Proton Structure from Basis Light-Front Quantization

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With

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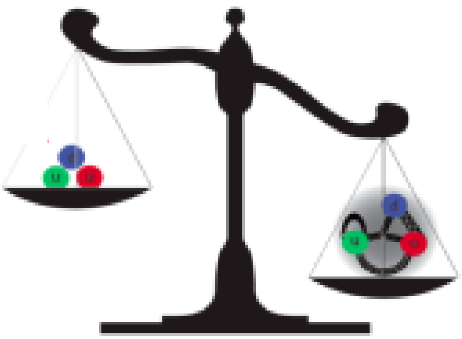
Outline

- Basis Light-front Quantization
- Application to proton
  - Electromagnetic Form factor
  - Parton distribution function (PDF)
  - TMD PDF + GPD
- Conclusion and outlook

Major Questions in Nuclear Physics

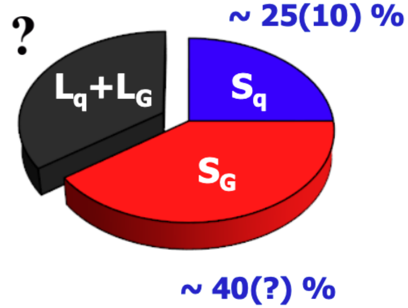
Origin of mass



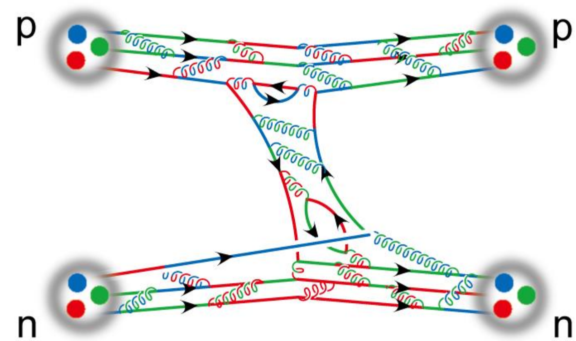
Spin puzzle

Ortibal angular momentum

$$\vec{L} = \vec{r} \times \vec{p}$$



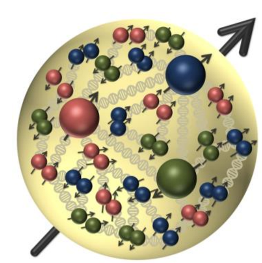
Nuclear force



We need to know the structure of nucleon beyond 1D and how it emerges from QCD

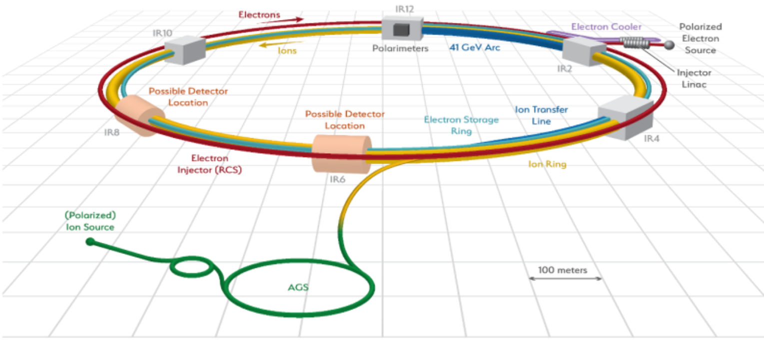
$$\mathcal{L}_{QCD} = \left(\bar{\psi}_q(i\not{D} - m_q)\psi_q\right) - \frac{1}{4}G^{\alpha}_{\mu\nu}G^{\mu\nu}_{\alpha}$$

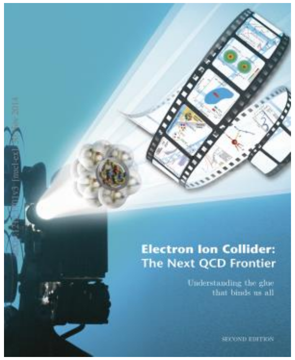
?

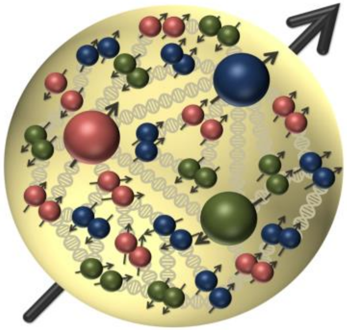


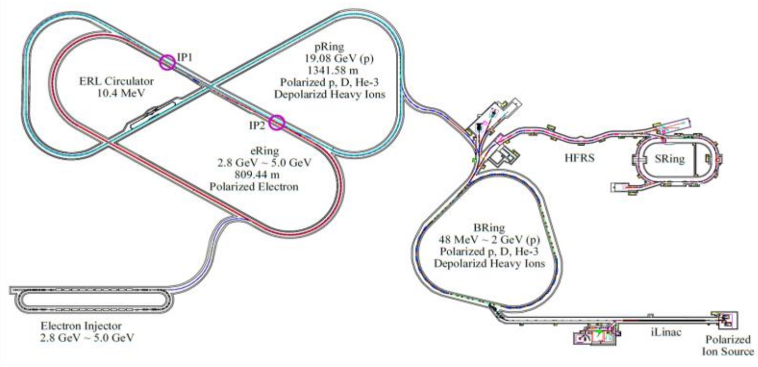
Electron–Ion Colliders

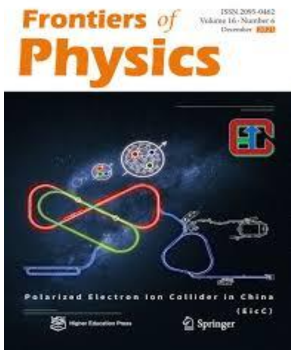
- Electron-Ion colliders with large collision energy and high luminosity











- EIC in the US is under construction by BNL@New York
- EicC in China is been planned by IMPCAS@Huizhou

Complimentarity

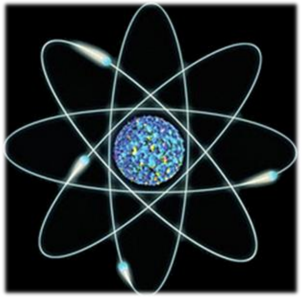
Nonperturbative Approach to Proton Structure

- Schrödinger equation universally describes bound-state structure

$$H|\psi\rangle = E|\psi\rangle$$

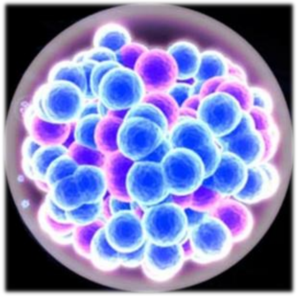
- Eigenstates $|\psi\rangle$ encode full information of the system

Nonrelativistic



atom

Nonrelativistic



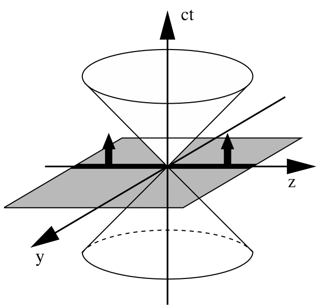
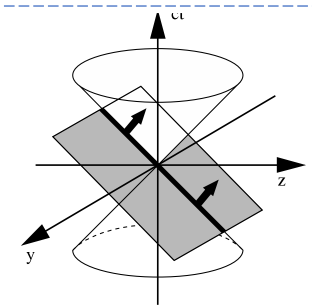
nucleus

Relativistic

nucleon

- Major challenges from relativity: frame dependence in wave function, particle number not conserving…

Light-front Quantization

Equal time quantization	Light-front quantization
$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$
	
x^1, x^2, x^3	$x^- = x^0 - x^3$, $x^\perp = x^{1,2}$
$P^0, \vec{P}$	$P^- = P^0 - P^3$, $P^+ = P^0 + P^3$, $P^\perp = P^{1,2}$
$i\frac{\partial}{\partial t}\lvert\phi(t)\rangle = H\lvert\phi(t)\rangle$	$i\frac{\partial}{\partial x^+}\lvert\phi(x^+)\rangle = \frac{1}{2}P^-\lvert\phi(x^+)\rangle$
$P^0 = \sqrt{m^2 + \vec{P}^2}$	$P^- = \frac{m^2 + P_\perp^2}{P^+}$

[Dirac, 1949]

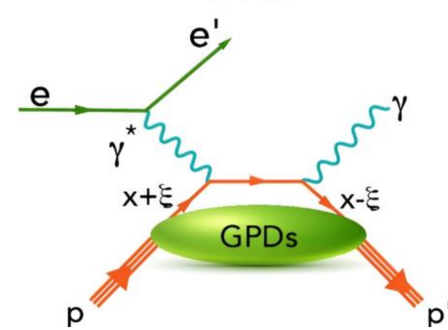
Main advantage:

- Parton distribution functions are defined on the light front
- Frame-independent light-front wave functions
- …

Eg., operator definition for GPD:

$$\Phi^{[\gamma^+]}(x, \Delta; Q^2) = \int \frac{dz^-}{8\pi} e^{ixP^+z^-/2} \langle P + \Delta, \Lambda | \bar{\psi}(z)\gamma^+\psi(0) | P, \Lambda \rangle$$

DVCS



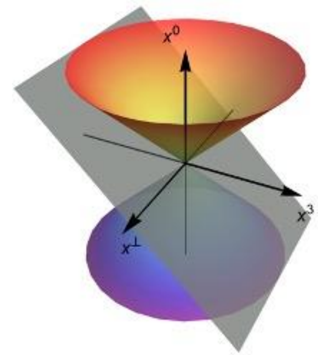
Basis Light-Front Quantization

- Hamiltonian eigenvalue equation:

[Vary, et.al, Phys.Rev.C '10]

$$P^-|\beta\rangle = P_\beta^-|\beta\rangle$$

- ○ $\boldsymbol{P^-}$: Light-Front Hamiltonian
- ○ $|\boldsymbol{\beta}\rangle$: Eigenstates
- ○ $\boldsymbol{P_\beta^-}$: Eigenvalues for eigenstates



- Basis setup:

Fock sector expansion: $|\beta_{\text{nucleon}}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\,q\bar{q}\rangle + \cdots\cdots$

Single particle basis $|\alpha\rangle = |n_1, m_1, n_2, m_2, n_3, m_3\rangle \otimes |k_1^+, k_2^+, k_3^+\rangle \otimes |\lambda_1, \lambda_2, \lambda_3, C\rangle$ in $|qqq\rangle$:

2-dimension harmonic oscillator; Discretized longitudinal momentum; Helicity and color

Basis truncation: $\sum_i (2n_i + |m_i| + 1) \le N_{\max}$ $\quad \sum_i k_i^+ = K_{\max}$ $\quad \Lambda = \sum_i (\lambda_i + m_i)$

- Advantages:

1. Rotational symmetry in transverse plane
2. Exact factorization between center-of-mass motion and intrinsic motion
3. Harmonic oscillator basis supplies infrared behavior for hadrons

Publications on Nucleon

$$|P,\Lambda\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, u\bar{u}\rangle + \left|qqq\, d\bar{d}\right\rangle + |qqq\, s\bar{s}\rangle + \cdots$$

- **Wave Functions:**

  [Chandan et al., Phys.Rev.D,102.016008] (2019)

  [Xu et al., Phys.Rev.D,108 9, 094002] (2023)

- GPDs: [Xu et al., Phys.Rev.D,104.094036] (2021)

  [Liu et al., Phys.Rev.D,105.094018] (2022)

  [Zhang et al.,Phys.Rev.D,109.034031] (2023)

  [Kaur et al., Phys. Rev. D 109, 014015] (2024)

  [Lin et al., Phys.Lett.B,847 138305] (2023)

  [Liu et al., Phys.Lett.B,855.138809] (2024)

- **TMDs:** [Zhi Hu et al., Phys.Lett.B,833.137360] (2022)

  [Zhimin Zhu et al., Phys.Rev.D,108.036009] (2023)

  [Zhimin Zhu et al., 1404.13720 [hep-ph]] (2024)

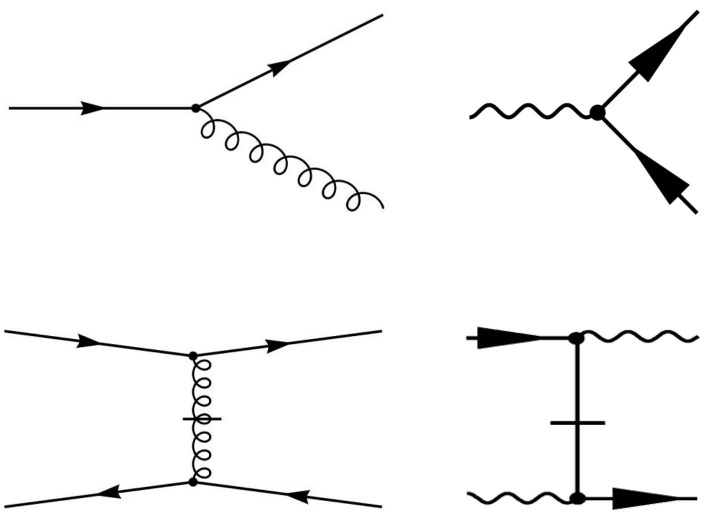
Light-front Hamiltonian

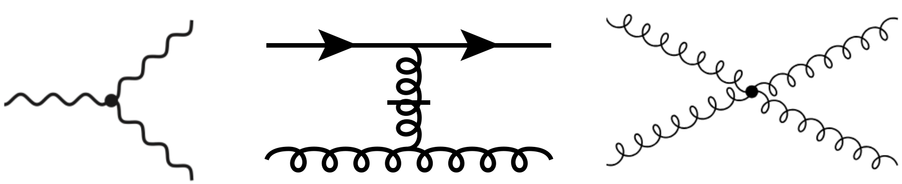
➢ QCD light-front Hamiltonian can be derived from QCD Lagrangian:

$$\mathcal{L}_{QCD} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} G^{\alpha}_{\mu\nu} G^{\mu\nu}_{\alpha} \quad \Rightarrow \quad P^{-}_{QCD} = H_K + H_I \qquad A^{+} = 0$$

$$H_K = \frac{1}{2}\int d^3x\, \bar{\psi}\gamma^{+}\frac{(i\partial^{\perp})^2 + m^2}{i\partial^{+}}\psi - \frac{1}{2}\int d^3x\, A^{i}_{a}(i\partial^{\perp})^2 A^{i}{}_{a}$$

$$\begin{aligned}
H_I = &+ g\int d^3x\, \bar{\psi}\gamma_{\mu}A^{\mu}\psi \\
&+ \frac{1}{2}g^2\int d^3x\, \bar{\psi}\gamma_{\mu}A^{\mu}\frac{\gamma^{+}}{i\partial^{+}}\gamma_{\nu}A^{\nu}\psi \\
&- ig^2\int d^3x\, f^{abc}\bar{\psi}\gamma^{+}T^{c}\psi\frac{1}{(i\partial^{+})^2}\left(i\partial^{+}A^{\mu}_{a}A_{\mu b}\right) \\
&+ \frac{1}{2}g^2\int d^3x\, \bar{\psi}\gamma^{+}T^{a}\psi\frac{1}{(i\partial^{+})^2}\bar{\psi}\gamma^{+}T^{a}\psi \\
&+ ig\int d^3x\, f^{abc} i\partial^{\mu}A^{\nu a}A^{b}_{\mu}A^{c}_{\nu} \\
&- \frac{1}{2}g^2\int d^3x\, f^{abc}f^{ade} i\partial^{+}A^{\mu}_{b}A_{\mu c}\frac{1}{(i\partial^{+})^2}\left(i\partial^{+}A^{\nu}_{d}A_{\nu e}\right) \\
&+ \frac{1}{4}g^2\int d^3x\, f^{abc}f^{ade}A^{\mu}_{b}A^{\nu}_{c}A_{\mu d}A_{\nu e}.
\end{aligned}$$





ψ: quark field operator

A^{a}_{μ}: gluon field operator

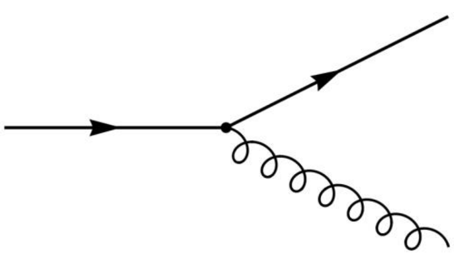
First Step: Up to $|qqqq\bar{q}\rangle$

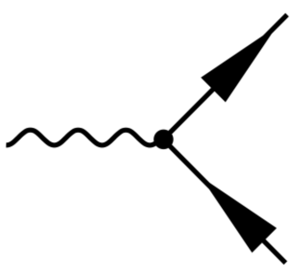
$$|P_{baryon}\rangle = \Psi_1|qqq\rangle + \Psi_2|qqqg\rangle + \Psi_{31}|qqq\,u\bar{u}\rangle + \Psi_{32}|qqq\,d\bar{d}\rangle + \Psi_{33}|qqq\,s\bar{s}\rangle$$

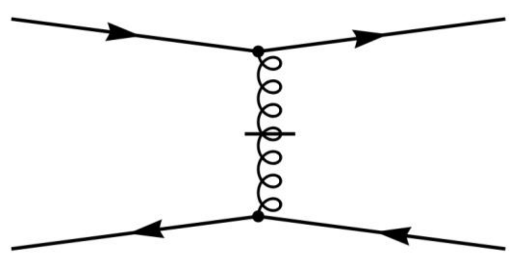
$$\boldsymbol{P^- = H_{K.E.} + H_{Interact}}$$

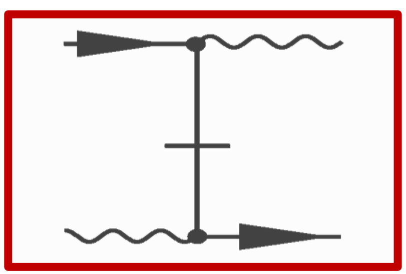
$$\boldsymbol{H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}}$$

$$\boldsymbol{H_{Interact} = g\bar{\psi}\,\gamma^\mu T^a\,\psi\,A_\mu^a + \frac{g^2 C_F}{2}\,j^+\frac{1}{(i\partial^+)^2}j^+ + \frac{g^2 C_F}{2}\,\bar{\psi}\gamma^\mu A_\mu \frac{\gamma^+}{i\partial^+} A_\nu \gamma^\nu \psi}$$









Fock Sector Decomposition

$$|P_{baryon}\rangle \rightarrow |qqq\rangle + |qqqg\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle$$

