



Twin paradox and chaos

Particles' trajectories in standing gravitational
waves field

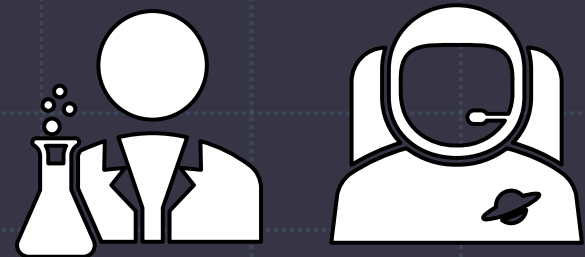
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Outline

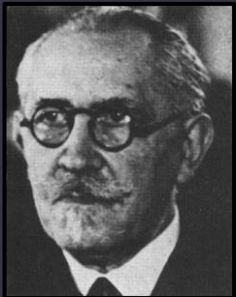


1. The twin paradox
2. (Standing) Gravitational waves
3. Chaos
4. Chaos vs standing gravitational waves
5. „Twin paradox“ and Chaos
6. Summary

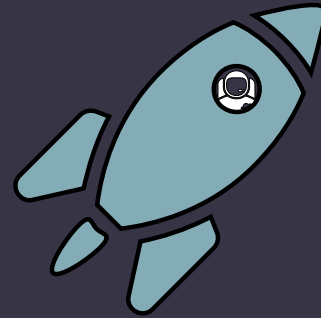
▶ The twin paradox



First formulation

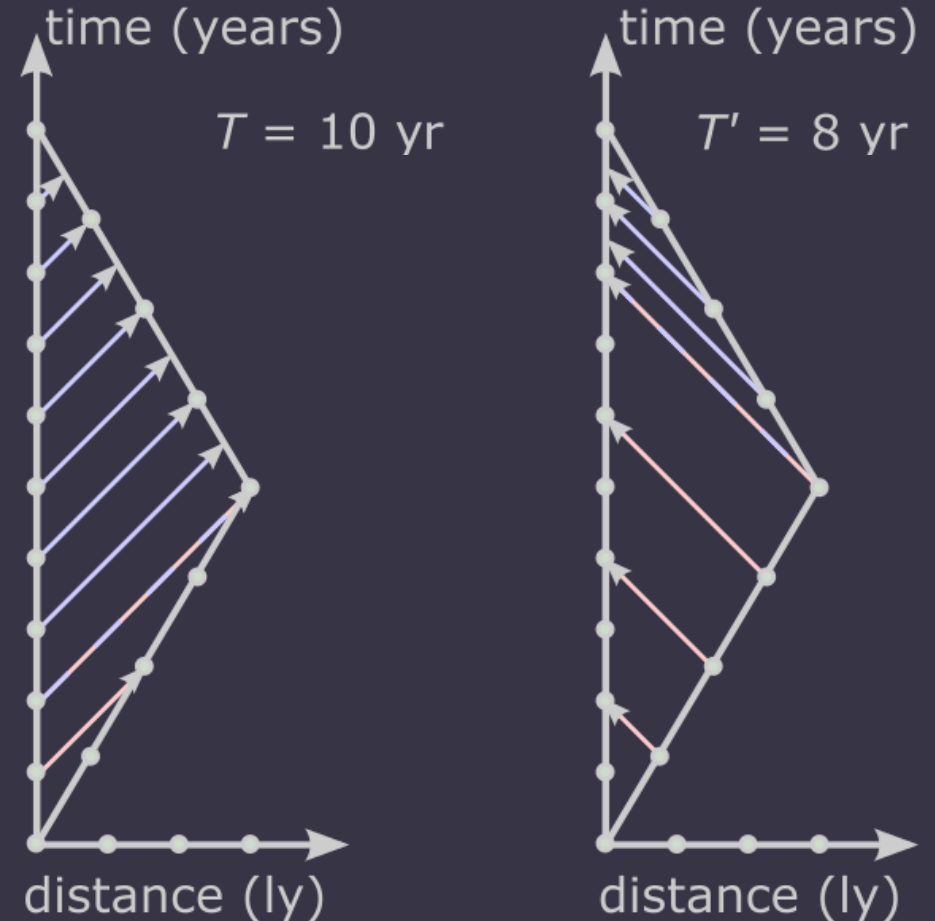


1. 1911 r. Paul Langevin
2. Thought experiment
3. Flat (no gravity) spacetime
4. First twin is setting off on a journey, while second is staying on the Earth



Why the „paradox“?

