Lattice extraction of the TMD soft function and CS kernel with the auxiliary field representation of the Wilson line

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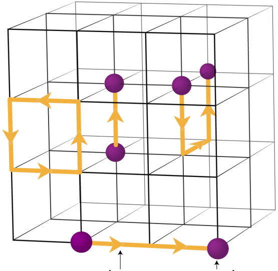
Outline

Long term goal

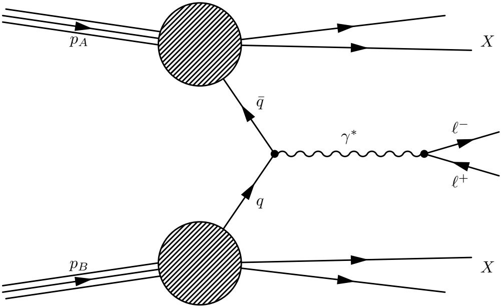


TMDs from lattice QCD

Nucleon Polarization		Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
	U	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
	L		g_1 = − Helicity	$h_{1L}^\perp$ = − Worm-gear
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − Worm-gear	h_1 = − Transversity; $h_{1T}^\perp$ = − Pretzelosity

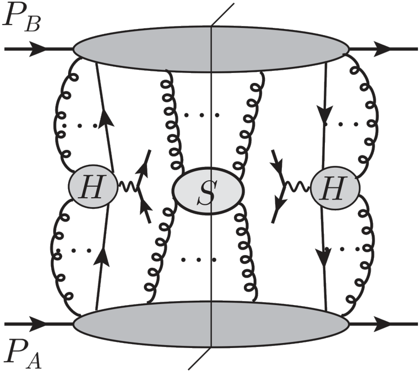
Drell-Yan process

- $\Lambda_{\mathrm{QCD}} \lesssim |\vec{q}_\perp| \ll Q$
- Collins-Soper (CS) scale ζ_a, ζ_b, where $\zeta_a \zeta_b = Q^4$
- Rapidity divergences



$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\mathrm{d}Y\mathrm{d}^2\vec{q}_\perp} = \sum_{i,j} H_{ij}\left(Q^2,\mu\right) \int \mathrm{d}^2\vec{b}_\perp e^{i\vec{b}_\perp\cdot\vec{q}_\perp} f_i\left(x_a,\vec{b}_\perp,\mu,\zeta_a\right) f_j\left(x_b,\vec{b}_\perp,\mu,\zeta_b\right)$$
$$\times\left[1+\mathcal{O}\left(\frac{q_\perp^2}{Q^2},\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right)\right]$$

Factorization



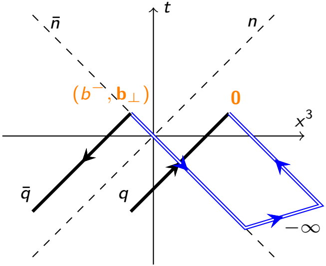
Drell-Yan leading region [Collins, 2011]

- Leading region has contribution from soft momentum states
- Need to regulate rapidity divergences present in beam and soft functions

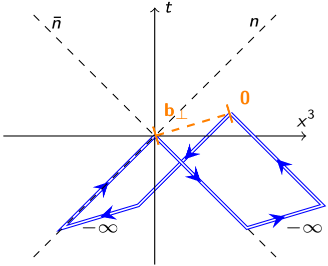
$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\mathrm{d}Y\mathrm{d}^2\vec{q}_\perp} = \sum_{i,j} H_{ij}(Q,\mu) \int \mathrm{d}^2\vec{b}_\perp e^{i\vec{b}_\perp\cdot\vec{q}_\perp} B_i\left(x_a, \vec{b}_\perp, \mu, \frac{\zeta_a}{\nu^2}\right) B_j\left(x_b, \vec{b}_\perp, \mu, \frac{\zeta_b}{\nu^2}\right)$$

$$\times S_i(b_\perp, \mu, \nu)\left[1 + \mathcal{O}\left(\frac{q_\perp^2}{Q^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right)\right]$$