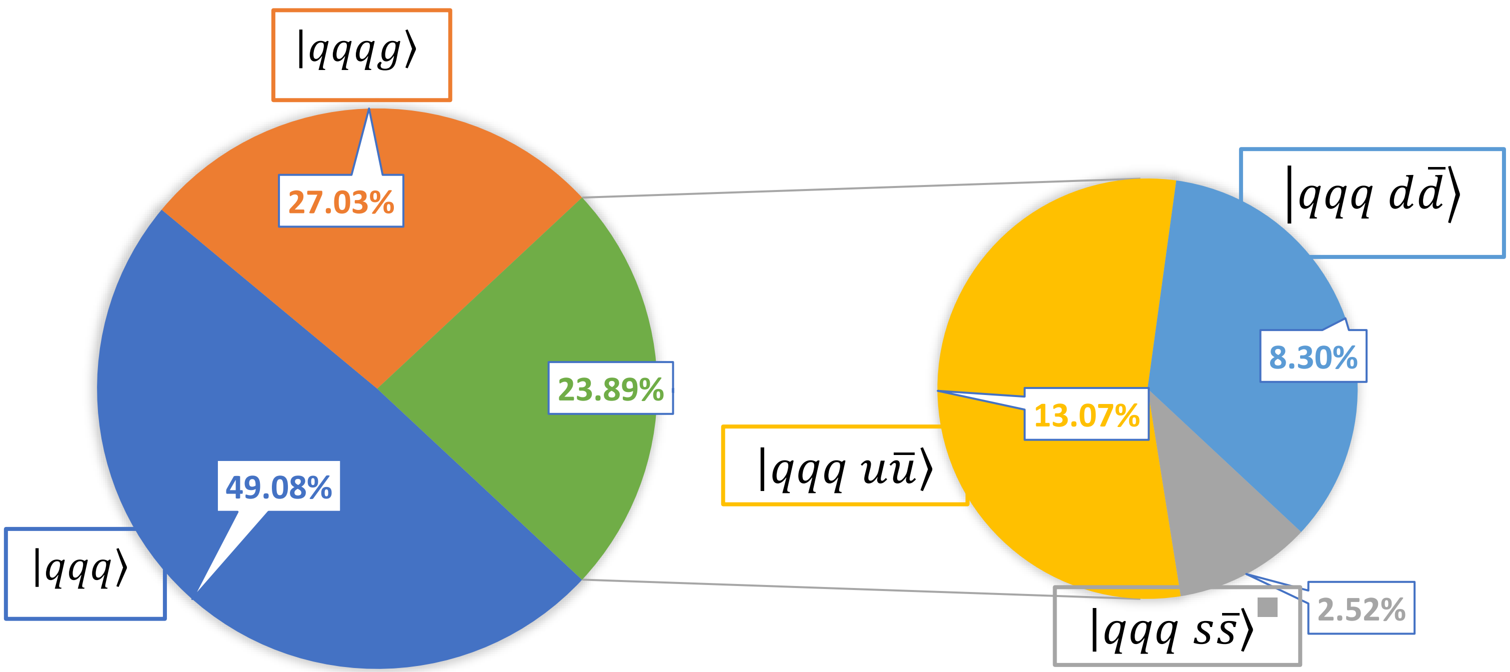
Parameter set:

Truncation parameter: $N_{\max} = 7$ and $K_{\max} = 16$

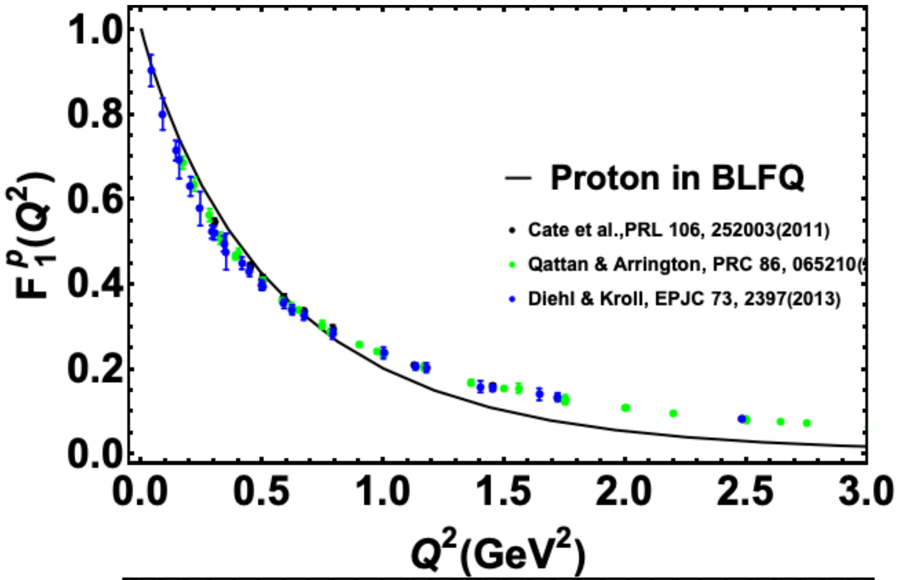
m_u	m_d	m_f	g	b	b_{inst}
1.0 GeV	0.9 GeV	5.8 GeV	3.0	0.7 GeV	2.8 GeV

In five quark Fock sector, we use current quark masses

$\mu_0^2 = 0.22\ \text{GeV}^2$



Form Factor

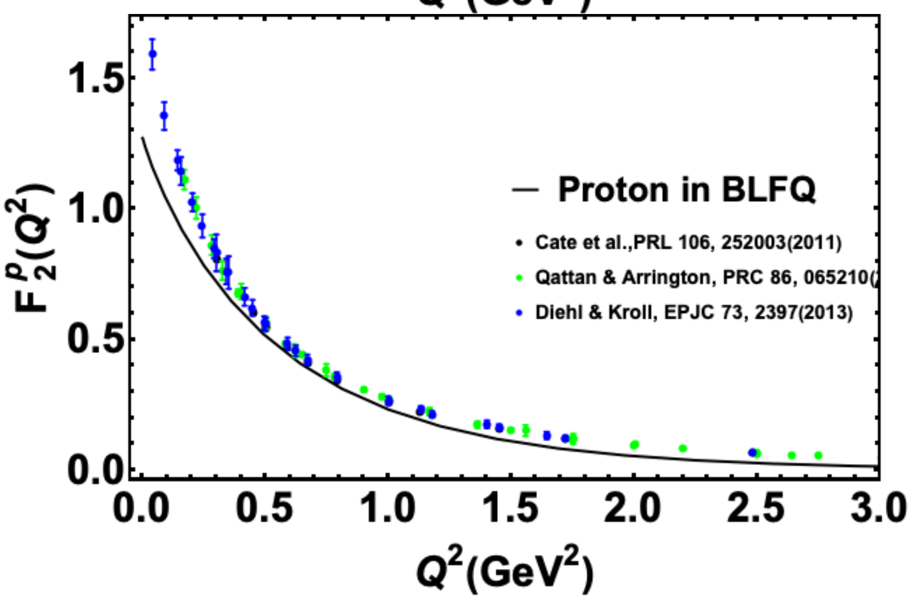


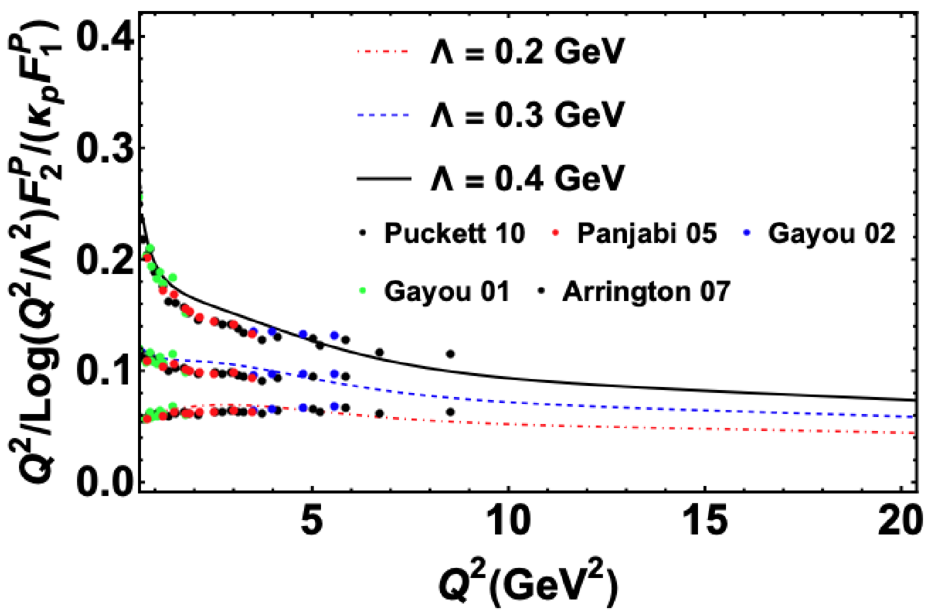
$$\langle N(p')|J^\mu(0)|N(P)\rangle$$
$$= \bar{u}(P')\left[\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}}{2M_P} q_\nu F_2(Q^2)\right] u(P)$$

$r_E = 0.786$ fm $\quad$ $r_E = 0.833 \pm 0.010$ fm

$r_M = 0.775$ fm $\quad$ $r_M = 0.851 \pm 0.026$ fm

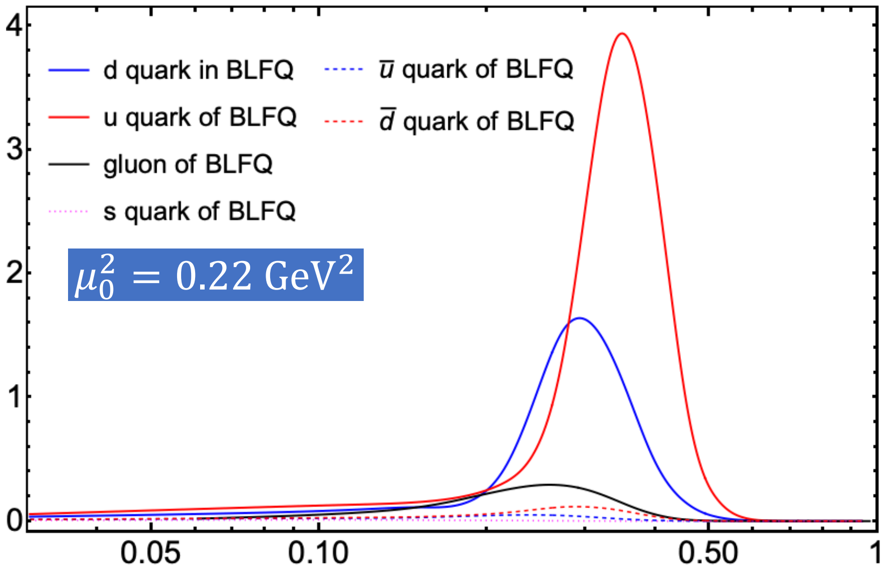
[Particle Data Group]

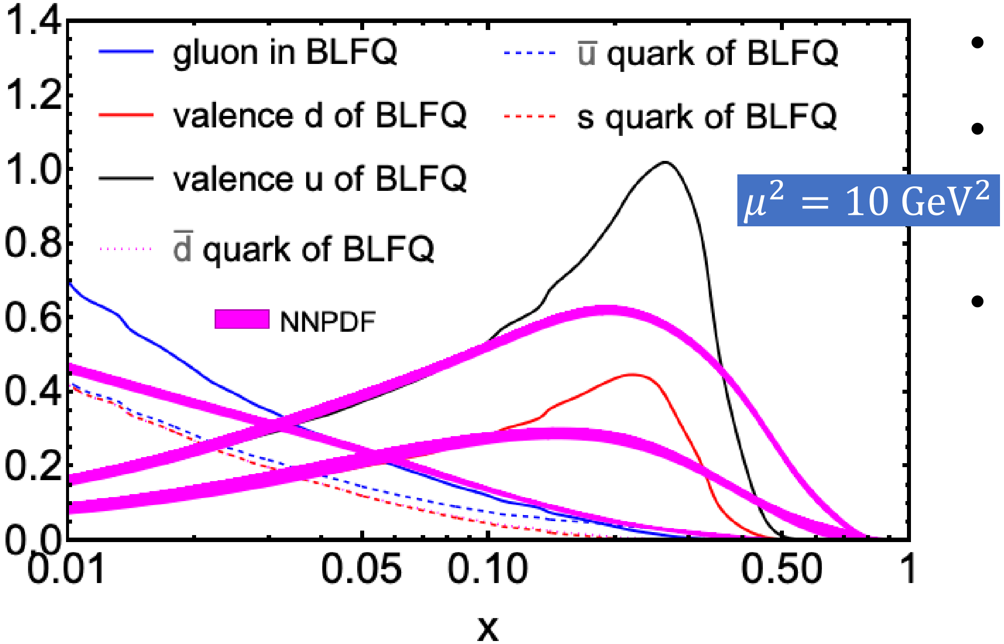




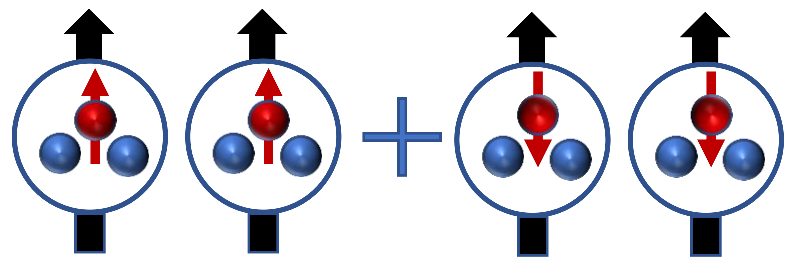
- Comparing to experimental data, BLFQ results show good agreement at small Q^2
- BLFQ results almost satisfy Sudakov FF relation at large Q^2

Unpolarized PDF



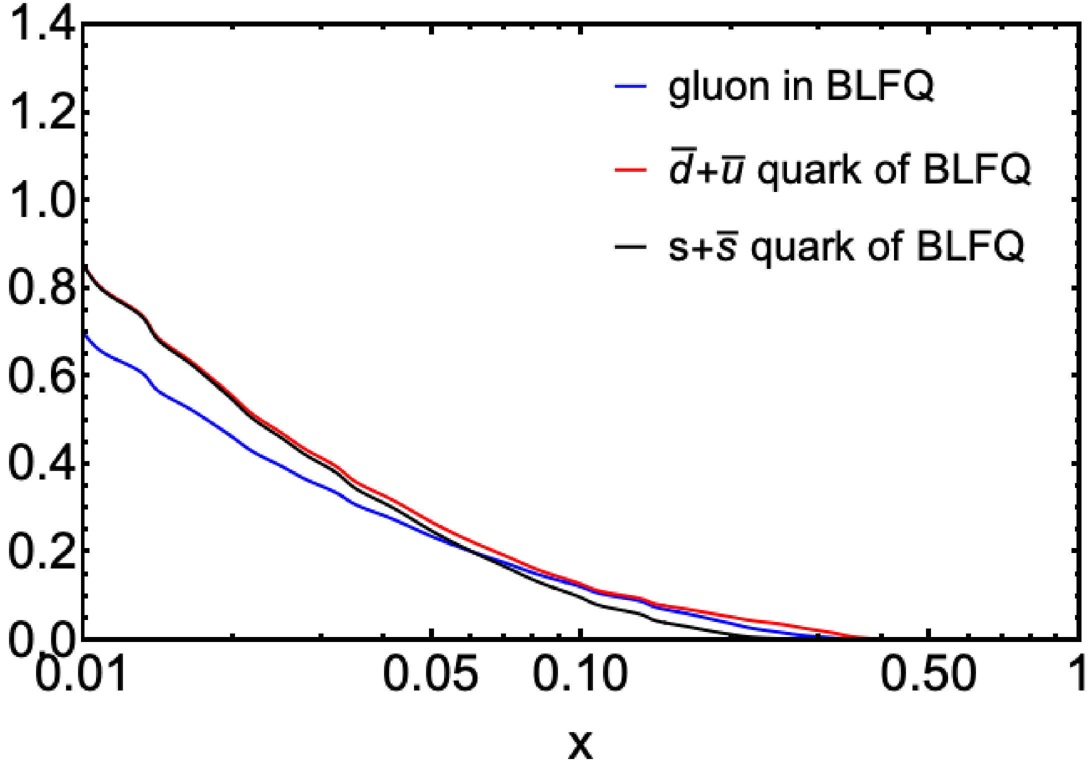


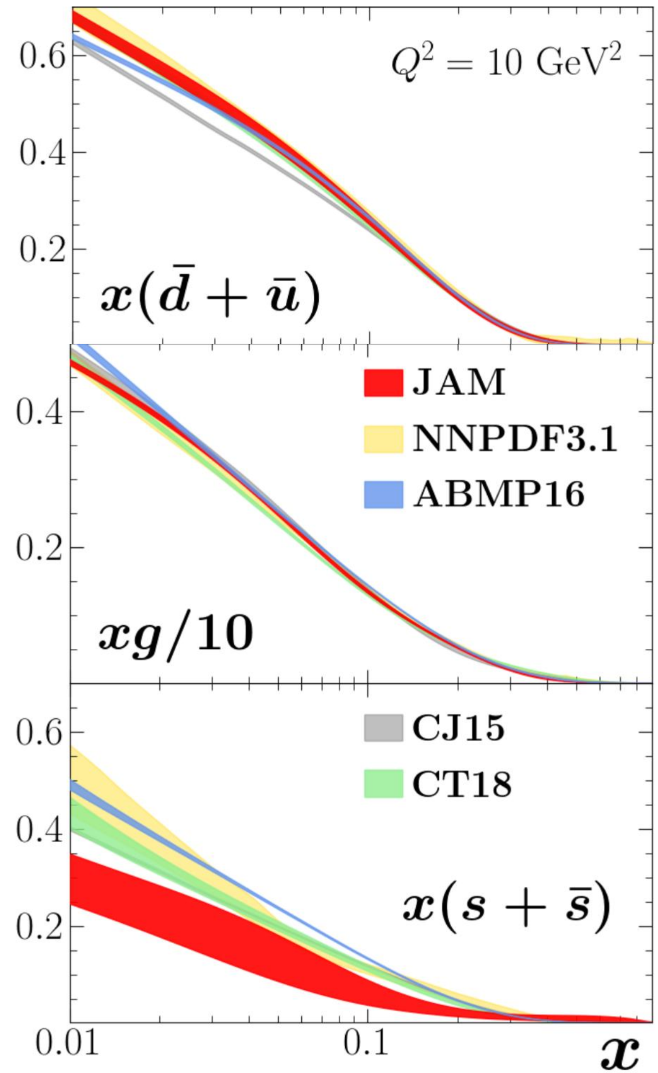
Unpolarized PDFs:



- Particle number density distribution
- Fitting the initial scale by comparing the second moment at experimental scale
- Qualitative agreement with global fitting

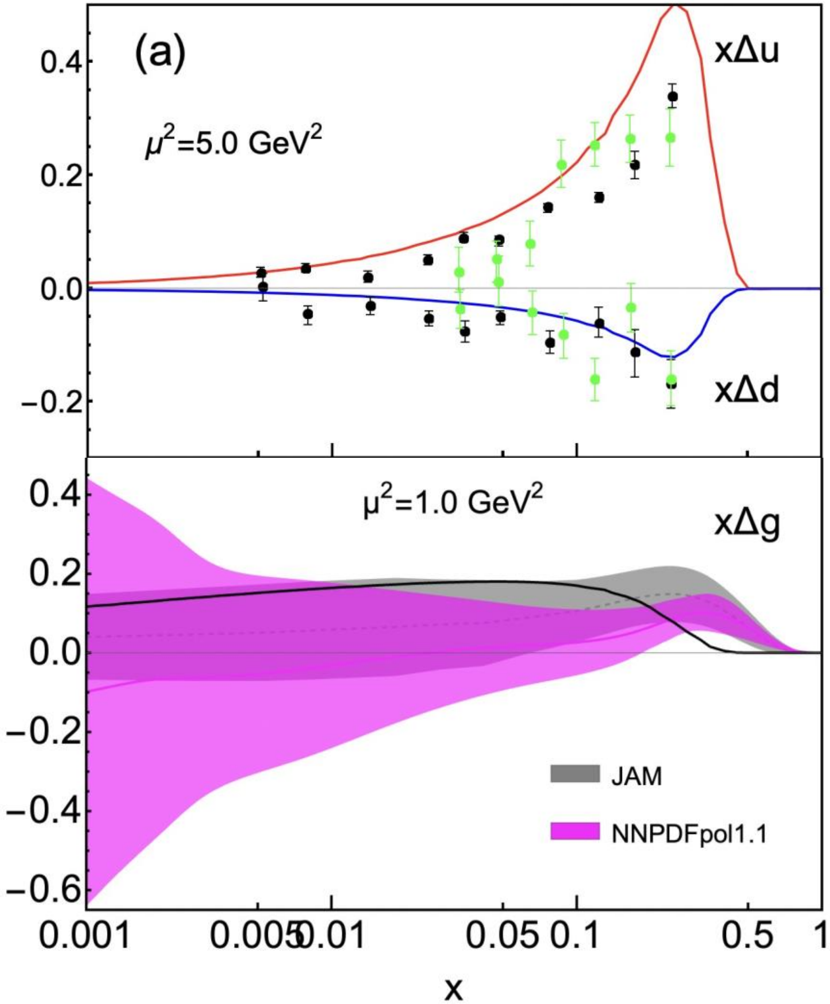
Unpolarized PDF

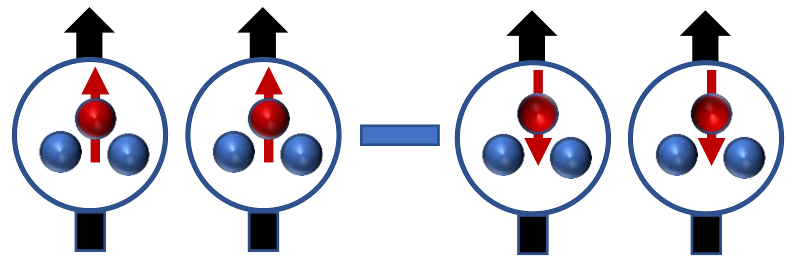




- Gluon and sea quark distribution are calculated
- Qualitative trend with various global fitting results