1. **Misunderstanding** of Einstein's theory of relativity
2. The twins are not equivalent
3. There is **no paradox** here
4. The „**Traveller**“ is always **younger** (in flat spacetime)

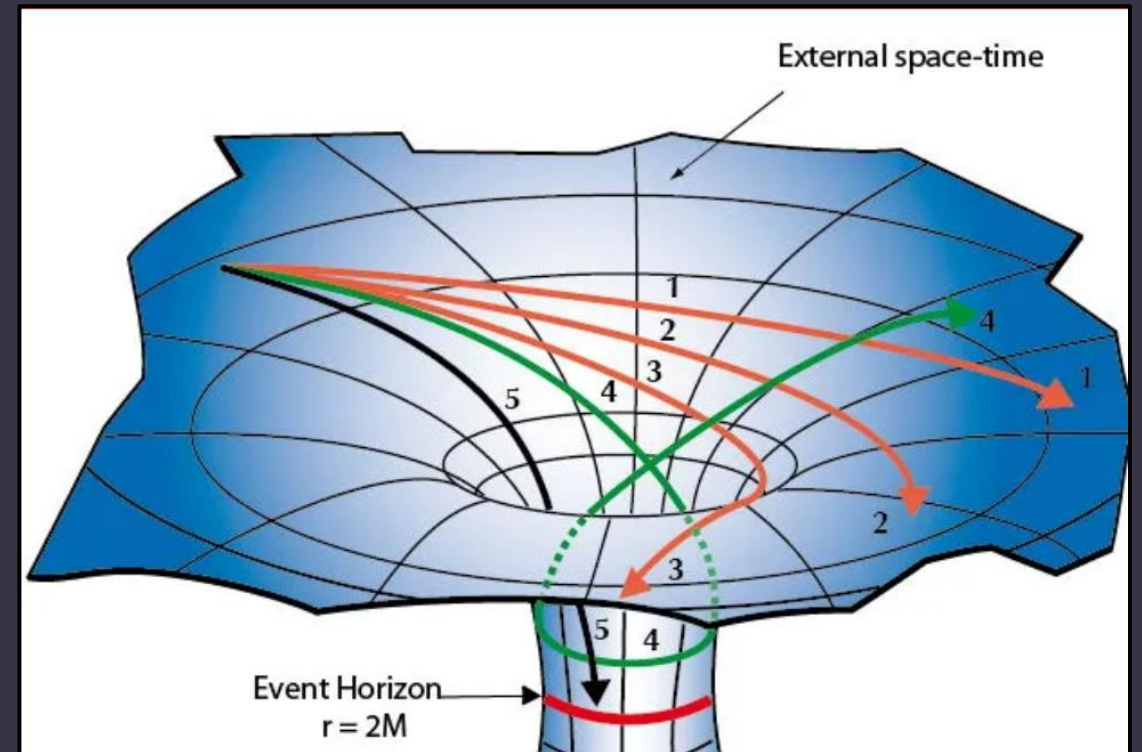


Schwarzschild spacetime



Image credit: solarseven via Getty Images

1. More complicated
2. Sokołowski (2012)
3. Trajectory matters (neither velocity nor acceleration)
4. The twin on radial geodesics ($\phi = \text{const}$) always the oldest



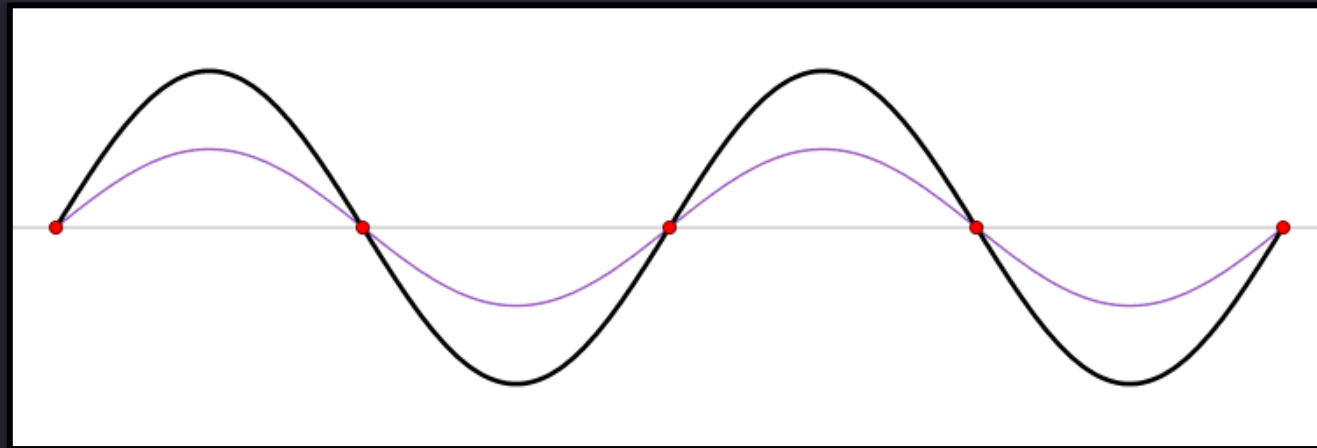
<https://weinsteing.wordpress.com/2017/12/14/the-geodesics-of-the-schwarzschild-metric-3/>



