

Down the rabbit hole with hierarchical autoregressive neural networks

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today



Neural network: what you think it is

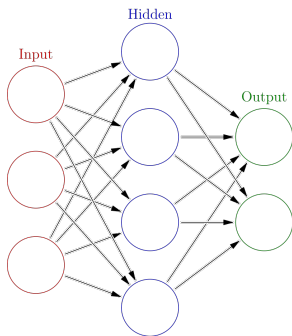


Figure: First Google Image search result for string "neural network"



Neural network: what you think it is

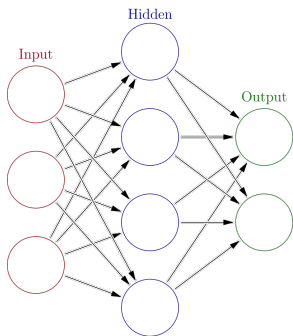


Figure: First Google Image search result for string "neural network"



Figure: Your reaction to that information

Neural network: what it really is

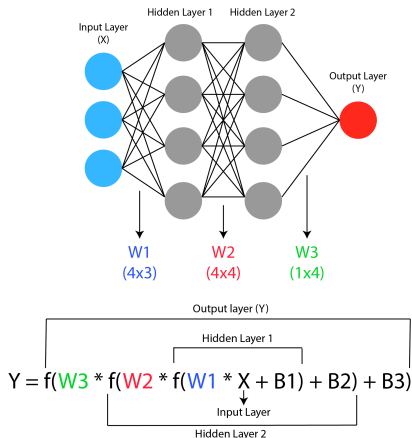


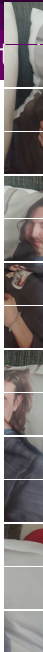
Figure: In physics, everything is a matrix



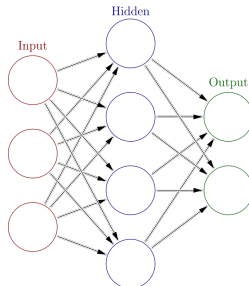
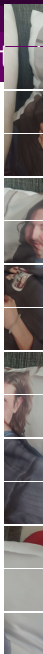
Neural network: what it does



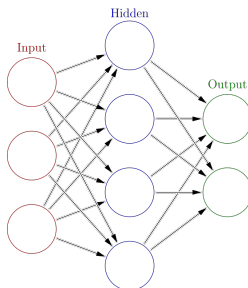
Neural network: what it does



Neural network: what it does



Neural network: what it does



Firlej	0.95
cushion	0.82
mug	0.53
hair	0.42
hand	0.34



Autoregression

$$X_t = \sum_{i=1}^p \varphi_i X_{t-i} + \varepsilon_t$$

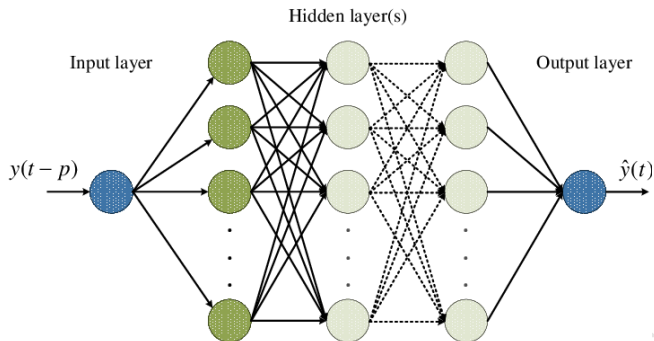


Figure: An example of autoregressive neural network



Hierarchy

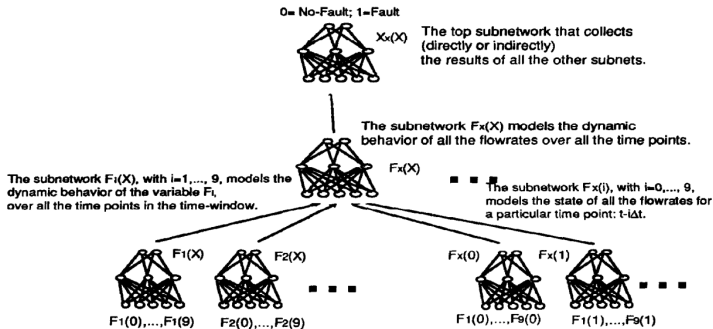
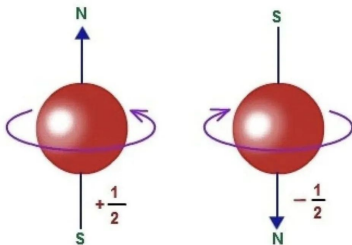


Figure: An example of hierarchical neural network



What is spin

Electron spin explained: imagine a ball that's rotating, except it's not a ball and it's not rotating



What is spin

Electron spin explained: imagine a ball that's rotating, except it's not a ball and it's not rotating