Helicity PDF





$\mu^2 = 5\ \mathrm{GeV}^2$

$\Delta\Sigma_u = 0.86$ $\Delta\Sigma_d = -0.16$

$\Delta\Sigma_{u-d} = 1.02 \sim 1.27$

$\Delta\Sigma_{u+d} = 0.7 > 0.35$

Extracted data

$$\Delta G = \int_{0.05}^{0.2} dx\, \Delta g(x) = 0.26 \sim 0.251(47)(16)$$

NNPDF

[COMPASS, Phys. Lett. B 693, 227 (2010)]

[HERMES, Phys. Rev. D 71, 012003 (2005)]

[NNPDF, Phys. Rev. Lett. 118, 102001 (2017)]

- Helicity PDFs encode information on spin contributions from parton to hadron
- At $x < 0.2$ region, helicity PDFs of u quark qualitatively agree with global fitting data
- Helicity PDF of d quark has a good agreement with global fitting results

Helicity PDF

- Spin decomposition

[Jaffe and Manohar, (1990)]

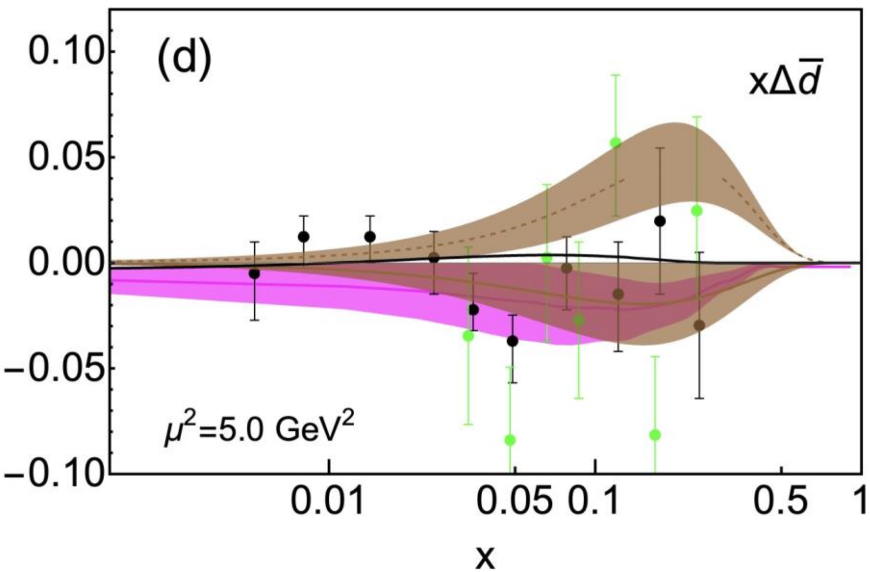
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

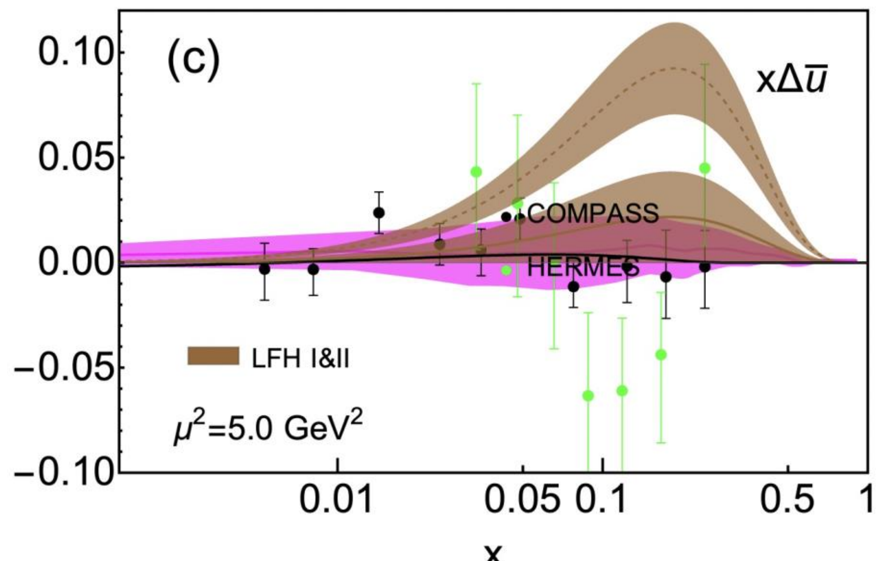
$\mu_0^2 = 0.22\ \mathrm{GeV}^2$

$$\Delta\Sigma_u = 0.86 \qquad \Delta\Sigma_{\bar{\mathrm{d}}} = 0.008$$

$$\Delta\Sigma_d = -0.16 \qquad \Delta\Sigma_{\bar{u}} = 0.008$$

$$\Delta\Sigma_s = \Delta\Sigma_{\bar{s}} = 0.007$$

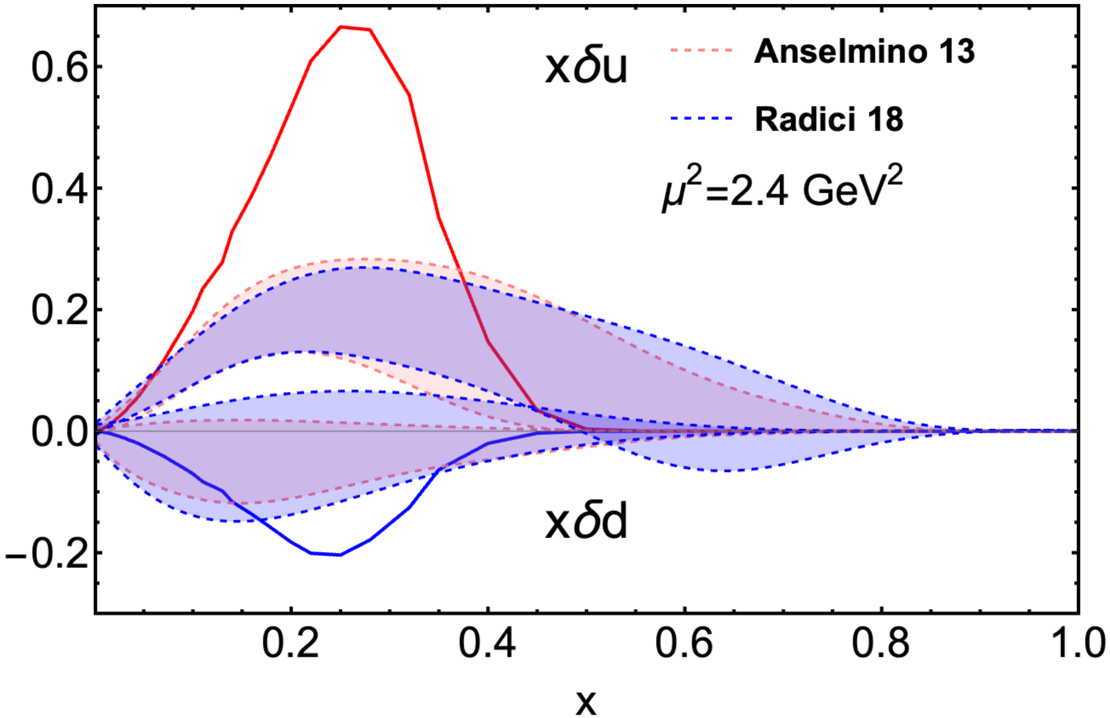


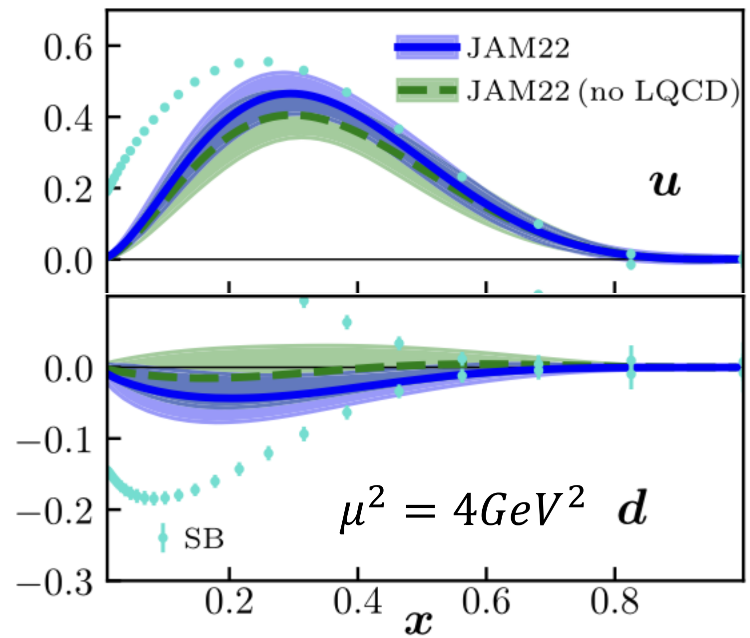


[JAM, Phys. Rev. Lett. 119, 132001 (2017)]

[NNPDF, Nucl. Phys. B 887, 276 (2014)]

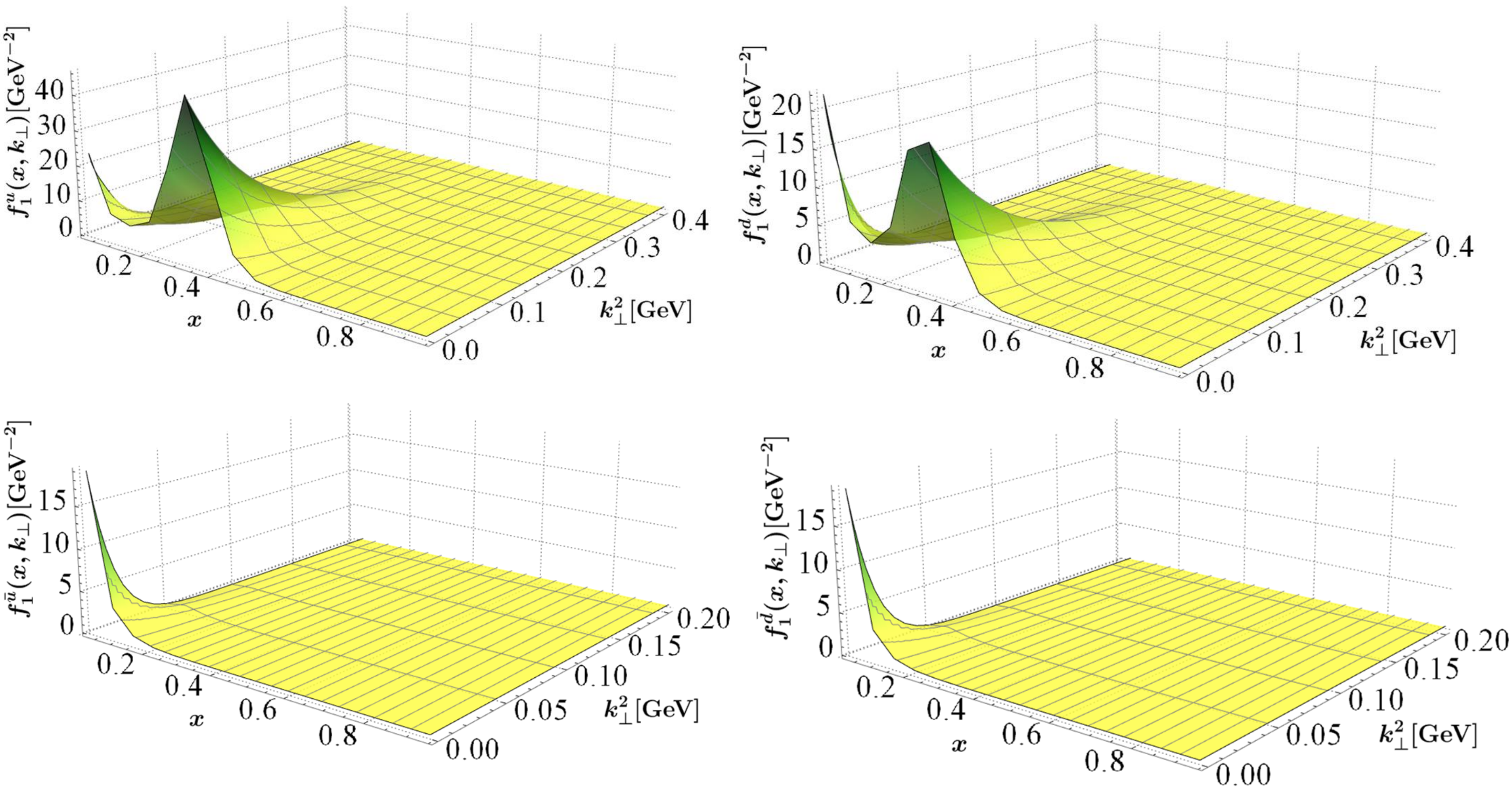
Transversity PDF





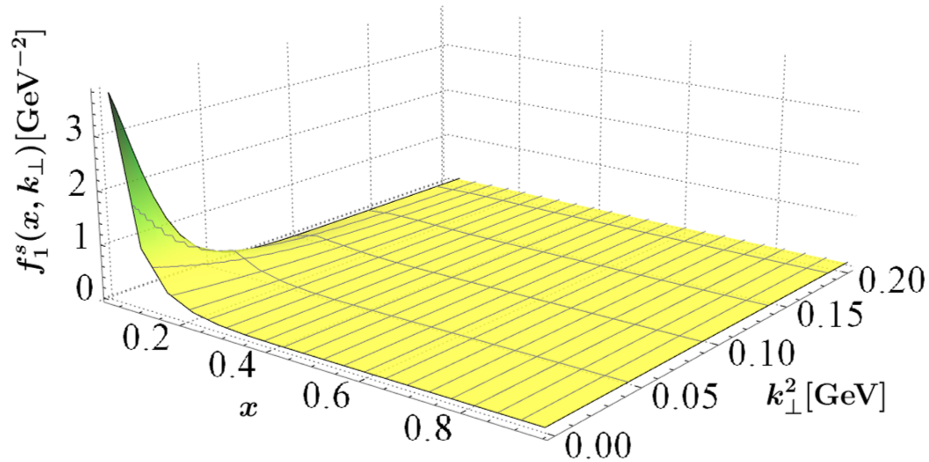
- BLFQ show approximately polarized symmetry between the transverse and longitudinal direction
- BLFQ results more close to the JAM data.
- Down quark's tensor charge agrees with the global fitting data
- Up quark's tensor charge is larger than global fitting data

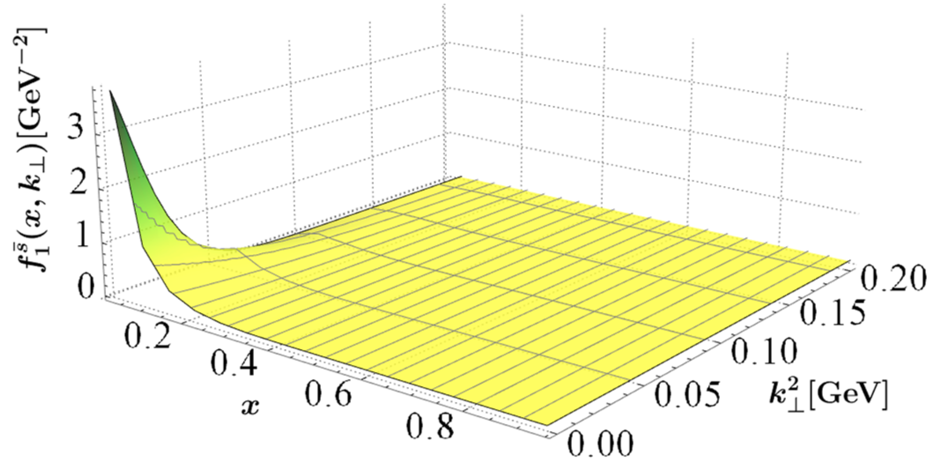
TMDs for Valence and Sea Quarks

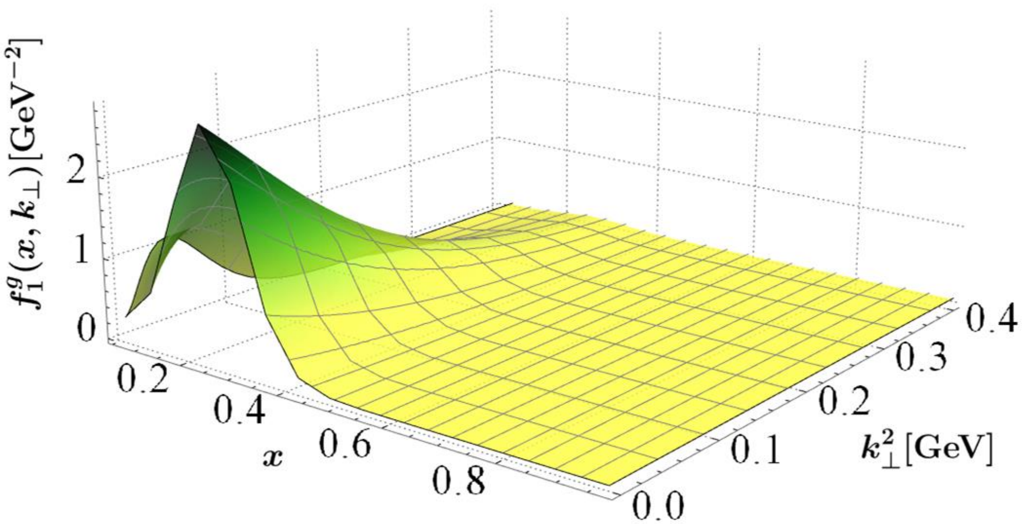


- T-even TMDs
- Gauge link ignored (=1)
- Small x behavior is contributed by $|qqqq\bar{q}\rangle$