▶ Standing gravitational waves

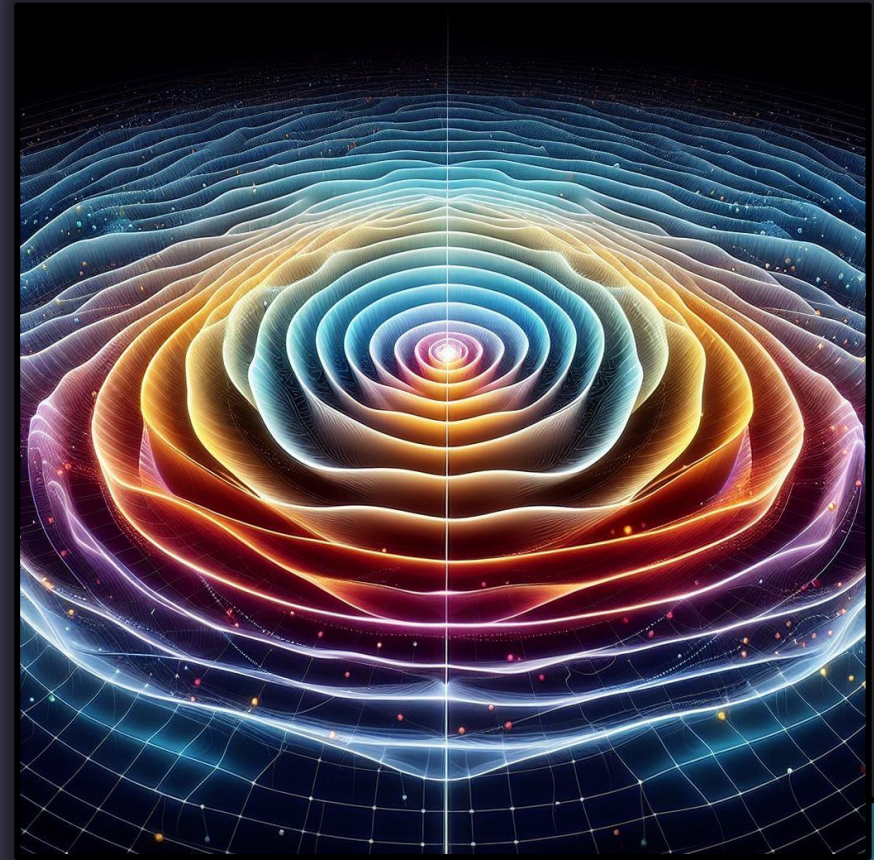
Standing mechanical waves

1. Not moving in space
2. Result of interference
3. Music (string) instruments



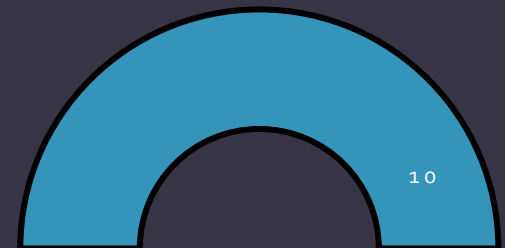
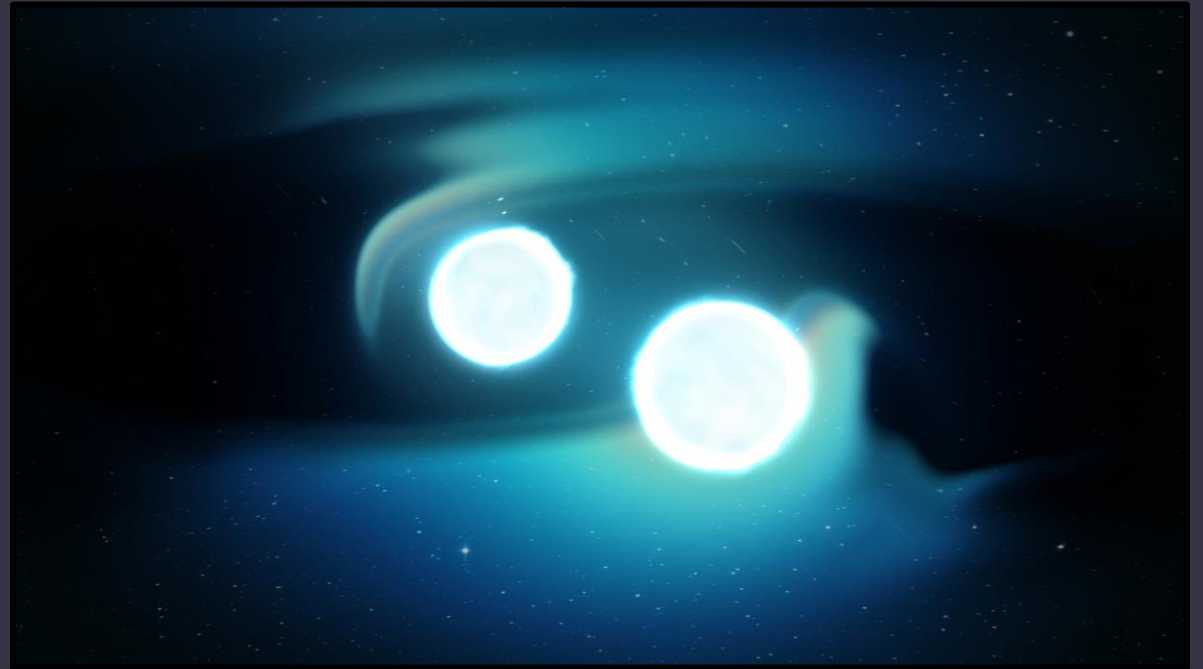
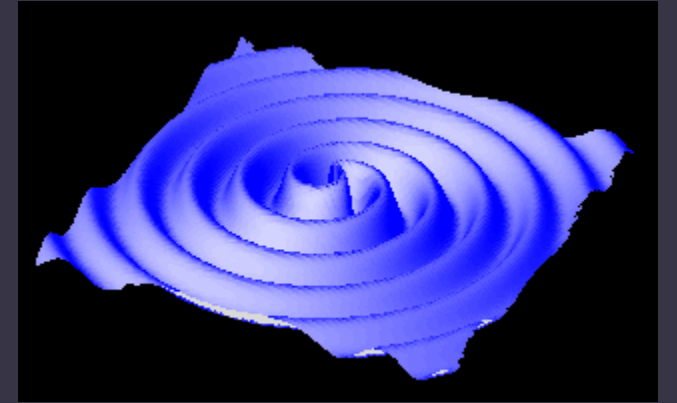
Standing Gravitational Waves (GW)

