



# Precision three-dimensional imaging of nuclei using recoil-free jets

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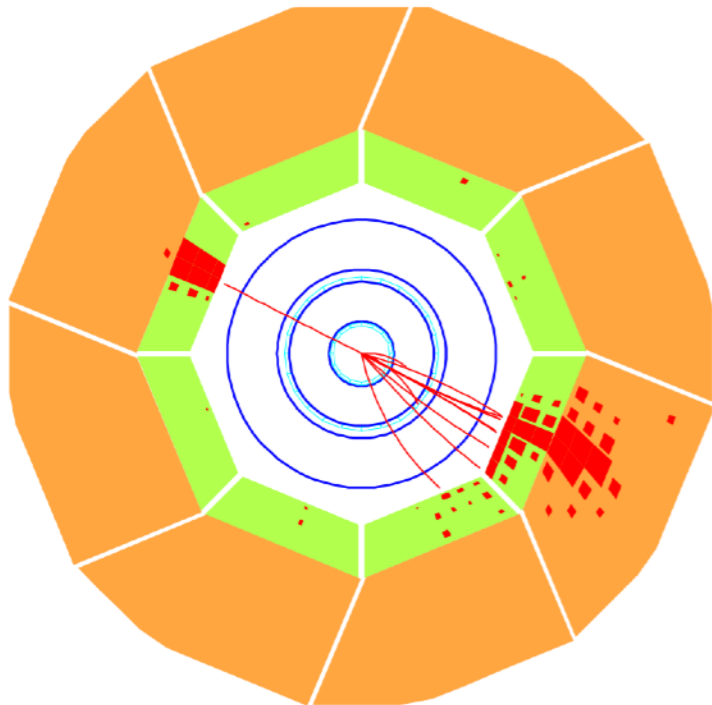
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Fudan University

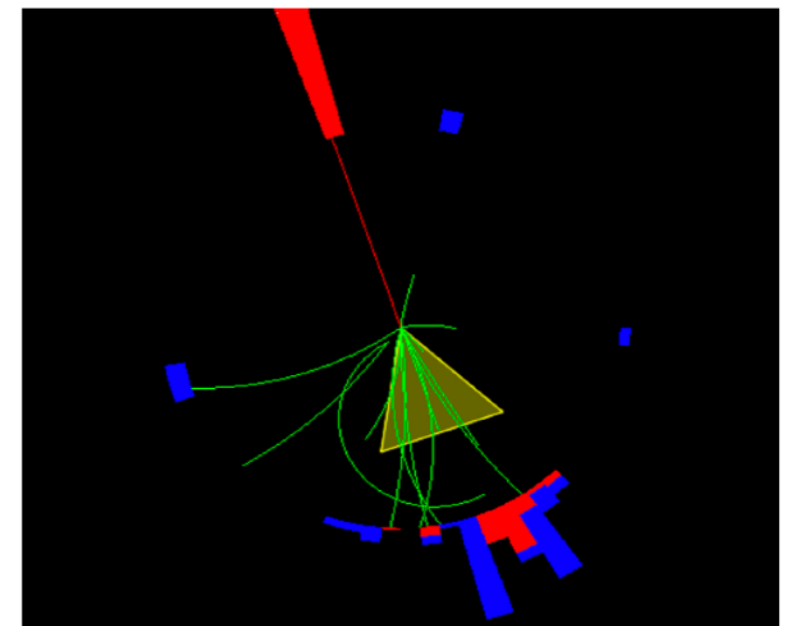
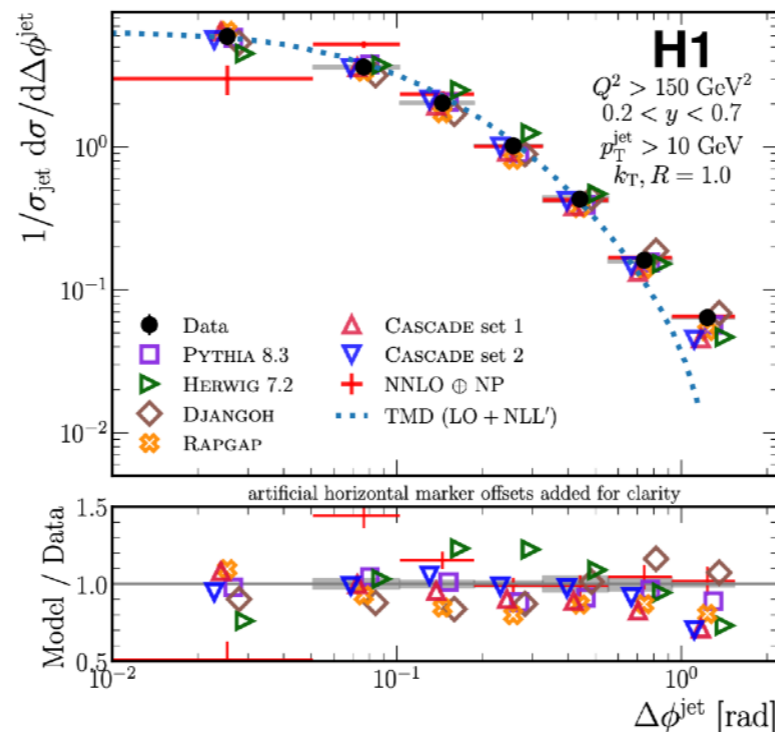
Jul 1, 2024

# Introduction

- In past DIS experiments, scientists mainly focused on jet behaviors in the Breit frame—the frame of the virtual photon and the nucleon.
- Recently, there has been many interests in studying observables in the lab frame of the incoming lepton and nucleus
  - **Event shape** Kang, Mantry, Qiu '12; Kang, Lee, Stewart '13; Li, Vitev, Zhu, '20
  - **Jet production** Liu, Ringer, Vogelsang, Yuan '19; Arratia, Kang, Prokudin, Ringer '19
  - **Hadron production** Gao, Michel, Stewart, Sun '22

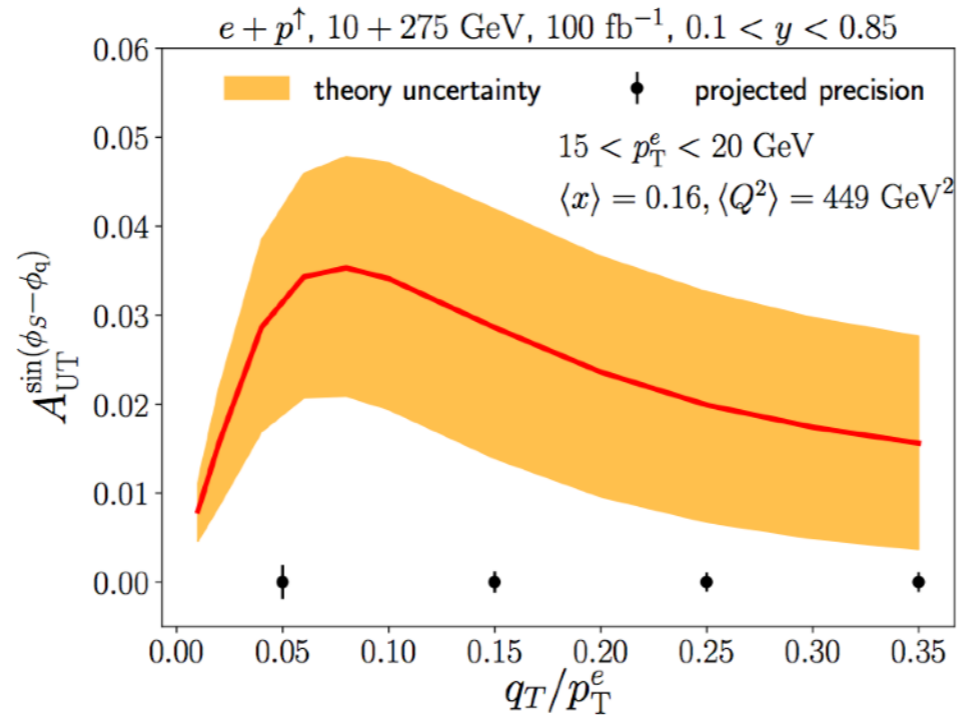
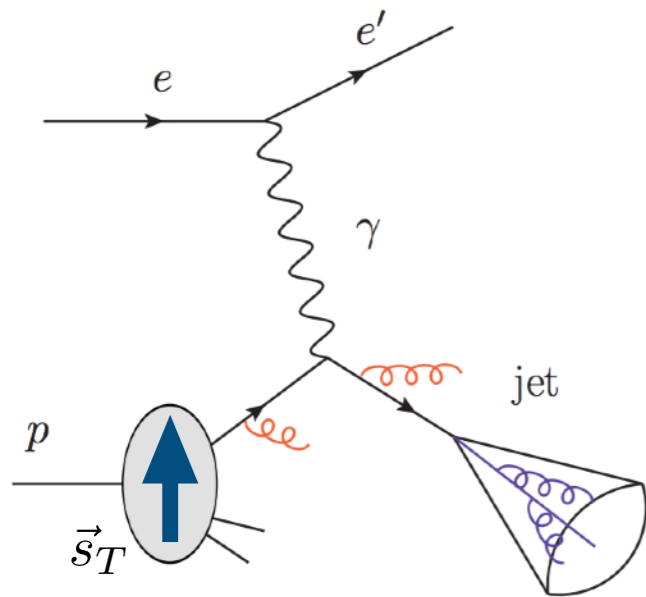


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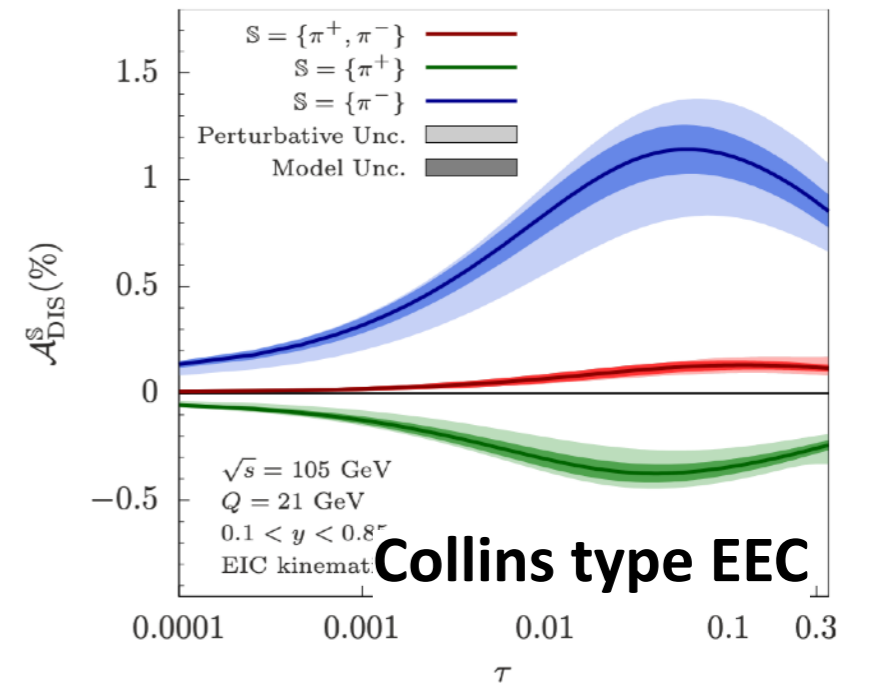


EIC Yellow Report

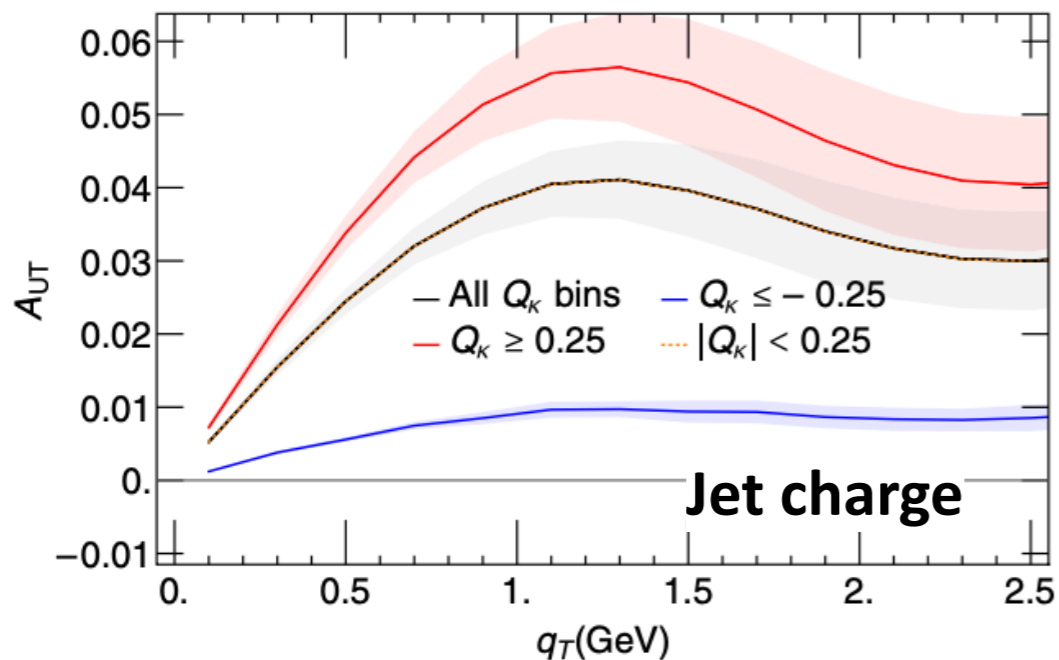
# Jets and 3D imaging



Arratia, Kang, Prokudin, Ringer '19  
Liu, Ringer, Vogelsang, Yuan '19



Kang, Lee, DYS, Zhao '23 JHEP



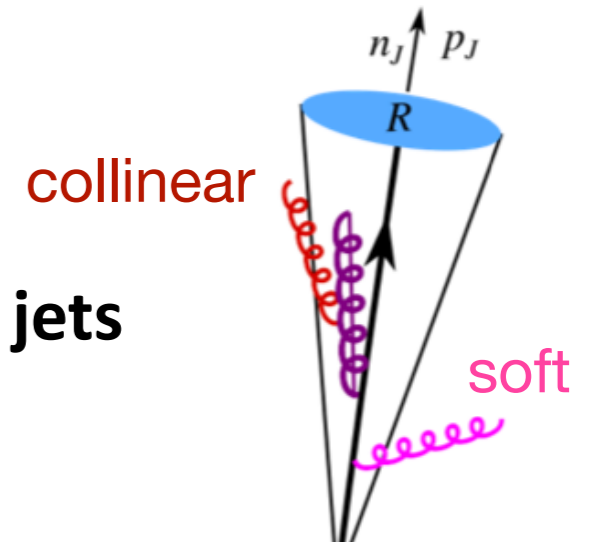
Kang, Liu, Mantry, DYS '20 PRL

- Jets are complementary to standard SIDIS extractions of TMDs [See Zhongbo's talk](#)
- Jet measurements allow independent constraints on TMD PDFs and FFs from a single measurement
- Azimuthal correlation between jet and lepton sensitive to TMD PDFs