Beam and soft function



Staple gauge link for beam function

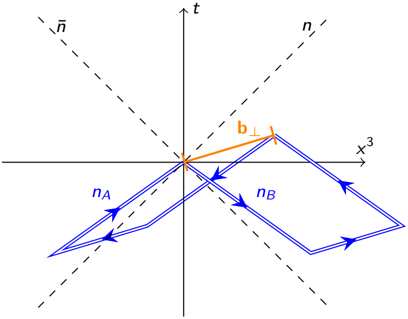


Wilson loop for soft function

$$B_i^{0(u)}\left(x, \vec{b}_\perp, \epsilon, \tau, xP^+\right) = \int \frac{\mathrm{d}b^-}{2\pi} e^{-ib^-(xP^+)} \langle P| \bar{\psi}_i^0\left(b^-, \vec{b}_\perp\right) W_{\bar{n}}\left(b^-, \vec{b}_\perp; -\infty, 0\right) \times W_\perp(-\infty\bar{n}; 0, b_\perp) W_{\bar{n}}(0; 0, -\infty) \frac{\gamma^+}{2} \psi_i^0(0) |P\rangle$$

$$S^0(b_\perp, \epsilon, \tau) = \frac{1}{N_c} \langle 0| W_n(b_\perp; 0, -\infty) W_{\bar{n}}(b_\perp; -\infty, 0) W_\perp(-\infty\bar{n}; 0, b_\perp) \times W_{\bar{n}}(0; 0, -\infty) W_n(0; -\infty, 0) W_\perp(-\infty n; b_\perp, 0) |0\rangle$$

Off lightcone regulator



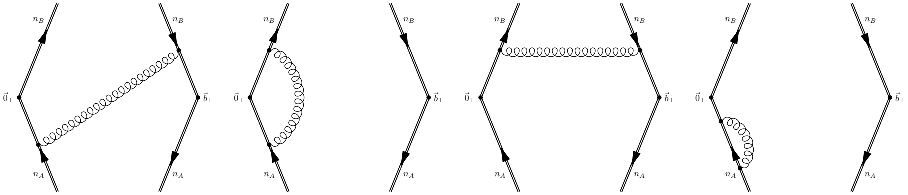
- Spacelike Wilson lines:

$$n_A \equiv n - e^{-y_A}\,\bar{n},$$
$$n_B \equiv \bar{n} - e^{+y_B}\,n$$

- Timelike direction also possible

$$f_i(x_b, \vec{b}_\perp, \mu, \zeta_b) = \lim_{\epsilon\to 0} Z^i_{\mathrm{UV}}\left(\mu, \epsilon, \zeta_b\right) \lim_{\substack{y_A\to+\infty \\ y_B\to-\infty}} B_i\left(x_b, \vec{b}_\perp, \epsilon, y_B, xP^+\right)$$
$$\times\sqrt{\frac{S_i\left(b_\perp, \epsilon, y_A - y_n\right)}{S_i\left(b_\perp, \epsilon, y_A - y_B\right) S_i\left(b_\perp, \epsilon, y_n - y_B\right)}}$$

One loop result in Minkowski space



$$
\begin{aligned}
&S(b_\perp, \epsilon, y_A, y_B) \\
&\quad = 1 + \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon} + \ln\left(\pi b_\perp^2 \mu_0^2 e^{\gamma_E}\right)\right)\left\{2 - 2|y_A - y_B|\frac{1 + e^{2(y_B - y_A)}}{1 - e^{2(y_B - y_A)}}\right\} + \mathcal{O}(\alpha_s^2)
\end{aligned}
$$

Collins-Soper kernel

- Collins-Soper (CS) kernel governs the rapidity evolution of the TMDPDF

$$K\left(b_{\perp},\mu\right)=\frac{\mathrm{d}f_{q}\left(x,b_{\perp},\mu,\zeta\right)}{\mathrm{d}\log\zeta}$$

- Can be obtained from the soft function

$$S_{q}\left(b_{\perp},y_{A},y_{B},\mu\right)=S_{I}\left(b_{\perp},\mu\right)e^{2K\left(b_{\perp},\mu\right)\left(y_{A}-y_{B}\right)}$$

