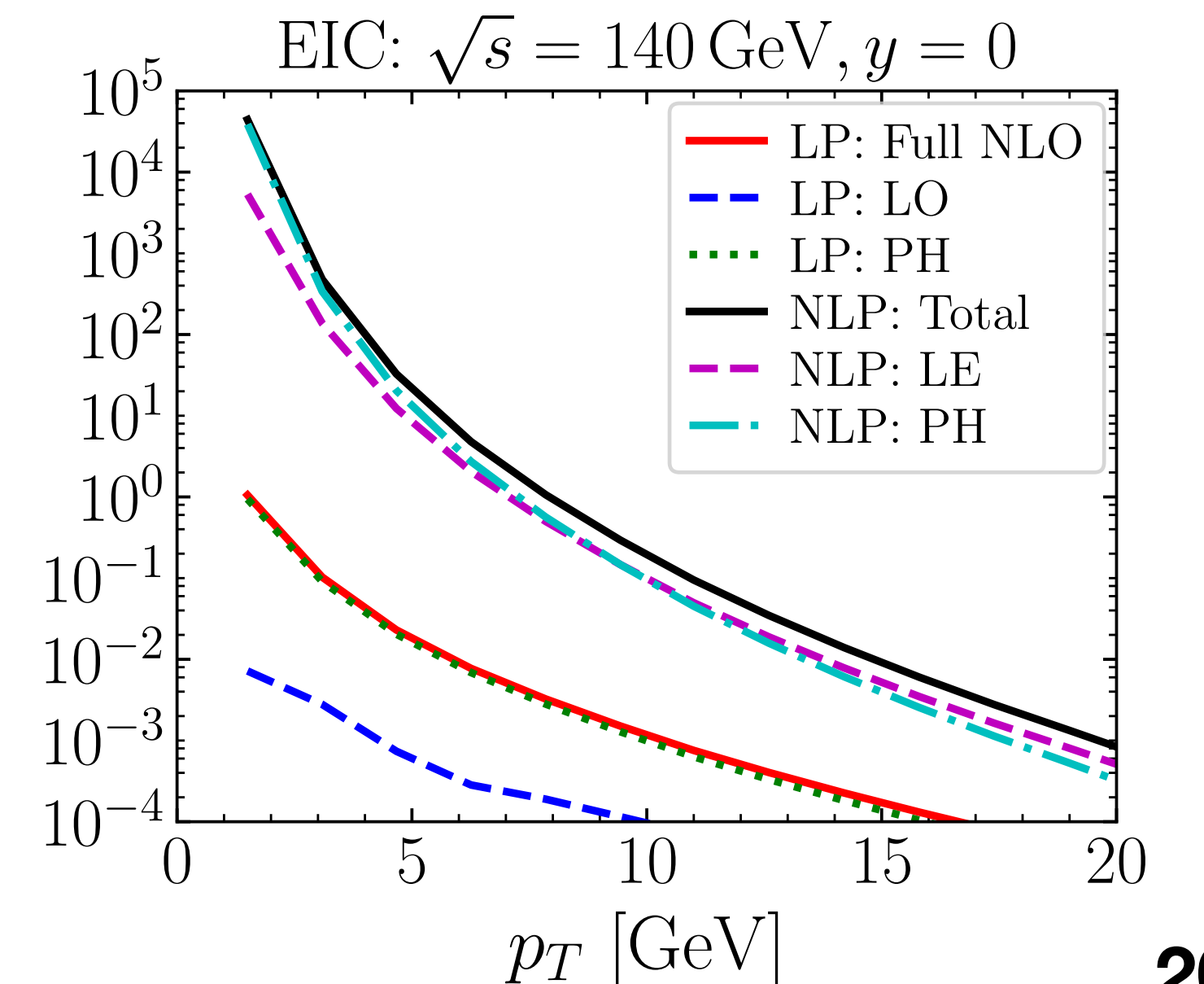
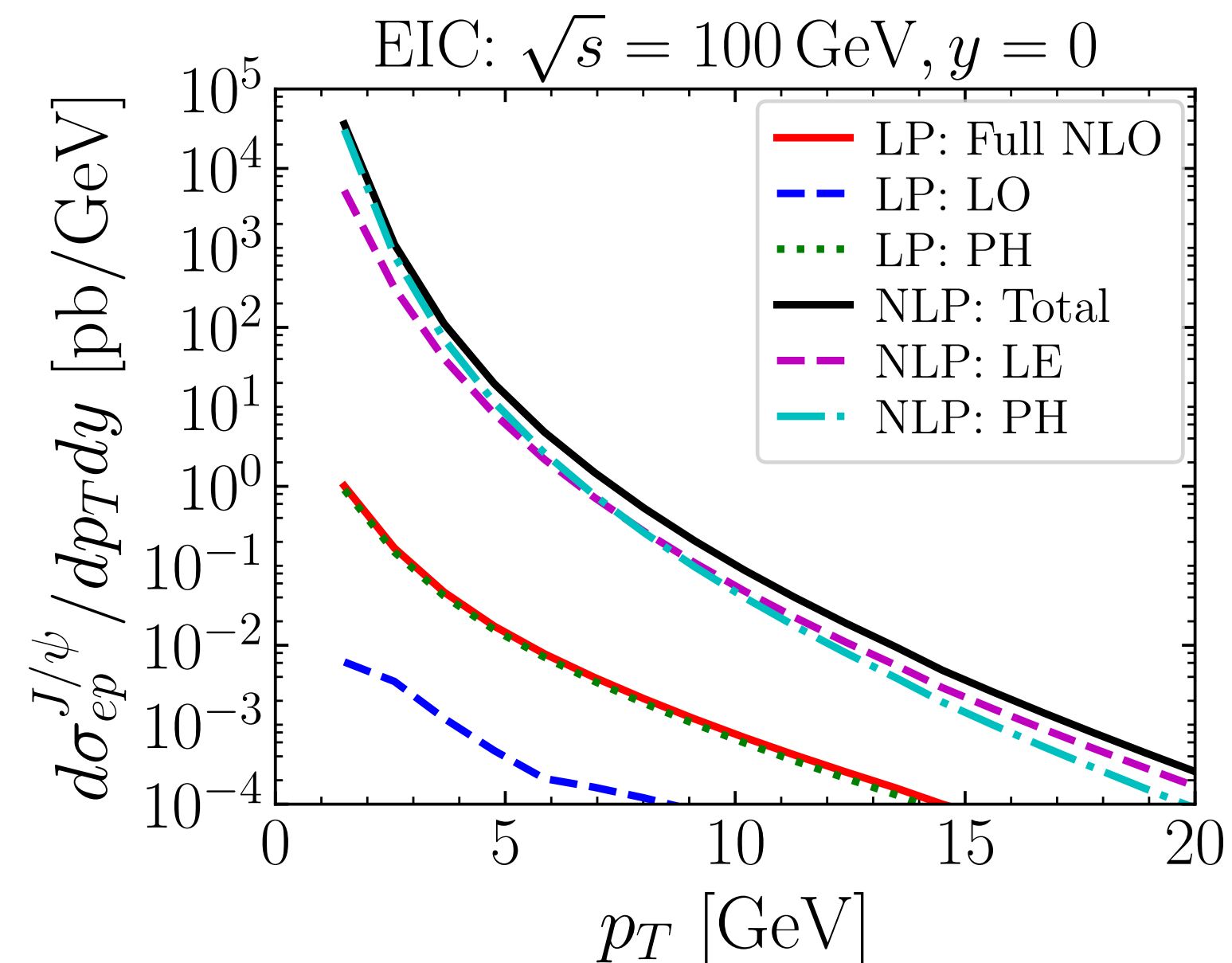