TMDs for Sea Quarks and Gluon

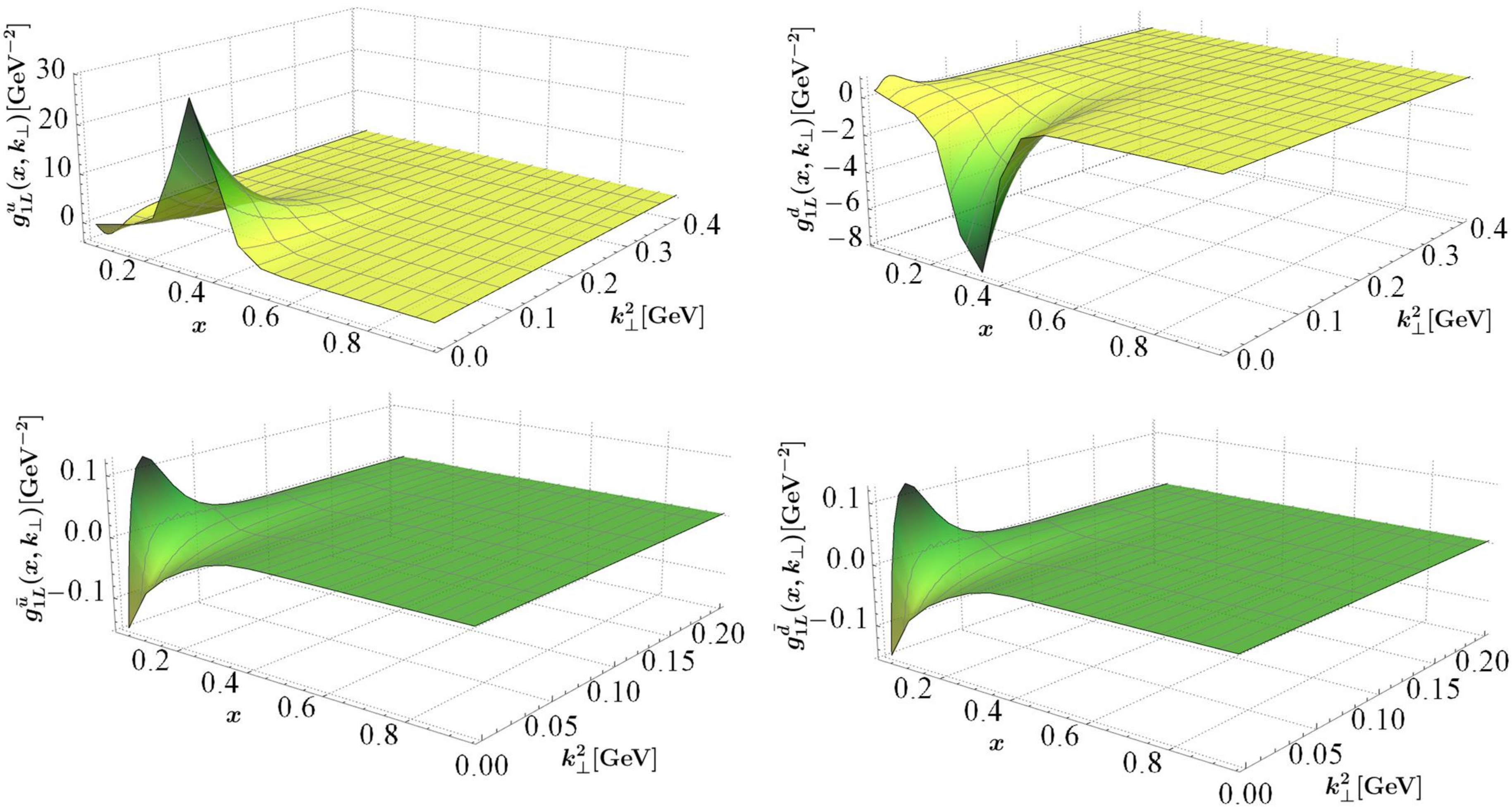






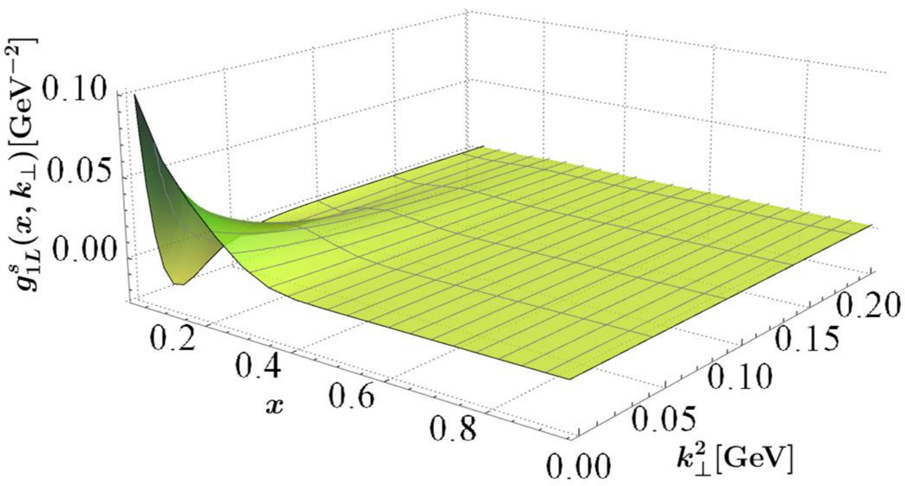
- Strange and anti-strange quarks are peaked in the small-x region

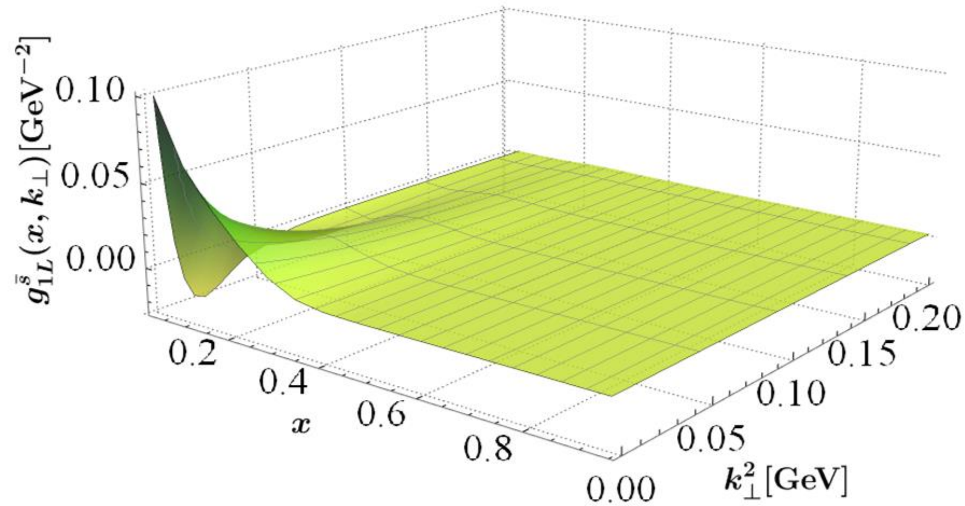
TMDs with Five Particle Fock Sector

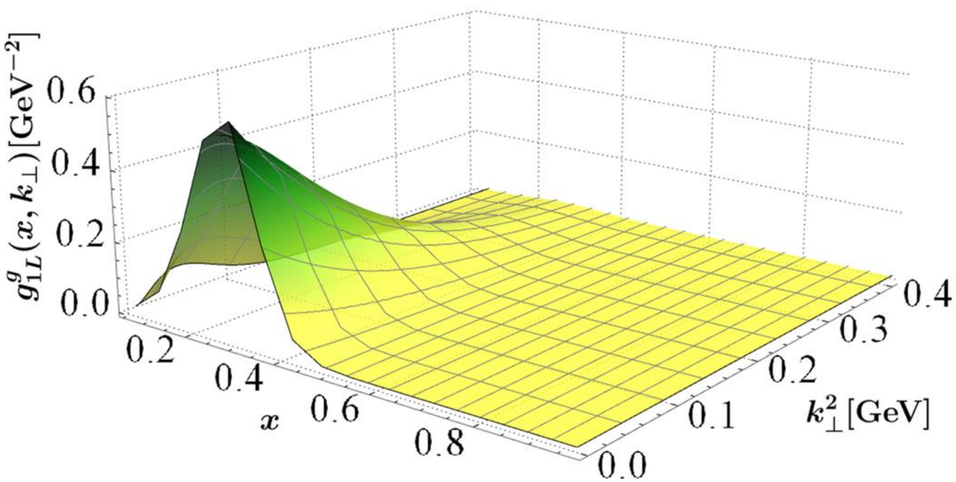


- Up and Down quarks have opposite sign
- Anti-up and anti-down quarks have the same sign

TMDs with Five Particle Fock Sectors

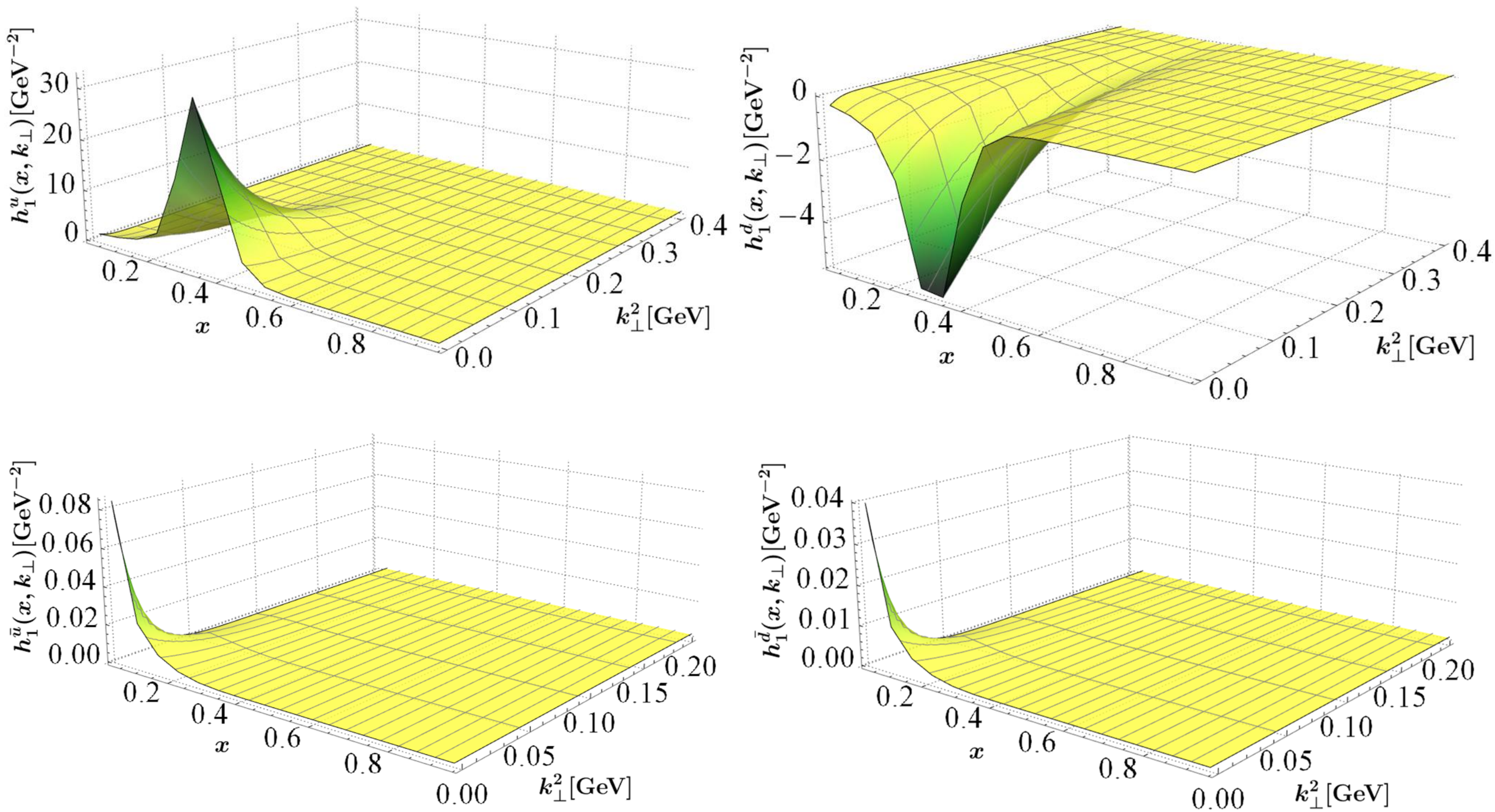






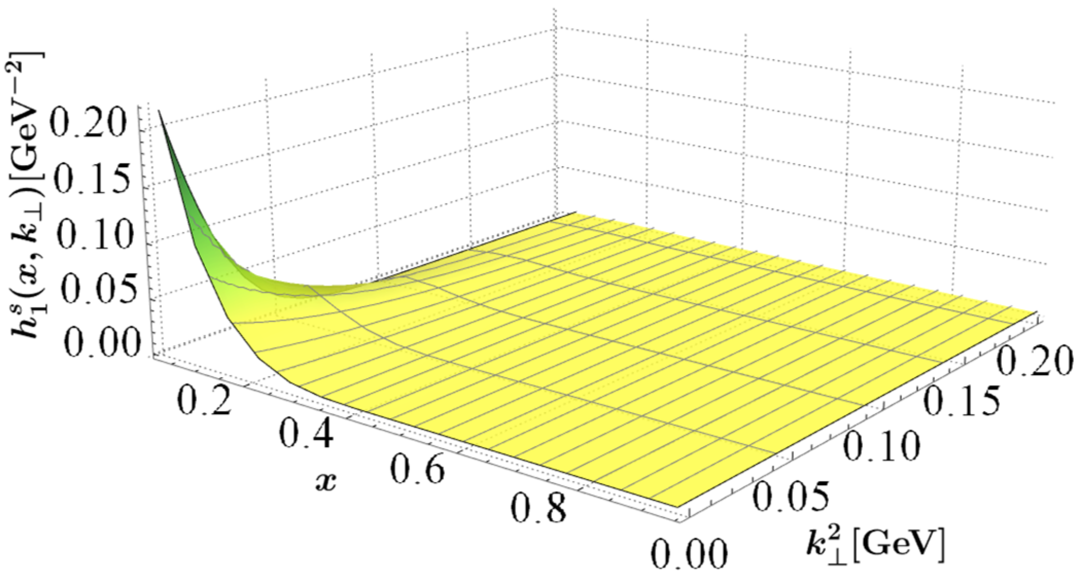
- Strange and anti-strange quark have the same distribution

TMDs with Five Particle Fock Sector

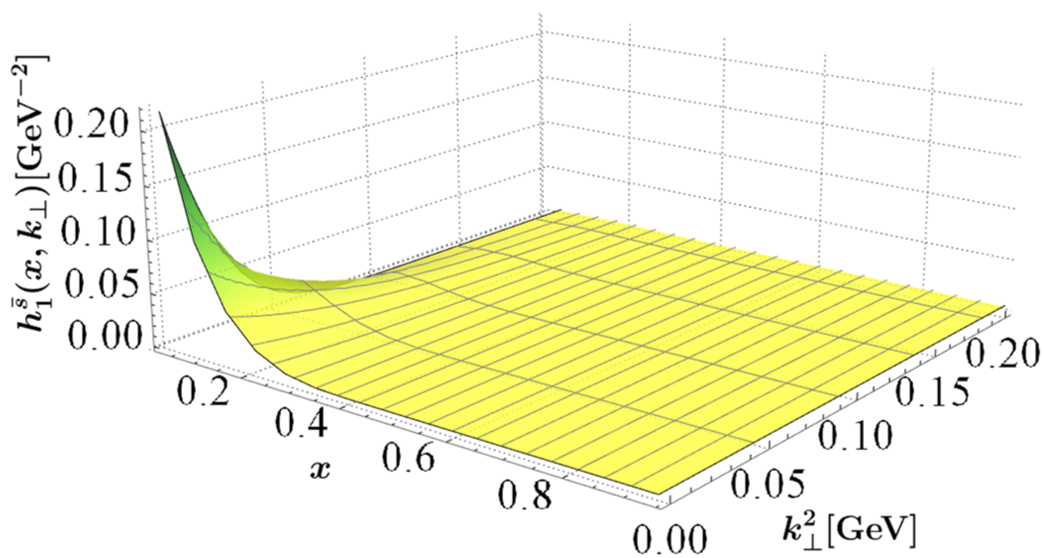


- Up and down quarks have opposite sign
- Anti-up and anti-down quarks have the same sign

TMDs with Five Particle Fock Sector



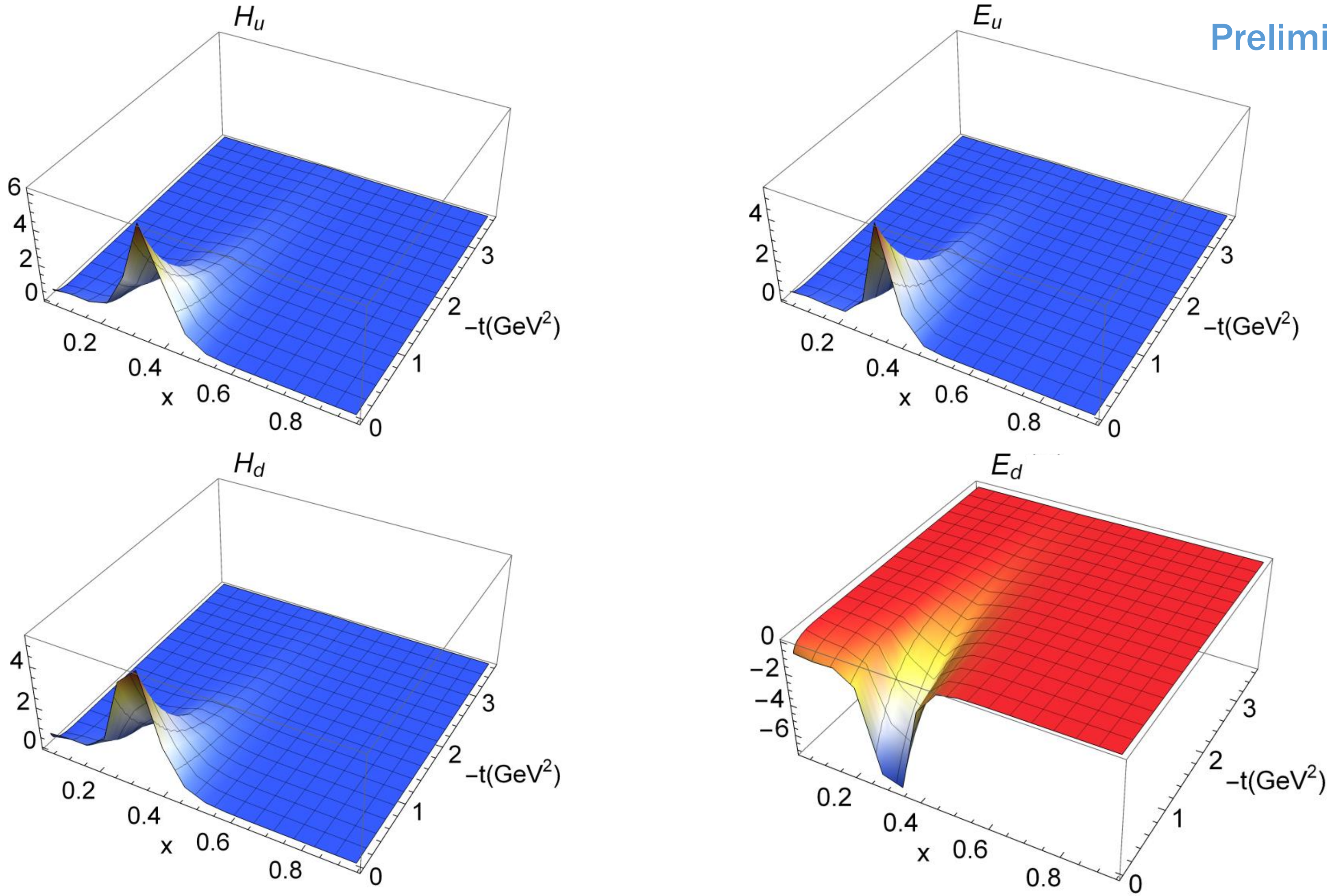
- There is no asymmetry between strange and anti-strange quark in the current Fock space truncation