- By direct computation:

$$K\left(b_{\perp},\mu\right)=\lim_{\substack{y_{A}\to+\infty\\ y_{B}\to-\infty}}\frac{1}{2}\frac{\partial}{\partial y_{n}}\log\left(\frac{S_{q}\left(b_{\perp},y_{n},y_{B},\mu\right)}{S_{q}\left(b_{\perp},y_{A},y_{n},\mu\right)}\right)$$

Lattice extraction of TMDPDFs

$$\tilde{f}_q\left(x,\vec{b}_\perp,\mu\tilde{\zeta},x\tilde{P}^z\right)=C_q\left(x\tilde{P}^z,\mu\right)\exp\left[\frac{1}{2}\gamma_\zeta^q(\mu,b_\perp)\log\frac{\tilde{\zeta}}{\zeta}\right]f_q\left(x,\vec{b}_\perp,\mu,\zeta\right)$$
$$\times\left\{1+\mathcal{O}\left(\frac{1}{(x\tilde{P}^z b_\perp)^2},\frac{\Lambda_{\mathrm{QCD}}^2}{(x\tilde{P}^z)^2}\right)\right\}$$

[Ebert, *et. al.*, 2019], [Ebert, *et. al*, 2022]

- C_q is a perturbatively calculable matching kernel
- $\tilde{\zeta}=x^2m_h^2e^{2(y_{\tilde{P}}+y_B-y_n)}$

quasi-TMDPDF

$$\tilde{f}_q\left(x,\vec{b}_\perp,\mu\tilde{\zeta},x\tilde{P}^z\right)=\tilde{f}_q^{\mathrm{naive}}\left(x,\vec{b}_\perp,\mu\tilde{\zeta},x\tilde{P}^z\right)\sqrt{\frac{\tilde{S}_q^{\mathrm{naive}}(b_\perp,\mu)}{S_q(b_\perp,\mu,2y_n,2y_B)}}$$

- $\tilde{f}_q^{\mathrm{naive}}$ and $\tilde{S}_q^{\mathrm{naive}}$ are lattice calculable objects
- S_q is the Collins soft function

Outline

1. Motivation
2. Existing methods
3. Our approach
4. Computational setup, and status

Intrinsic soft function

- Compute pion form factor

$$F\left(b_\perp, P^z\right) = \langle \pi(-P^z)|\, \bar{u}\Gamma u(b_\perp)\bar{d}\Gamma d(0)\, |\pi(P^z)\rangle$$

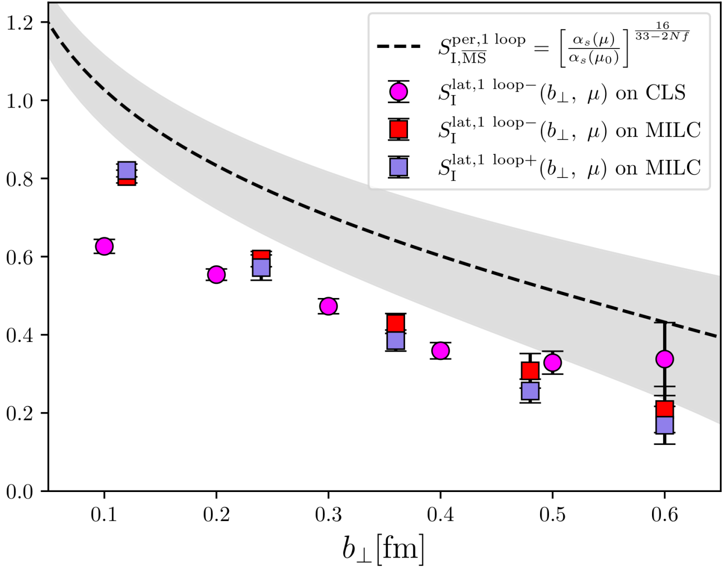
- Large P^z factorization:

$$F\left(b_\perp, P^z\right) = S_I\left(b_\perp, \mu\right) \int_0^1 \mathrm{d}x\mathrm{d}x' H_\Gamma\left(x, x', P^z, \mu\right) \times \Phi^\dagger\left(x', b_\perp, -P^z\right) \Phi\left(x, b_\perp, P^z\right) + \mathcal{O}\left(\frac{b_\perp}{P^z}, \frac{\Lambda_{\mathrm{QCD}}}{P^z}\right)$$