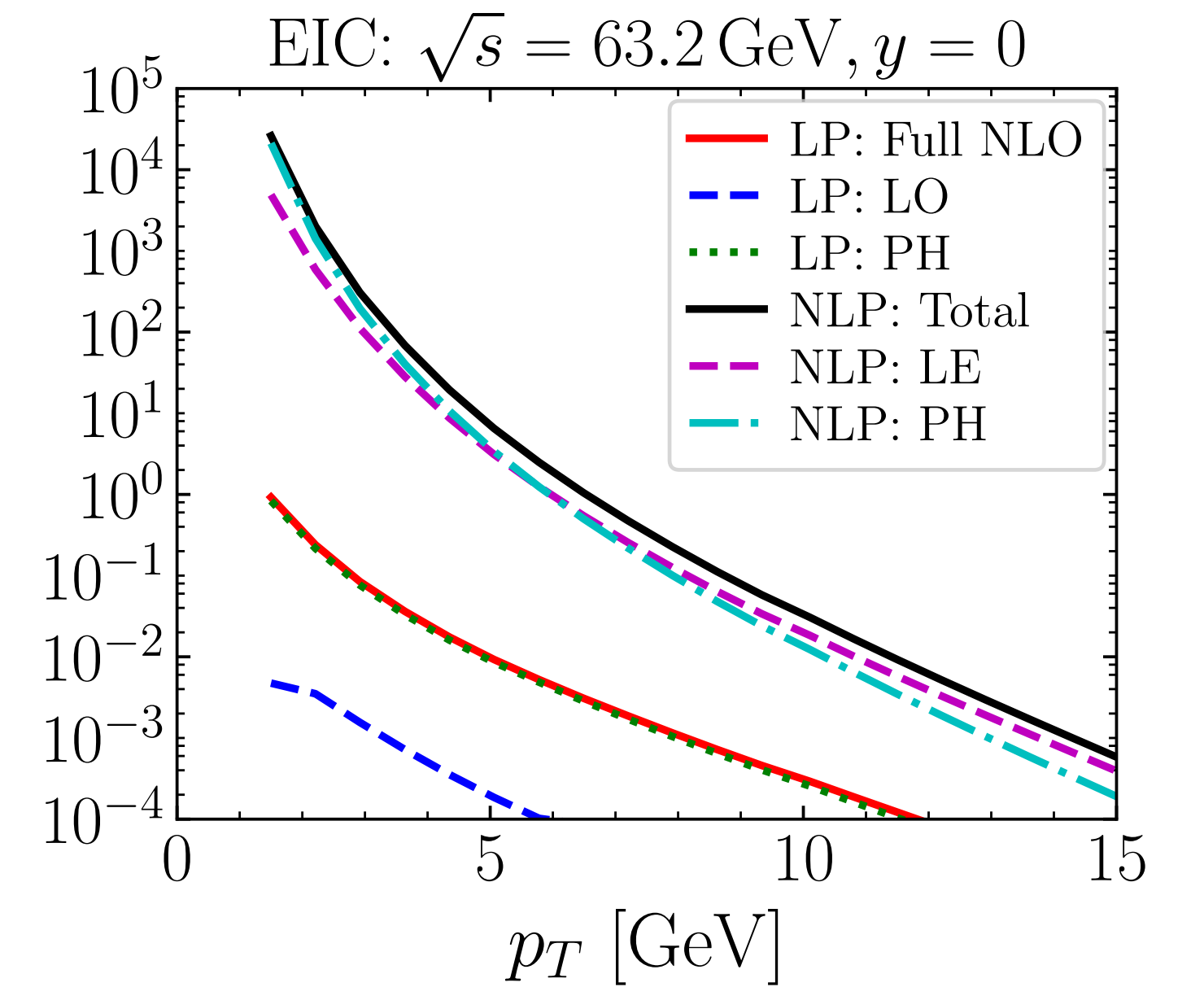
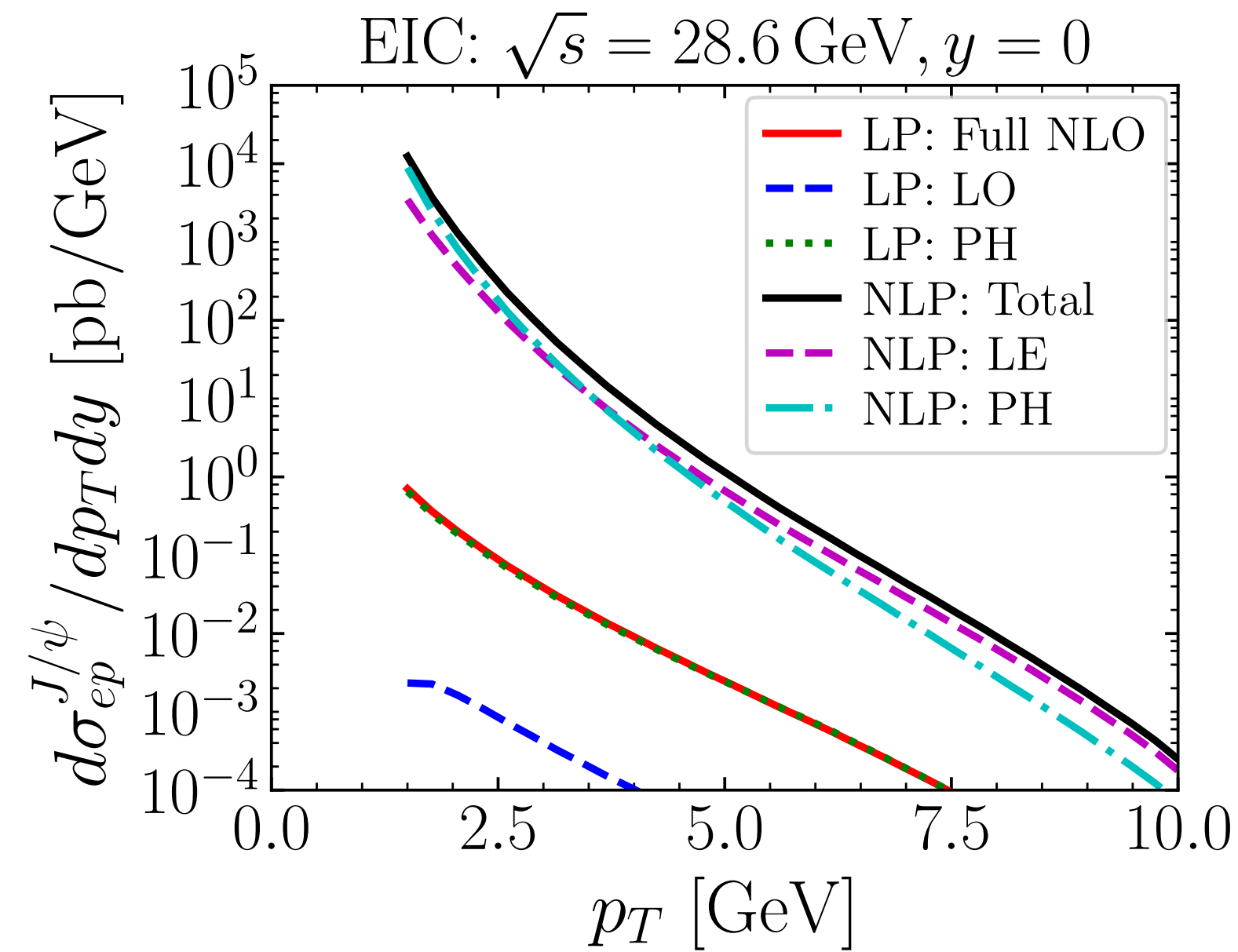