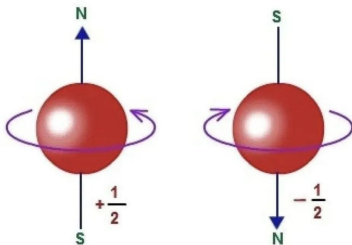


Figure: This is not what spin means



What is spin, again

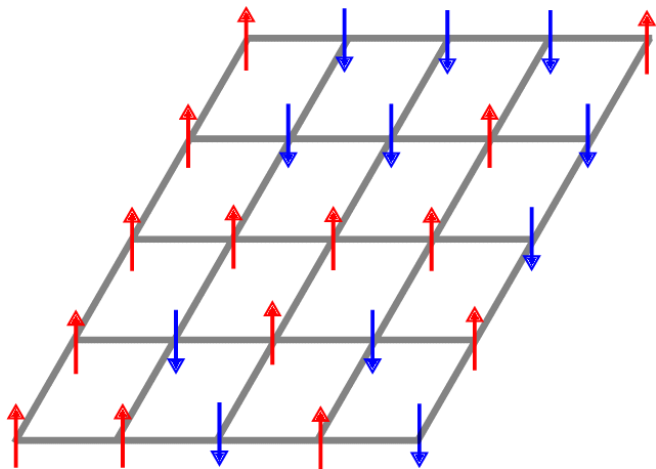


Figure: Illustration of 2D Ising model



Some physics now

$$M(\sigma) = \sum_{i \in \text{spins}} s_i$$

$$E(\sigma) = -\frac{1}{2} \sum_{i,j \in \text{neighbours}} s_i s_j$$

$$p(\sigma) = \frac{e^{-\beta E(\sigma)}}{Z(\sigma)}$$



Ising model

How to apply hierarchy

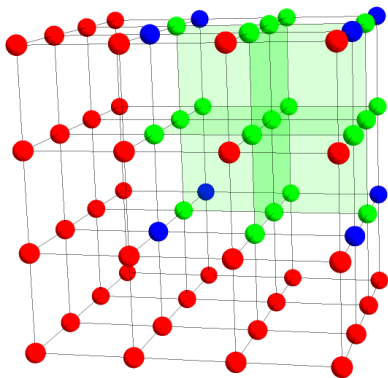


Figure: Hierarchy in 4x4x4 cube



How to apply autoregression

$$F = -\frac{1}{\beta} \log Z$$

$$F = \frac{1}{\beta} \sum_{i \in \text{configurations}} q_{\theta}(\sigma_i) [\beta E(\sigma_i) + \log q_{\theta}(\sigma_i)]$$



Some plots

Magnetization: Monte Carlo vs neural network

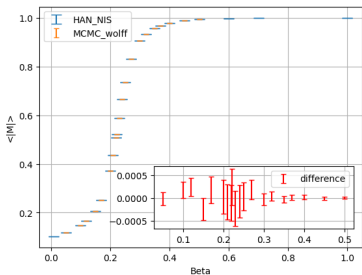
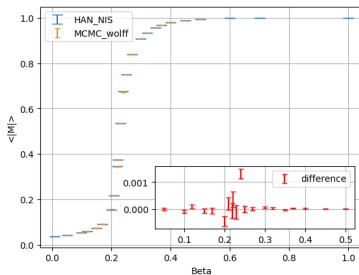
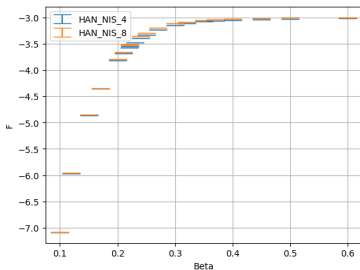
(a) $4 \times 4 \times 4$ (b) $8 \times 8 \times 8$

Figure: Magnetization vs temperature function β

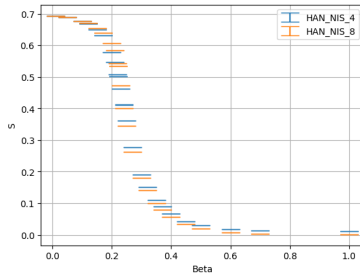


Some plots

Extensive properties: neural network



(a) free energy







(b) entropy

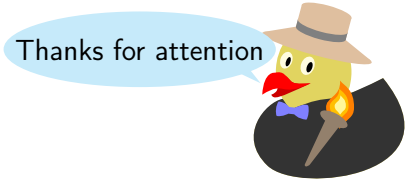
Figure: Extensive parameters vs temperature parameter β for 4x4x4 and 8x8x8 nets



Bibliography

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-  Hierarchical neural networks, [doi:10.1016/0098-1354\(92\)80053-C](https://doi.org/10.1016/0098-1354(92)80053-C)





<https://indico.cern.ch/event/1361918/contributions/5785806/attachments/2840617/4965149/title.pdf>