# Azimuthal correlations of QCD jets

- **All-order resummation of azimuthal correlation of QCD jets** was first studied by (Banfi, Dasgupta & Delenda '08)

$$q_T = \left| \sum_{i \notin \text{jets}} \vec{k}_{T,i} \right| + \mathcal{O}(k_T^2)$$



- **sum over all soft and collinear partons not combined with jets**
- **caused by particle flow outside the jet regions**
- **non-global observables** (Dasgupta & Salam '01)
- **CSS framework (indirect formalism, construct azimuthal angle from  $q_T$ )**
- **dijet** (Sun, Yuan & Yuan '14 & '15)

**Resummation formula:** 
$$\frac{d\sigma}{d\Delta\phi} = x_a f_a(x_a, \mu_b) x_b f_b(x_b, \mu_b) \frac{1}{\pi} \frac{d\sigma_{ab \rightarrow cd}}{d\hat{t}} b J_0(|\vec{q}_\perp| b) e^{-S(Q,b)}$$

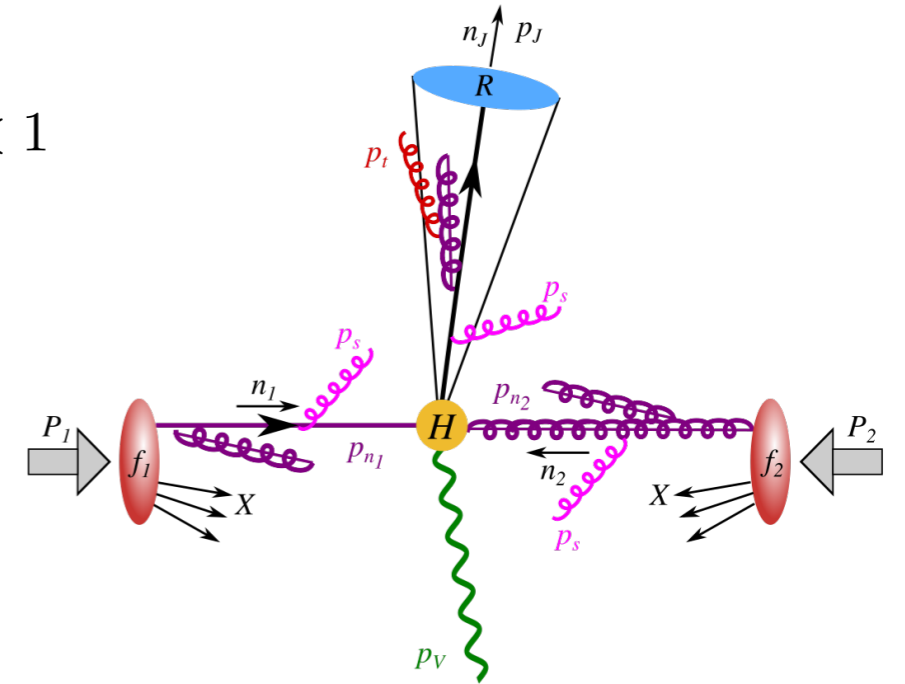
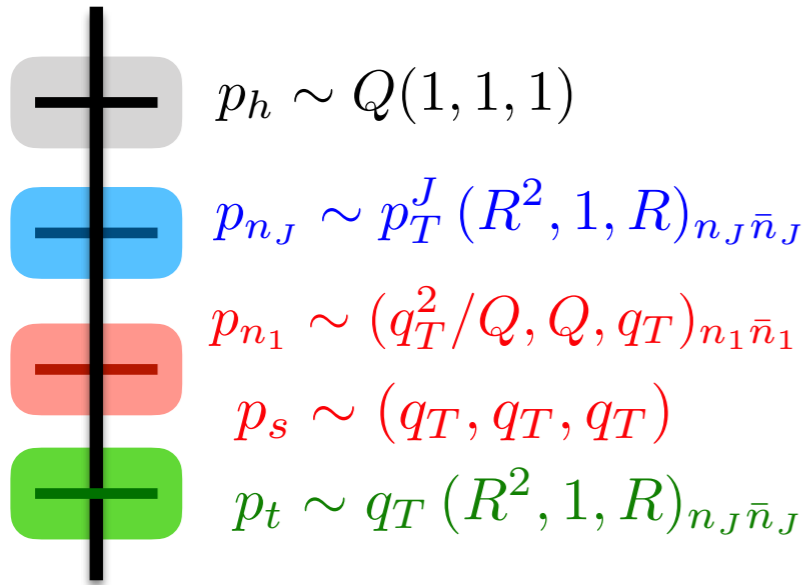
**Perturbative Sudakov factor:** 
$$S_P(Q, b) = \sum_{q,g} \int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[ A \ln \frac{Q^2}{\mu^2} + B + D \ln \frac{1}{R^2} \right]$$

# Jet radius and TMD joint resummation for boson-jet correlation in SCET

(Buffing, Kang, Lee, Liu '18; Chien, DYS & Wu '19 JHEP)

$$N_1(P_1) + N_2(P_2) \rightarrow \underbrace{\text{boson}(p_V) + \text{jet}(p_J)}_{q_T} + X$$

$$q_T \ll Q, R \ll 1$$



## Construction of the theory formalism

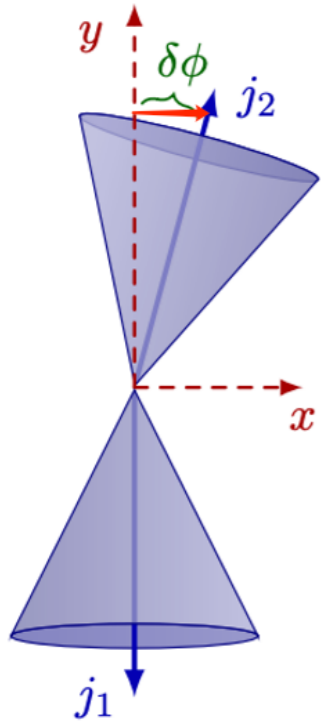
- Multiple scales:  $p_T, p_T R, q_T, q_T R$
- Theory tools: SCET + multi-Wilson formalism (Becher, Neubert, Rothen, DYS '16 PRL)

$$\frac{d\sigma}{d^2 q_T d^2 p_T d\eta_J dy_V} = \sum_{ijk} \int \frac{d^2 x_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{x}_T} \mathcal{S}_{ij \rightarrow V k}(\vec{x}_T, \epsilon) \mathcal{B}_{i/N_1}(\xi_1, x_T, \epsilon) \mathcal{B}_{j/N_2}(\xi_2, x_T, \epsilon)$$

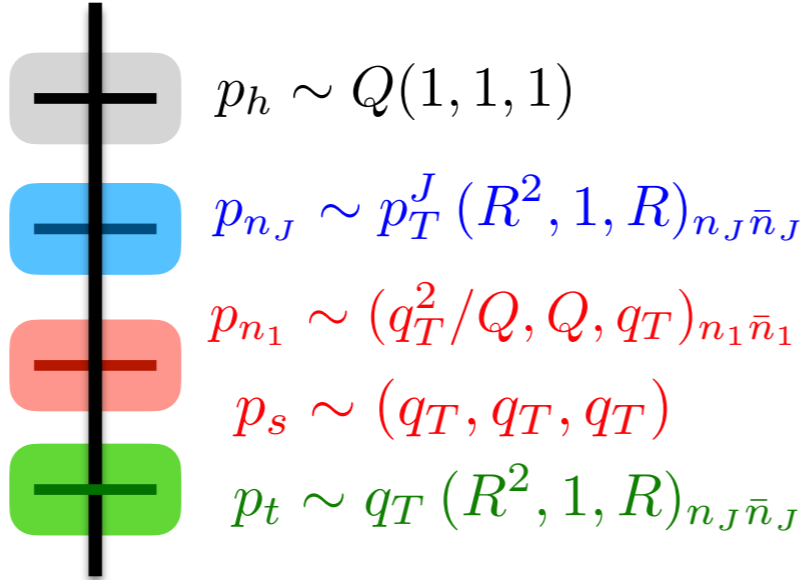
$$\times \mathcal{H}_{ij \rightarrow V k}(\hat{s}, \hat{t}, m_V, \epsilon) \sum_{m=1}^{\infty} \langle \mathcal{J}_m^k(\{\underline{n}_J\}, R p_J, \epsilon) \otimes \mathcal{U}_m^k(\{\underline{n}_J\}, R \vec{x}_T, \epsilon) \rangle$$

# Azimuthal decorrelation of QCD jets in pp, pA & UPC( $\gamma\gamma$ )

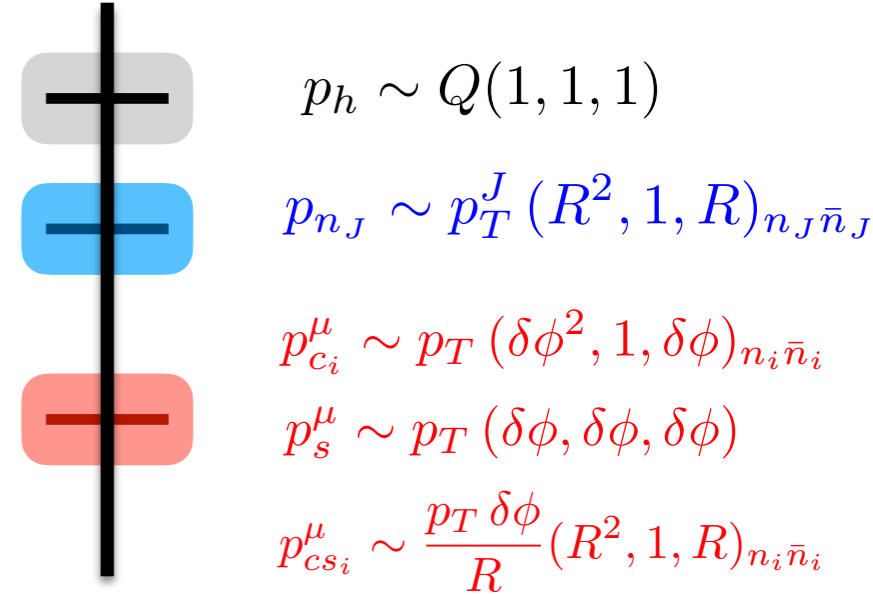
(Zhang, Dai, DYS, '22 JHEP, Gao, Kang, DYS, Terry, Zhang '23 JHEP)



## Indirect



## Direct



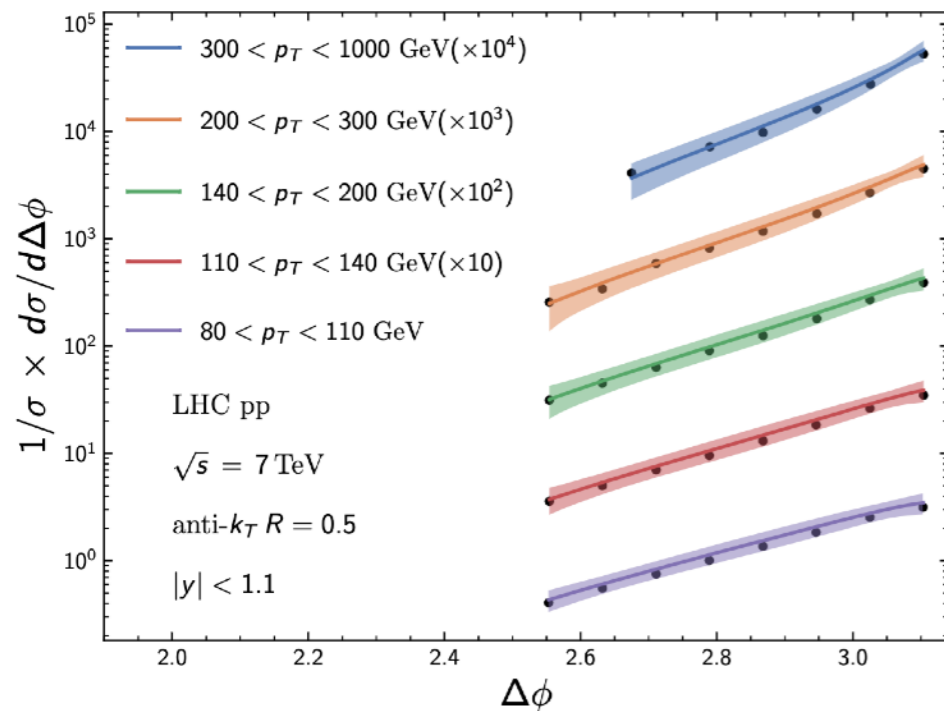
## The factorization formula

$$\frac{d^4\sigma_{pp}}{dy_c dy_d dp_T^2 dq_x} = \sum_{abcd} \frac{x_a x_b}{16\pi\hat{s}^2} \frac{1}{1 + \delta_{cd}} \mathcal{C}_x \left[ f_{a/p}^{\text{unsub}} f_{b/p}^{\text{unsub}} \mathbf{S}_{ab \rightarrow cd, IJ}^{\text{unsub}} S_c^{\text{cs}} S_d^{\text{cs}} \right] \mathbf{H}_{ab \rightarrow cd, JI}(\hat{s}, \hat{t}, \mu) J_c(p_T R, \mu) J_d(p_T R, \mu)$$