At NLP:  $p_T =$  Total transverse momentum of  $c\bar{c}$

# Lepto- and photo-production of $J/\psi$

Qiu, KW, soon-to-be-submitted

- ❖ **NLP is predominant.**
- ❖  $c\bar{c}$  easily forms a  $J/\psi$  bound state than single partons.
- ❖ At low  $p_T$ , Fixed order NRQCD should take over the QCD factorization approach with the FFs
- ❖ A matching is needed. (future work)



# Summary

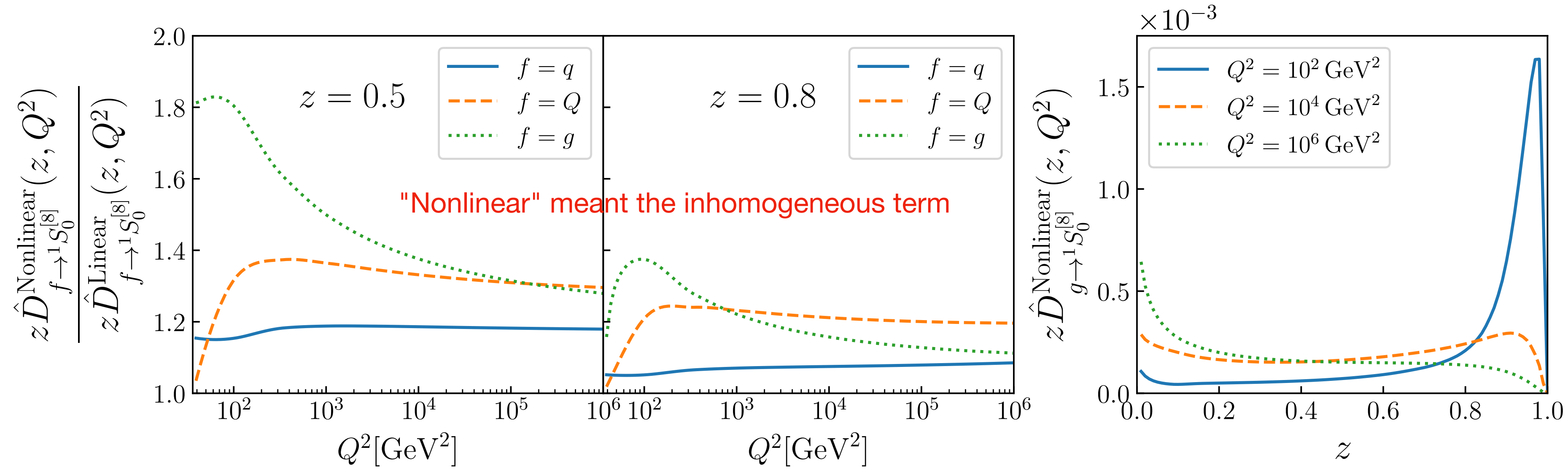
- ❖ Single-inclusive  $J/\psi$  production of high  $p_T$  in lepton-hadron collisions was studied using the hybrid factorization formalism in QED and QCD.
- ❖ For  $J/\psi$  production, the NLP contributions are important in ep collisions at EIC; future data comparison could further improve the shape of the FFs.
- ❖ The lepto-production and photo-production of  $J/\psi$  are equally treated in the factorization formalism without introducing an artificial cut in  $Q^2$ ;  $p_T$  is a unique scale in theory.
- ❖ **Outlook1**: A global fitting could systematically improve LDFs.
- ❖ **Outlook2**: Hadron production in SIDIS (e.g.  $e + p \rightarrow e' + J/\psi + X$ ), sensitive to TMDs.

***Thank you!***

# Appendix

# Quark pair corrections to SP FFs

Lee, Qiu, Sterman, KW, SciPost Phys. Proc.8, 143 (2022)



The quark pair corrections to DGLAP evolution remain significant even at high  $\mu^2 \sim p_T^2$ .

$$\frac{\partial D_{f \rightarrow H}}{\partial \ln \mu^2} = \gamma_{f \rightarrow f'} \otimes D_{f' \rightarrow H} + \frac{1}{\mu^2} \gamma_{f \rightarrow [Q\bar{Q}(\kappa)]} \otimes D_{[Q\bar{Q}(\kappa)] \rightarrow H}$$

$$\frac{\partial D_{f \rightarrow H}^{\text{Inhomogeneous}}}{\partial \ln \mu^2} \sim \frac{\partial D_{f \rightarrow H}^{\text{Homogeneous}}}{\partial \ln \mu^2}$$

$\mu^2 \rightarrow \infty$ : the slope of  $D_{f \rightarrow H}$  is the same as LP DGLAP.

Mueller and Qiu, NPB268, 427 (1986)

Qiu, NPB291, 746 (1987)

Eskola, Honkanen, Kolhinen, Qiu and Salgado, NPB660, 211 (2003)

The power corrections effect at low  $\mu^2$  does not go away fast: **analogous to nonlinear gluon recombination effects to gluon PDF at small- $x$  and large  $\mu^2$ .**

# Evolution of DP FFs in $u, v$ -space

Lee, Qiu, Sterman, KW, arXiv:2211.12648 [hep-ph]

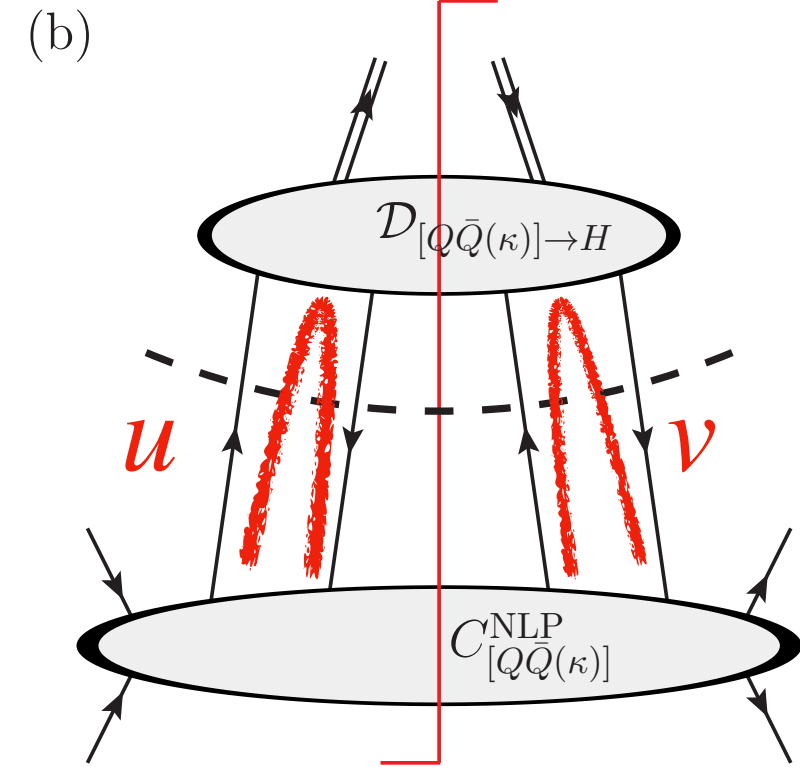
Consider the derivative of a test function:

$$D'_{\kappa \rightarrow n}(z, u, v) \equiv \frac{2\pi}{\alpha_s} \frac{dD_{\kappa \rightarrow n}(z, u, v)}{d \ln \mu^2},$$

$$D(z, u, v) \rightarrow D_z(z)D_u(u)D_v(v),$$

$$D_z(z, \alpha) = \frac{z^\alpha(1-z)^\beta}{B[1+\alpha, 1+\beta]},$$

$$D_{u,v}(x, \gamma) = \frac{x^\gamma(1-x)^\gamma}{B[1+\gamma, 1+\gamma]},$$

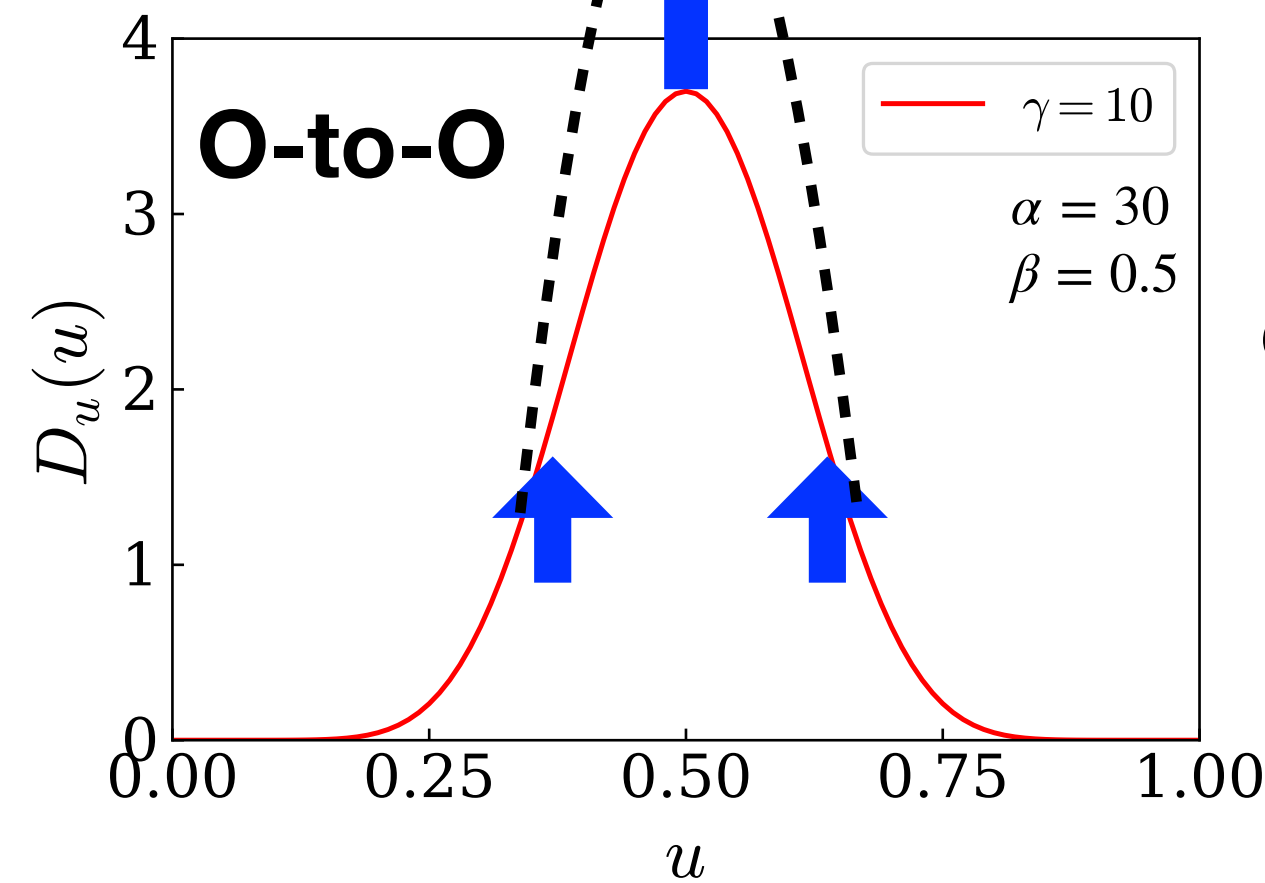
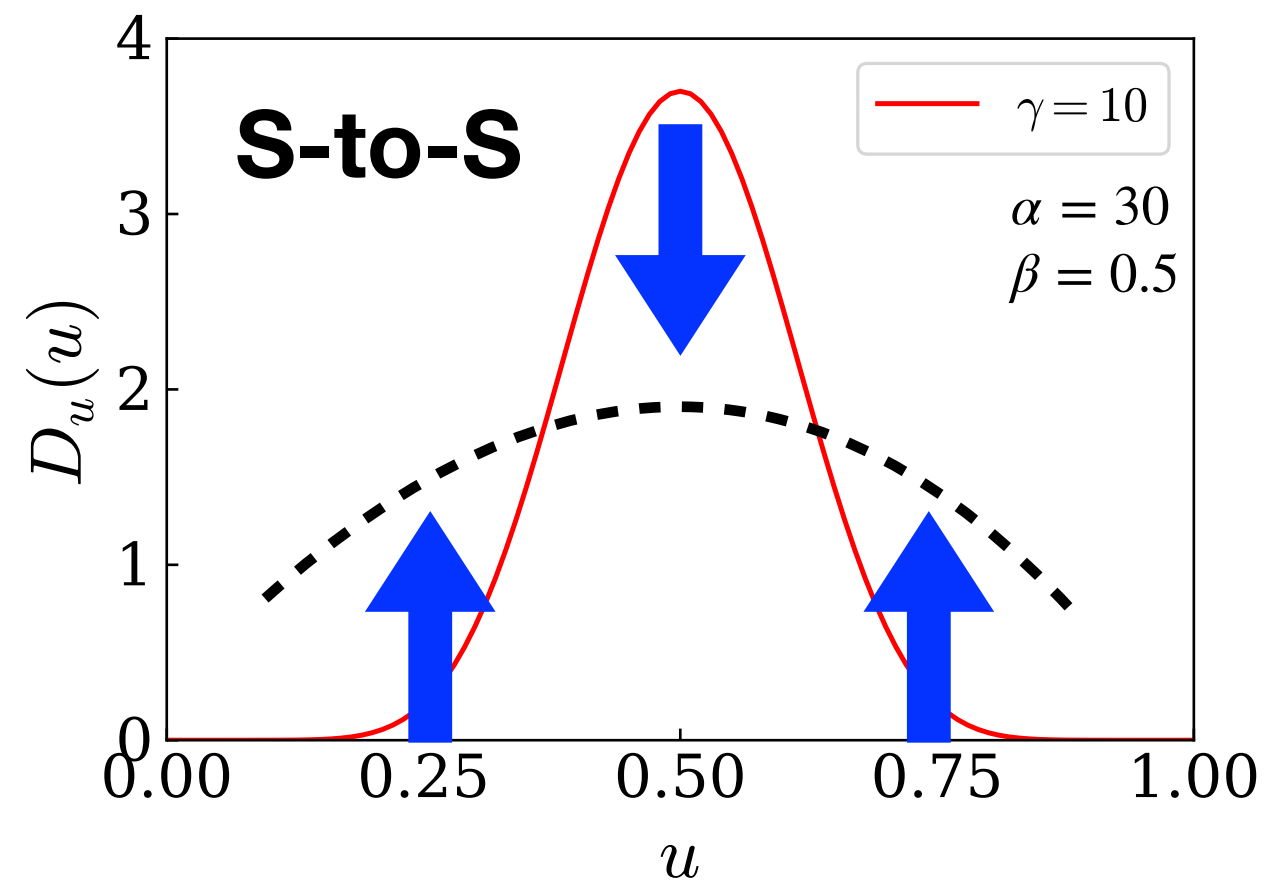
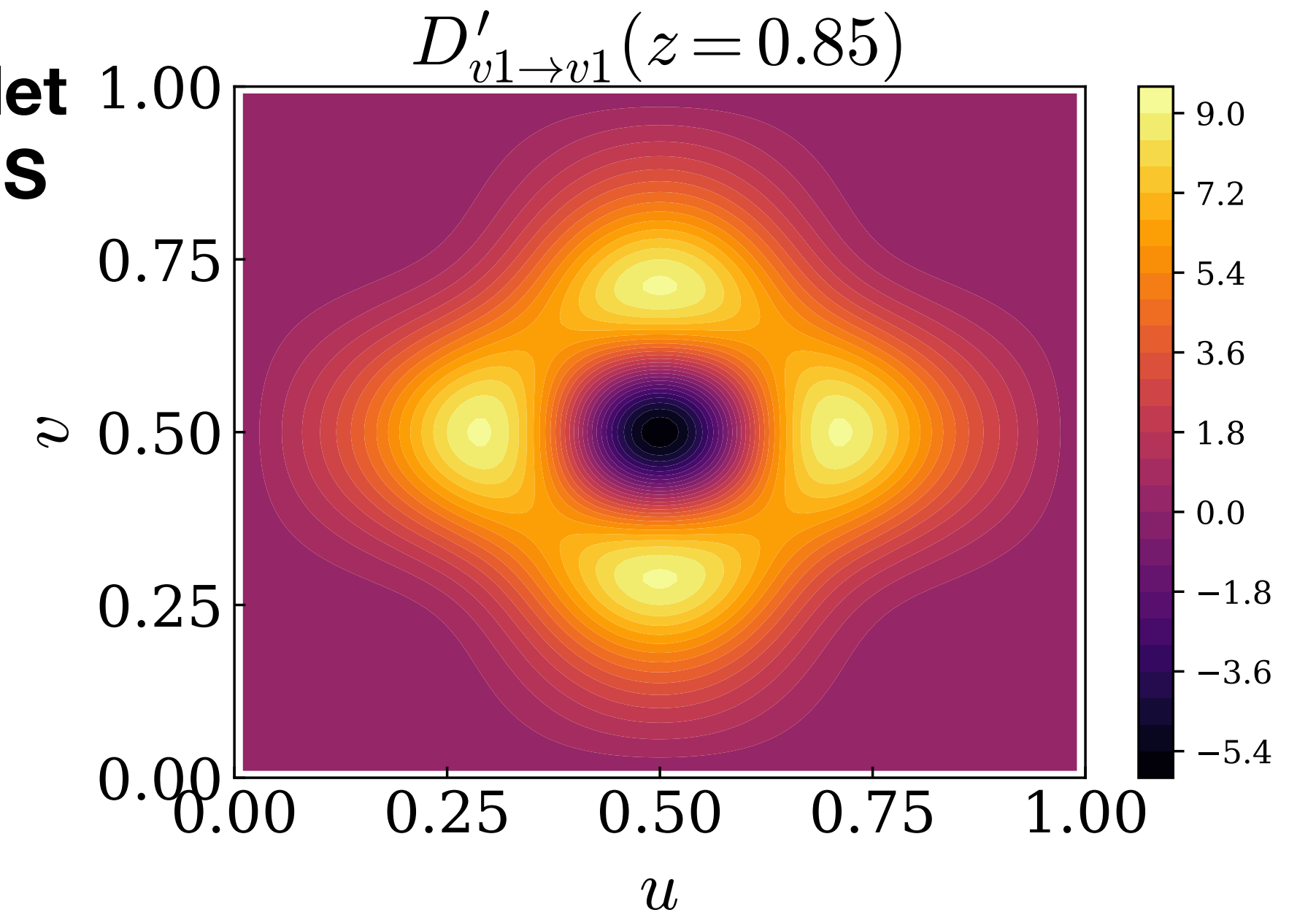