1. Similar to mechanical standing waves
2. Cylindrical symmetry
3. Vacuum solution of Einstein time-dependent equations



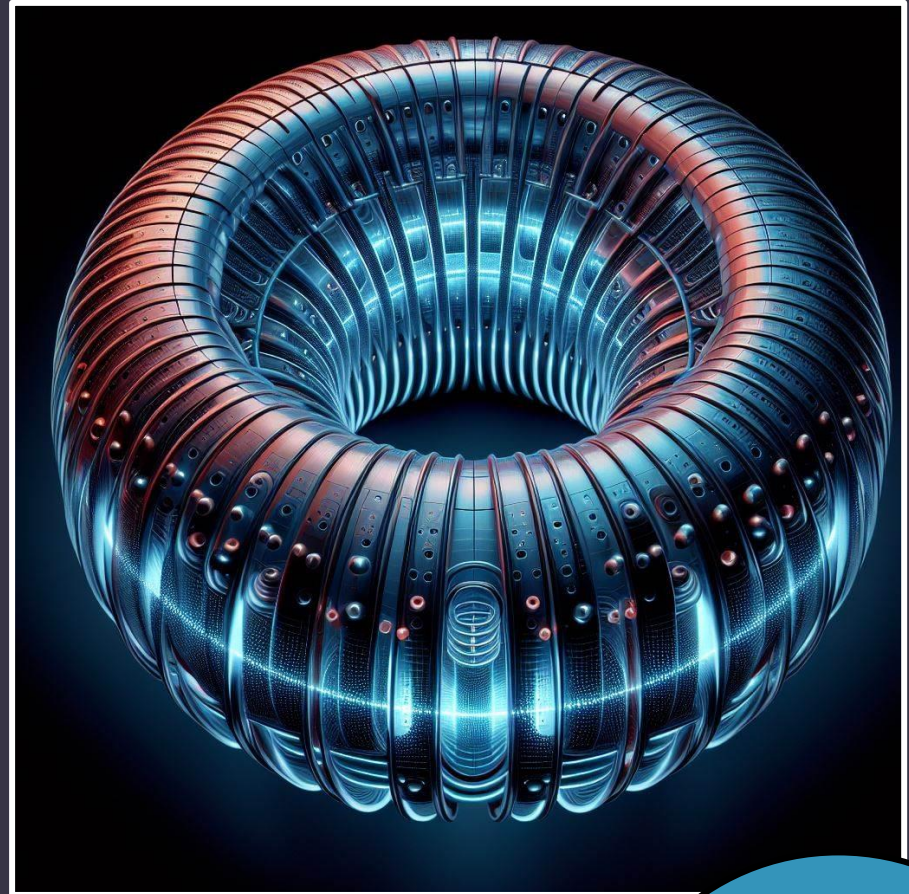
Gravitational wave sources

1. For example: Close binary systems of compact objects
2. What about standing gravitational wave sources?



Experiment

1. Production and detection
2. A source of GW: A Torus with excited altering EM field