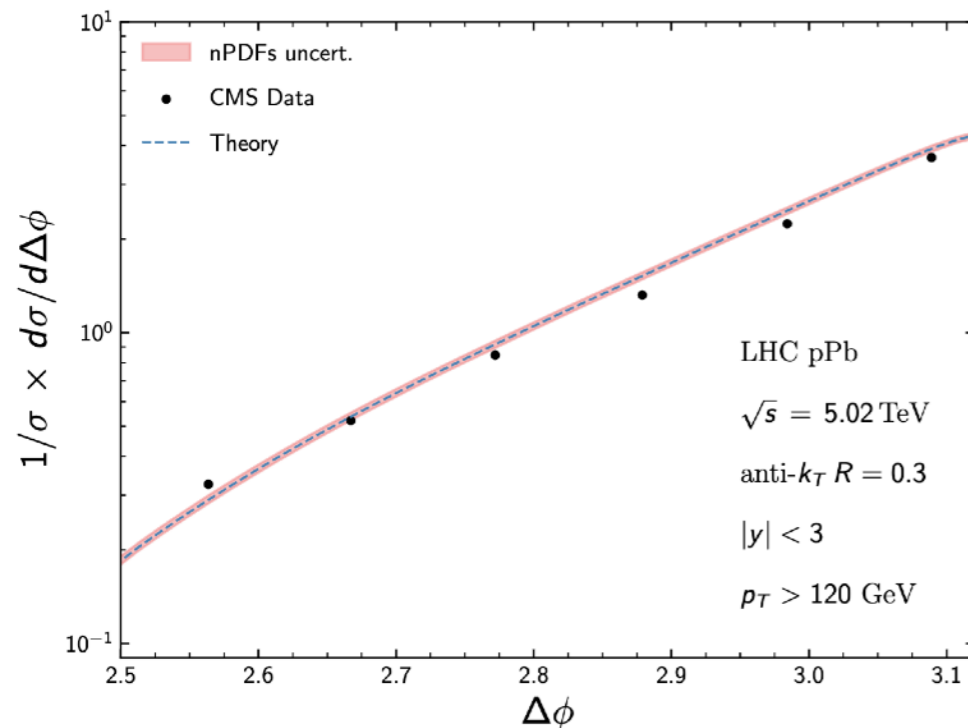
$$\begin{aligned} \mathcal{C}_x \left[ f_{a/p}^{\text{unsub}} f_{b/p}^{\text{unsub}} \mathbf{S}_{ab \rightarrow cd, IJ}^{\text{unsub}} S_c^{\text{cs}} S_d^{\text{cs}} \right] &= \int dk_{ax} dk_{bx} dk_{cx} dk_{dx} d\lambda_x \mathbf{S}_{ab \rightarrow cd, IJ}^{\text{unsub}}(\lambda_x, \mu, \nu) \\ &\times f_{a/p}^{\text{unsub}}(x_a, k_{ax}, \mu, \zeta_a/\nu^2) f_{b/p}^{\text{unsub}}(x_b, k_{bx}, \mu, \zeta_b/\nu^2) S_c^{\text{cs}}(k_{cx}, R, \mu, \nu) S_d^{\text{cs}}(k_{dx}, R, \mu, \nu) \\ &\times \delta(q_x - k_{ax} - k_{bx} - k_{cx} - k_{dx} - \lambda_x) . \end{aligned}$$

# Numerical results in pp, pA

(Zhang, Dai, DYS, '22 JHEP, Gao, Kang, DYS, Terry, Zhang '23 JHEP)



(also see Sun, Yuan, Yuan '14)



**Nuclear modified TMD PDFs**

(Alrashed, Anderle, Kang, Terry & Xing, '22 )

- **NLL resummation result is consistent with LHC data**
  - **Open questions:**
    - **Higher resummation accuracy? SIDIS is known at N3LL' accuracy**
    - **Better angular resolution?**
    - **Reduce contamination from UE?**
  - **One possible solution:**
    - **Recoil-free jet definition**
- E.g. anti- $k_T$  clustering algorithm +  $p_T^n$ -weighted recombination scheme**



# Recoil-free jet and all-order structure

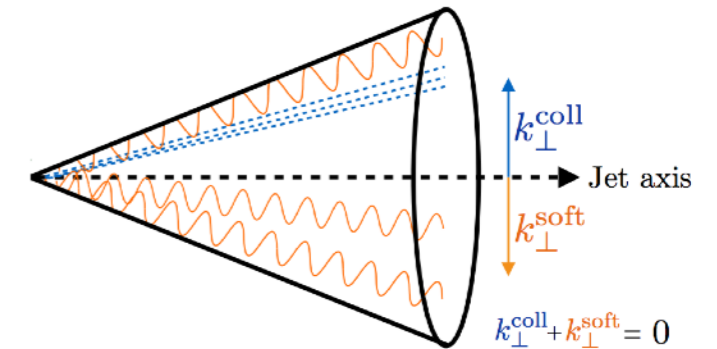
- Recoil absent for the  $p_T^n$ -weighted recombination scheme (Banfi, Dasgupta & Delenda '08)

$$p_{t,r} = p_{t,i} + p_{t,j},$$

$$\phi_r = (w_i \phi_i + w_j \phi_j) / (w_i + w_j)$$

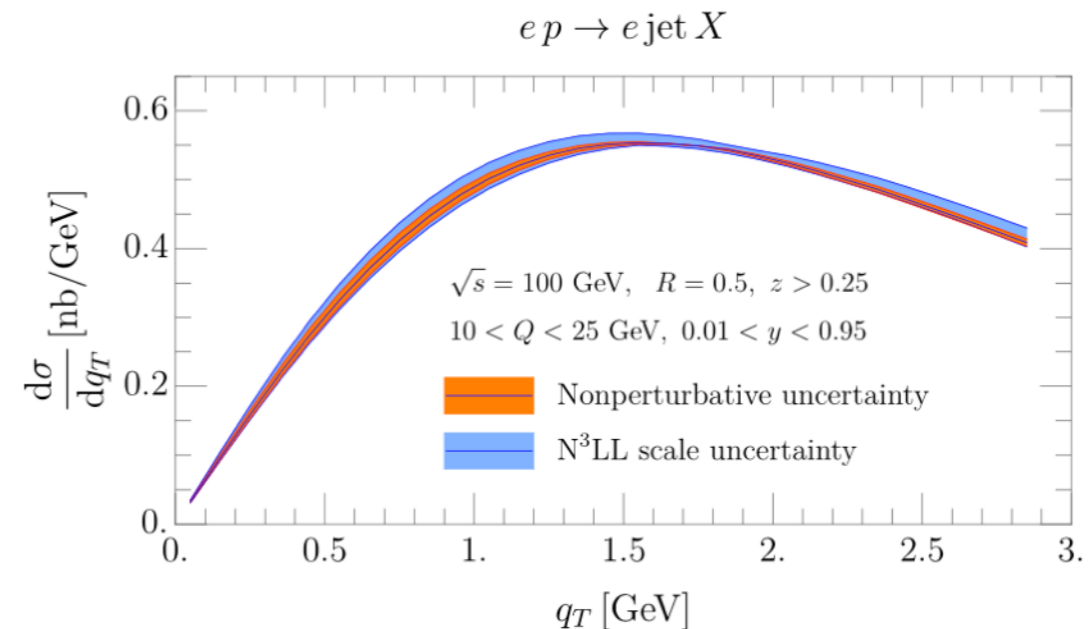
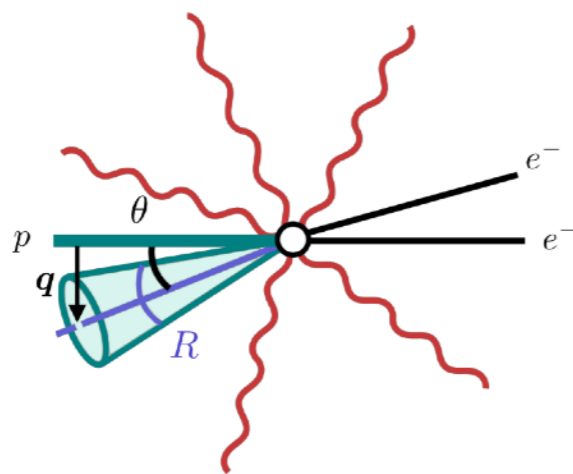
$$y_r = (w_i y_i + w_j y_j) / (w_i + w_j)$$

$$w_i = p_t^n$$



$n \rightarrow \infty$  Winner-take-all scheme (Bertolini, Chan, Thaler '13)

- N3LL resummation for jet  $q_T$  @ ee and ep (Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi '18 '19)



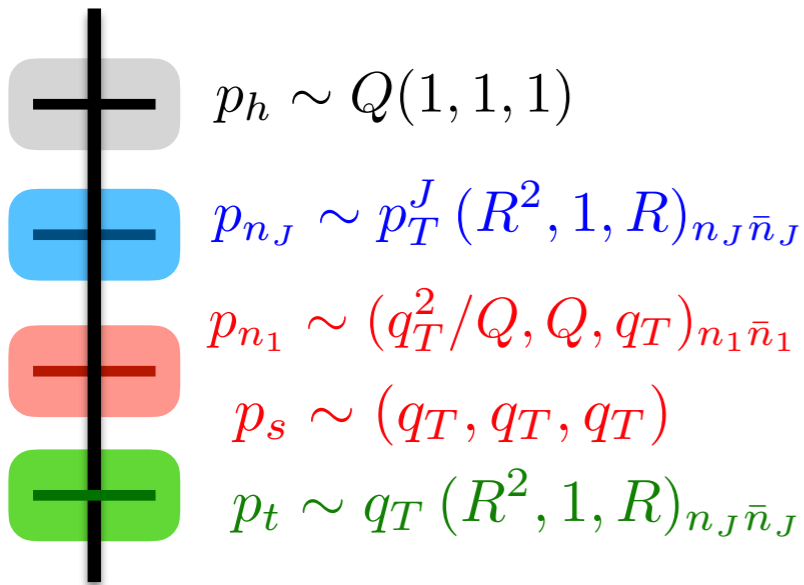
- NNLL resummation for  $\delta\phi$  @ pp (Chien, Rahn, DYS, Waalewijn & Wu '22 JHEP + Schrignder '21 PLB)
- NNLL resummation for  $\delta\phi$  @ ep & eA (Fang, Ke, DYS, Terry '23 JHEP)



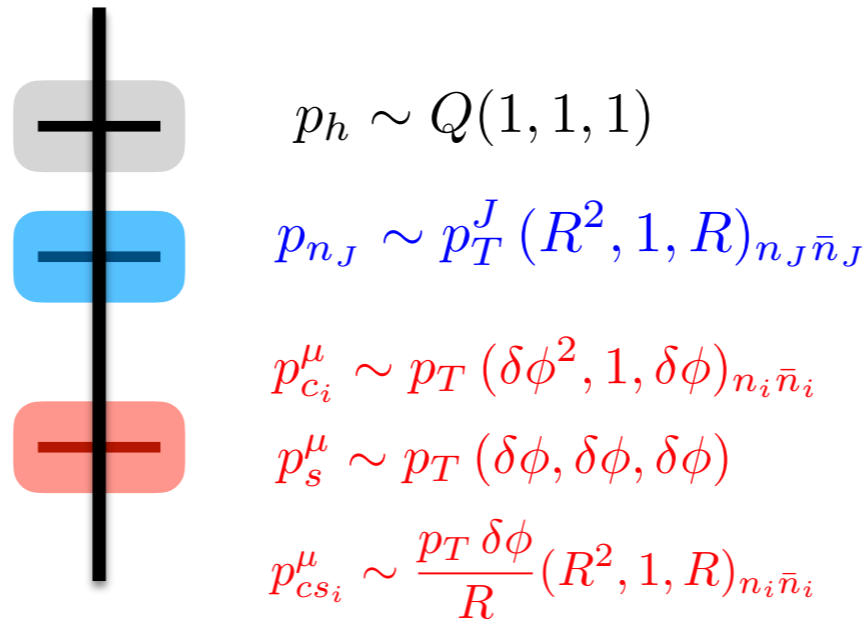
# Recoil-free azimuthal angle for boson-jet correlation

(Chien, Rahn, DYS, Waalewijn & Wu '22 JHEP + Schrignder '21 PLB)

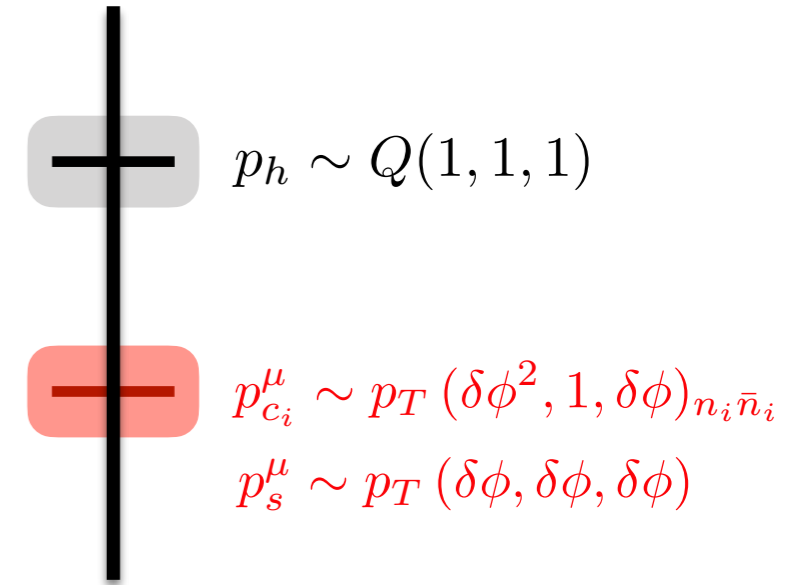
Indirect



Direct



Direct (recoil free)  $\delta\phi \ll \mathcal{O}(1)$



Following the standard steps in SCET2 we obtain the following factorization formula