**Diagonal singlet channel: S-to-S**

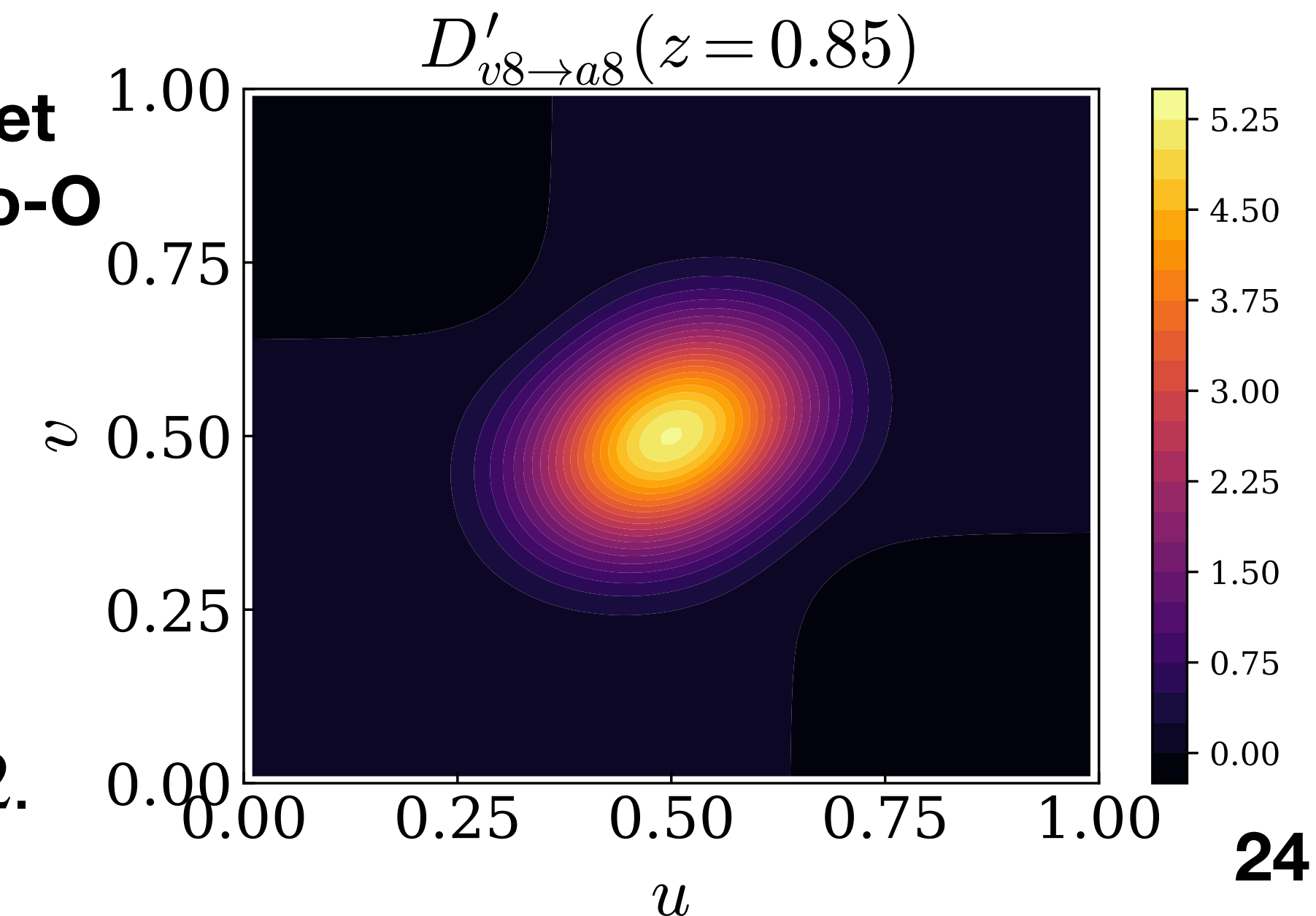
amplitude :  $p_Q = up_c$

C.C. :  $p_Q = vp_c$

with  $zp_c^+ = p^+$



**Diagonal octet channel: O-to-O**



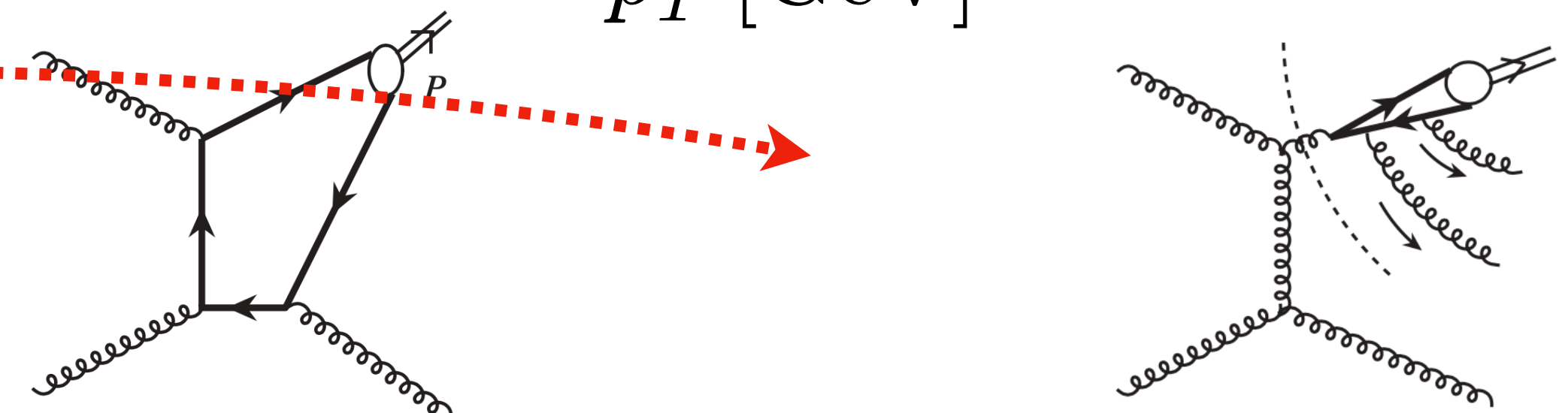
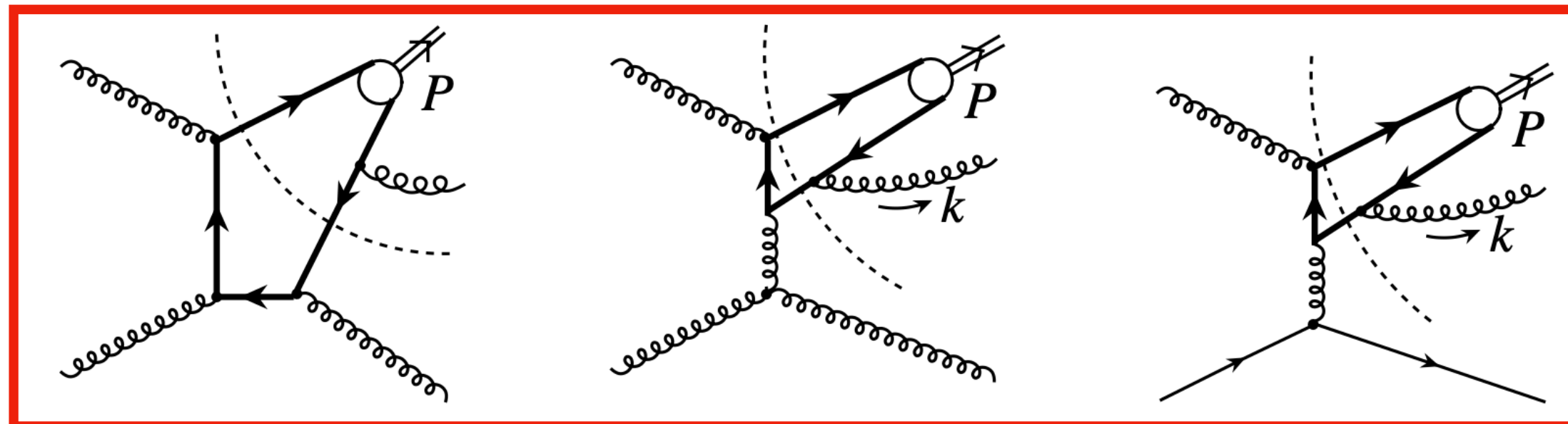