  - Intrinsic soft function
  - Perturbative kernel
  - quasi TMD wave functions

X. Ji, Y. Liu, Y.-S. Liu, Nucl. Phys. B955 (2020) 115054, Phys. Lett. B811 (2020) 135946

Intrinsic soft function

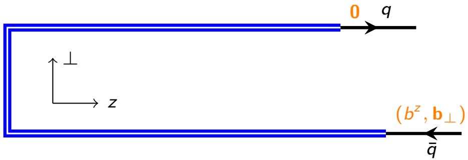


LPC Collaboration, JHEP 08 (2023) 172

Collins-Soper kernel

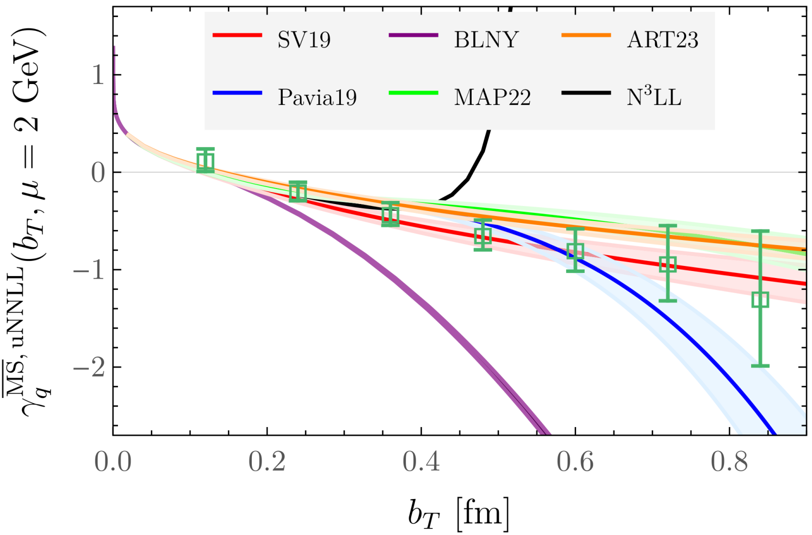
- Compute the ratio of qTMDPDFs or qTMDWFs at large P^z

$$K(b_\perp,\mu) = \frac{1}{\log(P_1^z/P_2^z)} \log \frac{C^{\mathrm{TMD}}(\mu, xP_1^z)\tilde{f}^{\mathrm{TMD}}\left(x, \vec{b}_\perp, \mu, P_1^z\right)}{C^{\mathrm{TMD}}(\mu, xP_2^z)\tilde{f}^{\mathrm{TMD}}\left(x, \vec{b}_\perp, \mu, P_2^z\right)}$$



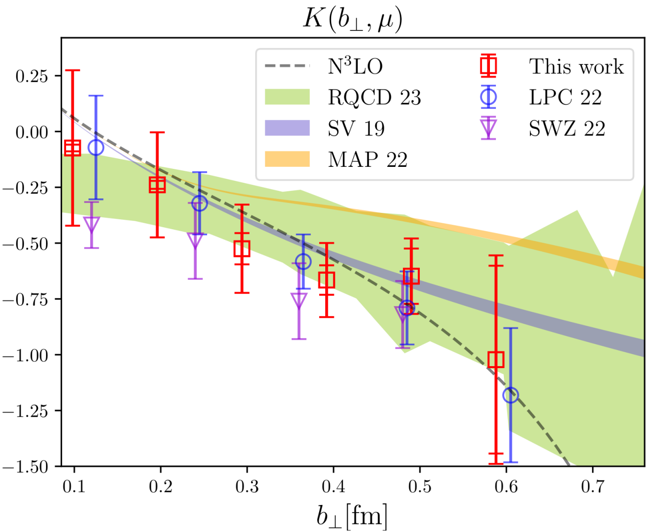
M. Ebert, I. Stewart, Y. Zhao, Phys. Rev., D99 (2019) 034505