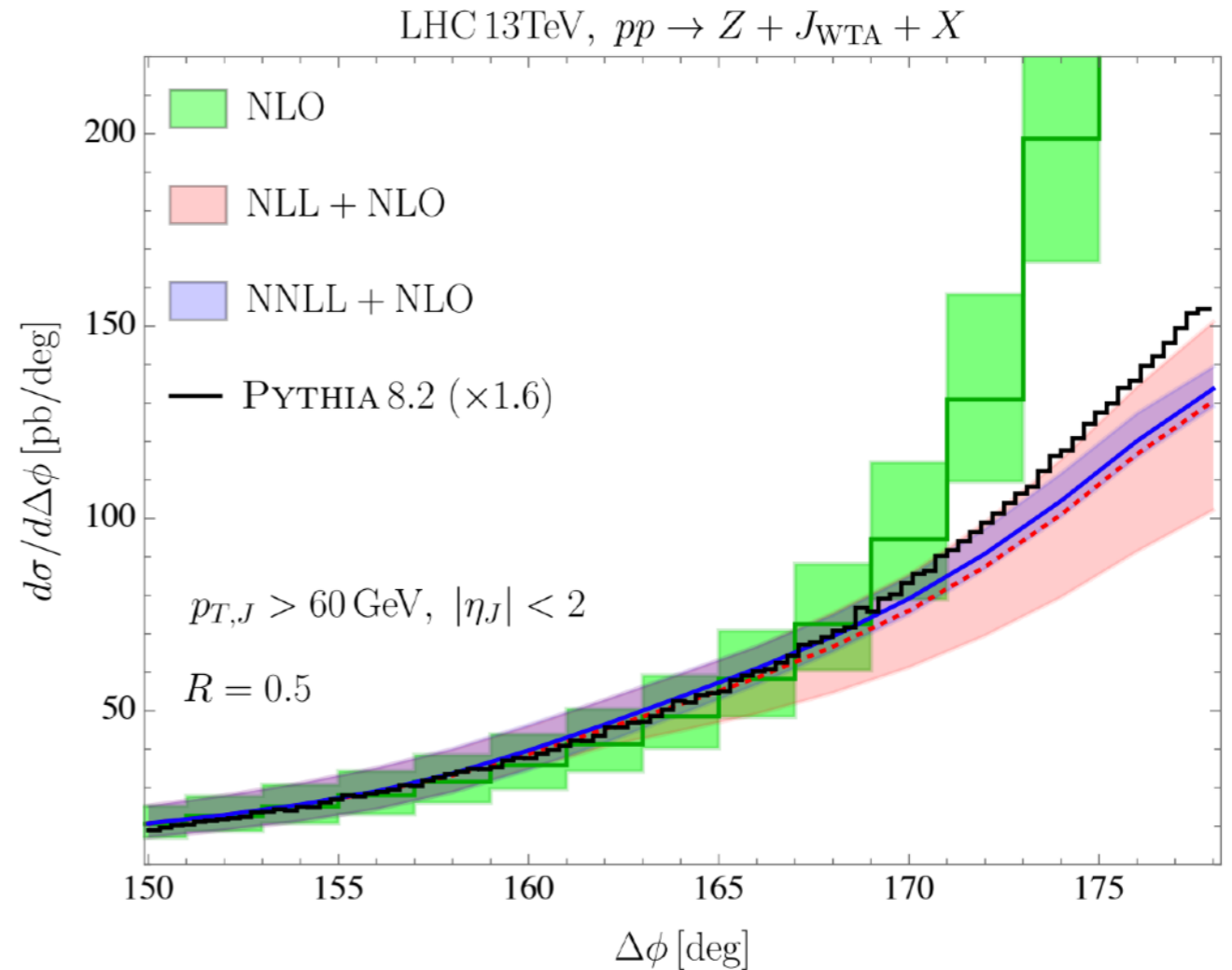
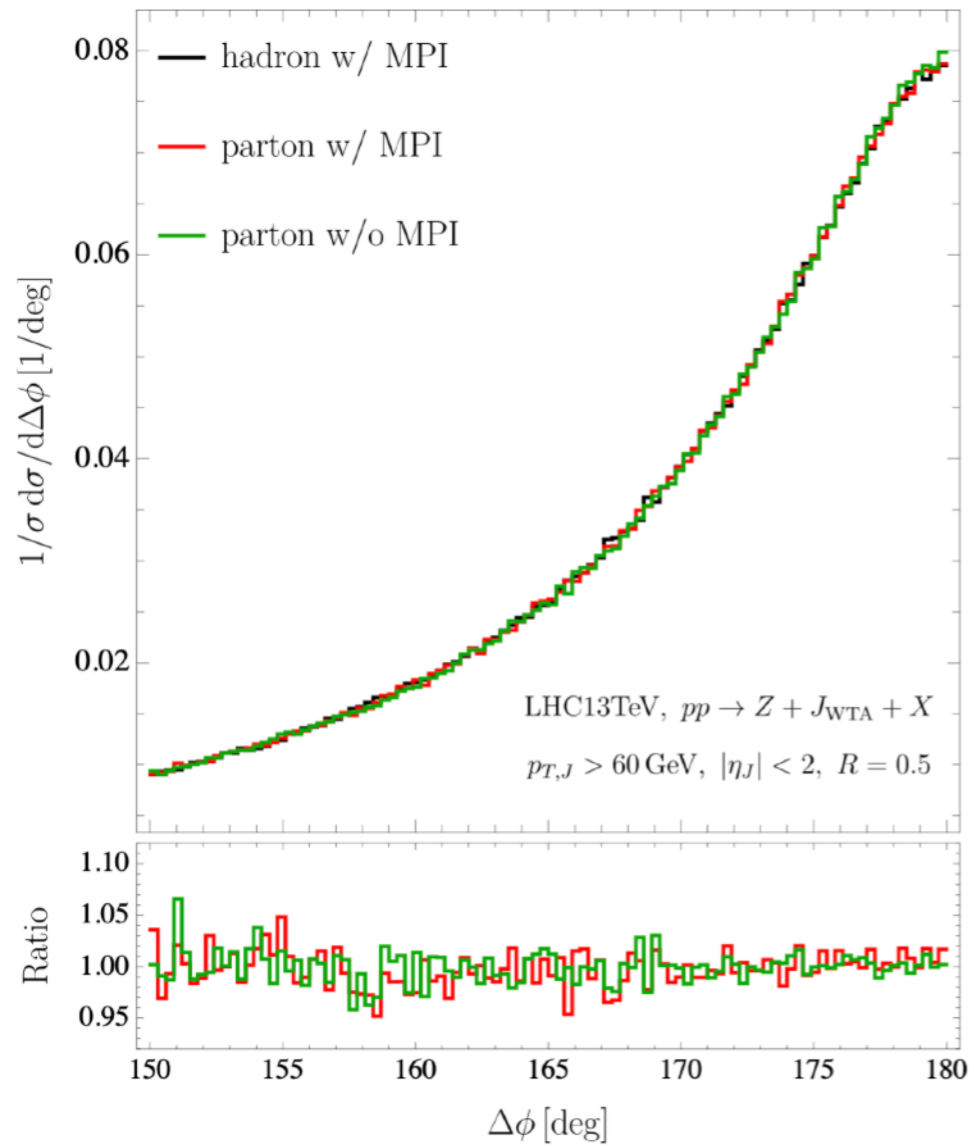
$$\frac{d\sigma}{dp_{x,V} dp_{T,J} dy_V d\eta_J} = \int \frac{db_x}{2\pi} e^{ip_{x,V} b_x} \sum_{i,j,k} B_i(x_a, b_x) B_j(x_b, b_x) S_{ijk}(b_x, \eta_J) H_{ij \rightarrow V k}(p_{T,V}, y_V - \eta_J) J_k(b_x)$$

**Fourier transformation in 1-dim**

Soft function can be obtained by boosted invariance  
(also see Gao, Li, Moult, Zhu '19,...)

# Numerical results

(Chien, Rahn, DYS, Waalewijn & Wu '22 JHEP + Schrignder '21 PLB )



- first NNLL resummation including full jet dynamics (anti- $k_T$  algorithm + WTA)
- non-perturbative effects (hadronization and MPI) are mild

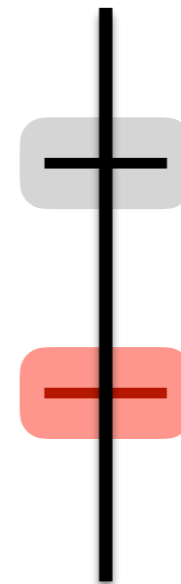
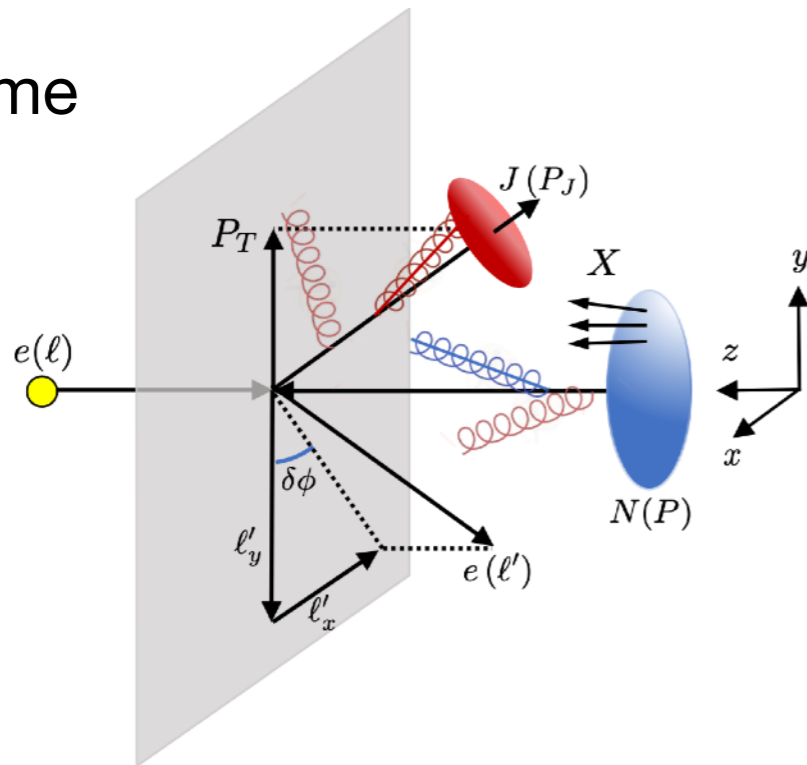
# Recoil-free azimuthal angle for electron-jet correlation

Fang, Ke, DYS, Terry '23 JHEP

$$e(\ell) + N(P) \rightarrow e(\ell') + J(P_J) + X$$

Standard TMD in back to back limit:  $Q \gg q_T \sim l_T \delta\phi$

Lab frame



$$p_h \sim Q(1, 1, 1)$$

$$p_{e_i}^\mu \sim l_T (\delta\phi^2, 1, \delta\phi)_{n_i \bar{n}_i}$$

$$p_s^\mu \sim l_T (\delta\phi, \delta\phi, \delta\phi)$$

Following the standard steps in SCET and CSS, we obtain the following resummation formula

$$\frac{d\sigma}{d^2 l'_T dy d\delta\phi} = \frac{\sigma_0 l'_T}{1-y} H(Q, \mu) \int_0^\infty \frac{db}{\pi} \cos(bl'_T \delta\phi) \sum_q e_q^2 f_{q/N}(x_B, b, \mu, \zeta_f) J_q(b, \mu, \zeta_J)$$

Hard factor

Fourier transformation  
in 1-dim

TMD PDF

Jet function

# Predictions in e-p

Fang, Ke, DYS, Terry '23

## TMD PDF (CSS treatment)

$$f_{q/N}(x_B, b, \mu, \zeta_f) = [C \otimes f]_{q/N}(x_B, b, \mu_f, \zeta_{fi}) U_{\text{NP}}^f(x_B, b, A, Q_0, \zeta_f) \times \exp \left[ \int_{\mu_f}^{\mu} \frac{d\mu'}{\mu'} \gamma_{\mu}^f(\mu', \zeta_f) \right] \left( \frac{\zeta_f}{\zeta_{fi}} \right)^{\frac{1}{2} \gamma_{\zeta}^f(b, \mu_f)},$$

## Jet function

$$J_q(b, \mu, \zeta_J) = J_q(b, \mu_J, \zeta_{Ji}) U_{\text{NP}}^J(b, A, Q_0, \zeta_J) \times \exp \left[ \int_{\mu_J}^{\mu} \frac{d\mu'}{\mu'} \gamma_{\mu}^J(\mu', \zeta_J) \right] \left( \frac{\zeta_J}{\zeta_{Ji}} \right)^{\frac{1}{2} \gamma_{\zeta}^J(b, \mu_J)}$$

## scale choice

$$\mu_H = Q, \quad \mu_f = \mu_J = \sqrt{\zeta_{fi}} = \sqrt{\zeta_{Ji}} = \mu_b = 2e^{-\gamma_E}/b$$