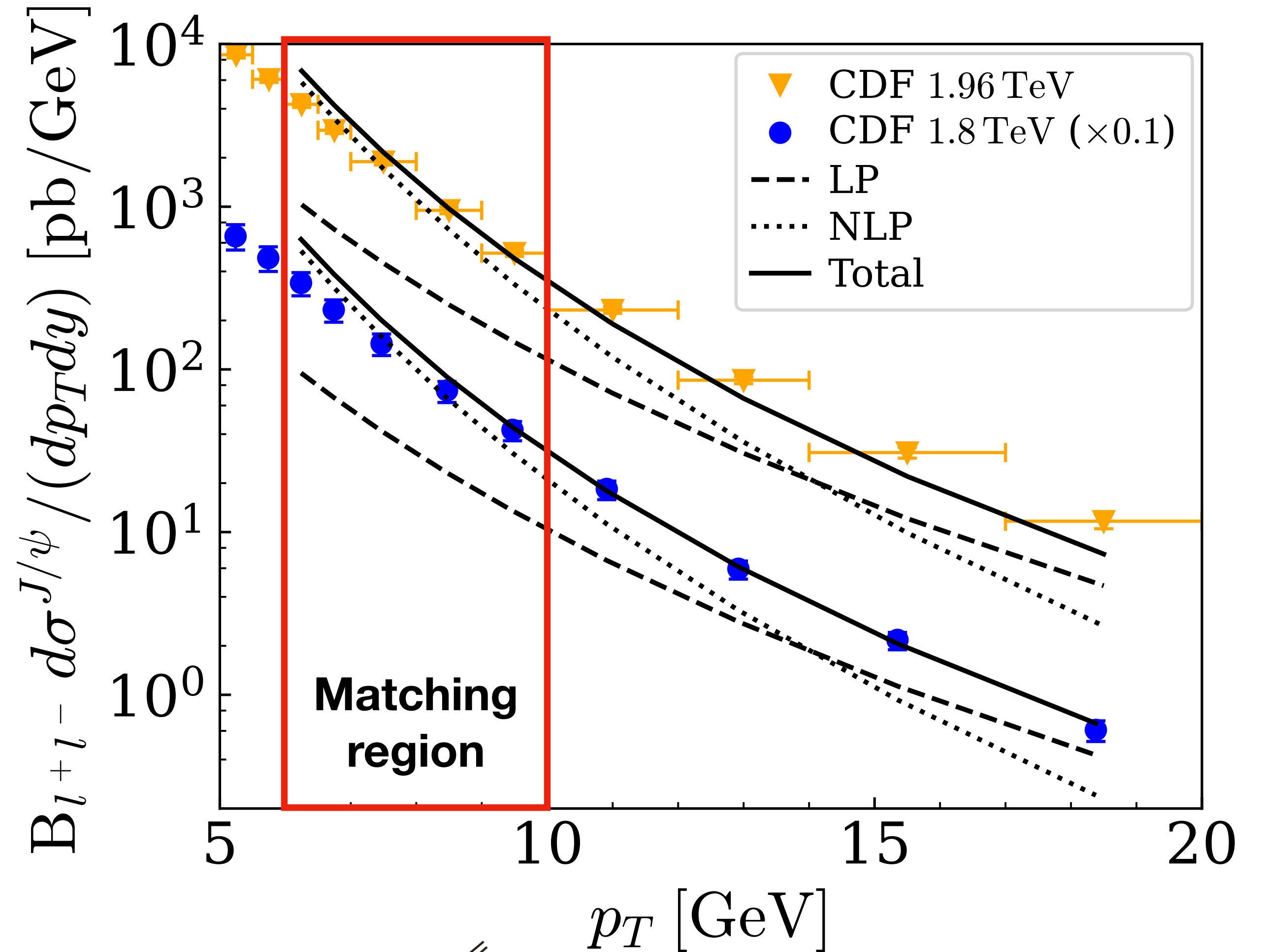
- S-to-S DP FFs get **broader** in  $u, v$ -space after evolution.
- O-to-O DP FFs become **narrower** with a large peak around  $u = v = 1/2$ .
- Off-diagonal channels: similar to O-to-O.