- Strange and anti-strange quark have the same distribution

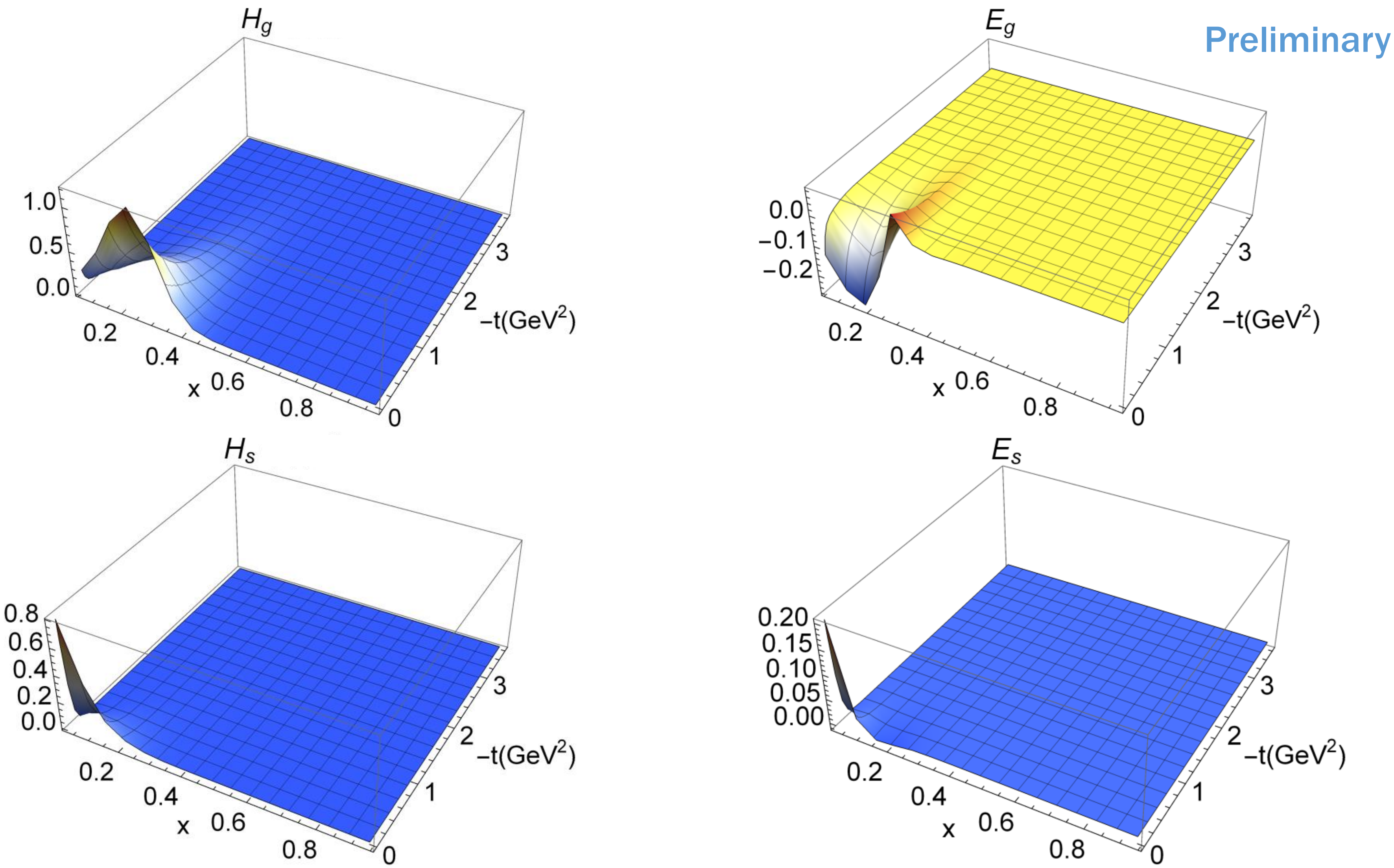
GPDs for Valence Quarks

Preliminary



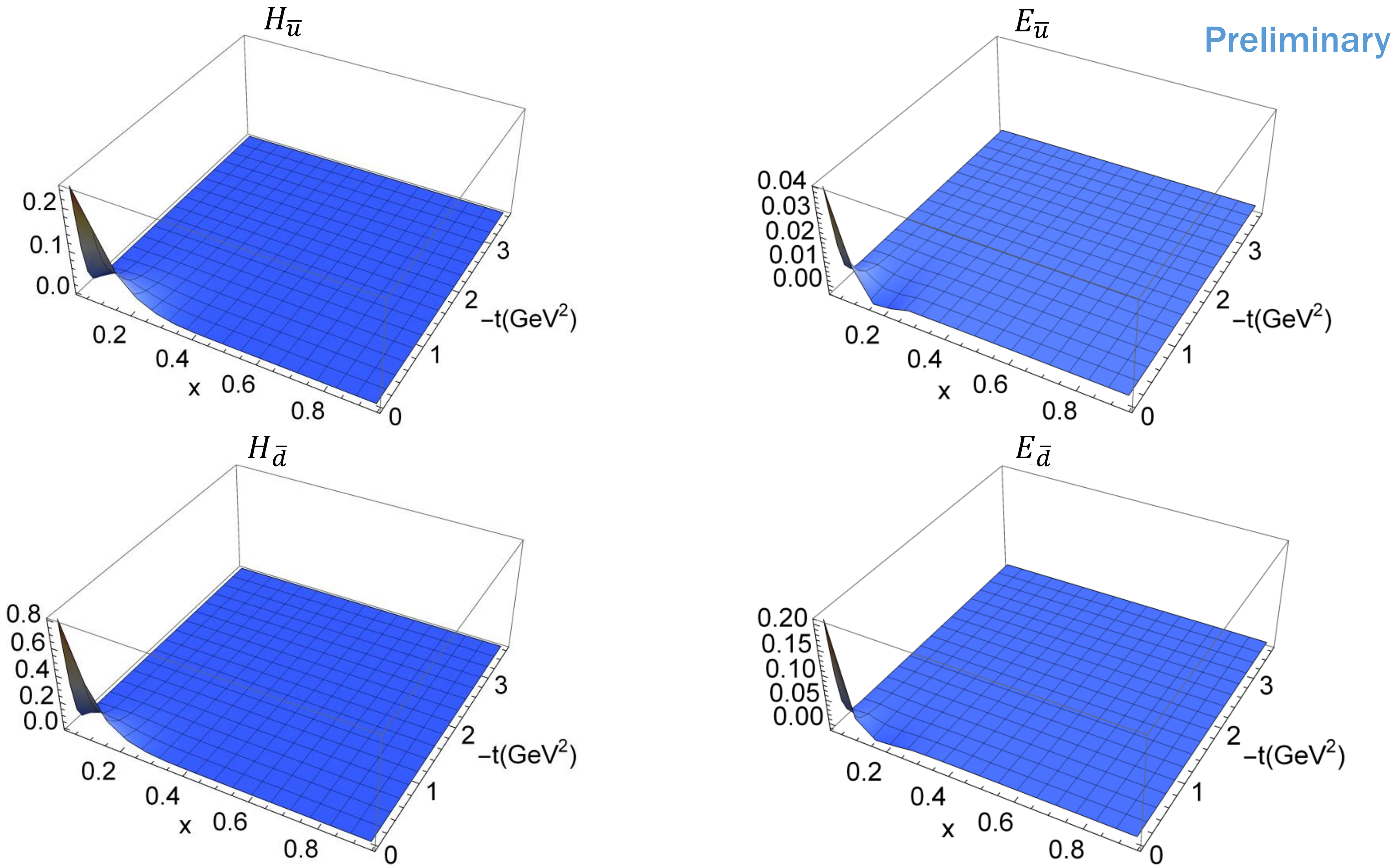
- u and d quark GPDs (contributions from all Fock sectors)
- E_u has positive distribution while E_d is negative, consistent with Dirac FF

GPDs for Gluon and Sea Quarks



- Gluon and s quark GPDs
- E_s mainly contributes at small x region

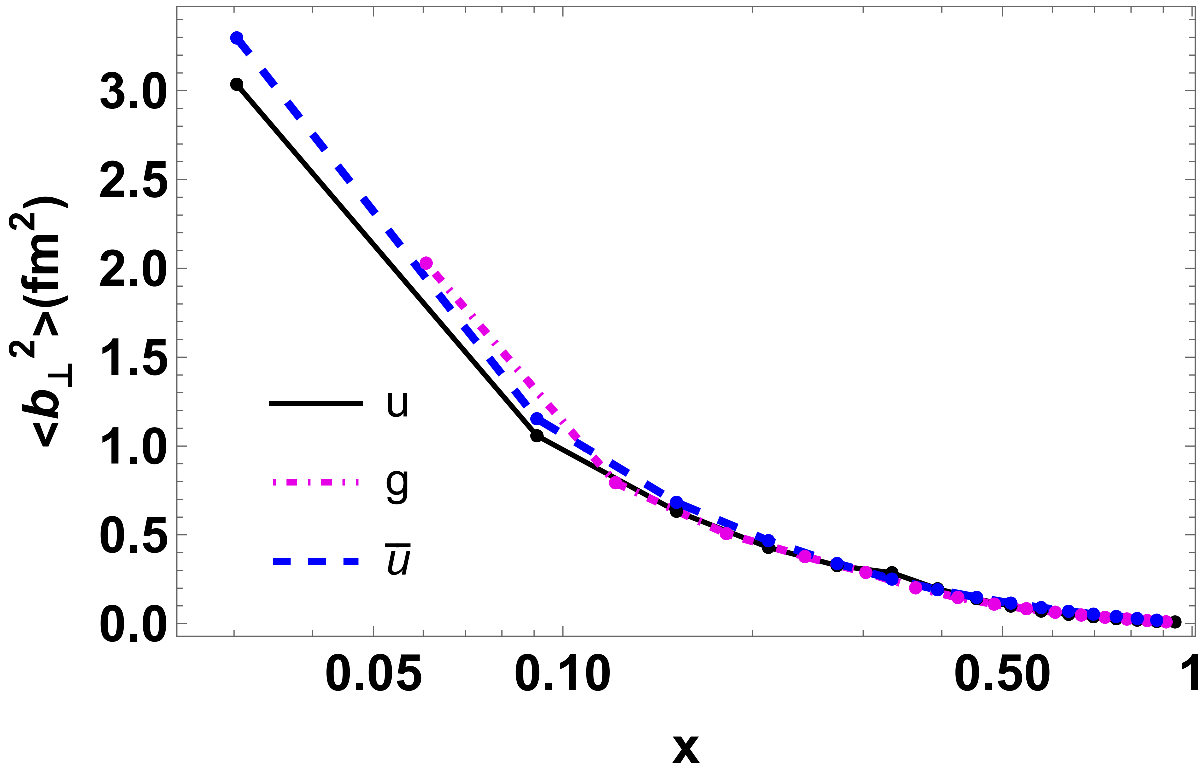
GPDs for Sea Quarks



- $\bar{u}$ and $\bar{d}$ GPDs are concentrated in small-x region
- $\bar{u}$ and $\bar{d}$ GPD E have small negative region around $x\sim0.2$

Transverse Radius

$$\left\langle b_\perp^2 \right\rangle^{q/g}(x) = \frac{\int \mathrm{d}^2 b_\perp \left(b_\perp\right)^2 \mathcal{H}\left(x, 0, b_\perp\right)}{\int \mathrm{d}^2 b_\perp \mathcal{H}\left(x, 0, b_\perp\right)}$$



- Valence quarks occupy core region
- Gluon radius > sea quark radius > valence quark radius
- As $x \to 1$, nucleon behaves like point particle

Conclusion

- Basis Light-front Quantization:
  - Nonperturbative approach to quantum field theory in Hamiltonian formalism
  - Full relativistic effects included
  - Access to 3D structure of hadrons: FFs, PDFs, GPDs, TMDs…
  - Access to parton correlation / higher twist distributions
- Clean input from QCD interactions
  - Correct overall behavior in both longitudinal and transverse directions
  - Almost correct spin structure
  - Insufficient large-x component (possibly due to missing confinement)

OutLook

- First principles calculation of proton structure
  - Full QCD light-front Hamiltonian implemented
  - More higher Fock sectors included
- Excited nucleon states/mesons/light nuclei
- Reaction dynamics through time-dependent light-front Schrödinger equation

Thanks you


LIGHT CONE 2024
Hadron Physics in the EIC era
The Institute of Modern Physics, Chinese Academy of Sciences, Huizhou Campus, China.
November 25-29, 2024
Physics Topics and Tools
» Physics of EIC and EicC
» Hadron spectroscopy and reactions
» Hadron/nuclear structure
» Spin physics
» Relativistic many-body physics
» QCD phase structure
» Light-front field theory
» AdS/CFT and holography
» Nonperturbative QFT methods
» Effective field theories
» Lattice field theories
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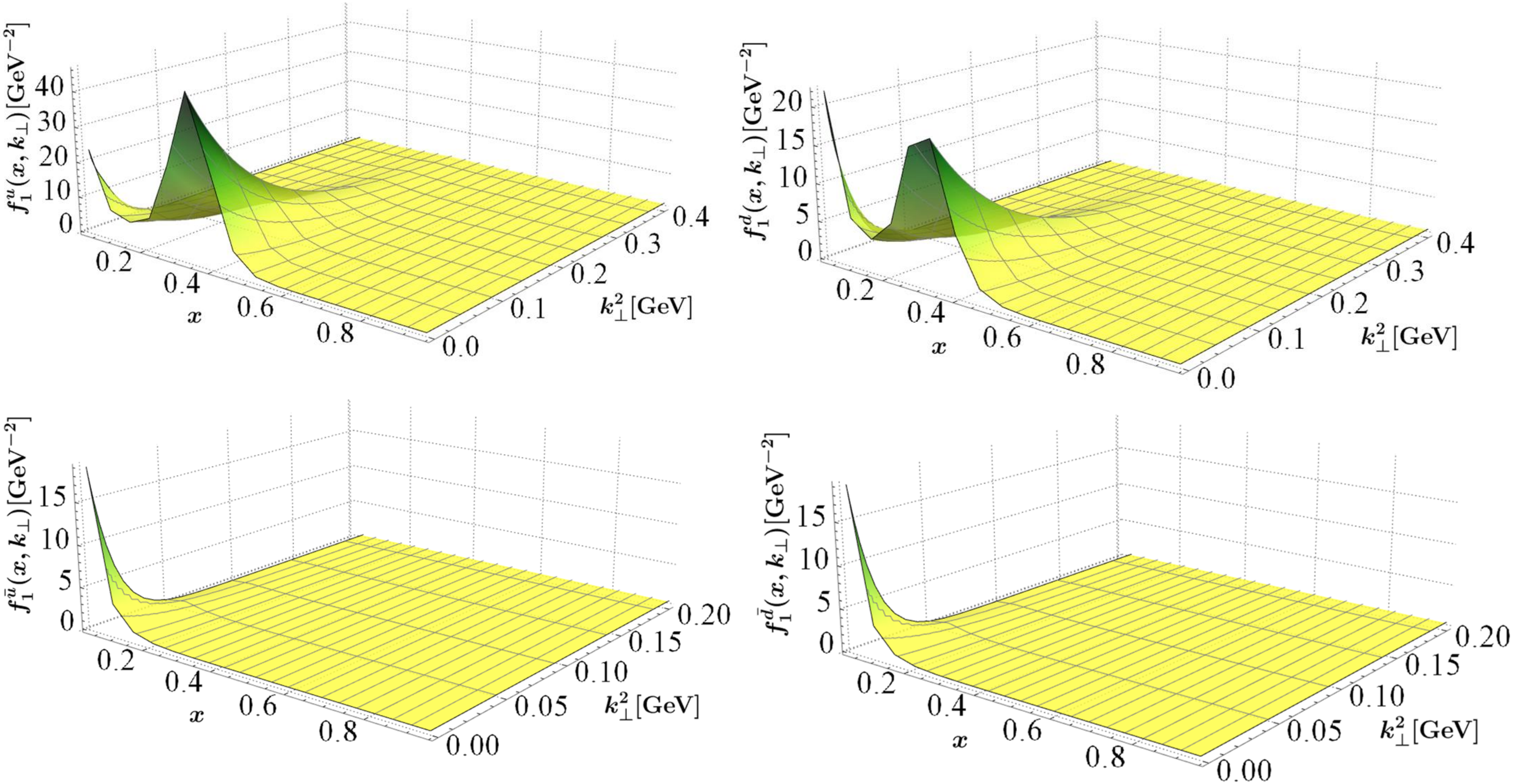
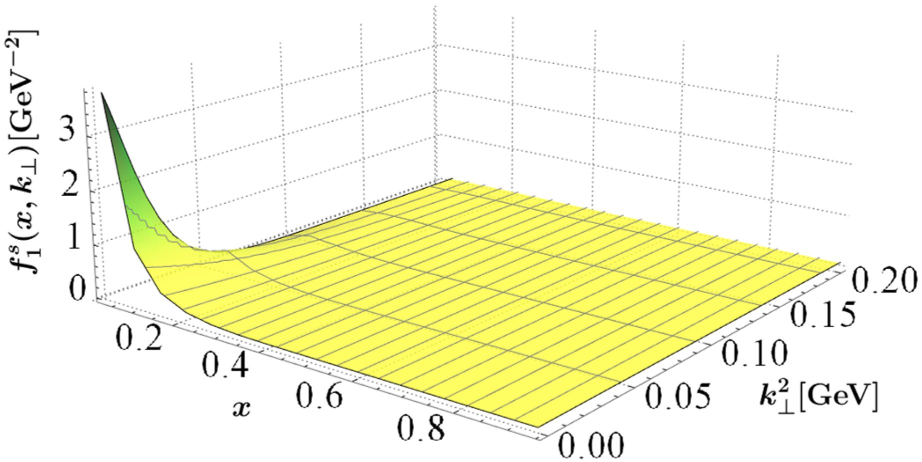
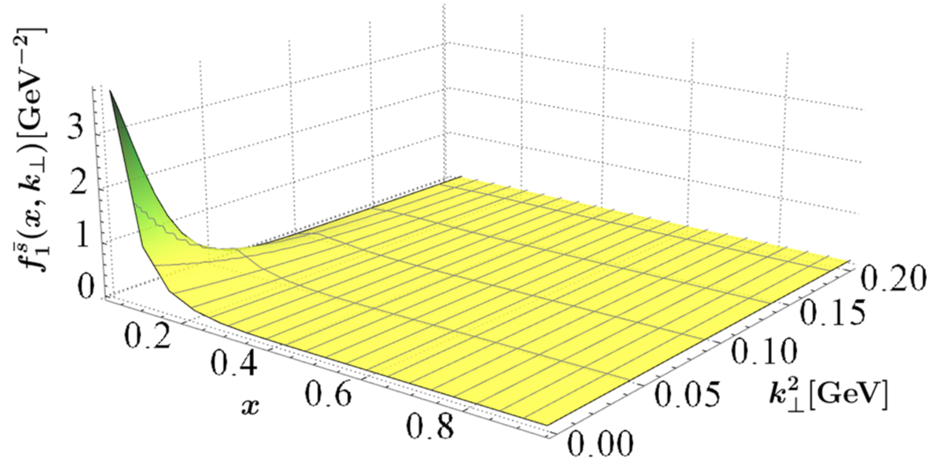
TMDs for Valence and Sea Quarks