## b\*-prescription to avoid Landau pole

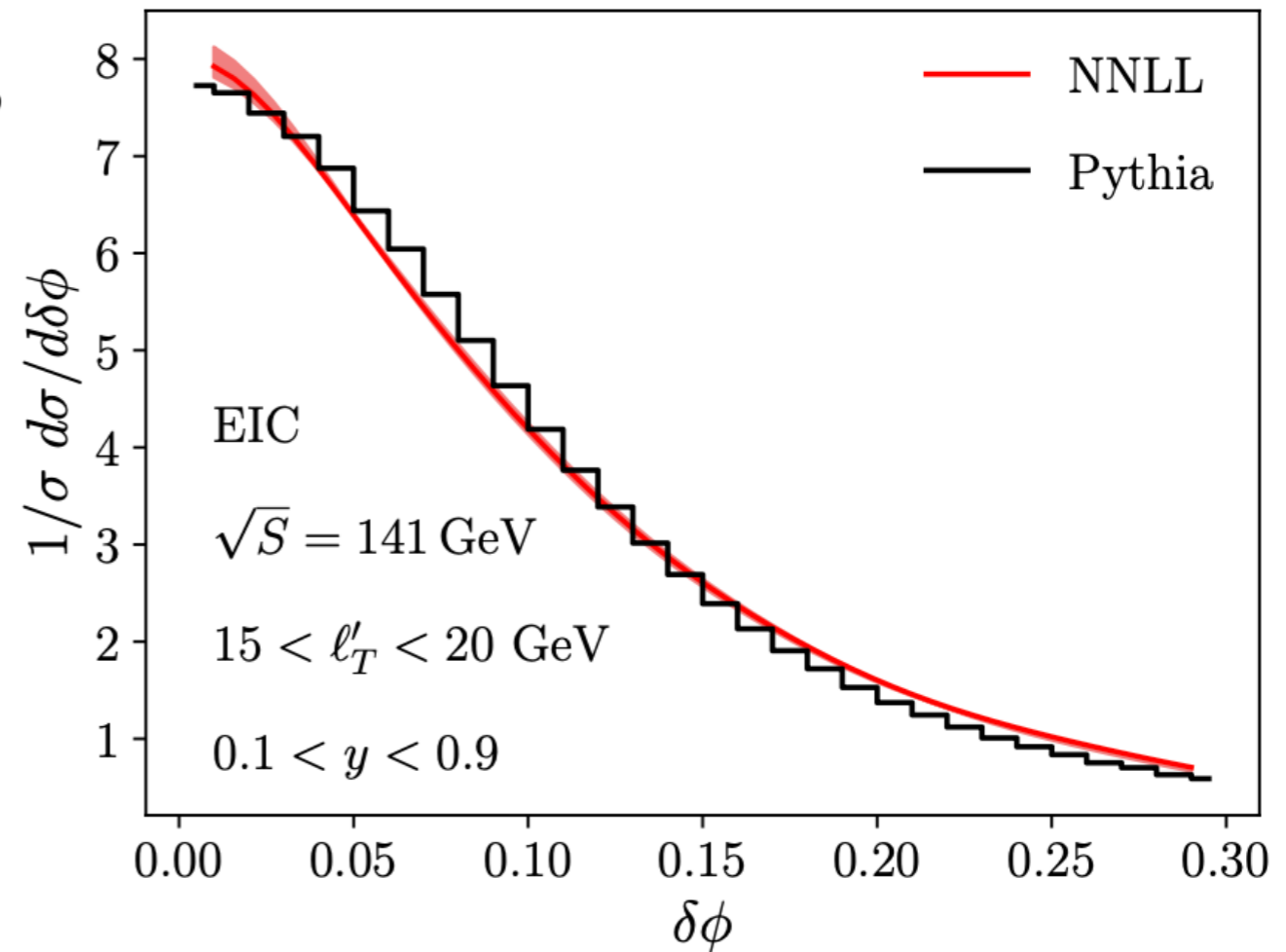
$$b_* = b / \sqrt{1 + b^2/b_{\text{max}}^2} \quad \mu_{b_*} = 2e^{-\gamma_E}/b_*$$

## non-perturbative model

$$U_{\text{NP}}^f = \exp \left[ -g_1^f b^2 - \frac{g_2}{2} \ln \frac{Q}{Q_0} \ln \frac{b}{b_*} \right]$$

$$U_{\text{NP}}^J = \exp \left[ -\frac{g_2}{2} \ln \frac{Q}{Q_0} \ln \frac{b}{b_*} \right]$$

Sun, Isaacson, Yuan, Yuan '14



$\mu_H$  varies between  $Q/2$  and  $2Q$ .  $\mu_b$  is fixed

# Predictions in e-A

Fang, Ke, DYS, Terry '23

We apply nuclear modified TMD PDFs

$$g_1^A = g_1^f + a_N(A^{1/3} - 1) \quad a_N = 0.016 \pm 0.003 \text{ GeV}^2$$

Collinear dynamics (nPDF) using EPPS16

(Alrashed, Anderle, Kang, Terry & Xing, '22)

We include LO momentum broadening of the jet within SCET<sub>G</sub>

$$J_q^A(b, \mu, \zeta_J) = J_q(b, \mu, \zeta_J) e^{\chi[\xi b K_1(\xi b) - 1]}$$

Opacity parameter  $\chi = \frac{\rho_G L}{\xi^2} \alpha_s(\mu_{b_*}) C_F$

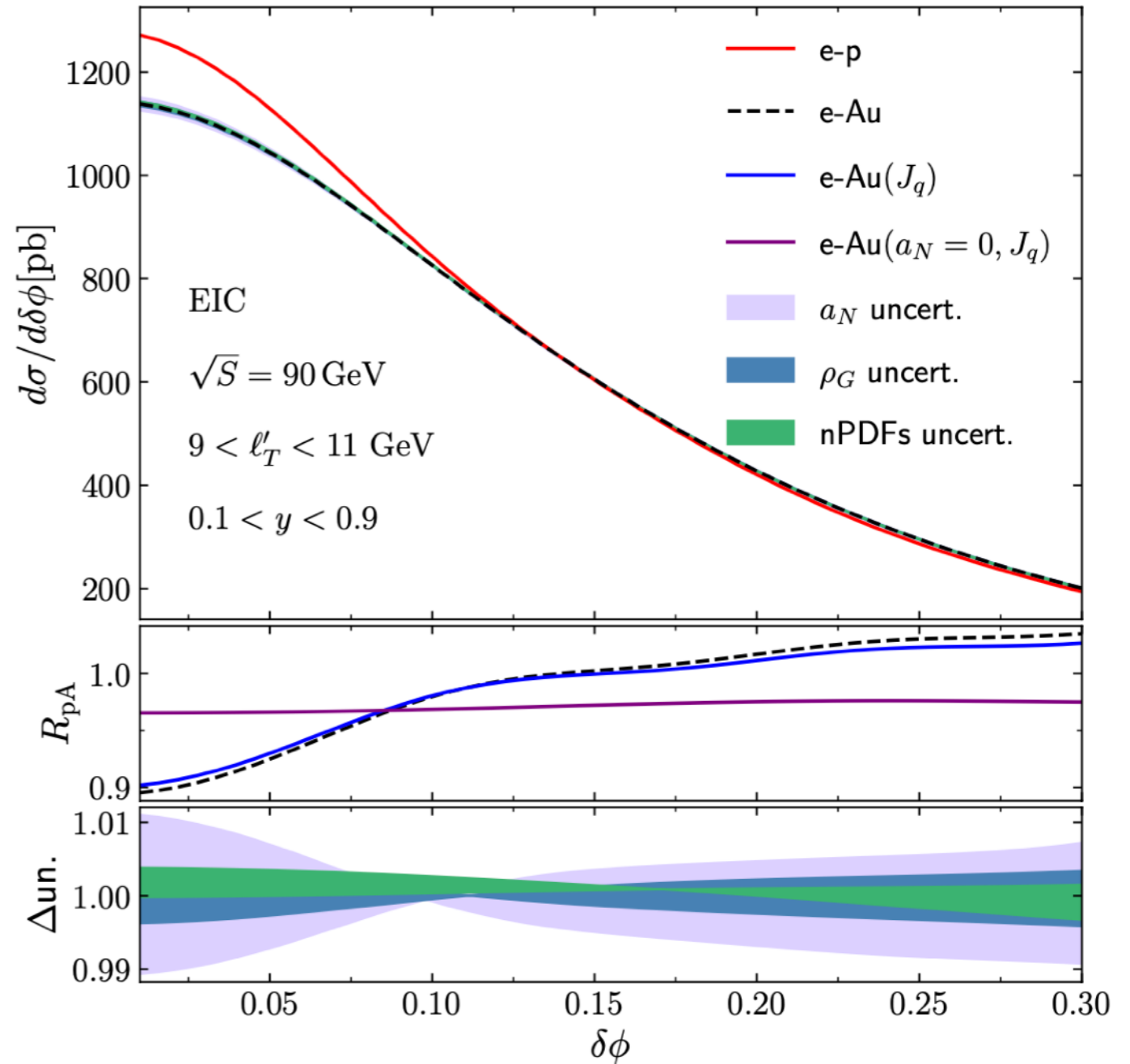
(Gyulassy, Levai, & Vitev '02)

$\rho_G$ : density of the medium

$\xi$ : the screening mass

L: the length of the medium

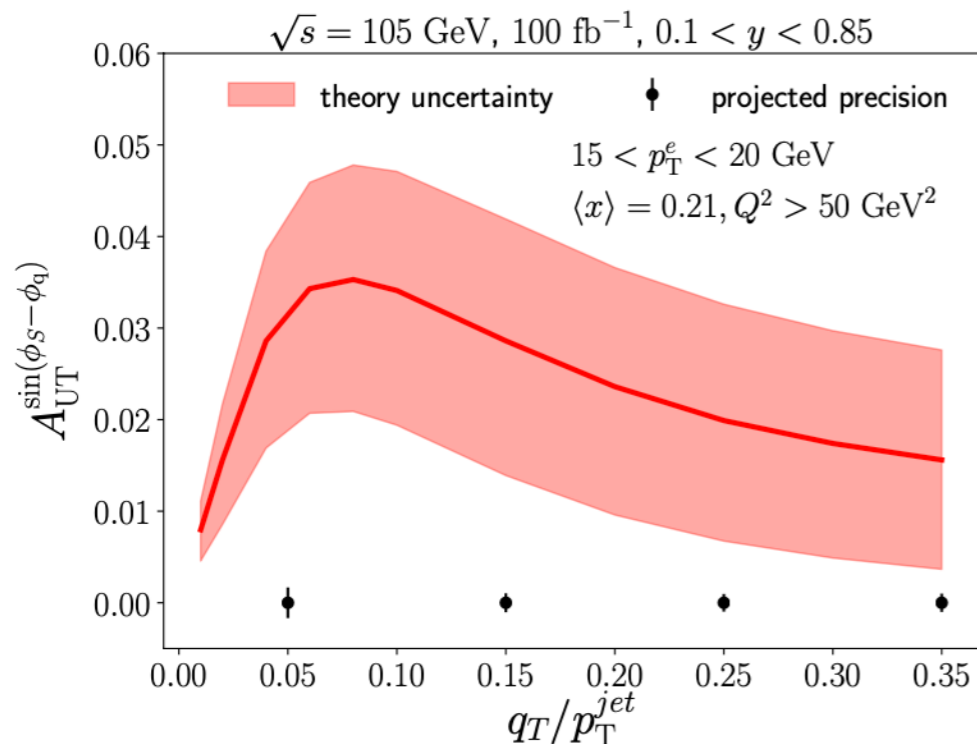
Parameter values are taken from a recent comparison between SCET<sub>G</sub> in e-A from the HERMES Ke and Vitev '23



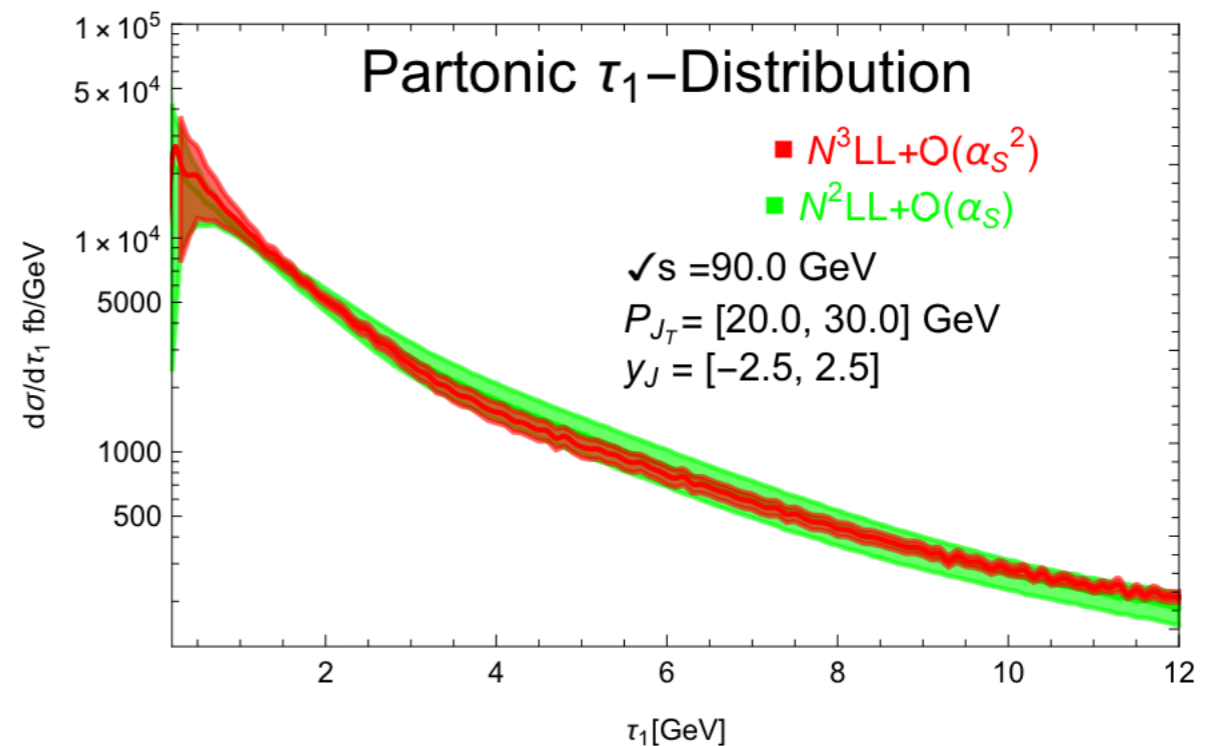
The process is primarily sensitive to the initial state's broadening effects, thereby serving as a clean probe of nTMD PDF

# Precision calculation for jets in DIS

- Precision calculations in DIS are essential for enhancing our understanding of partonic interactions and the internal structure of nucleons.
- The high-order calculation has reached N3LO accuracy for jet production in DIS [Currie, Gehrmann, Glover, Huss, Niehues, & Vogt '18](#)
- Several global event shape distributions in DIS are known at N3LL +  $\mathcal{O}(\alpha_s^2)$ 
  - **thrust** [Kang, Lee, & Stewart '15](#)
  - **(transverse) energy energy correlator** [Li, Vitev, & Zhu '20, Li, Makris, Vitev '21](#)
  - **1-jettiness** [Cao, Kang, Liu & Mantry '23](#)