# Toward the matching to NRQCD

Lee, Qiu, Sterman, KW, arXiv:2211.12648 [hep-ph]

1.  $\ln(p_T^2/m^2)$ -type logarithmically enhanced contributions start to dominate when  $p_T \gtrsim 5 \times (2m_c) \sim 15 \text{ GeV}$ , where the LP is significant, power corrections are small.
2. The NLP contribution is important at  $p_T \lesssim 10 \text{ GeV} = \mathcal{O}(2m_c)$ , where matching between QCD factorization and NRQCD factorization can be made.
3. Further exploration of the shape of the FFs at large- $z$  would help us understand the quarkonium production mechanism.



# Input LDFs

QED evolution starts at  $\mu = m_e \sim 0.5\text{MeV}$ :

$$f_{ele}(\xi) = \delta(1 - \xi) \longrightarrow \frac{\xi^\alpha (1 - \xi)^\beta}{B[1 + \alpha, 1 + \beta]}$$

$$f_{\bar{e}le}(\xi) = f_{\gamma le}(\xi) = 0$$

Full evolution starts at  $\mu = m_c \sim 1.3\text{GeV}$ :

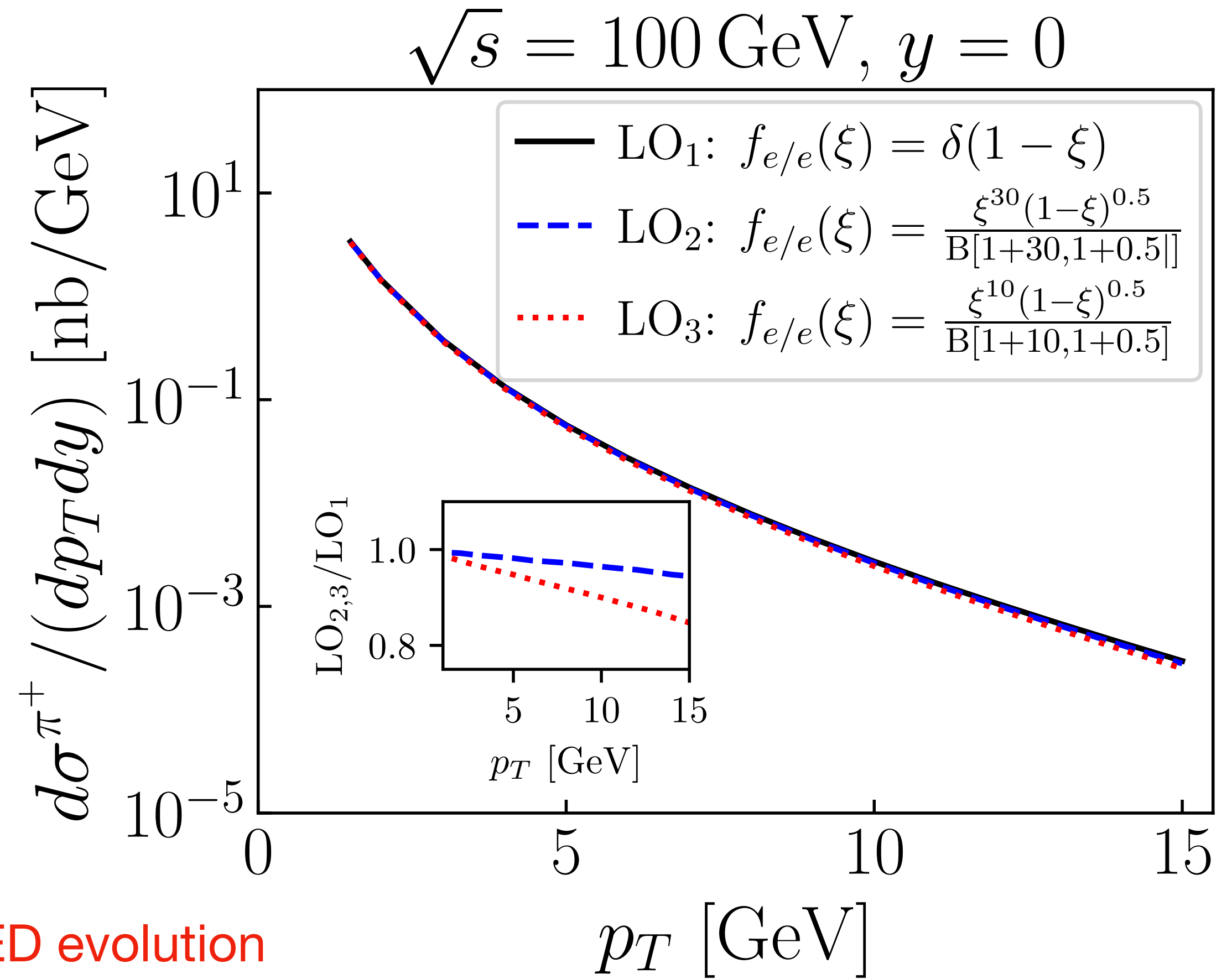
$$f_{qle}(\xi, \mu^2 = m_c^2) = f_{\bar{q}le}(\xi, \mu^2 = m_c^2) \approx f_{e+le}(\xi, \mu^2 = m_c^2)$$