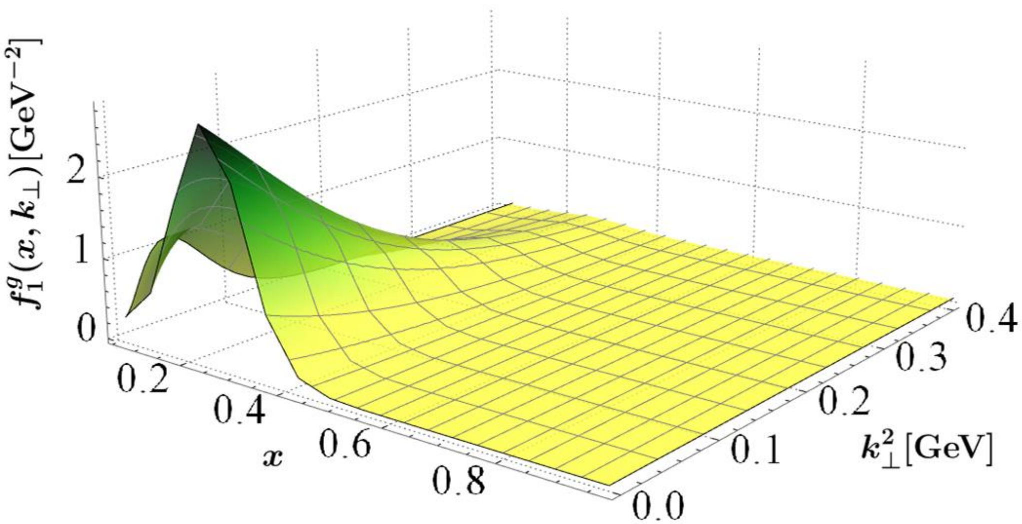


- Small x behavior is contributed by $|qqqq\bar{q}\rangle$

TMDs for Sea Quarks and Gluon

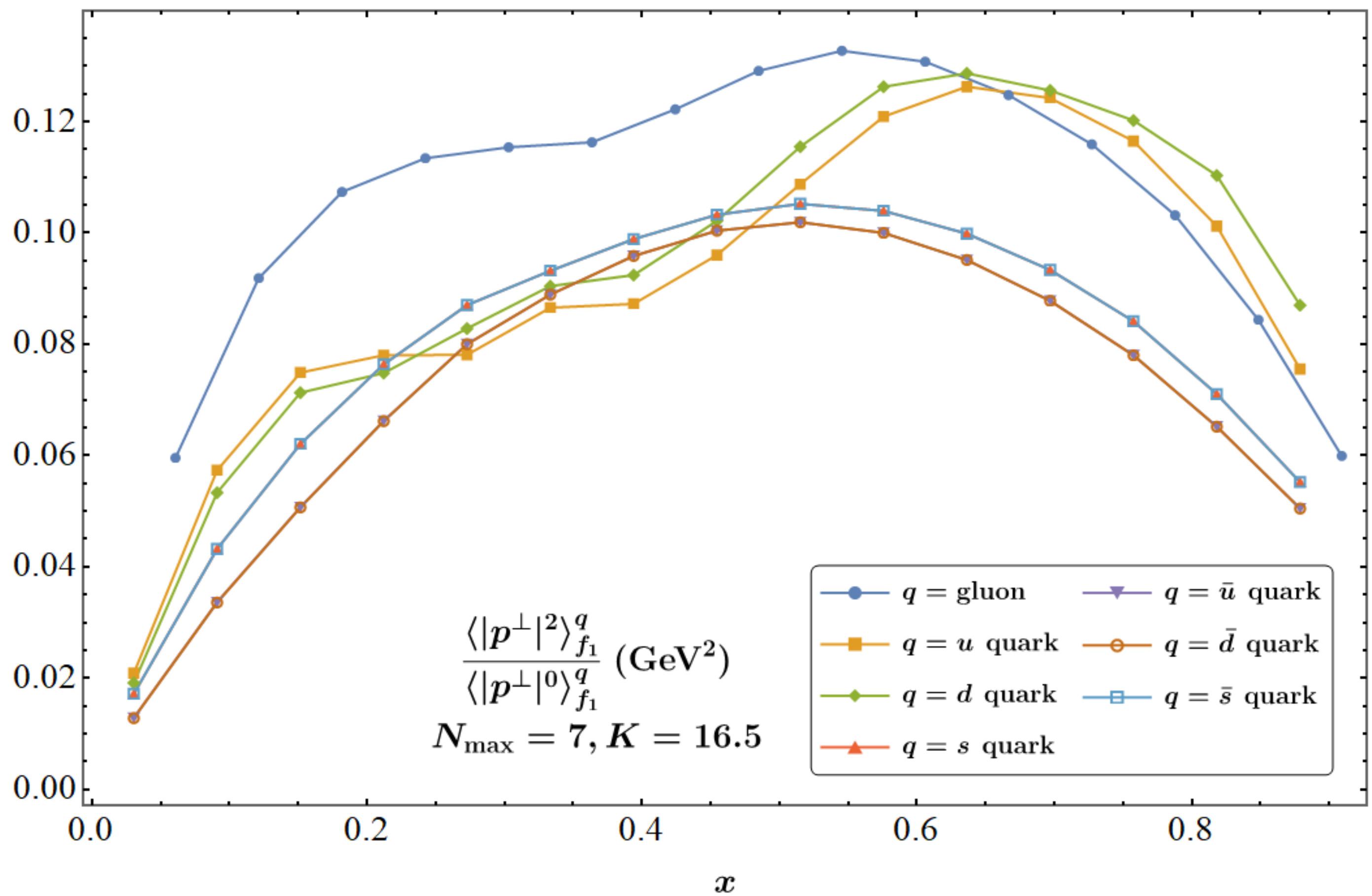






- Strange and anti-strange quarks are peaked in the small-x region

Average Transverse Momentum Distribution



- Total contributions from $|qqq\rangle + |qqq\ g\rangle + |qqqq\bar{q}\rangle$
- In small-x region, average transverse momentum of gluon is larger than quark
- In large-x region, gluon distribution is almost same with quark distributions

Publications on Nucleon GPDs

$$|P,\Lambda\rangle = |qqq\rangle + |qqqg\rangle + |qqq\,u\bar{u}\rangle + |qqq\,d\bar{d}\rangle + |qqq\,s\bar{s}\rangle + \cdots$$

- $|\boldsymbol{qqq}\rangle$:

  [Xu et al., Phys.Rev.D,104.094036] (2021)

  [Liu et al., Phys.Rev.D,105.094018] (2022)

  [Zhang et al., arXiv:2312.00667 [hep-th]] (2023)

  [Kaur et al., Phys. Rev. D 109, 014015] (2024)

- $|\boldsymbol{qqq}\rangle + |\boldsymbol{qqqg}\rangle$:

  [Xu et al., Phys.Rev.D,108 9, 094002] (2023)

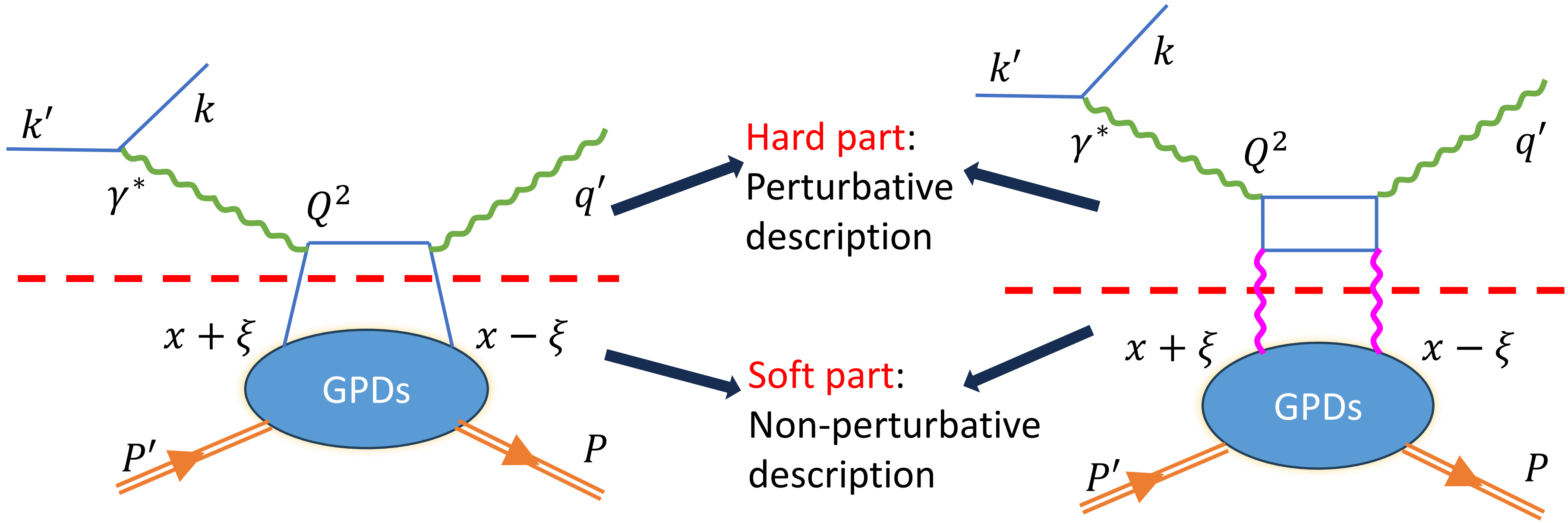
  [Lin et al., Phys.Lett.B,847 138305] (2023)

- $|\boldsymbol{qqq}\rangle + |\boldsymbol{qqqg}\rangle + |\boldsymbol{qqqq\bar{q}}\rangle$:

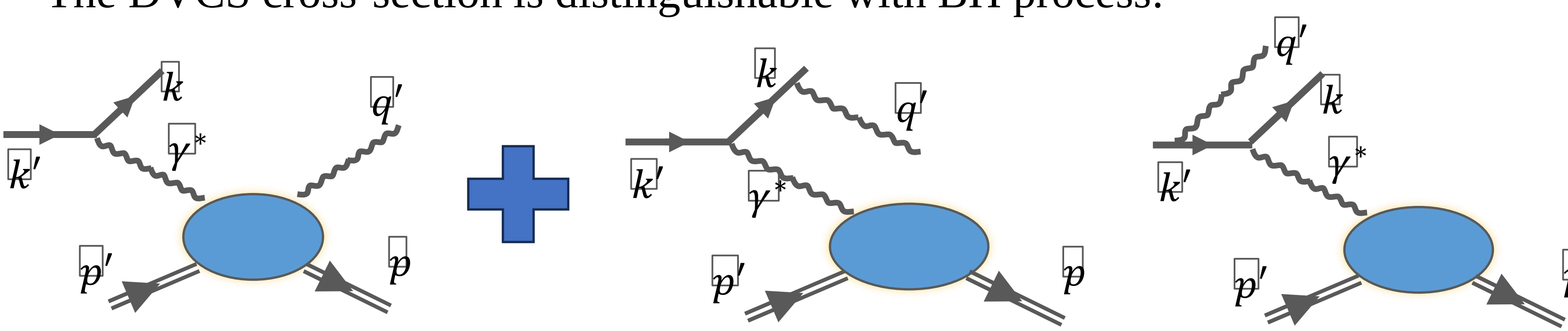
  Work in progress

Deeply Virtual Compton Scattering

- The deeply virtual Compton scattering describes the process: $e + p \rightarrow e + p + \gamma$



- The DVCS cross-section is distinguishable with BH process:



DVCS **Bethe-Heitler**

GPDs and Compton Form Factors

$$
\begin{aligned}
\frac{d^5\sigma_{\mathrm{DVCS}}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} &= \Gamma|T_{\mathrm{DVCS}}|^2 \\
&= \frac{\Gamma}{Q^2(1-\epsilon)}\Big\{F_{UU,T} + \epsilon F_{UU,L} + \epsilon\cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{\epsilon(\epsilon+1)}\cos\phi F_{UU}^{\cos\phi} + (2h)\sqrt{2\epsilon(1-\epsilon)}\sin\phi F_{LU}^{\sin\phi} \\
&+ (2\Lambda)\Big[\sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi} + \epsilon\sin 2\phi F_{UL}^{\sin 2\phi} + (2h)\Big(\sqrt{1-\epsilon^2}F_{LL} + 2\sqrt{\epsilon(1-\epsilon)}\cos\phi F_{LL}^{\cos\phi}\Big)\Big] \\
&+ (2\Lambda_T)\Big[\sin(\phi-\phi_S)(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}) + \epsilon\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)} + \epsilon\sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)} \\
&+ \sqrt{2\epsilon(1+\epsilon)}(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)})\Big] \\
&+ (2h)(2\Lambda_T)\Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_S F_{LT}^{\cos\phi_S} \\
&+ \sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\Big]\Big\}.
\end{aligned}
$$

Cross-section ⟷ Structure functions

GPDs and Compton Form Factors

➢ Cross sections to Compton form factors

$$|T|^2 = |T_{BH} + T_{DVCS}|^2 = |T_{BH}|^2 + |T_{DVCS}|^2 + T_{BH}^* T_{DVCS} + T_{DVCS}^* T_{BH}$$

Cross-section ⟷ Structure functions ⟷ Compton form factors

➢ GPDs to Compton form factors

CFFs ⟵

$$\mathcal{F}(\xi,-t) = \int_{-1}^{1} dx \left(\frac{1}{x-\xi-i\epsilon} - \frac{1}{x+\xi-i\epsilon} \right) F(x,\xi,-t)$$

$$\tilde{\mathcal{F}}(\xi,-t) = \int_{-1}^{1} dx \left(\frac{1}{x-\xi-i\epsilon} + \frac{1}{x+\xi-i\epsilon} \right) \tilde{F}(x,\xi,-t)$$

⟵ GPDs

- **Compton form factors are integrations of GPDs over x**
- **It is challenging to extract GPDs from CFFs**
- **Exploring GPDs from theory is interesting**

Semi-Inclusive DIS

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}\psi\mathrm{d}^2p_{h\perp}} = \frac{\alpha_{em}^2}{xyQ^2}\left(1+\frac{\gamma^2}{2x}\right)\Big(\mathcal{F}_{UU}+\lambda_l\mathcal{F}_{LU}+\lambda\mathcal{F}_{UL}+\lambda_l\lambda\mathcal{F}_{LL}+S_\perp\mathcal{F}_{UT}+\lambda_l S_\perp\mathcal{F}_{LT}\Big),$$

$$\mathcal{F}_{UU} = \frac{y^2}{1-\varepsilon}\Big(F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\varepsilon(1+\varepsilon)}F_{UU}^{\cos\phi_h}\cos\phi_h+\varepsilon F_{UU}^{\cos 2\phi_h}\cos 2\phi_h\Big),$$

$$\mathcal{F}_{UL} = \frac{y^2}{1-\varepsilon}\Big(\sqrt{2\varepsilon(1+\varepsilon)}F_{UL}^{\sin\phi_h}\sin\phi_h+\varepsilon F_{UL}^{\sin 2\phi_h}\sin 2\phi_h\Big),$$

$$\mathcal{F}_{LU} = \frac{y^2}{1-\varepsilon}\sqrt{2\varepsilon(1-\varepsilon)}F_{LU}^{\sin\phi_h}\sin\phi_h,$$

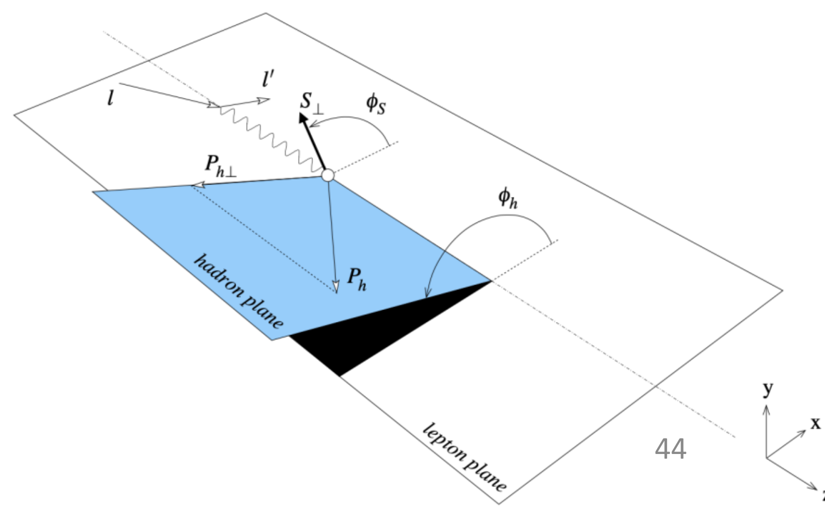
$$\mathcal{F}_{LL} = \frac{y^2}{1-\varepsilon}\Big(\sqrt{1-\varepsilon^2}F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}F_{LL}^{\cos\phi_h}\cos\phi_h\Big),$$

$$\begin{aligned}\mathcal{F}_{UT} = \frac{y^2}{1-\varepsilon}\Big[&\sqrt{2\varepsilon(1+\varepsilon)}F_{UT}^{\sin\phi_S}\sin\phi_S+\big(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\big)\sin(\phi_h-\phi_S)\\ &+\varepsilon F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S)+\sqrt{2\varepsilon(1+\varepsilon)}F_{UT}^{\sin(2\phi_h-\phi_S)}\sin(2\phi_h-\phi_S)+\varepsilon F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S)\Big],\end{aligned}$$

$$\begin{aligned}\mathcal{F}_{LT} = \frac{y^2}{1-\varepsilon}\Big[&\sqrt{2\varepsilon(1-\varepsilon)}F_{LT}^{\cos\phi_S}\cos\phi_S+\sqrt{1-\varepsilon^2}F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S)\\ &+\sqrt{2\varepsilon(1-\varepsilon)}F_{LT}^{\cos(2\phi_h-\phi_S)}\cos(2\phi_h-\phi_S)\Big],\end{aligned}$$

$$\varepsilon = \frac{1-y-\frac{1}{4}\gamma^2y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2y^2},$$

$$x=\frac{Q^2}{2\,P\cdot q}\qquad y=\frac{P\cdot q}{P\cdot l}\quad z=\frac{P\cdot P_h}{P\cdot q}\qquad \gamma=\frac{2Mx}{Q}$$



Spin Asymmetry

- Sivers Asymmetry:

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \langle \sin(\phi_h - \phi_S) \rangle_{UT} = \frac{F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}}{2(F_{UU,T} + \varepsilon F_{UU,L})},$$

- Collins Asymmetry:

$$A_{UT}^{\sin(\phi_h+\phi_S)} = \langle \sin(\phi_h + \phi_S) \rangle_{UT} = \frac{\varepsilon F_{UT}^{\sin(\phi_h+\phi_S)}}{2(F_{UU,T} + \varepsilon F_{UU,L})}.$$

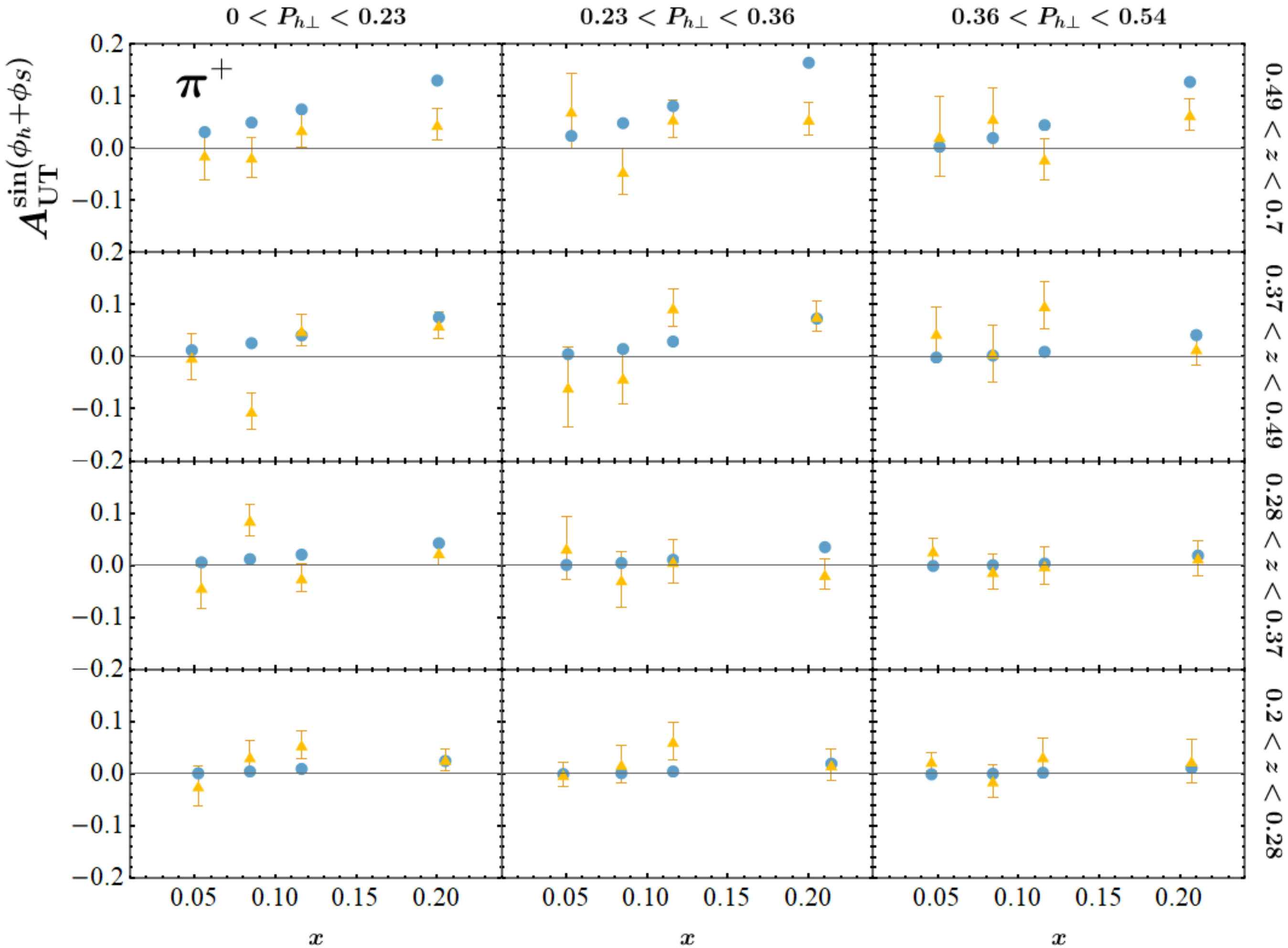
- Structure functions:

$$F_{UU,T} = \mathcal{C}\left[f_1 D_1\right] \quad F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right] \quad F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right]$$

$$F_{UU,L} = 0, \qquad F_{UT,L}^{\sin(\phi_h-\phi_S)} = 0,$$

$$\mathcal{C}\left[w\, f\, D\right] = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\right) w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2),$$