3. Interference of produced GW \rightarrow Standing waves
4. Cylindrical symmetry





Chaos

What is chaos?

1. Sensitivity for initial conditions
2. Deterministic
3. Can „imitate“ randomness after enough long time



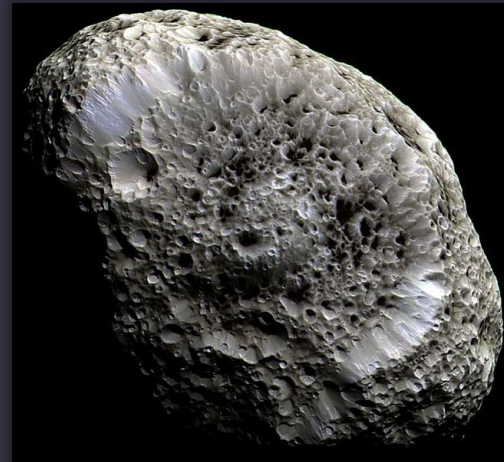
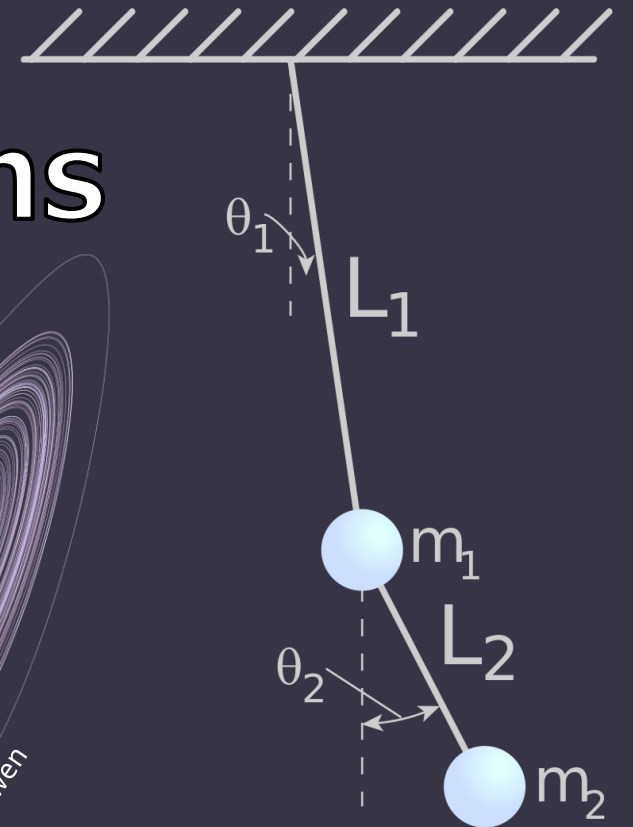
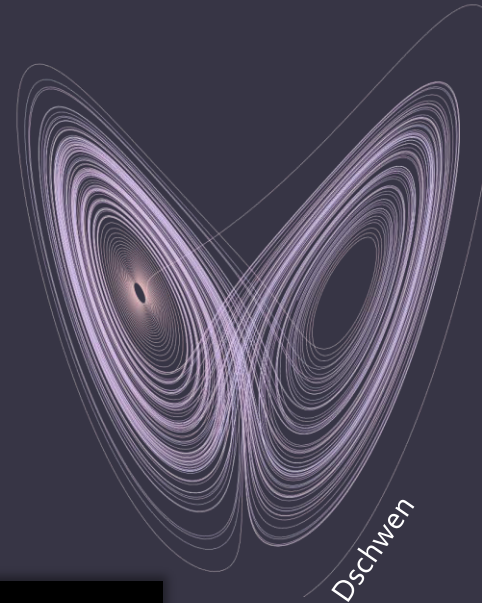
Examples of chaotic systems