$$f_{cle}(\xi, \mu^2 = m_c^2) = f_{\bar{c}le}(\xi, \mu^2 = m_c^2) = 0 \text{ estimated by QED evolution}$$

$$f_{g le}(\xi, \mu^2 = m_c^2) = 0$$

Bottom contribution joins the evolution at  $\mu = m_b \sim 4.2\text{GeV}$ :

$$f_{ble}(\xi, \mu^2 = m_b^2) = f_{\bar{b}le}(\xi, \mu^2 = m_b^2) = 0$$

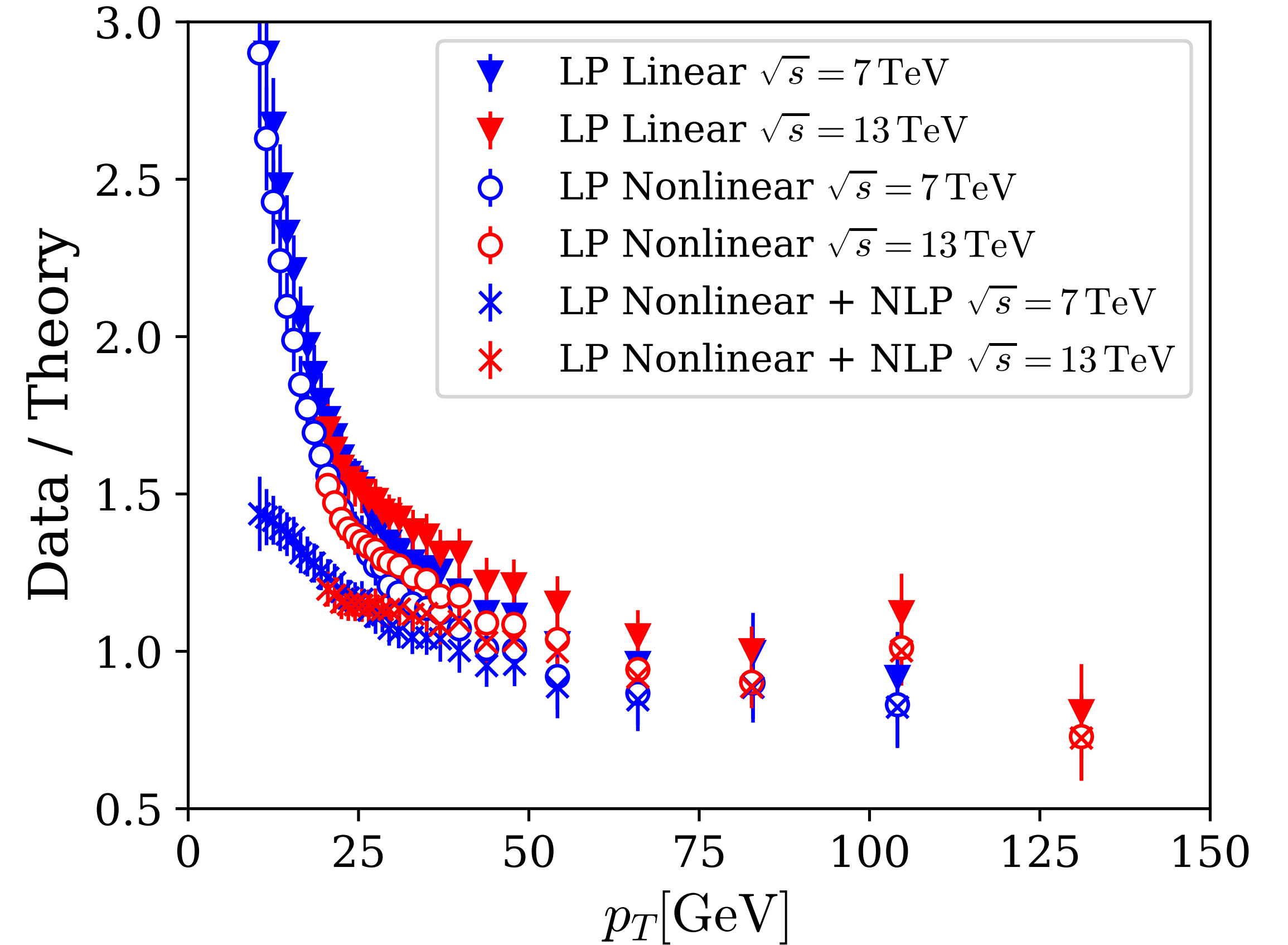


**Our choice:  $\alpha = 30, \beta = 0.5$**

# $^1S_0^{[8]}$ dominant scenario

Lee, Qiu, Sterman, KW, SciPost Phys. Proc.8, 143 (2022)

- Fitting the LP formalism with the linear DGLAP evolution eq. to CMS data on high  $p_T$  prompt  $J/\psi$  at  $\sqrt{s} = 7, 13$  TeV in the bin,  $|y| < 1.2$ .
- Only the  $^1S_0^{[8]}$  channel is considered, yielding unpolarized  $J/\psi$ . Combining LP and NLP could overshoot data for the other two color octet channels.
- $\langle \mathcal{O}(^1S_0^{[8]}) \rangle / \text{GeV}^3 = 0.1286 \pm 5.179 \cdot 10^{-3}$  fitted by high  $p_T$  data is similar to the one extracted using fixed order NRQCD at NLO. [Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 \(2012\)](#)



**The power corrections do not vanish even at the highest  $p_T$ , giving 10-30% corrections.**  
**At  $p_T = 30$  GeV and below, the NLP corrections become significant.**