[Arratia, Kang, Prokudin, Ringer '19](#)



[Cao, Kang, Liu & Mantry '23](#)



# N<sup>3</sup>LL + $\mathcal{O}(\alpha_s^2)$ predictions on lepton jet azimuthal correlation in DIS

Fang, Gao, Li, DYS 2407.XXXXX

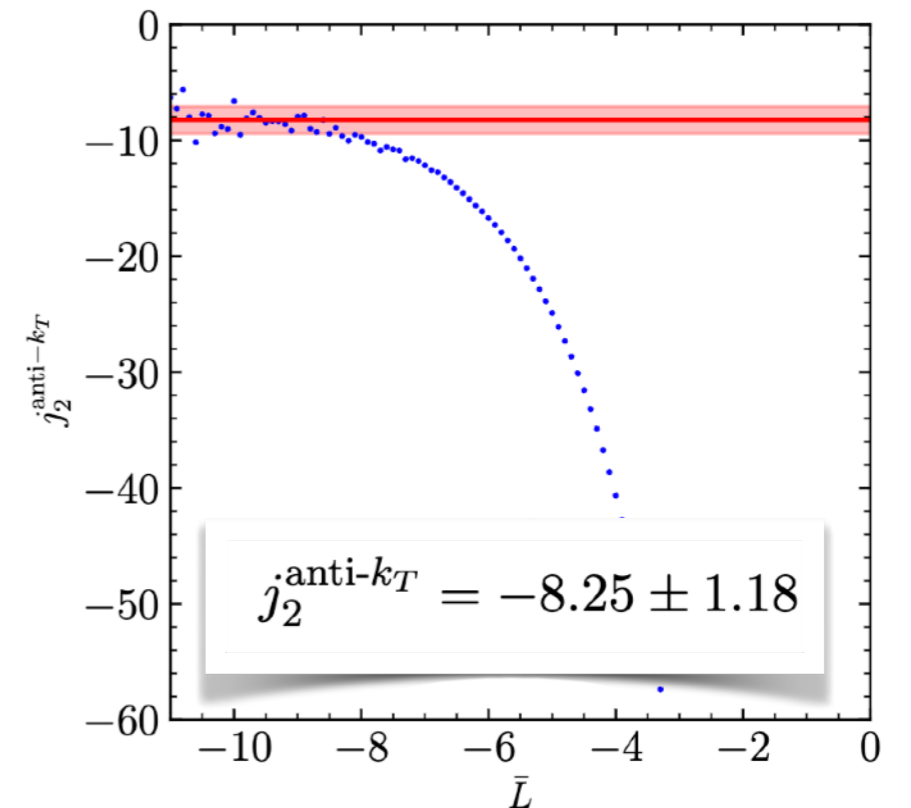
- All ingredients are known at N<sup>3</sup>LL+  $\mathcal{O}(\alpha_s^2)$ , except the two loop jet function  $j_2$ .
  - It was extracted numerically from the Event2 (Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi '19)
  - A preliminary numerical results are also calculated from SoftSERVE (Brune SCET2023)
- We study dijet production in e+e-, and compare two-loop singular cross section and  $\mathcal{O}(\alpha_s^2)$  predictions from NLOJET++ generator to extract  $j_2$

$$\frac{d\sigma}{dq_T} = \bar{\sigma}_0 H(Q, \mu_h) q_T \int_0^\infty b_T db_T J_0(q_T b_T) J_q(b_T, \mu_h, \zeta_f) J_{\bar{q}}(b_T, \mu_h, \zeta_f)$$

Integrated cross section:  $\sigma_L(Q_T) \equiv \int_0^{Q_T} dq_T \frac{d\sigma}{dq_T}$

Two-loop coefficient:

$$\begin{aligned}
 B = & C_F^2 \left[ \frac{\bar{L}^4}{2} + 3\bar{L}^3 + \bar{L}^2 \left( \frac{11}{2} - \frac{\pi^2}{3} + 6 \ln 2 \right) + \bar{L} \left( \frac{9}{4} + 18 \ln 2 - 4 \zeta_3 \right) - \frac{189}{16} + 5 \pi^2 \right. \\
 & \left. - \frac{173 \pi^4}{720} + 27 \ln 2 - \frac{9}{2} \pi^2 \ln 2 + 9 \ln^2 2 - 3 \zeta_3 \right] + C_F C_A \left[ \frac{11 \bar{L}^3}{9} + \bar{L}^2 \left( -\frac{35}{36} + \frac{\pi^2}{6} \right) \right. \\
 & \left. + \bar{L} \left( -\frac{57}{4} + \frac{11 \pi^2}{18} + 11 \ln 2 + 6 \zeta_3 \right) - \frac{51157}{1296} + \frac{1061 \pi^2}{216} - \frac{2 \pi^4}{45} + \frac{401 \zeta_3}{18} \right] \\
 & + C_F T_F n_f \left[ -\frac{4 \bar{L}^3}{9} + \frac{\bar{L}^2}{9} + \bar{L} \left( 5 - \frac{2 \pi^2}{9} - 4 \ln 2 \right) + \frac{4085}{324} - \frac{91 \pi^2}{54} - \frac{14 \zeta_3}{9} \right] \\
 & + \frac{j_2}{2},
 \end{aligned}$$

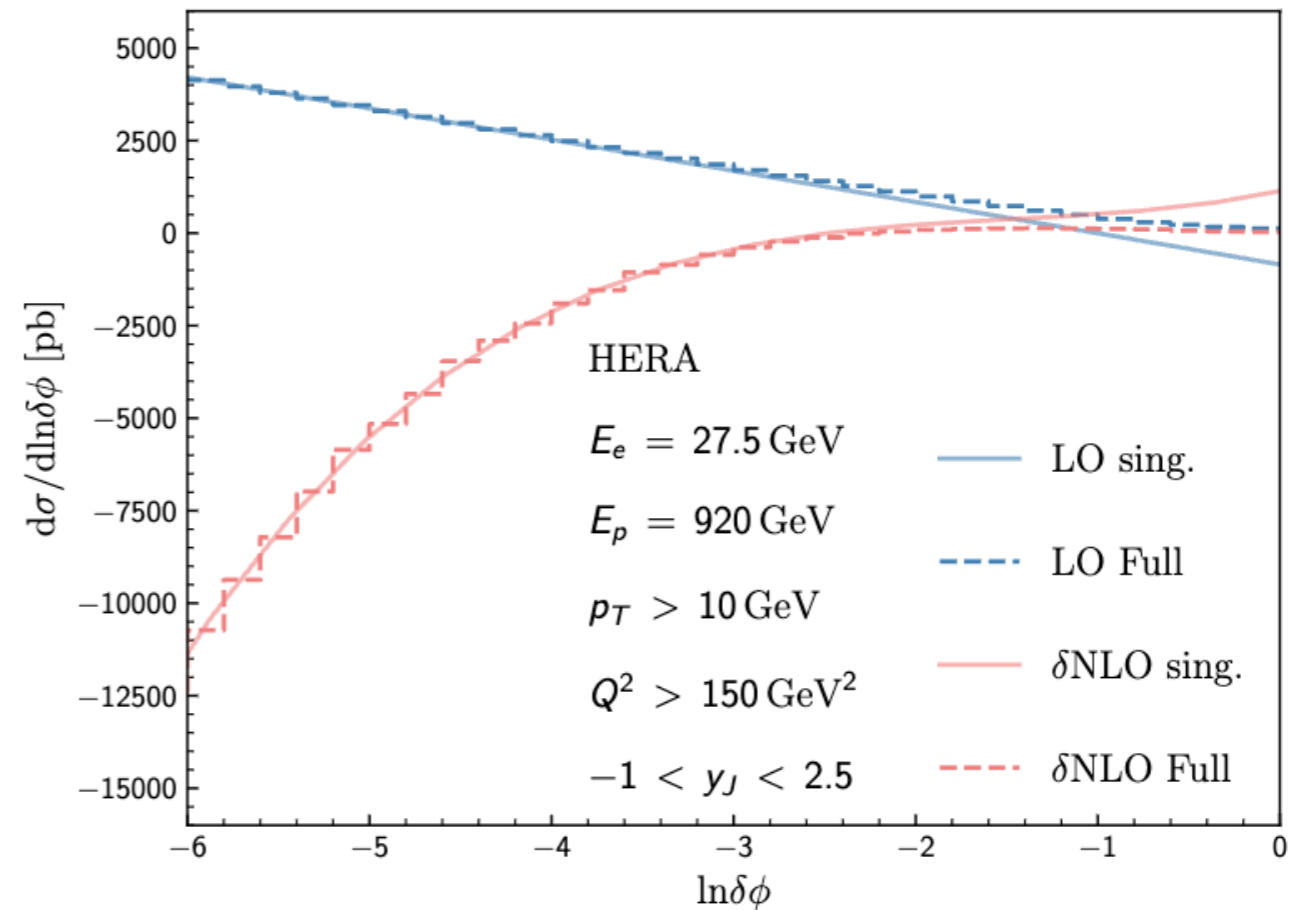
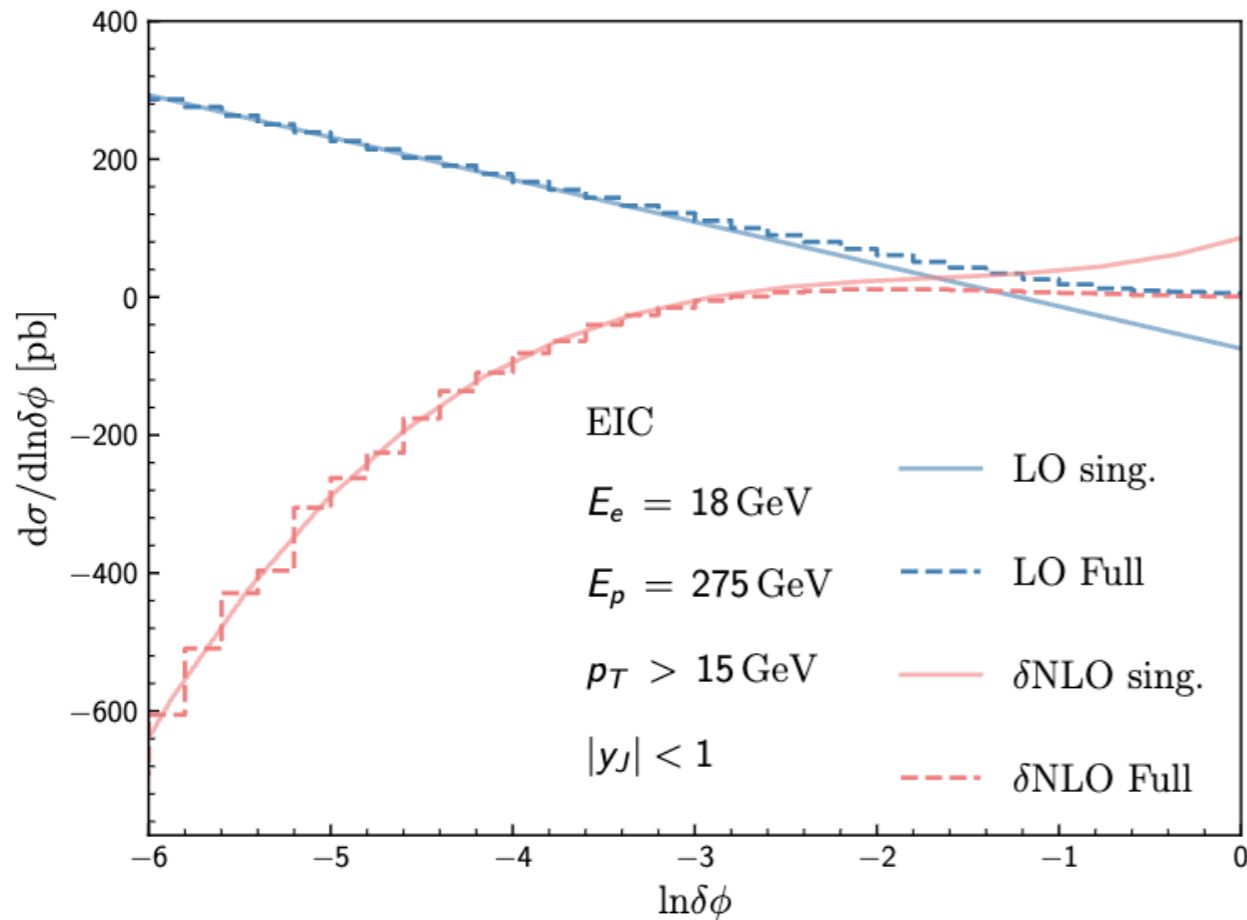