Spin Asymmetry



experiment data is HERMES 2020

$$A_{UT}^{\sin(\phi_h+\phi_S)} \propto \frac{\mathcal{C}[\omega_3 h_1 H_1^{\perp}]}{\mathcal{C}[f_1 D_1]} \qquad \omega_3 = -\frac{\hat{h}\cdot\vec{k}_T}{M}$$

M is the mass of nucleon

M_h is the mass of Hadron

Light-Front Hamiltonian

$$|P_{baryon}\rangle = \Psi_1|qqq\rangle + \Psi_2|qqqg\rangle$$

$$P^- = H_{K.E.} + H_{trans} + H_{longi} + H_{Interact}$$

$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

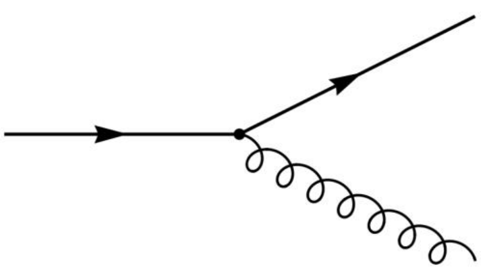
$$H_{trans} \sim \kappa_T^4 r^2$$

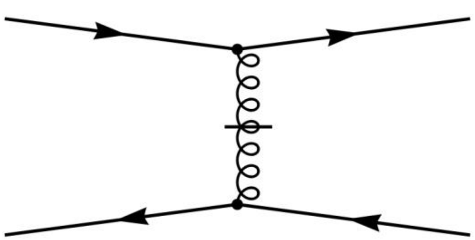
-- Brodsky, Teramond arXiv: 1203.4025

$$H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i}\left(x_i x_j \partial_{x_j}\right)$$

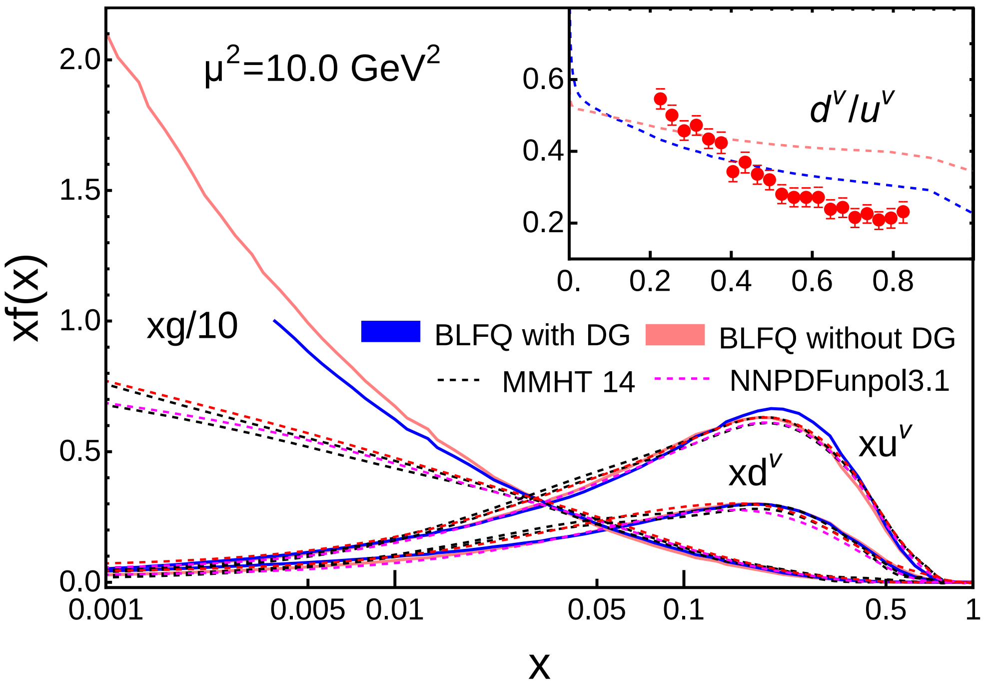
---Y Li, X Zhao , P Maris , J Vary, PLB 758(2016)

$$H_{Interact} = H_{Vertex} + H_{inst} = g\bar{\psi}\,\gamma^\mu T^a\,\psi\,A_\mu^a + \frac{g^2 C_F}{2}\,j^+\frac{1}{(i\partial^+)^2}j^+$$





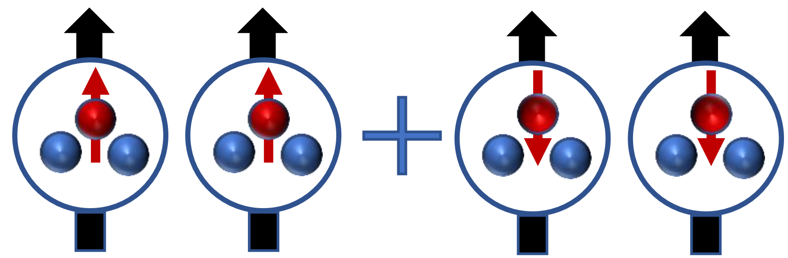
Unpolarized Parton Distribution Functions



$\mu_0^2 = 0.24 \pm 0.01\ \mathrm{GeV}^2$

$\mu_0^2 = 0.19 \pm 0.02\ \mathrm{GeV}^2$

Unpolarized PDFs:



- Interpret as particle number density distribution
- The data point are extracted from MARATHON data

Including the One Dynamical Gluon Fock Sector, the gluon distribution is closer to the global fit.

S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].

[EPJC 77 (2017) 663]

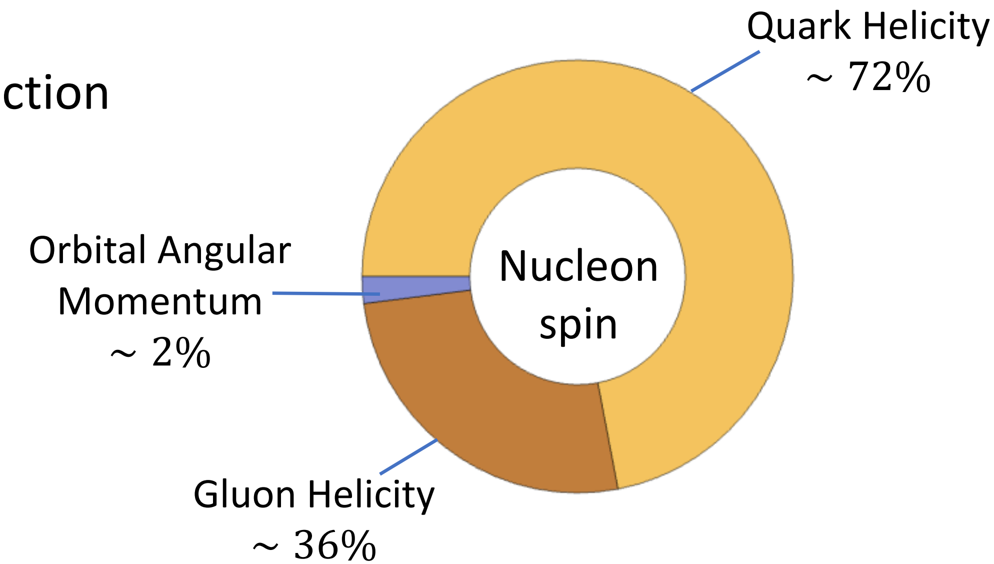
Nucleon Spin with BLFQ

- Obtain observables from wave function

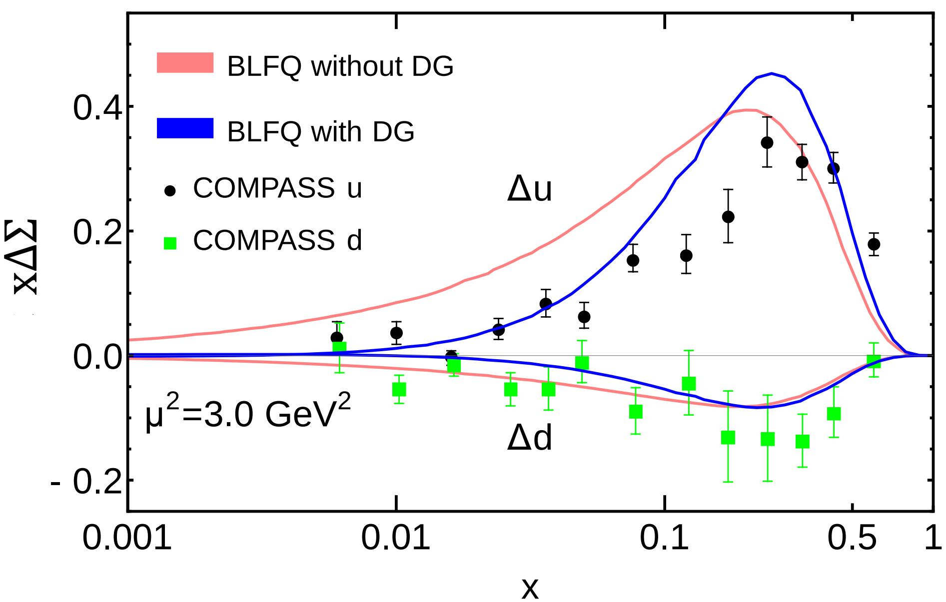
$$O \equiv \langle \beta', \Lambda' | \hat{O} | \beta, \Lambda \rangle$$

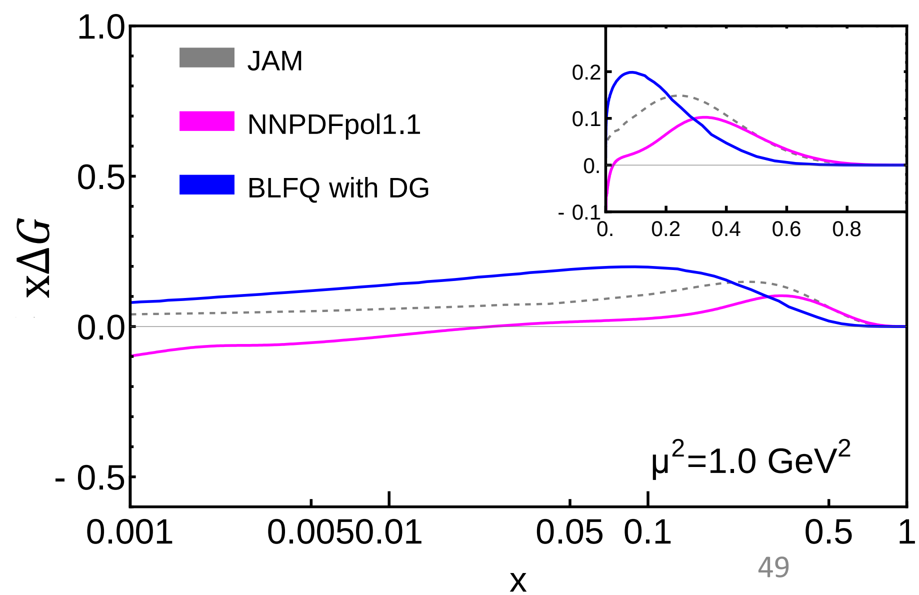
- Spin decomposition in BLFQ

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

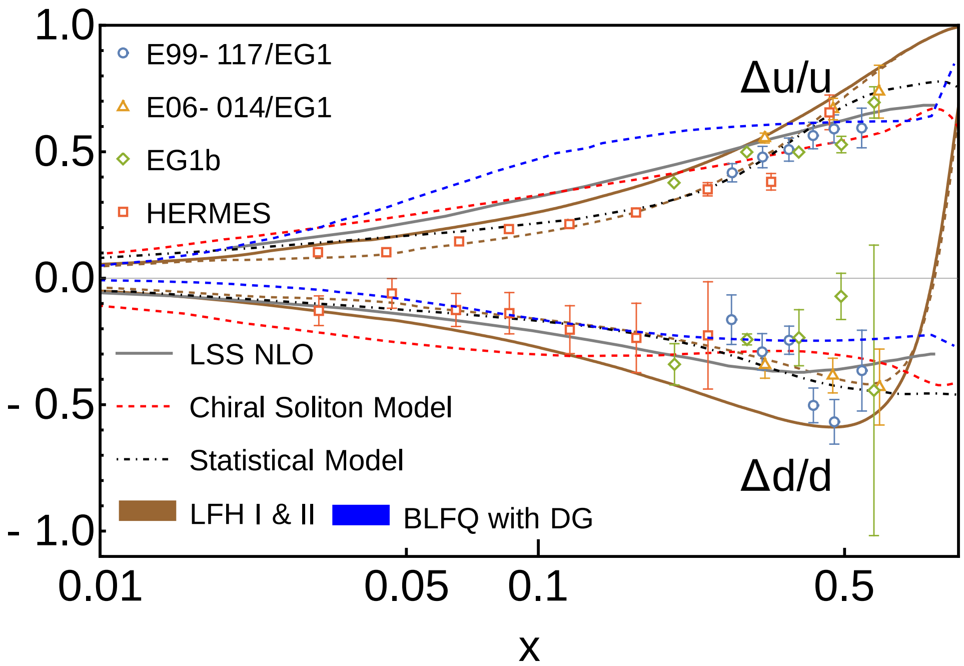


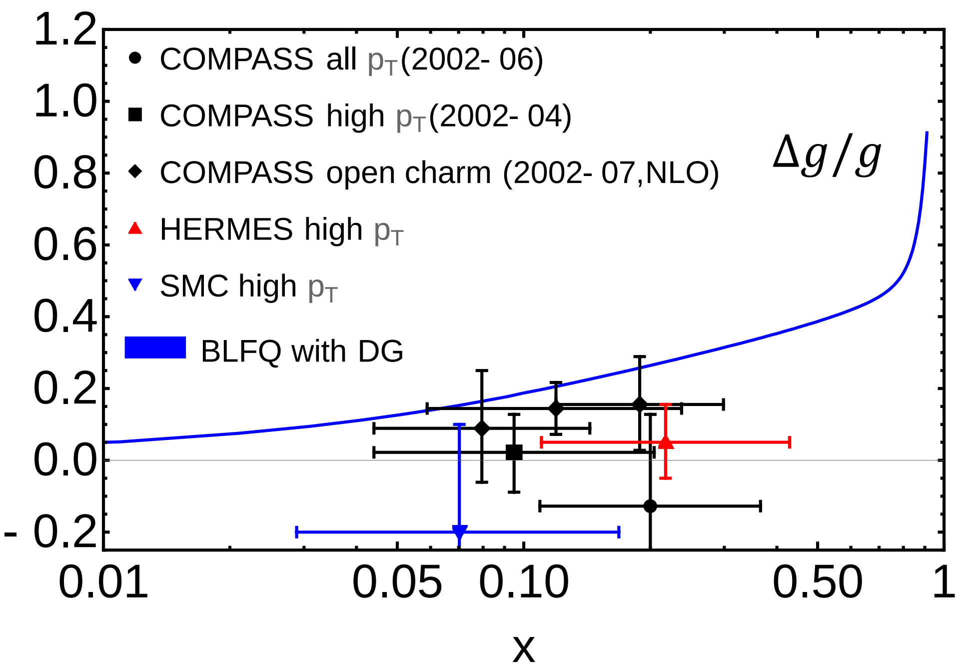
S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].





Helicity Parton Distribution Functions





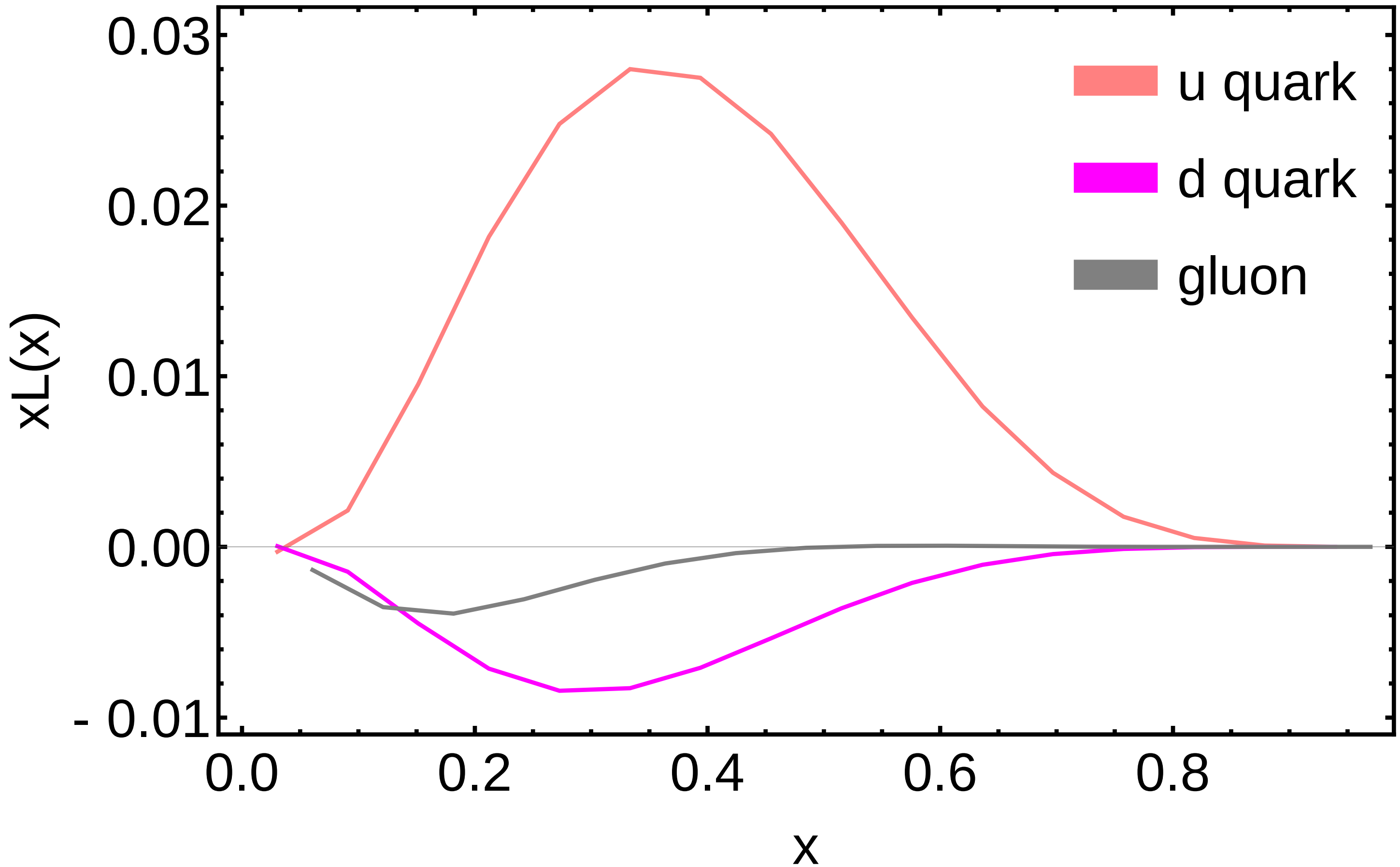
- $\Delta G = \int_0^1 dx \Delta g(x) = 0.131 \pm 0.003$, is sizeable to the proton spin.
- PHENIX Collaboration: $\Delta G^{[0.02,0.3]} = 0.2 \pm 0.1$ PRL 103 (2009) 012003

The sea quarks' contributions come from the DGLAP evolution

S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].