1. „Butterfly effect“ and weather forecast



2. Hyperion (Saturn's moon) trajectory

3. Double pendulum



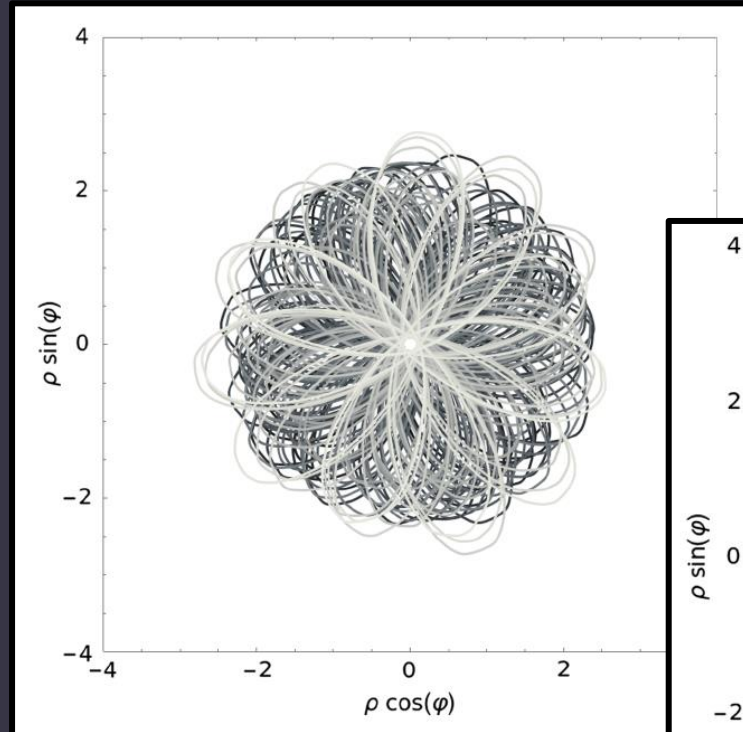
Credit: NASA/JPL/Space Science Institute



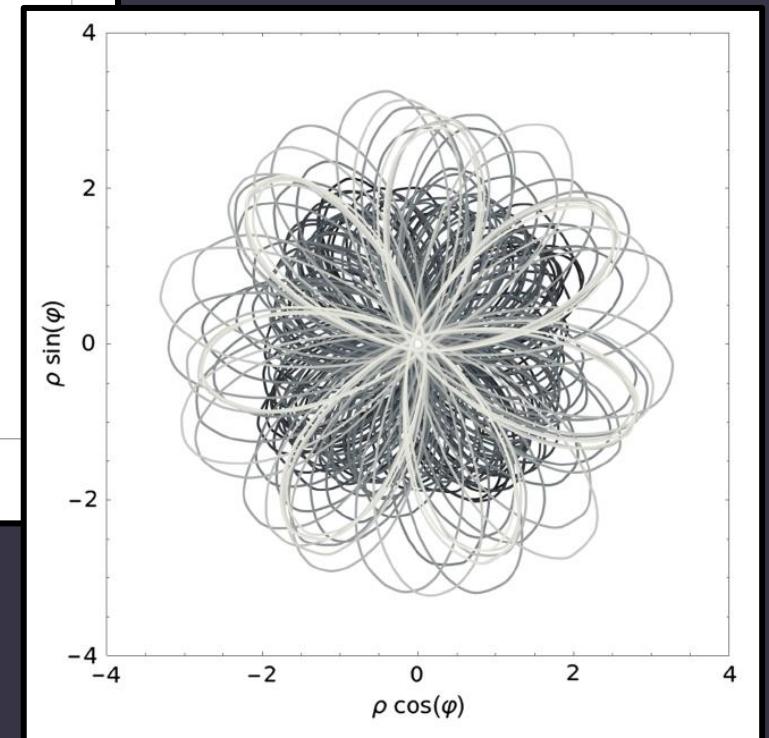
► Chaos and standing gravitational waves

„Chaos and Einstein-Rosen GW“

- Szybka, Naqvi, Phys. Rev. D, 108, L081501 (2023)
- $z = \text{const}$
- Trajectories for **freely falling particles**