# $N^3LL + \mathcal{O}(\alpha_s^2)$ predictions on lepton jet azimuthal correlation in DIS

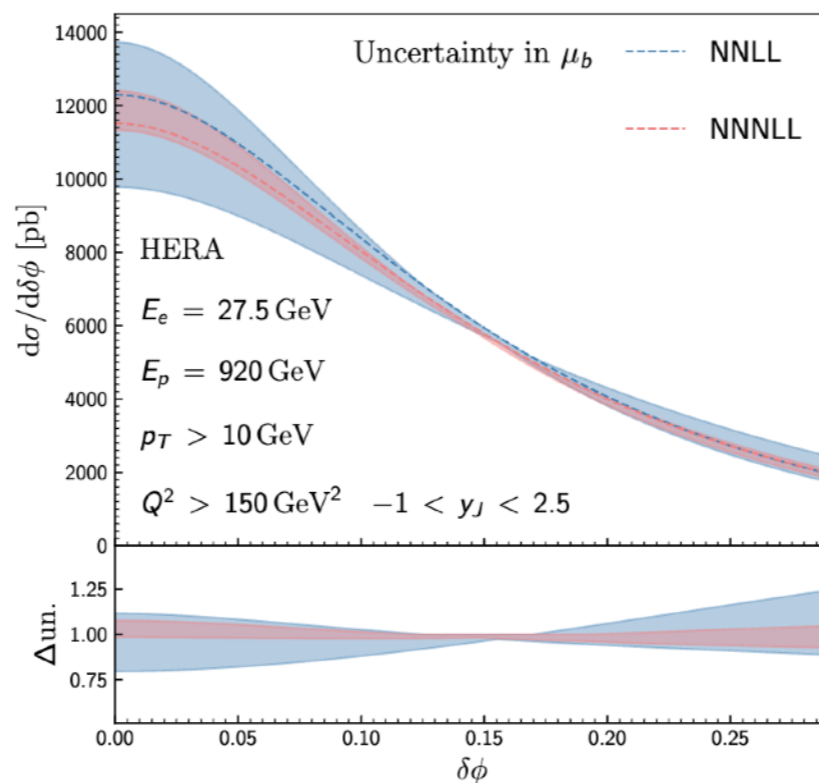
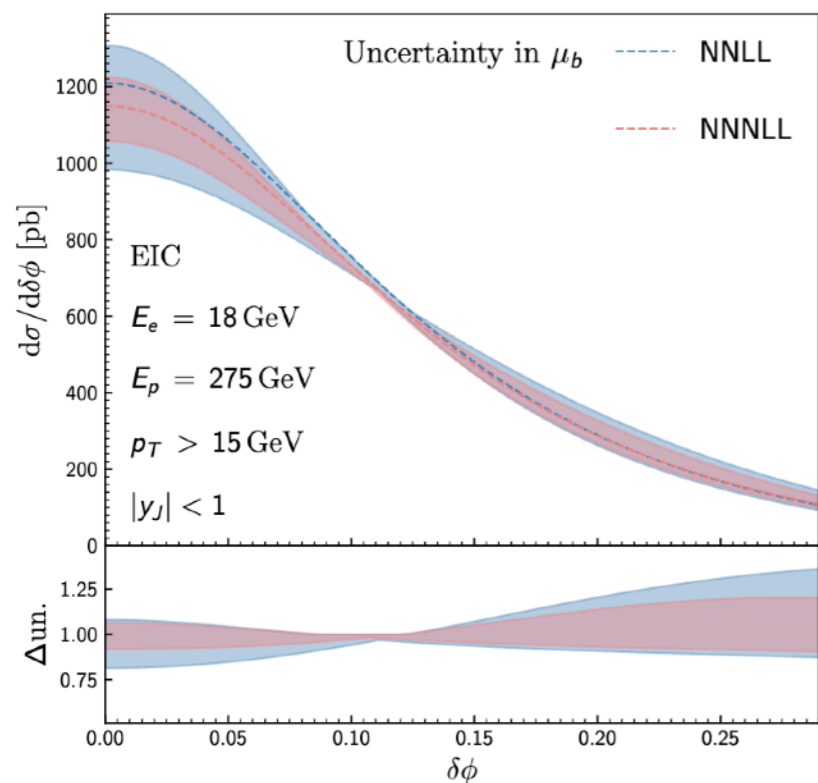
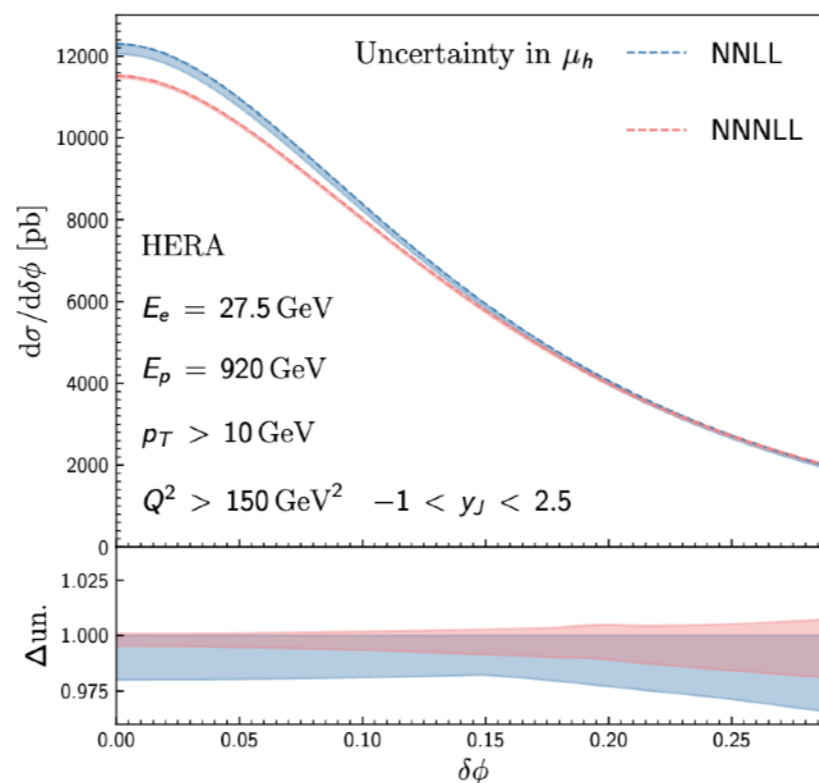
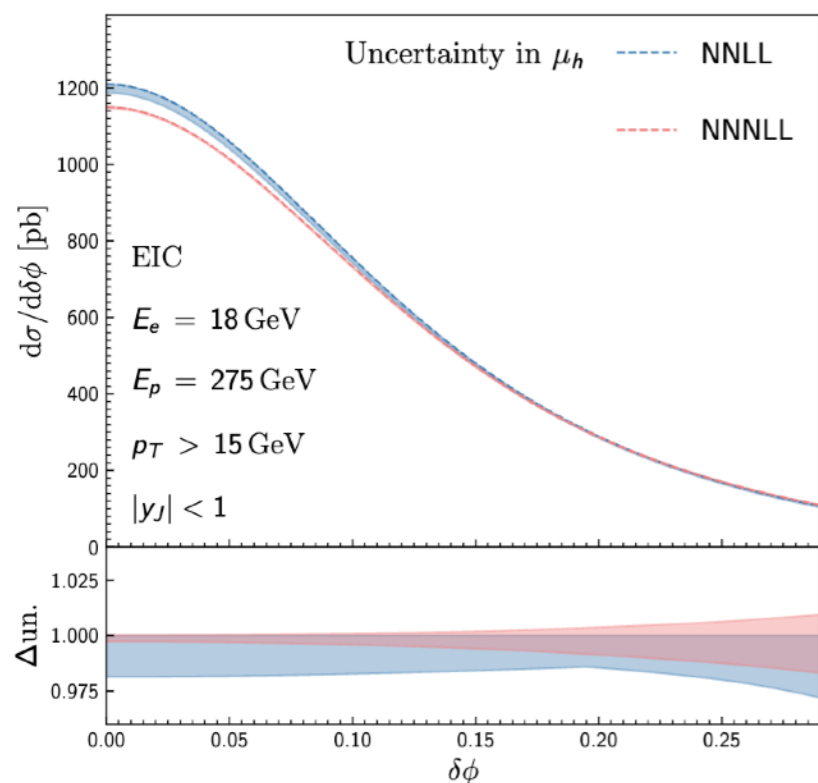
Fang, Gao, Li, DYS 2407.XXXXX

- We also compare the resummation expanded singular contribution in DIS with the full prediction from NLOJET++ up to  $\mathcal{O}(\alpha_s^2)$ .
- Good agreement in the back-to-back limit ( $\delta\phi \rightarrow 0$ ) is observed.
- Matching corrections (Y term) are important in the large  $\delta\phi$  region



# Comparison of resummation results at N2LL and N3LL

Fang, Gao, Li, DYS 2407.XXXXX



- The uncertainty bands are narrower at N3LL (red) compared to NNLL (blue)

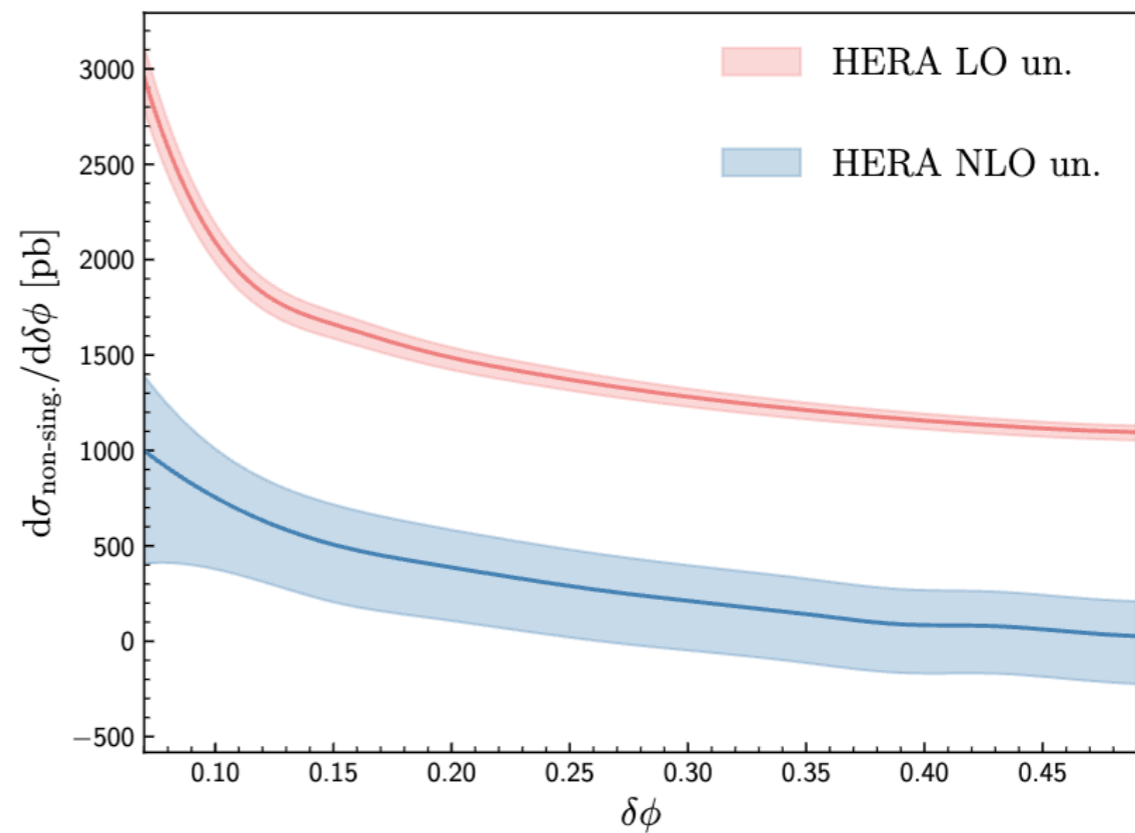
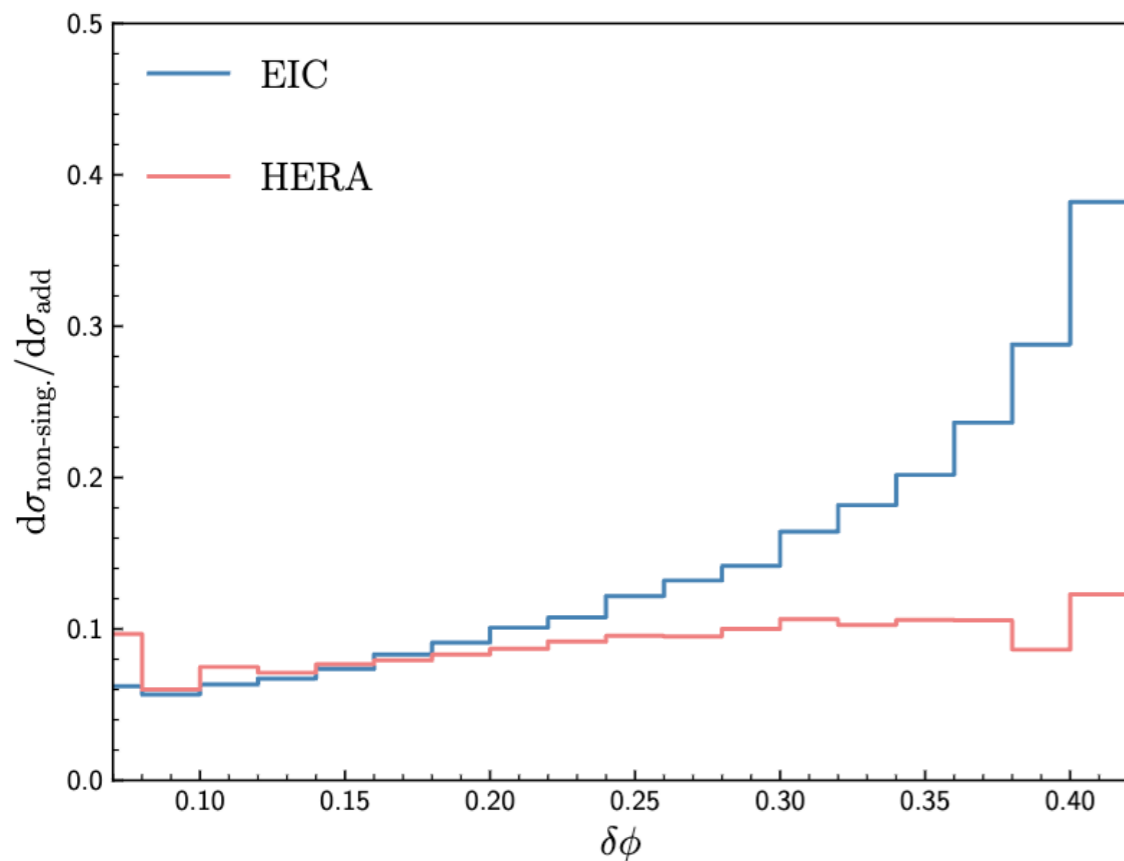
- At N3LL the dominant scale uncertainties are from  $\mu_b$  variation