CS kernel



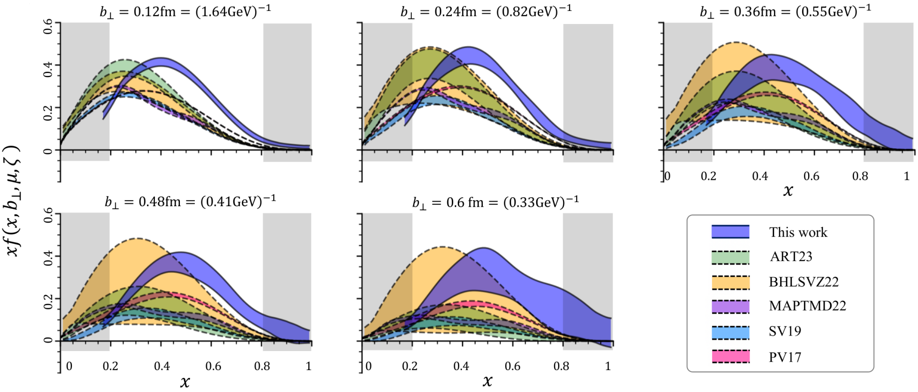
A. Avhadiev, P. Shanahan, M. Wagman, Y. Zhao, Phys. Rev. D198 (2023) 11, 114505

CS kernel



LPC Collaboration, JHEP 08 (2023) 172

Unpolarized TMDPDF



LPC Collaboration, J.-C. He et al., arXiv: 2211.02340

Need for alternative methods?

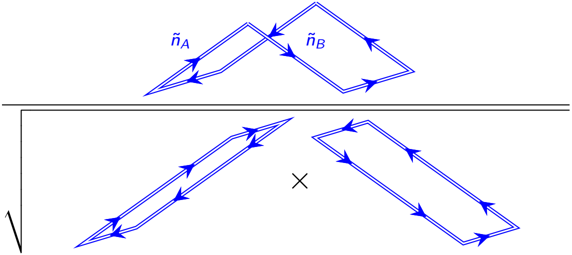
- Currently only have results for pion form factor
- Universality should give us same result for different hadrons
- Computation without hadron would be cheaper to perform

Outline

Our approach

$$\tilde{n}_A = \left(in_A^0, 0, 0, n_A^3\right), \quad \tilde{n}_B = \left(in_B^0, 0, 0, n_B^3\right)$$

$$S_{\text{ratio}}\left(\vec{b}_\perp, \tilde{n}_A, \tilde{n}_B, a, \tau\right) =$$



- Ratio gives correct dependence on $b_\perp$
- Removes linear divergences associated with finite length Wilson lines
- Ensures power counting in $b_\perp^4/\tau^4$
- Approaches lattice time τ independent result for large τ

Finite L Wilson lines

For $L \to \infty$ and $r_a, r_b \to 1$:

$$S\left(b_\perp, a, r_a, r_b, L\right) = 1 + \frac{\alpha_s C_F}{2\pi}\left(2 + \frac{(r_a r_b + 1)}{(r_a + r_b)}\log\left(\frac{(r_a - 1)(r_b - 1)}{(r_a + 1)(r_b + 1)}\right)\right)\log\left(\frac{b_\perp^2}{a^2}\right)$$
$$+ \frac{\alpha_s C_F}{2\pi}\left\{-4\log\left(\frac{b_\perp^2}{a^2}\right) + 2\frac{\pi b_\perp}{a} + 2\frac{\pi\left(|n_A| + |n_B|\right)L}{b_\perp}\right.$$
$$-2\frac{\pi\left(|n_A| + |n_B|\right)L}{a}$$
$$\left. + 2\frac{b_\perp^2}{L^2}\left(C_1 - \frac{1}{3}\right)\right\} + \mathcal{O}\left(\frac{b_\perp^4}{L^4}, \alpha_s^2\right)$$

$$C_1 = 1 - \frac{1}{2}\frac{1}{b_0^2(r_b^2 - 1)} - \frac{1}{2}\frac{1}{a_0^2(r_a^2 - 1)} \implies \frac{b_\perp^2}{L^2} \ll r_{a,b} - 1, \qquad r_{a,b} = \frac{n_{A,B}^3}{n_{A,B}^0}$$