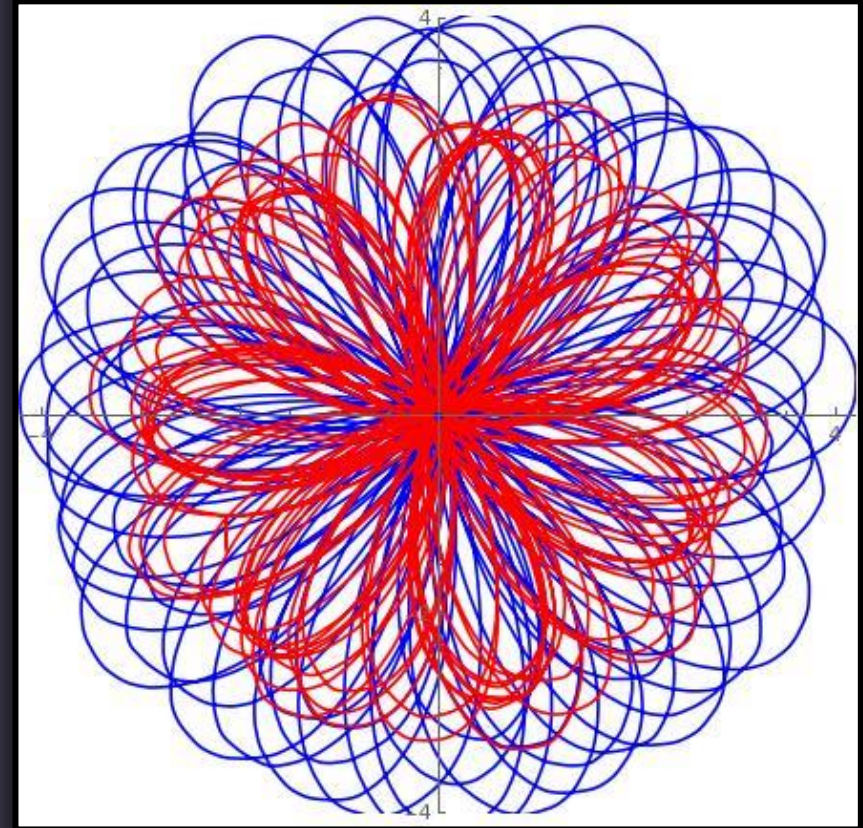


Change of $\dot{\rho}$ by 6×10^{-5}



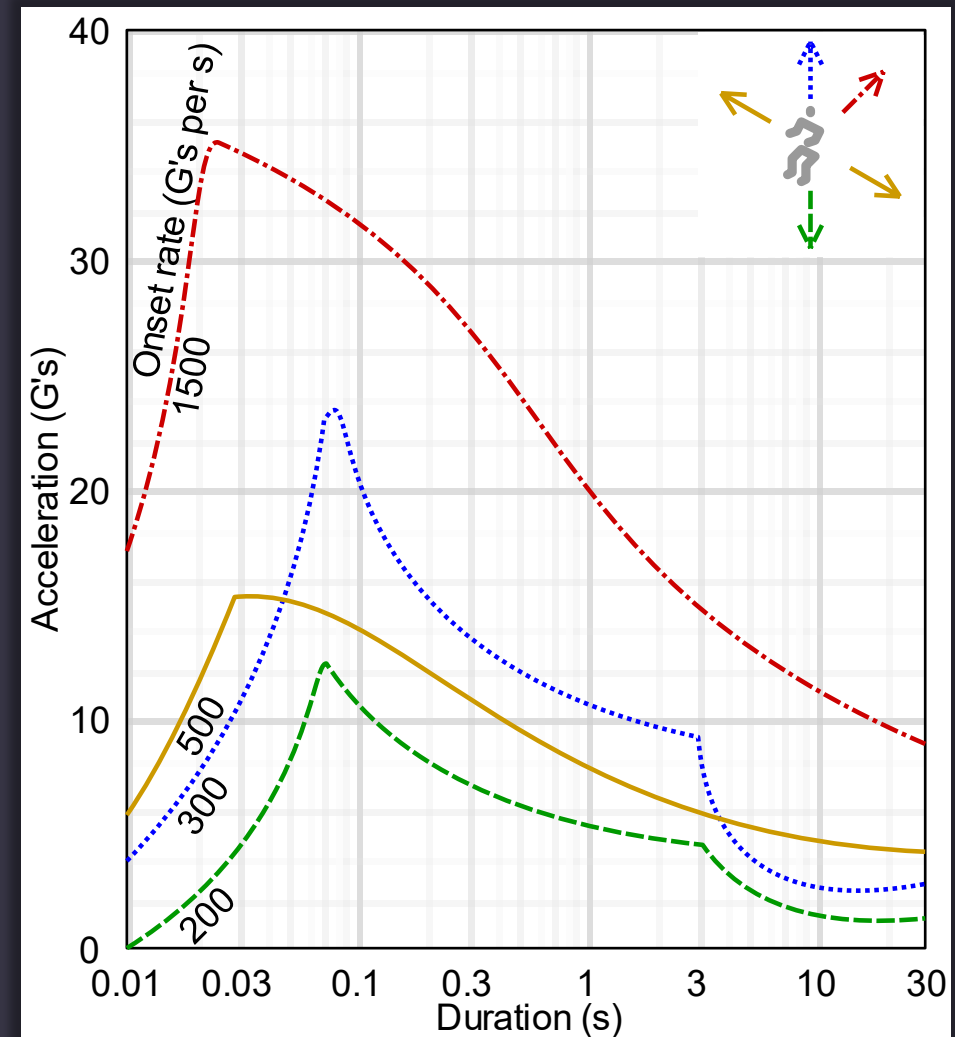
„Twin paradox“ and chaos

1. Particles start moving from very similar position: $\rho_A = 3.21$, $\rho_B = 3.22$
2. Trajectories = geodesics
3. Particles need to reunite
4. Comparing proper times („age“) of particles: particle B is 17% older



Particles → humans

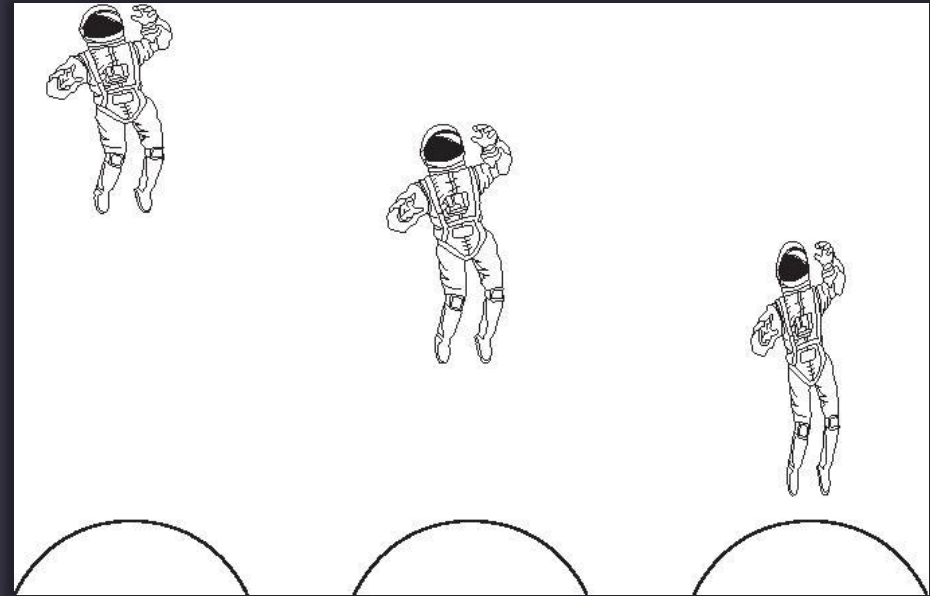
1. Safe conditions for people
2. Diagram: Acceleration vs duration
3. Different colours - different directions



Tidal forces

1. No external forces, just effects from standing gravitational waves
2. Nearby geodesics (trajectories) are becoming closer / more distant

↓
Tidal forces



Žr.: <https://www.rmg.co.uk/stories/topics/what-happens-if-you-fall-black-hole>

Summary

1. Spacetime with standing gravitational waves
2. Chaotic trajectories
3. „Twin paradox“ – age difference for freely falling particles
4. Tidal forces safe (e.g.) for humans → scale of the system

Literature

1. Sokolowski, Gen. Relativ. Gravit. (2012) 44:1267-1283
2. Szczególna teoria względności – wykłady prof. Sokołowskiego
3. „Elementy analizy tensorowej” L. M. Sokołowski
4. Szybka, Naqvi, Phys. Rev. D, 108, L081501 (2023)
5. Costa, Nataro (2015) „Inertial forces in GR”
6. Halilsoy, Nuovo Cimento B 102, 563 (1988)
7. „Chaos w układach dynamicznych” E. Ott
8. L. P. Grishchuk and M. V. Sazhin, Zh. Eksp. Teor. Fiz. 68, 1569 (1975)