# N<sup>3</sup>LL + $\mathcal{O}(\alpha_s^2)$ predictions on lepton jet azimuthal correlation in DIS

Fang, Gao, Li, DYS 2407.XXXXX

- In the large  $\delta\phi$  region the resummation formula receives significant matching corrections
- It is necessary to switch off the resummation and instead employ fixed-order calculations

$$d\sigma_{\text{add}} (\text{NNNLL} + \mathcal{O}(\alpha_s^2)) \equiv d\sigma(\text{NNNLL}) + \underbrace{d\sigma(\text{NLO}) - d\sigma(\text{NLO singular})}_{d\sigma(\text{NLO non-singular})}$$

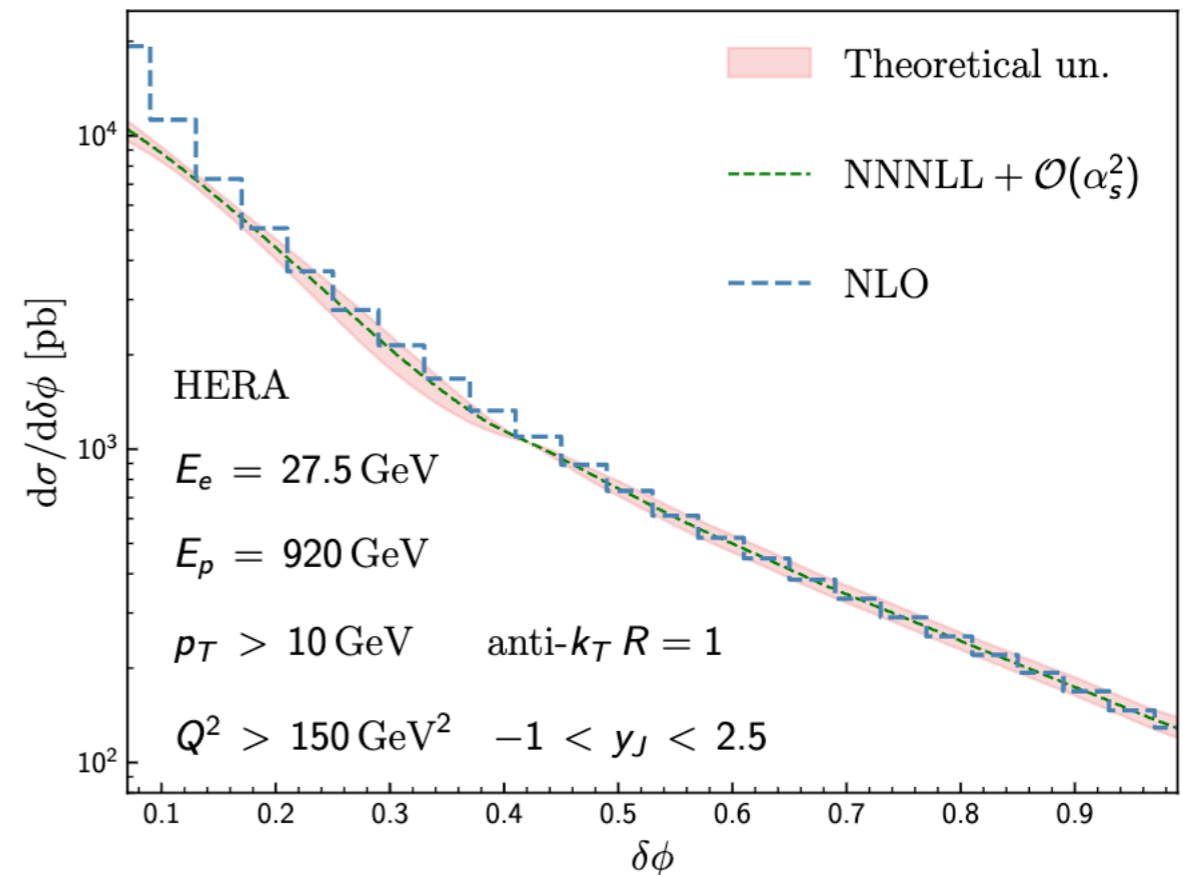
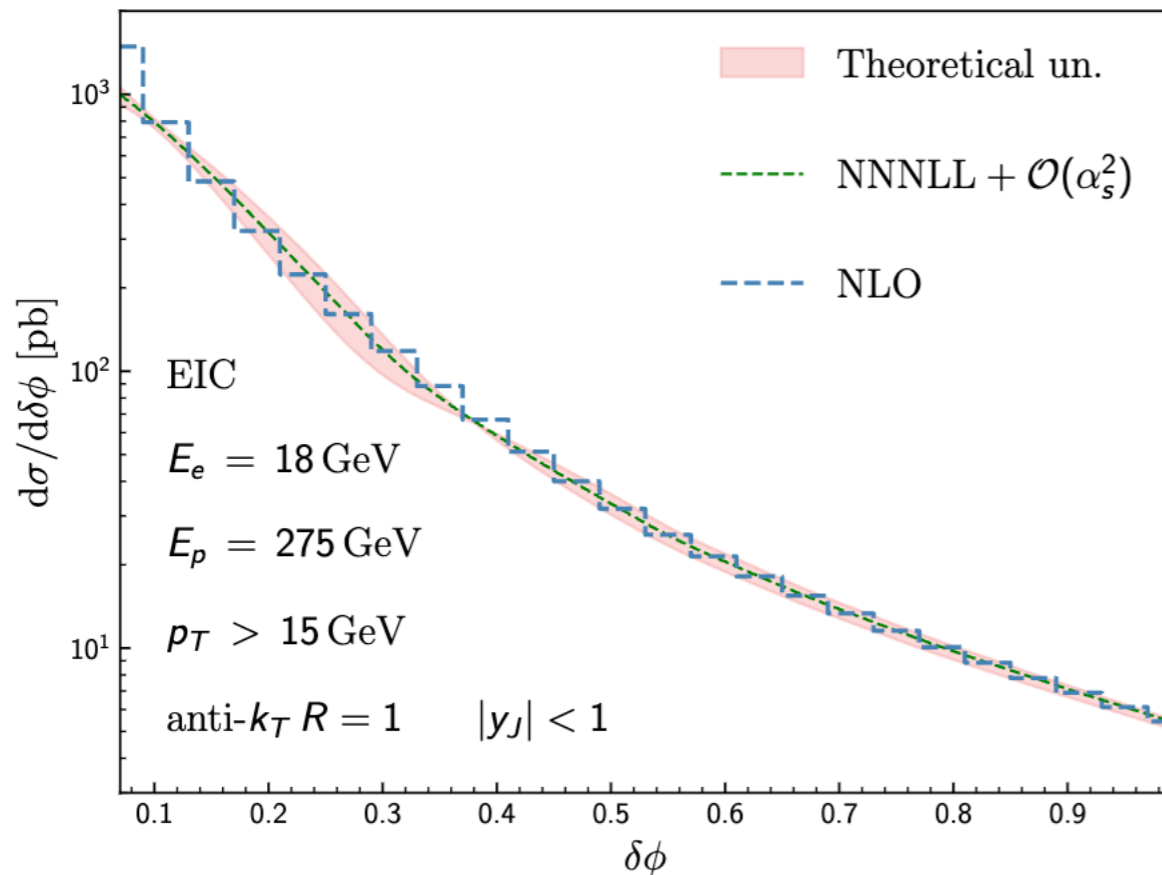


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# Summary

- **We have studied on the lepton-jet correlation in both e-p and e-A collisions. Utilizing SCET, we derived a factorization theorem for back-to-back lepton-jet configurations.**
- **In e-A collisions, we discussed the utility of our approach in disentangling intrinsic non-perturbative contributions from nTMDs and dynamical medium effects in nuclear environments. We find the process is primarily sensitive to the initial state's broadening effects.**
- **TMD resummation accuracy has been improved to N3LL +  $\mathcal{O}(\alpha_s^2)$  accuracy in e-p collisions. It is good to have the measurement at the HERA to make a comparison.**
- **Our work sets the groundwork for future experiments at the EIC, offering a robust framework for measuring nTMDs.**

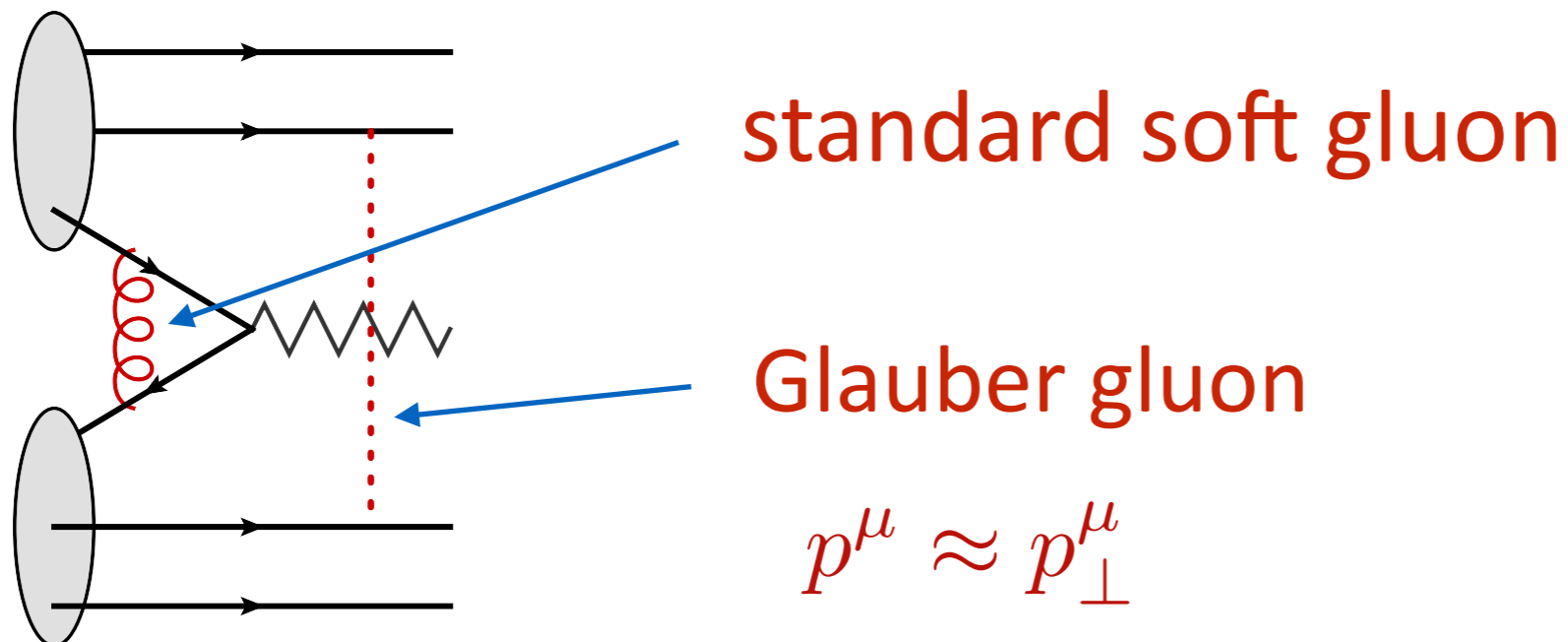
**Thank you**

# Backup

Quite nontrivial that the low-energy matrix element factorizes into a product

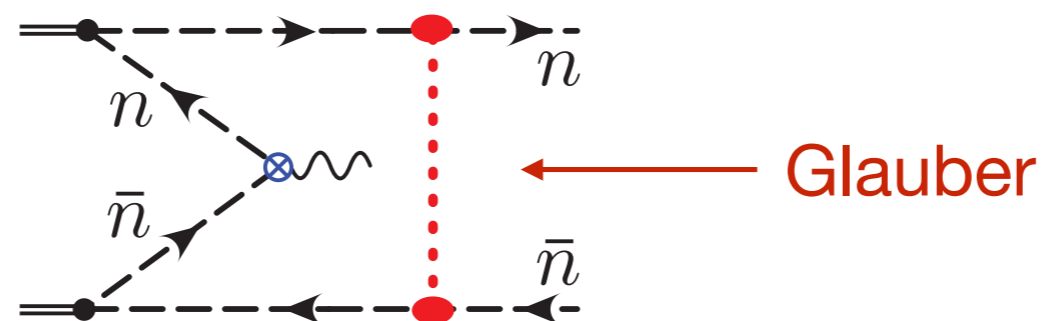
$$\langle P(p_1) | O_a(x_1) | P(p_1) \rangle \langle P(p_2) | O_b(x_2) | P(p_2) \rangle$$

One should be worried about long-distance interactions mediated by soft gluons





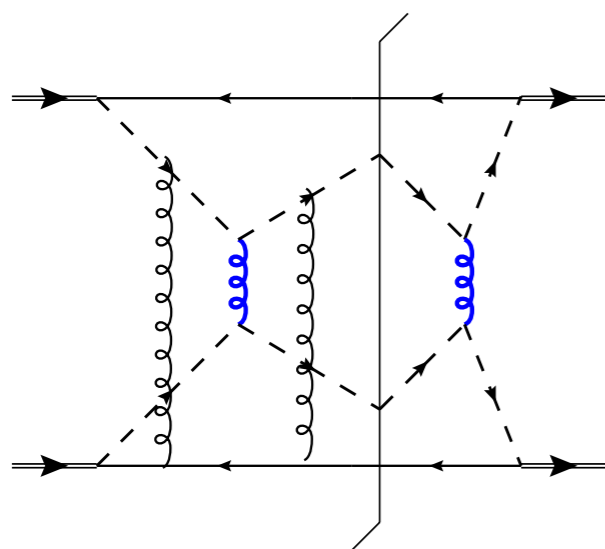
## All proton collisions include forward component (proton remnants)



Absence of factorization-violation due to Glauber gluons is important element of factorization proof for Drell-Yan process. Bodwin '85; Collins, Soper, Sterman '85 '88 ...

**e.g. Transverse momentum dependent (TMD) factorization is violated in di-jet production**

Collins, Qiu '07; Collins '07, Vogelsang, Yuan '07; Rogers, Mulders '10, ...



We remark that, because the TMD factorization breaking effects are due to the **Glauber region** where all components of gluon momentum are small, the interactions responsible for breaking TMD factorization are associated with large distance scales.

Rogers, Mulders '10

FIG. 8 (color online). The exchange of two extra gluons, as in this graph, will tend to give nonfactorization in unpolarized cross sections.