- Ratio removes problem terms

$$S_{\text{ratio}}\left(\vec{b}_\perp, r_a, r_b, a, L\right) = \frac{\tilde{S}\left(\vec{b}_\perp, r_a, r_b, a, L\right)}{\sqrt{\tilde{S}\left(\vec{b}_\perp, r_a, -r_a, a, L\right)\tilde{S}\left(\vec{b}_\perp, -r_b, r_b, a, L\right)}}$$

Connection to Minkowski space

- At large lattice time:

$$S_{\mathrm{ratio}}\left(b_\perp, \tilde{n}_A, \tilde{n}_B, a, \tau\right) = S_{\mathrm{lat}}\left(b_\perp, r_a, r_b, a\right) + \mathcal{O}\left(\frac{b_\perp^4}{\tau^4}\right)$$

- Direct mapping to rapidity variables in Collins' scheme:

$$r_a = \frac{n_A^3}{n_A^0} = \frac{1+e^{-2y_A}}{1-e^{-2y_A}}, \quad r_b = \frac{n_B^3}{n_B^0} = \frac{1+e^{2y_B}}{1-e^{2y_B}}$$

$$|r_a|, |r_b| > 1,\ n_A^0 n_B^0 (r_a r_b + 1) > 0$$

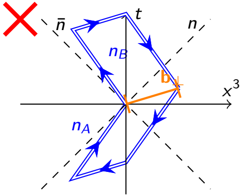
- Construct matching between lattice and continuum renormalization schemes

$$S\left(b_\perp, y_A, y_B, \mu\right) = C(r_a, r_b, \mu, a) \times S_{\mathrm{lat}}\left(b_\perp, r_a, r_b, a\right)$$

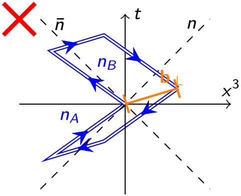
- Obtain CS kernel from:

$$S\left(b_\perp, y_A, y_B, \mu\right) = S_I\left(b_\perp, \mu\right) e^{2K(b_\perp,\mu)(y_A - y_B)}$$

What can we reconstruct in Minkowski space?



$|r_a| < 1, \quad |r_b| < 1, \quad n_A^0 n_B^0 (r_a r_b + 1) < 0$



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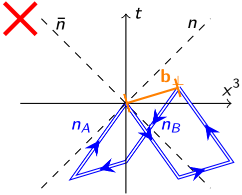
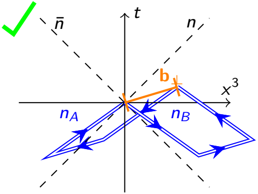


$|r_a| < 1, \quad |r_b| < 1, \quad n_A^0 n_B^0 (r_a r_b + 1) > 0$



$|r_a| > 1, \quad |r_b| > 1, \quad n_A^0 n_B^0 (r_a r_b + 1) > 0$

but it's okay: $S_{\mathrm{DY}} = S_{\mathrm{SIDIS}}$

Outline

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Auxiliary field definition of the Wilson line

Write Wilson line in terms of one dimensional 'fermions' that live along the path:

$$
\begin{aligned}
&P \exp\left\{-ig\int_{s_i}^{s_f} \mathrm{d}s n^\mu A_\mu(y(s))\right\} \\
&\quad = Z_\psi^{-1}\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\,\psi\bar{\psi}\exp\left\{ig\int_{s_i}^{s_f}\mathrm{d}s\bar{\psi}i\partial_s\psi - \bar{\psi}n\cdot A\psi\right\}
\end{aligned}
$$

[Gervais, Nevau 1980], [Aref'eva 1980]

Auxiliary field propagator:

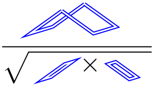
$$
in\cdot DH_n(y) = \delta(y) \xrightarrow{\text{Euclidean space}} -i\tilde{n}\cdot D_E H_{\tilde{n}}(y) = \delta(y), \qquad \tilde{n} = (in_0, \vec{n})
$$