N. Sato et al. [JAM], PRD93 (2016); E. R. Nocera et al. [NNPDF], NPB 887 (2014).

Orbital angular momentum distributions

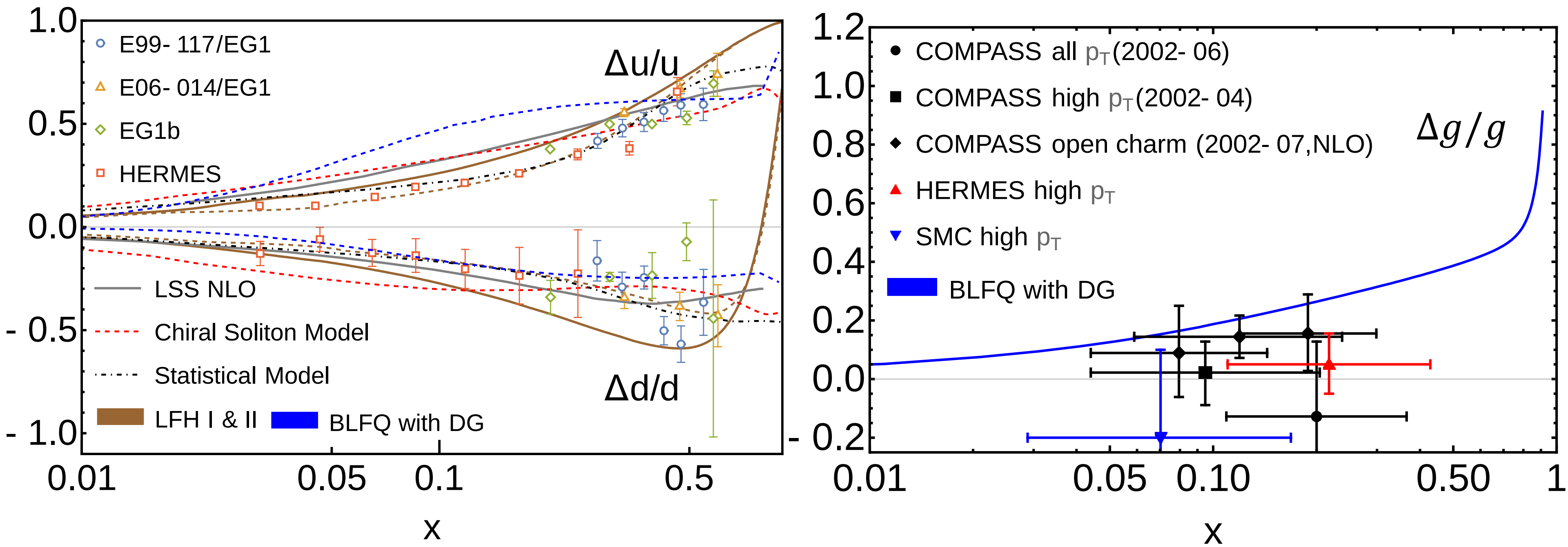


Canonical: $\ell_d = -0.0114 \pm 0.0004$ $\ell_u = 0.0327 \pm 0.0013$ $\ell_g = -0.0065 \pm 0.0005$

At the LC gauge : $\frac{1}{2}\Delta\Sigma = 0.359 \pm 0.002$ $\Delta G = 0.131 \pm 0.003$

[In preparation, Siqi Xu, C. Mondal *et.al*]

Helicity Parton Distribution Functions



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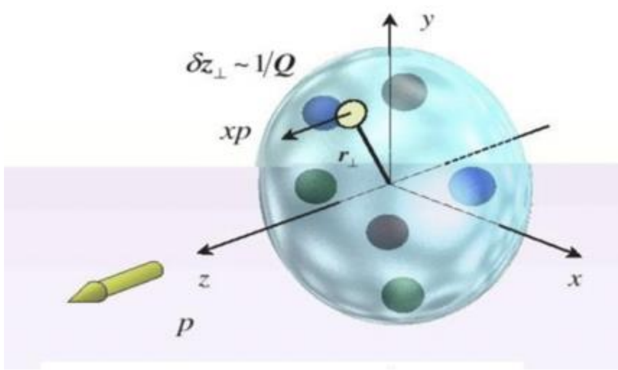
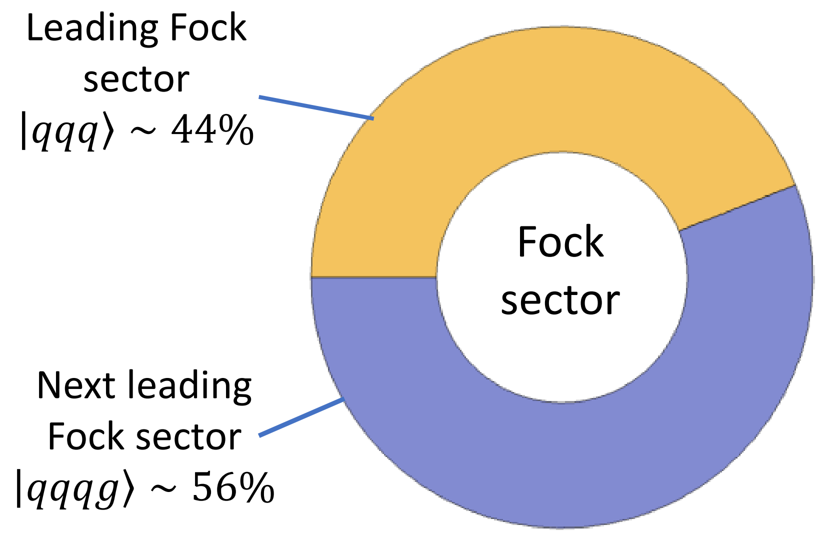
3-Dimension Structure of Nucleon

- Obtain observables from wave function

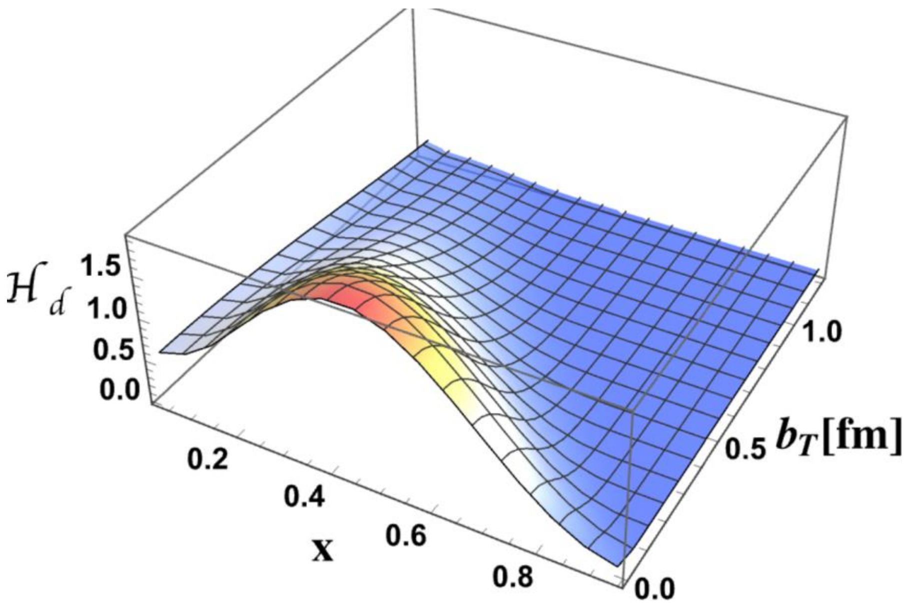
$$O \equiv \langle \beta | \hat{O} | \beta \rangle \qquad |\beta_{\text{nucleon}}\rangle = |qqq\rangle + |qqqg\rangle$$

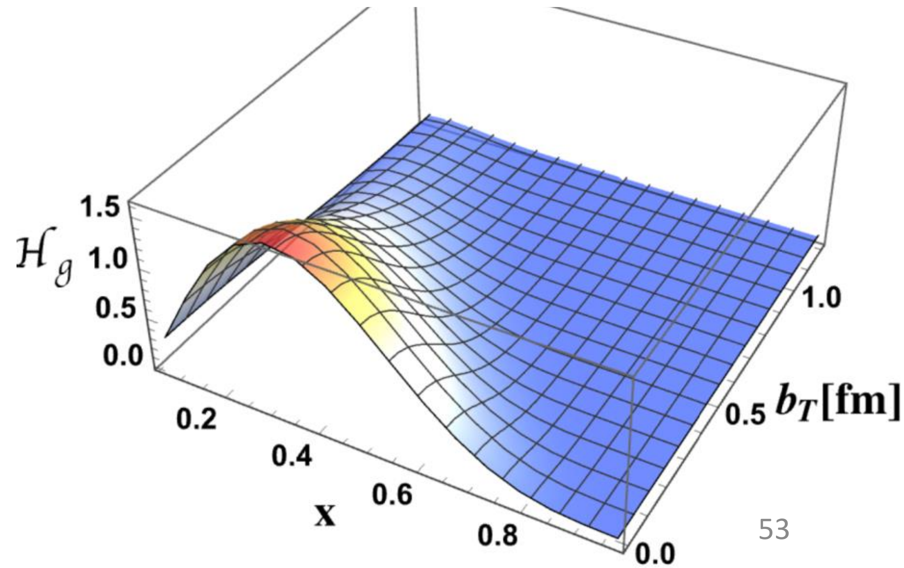




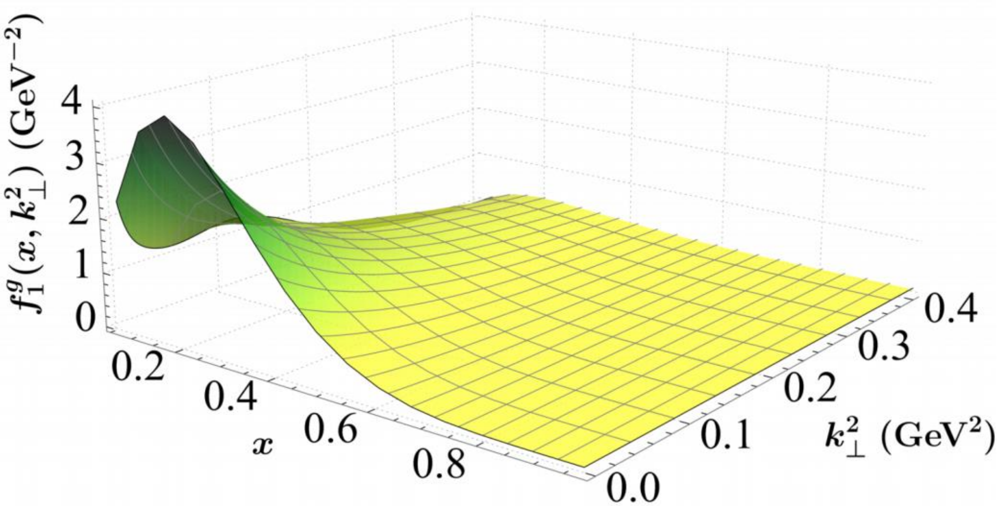
[arXiv:2209.08584 [hep-ph]]

[In preparation, Bolang Lin, Siqi Xu, C. Mondal *et.al*]

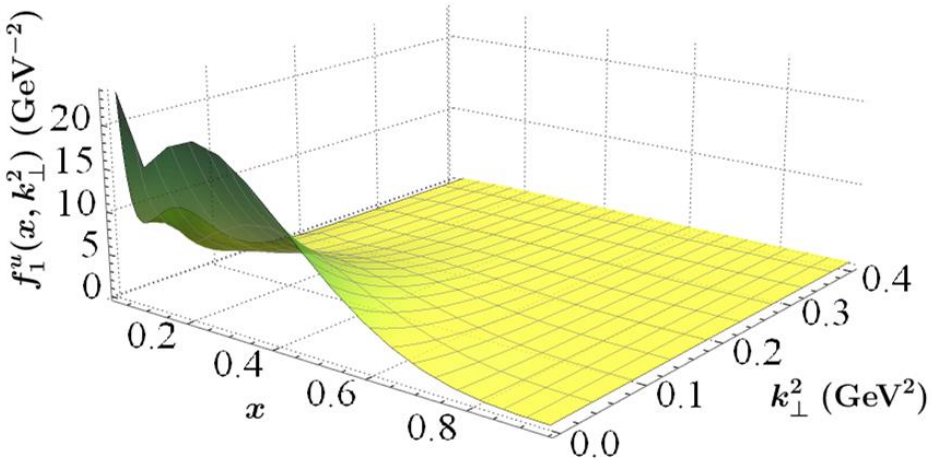


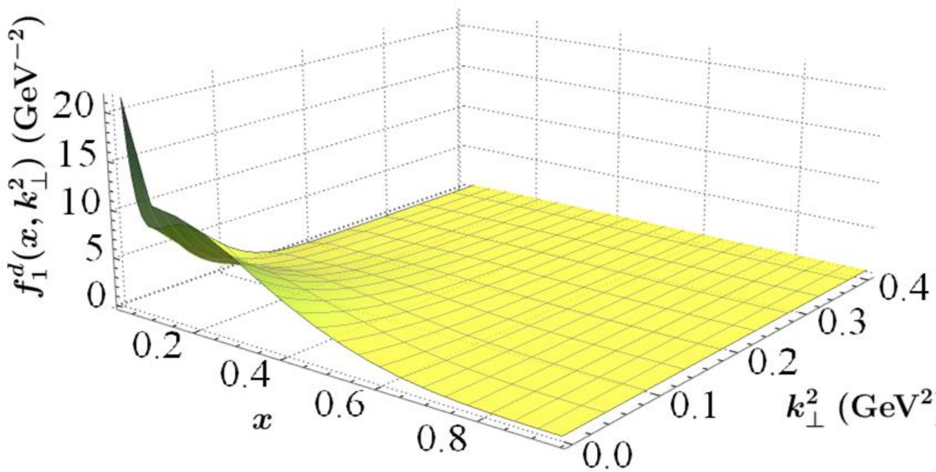


TMDs with One Dynamical Gluon

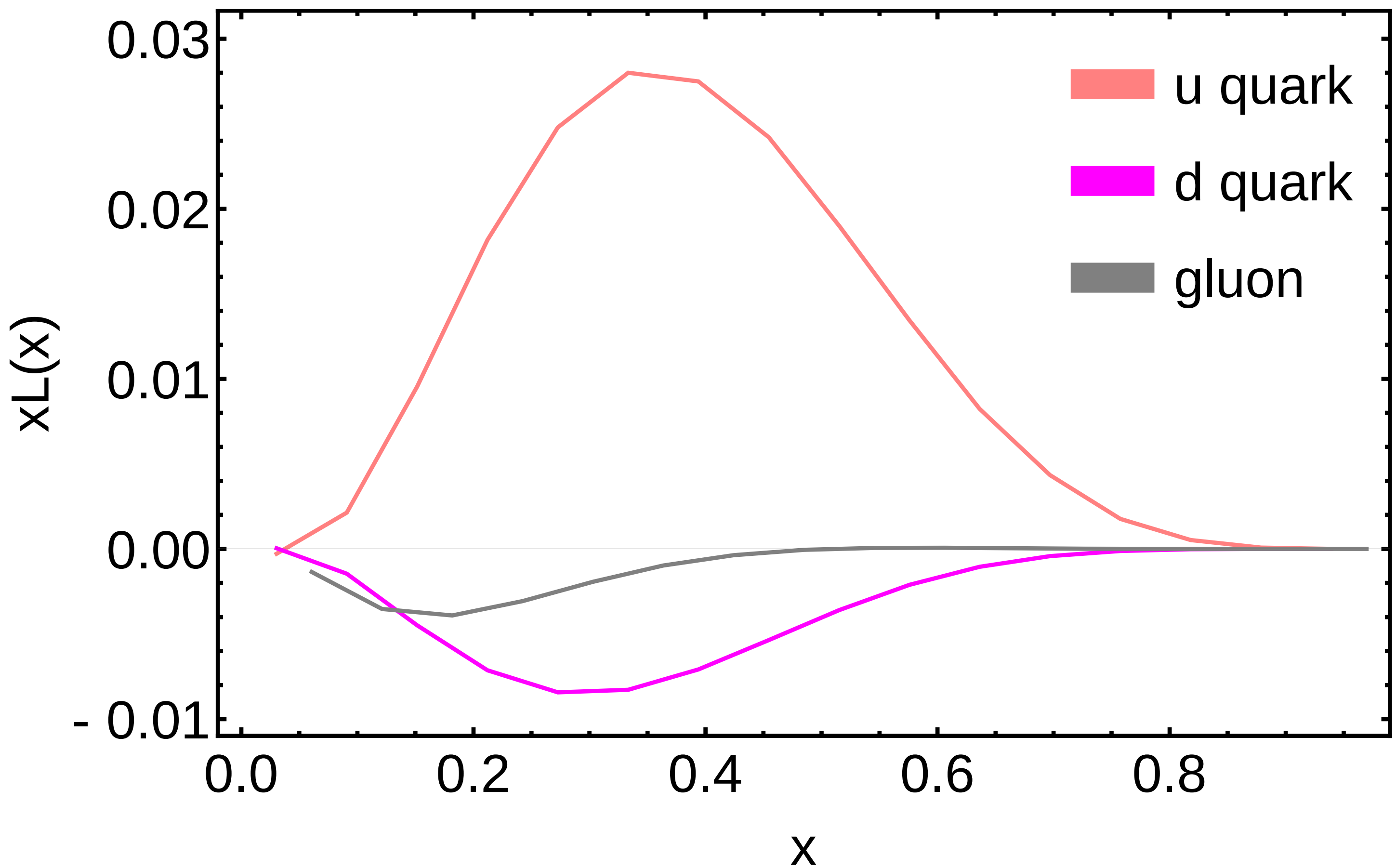


- Including $|qqq\rangle + |qqq\ g\rangle$ Fock sector, we calculate the gluon distribution in the proton
- Small x behaviour: higher Fock sector





Orbital angular momentum distributions



Canonical: $\ell_d = -0.0114 \pm 0.0004 \quad \ell_u = 0.0327 \pm 0.0013 \quad \ell_g = -0.0065 \pm 0.0005$

At the LC gauge : $\frac{1}{2}\Delta\Sigma = 0.359 \pm 0.002 \qquad \Delta G = 0.131 \pm 0.003$

[In preparation, Siqi Xu, C. Mondal *et.al*]