Auxiliary field propagator in Euclidean space

$$-i\tilde{n} \cdot D_E H_{\tilde{n}}(y) = \delta(y)$$

- Meaningful solution only obtained with a UV cutoff [Aglietti, *et. al.* 1992], [Agglietti, 1994]
- Result will grow exponentially in τ before taking ratio:

$$S_{\text{ratio}}\left(\vec{b}_\perp, \tilde{n}_A, \tilde{n}_B, a, \tau\right) =$$



- Use lattice as UV cutoff and construct discretized solution to equation of motions [Mandula, Ogilvie, 1992]

$$n_0\left[U(x, x+\hat{t})G(x+\hat{t}, y) - G(x, y)\right] + \sum_{\mu=1}^{3} \frac{-in_\mu}{2}\left[U(x, x+\hat{\mu})G(x+\hat{\mu}, y) - U(x, x-\hat{\mu})G(x-\hat{\mu}, y)\right] = \delta(x, y)$$

Conclusion and outlook

- Important to have multiple methods for computing the soft function
- Euclidean space calculation of soft function has a direct mapping to Minkowski space result
- Preliminary results should be available soon

Thank you!

Group members

Anthony Francis (NYCU), Issaku Kanamori (R-CCS, RIKEN), C.-J. David Lin (NYCU), WM (NYCU), Yong Zhao (Argonne)

Coordinate space

Perform integration in coordinate space:

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} e^{-ik(b+s\tilde{n}_A - t\tilde{n}_B)} \frac{1}{k^2} = \int_0^\infty \mathrm{d}u \int \frac{\mathrm{d}^d k}{(2\pi)^d} e^{-uk^2} e^{-(b+s\tilde{n}_A - t\tilde{n}_B)^2/4u}$$

$$= \frac{\Gamma(d/2-1)}{(4\pi)^{d/2}} \frac{1}{((b+s\tilde{n}_A - t\tilde{n}_B)^2/4)^{d/2-1}}$$

'u' integral only valid for

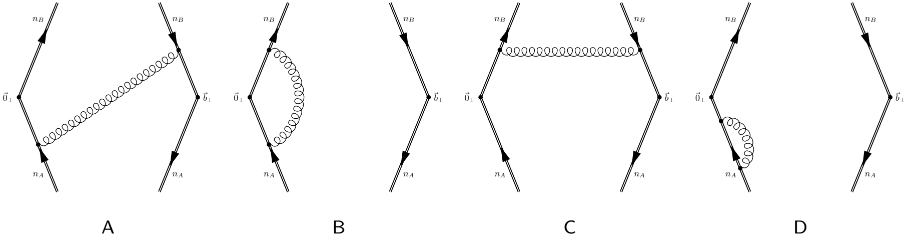
$$(s\tilde{n}_A - t\tilde{n}_B)^2 = s^2((n_A^3)^2 - (n_A^0)^2) + t^2((n_B^3)^2 - (n_B^0)^2) + st(n_A^3 n_B^3 + n_A^0 n_B^0) > 0$$

Euclidean space integral only finite when:

$$|n_A^3| > |n_A^0|, \quad |n_B^3| > |n_B^0|, \quad n_A^3 n_B^3 + n_A^0 n_B^0 > 0$$

$$\rightarrow \quad |r_a| > 1, \quad |r_b| > 1, \quad n_A^0 n_B^0 (r_a r_b + 1) > 0$$

Soft function in Euclidean space at one loop



Calculation in coordinate space at one loop:

$$S^{(1)}\left(b_\perp, \epsilon, r_a, r_b\right) = \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon} + \ln\left(\pi b_\perp^2 \mu_0^2 e^{\gamma_E}\right)\right)\left\{2 + \log\left|\frac{(r_a - 1)(r_b - 1)}{(r_a + 1)(r_b + 1)}\right| \frac{r_a r_b + 1}{r_a + r_b}\right\}$$

$$|r_a| > 1, \quad |r_b| > 1, \quad n_A^0 n_B^0 (r_a r_b + 1) > 0$$